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INTERNATIONAL COORDINATION OF FISCAL POLICY IN LIMITING ECONOMIES

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ABSTRACT

We examine the limiting behavior of cooperative and noncooperative fiscal policies as countries' market power goes to zero. We show that these policies converge if countries raise revenues through lump-sum taxation. However, if there are unremovable domestic distortions, such as distorting taxes, there can be gains to coordination even when a single country's policy cannot affect world prices. These results differ from the received wisdom in the optimal tariff literature. The key distinction is that, unlike in the tariff literature, the spending decisions of governments are explicitly modeled.

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I. Introduction

Writing on international economic interdependence, Frenkel and Razin (1985, p. 635) recently called for an analysis that "would determine the optimal pattern of government spending . . . along the lines of the optimal tariff literature." This paper is a first step in that direction. We consider a world economy composed of a number of countries in which governments choose policy to maximize the utility of their respective consumers. Given multiple policymakers, we need first to take a stand on how they interact. We contrast two polar regimes: In one regime, policymakers act in a coordinated fashion, choosing policy cooperatively to maximize world welfare. In the other regime, they choose policies noncooperatively to maximize their own country's welfare. As has long been recognized, the equilibria of these regimes may be quite different. In particular, the literature on optimal tariffs shows that substantial distortions and a reduction in world welfare can result if governments cannot commit to cooperation. In that literature, distortions arise from the monopoly power of large countries. A standard result is that if countries become small relative to the world economy, these distortions vanish and tariff policies in the two regimes converge.

In this paper we ask whether or not an analogous result holds for fiscal policy: Do cooperative and noncooperative fiscal policies converge as countries become small? Fiscal policies are modeled as choices of spending levels on public goods and their means of finance. Unlike the literature on optimal tariffs, this paper explicitly models the spending decisions of governments, and this difference turns out to be crucial to the results.

We begin by considering a model with lump-sum taxes. Expenditures on public goods affect world relative prices even though the revenues to finance them are raised through lump-sum taxes. As expected, the noncooperative equilibrium yields a lower level of welfare than the cooperative equilibrium. For this model we show that the analogue of the standard tariff result holds: as countries become small, the distortions vanish and policies in the two regimes converge.

We then consider a model with distorting taxes. In this case the tariff result does not hold: the cooperative and noncooperative policies are generally different, even in the limit. This suggests that if there are unremovable domestic distortions, countries can gain from international cooperation, even in markets where they have no monopoly power.[1] Since this result differs from the standard results reported in the tariff literature, it is important to understand the intuition behind it.

In the limiting noncooperative equilibrium, each government seeks to achieve two conflicting goals by using one instrument, the tax rate. On the one hand, governments seek to equate the marginal rate of substitution between consumption of private goods to the given world price. On the other, they seek to balance the welfare gains of providing public goods against the welfare losses from distorting taxes. Of course, the only way to achieve the first goal is to set the tax rate to zero and provide no public goods. The optimal tax rate in the limiting noncooperative equilibrium appropriately balances the tradeoffs in achieving these goals. In the limiting cooperative equilibrium, governments recognize that because of tax distortions, the world price does not signal the marginal rates of substitution between private goods of other countries' consumers. Therefore, governments do not seek to equate consumers' marginal rates of substitution to the world price; rather, they seek to equate consumers' marginal rates of substitution across countries.

This paper is related to several strands of literature.[2] First, it is related to other analyses of fiscal policy in a world economy. In terms of strategic analyses of fiscal policy, we unify the results of Backus, Devereux, and Purvis (1988), Devereux (1986), Hamada (1986), and Kehoe (1987). However, we limit our attention to static models to avoid issues concerning the time inconsistency of tax/spending policy of the type considered by Lucas and Stokey (1983) and Persson and Svensson (1986). Once these simple models are well understood, it would be interesting to explore dynamic models of policy in which a key ingredient is the interaction between time inconsistency and cooperation. Rogoff (1985) and Kehoe (1989) provide examples of this type of analysis.

The paper is organized as follows: Section II describes the basic model and establishes that noncooperative equilibria typically do not coincide with cooperative equilibria and that cooperative equilibria are optimal in a sense that noncooperative equilibria are not. Section III proves that in this model, the two solutions converge as the economy is replicated. Sections IV and V present economies in which these solutions diverge: in Section IV divergence occurs because the economy is not replicated, and in Section V it occurs because of a tax distortion. Section VI briefly summarizes our results and suggests how the analysis could be extended.

II. Monopoly Distortions

Consider a world economy composed of a finite number of countries. Equilibria of this economy are compared under two regimes: in the first, governments set policy cooperatively; in the second, governments play a noncooperative game. Under both regimes, governments optimally choose policy, taking as given that for each policy setting, private agents are in a competitive equilibrium. The noncooperative and cooperative equilibria can be easily computed. We solve first for the competitive equilibrium for an arbitrary setting of government policy. The competitive equilibrium allocations and prices are then used to express the governments' objective functions in terms of their policies. We then solve for the governments' policies under the two regimes.

A. Competitive Equilibria for Private Agents

Consider a world economy composed of a finite number of countries, I, with both private and public goods. Each country is populated by a large number of identical consumers, say L, and a government. For ease of notation, let L equal 1. Consumers in each country have endowments of two private goods. The government of each country has access to a production technology that transforms the first of these private goods into a public good which benefits only residents of the country. Each government pays for this public good by levying lump-sum taxes on its inhabitants.

In particular, a consumer of country i is endowed with a positive amount y_n^i of each (private) good n and is taxed τ^i units of good 1 for i = 1, ..., I, and n = 1, 2. This consumer chooses consumption levels of the private goods, denoted by c_n^i, for n = 1, 2, and receives g^i units of the country-i-specific public good. Consumer i's preferences over the consumption bundle (c_1^i, c_2^i, g^i) are given by $u^i(c_1^i, c_2^i, g^i)$. We assume that each u^i is monotone, strictly concave, and twice continuously differentiable, and that the marginal utility of each good goes to infinity as the amount of each good goes to zero. The consumer's budget constraint is

$$c_1^i + pc_2^i = y_1^i - \tau^i + py_2^i, \tag{1}$$

where p denotes the price of good 2 relative to good 1. The consumer, taking as given the price p and the tax/spending policy (τ^i, g^i) of the government of country i, chooses private good consumption c_1^i and c_2^i to maximize utility subject to (1). Let the demand functions for this consumer be denoted by $c_n^i(\tau^i, p)$ for n = 1, 2, where the dependence of these functions on the endowments is suppressed.

The government of country i has access to a production technology that converts private good 1 into a country-i-specific public good. For notational simplicity, we let this production function be linear with a unit coefficient. The budget constraint for the government of country i is $g^i = \tau^i$. Since government spending always equals taxes, government i's policy is summarized by τ^i and referred to either as spending or taxes.

Market clearing in markets for goods 1 and 2 requires

$$\sum_{i=1}^{I} c_1^i + \sum_{i=1}^{I} \tau^i = \sum_{i=1}^{I} y_1^i \tag{2}$$

and

$$\sum_{i=1}^{I} c_2^i = \sum_{i=1}^{I} y_2^i. \tag{3}$$

Let $\tau = (\tau^1,\ldots,\tau^I)$ and $c_n = (c_n^1,\ldots,c_n^I)$ for n = 1, 2.

A <u>competitive equilibrium</u> is an allocation of private consumption (c_1,c_2), a price p, and a vector of government tax/spending policies τ such that the following conditions hold: (i) the consumption and government-spending vectors satisfy (2) and (3), and (ii) the consumption allocations c_1^i and c_2^i maximize utility subject to (1), given τ^i and p for each i = 1, ..., I.

This equilibrium has three noteworthy features that we will use later. First, for any given vector τ, the market-clearing conditions together with the consumer demand functions implicitly define the equilibrium price as a function of τ, say $p = p(\tau)$. Second, given the government's budget constraint, we can express the maximized value of consumer i's utility as

$$V^i(\tau^i,p(\tau)) = u^i[c_1^i(\tau^i,p(\tau)),c_2^i(\tau^i,p(\tau)),\tau^i]. \tag{4}$$

Third, the private consumption allocations and prices in the above competitive equilibrium with public goods are identical to those in an economy with only private goods in which country-i consumers' private good endowments are $y_1^i - \tau^i$ and y_2^i, respectively, and τ enters the utility function as a fixed parameter. Because of this feature, the competitive equilibrium is clearly Pareto optimal in the class of allocations (c_1, c_2, τ) that satisfy (2) and (3) and take τ as given.

B. Noncooperative and Cooperative Equilibria

In Section I.A, government policies were arbitrary. In this section, however, we consider policies that are outcomes of either a noncooperative or a cooperative game among governments.

A <u>noncooperative equilibrium</u> is a vector of government policies τ, a competitive equilibrium price function $p(\tau)$, and vectors of competitive equilibrium allocation functions $c_1(\tau, p(\tau))$ and $c_2(\tau, p(\tau))$ such that (i) for each country i, τ^i maximizes (4) given $\tau^{-i} = (\tau^1, \ldots, \tau^{i-1}, \tau^{i+1}, \ldots, \tau^I)$ and (ii) for every τ, the resulting prices and allocations are a competitive equilibrium.

In a noncooperative equilibrium, each government chooses policy separately to maximize its country's objective function. In a cooperative equilibrium, governments instead choose policy jointly to maximize a world objective function. We assume that the world's objective function is a weighted average of the individual countries' objective functions. For an arbitrary vector λ of nonnegative weights, the world objective function is $\sum_i \lambda_i V^i(\tau^i, p(\tau))$.

A <u>cooperative equilibrium relative to λ</u> is a vector of government policies τ, a competitive equilibrium price function $p(\tau)$, and vectors of competitive equilibrium allocation functions $c_1(\tau, p(\tau))$ and $c_2(\tau, p(\tau))$ such that (i) the vector τ maximizes the world objective function and (ii) for every τ, the resulting prices and allocations are a competitive equilibrium.

Although we have just defined cooperative equilibria for arbitrary weights, we are more interested in cooperative equilibria relative to particular values of these weights. Such weights respect private ownership: they set to zero an excess savings function associated with a planning problem in which both private consumption and government spending are chosen. We show that cooperative equilibria relative to such weights solve a planning problem. To this end, consider the following planning problem: For a given vector $\lambda = (\lambda^1,\dots,\lambda^I)$ of nonnegative weights, let

$$W(\lambda) = \max_{\{c_1,c_2,\tau\}} \sum_{i=1}^{I} \lambda^i[u^i(c_1^i,c_2^i,\tau^i)] \tag{5}$$

subject to (2) and (3).

Let p_1 and p_2 denote the Lagrange multipliers on constraints (2) and (3), respectively, and let $p = p_2/p_1$ be the normalized Lagrange multiplier. Write the solution to this problem as $\{c_1(\lambda),c_2(\lambda),\tau(\lambda),p(\lambda)\}$, and call it a (world) social optimum relative to λ. For each country i, define the excess savings function $s^i(\lambda)$ to be

$$s^i(\lambda) = [y_1^i-c_1^i(\lambda)-\tau^i(\lambda)] + p(\lambda)[y_2^i-c_2^i(\lambda)]. \tag{6}$$

Let S denote the set of weights that yields excess savings of zero in each country; that is, $S = \{\lambda\epsilon R_+^I|s^i(\lambda)=0 \text{ for } i=1,\dots,I\}$. Call S the set of weights that respect private ownership, and call a cooperative equilibrium relative to some λ in S a cooperative equilibrium that respects private ownership. We then have

<u>Proposition 1</u>. A cooperative equilibrium that respects private ownership is a social optimum.

The proof of this proposition is given in Appendix A. Proposition 1 can be restated in a slightly more precise way. For any λ in S, the set of cooperative equilibria relative to λ coincides with the set of social optima relative to the same λ. The intuition behind the proposition runs something like this: The cooperative maximization problem is a search across policies (and therefore across competitive equilibria, given those policies) for the one that yields the highest value of the objective function. We know that the private consumption allocations of these competitive equilibria are optimal, given the government policy. Only one circumstance, then, could render the cooperative equilibria suboptimal; that is, government policy is not chosen optimally.

To understand how government spending could be chosen suboptimally, consider a cooperative equilibrium for an arbitrary vector of weights λ. Recall that in our cooperative equilibrium, the only choice that governments make is the level of government spending. Suppose, instead, that we consider a cooperative equilibrium in which governments not only choose spending but also make lump-sum transfers between residents of each country. In that equilibrium, for any vector of weights, the governments will set spending optimally and then use a separate set of instruments--the lump-sum transfers--to achieve the optimal income distribution across countries. In contrast, in our cooperative equilibrium these two goals must be achieved by a single set of instruments--the levels of government spending. If the weights chosen do not respect the initial distribution of income, the government spending decisions are distorted. Basically, countries assigned higher (or lower) weights than their endowments justify are compensated in utility terms by inefficiently high (or low) levels of government spending. In the proof of Proposition 1, we establish that the set of weights which respects this initial distribution of endowments is nonempty, and that the amount of government spending for a cooperative equilibrium relative to such weights is optimal.

We next show that with a fixed number of countries, the cooperative equilibria typically do not coincide with the noncooperative ones. To demonstrate this point, we compare the first-order conditions of the noncooperative equilibria with those of the cooperative equilibria. In a noncooperative equilibrium, the government of country k chooses spending τ^k to satisfy

$$\frac{\partial V^k}{\partial \tau^k} + \frac{\partial V^k}{\partial p}\frac{\partial p}{\partial \tau^k} = 0. \tag{7}$$

Using the envelope theorem, this condition is easily transformed into

$$(-1+u_3^k/u_1^k) + (y_2^k - c_2^k)\frac{\partial p}{\partial \tau^k} = 0. \tag{8}$$

We call the first term in equations (7) and (8) the <u>direct</u> effect of a change in policy and the second term the <u>indirect</u> (or general equilibrium) effect. The direct effect measures the impact of a change in policy by a government on that country's residents at a given world price p. Note, however, that with a finite number of countries, a change in spending by one government also affects this world price. The indirect effect measures the impact on residents of a change in government spending solely in terms of changing this world price.

Now a cooperative equilibrium that respects private ownership is a social optimum. Therefore the marginal rate of substitution between private and public consumption must be equated to the marginal rate of transformation. Hence, for each country k, government spending τ^k must satisfy $-1 + u_3^k/u_1^k = 0$. The wedge between these two first-order conditions is the term

$$(y_2^k - c_2^k)\ \frac{\partial p}{\partial \tau^k}, \tag{9}$$

which we call the <u>monopoly distortion</u>.

In the noncooperative allocation the monopoly distortion drives a wedge between the socially optimal decision and the noncooperative decisions. Basically, in the noncooperative allocation, each government takes into account its effect on world prices and chooses a policy to influence prices in a direction that benefits its residents. In particular, suppose that at the cooperative level of spending, country k is a net exporter of good 1. At this allocation a noncooperative government of country k would have an incentive to raise its spending a little. Doing so decreases the net private supply of private good 1 and raises the relative price of exports. In the process, country k makes itself better off. Likewise, if at the cooperative level of spending country k is a net importer of good 1, then a noncooperative government of this country would have an incentive to lower its spending a little. Doing so increases the net private supply of private good 1 and lowers the relative price of imports.

In general, then, when there is a finite number of countries the noncooperative and cooperative equilibria do not coincide because of monopoly distortions. Indeed, the only type of cooperative equilibrium that could also be a noncooperative equilibrium is one without trade. In this special case, monopoly distortions disappear and governments have no incentives to distort spending decisions to affect world prices.

III. Convergence in Replica Economies

In this section we show that if the economy of Section II is replicated, then the monopoly distortions go to zero and the noncooperative and cooperative allocations converge. Consider replicating the economy of Section II for a fixed number of times, say J. (Eventually, we let J go to infinity.) The J^{th} replica economy has countries indexed by ij for i = 1, ..., I and j = 1, ..., J, where i refers to the type of country and j refers to the replication number. All J consumers of type i have the same utility functions and endowments: that is, for all j = 1, ..., J, let $u^{ij} = u^{i1}$ and $y^{ij} = y^{i1}$. The demand function of consumer ij for good n is denoted by $c_n^{ij}(\tau^i,p)$ for n = 1,2. Market clearing for good 1 then requires

$$\sum_j \sum_i c_1^{ij}(\tau^{ij},p) + \sum_j \sum_i \tau^{ij} = \sum_j \sum_i y_1^{ij}. \tag{10}$$

Market clearing for good 2 is similarly defined. These conditions implicitly define the equilibrium price as a function of government spending. We write this function as $p = p(T^J)$, where $T^J = (\tau^{11},\ldots,\tau^{I1};\ldots;\tau^{1J},\ldots,\tau^{IJ})$. The objective function of the government of country ij is $V^{ij}(\tau^{ij},p(T^J))$, where V^{ij} is defined analogously to (4).

For the replica economy, noncooperative and cooperative equilibria are defined as in Section II. We focus on equilibria that are symmetric, in the sense that all countries of the same type choose the same policy; that is, $\tau^{ij} = \tau^{i1}$ for all i and j. From now on, this symmetry requirement is understood. We then have

Proposition 2. As the number of replications goes to infinity, the noncooperative equilibria converge to cooperative equilibria that respect private ownership.

The proof of this proposition is a straightforward application of the definition of a replica economy, together with a little price theory. For any given number of replications, the noncooperative solution clearly coincides with the cooperative solution if and only if the monopoly distortions are zero. In the J^{th} replica economy, the monopoly distortion for country k1 (the first replica of type k) is

$$(y_2^{k1}-c_2^{k1})\ \frac{\partial p(T^J)}{\partial \tau^{k1}}. \tag{11}$$

The proposition is proved by showing that this distortion goes to zero as J goes to infinity for each type-k country.

First, consider an economy with J equal to 1, that is, the original economy. In this economy, the market-clearing condition for good 1 is

$$\sum_{i=1}^{I} c_1^{i1}(\tau^{i1},p) + \sum_{i=1}^{I} \tau^{i1} = \sum_{i=1}^{I} y_1^{i1}. \tag{12}$$

The market-clearing conditions for the private goods define the equilibrium price function $p(T^1)$ and the private consumption allocations $\{c^{i1}|i=1,\ldots,I\}$. To evaluate how a spending change by the government of a type-k country affects the equilibrium price, differentiate (12) to obtain

$$\frac{\partial p(T^1)}{\partial \tau^{k1}} = \frac{1 - \partial c_1^{k1}/\partial \tau^{k1}}{\sum_{i=1}^{I} \partial c_1^{i1}/\partial p}. \tag{13}$$

Now consider an economy with J greater than 1. In such an economy, the market-clearing conditions (10) for good 1 define the equilibrium price function $p(T^J)$ and the private consumption allocations $\{c^{ij}|i=1,\ldots,I;j=1,\ldots,J\}$. To evaluate how a spending change by a government of a type-k country, say country k1, affects this price, differentiate (10) to obtain

$$\frac{\partial p(T^J)}{\partial \tau^{k1}} = \frac{1 - \partial c_1^{k1}/\partial \tau^{k1}}{\sum_{j=1}^{J} \sum_{i=1}^{I} \partial c_1^{ij}/\partial p}. \tag{14}$$

From the definition of a replica economy, $c_1^{ij}(p,\tau^{ij}) = c_1^{i1}(p,\tau^{i1})$ for all i and j, and by our symmetry assumption, $\tau^{ij} = \tau^{i1}$ for all i and j. Thus, in an equilibrium of the J^{th} replica economy, we can write (10) as

$$J\sum_{i=1}^{I} c_1^{i1}(\tau^{i1},p) + J\sum_{i=1}^{I} \tau^{i1} = J\sum_{i=1}^{I} y_1^{i1}, \tag{15}$$

which is equivalent to (12). That is, the competitive equilibria of the J^{th} replica economy are simply the competitive equilibria of the original economy replicated J times. In particular, with concave utility functions, all consumers of the same type get the same allocation. This fact about equilibrium allocations implies

$$\sum_{j=1}^{J}\sum_{i=1}^{I} \frac{\partial c_1^{ij}}{\partial p} = J\sum_{i=1}^{I} \frac{\partial c_1^{i1}}{\partial p}. \tag{16}$$

Combining (13), (14), and (16) gives

$$\frac{\partial p(T^J)}{\partial \tau^{k1}} = \frac{1}{J}\,\frac{\partial p(T^1)}{\partial \tau^{k1}}. \tag{17}$$

Using (17) and the fact that the equilibria in the replica economy are the replicated equilibria of the original economy, we have that as J goes to infinity, the monopoly distortion (11) goes to zero for each country. The noncooperative equilibria thus converge to the cooperative equilibria.

IV. Divergence in Nonreplica Economies

In Section III, replication was shown to cause the cooperative and noncooperative equilibria to converge. The process of replication implies that countries become small in two ways. First, each country's endowment, as a fraction of the world endowment, converges to zero. Second, each country's socially optimal level of government spending, as a fraction of the world endowment, converges to zero. In this section we present a parametric example of a nonreplica economy in which these conditions fail and the two solutions diverge. [This example is closely related to Devereux (1986).]

Let there be I countries (indexed $i=1,\ldots,I$) and I private goods (indexed $n=1,\ldots,I$). Consumers in country i own the world endowment of good i but own no other goods. Only the government of country i has access to a production technology that converts private good i into a country-specific good at a one-to-one rate. In addition, let c_n^i denote the consumption of private good n by consumers in country i; let y_i^i denote the country-i consumer endowment of good i; and let τ_i^i denote the amount of private good i that is converted by the government of country i into a public good. For each i, let $y_1^i = y_1^1$ and let the utility functions be given by

$$u^i(c_1^i,\ldots,c_I^i,\tau_i^i) = \sum_{n=1}^{I} \ln c_n^i/I + \ln \tau_i^i. \tag{18}$$

Let $p = (p_1,\ldots,p_I)$ denote the prices of the private goods. Consumers in country i solve the problem

$$V^i(\tau^i,p) = \max_{\{c^i\}} \sum_{n=1}^{I} \ln c_n^i/I + \ln \tau_i^i$$

subject to

$$\sum_{n=1}^{I} p_n c_n^i = p_i(y_i^i-\tau_i^i), \tag{19}$$

where $c^i = (c_1^i,\ldots,c_I^i)$ and the consumer's and the government's budget constraints are already combined. The resulting demand functions are $c_n^i = p_i(y_i^i-\tau_i^i)/Ip_n$. Market clearing requires

$$\sum_{i=1}^{I} c_n^i + \tau_n^n = y_n^n \quad \text{for } n = 1, \ldots, I. \tag{20}$$

Substituting the demand functions into the market-clearing conditions gives the equilibrium price functions $p_n(\tau) = (y_1^1-\tau_1^1)/(y_n^n-\tau_n^n)$, where we normalized prices by setting $p_1 = 1$. Given the price and demand functions, the first-order conditions for the noncooperative equilibrium can be rewritten as

$$\tau_i^i = I(y_i^i-\tau_i^i) \quad \text{for } i = 1, \ldots, I. \tag{21}$$

Given our symmetry assumption, (21) implies that the noncooperative level of government spending is $\tau_i^i = Iy_1^1/(I+1)$.

Now consider the cooperative solution. Given the symmetry of the example, any vector of weights that places an equal weight on each country will respect private ownership. The first-order conditions for this problem can be rewritten as

$$\tau_i^i = y_i^i - \tau_i^i \quad \text{for } i = 1, \ldots, I. \tag{22}$$

Imposing symmetry, we have that the cooperative level of government spending is given by $\tau_i^i = y_1^1/2$. Thus, as the number of countries goes to infinity, the cooperative and noncooperative solutions diverge.

Although in this example the number of countries goes to infinity, each type of country maintains monopoly power over a good. A given country i has two sources of monopoly power over private good i. First, it has monopoly power in endowments: it is the only country with endowments of good i. Second, it has monopoly power in production: it is the only country that can convert private good i into a public good. Neither of these two sources of monopoly power goes to zero as new types of countries are added. It is possible to construct examples in which either source alone causes the two solutions to diverge, but the algebra is somewhat tedious.

V. Divergence in an Economy With Tax Distortions

In this section we describe an economy with distortionary taxes and show that the two solutions do not necessarily converge even though monopoly distortions go to zero. Consider an economy identical to the one in Section II except that taxes are distortionary instead of lump sum. For simplicity, let all countries be identical. Since there is only one type of country, think of an economy with J such countries as the J^{th} replica of an original economy with one country. Let the distortionary tax be a linear tax on the consumption of good 1. A representative consumer in country j (indexed j=1,...,J) solves the problem

$$\max_{\{c^j\}} \; u^j(c_1^j, c_2^j, g^j) \tag{23}$$

subject to

$$(1+\tau^j)c_1^j + pc_2^j = y_1^j + py_2^j,$$

where τ^j is the consumption tax imposed by the government on its residents' consumption of good 1. This problem yields demand functions $c_n^j(\tau^j,p)$ for n = 1,2. The country-j government chooses taxes τ^j and government spending g^j to satisfy its budget constraint: $g^j = \tau^j c_1^j$. A competitive equilibrium is defined as in Section II. The market-clearing conditions implicitly define the equilibrium price as a function of the tax policies, say $p = p(\tau)$.

To define the government's objective function, first substitute the consumer's demand functions and the equilibrium value of the government's budget constraint into the consumer's utility function to obtain

$$V^j(\tau^j,p(\tau)) = u^j[c_1^j(\tau^j,p(\tau)), c_2^j(\tau^j,p(\tau)), \tau^j c_1^j(\tau^j,p(\tau))]. \tag{24}$$

The first-order conditions for the noncooperative level of taxes are

$$\frac{\partial V^k}{\partial \tau^k} + \frac{\partial V^k}{\partial p}\frac{\partial p}{\partial \tau^k} = 0 \quad \text{for } k = 1, \ldots, J. \tag{25}$$

Again, the first-order conditions are the sum of direct and indirect effects. The direct effects can be written as

$$\frac{\partial V^k}{\partial \tau^k} = u_1^k c_1^k \left[-\frac{1}{(1+\tau^k)} + \frac{u_3^k}{u_1^k}\left(1 + \frac{\tau^k}{c_1^k}\frac{\partial c_1^k}{\partial \tau^k}\right)\right] \tag{26}$$

and the indirect effects as

$$\frac{\partial V^k}{\partial p}\frac{\partial p}{\partial \tau^k} = \frac{u_1^k}{(1+\tau^k)}(y_2^k - c_2^k)\frac{\partial p}{\partial \tau^k} + u_3^k \tau^k \frac{\partial c_1^k}{\partial p}\frac{\partial p}{\partial \tau^k}. \tag{27}$$

From the market-clearing conditions, we have

$$\frac{\partial p}{\partial \tau^k} = \frac{-[c_1^k + (1+\tau^k)\partial c_1^k/\partial \tau^k]}{\sum_{j=1}^{J}(1+\tau^j)\partial c_1^j/\partial p}. \tag{28}$$

Recall that the direct effects measure how a change in government policy affects that country's residents at a given world price, whereas indirect effects measure how a policy change affects residents by affecting the world price. With distortionary taxes, both effects are changed. The direct effects no longer imply that the marginal rate of substitution should be equated to the marginal rate of transformation. Rather, these terms are modified by the elasticity of consumption with respect to the distortionary tax. The indirect effects are now composed of two terms. The first term in (27) is analogous to the indirect effect in (8); both represent monopoly distortions. The second term in (27), called the <u>tax distortion effect</u>, measures how much the price changes resulting from a tax change affect utility by changing the level of public goods provided. Note that if consumption of good 1 were completely inelastic with respect to its price, this second distortion would be zero, leaving only the monopoly distortion.

Compare this solution with the cooperative solution. Given the symmetry of the example, equal weights respect private ownership. We consider a symmetric solution in which all policies are the same. The first-order conditions for the cooperative allocation are

$$\frac{\partial V^k}{\partial \tau^k} + \sum_{j=1}^{J} \frac{\partial V^j}{\partial p} \frac{\partial p}{\partial \tau^k} = 0. \tag{29}$$

In contrast to the model in Section II, the extra distortion that results from taxes causes the indirect effects not to cancel. Indeed, the sum of indirect effects is

$$\sum_{j=1}^{J} \frac{\partial V^j}{\partial p} \frac{\partial p}{\partial \tau^k} = \sum_{j=1}^{J} \left[u_3^j \tau^j \frac{\partial c_1^j}{\partial p} \right] \frac{\partial p}{\partial \tau^k}. \tag{30}$$

This sum of the tax-induced distortions causes the two solutions to diverge. To see this, let $p(T^J)$ represent the equilibrium price function with J identical countries. As in Proposition 2, $\partial p(T^J)/\partial \tau^k = J^{-1} \partial p(T^1)/\partial \tau^k$. Using symmetry we have that the noncooperative solution is given by

$$\left[-\frac{1}{(1+\tau)} + \frac{u_3}{u_1} \left(1 + \frac{\tau}{c_1} \frac{\partial c_1}{\partial \tau} \right) \right] + \frac{1}{J} \left[\frac{u_3}{u_1} \frac{\tau}{c_1} \frac{\partial c_1}{\partial p} \right] \frac{\partial p(T^1)}{\partial \tau} = 0, \tag{31}$$

and the cooperative solution is given by

$$\left[-\frac{1}{(1+\tau)} + \frac{u_3}{u_1} \left(1 + \frac{\tau}{c_1} \frac{\partial c_1}{\partial \tau} \right) \right] + \left[\frac{u_3}{u_1} \frac{\tau}{c_1} \frac{\partial c_1}{\partial p} \right] \frac{\partial p(T^1)}{\partial \tau} = 0. \tag{32}$$

The wedge between these solutions is

$$\frac{(J-1)}{J}\left[\frac{u_3}{u_1}\,\frac{\tau}{c_1}\,\frac{\partial c_1}{\partial p}\right]\frac{\partial p(T^1)}{\partial \tau}. \tag{33}$$

We can use (28) to show that, in general, this wedge is nonzero and thus these two solutions diverge as the number of countries J goes to infinity. It is worth pointing out that in the special case of Cobb-Douglas utility, the relevant income and substitution effects cancel and this wedge is zero.

The intuition for this result is as follows: Substituting (28) into (29) and using (30) gives $u_3/u_1 = 1$; that is, in a cooperative equilibrium, the marginal rate of substitution between private and government consumption is equated to the marginal rate of transformation. Thus, the cooperative equilibrium with distortionary taxes has the same allocations as the cooperative equilibrium with lump-sum taxes. The equivalence follows from our symmetry assumptions.[3]

The noncooperative equilibrium allocations generally differ from the cooperative ones. To see the difference, consider the limiting noncooperative equilibrium. Since each government chooses its tax rate taking the world price as given, the last term on the left side of (31) is zero. This implies that in the limiting noncooperative equilibrium, the marginal rates of substitution between private and government consumption are <u>not</u> equated to the marginal rate of transformation. Governments choose not to equate these marginal rates because they have one instrument--the tax rate--and two conflicting goals. On the one hand, governments seek to equate the marginal rate of substitution between the private goods to the world price, while on the other, they seek to set u_3/u_1 equal to 1. The first goal can only be met by setting the tax rate equal to zero; achieving the second means distorting the marginal rates of substitution between private goods away from the world price. The optimal policy in the noncooperative equilibrium appropriately balances these two goals, achieving neither completely. In contrast, in the limiting cooperative equilibrium, governments recognize that because of tax distortions, the world price does not signal the marginal rates of substitution between the private goods of other countries' consumers. Thus, governments do not seek to equate the marginal rates of substitution to the world price; rather, they seek to equate consumers' marginal rates of substitution across countries. By appropriately adjusting all the tax rates, they can achieve these two goals.

There is an alternative way to see why the cooperative and noncooperative allocations differ. Suppose all governments are initially at the cooperative equilibrium. We show that even in the limit, a single government can deviate from this equilibrium and make itself better off. Suppose, for now, there exists a feasible policy change by a government which increases $c_1 + g$ by a small amount. Using a Taylor series expansion, the change in utility for that country is given by

$$\Delta u = u_1 dc_1 + u_2 dc_2 + u_3 dg. \tag{34}$$

To evaluate this expression, note that since we started at the cooperative equilibrium, $u_3 = u_1$, while from the consumer's budget constraint we have

$$dc_1 + dg + pdc_2 = 0 \tag{35}$$

and from the consumer's first-order condition we have $u_2/u_1 = p/(1+\tau)$. Substituting (34) into (35) and simplifying gives

$$\Delta u = u_1(dc_1+dg)[1-1/(1+\tau)]. \tag{36}$$

Since taxes are positive in the cooperative equilibrium, (36) implies that if there is a policy change which increases $c_1 + g$, then such a change increases utility. It is easy to show that such a policy change exists if the price elasticity of demand for good 1 is different from one, that is, if preferences are not Cobb-Douglas.[4] Thus, with any other preferences the cooperative equilibrium will not be a noncooperative equilibrium.

It is worth noting that all results in this section hold even if the instrument available to governments is a tariff rather than a consumption tax. We consider a consumption tax rather than a tariff for two reasons: First, for notational convenience, we want to examine a model with identical countries; obviously, a tariff cannot raise revenues if there is no trade. Second, with identical countries there is no monopoly distortion effect; consequently, the only source of distortion lies in the way taxes distort private decisions. Since we wanted to focus on this issue, we considered a consumption tax.

Consider the connection between our results and those in the tariff literature. There are two distinctions between our model and those in the tariff literature. First, the tariff literature assumes that governments can levy lump-sum taxes (and transfers) as well as distorting taxes. Second, in that literature, government spending is exogenous. In the limit, governments will not use distorting taxes if lump-sum taxes are available. Therefore, to make the comparison interesting, suppose that governments can only make non-negative lump-sum transfers but government spending is still exogenously fixed. Clearly, in the limit no government will levy a tax above that needed to finance government spending, so the noncooperative and cooperative equilibria coincide. However, when government spending is endogenous the two equilibria generally differ.

VI. Summary and Conclusions

In this extension of the analysis of tariff policy to models of fiscal policy, we have made two major points: First, if lump-sum taxes are available, then the basic results on tariff policy carry over to fiscal policy; as each country becomes small in the world economy, the noncooperative allocations converge to the cooperative allocations. Second, if revenues must be raised through distorting taxes, then these solutions generally do not converge.

We have made these points in simple models, but the intuition behind them is broader. In the limiting noncooperative equilibrium, each government uses a distorting tax to attempt to achieve two conflicting goals. Each government seeks to provide an optimal level of government spending and, at the same time, to equate the marginal rates of substitution of its consumers to the world price. Since other countries must also use distorting taxes, the world price does not, however, reflect the marginal rates of substitution of consumers in other countries. Thus, there is a loss of efficiency relative to the cooperative equilibrium. Similar results may hold for other types of distortions, such as incomplete markets.

Throughout the paper we restricted our analysis to static models to avoid problems associated with the time inconsistency of optimal policy. Rogoff (1985) and Kehoe (1989) have shown in dynamic settings that cooperative equilibria may be Pareto-dominated by noncooperative equilibria. An essential ingredient for this <u>nonoptimality result</u> is that policy in the cooperative equilibrium must be time inconsistent. In contrast, we attempt to isolate and understand factors that cause noncooperative equilibria to diverge from cooperative equilibria. Our main finding is that such a <u>divergence result</u> can hold in settings with distorting taxes. In particular, we show that the divergence result can hold even in a static model. Of course, in dynamic models with distortions, both of these results can hold simultaneously. Integrating these literatures would enable us to identify the benefits and costs of cooperation in policymaking.

Appendix A

Proof of Proposition 1

In proving Proposition 1, our basic line of argument is as follows: First, we consider a cooperative equilibrium in which governments are allowed to make transfers between countries. In Lemma 1, we show that for any vector of weights this equilibrium is a social optimum. In Lemma 2, we show that a nonempty set of weights exists for which the optimal transfers in such a cooperative equilibrium are zero. Combined, these two lemmas give us Proposition 1. (We view these equilibria with transfers simply as a convenient construct for proving Proposition 1, not as particularly interesting in their own right.)

To set up Lemma 1, we need several definitions. We must first define a cooperative equilibrium with transfers relative to any nonnegative vector of weights λ. For brevity, call this a λ-cooperative equilibrium with transfers. This equilibrium is composed of a competitive equilibrium for private agents and a cooperative equilibrium for governments. We begin with the competitive equilibrium. Let x^i denote the amount of good 1 that each agent in country i transfers to the rest of the world. Let $x = (x^1,\ldots,x^I)$, with $x^I = -\sum_{i=1}^{I-1} x^i$, be the vector of such transfers. For a given vector of government spending τ and transfers x, a competitive equilibrium is an allocation of private consumption (c_1,c_2) and a price p such that the allocations solve

$$V^i(\tau^i,x^i,p) = \max_{\{c_1^i,c_2^i\}} u^i(c_1^i,c_2^i,\tau) \tag{A1}$$

subject to

$$c_1^i + pc_2^i = y_1^i - \tau^i - x^i + py_2^i$$

and satisfy the market-clearing conditions (2) and (3). Substituting the demand functions into the market-clearing conditions gives the equilibrium price as a function of government spending and transfers, say $p = p(\tau,x)$. We then define a λ-cooperative equilibrium with transfers as a policy vector (τ,x), a price function $p(\tau,x)$, and allocation functions $c_1(\tau,x,p(\tau,x))$ and $c_2(\tau,x,p(\tau,x))$ that satisfy the following conditions: (i) the vector (τ,x) maximizes $\sum \lambda_i V^i(\tau^i,x^i,p(\tau,x))$ and (ii) for each vector (τ,x), the resulting prices and allocations constitute a competitive equilibrium. Next, a λ-social optimum is a vector (τ,c_1,c_2,p), where the allocations maximize (5) subject to (2) and (3) and where p denotes the normalized Lagrange multiplier for these constraints. Notice that any vector (τ,c_1,c_2,p) which is a λ-social optimum satisfies (2), (3), and

$$u_2^k/u_1^k = p \quad \text{for } k = 1, \ldots, I. \tag{A2}$$

With these definitions, it is straightforward to establish the first lemma.

Lemma 1. For any nonnegative λ, a λ-cooperative equilibrium with transfers is a social optimum.

Proof. The cooperative equilibrium allocations must satisfy all the conditions for a competitive equilibrium while the allocations in the social optimum must only satisfy market clearing. Thus, for any λ we have $W(\lambda) \geq \sum_i \lambda_i V^i(\tau^i,x^i,p(\tau,x))$ evaluated at the cooperative policies (τ,x). If we can choose transfers such that the λ-social optimum together with the transfers is a λ-cooperative equilibrium, we are done. To this end, let $(\hat{\tau},\hat{c}_1,\hat{c}_2,\hat{p})$ be a λ-social optimum. We claim $(\hat{\tau},\hat{x},\hat{c}_1,\hat{c}_2,\hat{p})$ is a λ-cooperative equilibrium, where

$$\hat{x}^k = y_1^k - \hat{c}_1^k - \hat{\tau}^k + \hat{p}(y_2^k - \hat{c}_2^k) \quad \text{for } k = 1, \ldots, I. \tag{A3}$$

To see this note that a λ-social optimum satisfies market clearing, the consumers' first-order conditions (A2), and by the definition of transfers, the private sector budget constraints. Hence, $(\hat{c}_1,\hat{c}_2,\hat{p})$ is a competitive equilibrium given $(\hat{\tau},\hat{x})$. Since $(\hat{\tau},\hat{x})$ are feasible choices in the cooperative environment, it follows that $\sum \lambda_i V^i(\hat{\tau}^i,\hat{x}^i,p(\hat{\tau},\hat{x})) \geq W(\lambda)$. Q.E.D.

In the next lemma, we show that the set of weights that respect private ownership is nonempty. These weights turn out to be exactly the set of weights for which the optimal transfers of Lemma 1 are zero.

Lemma 2. There exists a nonempty set S of nonnegative weights λ such that for each λ in S, the excess savings of each country is zero.

Proof. The proof is a fairly standard application of a fixed-point theorem along the lines of Negishi (1960) and Mantel (1971). Recall that the excess savings function of the i^{th} country is

$$s^i(\lambda) = [y_1^i - c_1^i(\lambda) - \tau^i(\lambda)] + p(\lambda)[y_2^i - c_2^i(\lambda)] \tag{A4}$$

and is defined for all λ in Δ, where $\Delta = \{\lambda \in R^I \mid \lambda^i \geq 0, \text{ and } \sum_{i=1}^{I} \lambda^i = 1\}$. These excess savings functions have three properties that are exploited in the proof. First, they are continuous functions of λ. Second, feasibility implies that they sum to zero. Third, these functions satisfy the condition that if $\lambda^i = 0$, then $s^i(\lambda) \geq 0$. That is, if consumer i receives a zero weight in the social optimum, then consumer i's excess saving is nonnegative.

Next, define the fixed-point map $g\colon \Delta \to \Delta$, where $g = (g^1,\ldots,g^I)$ and

$$g^i(\lambda) = \max[0,\lambda^i+s^i(\lambda)] / \sum_{j=1}^{I} \max[0,\lambda^j+s^j(\lambda)]. \tag{A5}$$

Notice that the denominator in (A5) is always positive. This is true because $\sum_j[\lambda^j+s^j(\lambda)] = 1$ implies $[\lambda^j+s^j(\lambda)] > 0$ for some j, which in turn implies that the denominator is positive. Since both the savings functions and the maximum function are continuous, the function g is continuous. Since the $g^i(\lambda)$ are nonnegative and sum to 1, we know that $g(\lambda)$ is in Δ. Thus, g is a continuous function that maps the compact, convex set Δ into itself. So, by Brouwer's theorem, we know there is a nonempty set S of weights such that $g(\lambda) = \lambda$ for all λ in S.

To finish the proof, we must show that a fixed point of g is a zero of s; that is, $g^i(\lambda) = \lambda^i$ for all i implies $s^i(\lambda) = 0$ for all i. If λ is a fixed point of g, then for all i, $a\lambda^i = \max[0,\lambda^i+s^i(\lambda)]$, where a is the denominator in (A5). This implies that $a\lambda^i = \lambda^i + s^i(\lambda)$ for all i, since we know that if $\lambda^i = 0$, then $s^i(\lambda) \geq 0$. Summing over all consumers gives $a\sum_i\lambda^i = \sum_i\lambda^i + \sum_i s^i(\lambda)$. Since the sum of these savings functions is zero, we have $a = 1$; thus, $s^i(\lambda) = 0$ for all i. Q.E.D.

Combining these two lemmas gives us

<u>Proposition 1</u>. A cooperative equilibrium that respects private ownership is a social optimum

<u>Proof</u>. A more precise statement of the proposition is that for any λ in S, a λ-cooperative equilibrium (without transfers) is a λ-social optimum. Comparing (A3) and (A4), we see that the transfers used to support a given λ-social optimum are simply the excess savings resulting from that optimum. Thus, by Lemma 2 for any λ in S, these optimal transfers are zero; so for such a λ, a λ-cooperative equilibrium with transfers is a λ-cooperative equilibrium (without transfers). Then, by Lemma 1, such a cooperative equilibrium is optimal. Q.E.D.

Notes

[1]Since our results with distorting taxes are at odds with received wisdom, it is natural to ask whether other sources of inefficiency lead to similar results. In an earlier version of the paper, we showed that for an overlapping generations economy with an inefficient competitive equilibrium, the noncooperative policies do not converge to the cooperative policies.

[2]This paper is also related to a literature in mathematical economics that characterizes Walrasian equilibria as the limit of noncooperative equilibria. [See, for example, the symposium in the Journal of Economic Theory (1980).] To clarify this relationship, consider the following two-stage manipulation game in an exchange economy inhabited only by private agents. In stage 1, the agents decide how much of their endowments to destroy. In stage 2, given their remaining endowments, they participate as price-takers in a competitive equilibrium. This manipulation game is closely related to the games we study here. Indeed, there may be a way to adapt the results in this literature to prove a more general version of some of our results.

[3]Note that the cooperative solution with proportional consumption taxes equals the cooperative solution with lump-sum taxes. This special feature of the cooperative equilibrium arises because there is symmetry and there is no production. If we change either of the assumptions, this result will not hold. However, the algebra for the rest of the derivations is somewhat tedious.

[4]To see this, note that with Cobb-Douglas preferences over the private goods, the demand function for good 1 is $c_1 = \alpha(y_1+py_2)/(1+\tau)$, where α is the share of private expenditure on good 1. Combining this demand function with the fact that $c_1 + g = (1+\tau)c_1$, we have that no change in tax rates can change $c_1 + g$.

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