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Reputation Spillover Across Relationships With Enduring and Transient Benefits: Reviving Reputation Models of Debt*

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ABSTRACT

A traditional explanation for why sovereign governments repay debts is that they want to keep a good reputation so they can easily borrow more. In our model, this explanation does not hold if a government has access to an adequate savings technology regardless of its past actions. Given such access, governments get only *transient benefits* from maintaining a good relationship with bankers, and such benefits cannot support borrowing. We argue that if a country is involved in a myriad of different trust relationships, there can be reputation spillover to a nondebt relationship, such as the government's relationship with domestic workers, which has *enduring benefits*. We show that such a spillover can allow a government's reputation to support a large amount of borrowing.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

The standard reputation argument for why sovereign governments repay their debts is because governments fear that if they default, their tarnished reputation will spillover to future debt relationships and make bankers less willing to lend to them. (See, for example, Jonathan Eaton and Mark Gersovitz 1981; Rodolfo Manuelli 1986; Herschel Grossman and John Van Huyck 1988; Harold Cole, James Dow, and William English 1989; and Andrew Atkeson 1991.¹) Recently, Jeremy Bulow and Kenneth Rogoff (1989b) have challenged this traditional explanation. In a provocative paper, they show that “under fairly general conditions, lending to small countries must be supported by the direct sanctions available to creditors, and cannot be supported by a country’s ‘reputation for repayment’” (1989, p. 43). A key reason for the difference between this result and the results in the rest of the literature is that Bulow and Rogoff assume that regardless of its past behavior, a government has a means of savings through which it can earn the market rate of return. The rest of the literature, either explicitly or implicitly, assumes that if a country defaults it cannot save. We argue that even under the Bulow and Rogoff assumption, reputation arguments for repayment of debt can be revived if there is a very different type of reputation spillover in which misbehavior in a relationship in the debt arena tarnishes a government’s reputation in other relationships in a different arena of behavior. For expositional purposes we first show that spillovers are necessary to revive these reputational arguments in a model with infinitely-lived governments and complete information. Since the assumption that governments are infinitely-lived is unnatural for our application, we then consider an environment with finitely-lived governments and incomplete information and demonstrate the necessity of spillovers.

We begin with a simple complete information infinite horizon model in which the government of one country is involved in a single relationship with the rest of the agents in the world, namely a debt relationship. Moreover, we assume the government has access to a savings technology which allows it to earn the market rate of return on its investments. We show the Bulow-Rogoff result

holds for this model: in equilibrium there can be no positive debt. The intuition for this result is as follows. Consider an outcome path with positive debt along which the government does not default. The government can do strictly better by deviating from this outcome path and defaulting at the point where the present value of the debt is largest. From this point on the government was, on net, paying back the bankers in each period. Thus it is feasible for the government to roll forward the funds it would have paid the bankers and use it to self-finance the original consumption and investment plans. Moreover, since it never has to pay back the debt it will have some excess funds in the future which it can use to increase consumption. Here the debt relationship has *transient benefits* in the sense that the value to the government of maintaining a good relationship with its creditors is positive at most up until the point where it receives its maximal loan in present value terms.

Next, we consider more interesting complete information models in which the government is involved in more than one relationship that involves some degree of trust. We first suppose that the other relationship is a second debt relationship involving another investment project. We show that even in an equilibrium with a spillover effect across these relationships, in which a default on a loan contract for either one of the investment projects leads to a denial of all future loans for both projects, no positive debt can be supported in equilibrium. More generally, we can show that spillover from the debt relationship to any other relationship with transient benefits will not support positive debt.

This analysis drives us to conclude that, in order for reputation to support good outcomes in a relationship which has transient benefits, such as a debt relationship, there needs to be a type of spillover not considered in the existing literature: misbehavior in the relationship with transient benefits must spillover to a relationship with *enduring benefits*. Such an enduring benefit relationship has the property that no matter what the past history, the value of maintaining a good relationship

for the future is large. In the context of our debt model this means that misbehavior in repaying debt must not only influence the actions of other bankers but must also influence the actions of agents in a entirely different arena of behavior. We illustrate our point by adding to the model a labor project in which domestic workers must trust the government before they will work for it in any period. The ensuing labor relationship viewed in isolation is a repeated game. In contrast to the debt relationship in which the value of maintaining a good relationship depends on the level of a state variable, the value of maintaining a good relationship in a repeated game is constant and thus enduring. We show that reputation spillover from the long-lived debt relationship, which has transient benefits, to the long-lived labor relationship, which has enduring benefits, can potentially support the full-commitment level of debt.

We argue that this type of spillover is likely to be important in the context of governments since governments are typically involved in a myriad of relationships which involve some degree of trust. International examples of such relationships include direct investments by foreigners or commitments to mutual defense pacts. Domestic examples include the implicit trust domestic firms and individuals have in the government when making such long-term commitments as human capital acquisition or the trust labor unions have in the government when they forestall strikes in exchange for the government's promise to carry out certain desirable policies. For all of these examples, the agents may substantially alter their behavior if they do not trust the government to honor its explicit or implicit contracts.

For expository purposes we have found it useful to begin with simple complete information models with infinitely-lived governments. Since governments (or more generally regimes) are finitely-lived, in order for our story about spillover effects to be relevant to our application we must show that such spillover effects both can arise and can support debt in a model with a finitely horizon. To do so we consider an incomplete information model in which there is a small amount

of uncertainty about whether the government attaches a large disutility to not honoring its contracts. (For some early work in industrial organization using repeated models with incomplete information see David Kreps and Robert Wilson 1982 or Paul Milgrom and John Roberts 1982.) We find several results. First, for any finite horizon there is an (essentially) unique equilibrium in which there are spillovers. Moreover, these spillovers are both a natural and necessary consequence of rational inference by the private agents. Second, for a long enough finite horizon positive debt can be supported in equilibrium. Third, we consider a sequence of finite horizon incomplete information economies with successively longer horizons. We show that both the finite horizon strategies and the equilibria of the incomplete information game converge naturally to the infinite horizon strategies and equilibria of the complete information game. Moreover, these limiting strategies are themselves a legitimate equilibrium to a (naturally-modified) infinite horizon game. Finally, we show that the same holds true if the uncertainty about the government's type converges to zero at a sufficiently slow rate as the horizon goes to infinity. These results imply that there is both a natural and close connection between the finite horizon incomplete information results and the infinite horizon complete information results. Indeed, we think of these results as providing one possible motivation for focusing on the equilibrium with spillover effects in the infinite horizon model.

In summary the point of our paper is the following. Standard models of debt produce dynamic games with physical state variables instead of the repeated games common in the industrial organization literature. In these dynamic games, the role that reputation spillover can play is subtle and it depends critically on the nature of underlying environment. The point of this paper is to clarify the nature of these interactions and to point out that in order to revive reputational models of debt, what is needed is a spillover different from the type considered in the literature, namely a spillover from the debt relationship to another type of relationship with enduring benefits as opposed to the standard spillovers across individual bankers within one debt relationship.

I. An Economy With a Single Relationship

Consider a world economy that consists of two countries. In each period t , $t = 0, \dots$, the economy has a single good. One country consists of a large number of risk-neutral agents called *bankers*. Each banker has a discount factor β and is endowed with a fixed amount of the good in each period. We assume bankers act competitively. The other country is represented by an agent, called the *government*, which has access to a country-specific investment project. In each period t , an investment of x_{t+1} units in the project produces output of Ax_{t+1} units in period $t + 1$. The project has a maximal size of 1; thus, $x_{t+1} \leq 1$. Assume that

$$(1.1) \quad \beta A > 1;$$

in other words, the project's gross return A is greater than the bankers' discount rate $\rho = 1/\beta$. The government also has access to a saving technology that allows it to earn the world rate of return ρ on its savings. This savings technology can be interpreted either as a domestic technology or as access to the world market for savings. The government has no endowments of the good in any period and zero borrowing or saving at the beginning of period 0, thus,

$$(1.2) \quad b_0 = 0.$$

The preferences of the government are given by

$$(1.3) \quad \sum_{t=0}^{\infty} \beta^t c_t.$$

We will soon consider a model in which the government cannot commit to honoring debt contracts made with the bankers. First, though, we build some intuition by considering a model in which there is a technology through which the government can commit to honoring its debt. From

the linearity of banker preferences and the assumption that they behave competitively, it is clear that in each period t the equilibrium gross rate of interest on loans is $1 + r_t = \rho$. From (1.1), the return on the project A is greater than ρ and hence with such an interest rate in each period it is optimal for the government to borrow so that it can fully fund the project. In period 0 the government borrows 1, invests it and consumes nothing. In period 1 and all future periods the government borrows 1 and invests it, repays ρ on the old loan and consumes output net of repayment, $A - \rho$. The discounted value of utility under commitment is thus

$$(1.4) \quad \beta(A-\rho) + \beta^2(A-\rho) + \dots = \frac{\beta(A-\rho)}{1-\beta}.$$

Of course, given the linear preferences of the government, and that the discount factor of the government β satisfies $\beta = 1/\rho$, there are a variety of ways the timing of consumption by the government can be structured to yield the same discounted value of utility.

Consider now a model without such a commitment technology. A precise description of the model is as follows. At the beginning of period t there are two state variables, the amount of debt either owed or saved, $(1+r_t)b_t$, and the amount of new output, Ax_t . If $b_t > 0$ the government decides whether to pay the old loan subject to the constraint

$$(1.5) \quad z_t(1+r_t)b_t \leq Ax_t.$$

If $b_t < 0$ the government receives a payment of ρb_t . Bankers, having seen the default decision, offer the government a new set S_{t+1} of loan contracts. Each contract s_{t+1} is a pair (r_{t+1}, b_{t+1}) which specifies an interest rate r_{t+1} and a loan amount. We also embed the savings possibilities in S_{t+1} by having S_{t+1} include savings contracts which specify $1 + r_{t+1} = \rho$ for any $b_{t+1} < 0$. The government then chooses some specific contract s_{t+1} , how much to consume and invest subject to the constraints

$$(1.6) \quad x_{t+1} \leq 1,$$

$$(1.7) \quad c_t + x_{t+1} - b_{t+1} = Ax_t - z_t(1+r_t)b_t.$$

In (1.7) if $b_t > 0$ then $z_t = 1$ corresponds to repayment and $z_t = 0$ to default while if $b_t < 0$, then $z_t = 1$, and $1 + r_t = \rho$.

We begin by showing that the full commitment allocations cannot be supported as equilibrium allocations. To see this consider these allocations and consider the decision to repay at the end of some period $t \geq 1$. If the government continues with the full commitment policy it gets $A - \rho$ in all periods. Consider the following deviation. The government defaults at t , invests 1 and consumes $A - 1$. In period $t + 1$ and all periods thereafter it invests 1 in the project, and consumes $A - 1$. Thus under commitment the government gets $(A - \rho)/(1 - \beta)$ while under this deviation it gets $(A - 1)/(1 - \beta)$. Since $\rho > 1$ this deviation is strictly preferred for all discount factors $\beta \in (0, 1)$. Thus the full commitment allocations cannot be supported as equilibrium allocations.

The intuition for this result is simply that once the government has one unit on hand it has no need to borrow any more and thus the value of maintaining a good relationship with the bankers is zero. Moreover, if it breaks this relationship by defaulting it saves the funds it owed and thus defaulting dominates maintaining a good relationship. More generally, in the spirit of Bulow and Rogoff, we can show,

Proposition 1. In the economy with a single relationship the unique equilibrium allocations are the autarky allocations.

Proof. Consider an equilibrium outcome. In any equilibrium bankers break-even so

$$(1.8) \quad [(1+r_t)z_t - \rho]b_t = 0.$$

It follows that, if there are any loans made, the gross interest rate is ρ , that is, if $z_t = 1$ and $b_t \neq 0$ then $1 + r_t = \rho$. If $z_t = 0$, then no loans are made, so $b_t \leq 0$. Clearly b_t cannot be greater than

or equal to 1 in any equilibrium. If it were then the government would certainly prefer to deviate by defaulting on the amount owed, ρb_t , and then consuming $\rho b_t - 1$ in extra consumption at date t . In all future periods it would fund the project by investing 1 and it would consume $A - 1$ in each period. Since b_t is bounded in equilibrium it follows that

$$(1.9) \quad \lim_{t \rightarrow \infty} \beta^t b_t = 0.$$

Next we show that b_t cannot be any strictly positive number between 0 and 1. By way of contradiction suppose that at some date, say date v , that $b_v > 0$. Let

$$(1.10) \quad \beta^r b_r = \max_t \beta^t b_t.$$

Thus r is the date at which the present value of borrowing is the largest. Clearly r is finite since $b_t \leq 1$ for all t . If there are multiple dates which satisfy (1.11) let r be the largest such date. Consider, for now, deviating at date r by defaulting at date r and then continuing with the original consumption and investment levels. To see that this is feasible first note that the original debt sequence must satisfy

$$(1.11) \quad \beta^t b_t = \sum_{s=t}^{\infty} \beta^{s+1} (Ax_s - c_s)$$

where we have obtained this by iterating on (1.7) and using (1.8) and (1.9) gives that for any t .

Under our deviation the new debt sequence, denoted by \hat{b}_t , satisfies for $t \geq r$,

$$(1.12) \quad \beta^t \hat{b}_t = \sum_{s=t}^{\infty} \beta^{s+1} (Ax_s - c_s) - \beta^r b_r.$$

Since $\beta^t b_t < \beta^r b_r$ for all $t > r$, it follows that \hat{b}_t is nonpositive. Under this deviation the government never borrows again from the bankers but rather self-finances the investment and consumption stream by saving. Note also that since under the original allocation (1.9) and (1.10) hold it follows that

$$(1.13) \lim_{t \rightarrow \infty} \beta^t b_t = -\beta^r b_r.$$

Clearly then there is some sufficiently large date T such that it is possible to increase consumption at T and still have the new debt sequence be nonpositive. Since this deviation is preferred to the conjectured outcome no such equilibrium can exist. \square

The intuition for this proposition is similar to the intuition for why the full commitment allocations are not supportable as equilibrium allocations. Consider any equilibrium and consider the period in which the present value of debt owed by the government is maximal. Since this value of the debt is the largest it will ever be, this means that in each period after this the government is, on net, paying back the bankers. If the government instead defaults and saves with the bankers it can finance its original investment pattern and increase consumption.

Notice that our model works quite differently than the models of Eaton and Gersovitz and others. In those models trigger strategies, which specify that if the government ever defaults bankers refuse to lend to it from then on, will support positive debt. A key reason for this is that in those models the government had no way to safely save. Thus if the government ever defaulted it could neither borrow nor save and hence it was stuck in autarky forever. In such models if there is no way to save then the debt relationship has *enduring benefits*: no matter what the past history, the value of maintaining a good relationship for the future is large. For a sufficiently patient government the present value of the loss of a good relationship with bankers is enormous and it gives the government a big incentive to maintain a good relationship with bankers by not defaulting. Bankers understand this incentive and lend to the government the full commitment amount. In contrast, in our economy if the government defaults, it may well never get to borrow again, but its ability to safely save undercuts the force of these borrowing restrictions. In this setup, if there were positive debt, a government could do better by defaulting when the debt is relatively large and then saving and self-financ-

ing the investment project than it could by repaying and maintaining a good relationship with the bankers. Here the debt relationship has only *transient benefits* in the sense that if the debt was ever positive then within some finite time the value of maintaining a good relationship for the future becomes trivial. Even with little discounting within some finite time the gains from defaulting outweigh the losses from losing the transient benefits and it is not possible to support any borrowing in equilibrium.

Bulow and Rogoff's model also includes a savings option for the government. In their model the government can always safely save abroad with foreign bankers. One interpretation of this setup is that there are legal institutions which effectively prohibit foreign bankers from defaulting on the repayment of the principal plus interest on the savings of the government. Bulow and Rogoff argue that this institutional setup is reasonable and document historical support for it. From a theoretical point of view it makes little difference if the savings option arises because of a domestic technology or because of a set of foreign legal institutions. What matters is that if a government defaults it has some means of saving and earning a rate of return as good as the rate it gets when it does not default. As long as this is true than the debt relationship has only transient benefits and it is impossible to support positive debt.

II. Spillover with Multiple Relationships

In this section we add to the model of Section I other relationships that involve trust. We first show that if we add another debt relationship our earlier results are unchanged: no positive debt can be supported in equilibrium. We then show that if we add a different type of relationship, which we call a labor relationship, then large amounts of borrowing can be supported. The key distinction between these relationships is that the debt relationship has only transient benefits while the labor relationship has enduring benefits.

A. Adding Another Debt Relationship

Consider adding to the model of Section 2 another debt relationship. Let the government have a second project which is the same as before, namely it has return A as in (1.1) and there is a maximal size of 1. In order to make our point as sharply as possible assume that there is a third country inhabited with another set of bankers of the same type as before, and that for some reason, the government can only use funds from the first lending country to fund the first debt project and funds from the second lending country to fund the second debt project. For example, one could imagine that the two projects use different types of project-specific physical goods and that bankers in the first lending country have sole access to the first type of goods while those in the second have sole access to the second type of goods and that there is no market in which the different types of goods can be exchanged for each other. Let the government be endowed with zero units of either type of good in period 0.

Consider first strategies in which a government's misbehavior in one debt relationship spillovers to affect its treatment in another debt relationship. Specifically, considers strategies for the bankers which specify that if the government breaks a contract with any banker in either country then no banker will lend it any funds again. Faced with such strategies the government will either simultaneously honor both types of debt contracts or break both since breaking either one causes both groups of bankers to stop lending.

After a moment's reflection it should be clear that even though there is a spillover across the debt relationships it is still impossible to support positive debt. Given such strategies by the bankers the optimal behavior of the government is to treat both projects symmetrically. Consider the incentive for the government to default in this model with two projects versus the incentive to default in the model with a single project. It should be clear that both the benefits and the losses from defaulting in the model with two projects are simply twice what they are in the model with one

project and thus, the default decisions are unchanged. More generally we can use similar logic to that in Proposition 1 to show that no positive debt can be supported in equilibrium.

B. Adding a Labor Relationship

Consider adding to the model of Section I a labor relationship. This labor relationship emanates from a project which is available in each period. If the number of workers hired is N_t the project's output is AN_t . The project has a maximal size N . (The assumption that the labor project has the same productivity as the debt project is for notational simplicity only.) The economy has a large number of domestic agents who have the specialized skills the government needs to run the project. Each of these workers is risk-neutral and has an alternative employment opportunity that earns a worker ω units with certainty in each period. We assume that

$$(2.1) \quad \beta A \geq \omega.$$

We will model the government as maximizing its utility subject to its resource constraints. With a little more notation, we could reinterpret the model as one in which the government maximizes the welfare of its citizens, provides public goods by using specialized resources, and taxes in a distorting way.

Now if there were full commitment, then the equilibrium would be as follows. In period 0 the government would take a loan of 1 from the bankers and invest it. It would hire N workers at wage ω , pay them ωN and consume output net of repayments, namely $(A - \omega)N$. In any period t after that it would take a new loan of 1 from the banker, invest it and hire N workers at wage ω . It would then repay the bankers ρ for the old loan from period $t - 1$, pay the workers ωN and consume the output net of repayments, namely $A - \rho + (A - \omega)N$.

Consider the model without commitment. The timing of the model is the same as before, with these additions. In the beginning of each period, each of the large number of workers offers an employment schedule. Each worker i offers to supply $n_t(i, w_t)$ units of labor to the government for a promise of w_t units of pay, where n_t is either zero or one. Confronted with a continuum of such wage schedules, all of which are identical, the government announces some particular wage w_t together with an employment cap N_t . At the end-of-period t when the output of the two projects is realized, the government decides whether or not to honor its contracts with the bankers and the workers. The constraints faced by the government are

$$(2.2) \quad z_t^b(1+r)b_t + z_t^a \omega_t N_t \leq Ax_t + AN_t$$

together with

$$(2.3) \quad c_t + x_{t+1} - b_{t+1} = Ax_t + AN_t - z_t^b(1+r)b_t - z_t^a w_t N_t$$

and

$$(2.4) \quad x_{t+1} \leq 1 \text{ and } N_t \leq N.$$

In (2.3) we have assumed that the number of workers is N_t . Also, in (2.3) $z_t^a = 1$ and $z_t^b = 1$ correspond to honoring the debt and labor contracts.

Consider strategies in which misbehavior by the government in the debt relationship spills over to the labor relationship and vice-versa. Specifically, suppose that the bankers' and workers' strategies specify that if the government ever breaks either the debt contract or the labor contract then they will never trust it again: bankers will never lend to it and workers never work for it. We will show that with such a spillover positive borrowing can be supported in equilibrium. Indeed, if the government is sufficiently patient then the full commitment allocations can be supported.

More formally let the bankers strategies at time t specify that for histories of the game with no previous default, namely $z_s^b = z_s^a = 1$ for all $s < t$, then

$$S_t = \{(r_t, b_t) \mid 1 + r_t = \rho, b_t \leq 1\}.$$

That is, the bankers will lend at rate ρ any amount up to one. For any history in which there has been a default $S_t = \phi$, and thus bankers do not lend. Let the workers strategies specify that for histories with no previous default $n_t(i, w_t) = 1$ if $w_t \geq \omega$, and zero otherwise. For any history in which there has been a default $n_t(i, w_t) = 0$. The government's strategy specifies its full commitment allocations if it never defaulted in the past. If it has defaulted the government's strategies specify self-finance the investment project, borrow nothing and pay the workers nothing. Call these strategies the *spillover strategies*. We then have

Proposition 2. There exists a $\underline{\beta} \in (0, 1)$ such for all $\beta \in (\underline{\beta}, 1)$ the full commitment allocations are supportable as equilibrium outcomes.

Proof. Consider the spillover strategies defined above. Consider first histories in which there have been no defaults before time t . It is optimal for the bankers to lend 1 and the workers to work if $w_t \geq \omega$. It is optimal for the government to borrow 1, fully fund the investment project, offer a wage of ω and an employment cap of N . Consider now the decision of whether or not to default. If it defaults on both contracts the government saves the current payments to bankers and workers, $\rho + \omega N$. However, it loses the surplus from the labor project, $(A - \omega)N$, from $t + 1$ onwards. Thus, sticking with full commitment is at least as good as the deviation if

$$(2.5) \quad \rho + \omega N \leq \frac{\beta(A - \omega)}{1 - \beta}.$$

As β increases to 1 the left-hand side monotonically decreases to $1 + \omega N$ while the right-hand side monotonically increases to infinity. Thus there is some $\underline{\beta} \in (0, 1)$ such that (2.5) holds for all $\beta \in (\underline{\beta}, 1)$.

For histories after deviations the strategies are clearly optimal. Thus the above strategies constitute a perfect equilibrium if $\beta \in (\underline{\beta}, 1)$. \square

So far we have investigated one particular type of trigger strategies in which there is spillover across the two types of relationships. Of course, there are many other types of strategies without such a spillover. In particular, consider strategies in which misbehavior in one relationship only affects the actions of agents in that relationship and doesn't spillover to the other relationships. Specifically, suppose that workers will continue to work as long as the government doesn't default on the labor contract and that bankers will continue to lend as long as the government doesn't default on its debt contract. These nonspillover strategies can clearly support an equilibrium with workers working positive amounts but they can't support any positive borrowing for the same reasons as before.

III. Reputation Spillover Across Relationships in a Finite Horizon

We consider now an incomplete information reputation model. It is easy to show that if there is a single debt relationship or a spillover to another debt relationship then reputation can support, effectively, zero debt. (For details see the working paper version of this paper.) More interesting, for our application to sovereign debt is showing that even with a finitely-lived government, spillover to a labor relationship can support debt. To do so we add incomplete information to a finite-horizon version of the model with debt and labor relationships. We will compute the unique equilibrium using standard backward induction arguments. We then show that the natural limit of both the equilibrium strategies and the equilibrium outcomes are the spillover strategies used in Proposition 2. We thus think of this model as one motivation for why it may be natural for such spillovers to occur.

Consider the reputation model. It has two types of government: a normal government and an honest government. The *normal* government is risk-neutral and discounts the future at rate β . The *honest* government evaluates consumption streams the same way the normal government does, but the honest government also assigns a large disutility to breaking any contract it has signed. In particular, we can write the preferences of the honest government as

$$\sum_{t=0}^T \beta [c_t - (1-z_t^b)M^b - (1-z_t^a)M^a]$$

where M^b and M^a are large positive numbers. (Recall that $z_t^b = 1$ and $z_t^a = 1$ correspond to repayment of the debt and labor projects and $z_t^b = 0$ and $z_t^a = 0$ correspond to default on these projects.) The type of the government is private information. Bankers hold subjective beliefs about what type the government is, and they update these beliefs after seeing the actions of the government.

We will consider a Bayesian equilibrium for this model. It should be clear that in any such equilibrium the honest government will honor all debt and labor contracts. Thus, if a private agent, either a banker or a worker, sees the government break either type of contract, the agent will know for sure that the government is not honest. A simple backward induction argument implies that workers will never work for, or bankers lend to, a government that they know is not honest. Hence, the normal government will either honor both contracts or break both, since breaking either one causes the government to lose its reputation. Thus, the reputation from the trust involved in the debt relationship spills over to the trust involved in the labor relationship and vice versa.

One way to analyze the model's implications for government borrowing is to compute the equilibrium directly by backward induction. However, we find it more convenient to develop some simple properties of the equilibrium indirectly. To do that, we first backtrack a little: we consider a model that has only a labor project and develop an expression for the value of reputation in the labor relationship alone. We then consider the model with both the debt and labor relationships.

We use the result that breaking trust in the debt relationship spills over into the labor relationship to show how the full commitment level of debt can be supported.

Consider the model with $x_t = b_t = 0$ for all t . In the Bayesian equilibrium we consider the workers' beliefs about the government are summarized by a conditional probability that the government is honest. We denote this conditional probability at t by p_t and call it the *reputation* of the government. In this equilibrium, the history of past events at the beginning of period t is summarized by the conditional probability p_t . The strategies of agents are allowed to depend on p_t together with the actions that have already occurred within the period. Thus, the work decision of a worker can be written as $n_t(i, w_t, p_t)$. The model's interesting equilibria turn out to involve the government randomizing, or *mixing*, over whether to repay or not. Let σ_t denote the probability that the government repays. The equilibrium we compute has two requirements: At every possible state, each agent acts optimally given the strategies and beliefs of other agents, and beliefs are updated according to Bayes' rule wherever possible. Since the honest government never defaults, Bayes' rule implies that the probability that the government is honest at $t + 1$ conditional on repayment at t (and all previous periods) is

$$(3.1) \quad p_{t+1} = \frac{p_t}{p_t + (1-p_t)\sigma_t}.$$

In each period t , each of the N workers will supply one unit of labor if the expected wage is greater than or equal to ω . If the honest government honors its contract and the normal government repays with probability σ_t , then at an offered wage of

$$(3.2) \quad w_t = \frac{\omega}{p_t + (1-p_t)\sigma_t}$$

the workers will receive an expected wage of ω . Hiring workers will only be profitable for the honest government if the wage rate is less than the return on the project, that is,

$$(3.3) \quad w_t \leq A.$$

Backward induction in this model works exactly like it does in the standard Kreps-Wilson (1982) and Milgrom-Roberts (1982) models of the chain store paradox. Solving backward until period zero gives this value function:

$$(3.4) \quad V_0(p_0, T) = \left\{ \begin{array}{ll} \sum_{s=0}^{T-1} \beta^s (A - \omega)N + \beta^T AN & \text{if } \omega/A \leq p_0 \leq 1 \\ \sum_{s=0}^{T-2} \beta^s (A - \omega)N + \beta^{T-1} AN & \text{if } (\omega/A)^2 \leq p_0 < \omega/A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ (A - \omega)N + \beta AN & \text{if } (\omega/A)^T \leq p_0 < (\omega/A)^{T-1} \\ AN & \text{if } (\omega/A)^{T+1} \leq p_0 < (\omega/A)^T \\ 0 & \text{if } p_0 < (\omega/A)^{T+1} \end{array} \right\}.$$

The value function corresponds to the following equilibrium behavior. In the lowest region, defined by $p_0 \leq (\omega/A)^{T+1}$, there is no mixing probability that is both high enough to give a wage $w_0 < A$ and low enough to push up next period's prior into the next highest region. Hence, workers do not work at period zero. Workers do not revise their priors, and then $p_1 = p_0$ and the workers do not work at period 1, and so on. The value of utility is thus zero. In the next highest region, with $(\omega/A)^{T+1} \leq p_0 < (\omega/A)^T$, workers work and the government is just indifferent between currently defaulting and repaying. Thus, the value of utility is AN . In the next region, with $(\omega/A)^T \leq p_0 < (\omega/A)^{T-1}$, the government strictly prefers to repay this period and will start mixing the next period.

Thus, today it gets $(A-\omega)N$ units of consumption, from tomorrow on the value of its utility is AN , and its total utility is $(A-\omega)N + \beta AN$. For the next highest region, the government strictly prefers to repay for two periods and then starts mixing, which gives it a total utility of $(A-\omega)N + \beta(A-\omega)N + \beta^2 AN$, and so on.

Consider now the equilibrium behavior for a fixed prior p_0 as the time horizon lengthens. Suppose p_0 is such that

$$(3.5) \quad (\omega/A)^{k+1} \leq p_0 < (\omega/A)^k.$$

Then the government will honor its contracts for sure in the first $T - k$ periods. In periods $T - k + 1$ through period $T - 1$, it will mix, and in period T , it will default. As T gets longer, so does the length of time that the government repays for sure, and in the limit the government never defaults.

There is a feature of this equilibrium which we will use later. Given any initial prior $p_0 > 0$, let $k(p_0)$ be the integer such that (3.5) holds. Then in period $T - k(p_0)$ the government is just indifferent between defaulting and repaying, the value of either is AN . One period before this at $T - k(p_0) - 1$ the value of repaying is $(A-\omega)N + \beta AN$ or

$$(3.6) \quad AN + (\beta A - \omega)N.$$

By assumption $\beta A > 1 > \omega$, so that in moving back one period the value of repaying strictly increases while the value of defaulting remains fixed at AN . Thus there is a strictly positive gap of $(\beta A - \omega)N$ between these values. Continuing in this manner it is clear that from (3.4) that j periods before $T - k(p_0)$ the value of repaying can be rewritten as

$$(3.7) \quad AN + \frac{(1-\beta^j)}{1-\beta} (\beta A - \omega)N$$

while the value of defaulting is simply AN . Thus the gap between the value of repaying and the value of defaulting monotonically increases as we move back in time from $T - k(p_0)$.

We will construct borrowing, investing, and repaying allocations which coincide with the full commitment levels for all but a fixed finite number of periods at the end of the game. Given some k' let these allocations specify: follow the full commitment allocations before $T - k'$, at $T - k'$ switch to self-financing the investment project from then on. More precisely, given k' let these allocations specify borrow 1, invest 1, and repay ρ for $t < T - k'$. For $t = T - k'$ borrow 0, invest 1, and repay ρ . For $t > T - k'$ borrow 0, invest 1, repay 0 and save 1.

The question of interest is for what k' can these allocations be supported in the reputation model with spillovers. To compute this number ignore spillovers for a moment and consider only the direct costs and benefits of defaulting on the debt. At any date $t \leq T - k'$ the net benefits of defaulting on the debt are simply the saving of funds used to repay the loan, namely ρ . For $t > T - k'$ there are no loans so the net benefits of defaulting are zero. Now add back in the spillover effects. For some given prior p_0 if we set $k' = k(p_0) + 1$, the difference between the value of honoring both debt and labor contracts and defaulting at date $T - k'$ is $(\beta A - \omega)N - \rho$. At any date before this the net benefits to defaulting on the debt are the same, namely ρ , while the gap between repaying and defaulting on the labor project increases. At any date after this there is no debt so the net benefits of defaulting on it are obviously zero. Thus if

$$(3.8) \quad \rho < (\beta A - \omega)N$$

then it is possible to support the full commitment allocations from date 0 through date $T - k'$ with $k' = k(p_0) + 1$. More generally for any given β , let $j(\beta)$ be the first integer such that the gap between the net benefits of defaulting on the debt are smaller than the net gains from repaying the workers, namely

$$(3.9) \quad \rho < \frac{(1-\beta^j)}{1-\beta} (\beta A - \omega) N.$$

If we let $k' = k(p_0) + j(\beta)$ then using the same reasoning as before we can support the full commitment allocations up to period $T - k'$.

Next consider the strategies in the equilibrium. For computational reasons we have found it convenient to express these strategies as functions of the state variable p_t . Of course, the more general way of defining strategies is to define them as functions of the entire history of past actions of the players. It is obvious that for $t < T - k'$ the strategies of the normal government, the bankers, and the workers are simply the truncation of the spillover strategies. In particular, if the government has never defaulted on either contract, the lenders and workers continue with their full commitment actions while if it has defaulted bankers lend and the workers do not work. (Of course, for $t \geq T - k'$ these strategies do not coincide with the truncation of the infinite horizon spillover strategies and moreover they are somewhat complicated to write down as functions of the history.)

We summarize our discussion with

Proposition 3. There exists a $\underline{\beta} \in (0,1)$ such that for all $\beta \in (\underline{\beta},1)$ the equilibrium allocations coincide with those under full commitment from period 0 up through period $T - k'$ where $k' = k(p_0) + j(\beta)$. Moreover, during these same periods the strategies of the normal government, the bankers, and the workers coincide with the truncation of the infinite spillover strategies of Proposition 2.

Now there is a natural sense in which both the equilibrium allocations and strategies converge to those of Proposition 2. We have

Corollary 1. Consider a sequence of economies in which T converges to infinity. Given the $\underline{\beta}$ of Proposition 3, for any $\beta \in (\underline{\beta},1)$ the equilibrium allocations and strategies converge to those of

Proposition 2. Moreover, these limiting strategies are themselves an equilibrium of the natural infinite horizon game.

To be precise the infinite horizon game of Section 2 must be trivially modified to include the honest government. Clearly the limiting strategies for the normal government, the lenders and workers namely, the spillover strategies, plus the limiting strategies of the honest government constitute an equilibrium of this modified infinite horizon game. Now the limiting economy discussed above is an infinite horizon incomplete information economy with probability p_0 of an honest government. In such a limit economy there are, of course, a large set of other trigger-type equilibrium. Here we have shown that the natural limit of our finite horizon economy goes smoothly to one member of this set, namely the one with spillover strategies.

Finally, we consider a sequence of economies in which the amount of incomplete information is converging to zero as the horizon length grows. To that end consider the following:

Condition 1. The sequence of priors $p(T)$ satisfy

$$(3.11) \quad (\rho/A)^{k(T)+1} \leq p_0(T) < (\rho/A)^{k(T)},$$

where (i) $k(T)$ converges monotonically to infinity as T does and (ii) $T - k(T)$ converges to infinity. Since $\rho/A < 1$ clearly $p_0(T)$ monotonically converges to zero as T converges to infinity. Now consider a sequence of economies indexed by T and $p_0(T)$. Along such a sequence the importance of the honest government is monotonically declining to zero. Notice that in a sense the prior is shrinking to zero more slowly than the horizon length is growing to infinity.

Consider a sequence of economies indexed by T and $p_0(T)$ where $p_0(T)$ satisfies Condition 1. Let $\underline{\beta}$ be such that (3.10) is satisfied for some finite $j(\underline{\beta})$. In this setup the sequence of finite horizon economies with incomplete information converge in a natural sense (in the space of economies) to

the infinite horizon economy with complete information. Using (3.4) and the above analysis one can show that the equilibrium value of utility converges to the full commitment level. Moreover, there is a natural sense in which the equilibrium strategies converge to the infinite horizon spillover strategies. Finally these limiting strategies themselves constitute an equilibrium of the limiting economy, namely the infinite horizon economy with complete information.

IV. Scope and Limitations of the Analysis

The main point of the paper is to show that if a government always has an adequate means to save, reputation can support debt only if there is reputation spillover to a nondebt relationship with enduring benefits. We set up the model to capture the idea that an agent's behavior in one type of relationship may provide information as to his behavior in other types of relationships. The model captures this idea in a stark fashion: if the government does not honor contracts in one relationship then all agents know for sure that it will not honor them in any other relationship.

Now if we amend the model so as to weaken the connection between behavior across relationships then the reputation spillover diminishes and so does the mutually reinforcing effects of different relationships. To see this suppose that we weaken the correlation of honest behavior across relationships by adding to the model types of governments that are honest in only the labor relationship or types that are honest in only the debt relationship. In such a model if a worker sees the government default on a debt he knows the government is not honest in the debt relationship but he doesn't know whether the government is normal or simply honest only in the labor relationship. If the prior on the type of government which is honest only in the labor relationship is high enough so that workers will work based on that prior alone, then the spillover across relationships disappears: a normal government would find it optimal to default, the lenders would realize this and not lend. Of course, if the prior on the honest-in-labor government were smaller than this critical

level but the sum of the priors on the honest-in-labor and honest-in-both were higher then there would be a spillover effect. This result is intuitive: the only way that misbehavior in a relationship with transient benefits can have large costs is for this misbehavior to have enough information content so that agents in relationships with enduring benefits are put off by it.

It is interesting to contrast how this model works when more types of agents are added with the standard reputation model of Kreps-Wilson and Milgrom-Roberts. In the chain store model if more types of incumbents are added in addition to the tough incumbent basically the same equilibrium goes through. Indeed Fudenberg and Levine (1989) show that regardless of the number and probabilities of types that are added, as long as the normal type and the tough type have strictly positive priors then any equilibrium has payoffs which are close, in the sense of average payoffs, to the full commitment payoffs. (For a detailed discussion of the general results in this area see Fudenberg and Levine 1989.)

Now in Section III we showed that adding another debt relationship did not lead to mutually reinforcing effects but that adding a labor relationship did. There is a connection between this type of result and the results in Fudenberg and Kreps (1987) who investigate various versions of a concession game. They show that multiple relationships may or may not be mutually reinforcing depending on what they assume about whether entrants that concede can later reenter. If such entrants can reenter then multiple relationships are mutually reinforcing while if they cannot then the relationships are not. In their model there is a sense in which if entrants can reenter then each relationship has enduring benefits while if they cannot then each relationship has transient benefits.

We turn next to our notions of enduring and transient benefits. In the infinite horizon complete information game with a single debt relationship the value of the relationship depended on the value of outstanding debt. Along any outcome path once the government received the largest loan in present value terms it had no need for the lenders. We termed such a relationship one with

transient benefits basically because its benefits only lasted up until the point at which the government received all it wanted from the lenders. In the model this occurred within finite time. In contrast, in the infinite horizon games with relationships with a repeated relationship like the labor relationship, no matter what happened in the past, the government never receives all its wants from the workers, instead it always needs them in the future. We termed this type of relationship one with enduring benefits because the value of the relationship stays high forever.

In finite horizon models all benefits necessarily terminate at the end of the horizon. With complete information this termination unwinds all equilibria other than autarky. With incomplete information, however, it does not and a more subtle distinction between debt-type relationships and labor-type relationships must be made. In a repeated relationship, like the labor-relationship the value of maintaining a good reputation in the relationship in any one period is constant, regardless of the history of play. Thus as we move back from the end of the game the value of the relationship grows by a constant amount, in discounted terms, every time a period is added. After enough periods the value of the relationship outweighs the benefits of default and good outcomes can be supported. In the debt relationship the value of maintaining it depends crucially on the level of the state variables, the new output. Bankers realize that for the normal government as the size of the new output approaches one the value of the relationship approaches zero. Because of this bankers lend very little in the beginning and let this value grow slowly over time only as they become more convinced that the government is honest. This allows the government to establish a reputation without giving it enough funds so that it can become self-sufficient. For low values of the prior this means that the government doesn't do much better than in autarky. (For details see Cole and Kehoe 1991.)

V. Conclusion

We have shown that if a government always has access to a means of earning the market rate of return on its savings, debt relationships have only transient benefits. Because of this, reputation spillover across a sequence of bankers in a single long-lived debt relationship is not sufficient to support much borrowing. Moreover, even with multiple long-lived debt relationships and spillover across them, reputation cannot support much borrowing. What is needed to revive reputation models of debt is a spillover from one long-lived relationship to a very different type of long-lived relationship, namely, one with enduring benefits.

The basic idea that an agent's reputation in one relationship may spill over into other relationships is not new. However, in the literature, the spillover is such that actions of agents in one arena of behavior affect reputation in that arena only. For example, in the debt literature, if a government defaults, it ruins the government's reputation in the debt arena; in the industrial organization literature on entry deterrence, if an incumbent doesn't fight entry then it ruins the incumbent's reputation in the entry deterrence arena. Here we have shown that when spillovers stay within the debt arena, reputation cannot support lending. For that, a government's actions in the debt arena must spill over to a different type of arena, such as labor relationships. Viewed this way, the benefits of maintaining a good relationship in one arena cannot be calculated simply by looking at that arena alone. Instead, account must be taken of the ramifications in a variety of other arenas, which, at least on the surface, may not seem to be directly connected to the arena in which the misbehavior originates.

This basic idea can be applied in many contexts. It might explain why countries honor some commitments, like treaties, when a narrow cost/benefit analysis would recommend breaking them. Consider, for example, a fishing treaty between the United States and Canada. Suppose that at the time the treaty was signed it seemed like a good idea, but later developments reveal that the treaty is costing the United States a lot. Nonetheless, the United States might honor the treaty because

breaking it would damage its reputation with Canada in other relationships that involve trust. Moreover, breaking that treaty might cause a negative reputation spillover with, say, the Japanese in a completely different arena which involves a trust relationship, such as a mutual defense pact.

Finally, note that Bulow and Rogoff (1989b) themselves briefly mention an idea similar to ours. Nevertheless, their punchline is that lending must be supported by direct sanctions. Ours is quite different. We think the notion of reputation spillover is an important one and may well play a significant role in complex international environments with a variety of interconnected agents and relationships. If reputation spillovers do play such a role, they may well greatly lessen the need for direct sanctions.

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