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A CONTRIBUTION TO THE PURE THEORY OF MONEY

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ABSTRACT

We analyze a general equilibrium model with search frictions and differentiated commodities. Because of the many differentiated commodities, barter is difficult because it requires a double coincidence of wants, and this provides a medium of exchange role for fiat money. We prove the existence of equilibrium with valued fiat money and show it is robust to certain changes in the environment, including imposing transactions costs, storage costs, and taxes on the use of money. Rate of return dominance, liquidity, and the potential welfare improving role of fiat money are discussed.

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I. Introduction

One of the oldest questions in economics asks why trade so often takes place using money. Of particular interest is the phenomenon of fiat currency (an unbacked, intrinsically useless asset) circulating as medium of exchange.¹ This paper adopts the venerable notion that agents accept fiat money in trade because they expect that others will do the same. We prove the existence of Nash equilibrium in which this is indeed the case and further show that such equilibria are robust. That is, even if fiat money has certain properties that reduce its desirability as an asset, it can maintain its value due to its medium of exchange function.² We also use the model to address some issues in monetary economics, including liquidity, rate of return dominance, and the welfare implications of using fiat currency.

The approach here is similar in spirit to Kiyotaki and Wright [11]. Agents have idiosyncratic tastes for differentiated commodities and meet randomly over time in a way that implies trade must be bilateral and quid pro quo. That model assumed a great deal of asymmetry in goods and in agents, as we were interested in showing how certain commodities could endogenously become media of exchange, or commodity money, depending both on their intrinsic properties and on extrinsic beliefs. Even though we were also able to demonstrate the existence of equilibria with circulating fiat money, the analysis was rather complicated, and we never did check the robustness of those equilibria--we always assumed fiat currency had intrinsic properties that make it the best available asset. Here, goods and agents will both display a certain symmetry, meaning there is no natural candidate for commodity money but highlighting a potential role for, and drastically simplifying the analysis of, fiat currency.

Our environment is similar to, but also different in important ways from, the existing search or matching framework, as popularized by Diamond [3]. The key difference is that Diamond's models have only one type of commodity, while we assume that different people have different tastes over a large number of differentiated goods. This is what makes pure barter difficult in our economy and is, therefore, what allows for a medium-of-exchange role for money. Now Diamond [4] also presents a "monetary" version of his economy, but in that model money is used because barter is ruled out exogenously via a cash-in-advance constraint. The goal here is to capture monetary exchange as an equilibrium phenomenon and not to force it onto the system exogenously.³

We introduce the basic model without fiat money in Section II and prove there exists a unique nondegenerate steady-state, symmetric equilibrium. In Section III, we prove there also exists an equilibrium with valued fiat money, although we note that some barter always co-exists along side of monetary exchange. The liquidity of money and real commodities is examined, and we show that an individual holding money acquires a desired commodity faster than one holding some other good even though the latter could barter directly. In Section IV, we look at the robustness of monetary equilibrium by endowing fiat currency with a relatively high transactions cost, assuming it is dominated in rate of return or storage cost, and imposing taxes on its usage. As long as the transactions cost, rate of return dominance, or tax is not too severe, the value of fiat money can survive. This section also examines if the use of money may improve welfare. We conclude the paper in Section V.

II. The Nonmonetary Economy

Consider a model with a continuum of differentiated commodities identified by points around a circle with a circumference of 2. There is also a continuum of infinite-lived agents with unit mass identified by points on the same circle. The agents have diverse tastes for the goods: the agent indexed by point i has as his most preferred commodity the good indexed by point i , and he derives utility $u(z)$ from consuming 1 unit of a good that is distance z from i , where $u'(z) < 0$.⁴ We also assume $u'(z) + zu''(z) \leq 0$ (which is automatically satisfied if u is concave). The distance between a randomly selected good and a given agent's ideal good is distributed uniformly on $[0,1]$. Except for this matching of different individuals with different ideal commodity types, the set of agents and the set of goods are homogeneous.

Goods are technologically indivisible, come in units of size one, and may be stored one unit at a time. There is no direct storage cost or benefit for now, but this is relaxed below. Goods are acquired in a production process, where potential production opportunities arrive in continuous time according to a Poisson process with fixed arrival rate α . These opportunities, or projects, are characterized by a commodity type drawn randomly from the circle and a cost of production in terms of disutility, c , independently drawn from a cumulative distribution function $F(c)$, with a greatest lower bound of zero. We assume agents cannot consume their own output: once a good is produced, the agent must take it to a trading sector and attempt to exchange it for something else that he can consume.⁵

In the trading process, agents are randomly matched in pairs over time according to a Poisson process with fixed arrival rate β ,

unless there are no other agents attempting to trade, in which case there are never any meetings. Letting N_1 be the measure of traders, we denote the arrival rate of partners for an individual in the exchange sector as $B = B(N_1)$, where $B(0) = 0$ and $B(N_1) = \beta$ for $N_1 > 0$.⁶ In a given meeting, trade occurs if and only if both partners agree and entails a one-for-one swap (because goods are indivisible and can be stored only 1 unit at a time). Once an exchange takes place, agents can consume and proceed back to production whenever they wish. However, there is a disutility cost ϵ that must be incurred any time a good is accepted in trade. Letting $u_\epsilon(z) = u(z) - \epsilon$, we assume $u_\epsilon(0) > 0$ and that there exists $z_\epsilon < 1$ such that $u_\epsilon(z_\epsilon) = 0$, where 0 is also the utility of not consuming.

The agents in this economy choose strategies for determining when to produce, trade, and consume in order to maximize expected discounted utility from consumption net of production and transactions costs, given the strategies of others and the matching process. In this paper we will only consider outcomes where all agents are anonymous (i.e., the names of individuals do not matter) and all agents and all goods play symmetric roles. We also concentrate exclusively on steady states, where the strategies and distribution of agents are unchanging over time.⁷ Before defining our notion of equilibrium precisely, it is possible and useful to discuss certain general features of individual behavior.

Consider an agent in the trading sector with good i , and let $\theta(i)$ denote the probability with which he believes that a randomly encountered agent is willing to accept i . Suppose he believes $\theta(i) = \theta_1$ is independent of i (no good is more acceptable than any other). Then

we claim that if he is offered good i' in exchange for i , he will not trade unless he is going to consume it. If he does not consume i' he will only have to trade it again. But since it is no easier to trade i' than it was to trade i , when $\theta(i') = \theta(i) = \theta_1$, he would not be willing to pay the transactions cost ϵ to make the exchange. Hence, trade only occurs here when two partners both have something the other wants to consume, which is Jevons' [8] deservedly famous "double coincidence of wants" problem with pure barter. Further, in a steady state, once an agent gets a good he is going to consume he consumes it and moves back to production immediately, as long as he discounts the future, as there is no gain to delaying.⁸

Based on these observations, we can formulate the problem faced by an individual in this economy as a dynamic program. Let j denote the agent's state: $j = 0$ if he is in the production sector, and $j = 1$ if he is in the exchange sector. Let V_j be the optimal value function, which is independent of time and history (except as summarized by j) and is independent of the type of good one has in hand or the type of good one is trying to acquire for the same reason that θ_1 is independent of these factors. If r is the constant discount rate, the value functions satisfy

$$\begin{aligned} rV_0 &= \alpha \int_0^{\infty} \max(0, V_1 - V_0 - c) dF(c) \\ rV_1 &= B\theta_1 \int_0^1 \max[0, V_0 - V_1 + u_{\epsilon}(z)] dz. \end{aligned} \tag{2.1}$$

The first equation describes the return to search in the production sector, which is the arrival rate of projects times the expected gain from accepting if the gain is positive. The second describes the return

to search in the exchange sector, which equals the rate at which partners arrive times the probability they are willing to trade times the expected gain from accepting if the gain is positive.⁹

The maximization problems in (2.1) involve decision rules mapping the supports of c and z into {Accept,Reject} and are clearly solved by reservation strategies: (i) accept a production project iff $V_1 - V_0 - c > 0$ or, equivalently, iff $c < k$, where $k \equiv V_1 - V_0$; (ii) accept a trade iff $V_0 - V_1 + u_\varepsilon(z) > 0$ or, equivalently, iff $z < x$, where $u_\varepsilon(x) \equiv V_1 - V_0$. Equations (2.1) can therefore be rewritten as

$$rV_0 = \alpha \int_0^k (V_1 - V_0 - c)dF(c) = \alpha \int_0^k (k-c)dF(c)$$

$$rV_1 = B\theta_1 \int_0^x [V_0 - V_1 + u_\varepsilon(z)]dz = B\theta_1 \int_0^x [u(z) - u(x)]dz.$$

To reduce notation, we write these as

$$rV_0 = \alpha s_0(k)$$

$$rV_1 = B\theta_1 s_1(x)$$
(2.2)

where $s_0(k) = \int_0^k (k-c)dF(c)$ and $s_1(x) = \int_0^x [u(z) - u(x)]dz$. One easily shows that $s_0(k)$ is increasing for $k \in [0, \infty)$, while $s_1(x)$ is strictly increasing and weakly convex (due to the assumption $u'(z) + zu''(z) \leq 0$) for $x \in [0, z_\varepsilon]$.¹⁰

Now suppose all agents use symmetric reservation trading strategies, and let z be the distance along the circle between a fixed good i and the ideal commodity type of a randomly sampled agent. Then z is distributed uniformly on $[0,1]$, and so the probability that a randomly encountered trader is willing to accept good i is $\text{Prob}(z < x) = x$, which is independent of i . Therefore, it is indeed rational for indi-

viduals to believe that $\theta(i) = \theta_1$ will not depend on i under the assumption of symmetric behavior. Further, given x and k , the steady-state number of agents in the exchange sector N_1 is a solution to

$$\dot{N}_1 = \alpha F(k)N_0 - Bx^2N_1 = 0 \quad (2.3)$$

where $N_0 = 1 - N_1$. The first term is the flow into the exchange sector from production, which is the arrival rate of projects times the probability they are acceptable times the number of producers. The second term is the flow back, which is the arrival rate of trading partners times the double coincidence probability times the number of traders (see Figure 1).

We define a symmetric, steady-state equilibrium for the non-monetary model as follows.

Definition 1: An equilibrium for the nonmonetary economy is a list (k, x, V_0, V_1) plus a distribution that solves (2.3), satisfying

- (a) (2.2) and $u_\varepsilon(x) = k = V_1 - V_0$ (maximization)
- (b) $\theta_1 = x$ (rational expectations).

Notice $N_1 = 0$ (along with $V_0 = V_1 = k = 0$ and $x = z_\varepsilon$) will always constitute a degenerate equilibrium, because when there is no one with whom to trade, the best response is to not produce. We also have the following, more interesting result.

Proposition 1: There exists a unique nondegenerate equilibrium.

Proof: A nondegenerate equilibrium satisfies (2.2), with $\theta_1 = x$, $B = \beta$, and $k = u_\varepsilon(x) = V_1 - V_0$. These conditions reduce to one equation in x :

$$T(x) = ru_{\epsilon}(x) + \alpha s_0 [u_{\epsilon}(x)] - \beta x s_1(x) = 0.$$

Any solution to $T(x) = 0$ (along with the implied values for the other variables) constitutes an equilibrium, which is nondegenerate iff $k = u_{\epsilon}(x) > 0$, which holds true iff $x < z_{\epsilon}$. It is easy to verify that $T(0) > 0 > T(z_{\epsilon})$ and $T'(x) < 0$. Hence, there exists a unique solution, $x \in (0, z_{\epsilon})$, to $T(x) = 0$. \square

Figure 1 depicts activities in this nondegenerate equilibrium. An agent in the production sector looks for projects and utilizes them if and only if they cost less than k . When production occurs, the agent moves to the exchange sector, where he looks for other traders. When two traders meet, if $z < x$ for both they trade, otherwise they part company. When a trade occurs, the agent consumes and enjoys utility $u(z)$ minus transactions cost ϵ and then moves back to production. The key aspect of the model, and that which departs from the models of Diamond [3,4], for example, is that here the agents have heterogeneous tastes over differentiated goods. This is what makes pure barter exchange difficult: the probability of trade in a given meeting is the probability that $z < x$ for both traders, x^2 , where x squared represents the double coincidence. In the next section we ask if this might give rise to a role for fiat currency.

III. The Monetary Economy

We hypothesize now the existence of a new object--one that does not provide utility to any agent, does not aid in production (intrinsic uselessness), and is not a convertible claim to something else that does provide utility or aid in production. These are the defining characteristics of fiat money (see Wallace [19]). The new

object could be thought of as a particular type of paper, or perhaps a shell, with its supply fixed at M units. In order to maintain the same basic framework used above, assume that this shell is indivisible and that each agent can carry no more than 1 unit of either shells or "real" commodities at a time. Thus, $M \leq 1$ units are held by the fraction M of the agents.¹¹

The question we ask here is simple: Can this worthless shell or paper take on value, in the sense that agents willingly accept it in trade for their commodities merely because they believe that others will do the same for them in the future? We generally assume there is a transactions cost η to accepting a shell, analogous to the cost ϵ of accepting goods, and we will eventually allow η to be greater or less than ϵ because we are interested in varying the relative intrinsic properties of the objects. However, it facilitates the presentation considerably to set $\eta = 0$ for now and to return to the general case in the next section.

Again, V_j is the value function, but now j denotes one of three states: $j = 0, 1, \text{ or } m$ indicates, respectively, an agent is a producer, a commodity trader (i.e., in the trading sector with one of the consumption goods), or a money trader (i.e., in the trading sector with fiat money). The proportions of the population in the three states are (N_0, N_1, N_m) . Also, let the probability of a good being accepted by a random commodity trader be θ_1 , the probability of a good being accepted by a random money trader be θ_2 , and the probability of fiat money being accepted by a random commodity trader be θ_3 . We only consider symmetric, steady-state outcomes, which means that all of the real commodities are equally acceptable, and therefore the V_j and θ_j do not depend

on the good one has in hand or the good one is trying to acquire, by an argument analogous to that in the previous section. As a final piece of notation, it will be useful to define $m = N_m / (N_m + N_1)$ to be the fraction of agents in the trading sector with money, or the probability that a randomly sampled trader is a money trader.

The value functions now satisfy the following conditions:

$$\begin{aligned} rV_0 &= \alpha s_0(k) \\ rV_1 &= B(1-m)\theta_1 s_1(x) + Bm\theta_2 \max(0, V_m - V_1) \\ rV_m &= \max\{B(1-m)\theta_3 s_1(y), rV_0\}. \end{aligned} \tag{3.1}$$

The first equation describes the return to search for a producer, as in the previous section, where $k = V_1 - V_0$. The second describes the return to search in the exchange sector for a commodity trader, where $u_\epsilon(x) = V_1 - V_0$: the first term is the expected gain from meeting a willing commodity trader, while the second term is the gain from meeting a willing money trader, whereupon the commodity trader has the option of switching from goods to money. Finally, the return for a money trader in the third equation is one of two things. If $V_m > V_0$, then $rV_m = B(1-m)\theta_3 s_1(y)$, where $u_\epsilon(y) = V_m - V_0$ defines the reservation good for a money trader; if we do not have $V_m > V_0$, we assume that shells will be freely disposed of and agents with money will switch to production.

The symmetric use of the strategies described above implies $\theta_1 = x$ and $\theta_2 = y$. All we say about θ_3 for now is that $V_m < V_1$ implies $\theta_3 = 0$, and $V_m > V_1$ implies $\theta_3 = 1$. In the former case, agents holding fiat money freely dispose of it, and the steady state is determined as in the nonmonetary economy by (2.3). As long as $\theta_3 > 0$, however, the steady-state distribution is determined by solving

$$\dot{N}_1 = \alpha F(k)N_0 - B[(1-m)x^2 + m\theta_3]N_1 = 0 \quad (3.2)$$

where $m = M/(M + N_1)$ and $N_0 = 1 - N_1 - M$. The first term in (3.2) is the flow into state 1 from production, while the second term is the flow out, which consists of commodity traders who barter with other commodity traders, plus commodity traders who meet money traders and accept their currency (see Figure 2).

We now define equilibrium for this economy with M fixed exogenously, where M is the number of shells or the supply of fiat money.¹²

Definition 2: Given M , an equilibrium for the monetary economy is a list (k, x, y, V_0, V_1, V_m) plus a distribution solving (3.2), satisfying

- (a) (3.1), $k = u_\varepsilon(x) = V_1 - V_0$, and $u_\varepsilon(y) = V_m - V_0$
- (b) $\theta_1 = x$, $\theta_2 = y$, and $\theta_3 = 1$ if $V_m > V_1$ or $\theta_3 = 0$ if $V_m < V_1$.

It is convenient to conduct the analysis in two steps. First we describe what happens for a fixed value of m , with the money stock $M = M(m)$ chosen after the fact to be consistent with this initial m . Then the mapping from m to M is shown to be onto--for any $M \in [0, 1]$ there exists (at least one) m such that $M = M(m)$. Therefore any exogenous M implies a value of m , and therefore it implies the existence of equilibrium in our desired sense.

Definition 3: Given an exogenous m (instead of M), the same variables satisfying the conditions in Definition 2 define an m-equilibrium.

We say that an equilibrium, or m -equilibrium, is nondegenerate if $N_1 > 0$ (which requires production, $k = V_1 - V_0 > 0$). We say that an equilibrium is monetary (or that money has value) if $\theta_3 > 0$ and non-

monetary otherwise. It is a pure monetary equilibrium if $\theta_3 = 1$, in which case money is universally accepted (as we demonstrate below, it is possible for money to be accepted sometime or by some agents, but not all the time or by all the agents, in mixed-strategy equilibria).

The first thing to note is that when $\theta_3 = 0$, the system describing the monetary economy reduces exactly to that of the nonmonetary economy. When agents believe fiat money will be valueless, the monetary economy will have only nonmonetary equilibria (the two that were described in Section II). A minimal requirement for money to circulate is that individuals believe in it. We now show there exists pure monetary equilibria, where such beliefs are rational.

Proposition 2: For any $m \in (0,1)$, there exists a unique nondegenerate pure monetary m -equilibrium.

Proof: Given $m \in (0,1)$, a nondegenerate pure monetary m -equilibrium satisfies (3.1) with $(\theta_1, \theta_2, \theta_3) = (x, y, 1)$ and $B = \beta$,

$$\begin{aligned} rV_0 &= \alpha s_0(k) \\ rV_1 &= \beta(1-m)x s_1(x) + \beta m y (V_m - V_1) \end{aligned} \tag{3.3}$$

$$rV_m = \beta(1-m)s_1(y)$$

plus $k = u_\varepsilon(x) = V_1 - V_0$ and $u_\varepsilon(y) = V_m - V_0$. We will reduce these to a single equation and show it has a unique solution. Begin by combining the first and third equations of (3.3) into

$$\beta(1-m)s_1(y) - r u_\varepsilon(y) = \alpha s_0[u_\varepsilon(x)]. \tag{3.4}$$

Since the left hand side of (3.4) is increasing in y and the right hand side is decreasing in x , this implicitly defines a function $x = \phi(y)$, given the following lemma (the proof is left for the reader).

Lemma 1: The function $\phi: [y_1, \bar{y}] \rightarrow [0, z_\epsilon]$ is well-defined and differentiable, where y_1 and \bar{y} are given as follows: y_1 is the unique solution to $\beta(1-m)s_1(y_1) = ru_\epsilon(y_1)$, and \bar{y} is either z_ϵ or the value of y that satisfies (3.4) with $x = 0$, if such a y exists and is less than z_ϵ .

As (3.4) implies $\phi'(y) < 0$, this function is downward sloping in the (y, x) plane as shown in Figure 3.

Next, combine the second two equations of (3.3) into

$$\phi(y, x) = (r + \beta my)[u(y) - u(x)] - \beta(1-m)[s_1(y) - xs_1(x)] = 0. \quad (3.5)$$

The value of y in m -equilibrium satisfies $T(y) \equiv \phi[y, \phi(y)] = 0$. Define y_1 as in Lemma 1 as the solution to $\beta(1-m)s_1(y_1) = ru_\epsilon(y_1)$, and observe $y > y_1$ iff $x < z_\epsilon$; that is, $y > y_1$ iff $k = u_\epsilon(x) > 0$, which means $N_1 > 0$. Also, define y_2 by $y_2 = \phi(y_2)$, where $y_1 < y_2 < \bar{y}$ from (3.4) and Lemma 1, and observe $V_m - V_1 = u(y) - u[\phi(y)]$ is positive iff $y < y_2$. In other words, necessary and sufficient conditions for a nondegenerate pure monetary equilibrium is that the y that solves $T(y) = 0$ is between y_1 and y_2 . Since

$$T(y_1) = \phi(y_1, z_\epsilon) = \beta my_1 u_\epsilon(y_1) + \beta(1-m)z_\epsilon s_1(z_\epsilon) > 0$$

$$T(y_2) = \phi(y_2, y_2) = -\beta(1-m)s_1(y_2)(1-y_2) < 0$$

there exists $y \in (y_1, y_2)$ such that $T(y) = 0$. As it is straightforward to show $T'(y) < 0$ when $T(y) = 0$, this y is unique (Figure 3). \square

Proposition 3: For $M \in (0,1)$, there exists a nondegenerate pure monetary equilibrium.

Proof: For any $m \in (0,1)$ there exists a unique nondegenerate pure monetary m -equilibrium by Proposition 2. In this m -equilibrium, the steady-state equations yield the unique stock of fiat money,

$$M = M(m) = \frac{\alpha F[u_{\epsilon}(x)]m}{\alpha F[u_{\epsilon}(x)] + \beta x^2(1-m)^2 + \beta ym(1-m)}. \quad (3.6)$$

The m -equilibrium is an equilibrium when $M = M(m)$. Clearly, $M(m)$ is continuous on $(0,1)$, $M(m) \rightarrow 0$ as $m \rightarrow 0$, and $M(m) \rightarrow 1$ as $m \rightarrow 1$. Hence, for any $M \in (0,1)$ there exists (at least one) $m \in (0,1)$ satisfying $M = M(m)$. Since there exists an m -equilibrium for this m , there exists an equilibrium for the original M . \square

This demonstrates that there is a solution to the model for any $M \in (0,1)$ with $V_m > V_1$ and, therefore, with $\theta_3 = 1$, which means commodity traders universally surrender goods for fiat currency.¹³ The reason agents are always willing to trade goods for money is because $V_m > V_1$. The reason $V_m > V_1$ is because agents are always willing to trade goods for money. Through this beautiful circularity the value of fiat money supports itself. Notice, however, that in our model some barter always co-exists with monetary exchange in equilibrium--when two commodity traders meet who happen to have a double coincidence, the model naturally predicts they will trade. There is no constraint that says agents have to use money, as there is in some models, and so we think there is a clear sense in which the use of money is endogenous in our model.

Of course, the double coincidence problem makes monetary trade easier than barter: the chance of a commodity trader's partner being willing to trade is only $\theta_1 = x < 1$, while the chance of a money trader's partner being willing to trade is $\theta_3 = 1$. When two commodity traders meet they exchange with probability $\theta_1^2 = x^2$, while when a commodity and money trader meet they exchange with probability $\theta_2\theta_3 = y$. The next result shows that $y > x^2$, so the probability of trade is greater in the latter case. This is true even though $y < x$, so that money traders are more demanding because their reservation good is closer to their ideal good, and thus they make better trades on average.

Lemma 2: In pure monetary equilibrium, $x^2 < y < x$.

Proof: The equations $u_e(x) = V_1 - V_0$ and $u_e(y) = V_m - V_1$ imply $x > y$ iff $V_m > V_1$, so in pure monetary equilibrium $x > y$. The other inequality, which depends on the condition $u'(z) + zu''(z) \leq 0$, is derived in Appendix A. \square

The above results are closely related to the concept of liquidity. Let D_j be the random duration of time it takes to acquire a consumption good for an agent in state j , $j = 1$ or m . To capture the notion that objects are less liquid the longer it takes to exchange them for something you want, define the liquidity of real commodities or money by $\ell_j = 1/ED_j$, $j = 1$ or m .¹⁴ For a money trader, D_m is exponentially distributed with $ED_m = 1/[\beta(1-m)y]$, so the liquidity of money is $\ell_m = \beta(1-m)y$. The liquidity of commodities is more difficult to compute because commodity traders can acquire consumption goods directly via barter or indirectly by trading goods for money and then money for other goods. In Appendix B we derive $\ell_1 = \beta(1-m)[(1-m)x^2 + my]$, hence,

$\ell_m - \ell_1 = \beta(1-m)^2(y-x^2) > 0$ since $y > x^2$ by Lemma 2. A higher probability of acceptability for money is reflected in a greater liquidity of money.

We now turn to a discussion of mixed-strategy monetary equilibria--that is, equilibria with θ_3 strictly between 0 and 1. This can only hold if $V_1 = V_m$, and so the conditions $u_e(x) = V_1 - V_0$ and $u_e(y) = V_m - V_0$ immediately imply $x = y$. Then the second and third equations in (3.3) imply

$$rV_1 = \beta(1-m)x s_1(x) = \beta(1-m)\theta_3 s_1(y) = rV_m$$

which entails $\theta_3 = x$. Also, since $V_1 = V_m$, one equation drops out of (3.3) and the remaining conditions combine to yield

$$T(x) = ru_e(x) + \alpha s_0[u_e(x)] - \beta(1-m)x s_1(x) = 0.$$

For any fixed $m \in (0,1)$, this has a unique solution $x \in (0, z_e)$ (the argument is exactly the same as the one in the proof of Proposition 1).

This x , together with $k = u_e(x)$, $y = x$, and the implied values of V_j and N_j , constitute an m -equilibrium where money has value but is not universally accepted. In fact, it is essentially the same as the unique nondegenerate equilibrium of the nonmonetary economy, with the arrival rate of trading partners reduced from β to $\beta(1-m)$. Further, (3.6) simplifies in this case to

$$M = M(m) = \frac{\alpha F(k)m}{\alpha F(k) + \beta(1-m)x^2}.$$

If we set $M = M(m)$, we turn the m -equilibrium into an equilibrium for that value of M . As with equilibria in pure strategies, since $M(m)$ is continuous on $(0,1)$, $M(m) \rightarrow 0$ as $m \rightarrow 0$ and $M(m) \rightarrow 1$ as $m \rightarrow 1$; for any

$M \in (0,1)$ there exists (at least one) $m \in (0,1)$ satisfying $M = M(m)$. Hence, there exists an equilibrium with $\theta_3 \in (0,1)$ for that value of M .

Of course, rather than agents using mixed strategies where they accept money with probability θ_3 , it is equivalent here for a proportion of the population equal to θ_3 to accept money with probability 1, while the rest reject money with probability 1. In this case, some, but not all, agents use fiat money. Under either interpretation, in monetary equilibria where money is not universally accepted, the money does nothing to stimulate trade. Thus, the probability of randomly sampled commodity and money traders exchanging is $y\theta_3 = x^2$, which just equals the double coincidence probability. Nevertheless, since money is no less acceptable than barter, agents have nothing to lose by accepting it, and so they sometimes do.¹⁵

Finally, we briefly discuss the total number of symmetric, steady-state equilibria in this model, given an exogenous setting for the money supply, M_0 . For a set of values for M_0 with measure 1, the inverse image $M^{-1}(M_0) = \{m: M_0 = M(m)\}$ is a set with an odd number of elements, say $n(M_0) < \infty$. For each of the elements $m \in M^{-1}(M_0)$, there is a unique pure monetary m -equilibrium plus a unique mixed-strategy monetary m -equilibrium. Hence, there exist exactly $2n(M)$ monetary equilibria for almost all M , $n(M)$ pure and $n(M)$ mixed strategy. Of course, there also exists a unique nonmonetary equilibrium, so that generically there is an odd number of nondegenerate equilibria and a unique degenerate outcome. The main point is that, generically, there will be a finite number of equilibria in the monetary economy.

IV. Robustness

In this section we demonstrate that not only can fiat money take on value in this economy, it is robust. We begin by allowing money to have a positive transactions cost, η . Although it may well be reasonable to assume $\eta < \epsilon$ (a lower transactions cost on monetary than on barter exchange), we want to emphasize that the value of fiat money follows because of its medium-of-exchange function, not because we have somehow endowed it with desirable intrinsic properties. In fact, because money plays a nontrivial, medium-of-exchange role in our economy, its value can survive even if $\eta > \epsilon$, as long as η is not too great.

The generalized version of (3.3), allowing for a nonzero η , that must hold for a nondegenerate pure monetary equilibrium is

$$\begin{aligned} rV_0 &= \alpha s_0(k) \\ rV_1 &= \beta(1-m)xs_1(x) + \beta my(V_m - V_1 - \eta) \\ rV_m &= \beta(1-m)s_1(y). \end{aligned} \tag{4.1}$$

The difference between (3.3) and (4.1) is that now the second equation accounts for the disutility η from accepting a shell and implies that $\theta_3 = 1$ now requires $V_m > V_1 + \eta$. The equilibrium function constructed from (3.4) and (3.5) in Proposition 2 now becomes

$$\begin{aligned} T(y; \eta) &= (r + \beta my)[u(y) - u(\phi(y))] - \eta \beta my \\ &\quad - \beta(1-m)[s_1(y) - \phi(y)s_1(\phi(y))]. \end{aligned}$$

A nondegenerate pure monetary equilibrium is a solution to $T(y;\eta) = 0$ with $y_1 < y < y_2(\eta)$, where y_1 is the same as before and now $y_2(\eta)$ solves $u(y_2) = u[\phi(y_2)] + \eta$. Notice $y_2(\eta) > y_1$ as long as η is not too large.

If $\eta = 0$ then $T(y_1;0)$ and $T[y_2(0);0]$ are strictly positive and strictly negative, respectively, and therefore there exists a solution to $T(y,0) = 0$ with $y_1 < y < y_2(0)$. Hence, for all η in some open neighborhood of the origin there still exists a solution to $T(y,\eta) = 0$ with $y_1 < y < y_2(\eta)$, and therefore there exists a nondegenerate pure monetary equilibrium for that η . However if η gets too large, and in particular, if $\eta > u_\epsilon(y_1)$, then $y_1 > y_2(\eta)$ and monetary equilibrium cannot exist. Fix $\eta > 0$ so that a monetary equilibrium does exist, and note that we can shift $u(\cdot)$ and ϵ together so that $u_\epsilon(\cdot) = u(\cdot) - \epsilon$ is the same, without affecting the system. Hence, if we shift $u(\cdot)$ and ϵ down until $\epsilon < \eta$, we have a pure monetary equilibrium where monetary exchange has a higher transactions cost than barter. In this case agents use money simply because it is universally acceptable, even though it is intrinsically inefficient as an exchange medium, relative to barter.

We now turn to "rate of return dominance"--a phenomenon Hicks [5, p. 5] called the "central issue in the pure theory of money." Why do individuals voluntarily accept, or hold, barren money over other assets that yield interest? More generally, is it possible to have a model where rational agents use one thing for a medium of exchange when another appears, based on objective data, to have a greater flow yield?¹⁶ Introduce w_j as the instantaneous flow reward (if positive) or cost (if negative), in terms of utility, to being in state j , where $j = 0, 1, \text{ or } m$. Conditions (4.1) now become

$$rV_0 = w_0 + \alpha s_0(k)$$

$$rV_1 = w_1 + \beta(1-m)x s_1(x) + \beta m y (V_m - V_1 - \eta) \quad (4.2)$$

$$rV_m = w_m + \beta(1-m)s_1(y).$$

Continuity again establishes the existence of a nondegenerate pure monetary equilibrium for all η and w_j in some open neighborhood of the origin. The key result is that we can have $V_m > V_1$, even if $w_m < w_1$. Hence, money can be a universally acceptable medium of exchange even if its rate of return is less than the rate of return on storing real assets. However, if w_m is too much less than w_1 then we cannot have valued fiat money. Of course, the subjective return to holding currency is measured by V_m , and this cannot fall short of the return to storing real commodities, V_1 , by the very definition of a monetary equilibrium. But the observed rates of return are the flow yields, w_j (there are no capital gains associated with a change in exchange rates in our model). This does seem to genuinely capture the phenomena of agents using one thing for a medium of exchange, even when it objectively appears to be an inferior asset.

Next, we turn to taxing the holders of fiat money. While there are many ways to do this in actual economies, we assume that according to a Poisson process with arrival rate δ , a government revenue agent confronts each money holder and simply confiscates his currency, whereupon the agent moves back to production without consuming. Also, government purchasing agents with money meet commodity traders, according to a Poisson process with arrival rate γ , and buy their goods. In order to keep the money stock M constant, we require $\delta M = \gamma N_1$, or, dividing by $N_1 + M$, we require $\delta m = \gamma(1-m)$. The equilibrium conditions become

$$\begin{aligned}
 rV_0 &= \alpha s_0(k) \\
 rV_1 &= \beta(1-m)xs_1(x) + \left[\beta my + \frac{\delta m}{(1-m)} \right] (V_m - V_1) \\
 rV_m &= \beta(1-m)s_1(y) + \delta(V_0 - V_m)
 \end{aligned} \tag{4.3}$$

where we have set $\eta = w_j = 0$ for simplicity.

Again, similar arguments can be used to verify that there exists a pure monetary equilibrium if the tax rate δ is not too big, but for large δ money simply cannot have value. As an illustration, assume that $u_\varepsilon(z) = A - z$ and $c = 0$ with probability 1 (production is free, although it still takes time to locate projects). In this case, it is not difficult to show that a pure monetary equilibrium exists for all $m \in (0,1)$ as long as $\delta < \delta_1 \equiv (\alpha+r)(1-A)/A$. But for $\delta > \delta_1$, the maximum value of m that allows monetary equilibrium, say m^* , is given by the following function of δ ¹⁷

$$m^* = 1 - 2\beta^{-1} [A\delta - (1-A)(\alpha+r)] \left[1 + \frac{\delta}{(\alpha+r)} \right]^2.$$

Notice m^* is decreasing in δ , and eventually $m^* = 0$. The use of money can be taxed, but if the tax rate becomes too high, eventually the use of money will be abandoned.

Finally, we examine welfare. Consider an example using $u(z) = A - z$ and $c = 0$ with probability 1 and with parameter values $A = 0.6$, $\varepsilon = \eta = 0.01$, $r = 0.001$, $\alpha = 0.5$, and $\beta = 10$. Figure 4 depicts x , y , and welfare W , where welfare is defined as (proportional to) steady-state utility, $\sum_j N_j V_j$, against the supply of money, M . The most interesting feature is that W is maximized at a strictly positive value of M . Hence, the introduction of fiat money into a barter economy can improve welfare, although when M becomes too large, W must fall because there

are too many money traders and not enough commodity traders with whom to trade (too much money chasing too few goods, as they say). The optimum quantity of money is strictly between 0 and 1 in this example. In fact, V_0 , V_1 , and V_m can all be maximized at $M > 0$, and so welfare is greater in a monetary than in a nonmonetary steady state according to the strict Pareto criterion (even if $\eta > \epsilon$, $w_m > w_1$, etc.) in some examples we have looked at. In other examples, however, such as those those with large values for β , the optimum quantity of money is $M = 0$.

The point is that money can, but need not, improve welfare in the model. This potential welfare-improving function can exist despite the fact that introducing fiat money requires reducing the number of real commodities circulating in steady state.¹⁸ Furthermore, it does not depend on money having superior intrinsic properties, such as a smaller transactions cost than barter or a lower storage cost than real commodities, as was the case in Kiyotaki and Wright [11]. The role of money in this model derives from the fact that it speeds up trade by helping to overcome the double coincidence problem. Even though the number of commodity traders falls as M grows, at least for some parameter values, agents find acceptable trades more quickly for larger M , and this additionally encourages them to only accept goods closer to their ideal. In this way, the use of fiat money as a medium of exchange can improve welfare.

V. Conclusion

A model with search type frictions and many commodities highlights the double coincidence problem with pure barter and therefore provides a natural framework within which to think about money as a medium of exchange. We have shown that both pure barter and monetary

equilibria exist in our model and have characterized the role of money in terms of liquidity. We have further demonstrated that these monetary equilibria are robust. Even if fiat money is endowed with properties making it less than an ideal asset (e.g., a relatively low rate of return) or less than an ideal medium of exchange (e.g., a relatively high transactions cost), it can continue to circulate and to play a role in facilitating trade and improving welfare. Of course, if the intrinsic properties of money become too unfavorable then it simply cannot circulate.

The model presented here is obviously special, and there is room for much work. We think the key assumptions underlying the results are the following. First, the transactions cost ϵ was critical in reducing the nonmonetary economy to a pure barter economy; with $\epsilon = 0$, all trades would be acceptable, so all goods would serve as media of exchange, and there would be little reason to use fiat money. Second, symmetry in the sets of goods and agents reduced the possibility of commodity money surfacing and thus made a role for fiat money seem natural. Third, the indivisibility of commodities, combined with the restriction on storage (implying inventories always consist of one unit of one thing), kept the model tractable but precluded a potentially interesting analysis of the distribution of prices. Our hope is to convince the reader that results from models like this and their potential generalizations are worth pursuing.

Notes

¹As Menger [14, p. 239] put it, "It is obvious even to the most ordinary intelligence, that a commodity should be given up by its owner in exchange for another more useful to him. But that every economic unit in a nation should be ready to exchange his goods for little metal disks apparently useless as such, or for documents representing the latter, is a procedure so opposed to the ordinary course of things, that ... [it is] downright 'mysterious.'"

²This is a robustness that does not appear, for example, in the overlapping generations model of fiat money--the presence of storage opportunities, or of other assets, with dominating rates of return necessarily drives money out of the system without auxiliary assumptions such as legal restrictions. This follows because money does not serve as a medium of exchange in those models. Cash-in-advance models, or money-in-the-utility-function models, while clearly useful for some purposes, do not explain the existence of valued fiat currency at all; both appeal to implicit features of their underlying environment in order to motivate the imposed role for money, but the problem is that once we make these features explicit, they may well have other implications that should not be ignored.

³Jones [9], Oh [15], Iwai [7], and Ariga [1] have also analyzed money using models of bilateral trade. Somewhat similar are the spatial separation models studied by Townsend [17] or Townsend and Wallace [18]. A survey of other efforts along these lines is provided by Ostroy and Starr [16].

⁴For instance, each consumer has an ideal color and derives utility from consuming goods that decreases with the difference between

the goods' color and his ideal color. When there is no risk of confusion, we refer to a good that is distance z from an agent's ideal as good z .

⁵Diamond's models [3,4] also impose a "taboo" against consuming one's own output, which Diamond [4, p. 2] motivates by saying "The assumed taboo plays the role of the advantage of specialization and trade over self-sufficiency in a modern economy." Since our model has differentiated goods, we can adopt the following alternative story that leads to exactly the same formal structure. Assume production projects yield two goods of the same type, agents also must consume goods in pairs, and they enjoy utility $u(z)$ where z is the distance between the two with $u' > 0$ (i.e., they like diversity). Now, once a pair of goods is produced, the agent could consume both but typically prefers to trade one for something else. This leads to exactly the same mathematical structure as in the text (except that u' changes sign).

⁶This is referred to as a constant returns-to-scale matching technology. To see why, let $\mu(N)$ be the number of meetings per unit of time when N agents are searching for a partner. Then $B(N) = \mu(N)/N$ is the probability of a given agent finding a match, and when $\mu(\cdot)$ displays constant returns, i.e., when $\mu(N) = \beta \cdot N$, we have $B(N) = \beta$ as long as $N > 0$. In Kiyotaki and Wright [12], we analyze pure barter with a general matching technology, but the messages concerning fiat money are better made without such complications.

⁷See Jovanovic and Rosenthal [10] for a general description of anonymous sequential games into which our model fits.

⁸We still need to show the assumption $\theta(i) = \theta_i \psi_i$ is consistent with rational beliefs, which it will be in the symmetric,

steady-state equilibrium to be described below. We do not claim that other types of equilibria could not exist, by the way. Equilibria where commodities play asymmetric roles could lead one to identify some of them as commodity monies, as in Kiyotaki and Wright [11], but the purpose here is to concentrate exclusively on fiat money.

⁹In discrete time periods of length h , Bellman's equations are

$$V_0 = e^{-rh} \left\{ (1-ah)V_0 + ah \int_0^{\infty} \max(V_0, V_1 - c) dF(c) \right\} + o_0(h)$$

$$V_1 = e^{-rh} \left\{ (1-Bh\theta)V_1 + Bh\theta \int_0^1 \max[V_1, V_0 + u_\varepsilon(z)] dz \right\} + o_1(h)$$

where $o_j(h)$ satisfies $o_j(h)/h \rightarrow 0$ as $h \rightarrow 0$ and represents the possibility of more than one arrival in $[t, t+h]$ --recall that for a Poisson process with parameter γ , the probability of n arrivals in an interval of length h is $p(n, h) = e^{-\gamma h} (\gamma h)^n / n!$, so as $h \rightarrow 0$, $p(n, h)/h \rightarrow \gamma$ for $n = 1$ and $p(n, h)/h \rightarrow 0$ for $n > 1$. If we rearrange these we get

$$[1 - e^{-rh}]V_0 = e^{-rh} ah \int_0^{\infty} \max(0, V_1 - V_0 - c) dF(c) + o_0(h)$$

$$[1 - e^{-rh}]V_1 = e^{-rh} Bh\theta \int_0^1 \max[0, V_0 - V_1 + u_\varepsilon(z)] dz + o_1(h).$$

Dividing both sides by h and letting $h \rightarrow 0$ now yields the continuous time recursive equations (2.1) in the text.

¹⁰Roughly speaking, $s_0(k)$ is the expected gain from production, or the expected producer surplus, and $s_1(x)$ is the expected gain from trade or the expected consumer surplus.

¹¹It actually does not matter if the shell is divisible as long as we maintain the assumption that agents cannot store both it and any other commodity at the same time. Now this is unrealistic because

it seems feasible in the real world to hold both money and other objects simultaneously. But the puzzle in monetary economics is to explain why people voluntarily surrender goods for worthless objects, like paper or shells, and this would be a puzzle whether or not goods and money could be held simultaneously, whether or not they were divisible, etc. Indeed, our restrictions imply that agents must give up all of their goods in order to acquire and store money, and this makes valued fiat currency seem all the more puzzling.

¹²We want to emphasize that M is the quantity of real balances in the economy. Since it clearly does not matter if a unit of any good trades for 1 shell/dollar or P shells/dollars, only $M = S/P$ matters, where S is the total number of shells/dollars in circulation and can be thought of as nominal balances. For any equilibrium with S shells/dollars exchanging for goods at rate P , there is an equilibrium with $S' = \lambda S$ and $P' = \lambda P$, for all $\lambda > 0$, that produces the same real outcome.

¹³When $M = 1$, there is also a monetary equilibrium in which everyone holds money and no one ever consumes. Even though a money trader has no one to trade with, in principle, a commodity trader would accept money since he has no one to barter with either. Money is still no less marketable than goods, so agents will not dispose of it to return to production. Also, note that although we have shown that m -equilibrium is unique, each m -equilibrium is supported by a unique $M(m)$, and if we start with a value of M there exists at least one m -equilibrium, we have not ruled out the possibility that for a given value of M , there might be multiple equilibria. Uniqueness would hold if $M = M(m)$ was invertible, which we have not been able to verify in

general (although it was in all of the examples we studied). Finally, we point out that our proof of the uniqueness of m -equilibrium, which follows from $T'(y) < 0$ when $T(y) = 0$, uses the assumption that $u'(z) + zu''(z) \leq 0$.

¹⁴For a general discussion of first passage times, like D_j , see Heyman and Sobel [6]. See Lippman and McCall [13] for a discussion of some related notions of liquidity.

¹⁵Since mixed-strategy equilibrium is formally just like equilibrium in the nonmonetary economy, one might expect that a second money (say a red money as well as the existing blue money) that was universally acceptable could be introduced into it. The result would be an economy with dual fiat currencies having different rates of acceptability. We are currently working on these issues.

¹⁶As Hicks [5, p. 6] noted in 1935, economists "would have put it down to 'frictions,' and since there was no adequate place for frictions in the rest of their economic theory, a theory of money based on frictions did not seem to them a promising field for economic analysis." The difficulty of modeling frictions explicitly has led some, like Bryant and Wallace [2, p. 1], who agree that dominance is the crucial anomaly, to suggest instead that it is "... to be explained by deviations from laissez-faire, for example, legal restrictions"

¹⁷This is derived by calculating the maximum m for which the equilibrium function $T(y)$ in Proposition 2 is negative at $y = y_2$.

¹⁸Mixed-strategy monetary equilibria are never efficient. As $\theta_3 = x$ in such equilibria, the partially accepted fiat money does nothing to increase the frequency of trade; it simply reduces the number of real commodities in circulation. Alternatively, the mixed-strategy

equilibrium is like pure barter with a reduced arrival rate, $\beta(1-m)$ instead of β , and a reduced arrival rate can only hurt welfare.

Appendix A

We complete the proof of Lemma 2 by showing that $y > x^2$. Evaluate the function $T(y)$ in the proof of Proposition 2 at $y = x^2$,

$$T(x^2) = (r + \beta mx^2)[u(x^2) - u(x)] + \beta(1-m)[xs_1(x) - s_1(x^2)]$$

where $x = \phi(y) = \phi(x^2)$. The first term is positive since $x < 1$. To check the second term, we use a Taylor's expansion of $s_1(x)$ around x^2 : for some $x_0 \in (x^2, x)$,

$$\begin{aligned} xs_1(x) - s_1(x^2) &= x[s_1(x^2) + (x-x^2)s'(x^2) + 0.5(x-x^2)^2s''(x_0)] - s_1(x^2) \\ &\geq x[s_1(x^2) + (x-x^2)s'(x^2)] - s_1(x^2) \\ &= (1-x)[x^2s'(x^2) - s_1(x^2)] \geq 0. \end{aligned}$$

Both inequalities follow from the convexity of $s_1(x)$, which follows easily from the assumption $u'(z) + zu''(z) \leq 0$. Hence, $T(x^2) > 0$, and so the solution to $T(y) = 0$ must lie to the right of x^2 , as seen in Figure 3. \square

Appendix B

We begin to derive the expression for λ_1 by letting p_{ij} be the instantaneous transition rate from state i to state j . Then, in particular,

$$p_{10} = \beta(1-m)x^2, p_{1m} = \beta my, p_{m0} = \beta(1-m)y. \quad (B.1)$$

Now starting in state 1, let T be the time of the first transition, a random variable with cumulative distribution function

$$F(t) = \text{pr}(T \leq t) = 1 - \exp[-(p_{10} + p_{1m})t].$$

The transition at t can be either to state 0 or to state m . If we let T_0 and T_m be independent random variables with cumulative distribution functions

$$F_j(t) = \text{pr}(T_j \leq t) = 1 - \exp[-p_{1j}t], \quad j = 0, m$$

then the actual transition is to state m iff $T_m < T_0$. This occurs with probability

$$\begin{aligned} \text{pr}(T_m < T_0) &= \int_0^{\infty} \text{pr}(T_0 > t | T_m = t) dF_m(t) \\ &= \int_0^{\infty} [1 - F_0(t)] dF_m(t) = \frac{p_{1m}}{p_{1m} + p_{10}}. \end{aligned}$$

Let D_1 be the random time it takes to get from state 1 to state 0. Given the first transition is at time t , then conditional on knowing whether the transition was to m or to 0, we either have $E[D_1 | T_0 > T_m = t] = t + 1/p_{m0}$ or $E[D_1 | T_m > T_0 = t] = t$. Thus,

$$\begin{aligned} E[D_1|T = t] &= \text{pr}(T_m < T_0) \left[t + \frac{1}{p_{m0}} \right] + \text{pr}(T_0 < T_m) t \\ &= t + \frac{p_{1m}}{(p_{10} + p_{1m})p_{m0}}. \end{aligned}$$

Hence, the unconditional expectation is

$$ED_1 = \int_0^{\infty} E[D_1|T = t]dF(t) = \frac{p_{m0} + p_{1m}}{(p_{10} + p_{1m})p_{m0}}. \quad (\text{B.2})$$

Substituting the values for p_{ij} given by (B.1) into (B.2) and using

$\lambda_1 = 1/ED_1$ yields the desired formula. \square

References

1. K. Ariga, Transaction demand for money, mimeo, Kyoto University, 1988.
2. J. Bryant and N. Wallace, "A Suggestion for Further Simplifying the Theory of Money," Research Department Staff Report 62, Federal Reserve Bank of Minneapolis, 1980.
3. P. A. Diamond, Aggregate demand management in search equilibrium, Journal of Political Economy 90 (1982), 881-894.
4. P. A. Diamond, Money in search equilibrium, Econometrica 52 (1984), 1-20.
5. J. R. Hicks, A suggestion for simplifying the theory of money, Economica 2 (1935), 1-19.
6. D. P. Heyman and M. J. Sobel, "Stochastic Models in Operations Research, Vol. 1: Stochastic Processes and Operating Characteristics," McGraw-Hill, New York, 1982.
7. K. Iwai, "The Evolution of Money: A Search Theoretic Foundation of Monetary Economics," CARESS Working Paper 88-03, University of Pennsylvania, 1988.
8. W. S. Jevons, "Money and the Mechanism of Exchange," D. Appleton and Company, London, 1875.
9. R. A. Jones, The origin and development of media of exchange, Journal of Political Economy 84 (1976), 757-775.
10. B. Jovanovic and R. W. Rosenthal, Anonymous sequential games, Journal of Mathematical Economics 17 (1988), 77-87.
11. N. Kiyotaki and R. Wright, On money as a medium of exchange, Journal of Political Economy 97 (1989), 927-954.

12. N. Kiyotaki and R. Wright, A search equilibrium model of production and trade: Equilibrium, welfare, and consumption statics, mimeo, University of Pennsylvania and University of Wisconsin, 1989.
13. S. A. Lippman and J. J. McCall, An operational measure of liquidity, American Economic Review 76 (1986), 43-55.
14. K. Menger, On the origin of money, Economic Journal 2 (1892), 239-255.
15. S. Oh, A theory of a generally acceptable medium of exchange and barter, Journal of Monetary Economics 23 (1989), 101-120.
16. J. M. Ostroy and R. M. Starr, "The Transactions Role of Money," Department of Economics Working Paper 505, University of California - Los Angeles, 1988. Forthcoming in "Handbook of Monetary Economics."
17. R. M. Townsend, Models of money with spatially separated agents, in "Models of Monetary Economies," (J. H. Kareken and N. Wallace, Eds.) Federal Reserve Bank of Minneapolis, Minneapolis, MN, 1980.
18. R. M. Townsend and N. Wallace, Circulating private debt: An example with a coordination problem, in "Contractual Arrangements for Intertemporal Trade," (E. C. Prescott and N. Wallace, Eds.) University of Minnesota Press, Minneapolis, MN, 1987.
19. N. Wallace, The overlapping generations model of fiat money, in "Models of Monetary Economies," (J. H. Kareken and N. Wallace, Eds.) Federal Reserve Bank of Minneapolis, Minneapolis, MN, 1980.

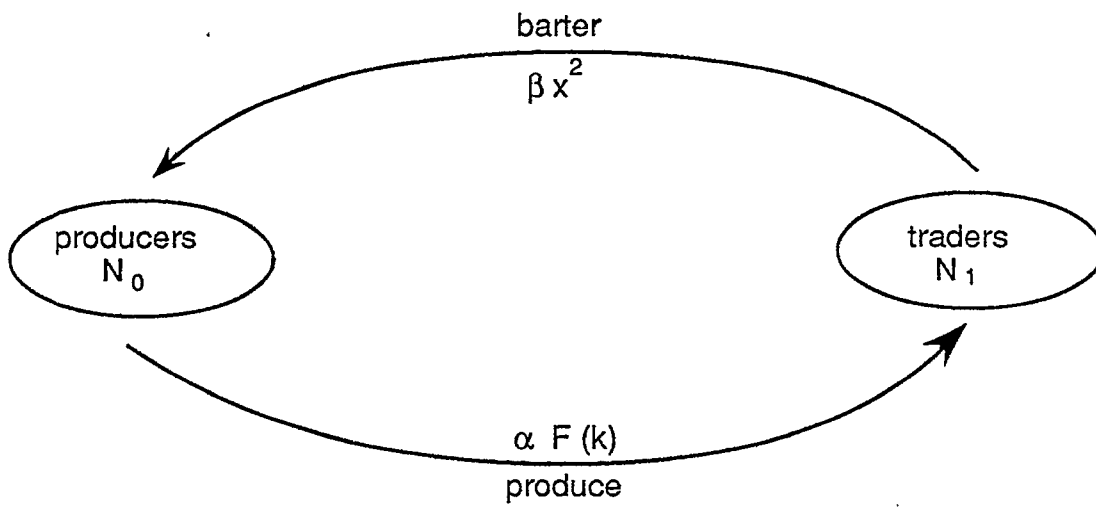


FIGURE 1

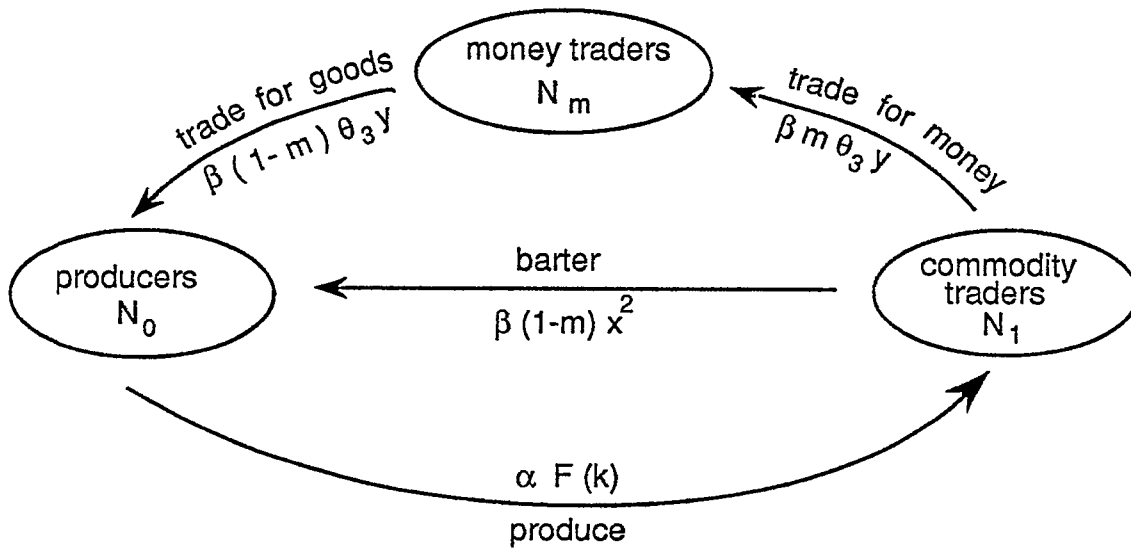


FIGURE 2

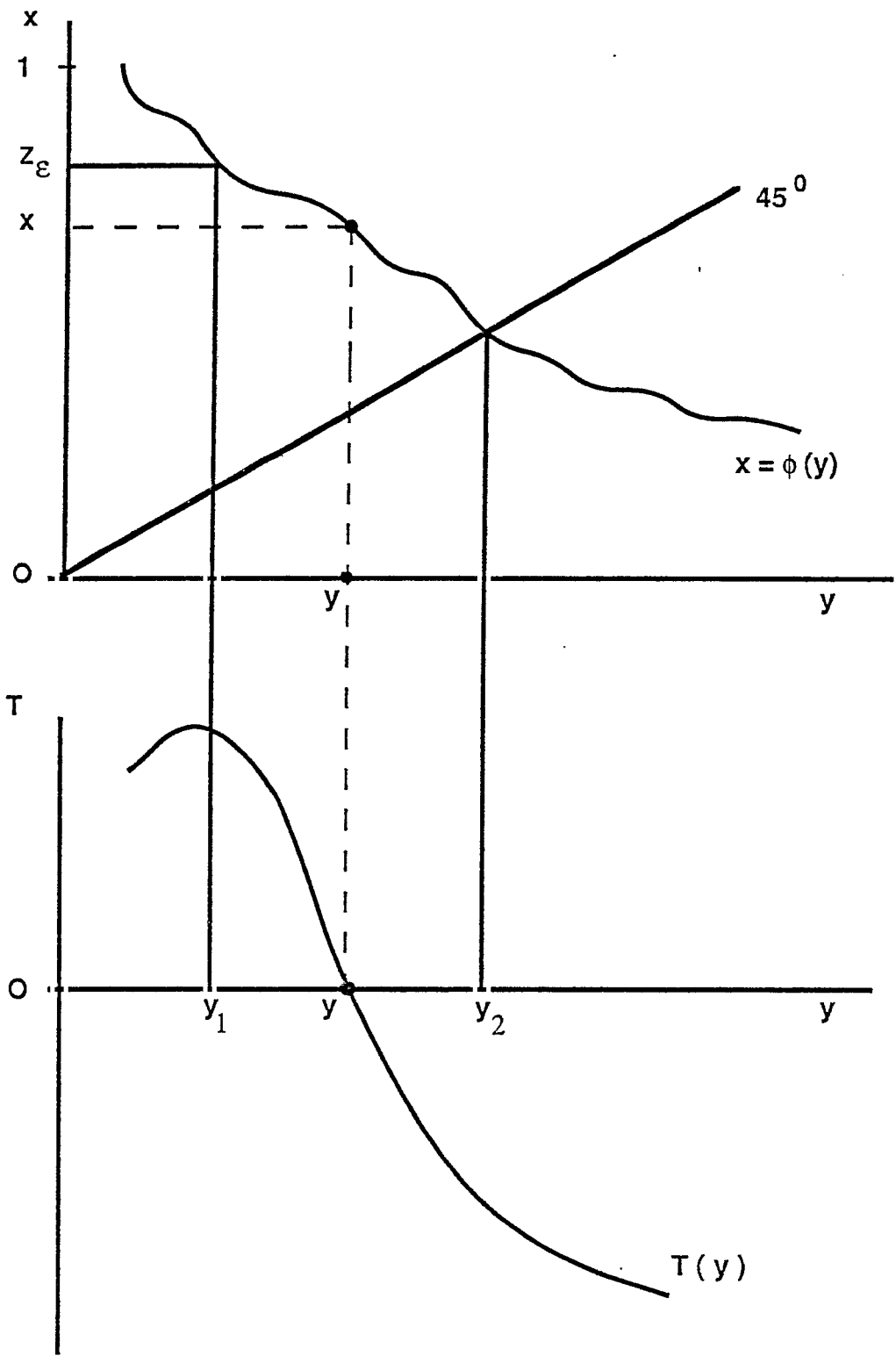


FIGURE 3

W, x, and y vs. Money Supply

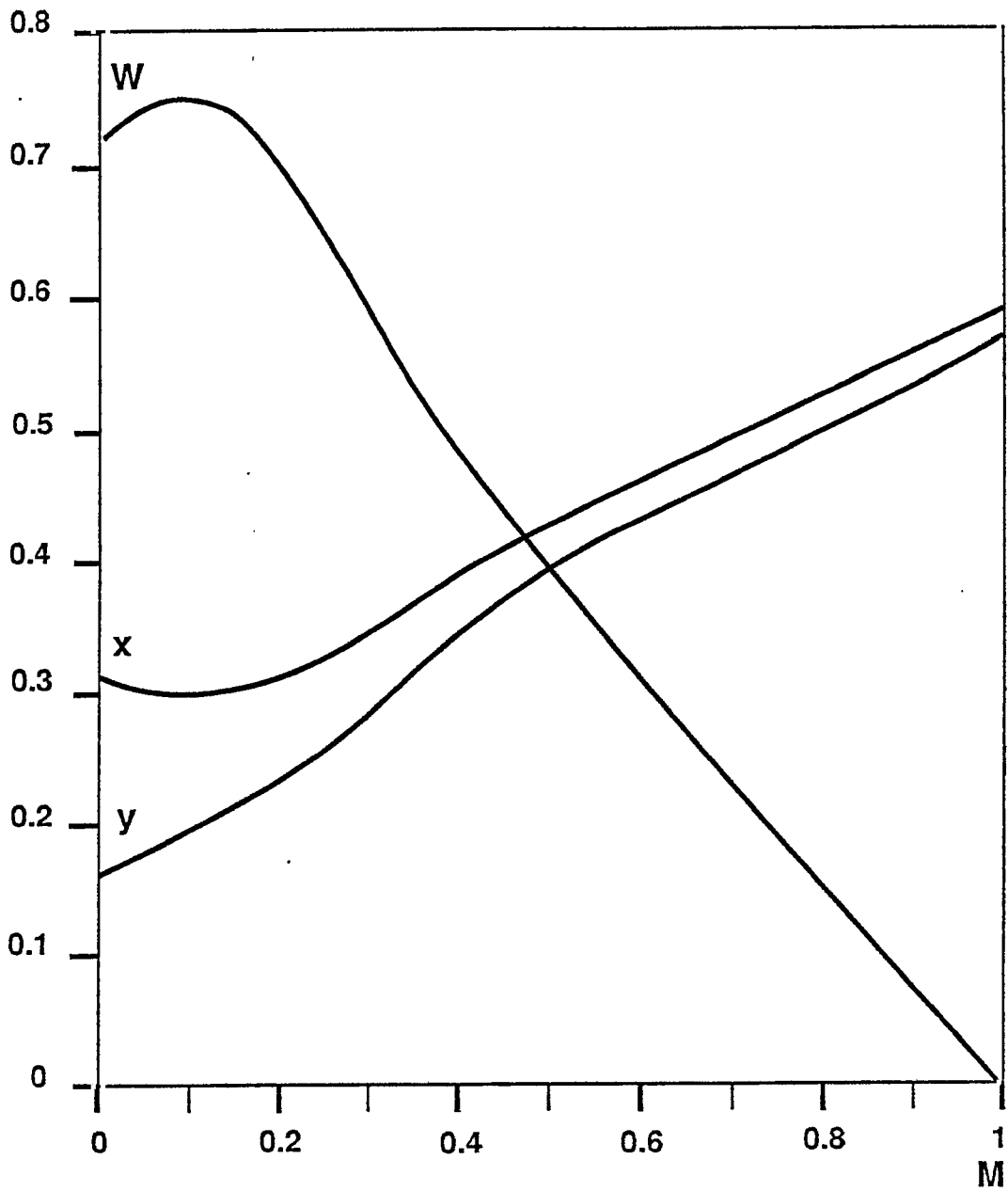


FIGURE 4