Theory Ahead of Business Cycle Measurement*

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Economists have long been puzzled by the observations that during peacetime industrial market economies display recurrent, large fluctuations in output and employment over relatively short time periods. Not uncommon are changes as large as 10 percent within only a couple of years. These observations are considered puzzling because the associated movements in labor's marginal product are small.

These observations should not be puzzling, for they are what standard economic theory predicts. For the United States, in fact, given people's ability and willingness to intertemporally and intratemporally substitute consumption and leisure and given the nature of the changing production possibility set, it would be puzzling if the economy did not display these large fluctuations in output and employment with little associated fluctuations in the marginal product of labor. Moreover, standard theory also correctly predicts the amplitude of these fluctuations, their serial correlation properties, and the fact that the investment component of output is about six times as volatile as the consumption component.

This perhaps surprising conclusion is the principal finding of a research program initiated by Kydland and me (1982) and extended by Kydland and me (1984), Hansen (1985a), and Bain (1985). We have computed the competitive equilibrium stochastic process for variants of the constant elasticity, stochastic growth model. The elasticities of substitution and the share parameters of the production and utility functions are restricted to those that generate the growth observations. The process governing the technology parameter is selected to be consistent with the measured technology changes for the American economy since the Korean War. We ask whether these artificial economies display fluctuations with statistical properties similar to those which the American economy has displayed in that period. They do.[1]

I view the growth model as a paradigm for macro analysis—analogous to the supply and demand construct of price theory. The elasticities of substitution and the share parameters of the growth model are analogous to the price and income elasticities of price theory. Whether or not this paradigm dominates, as I expect it will, is still an open question. But the early results indicate its power to organize our knowledge. The finding that when uncertainty in the rate of technological change is incorporated into the growth model it

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[1]Others [Barro (1981) and Long and Plosser (1983), for example] have argued that these fluctuations are not inconsistent with competitive theory that abstracts from monetary factors. Our finding is much stronger: standard theory predicts that the economy will display the business cycle phenomena.

displays the business cycle phenomena was both dramatic and unanticipated. I was sure that the model could not do this without some features of the payment and credit technologies.

The models constructed within this theoretical framework are necessarily highly abstract. Consequently, they are necessarily false, and statistical hypothesis testing will reject them. This does not imply, however, that nothing can be learned from such quantitative theoretical exercises. I think much has already been learned and confidently predict that much more will be learned as other features of the environment are introduced. Prime candidates for study are the effects of public finance elements, a foreign sector, and, of course, monetary factors. The research I review here is best viewed as a very promising beginning of a much larger research program.

The Business Cycle Phenomena

The use of the expression *business cycle* is unfortunate for two reasons. One is that it leads people to think in terms of a time series' business cycle component which is to be explained independently of a growth component; our research has, instead, one unifying theory of both of these. The other reason I do not like to use the expression is that it is not accurate; some systems of low-order linear stochastic difference equations with a nonoscillatory deterministic part, and therefore no cycle, display key business cycle features. (See Slutzky 1927.) I thus do not refer to business cycles, but rather to business cycle *phenomena*, which are nothing more nor less than a certain set of statistical properties of a certain set of important aggregate time series. The question I and others have considered is, Do the stochastic difference equations that are the equilibrium laws of motion for the stochastic growth display the business cycle phenomena?

More specifically, we follow Lucas (1977, p. 9) in defining the business cycle phenomena as the recurrent fluctuations of output about trend and the co-movements among other aggregate time series. Fluctuations are by definition deviations from some slowly varying path. Since this slowly varying path increases monotonically over time, we adopt the common practice of labeling it *trend*. This trend is neither a measure nor an estimate of the unconditional mean of some stochastic process. It is, rather, defined by the computational procedure used to fit the smooth curve through the data.

If the business cycle facts were sensitive to the detrending procedure employed, there would be a problem. But the key facts are not sensitive to the procedure if the trend curve is smooth. Our curve-fitting method is to take the logarithms of variables and then select the trend path $\{\tau_t\}$ which minimizes the sum of the squared deviations from a given series $\{Y_t\}$ subject to the constraint that the sum of the squared second differences not be too large. This is

$$\min_{\{\tau_t\}_{t=1}^{T}} \sum_{t=1}^{T} (Y_t - \tau_t)^2$$

subject to

$$\sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \leq \mu.$$

The smaller is μ, the smoother is the trend path. If $\mu = 0$, the least squares linear time trend results. For all series, μ is picked so that the Lagrange multiplier of the constraint is 1600. This produces the right degree of smoothness in the fitted trend when the observation period is a quarter of a year. Thus, the sequence $\{\tau_t\}$ minimizes

$$\sum_{t=1}^{T} (Y_t - \tau_t)^2 + 1600 \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2.$$

The first-order conditions of this minimization problem are linear in Y_t and τ_t, so for every series, $\tau = AY$, where A is the same $T \times T$ matrix. The deviations from trend, also by definition, are

$$Y_t^d = Y_t - \tau_t \quad \text{for } t = 1, \ldots, T.$$

Unless otherwise stated, these are the variables used in the computation of the statistics reported here for both the United States and the growth economies.

An alternative interpretation of the procedure is that it is a high pass linear filter. The facts reported here are essentially the same if, rather than defining the deviations by $Y^d = (I - A)Y$, we filtered the Y using a high pass band filter, eliminating all frequencies of 32 quarters or greater. An advantage of our procedure is that it deals better with the ends of the sample problem and does not require a stationary time series.

To compare the behaviors of a stochastic growth economy and an actual economy, only identical statistics for the two economies are used. By definition, a *statistic* is a real valued function of the raw time series. Consequently, if a comparison is made, say, between the standard deviations of the deviations, the date t deviation for the growth economy must be the same function of the data generated by that model as the date t deviation for the American economy is of that economy's data. Our definitions of the deviations satisfy this criterion.

Figure 1 plots the logs of actual and trend output for the U.S. economy during 1947–82, and Figure 2 the corresponding percentage deviations from trend of output and hours of market employment. Output and hours clearly move up and down together with nearly the same amplitudes.

Table 1 contains the standard deviations and cross serial correlations of output and other aggregate time series for the American economy during 1954–82. Consumption appears less variable and investment more variable than output. Further, the average product of labor is procyclical but does not vary as much as output or hours.

The Growth Model

This theory and its variants build on the neoclassical growth economy of Solow (1956) and Swan (1956). In the language of Lucas (1980, p. 696), the model is a "fully articulated, artificial economic system" that can be used to generate economic time series of a set of important economic aggregates. The model assumes an aggregate production function with constant returns to scale, inputs labor n and capital k, and an output which can be allocated either to current consumption c or to investment x. If t denotes the date, $f\colon R^2 \to R$ the production function, and z_t a technology parameter, then the production constraint is

$$x_t + c_t \leq z_t f(k_t, n_t)$$

where $x_t, c_t, k_t, n_t \geq 0$. The model further assumes that the services provided by a unit of capital decrease geometrically at a rate $0 < \delta < 1$:

$$k_{t+1} = (1-\delta)k_t + x_t.$$

Solow completes the specification of his economy by hypothesizing that some fraction $0 < \sigma < 1$ of output is invested and the remaining fraction $1 - \sigma$ consumed and that n_t is a constant—say, $\bar{n}$—for all t. For this economy, the law of motion of capital condition on z_t is

$$k_{t+1} = (1-\delta)k_t + \sigma z_t f(k_t, \bar{n}).$$

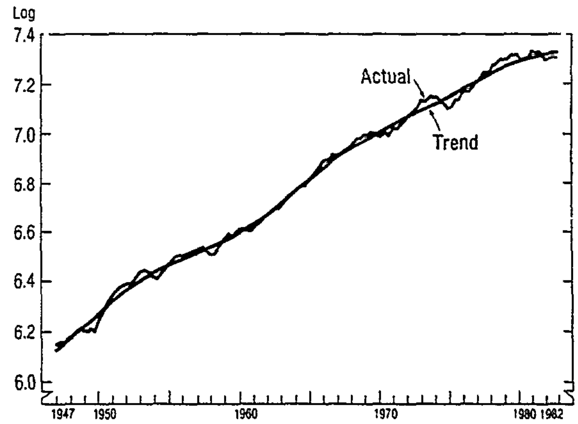
Once the $\{z_t\}$ stochastic process is specified, the stochastic process governing capital and the other economic aggregates are determined and realizations of the stochastic process can be generated by a computer.

This structure is far from adequate for the study of the business cycle because in it neither employment nor the savings rate varies, when in fact they do. Being explicit about the economy, however, naturally leads to

Figure 1

Actual and Trend Logs of U.S. Gross National Product

Quarterly, 1947–82

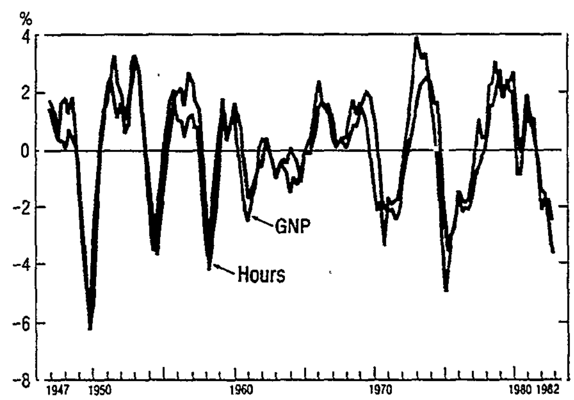


Source of basic data: Citicorp's Citibase data bank

Figure 2

Deviations From Trend of Gross National Product and Nonfarm Employee Hours in the United States

Quarterly, 1947–82



Source of basic data: Citicorp's Citibase data bank

Table 1

Cyclical Behavior of the U.S. Economy

Deviations From Trend of Key Variables, 1954:1–1982:4

Variable x	Standard Deviation	Cross Correlation of GNP With $x(t-1)$	$x(t)$	$x(t+1)$
Gross National Product	1.8%	.82	1.00	.82
Personal Consumption Expenditures				
Services	.6	.66	.72	.61
Nondurable Goods	1.2	.71	.76	.59
Fixed Investment Expenditures	5.3	.78	.89	.78
Nonresidential Investment	5.2	.54	.79	.86
Structures	4.6	.42	.62	.70
Equipment	6.0	.56	.82	.87
Capital Stocks				
Total Nonfarm Inventories	1.7	.15	.48	.68
Nonresidential Structures	.4	−.20	−.03	.16
Nonresidential Equipment	1.0	.03	.23	.41
Labor Input				
Nonfarm Hours	1.7	.57	.85	.89
Average Weekly Hours in Mfg.	1.0	.76	.85	.61
Productivity (GNP/Hours)	1.0	.51	.34	−.04

Source of basic data: Citicorp's Citibase data bank

the question of what determines these variables, which are central to the cycle.

That leads to the introduction of a stand-in household with some explicit preferences. If we abstract from the labor supply decision and uncertainty (that is, $z_t = \bar{z}$ and $n_t = \bar{n}$), the standard form of the utility function is

$$\Sigma_{t=0}^{\infty} \beta^t u(c_t) \text{ for } 0 < \beta < 1$$

where β is the subjective time discount factor. The function $u: R_+ \rightarrow R$ is twice differentiable and concave. The commodity space for the deterministic version of this model is l_∞, infinite sequences of uniformly bounded consumptions $\{c_t\}_{t=0}^{\infty}$.

The theorems of Bewley (1972) could be applied to establish existence of a competitive equilibrium for this l_∞ commodity-space economy. That existence argument, however, does not provide an algorithm for computing the equilibria. An alternative approach is to use the competitive welfare theorems of Debreu (1954). Given local nonsaturation and no externalities, competitive equilibria are Pareto optima and, with some additional conditions that are satisfied for this economy, any Pareto optimum can be supported as a competitive equilibrium. Given a single agent and the convexity, there is a unique optimum and that optimum is the unique competitive equilibrium allocation. The advantage of this approach is that algorithms for computing solutions to concave programming problems can be used to find the competitive equilibrium allocation for this economy.

Even with the savings decision endogenous, this economy has no fluctuations. As shown by Cass (1965) and Koopmans (1965), the competitive equilibrium path converges monotonically to a unique rest point or, if z_t is growing exponentially, to a balanced growth path. There are multisector variants of this model in which the equilibrium path oscillates. (See Benhabib and Nishimura 1985 and Marimon 1984.) But I know of no multisector model which has been restricted to match observed factor shares by sector, which has a value for β consistent with observed interest rates, and which displays oscillations.

When uncertainty is introduced, the household's objective is its expected discounted utility:

$$E\left\{\sum_{t=0}^{\infty} \beta^t u(c_t)\right\}.$$

The commodity vector is now indexed by the history of shocks; that is, $\{c_t(z_1, \ldots, z_t)\}_{t=0}^{\infty}$ is the commodity point. As Brock and Mirman (1972) show, if the $\{z_t\}$ are identically distributed random variables, an optimum to the social planner's problem exists and the optimum is a stationary stochastic process with $k_{t+1} = g(k_t, z_t)$ and $c_t = c(k_t, z_t)$. As Lucas and Prescott (1971) show, for a class of economies that include this one, the social optimum is the unique competitive equilibrium allocation. They also show that for these homogeneous agent economies, the social optimum is also the unique sequence-of-markets equilibrium allocation. Consequently, there are equilibrium time-invariant functions for the wage $w_t = w(k_t, z_t)$ and the rental price of capital $r_t = r(k_t, z_t)$, where these prices are relative to the date t consumption good. Given these prices, the firm's period t problem is

$$\max_{k_t, n_t \geq 0} \{y_t - r_t k_t - w_t n_t\}$$

subject to the output constraint

$$y_t \leq z_t f(k_t, n_t).$$

The household's problem is more complicated, for it must form expectations of future prices. If a_t is its capital stock, its problem is

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + x_t \leq w_t \bar{n} + r_t a_t$$

$$a_{t+1} \leq (1-\delta) a_t + x_t$$

and given $a_0 - k_0$. In forming expectations, a household knows the relation between the economy's state (k_t, z_t) and prices, $w_t = w(k_t, z_t)$ and $r_t = r(k_t, z_t)$. Further, it knows the process governing the evolution of the per capita capital stock, a variable which, like prices, is taken as given.

The elements needed to define a *sequence-of-markets equilibrium* are the firm's policy functions $y(k_t, z_t)$, $n(k_t, z_t)$, and $k(k_t, z_t)$; the household's policy functions $x(a_t, k_t, z_t)$ and $c(a_t, k_t, z_t)$; a law of motion of per capita capital $k_{t+1} = g(k_t, z_t)$; and pricing functions $w(k_t, z_t)$ and $r(k_t, z_t)$. For equilibrium, then,

- The firm's policy functions must be optimal given the pricing functions.
- The household's policy functions must be optimal given the pricing functions and the law of motion of per capita capital.
- Spot markets clear; that is, for all k_t and z_t

$$\bar{n} = n(k_t, z_t)$$

$$k_t = k(k_t, z_t)$$

$$x(k_t, k_t, z_t) + c(k_t, k_t, z_t) = y(k_t, z_t).$$

(Note that the goods market must clear only when the representative household is truly representative, that is, when $a_t = k_t$.)

- Expectations are rational; that is,

$$g(k_t, z_t) = (1-\delta) k_t + x(k_t, k_t, z_t).$$

This definition still holds if the household values productive time that is allocated to nonmarket activities. Such time will be called *leisure* and denoted l_t. The productive time endowment is normalized to 1, and the household faces the constraints

$$n_t + l_t \leq 1$$

for all t. In addition, leisure is introduced as an argument of the utility function, so the household's objective becomes the maximization of

$$E \sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

Now leisure—and therefore employment—varies in equilibrium.

The model needs one more modification: a relaxation of the assumption that the technology shocks z_t are identically and independently distributed random variables. As will be documented, they are not so distributed. Rather, they display considerable serial correlation, with their first differences nearly serially uncorrelated. To introduce high persistence, we assume

$$z_{t+1} = \rho z_t + \epsilon_{t+1}$$

where the $\{\epsilon_{t+1}\}$ are identically and independently distributed and ρ is near 1. With this modification, the recursive sequence-of-markets equilibrium definition continues to apply.

Using Data to Restrict the Growth Model

Without additional restrictions on preferences and technology, a wide variety of equilibrium processes are consistent with the growth model. The beauty of this model is that both growth and micro observations can be used to determine its production and utility functions. When they are so used, there are not many free parameters that are specific to explaining the business cycle phenomena and that cannot be measured independently of those phenomena. The key parameters of the growth model are the intertemporal and intratemporal elasticities of substitution. As Lucas (1980, p. 712) emphasizes, "On these parameters, we have a wealth of inexpensively available data from census cohort information, from panel data describing the reactions of individual households to a variety of changing market conditions, and so forth." To this list we add the secular growth observations which have the advantage of being experiments run by nature with large changes in relative prices and quantities and with idiosyncratic factors averaged out.[2] A fundamental thesis of this line of inquiry is that the measures obtained from aggregate series and those from individual panel data must be consistent. After all, the former are just the aggregates of the latter.

Secularly in the United States, capital and labor shares of output have been approximately constant, as has r, the rental price of capital. However, the nation's real wage has increased greatly—more than 100 percent since the Korean War. For these results to hold, the model's production function must be approximately Cobb-Douglas:

$$z_t f(k_t, n_t) = z_t k_t^{1-\theta} n_t^{\theta}.$$

The share parameter θ is equal to labor's share, which has been about 64 percent in the postwar period, so $\theta = 0.64$. This number is smaller than that usually obtained because we include services of consumer durables as part of output. This alternative accounting both reduces labor's share and makes it more nearly constant over the postwar period.

The artificial economy has but one type of capital, and it depreciates at rate δ. In fact, different types of capital depreciate at different rates, and the pattern of depreciation over the life of any physical asset is not constant. Kydland and I (1982, 1984) simply pick $\delta = 0.10$. With this value and an annual real interest rate of 4 percent, the steady-state capital–annual output ratio is about 2.6. That matches the ratio for the U.S. economy and also implies a steady-state investment share of output near the historically observed average. Except for parameters determining the process on the technology shock, this completely specifies the technology of the simple growth model.

A key growth observation which restricts the utility function is that leisure per capita l_t has shown virtually no secular trend while, again, the real wage has increased steadily. This implies an elasticity of substitution between consumption c_t and leisure l_t near 1. Thus, the utility function restricted to display both constant intertemporal and unit intratemporal elasticities of substitution is

$$u(c_t, l_t) = ([c_t^{1-\phi} l_t^{\phi}]^{1-\gamma} - 1)/(1 - \gamma)$$

where $1/\gamma > 0$ is the elasticity of substituting between different date composite commodities $c_t^{1-\phi} l_t^{\phi}$. This leaves γ and the subjective time discount factor β [or, equivalently, the subjective time discount rate $(1/\beta) - 1$] to be determined.

The steady-state interest rate is

$$i = (1/\beta) - 1 + \gamma(\dot{c}/c).$$

As stated previously, the average annual real interest rate is about 4 percent, and the growth rate of per capita consumption $\dot{c}/c$ has averaged nearly 2 percent. The following studies help restrict γ. Tobin and Dolde (1971) find that a γ near 1.5 is needed to match the life cycle consumption patterns of individuals. Using individual portfolio observations, Friend and Blume (1975) estimate γ to be near 2. Using aggregate stock market and consumption data, Hansen and Singleton (1983) estimate γ to be near 1. Using international data, Kehoe

[2]See Solow 1970 for a nice summary of the growth observations.

(1984) also finds a modest curvature parameter γ. All these observations make a strong case that γ is not too far from 1. Since the nature of fluctuations of the artificial economy is not very sensitive to γ, we simply set γ equal to 1. Taking the limit as $\gamma \rightarrow 1$ yields

$$u(c_t, l_t) = (1-\phi) \log c_t + \phi \log l_t.$$

This leaves β and ϕ still to be determined.

Hansen (1985b) has found that growing economies—that is, those with z_t having a multiplicative, geometrically growing factor $(1+\lambda)^t$ with $\lambda > 0$—fluctuate in essentially the same way as economies for which $\lambda = 0$. This justifies considering only the case $\lambda = 0$. If $\lambda = 0$, then the average interest rate approximately equals the subjective time discount rate.[3] Therefore, we set β equal to 0.96 per year or 0.99 per quarter.

The parameter ϕ is the leisure share parameter. Ghez and Becker (1975) find that the household allocates approximately one-third of its productive time to market activities and two-thirds to nonmarket activities. To be consistent with that, the model's parameter ϕ must be near two-thirds. This is the value assumed in our business cycle studies.

Eichenbaum, Hansen, and Singleton (1984) use aggregate data to estimate this share parameter ϕ, and they obtain a value near five-sixths. The difference between two-thirds and five-sixths is large in the business cycle context. With $\phi = 2/3$, the elasticity of labor supply with respect to a temporary change in the real wage is 2, while if $\phi = 5/6$, it is 5. This is because a 1 percent change in leisure implies a $\phi/(\phi-1)$ percent change in hours of employment.

We do not follow the Eichenbaum-Hansen-Singleton approach and treat ϕ as a free parameter because it would violate the principle that parameters cannot be specific to the phenomena being studied. What sort of science would economics be if micro studies used one share parameter and aggregate studies another?

The Nature of the Technological Change

One method of measuring technological change is to follow Solow (1957) and define it as the changes in output less the sum of the changes in labor's input times labor share and the changes in capital's input times capital share. Measuring variables in logs, this is the percentage change in the technology parameter of the Cobb-Douglas production function. For the U.S. economy between the third quarter of 1955 and the first quarter of 1984, the standard deviation of this change is 1.2 percent.[4] The serial autocorrelations of these changes are $\rho_1 = -0.21$, $\rho_2 = -0.06$, $\rho_3 = 0.04$, $\rho_4 = 0.01$, and $\rho_5 = -0.05$. To a first approximation, the process on the percentage change in the technology process is a random walk with drift plus some serially uncorrelated measurement error. This error produces the negative first-order serial correlation of the differences.

Further evidence that the random walk model is not a bad approximation is based on yearly changes. For the quarterly random walk model, the standard deviation of this change is 6.63 times the standard deviation of the quarterly change. For the U.S. data, the annual change is only 5.64 times as large as the quarterly change. This, along with the negative first-order serial correlation, suggests that the standard deviation of the persistent part of the quarterly change is closer to $5.64/6.63 = 0.85$ than to 1.2 percent. Some further evidence is the change over four-quarter periods—that is, the change from a given quarter of one year to the same quarter of the next year. For the random walk model, the standard deviation of these changes is 2 times the standard deviation of the quarterly change. A reason that the standard deviation of change might be better measured this way is that the measurement noise introduced by seasonal factors is minimized. The estimate obtained in this way is 0.95 percent. To summarize, Solow growth accounting finds that the process on the technology parameter is highly persistent with the standard deviation of change being about 0.90.[5]

The Solow estimate of the standard deviation of technological change is surely an overstatement of the variability of that parameter. There undoubtedly are non-negligible errors in measuring the inputs. Since the capital input varies slowly and its share is small, the most serious measurement problem is with the labor input. Fortunately there are two independent measures of the aggregate labor input, one constructed from a survey of employers and the other from a survey of households. Under the assumption of orthogonality of their measurement errors, a reasonable estimate of the variance of the change in hours is the covariance between the changes in the two series. Since the household survey is not used to estimate aggregate output, I

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[3]Actually, the average interest rate is slightly lower because of risk premia. Given the value of γ and the amount of uncertainty, the average premium is only a fraction of a percent. See Mehra and Prescott 1985 for further details.

[4]I use Hansen's (1984) human capital–weighted, household hour series. The capital stock and GNP series are from Citicorp's Citibase data bank.

[5]The process $z_{t+1} = .9z_t + \epsilon_{t+1}$ is, like the random walk process, highly persistent. Kydland and I find that it and the random walk result in essentially the same fluctuations.

use the covariance between the changes in household hours and output as an estimate of the covariance between aggregate hours and output. Still using a share parameter of $\theta = 0.75$, my estimate of the standard deviation of the percentage change in z_t is the square root of $\mathrm{var}(\Delta\hat{y}) - 2\theta\,\mathrm{cov}(\Delta\hat{h}_1, \Delta\hat{y}) + \theta^2\,\mathrm{cov}(\Delta\hat{h}_1, \Delta\hat{h}_2)$, where the caret ($\hat{}$) denotes a measured value. For the sample period my estimate is 0.763 percent. This is probably a better estimate than the one which ignores measurement error.

Still, my estimate might under- or overstate the variance of technological change. For example, the measurement of output might include significant errors. Perhaps measurement procedures result in some smoothing of the series. This would reduce the variability of the change in output and might reduce the covariance between measured hours and output.

Another possibility is that changes in hours are associated with corresponding changes in capital's utilization rate. If so, the Solow approach is inappropriate for measuring the technology shocks. To check whether this is a problem, I varied θ and found that $\theta = 0.85$ yields the smallest estimate, 0.759, as opposed to 0.763 for $\theta = 0.75$. This suggests that my estimate is not at all sensitive to variations in capital utilization rates.

To summarize, there is overwhelming evidence that technological shocks are highly persistent. But tying down the standard deviation of the technology change shocks is difficult. I estimate it as 0.763. It could very well be larger or smaller, though, given the accuracy of the measurements.

The Statistical Behavior of the Growth Models

Theory provides an equilibrium stochastic process for the growth economy studied. Our approach has been to document the similarities and differences between the statistical properties of data generated by this stochastic process and the statistical properties of American time series data. An alternative approach is to compare the paths of the growth model if the technological parameters $\{z_t\}$ were those experienced by the U.S. economy. We did not attempt this because theory's predictions of paths, unlike its predictions of the statistical properties, are sensitive to what Leamer (1983, p. 43) calls "whimsical" modeling assumptions. Another nontrivial problem is that the errors in measuring the innovations in the z_t process are as large as the innovations themselves.

The Basic Growth Model

With the standard deviation of the technology shock equal to 0.763, theory implies that the standard deviation of output will be 1.48 percent. In fact, it is 1.76 percent for the post–Korean War American economy. For the output of the artificial economy to be as variable as that, the variance of the shock must be 1.0, significantly larger than the estimate. The most important deviation from theory is the relative volatility of hours and output. Figure 3 plots a realization of the output and employment deviations from trend for the basic growth economy. A comparison of Figures 2 and 3 demonstrates clearly that, for the American economy, hours in fact vary much more than the basic growth model predicts. For the artificial economy, hours fluctuate 52 percent as much as output, whereas for the American economy, the ratio is 0.95. This difference appears too large to be a result of errors in measuring aggregate hours and output.

The Kydland-Prescott Economy

Kydland and I (1982, 1984) have modified the growth model in two important respects. First, we assume that a distributed lag of leisure and the market-produced good combine to produce the composite commodity good valued by the household. In particular,

$$u\left(c_t, \textstyle\sum_{i=0}^{\infty} \alpha_i l_{t-i}\right) = (1/3) \log c_t + (2/3) \log \textstyle\sum_{i=0}^{\infty} \alpha_i l_{t-i}$$

where $\alpha_{i+1}/\alpha_i = 1 - \eta$ for $i = 1, 2, \ldots$ and $\sum_{i=0}^{\infty} \alpha_i = 1$.

Figure 3

Deviations From Trend of GNP and Hours Worked in the Basic Growth Economy

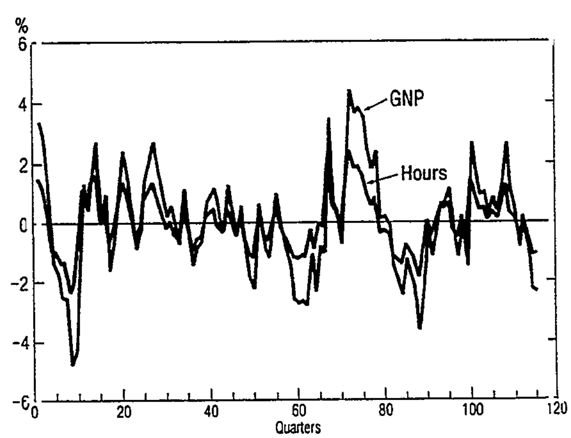


Table 2
Cyclical Behavior of the Kydland-Prescott Economy*

Variable x	Standard Deviation	Cross Correlation of GNP With $x(t-1)$	$x(t)$	$x(t+1)$
Gross National Product	1.79%	.60	1.00	.60
	(.13)	(.07)	(—)	(.07)
Consumption	.45	.47	.85	.71
	(.05)	(.05)	(.02)	(.04)
Investment	5.49	.52	.88	.78
	(.41)	(.09)	(.03)	(.03)
Inventory Stock	2.20	.14	.60	.52
	(.37)	(.14)	(.08)	(.05)
Capital Stock	.47	−.05	.02	.25
	(.07)	(.07)	(.06)	(.07)
Hours	1.23	.52	.95	.55
	(.09)	(.09)	(.01)	(.06)
Productivity (GNP/Hours)	.71	.62	.86	.56
	(.06)	(.05)	(.02)	(.10)
Real Interest Rate (Annual)	.22	.65	.60	.36
	(.03)	(.07)	(.20)	(.15)

*These are the means of 20 simulations, each of which was 116 periods long. The numbers in parentheses are standard errors.
Source: Kydland and Prescott 1984

Kydland (1983) provides justification for this preference ordering based on an unmeasured, household-specific capital stock that, like c_t and l_t, is an input in the production of the composite commodity. The economy studied has $\alpha_0 = 0.5$ and $\eta = 0.1$. This increases the variability of hours.

The second modification is to permit the workweek of capital to vary proportionally to the workweek of the household. For this economy, increases in hours do not reduce the marginal product of labor as much, so hours fluctuate more in response to technology shocks of a given size.

The statistical properties of the fluctuations for this economy are reported in Table 2. As is clear there, hours are now about 70 percent as variable as output. This eliminates much of the discrepancy between theory and measurement. If the standard deviation of the technology shock is 0.72 percent, then fluctuations in the output of this artificial economy are as large as those experienced in the U.S. economy.

A comparison of Tables 1 and 2 shows that the Kydland-Prescott economy displays the business cycle phenomena. It does not quite demonstrate, however, that there would be a puzzle if the economy did not display the business cycle phenomena. That is because the parameters α_0 and η have not been well tied down by micro observations.[6] Better measures of these parameters could either increase or decrease significantly the amount of the fluctuations accounted for by the uncertainty in the technological change.

[6]Hotz, Kydland, and Sedlacek (1985) use annual panel data to estimate α_0 and η and obtain estimates near the Kydland-Prescott assumed values.

The Hansen Indivisible Labor Economy

Labor economists have estimated labor supply elasticities and found them to be small for full-time prime-age males. (See, for example, Ashenfelter 1984.) Heckman (1984), however, finds that when movements between employment and nonemployment are considered and secondary workers are included, elasticities of labor supply are much larger. He also finds that most of the variation in aggregate hours arises from variation in the number employed rather than in the hours worked per employed person.

These are the observations that led Hansen (1985a) to explore the implication of introducing labor indivisibilities into the growth model. As shown by Rogerson (1984), if the household's consumption possibility set has nonconvexities associated with the mapping from hours of market production activities to units of labor services, there will be variations in the number employed rather than in the hours of work per employed person. In addition, the aggregate elasticity of labor supply will be much larger than the elasticity of those whose behavior is being aggregated. In this case aggregation matters, and matters greatly.

There certainly are important nonconvexities in the mapping from hours of market activities to units of labor services provided. Probably the most important nonconvexity arises from the considerable amount of time required for commuting. Other features of the environment that would make full-time workers more than twice as productive as otherwise similar half-time workers are not hard to imagine. The fact that part-time workers typically are paid less per hour than full-time workers with similar human capital endowments is consistent with the existence of important nonconvexities.

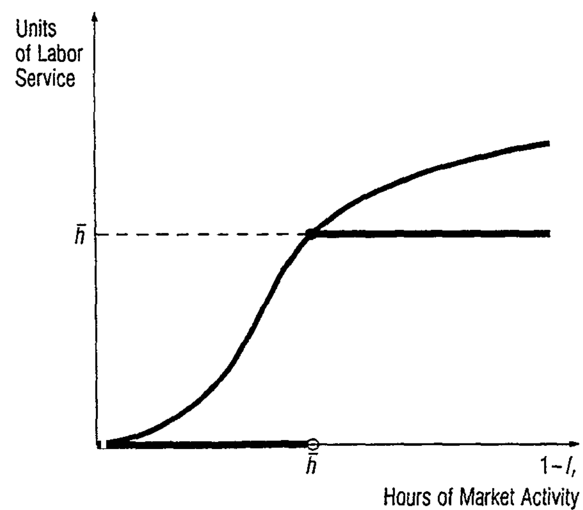
Hansen (1985a) restricts each identical household to either work $\bar{h}$ hours or be unemployed. His relation is as depicted by the horizontal lines in Figure 4. This assumption is not as extreme as it appears. If the relation were as depicted by the curved line, the behavior of the economy would be the same. The key property is an initial convex region followed by a concave region in the mapping from hours of market activity to units of labor service.

With this modification, lotteries that specify the probability of employment are traded along with market-produced goods and capital services. As before, the utility function of each individual is

$$u(c, l) = (1/3) \log c + (2/3) \log l.$$

Figure 4

Relation Between Time Allocated to Market Activity and Labor Service



If an individual works, $l = 1 - \bar{h}$; otherwise, $l = 1$. Consequently, if π is the probability of employment, an individual's expected utility is

$$E\{u(c, l)\} = (1/3) \log c + (2/3)\ \pi \log (1-\bar{h}).$$

Given that per capita consumption is $\bar{c}$ and per capita hours of employment $\bar{n}$, average utility over the population is maximized by setting $c = \bar{c}$ for all individuals. If $\bar{l}$, which equals $1 - \pi\bar{h}$, denotes per capita leisure, then maximum per capita utility is

$$U(\bar{c}, \bar{l}) = (1/3) \log \bar{c} + (2/3)\ [(1-\bar{l})/\bar{h}] \log (1-\bar{h}).$$

This is the utility function which rationalizes the per capita consumption and leisure choices if each person's leisure is constrained to be either $1 - \bar{h}$ or 1. The aggregate intertemporal elasticity of substitution between different date leisures is infinity independent of the value of the elasticity for the individual (in the range where not all are employed).

Hansen (1985a) finds that if the technology shock standard deviation is 0.71, then fluctuations in output for his economy are as large as those for the American economy. Further, variability in hours is 77 percent as large as variability in output. Figure 5 shows that aggregate hours and output for his economy fluctuate together with nearly the same amplitude. These theoretical findings are the basis for my statement in the introduction that there would be a puzzle if the economy did not display the business cycle phenomena.

Empirical Labor Elasticity

One important empirical implication of a shock-to-technology theory of fluctuations is that the empirical labor elasticity of output is significantly larger than the true elasticity, which for the Cobb-Douglas production function is the labor share parameter. To see why, note that the capital stock varies little cyclically and is nearly uncorrelated with output. Consequently, the deviations almost satisfy

$$y_t = \theta h_t + z_t$$

where y_t is output, h_t hours, and z_t the technology shock. The empirical elasticity is

$$\eta = \text{cov}(h_t, y_t)/\text{var}(h_t)$$

which, because of the positive correlation between h_t and z_t, is considerably larger than the model's θ, which is 0.64. For the basic, Kydland-Prescott, and Hansen growth economies, the values of η are 1.9, 1.4, and 1.3, respectively.

Because of measurement errors, the empirical elasticity for the American economy is not well-estimated by simply computing the ratio of the covariance between hours and output and dividing by the variance of hours. The procedure I use is based on the following probability model:

$$\hat{y}_t = y_t + \epsilon_{1t}$$

$$\hat{h}_{1t} = h_t + \epsilon_{2t}$$

$$\hat{h}_{2t} = h_t + \epsilon_{3t}$$

where the caret ($\hat{}$) denotes a measured value. The ϵ_{it} are measurement errors. Here, the $\hat{h}_{1t}$ measure of hours uses the employer survey data while the $\hat{h}_{2t}$ measure uses the household survey data. Since these are independent measures, a maintained hypothesis is that ϵ_{2t} and ϵ_{3t} are orthogonal. With this assumption, a reasonable estimate of $\text{var}(h_t)$ is the sample covariance between $\hat{h}_{1t}$ and $\hat{h}_{2t}$. Insofar as the measurement of output has small variance or ϵ_{1t} is uncorrelated with the hours measurement errors or both, the covariance between measured output and either measured hours series is a reasonable estimate of the covariance between output and hours. These two covariances are 2.231×10^{-4} and 2.244×10^{-4} for the sample period, and I take the average as my estimate of $\text{cov}(h_t, y_t)$ for the American economy. My estimate of the empirical labor elasticity of output is

$$\hat{\eta} = [\text{cov}(\hat{h}_{1t}, \hat{y}_t) + \text{cov}(\hat{h}_{2t}, \hat{y}_t)]/2\ \text{cov}(\hat{h}_{1t}, \hat{h}_{2t})$$

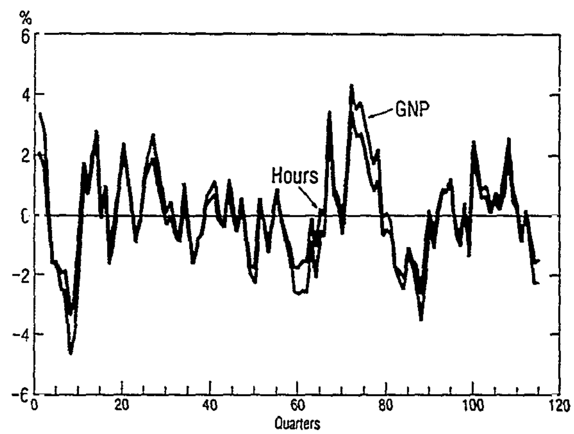
$$= 1.1.$$

This number is considerably greater than labor's share, which is about 0.70 when services of consumer durables are not included as part of output. This number strongly supports the importance of technological shocks in accounting for business cycle fluctuations. Nevertheless, the number is smaller than those for the Kydland-Prescott and Hansen growth economies.

One possible reason for the difference between the U.S. economy and the growth model empirical labor elasticities of output is cyclical measurement errors in output. A sizable part of the investment component of output is hard to measure and therefore not included in

Figure 5

Deviations From Trend of GNP and Hours Worked in Hansen's Indivisible Labor Economy



Source: Gary D. Hansen, Department of Economics, University of California, Santa Barbara the U.S. National Product Accounts measure of output, the gross national product (GNP). In particular, a firm's major maintenance expenditures, research and development expenditures, and investments in human capital are not included in GNP. In good times—namely, when output is above trend—firms may be more likely to undertake major repairs of a not fully depreciated asset, such as replacing the roof of a 30-year-old building which has a tax life of 35 years. Such an expenditure is counted as maintenance and therefore not included in GNP even though the new roof will provide productive services for many years. The incentive for firms to do this is tax savings: by expensing an investment rather than capitalizing it, current tax liabilities are reduced. Before 1984, when a railroad replaced its 90-pound rails, the expenditure was treated as a maintenance expense rather than an investment expenditure. If these and other types of unmeasured investment fluctuate in percentage terms more than output, as do all the measured investment components, the volatility of GNP is larger than measured. We do know that investment in rails was highly procyclical and volatile in the postwar period. A careful study is needed to determine whether the correction for currently unmeasured investment is small or large.

Another reason to expect the American economy's labor elasticity to be less than the model's is that the model shocks are perfectly neutral with respect to the consumption and investment good transformation. Persistent shocks which alter the product transformation frontier between these goods would cause variation in output and employment but not in the productivity parameters. For fluctuations so induced, the empirical labor elasticity of output would be the true elasticity. Similarly, relatively permanent changes in the taxing of capital—such as altering depreciation rates, the corporate income tax rate, or the investment tax credit rate—would all result in fluctuations in output and employment but not in the productivity parameters.

A final reason for actual labor elasticity to be less than the model's is the way imports are measured. An increase in the price of imported oil, that is, an increase in the quantity of output that must be sacrificed for a given unit of that input, has no effect on measured productivity. From the point of view of the growth model, however, an oil price increase is a negative technology shock because it results in less output, net of the exports used to pay for the imported oil, available for domestic consumption and investment. Theory predicts that such shocks will induce variations in employment and output, even though they have no effect on the aggregate production function. Therefore, insofar as they are important, they reduce the empirical labor elasticity of output.

Extensions

The growth model has been extended to provide a better representation of the technology. Kydland and I (1982) have introduced a technology with more than one construction period for new production capacity.[7] We have also introduced inventory as a factor of production. This improves the match between the model's serial correlation properties and the U.S. postwar data, but has little effect on the other statistics.

Kydland (1984) has introduced heterogeneity of labor and found that if there are transfers from high human capital people to low human capital people, theory implies that hours of the low fluctuate more than hours of the high. It also implies a lower empirical labor elasticity of output than the homogeneous household model.

Bain (1985) has studied an economy that is richer in sectoral detail. His model has manufacturing, retailing, and service-producing sectors. A key feature of the technology is that production and distribution occur sequentially. Thus there are two types of inventories—those of manufacturers' finished goods and those of final goods available for sale. With this richer detail, theory implies that different components of aggregate inventories behave in different ways, as seen in the data. It also implies that production is more volatile than final sales, an observation considered anomalous since inventories can be used to smooth production. (See, for example, Blinder 1984.)

Much has been done. But much more remains to be explored. For example, public finance considerations could be introduced and theory used to predict their implications. As mentioned above, factors which affect the rental price of capital affect employment and output, and the nature of the tax system affects the rental price of capital. Theory could be used to predict the effect of temporary increases in government expenditures such as those in the early 1950s when defense expenditures increased from less than 5 to more than 13 percent of GNP. Theory of this type could also be used to predict the effect of terms-of-trade shocks. An implication of such an exercise most likely will be that economies with persistent terms-of-trade shocks fluctu-

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[7] Altug (1983) has introduced two types of capital with different gestation periods. Using formal econometric methods, she finds evidence that the model's fit is improved if plant and equipment investment are not aggregated.

ate differently than economies with transitory shocks. If so, this prediction can be tested against the observations.

Another interesting extension would be to explicitly model household production. This production often involves two people, with one specializing in market production and the other specializing in household production while having intermittent or part-time market employment. The fact that, cyclically, the employment of secondary wage earners is much more volatile than that of primary wage earners might be explained.

A final example of an interesting and not yet answered question is, How would the behavior of the Hansen indivisible labor economy change if agents did not have access to a technology to insure against random unemployment and instead had to self-insure against unemployment by holding liquid assets? In such an economy, unlike Hansen's, people would not be happy when unemployed. Their gain of more leisure would be more than offset by their loss as an insurer. Answering this question is not straightforward, because new tools for computing equilibria are needed.

Summary and Policy Implications

Economic theory implies that, given the nature of the shocks to technology and people's willingness and ability to intertemporally and intratemporally substitute, the economy will display fluctuations like those the U.S. economy displays. Theory predicts fluctuations in output of 5 percent and more from trend, with most of the fluctuation accounted for by variations in employment and virtually all the rest by the stochastic technology parameter. Theory predicts investment will be three or more times as volatile as output and consumption half as volatile. Theory predicts that deviations will display high serial correlation. In other words, theory predicts what is observed. Indeed, if the economy did not display the business cycle phenomena, there would be a puzzle.

The match between theory and observation is excellent, but far from perfect. The key deviation is that the empirical labor elasticity of output is less than predicted by theory. An important part of this deviation could very well disappear if the economic variables were measured more in conformity with theory. That is why I argue that theory is now ahead of business cycle measurement and theory should be used to obtain better measures of the key economic time series. Even with better measurement, there will likely be significant deviations from theory which can direct subsequent theoretical research. This feedback between theory and measurement is the way mature, quantitative sciences advance.

The policy implication of this research is that costly efforts at stabilization are likely to be counterproductive. Economic fluctuations are optimal responses to uncertainty in the rate of technological change. However, this does not imply that the amount of technological change is optimal or invariant to policy. The average rate of technological change varies much both over time within a country and across national economies. What is needed is an understanding of the factors that determine the average rate at which technology advances. Such a theory surely will depend on the institutional arrangements societies adopt. If policies adopted to stabilize the economy reduce the average rate of technological change, then stabilization policy is costly. To summarize, attention should be focused not on fluctuations in output but rather on determinants of the average rate of technological advance.

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