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TAX ANALYSIS IN A REAL BUSINESS CYCLE MODEL: ON MEASURING HARBERGER TRIANGLES AND OKUN GAPS

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ABSTRACT

A tax distorted real business cycle model is parameterized, calibrated, and solved numerically in an attempt to measure the size of Harberger Triangles relative to Okun Gaps. In particular, the model constructed is used to study, quantitatively, the impact of various distortional government tax and subsidy schemes. It is shown that the government can use tax policy to stabilize cyclical fluctuations, and this is done for the economy being studied. The benefits of implementing such a stabilization policy are calculated and compared with the size of the welfare gains realized from reducing various tax distortions.

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"It takes a heap of Harberger Triangles to fill an Okun Gap."

Tobin (1977, p. 468)

"Our task as I see it . . . is to write a FORTRAN program that will accept specific economic policy rules as 'input' and will generate as 'output' statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies."

Lucas (1980, p. 709–710)

I. INTRODUCTION

In a set of important papers, Kydland and Prescott (1982) and Long and Plosser (1983) operationalized the neoclassical growth model for the study of business cycle phenomena. The prototypical dynamic stochastic computational general equilibrium models constructed by these researchers were, in various forms, able to capture many features of U.S. business cycles remarkably well, such as the postwar correlation structure between output, consumption, investment, and productivity. The subsequent research spawned by this work has also met with much success. For instance Hansen (1985), simulating a version of the Kydland and Prescott (1982) model with indivisible labor, found that it could mimic quite well some U.S. labor market stylized facts, particularly the large fluctuations in hours worked relative to productivity. It is easy to incorporate growth into the framework, as has been illustrated by King, Plosser, and Rebelo (1988). Greenwood, Hercowitz, and Huffman (1988) present evidence that it may be fruitful to incorporate shocks to the marginal efficiency of newly produced capital into the basic paradigm and allow for an endogenous utilization rate on previously installed capital. Finally, this modeling strategy can be employed to address questions concerning open economies. Mendoza (1989) simulates an open economy variant of the neoclassical growth model and finds that it can duplicate Canadian trade balance statistics well.

The next step in the development of this research strategy is the employment of these (real business cycle) models for policy analysis. An obvious place to start is the area of public finance. There is a voluminous public finance literature that studies the *quantitative* effects of distortional taxation. This work has led to important insights into the size of welfare losses associated with distortional taxation. For instance, Stuart (1981) illustrates the tremendous disincentive effect that income taxation has had on Swedish labor supply. Most of this type of work, though, has been undertaken within the context of deterministic static models. There are, however, issues best handled within dynamic settings. An example of such an issue would be the impact of an investment tax credit. At the heart of investment decision–making is an intertemporal tradeoff—the sacrifice of consumption today for consumption tomorrow—from which it is difficult to abstract while retaining the essence of the problem. The use of dynamic perfect foresight models to address such issues has recently been advanced in pioneering work by Auerbach, Kotlikoff, and Skinner (1983), Jorgenson and Yun (1986), and Judd (1987). In particular, Judd (1987) examines the quantitative effects of capital income taxation and investment tax subsidies. Here a useful technique is proposed for examining the effects of marginal tax changes via a linearization of the dynamic economy around its steady–state. This work builds on earlier theoretical research by Brock and Turnovsky (1981). Auerbach, Kotlikoff, and Skinner (1983) investigate, using a variation of the multiple–shooting simulation technique, the dynamic effects of a tax reform that switches from income to consumption or wage taxation. Finally, similar methods are used by Jorgenson and Yun (1986) to study the bias in the U.S. tax system favoring the accumulation of household capital at the expense of business capital.

There are plenty of other economic issues that require the use of dynamic stochastic models. For example, it may be desirable to know how a proposed fiscal program may impact on the variability of macroaggregates over the course of the business cycle, or for that matter whether the business cycle could be stabilized by some proposed fiscal program. Dynamic stochastic computational general equilibrium models to date do not seem to have been used extensively in the field of public finance. The task set for this paper will be to analyze the cost of distortional fiscal policies relative to the cost of fluctuations in aggregates over the business cycle. That is, it attempts to measure the size of Harberger Triangles relative to Okun Gaps. In a sense then, this study blends ingredients from the real business cycle and public finance literatures.[1] Motivation for this work was provided by Lucas (1987) where some measurements of the costs of fluctuations in consumption over the business cycle are presented. Also, the paper is complementary to Cooley and Hansen's (1989) analysis in a dynamic cash–in–advance economy of the cost of the inflation tax.

The remainder of this paper is organized as follows: In the next section a tax distorted real business cycle model is constructed. In Section III this economy is calibrated and parameterized so that it roughly mimics the salient features of the business cycle displayed by the post war U.S. economy. The welfare costs of distortional taxation for the model economy are calculated in Section IV. The tax and subsidy rates are varied to obtain a feel for how the dynamic stochastic properties of the economy change when government policies are altered. In Section V a tax program that stabilizes output for the model economy is implemented, and the welfare gains (costs) from undertaking such a policy are evaluated. Some final remarks are made in Section VI.

II. THE ECONOMY

Consider an economy inhabited by a representative household, a firm, and a government. The firm produces output using factor inputs, namely capital and labor services, hired from the household. The household in turn uses the income derived from supplying these services to purchase either consumption or investment goods from the firm. Last, a set of distortional taxes is levied on private agents by the government with the tax revenue raised being rebated to the private sector via lump–sum transfer payments.

A. Firms

Output in any given period t, or y_t, is governed by the following constant–returns–to–scale production function

$$y_t = f(k_t h_t, \ell_t), \tag{1}$$

where $k_t h_t$ represents the input of capital *services* in this period and ℓ_t is period–t labor input. Here k_t represents the capital stock and h_t the rate at which it is utilized. Given a rental rate for capital services, r_t, and a wage rate for labor, w_t, the firm chooses $k_t h_t$ and ℓ_t so as to maximize profits, π_t. Specifically, the firm solves the following problem:

$$\max_{k_t h_t, \ell_t} \quad \pi_t = f(k_t h_t, \ell_t) - r_t k_t h_t - w_t \ell_t, \tag{P1}$$

which has as first–order conditions

$$f_1(k_t h_t, \ell_t) = r_t \tag{2}$$

$$f_2(k_t h_t, \ell_t) = w_t. \tag{3}$$

Firms will make zero profits in each period t due to the constant–returns–to–scale assumption; in other words $\pi_t = 0$ for all t.

B. Households

The representative household's goal in life is to maximize his expected lifetime utility as given by

$$\underset{0}{\mathrm{E}}\left[\sum_{t=0}^{\infty} \beta^{t}\, \mathrm{U}(c_t,\ell_t)\right] \qquad 0 < \beta < 1, \tag{4}$$

where c_t and ℓ_t represent his period–t consumption and labor supply. The momentary utility function $U(\cdot)$ is assumed to have the usual properties. The household has three primary sources of income: first, the income derived from selling capital services, $r_t k_t h_t$, second, labor income, $w_t \ell_t$, and third, the lump–sum transfer payment, τ_t, it receives from the government. Capital and labor income are taxed at the rates λ_k and λ_ℓ, respectively.

The household faces a nontrivial decision concerning the supply of capital services. Specifically, a given stock of capital, k_t, can be utilized at a variable rate, h_t. On the one hand a higher level of utilization, h_t, allows for a higher level of capital services, $k_t h_t$, to be obtained from a given stock of capital, k_t, while on the other hand it causes the capital stock to depreciate at a higher rate of δ_t given by $\delta_t = \delta(h_t)$, with the depreciation rate function satisfying $0 \leq \delta \leq 1$, $\delta' > 0$ and $\delta'' > 0$.

The household can either consume or save its after–tax income. Savings is undertaken in the form of physical capital accumulation. The evolution of the household's capital stock is described by

$$k_{t+1} = k_t(1 - \delta(h_t)) + i_t(1+\epsilon_t). \tag{5}$$

Here i_t units of output invested in period t increases the period–t+1 capital stock by $i_t(1+\epsilon_t)$ units, where $\epsilon_t \in E \subset (-1,\infty)$ is a disturbance term (known at time t) drawn from the distribution function $\Phi(\epsilon_t \mid \epsilon_{t-1})$ defined on $E \times E$. The shock ϵ_t operates as technological shift factor affecting the efficiency of "newly" produced capital. As is discussed in Greenwood, Hercowitz and Huffman (1988), a high realization of ϵ_t operates to stimulate the formulation of "new" capital and promote the more intensive utilization and accelerated depreciation of "old" capital. *Gross* investment, i_t, is subsidized by the government at the rate λ_i.

The representative household's dynamic programming problem can now be cast. Formally, the problem is

$$V(k_t;s_t) = \max_{(c_t,k_{t+1},h_t,\ell_t)} \left\{U(c_t,\ell_t) + \beta \int V(k_{t+1};s_{t+1})\, d\Psi(s_{t+1}|s_t)\right\} \tag{P2}$$

subject to

$$c_t + (1-\lambda_i)k_{t+1}/(1+\epsilon_t) = (1-\lambda_k)\, r_t k_t h_t + (1-\lambda_\ell)w_t\ell_t + (1-\lambda_i)[1-\delta(h_t)]k_t/(1+\epsilon_t) + \tau_t, \tag{6}$$

with the state vector s_t being governed by distribution function $\Psi(s_t|s_{t-1})$. Needless to say the household takes the aggregate state-of-the-world, s_t, as given. Additionally, in equilibrium the wage and rental rates, w_t and r_t, and the level of transfer payments from the government, τ_t, will all be functions of s_t. A precise definition of s_t will be deferred until the model's general equilibrium is discussed. The upshot of the above optimization problem is summarized by the following set of efficiency conditions — in addition to the budget constraint (6):

$$\frac{(1-\lambda_i)U_1(c_t,\ell_t)}{(1+\epsilon_t)} = \beta \int U_1(c_{t+1},\ell_{t+1})\Big[(1-\lambda_k)r_{t+1}h_{t+1} + \frac{(1-\lambda_i)(1-\delta(h_{t+1}))}{(1+\epsilon_{t+1})}\Big] d\Psi(s_{t+1}|s_t) \tag{7}$$

$$(1-\lambda_k)r_t = (1-\lambda_i)\delta'(h_t)/(1+\epsilon_t) \tag{8}$$

$$U_1(c_t,\ell_t)(1-\lambda_\ell)w_t = -U_2(c_t,\ell_t). \tag{9}$$

The first equation (7) is a standard optimality condition governing investment. The left–hand side of this equation represents the loss in current utility which is realized when an extra unit of current investment is undertaken. Note that an efficiency unit of period–t+1 capital costs $1/(1+\epsilon_t)$ units of current output to produce. Given the investment tax credit, the cost to private agents of acquiring an extra unit of period–t+1 capital is only $(1-\lambda_i)/(1+\epsilon_t)$. The right–hand side portrays the discounted expected future utility obtained from an extra unit of investment today. Observe that the term in brackets represents the realized marginal after–tax return on investment. This realized return has two components. The first, $(1-\lambda_k)r_{t+1}h_{t+1}$, is the after–tax return earned on capital from production in period t+1. The second is the undepreciated portion of the capital stock $(1-\delta(h_{t+1}))$, which has a resale value then of $(1-\lambda_i)(1-\delta(h_{t+1}))/(1+\epsilon_{t+1})$.

The next equation (8) characterizes efficient capital utilization. It states capital should be utilized at the level h_t which sets the marginal benefit of capital services equal to the marginal user cost. The marginal user cost of capital is made up of two components. Specifically, $\delta'(h_t)$, represents the marginal cost in terms of increased depreciation from utilizing capital at a higher level, while $(1-\lambda_i)/(1+\epsilon_t)$ is the current replacement cost of old in terms of new capital. Finally, equation (9) sets the after–tax marginal benefit from working equal to the marginal disutility of labor.

C. Government

The government, like any other entity in the economy, must satisfy a budget constraint. Since the focus of the following analysis will be on the impact of distortional taxes, it will be assumed that the revenue collected by the government is rebated back to agents in a lump–sum manner. Thus, the analysis abstracts away from the wealth effects associated with using distortional taxation to finance government spending on goods and services. Specifically, in equilibrium the government's lump–sum transfer payments will be equal to

$$\tau_t = \lambda_k r_t K_t H_t + \lambda_\ell w_t L_t - \lambda_i [K_{t+1} - (1-\delta(H_t))K_t]/(1+\epsilon_t), \tag{10}$$

where upper case letters denote the *equilibrium* or *aggregate* values of the household's decision–variables.

D. Competitive Equilibrium

The above description of the economy under study is now completed with the following definition of a competitive equilibrium where the aggregate state–of–the–world, s_t, is now defined to be given by the vector $s_t = (K_t, \epsilon_t)$.

Definition: A competitive equilibrium is a set of laws of motion, $K_{t+1} = K(K_t,\epsilon_t)$, $H_t = H(K_t,\epsilon_t)$, $L_t = L(K_t,\epsilon_t)$, a set of pricing and transfer functions, $r_t = r(K_t,\epsilon_t)$, $w_t = w(K_t,\epsilon_t)$, $\tau_t = \tau(K_t,\epsilon_t)$, and a probability distribution function, $\psi(K_{t+1},\epsilon_{t+1}|K_t,\epsilon_t)$, such that[2]

(i) Firms solve problem (P1), given $r(K_t,\epsilon_t)$ and $w(K_t,\epsilon_t)$, with the solution to this problem having the form $k_t h_t = K_t H(K_t,\epsilon_t)$ and $\ell_t = L(K_t,\epsilon_t)$.

(ii) Households solve (P2), given $r(K_t,\epsilon_t)$, $w(K_t,\epsilon_t)$, $\tau(K_t,\epsilon_t)$, and the probability distribution function $\psi(K_{t+1},\epsilon_{t+1}|K_t,\epsilon_t)$, with the solution to this problem having the form $k_{t+1} = K(K_t,\epsilon_t)$, $h_t = H(K_t\epsilon_t)$ and $\ell_t = L(K_t,\epsilon_t)$.

(iii) The government's budget constraint (10) holds.

(iv) The distribution function $\psi(K_{t+1},\epsilon_{t+1}|K_t,\epsilon_t)$ governing the evolution of the aggregate state–of–the–world is consistent with the distribution function $\Phi(\epsilon_{t+1}|\epsilon_t)$ and the law of motion $K_{t+1} = K(K_t,\epsilon_t)$.

Specifically,

$$\psi(K_{t+1},\epsilon_{t+1}|K_t,\epsilon_t) = \text{prob}\,\{K(K,\epsilon) \leq K_{t+1}, \epsilon' \leq \epsilon_{t+1}|K=K_t,\epsilon=\epsilon_t\}$$

$$= I(K_{t+1}-K(K_t,\epsilon_t))\int^{\epsilon_{t+1}}\Phi(d\epsilon'|\epsilon_t),$$

where $I(x) = 1$ if $x \geq 0$ and $I(x) = 0$ otherwise.

Finally, observe that in equilibrium the economy's resource constraint will always hold, a fact readily verified by substituting (10) into (6)—while applying (2) and (3)—to obtain (11) below

$$C_t + \frac{K_{t+1}}{(1+\epsilon_t)} = f(K_t H_t, L_t) + \frac{(1-\delta(H_t))K_t}{(1+\epsilon_t)}. \tag{11}$$

III. MODEL PARAMETERIZATION AND CALIBRATION

A quantitative analysis of the macroeconomic ramifications and welfare costs of distortional taxation and business cycle stabilization will now be undertaken. To undertake such an investigation, both tastes and technology must suitably parameterized. To this end, let

$$U(c,\ell) = \frac{1}{1-\gamma}\left[\left[c^{1-\theta}(1-\ell)^{\theta}\right]^{1-\gamma} - 1\right],$$

$$f(kh,\ell) = (kh)^{\alpha}\,\ell^{1-\alpha},$$

and

$$\delta(h) = \frac{1}{\omega}h^{\omega},$$

where $\gamma > 0$, $0 < \alpha, \theta < 1$, and $\omega > 1$.

The technology shock is assumed to follow a two–state Markov process. Specifically,

$$\epsilon_t \in E \equiv \{e^{\xi_1} - 1,\ \epsilon^{\xi_2} - 1\},$$

with

$$\text{prob}\,[\epsilon_{t+1} = e^{\xi_s} - 1 \mid \epsilon_t = e^{\xi_r} - 1] \equiv \pi_{rs},$$

and where $0 \leq \pi_{rs} \leq 1$, $\pi_{r1} + \pi_{r2} = 1$ and r, s = 1,2. It is additionally assumed that $\pi_{11} = \pi_{22} = \pi$ and $\xi_1 = -\xi_2 = \xi > 0$. Given this representation, it is easy to show that the technology shock's stochastic structure is conveniently summarized by the standard deviation, σ, and autocorrelation coefficient, ρ, of the random variable $\ln(1 + \epsilon)$; in particular $\sigma = \xi$ and $\rho = 2\pi - 1$.

In order to simulate the model values must be assigned to the parameters shown below:

Utility: β, γ, θ

Technology: α, ω, ρ, σ

Taxes: λ_k, λ_ℓ, λ_i.

So as to impose some discipline on the simulation experiments being conducted, the calibration procedure advanced by Kydland and Prescott (1982) is adopted. In line with this approach, as many model parameter values as possible are set in advance based upon one of the following: (a) a priori information about their magnitudes, (b) so that in the model's deterministic steady–state the values for various endogenous variables assume their average values for the postwar U.S. economy based upon annual data for the 1948–1985 sample period, or (c) so that population second moments for various variables in the model's stochastic steady–state match the corresponding sample moments from the U.S. data.

To begin with, since the length of a period in the model is taken to be one year, β, the discount factor, is set to 0.96. Next, capital's share of income or α was chosen to be 0.29, its average annual value over the 1948–1985 period. The capital and labor income tax rates, λ_k, and λ_ℓ, were both initially set at 0.35. This is roughly in the range of rates quoted by Auerbach, Kotlikoff and Skinner (1983, p. 97). The investment subsidy has varied a great deal in the post war era. It has ranged from 0 percent to 10 percent after 1975 [see Fullerton and Gordon (1983), p. 384]. A value of 0.07 was picked as the benchmark for λ_i.

Next, the parameters θ and ω were chosen so that the model's deterministic steady–state satisfies two restrictions. The first constrains the ratio of working to total hours to be 0.26. This number corresponds to the average ratio of total hours worked to total nonsleeping hours of the working age population observed in the U.S. data. The second sets the steady–state depreciation rate to be 0.10, the value used by Kydland and Prescott (1982). The implied values for θ and ω are 0.61 and 1.42, respectively.[3]

Finally, the parameters σ and ρ were picked so that the model exhibits the same percentage standard deviation and first–order serial correlation for output as is observed in the data. This necessitated picking $\sigma = 0.058$ and $\rho = 0.60$. One parameter remains to be specified, the coefficient of relative risk aversion of γ. The value of this parameter is somewhat controversial with estimates ranging from 1 [Kydland and Prescott (1982)] to infinity [Hall (1988)]. Here γ was assigned a value of 15, which leads to the correlation between consumption and output in the model being roughly in line with the stylized facts for the postwar U.S. economy. Note that this relatively high value for the coefficient of relative aversion (or low one for the intertemporal elasticity of substitution) tends to bias the measurement in favor of finding large Okun Gaps and small Harberger Triangles.

IV. WELFARE COSTS OF DISTORTIONAL TAXATION

In this section an analysis of the impact of taxes on economic aggregates is conducted. The economy specified in the previous section will operate as a benchmark from which the properties of economies with different tax structures will be gauged. Thus, the behavior of otherwise identical economies with different tax structures will be compared. No attempt is made to characterize the path that the economy would follow in leaving one stochastic steady–state to move to another. The experiments performed are intended to serve only as exercises that are illustrative of the effect of changes in tax parameters.

Panel II of Table 1 summarizes the dynamic behavior of the benchmark model. A discussion of the discrete state space method used to solve numerically the model is contained in the Appendix. The corresponding statistics from the U.S. data for the 1948 to 1985 sample period are shown in Panel I of the table, these numbers being taken from Greenwood, Hercowitz, and Huffman (1988). The statistics generated by the model, shown in Table 1, are largely consistent with the corresponding statistics garnered from the data. For example, consumption is less volatile than total output whereas investment is much more so. No corresponding statistics for the capital stock are presented because the capital stock in the model is measured in efficiency units. No such measure is available for the data. It is interesting to note that the presence of taxes in the model tends to *amplify* rather than stabilize the variability and persistence effects of technological shocks. If all taxes and subsidies are set to zero, the variability of macroaggregates drops dramatically. For example, the percentage standard deviations of output, consumption and hours drop from 3.5, 1.7, and 2.0 to 2.9, 1.2, and 1.4, respectively. The autocorrelations coefficients for these variables fall from 0.66, 0.96, and 0.57 to 0.63, 0.92, and 0.57.

Table 2 presents comparisons of statistics from the benchmark economy with those from an otherwise identical economy where the tax (subsidy) parameters have been changed. Panel I reports statistics for the benchmark economy. Statistics for an economy which is identical to the benchmark except for the fact that the capital income tax is cut from 35 percent to 25 percent, are presented in Panel II. The average level of output increases 9 percent due to this policy with total hours rising by 2 percent. Productivity of labor increases by 7 percent and the average capital stock increases by 30 percent. As an illustration of the volatility induced by the tax system, the percentage standard deviations of the aggregates are all at least as large for Panel I as for Panel II. Additionally, this policy has the effect of increasing the correlation of investment with output and decreasing the correlation of consumption with output.

The effects on the economy of cutting the labor income tax rate from 35 percent to 25 percent, while holding all other tax (subsidy) rates at their benchmark levels are illustrated in Panel III. In this case output and hours increase by 10 percent. Average labor productivity actually declines slightly, but this is due to the fact that there is a very large increase in the quantity of labor employed. Again, Panel III illustrates that a cut in the labor income tax rate lowers the volatility of almost all the aggregates. This policy causes consumption to be more procyclical.

Panel IV shows the impact of increasing the investment tax credit from 7 percent to 14 percent. This raises the average level of output by 4 percent, with hours and productivity rising by 1 percent and 3 percent respectively. An increase (decrease) in the investment tax credit has the impact of raising (lowering) the variability of macroaggregates. This may seem somewhat surprising in light of the results obtained from the previous two experiments. These results are easily reconciled, though, by observing that the benchmark equilibrium capital and labor income taxation have depressing effects on capital accumulation and hours worked, while the investment tax credit stimulates them. Thus, a hike in the investment tax credit operates on macroaggregates in the same way as a cut in capital and labor income taxation do.

An attempt to measure the gain (or loss) in welfare from these policy changes is reported in the row at the bottom of Table 2, entitled "Welfare Gain." This measure is constructed as follows: Associated with the dynamic programming problem (P2) is a set of equilibrium *decision–rules* specifying the agent's consumption and labor effort for each state–of–the–world.[4] By using these decision–rules in conjunction with the economy's invariant probability distribution across states, the expected value of the agent's lifetime utility can be computed. This calculation can also be done for the other economies listed in Table 2, for which the taxes and subsidies are changed. For each of the experiments one can calculate the *constant* amount of consumption that could be given to or taken away from each agent in each state under the new policy regime that would leave the agent just as well off as in the benchmark economy. This constant amount of consumption is then divided by the average level of output under the new policy regime. The resulting statistic could be interpreted as the percentage change in the level of output under the new policy regime that would be needed to leave the agent just equally as well–off as in the benchmark economy.

Panel II shows that the cut in the capital income tax from 35 percent to 25 percent results in an improvement in welfare that is equivalent to a 4.05 percent increase in GNP. Similarly, a cut in the labor income tax results in a welfare gain of 3.10 percent. It is interesting to note that an increase in the investment tax credit improves welfare. This is an illustration of the theory of the second–best. Raising the investment tax credit works to counteract the depressing effects on capital accumulation that labor and capital income taxation have. Specifically, increasing the investment tax credit from 7 percent to 14 percent results in a welfare gain of 1.91 percent.

Table 2 helps one to obtain a handle on the welfare and quantitative impact of changing some observed distortional government policies. It would seem that the Harberger Triangles associated with the distortional taxation are quite large. For example, the cut in the capital income tax from 35 percent to 25 percent results in an increase in the measure of welfare by what amounts to 4.05 percent of total output. To get a grip on what this means in terms of current levels of output, this would amount to 212 billion current dollars of 1989 GNP![5]

V. WELFARE GAINS FROM BUSINESS CYCLE STABILIZATION

Few topics in macroeconomics have had as much attention devoted to them as business cycle stabilization. Two questions at the heart of the debate are: (1) Is it desirable to pursue business cycle stabilization policies and (2) is it feasible, both theoretically and practically speaking, to stabilize economic fluctuations? Advocates of business cycle stabilization feel that the potential benefits are large with Tobin (1977) asserting: "It takes a heap of Harberger Triangles to fill an Okun Gap." Opponents feel that either the potential benefits are small [Lucas (1987)] or that business cycle stabilization is either infeasible or impractical [Friedman (1953)]. The artificial economy developed here can be used to cast some further light on these issues.

To begin with, is business cycle stabilization feasible? The notion of business cycle stabilization must be formalized somehow, so suppose that the government desires to eliminate recessions from the economy, defined simply here as those realizations of income which fall below the mean level of output, $\overline{y}$, for the benchmark economy. How should the government do this? The standard prescription would be that the government should attempt to control the flow of economic activity at the point where maximum influence can be exerted on the production of output. A tax or subsidy on the firm's production of output satisfies this prescription.

Specifically, let a state–contingent subsidy be paid on firms' period–t production in the amount $\lambda_{st} = \lambda_s(K_t,\epsilon_t) \geq 0$ for state (K_t,ϵ_t).[6] The firm's problem now becomes [cf.(P1)]

$$\max_{k_t h_t, \ell_t} \quad \pi = (1+\lambda_{st})\, f(k_t h_t,\ell_t) - r_t k_t h_t - w_t \ell_t, \tag{P3}$$

with the associated first–order conditions

$$(1+\lambda_{st}) f_1(k_t h_t,\ell_t) = r_t \tag{12}$$

and

$$(1+\lambda_{st}) f_2(k_t h_t,\ell_t) = w_t. \tag{13}$$

By adding to these conditions the complementary slackness conditions (14) associated with the government's stabilization target one obtains a system of three equations in three unknowns determining solutions for $k_t h_t$, ℓ_t and λ_{st} as functions of r_t and w_t.

$$\lambda_{st}[f(k_t h_t \ell_t)-\bar{y}] = 0 \text{ with } \lambda_{st} \geq 0 \text{ and } f(k_t h_t,\ell_t) \geq \bar{y}. \tag{14}$$

Intuitively one would expect that the government's stabilization policy requires that the output subsidy move countercyclically in response to investment shocks so as to entice production in recessions. The *impact* effect of a high realization of ϵ_t operates to reduce the user cost of capital $r_t = [(1-\lambda_i)/(1-\lambda_k)]\delta'(H_t)/(1+\epsilon_t)$, because "old" capital can now be replaced with more efficient "new" capital. In response, the firm hires more capital services, ceteris paribus. Given this increase in the employment of capital services, less labor needs to be hired to meet the output target. The production subsidy is cut as a consequence. Formally, by performing the standard comparative statics exercise on (12), (13), and (14), it is easy to demonstrate that

$\frac{d\lambda_{st}}{dr_t} > 0$, $\frac{dk_t h_t}{dr_t} < 0$, and $\frac{d\ell_t}{dr_t} > 0$.

The definition of a competitive equilibrium with the stabilization scheme in place reads:

Definition: A competitive equilibrium is a set of laws of motion, $K_{t+1} = K(K_t,\epsilon_t)$, $H_t = H(K_t,\epsilon_t)$, $L_t = L(K_t,\epsilon_t)$, a set of pricing, tax, and transfer functions, $r_t = r(K_t,\epsilon_t)$, $w_t = w(K_t,\epsilon_t)$, $\lambda_{st} = \lambda_s(K_t,\epsilon_t)$, $\tau_t = \tau(K_t,\epsilon_t)$, and a probability distribution function $\psi(K_{t+1},\epsilon_{t+1}|K_t,\epsilon_t)$, such that[7]

(i) Firms solve problem (P3), given $r(K_t,\epsilon_t)$, $w(K_t,\epsilon_t)$ and $\lambda_s(K_t,\epsilon_t)$, with the solution to this problem having the form $k_t h_t = K_t H(K_t,\epsilon_t)$ and $\ell_t = L(K_t,\epsilon_t)$.

(ii) Households solve problem (P2), given $r(K_t,\epsilon_t)$, $w(K_t,\epsilon_t)$, $\tau(K_t,\epsilon_t)$, and the probability distribution function $\psi(K_{t+1},\epsilon_{t+1}|K_t,\epsilon_t)$, with the solution to this problem having the form $k_{t+1} = K(K_t,\epsilon_t)$, $h_t = H(K_t,\epsilon_t)$ and $\ell_t = L(K_t,\epsilon_t)$.

(iii) The stabilization conditions (14) and the government's budget constraint (15) hold.

$$\tau_t = \lambda_k r_t K_t H_t + \lambda_\ell w_t L_t - \lambda_i[K_{t+1}-(1-\delta(H_t))K_t]/(1+\epsilon_t) - \lambda_{st}F(K_t H_t,L_t). \tag{15}$$

(iv) The distribution function $\psi(K_{t+1},\epsilon_{t+1}|K_t,\epsilon_t)$ governing the evolution of the aggregate state-of-the-world is consistent with the distribution function $\Phi(\epsilon_{t+1}|\epsilon_t)$ and the law of motion $K_{t+1} = K(K_t,\epsilon_t)$.

Specifically,

$$\psi(K_{t+1},\epsilon_{t+1}|K_t,\epsilon_t) = \text{prob}\ \{K(K,\epsilon) \leq K_{t+1},\epsilon' \leq \epsilon_{t+1}|K=K_t,\epsilon=\epsilon_t\}$$

$$= I(K_{t+1}-K(K_t,\epsilon_t))\int^{\epsilon_{t+1}}\Phi(d\epsilon'|\epsilon_t),$$

where $I(x) = 1$ if $x \geq 0$ and $I(x) = 0$ otherwise.

As is well known, in a distortion–free competitive equilibrium such a stabilization scheme can only reduce welfare. Given the presence of distortional taxation, however, the competitive equilibrium modeled above is not distortion–free. As has been seen such taxation has a depressing effect on work effort, capital accumulation and output. These effects may be especially unwelcome during recessions. Eliminating recessions could operate to bring output closer on average to the ideal level that would prevail in a distortion–free environment. The results of this experiment are presented in Panel III of Table 1. Such a stabilization scheme operates to increase welfare by an equivalent of about 0.67 percent of aggregate output. If it is permissible to interpret this as an Okun Gap, then it seems small when compared with the Harberger Triangles for this economy. Such a stabilization policy also reduces the variance of output, consumption, investment, and hours, as can be seen by comparing column (1) in Panels II and III of Table 1. This latter fact is also reflected in Figure 1 which shows how the marginal distribution for the artificial economy's capital stock is condensed and skewed to the right by the stabilization scheme.[8] This transpires since the stabilization policy operates to induce more investment in those states where the technology shock is bad by making the private return on capital less procyclical. Figure 2 illustrates the state contingent schedule of subsidies needed to implement the stabilization program.[9] No output subsidies were needed when the technology was high (i.e., when $\epsilon = e^{\xi_1} - 1$). Thus the diagram portrays the required output subsidy in the low shock states as a decreasing convex function of the aggregate capital stock. The mean and standard deviation of the required output subsidy rate over the business cycle are 0.013 and 0.014 respectively. These numbers are small, but this shouldn't be all that surprising. Postwar fluctuations in U.S. output have in fact been small with the standard deviation of the logarithm of U.S. output being only 0.035. The required subsidy for the worst case scenario in the model ($K = K_1$, $\epsilon = e^{\xi_2} - 1$) is only 0.044 and the probability of this event occurring is infinitesimal. Also whether such plans would be practical in reality is questionable. If the execution of such plans requires that the government identify and adjust taxes quickly to the underlying state of nature, then achieving success could be problematic especially if mistakes are costly. Friedman (1953) argues against implementing discretionary stabilization exactly because of such practical considerations. Note that in the case under consideration taxes could not just be linked to output, say as under some traditional progressive income taxation scheme, because by design output remains constant over those states of the world where the stabilization scheme is in effect.

The benefits of business cycle stabilization reported here are somewhat large when compared with those that Lucas (1987) reports. (His number for a comparable economy would lie below 0.38% of GNP.) This is because business cycle stabilization operates here to ameliorate some of the deleterious effects of distortional taxation. In particular, it helps to counteract the depressing effects that distortional taxation has on work effort and capital accumulation. Unlike in Lucas (1987), business cycle stabilization raises the mean level of output. One might argue that the distortions should be attacked directly rather than indirectly through business cycle stabilization. This question rings just as loudly whether the source of the distortion is sticky prices, monopolistic competition, efficiency wages or distortional taxation to name a few possibilities. Clearly, any optimal monetary–cum–fiscal policy should be characterized by an optimal taxation program designed to simultaneously undertake government spending, raise revenue, and correct distortions in an efficient manner; in such a scheme there would be no role for business cycle stabilization, per se. The argument for business cycle stabilization must rest on improving on an initial set of non–optimal allocations in a world of imperfect government policymaking. If the notion of business cycle stabilization now seems somewhat imprecise and a little dubious, perhaps indeed it actually is so. The virtue (or some may say the vice) of the modeling approach adopted here is precisely that it forces the modeler to model explicitly the underlying distortion affecting the economy and explain why the proposed interventions dominate other schemes in the set of feasible government policies.

VI. CONCLUSION

A tax distorted real business cycle model was parameterized, calibrated and simulated in an attempt to measure the size of Harberger Triangle relative to Okun Gaps. It was shown that distortional taxes, similar to those observed in the U.S. economy, appear to exacerbate the volatility and serial correlation properties of macroaggregates, relative to what would happen if there were no such taxes. The model was also capable of analyzing how the average *levels* of the aggregates would change when the distortional taxes have changed. The experiments undertaken indicated that the "Harberger Triangles" associated with distortional taxation were of substantial size. Additionally, a set of state–contingent taxes was constructed which could stabilize output at any particular level. Such a policy of business cycle stabilization may or may not be welfare improving in general, but in the case of the model studied here it improved welfare. The benefits of eliminating "Okun Gaps", however, were small when compared with those obtained from reducing "Harberger Triangles".

Given that in the framework employed here there was considerable disparity between the allocations in the economy with distortional taxation and the optimal allocations, there would appear to be some latitude for the government to minimize this distance. Missing from the analysis was any attempt to characterize the costs or benefits that arise from government spending programs which, presumably, explain the need for taxation in the first place. Such spending can influence the production technology [as studied by Aschauer (1989)] or the utility function through government provided services [Aschauer (1985)]. A study that attempted to integrate the analysis of government spending with that of distortional taxation may well come to different conclusions, with regard to the welfare costs of government behavior, than does the present study. Furthermore, the conclusions obtained could be changed if factors concerning economic growth were introduced into the analysis. The welfare effects that tax distortions have in a stationary economy via their effects on the levels of various macroaggregates could be amplified if the analysis were conducted within a model that allowed government policy to affect the rate of economic growth. This observation brings to the fore the ultimate goal of analyses such as this. Ideally, one would like to calculate the properties of an optimal fiscal program designed to simultaneously provide government services, raise revenue and correct distortions in an efficient manner for a growing economy. This is a large and demanding task but progress, however slow, along this road is being made.

Table 1

	I Annual U.S. data 1948–1985			II Benchmark Model			III Benchmark Model —Stabilized		
Variables	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Output	3.5	0.66	1.00	3.5	0.66	1.00	1.7	0.65	1.00
Consumption	2.2	0.72	0.74	1.7	0.96	0.67	0.9	0.91	0.08
Investment	10.5	0.25	0.68	20.0	0.57	0.92	12.3	0.56	0.90
Hours	2.1	0.39	0.81	2.0	0.57	0.88	0.7	0.74	0.31
Productivity	2.2	0.77	0.82	2.0	0.85	0.88	1.6	0.81	0.91
Welfare Gain, %								0.67	

Note: The U.S. original data was divided by the 16+ population, then logged and detrended by a linear–quadratic time trend. Output is GNP, and consumption and (gross) investment are the totals from the national income accounts, all in 1982 dollars. Hours data are from the *Current Population Survey* (which is a survey of households) and was calculated by multiplying total employment by average weekly hours.

(1) standard deviations, measured in percent

(2) first–order autocorrelations

(3) correlations with output

Table 2

	Benchmark Model			Capital Income Tax cut from 35% to 25%			Labor Income Tax cut from 35% to 25%			Investment Tax Credit Increased 7% to 14%		
	(I)			(II)			(III)			(IV)		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Output	0.1715	3.5	1.00	0.1869	3.5	1.00	0.1890	3.3	1.00	0.1789	3.5	1.00
Consumption	0.1472	1.7	0.67	0.1563	1.5	0.65	0.1622	1.7	0.69	0.1514	1.6	0.66
Investment	0.0243	19.6	0.92	0.0306	17.3	0.94	0.0268	18.1	0.92	0.0274	18.3	0.93
Hours	0.2596	2.0	0.88	0.2646	2.0	0.91	0.2881	1.8	0.87	0.2622	2.0	0.90
Productivity	0.6605	2.0	0.88	0.7060	1.8	0.89	0.6559	2.0	0.90	0.6819	1.9	0.89
Capital	0.2500	11.5	0.40	0.3358	9.5	0.34	0.2599	11.2	0.42	0.2814	10.7	0.38
Welfare Gain, %					4.05			3.10			1.91	

(1) mean

(2) standard deviation, %

(3) correlation with output

Fig. 1. Density functions for capital stock

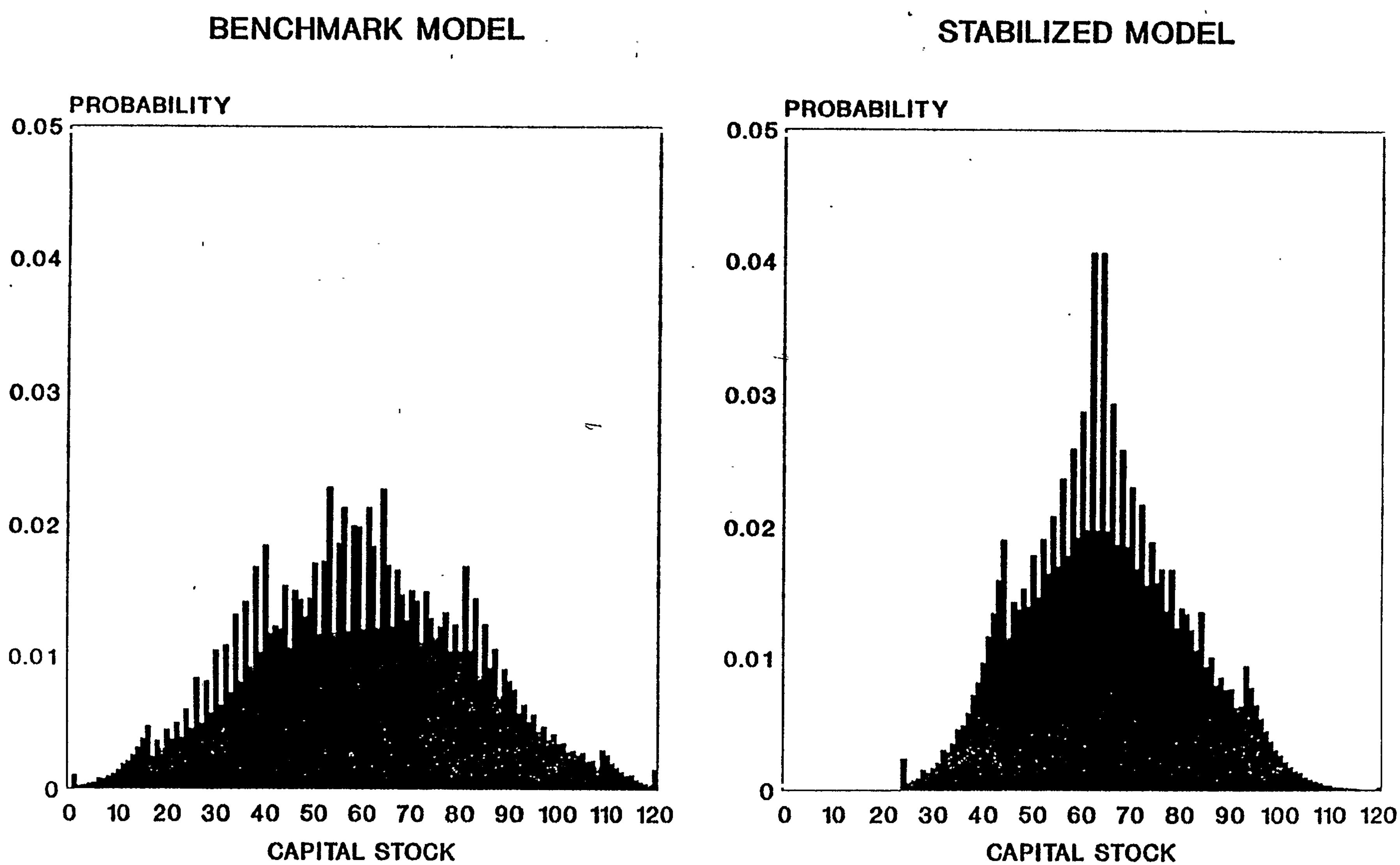
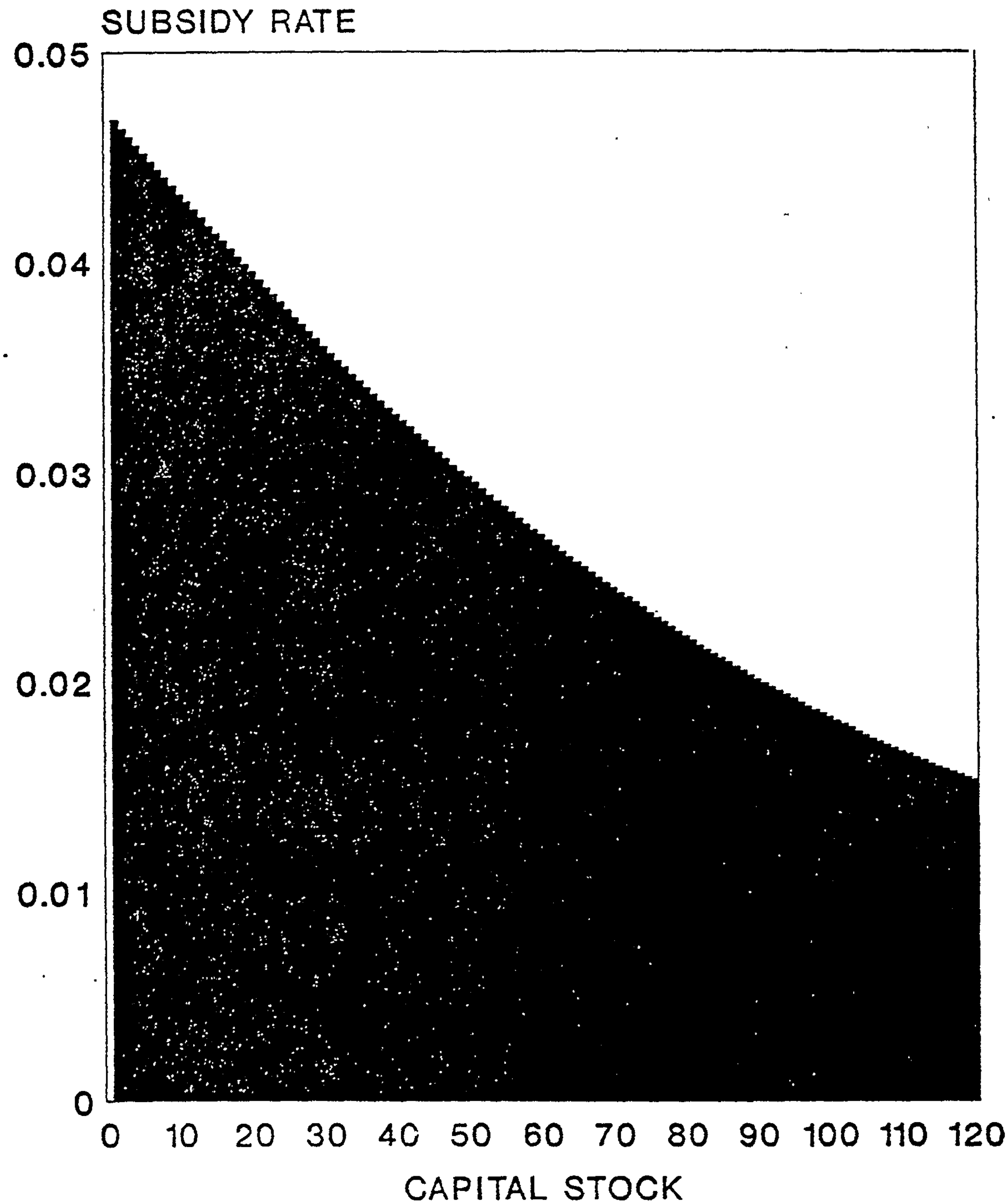


Fig. 2. Subsidy rates in recessions: Shock 2



APPENDIX

A. Computation of Equilibrium

In light of the definition for a competitive equilibrium presented in Section II.D and equations (2), (3), (6), (7), (8), (9) and (10) the representative household's dynamic programming problem (P2) will be recast as

$$V(k_t;K_t,\epsilon_t) = \max_{k_{t+1}} \left\{ U(c_t,L_t)+\beta \int_E V(k_{t+1};K_{t+1},\epsilon_{t+1})d\Omega(\epsilon_{t+1} \mid \epsilon_t) \right\} \quad (P4)$$

subject to

$$c_t + (1-\lambda_i)k_{t+1}/(1+\epsilon_t) = (1-\lambda_k)f_1(K_tH_t,L_t)k_tH_t$$

$$+ (1-\lambda_\ell)f_2(K_tH_t,L_t)L_t + (1-\lambda_i)[1-\delta(H_t)]k_t/(1+\epsilon_t) + \tau_t,$$

with the aggregate capital stock K_t following the law of motion $K_{t+1} = K(K_t,\epsilon_t)$, and where $H_t = H(K_t,\epsilon_t)$, $L_t = L(K_t,\epsilon_t)$ and $\tau_t = \tau(K_t,\epsilon_t)$ are implicitly defined by

$$(1-\lambda_k)f_1(K_tH_t,L_t) = (1-\lambda_i)\delta'(H_t)/(1+\epsilon_t),$$

$$U_1(f(K_tH_t,L_t) - [K(K_t,\epsilon_t) - (1-\delta(H_t))K_t]/(1+\epsilon_t),L_t)(1-\lambda_\ell)f_2(K_tH_t,L_t)$$

$$= - U_2(f(K_tH_t,L_t)-[K(K_t,\epsilon_t)-(1-\delta(H_t))K_t]/(1+\epsilon_t),L_t),$$

and

$$\tau_t = \lambda_k f_1(K_tH_t,L_t)K_tH_t + \lambda_\ell f_2(K_tH_t,L_t)L_t$$

$$- \lambda_i(K_{t+1}-(1-\delta(H_t))K_t)/(1+\epsilon_t).$$

Note that the tax distorted economy's general equilibrium cannot be directly computed from problem (P4) using traditional dynamic programming algorithms. Solving this programming problem requires knowing the equilibrium law of motion for the aggregate capital stock, but this in turn requires knowing the individual's decision–rule governing capital accumulation. The following iterative procedure, which is a variation on the standard dynamic programming algorithm, is proposed here: To begin with, an initial guess is made for *both* the value function on the right–hand side of (P4) and the equilibrium law of motion for the capital stock. Denote these guesses by $V^0(k_{t+1};K_{t+1},\epsilon_{t+1})$ and $K^0(K_t,\epsilon_t)$. Next problem (P4) is solved using these guesses. The optimized value of the maximand, which represents the left–hand side of the functional equation, is used as a revised guess for the value function, or $V^1(\cdot)$. As part of the solution to this problem, the individual's decision–rule for capital accumulation is obtained; it has the form $k_{t+1} = k^0(k_t,K_t,\epsilon_t)$. Since in *equilibrium* capital accumulation at the individual and aggregate levels must coincide, or $k_t = K_t$, this decision–rule forms the basis for the revised guess for the law of motion for the aggregate capital stock, $K^1(K_t,\epsilon_t)$. Specifically, $K^1(K_t,\epsilon_t) = k^0(K_t,K_t,\epsilon_t)$. These revised guesses for $V(\cdot)$ and $K(\cdot)$ are used as the foundation for the next round in the iterative scheme, the procedure being repeated until the decision rules have converged. Note that Baxter (1988), Bizer and Judd (1989), Coleman (1988), Cooley and Hansen (1989), Danthine and Donaldson (1988) and Kydland (1987) also tackle the problem of computing sub–optimal equilibria in various different ways.

The iterative scheme discussed above is straightforward to operationalize. To do this the individual and aggregate shocks for the economy are constrained to be elements of the finite time–invariant set $K = \{K_1,...,K_n\}$. Thus, the aggregate state space of the economy, $K \times E$ is discrete.[10] At the t + 1 stage of the algorithm the controller will be in possession of both a guess for the value function, $V^t(\cdot)$, and the aggregate law of motion for the capital stock, $K^t(\cdot)$. These guesses take the form of a value for $V^t(k_h;K_i,\epsilon_r)$ for each of the $2n^2$ possible combinations of (k_h,K_i,ϵ_r) in the state space $K \times K \times E$ and a value for $K^j(K_i,\epsilon_r)$ for each of the 2n potential combinations of (K_i,ϵ_r) in $K \times E$. Problem (P4) is then solved using these guesses. The optimized value of the maximand, or the righthand side of the value function, is used for the updated guess of the value function, or $V^{t+1}(\cdot)$. As part of the solution the problem (P4), a decision–rule governing the individual agent's capital accumulation is computed. This rule, $k_j' = k^t(k_h,K_i,\epsilon_r)$, specifies an optimal value for the agent's capital stock next period $k_j' \in K$ for each $(k_h,K_i,\epsilon_r) \in K \times K \times E$. The revised guess for the aggregate law of motion K^{t+1} is given by $K_j' = K^{t+1}(K_i,\epsilon_r) = k^{t+1}(K_i,K_i,\epsilon_r)$: $K \times E \to K$. Thus, the algorithm forms a mapping T such that $K^{t+1} = TK^t$. This iterative scheme is repeated until the individual and aggregate laws of motion for capital accumulation have converged, or until $\max_{K_i,\epsilon_r} |k^t(K_i,K_i,\epsilon_r) - K^t(K_i,\epsilon_r)| = 0$.[11] A proof that this algorithm converges for a similar model is contained in Greenwood and Huffman (1988).

The discretization of the model's state space allows for the exact joint stationary distribution of the sample economy's state variables—the aggregate capital stock and technology shock—to be numerically computed in a straightforward manner. Once the joint stationary distribution governing the model's state variables is obtained, it is easy to calculate population moments of interest for the model's various endogenous variables since these are all functions of the current state of the world. [The discussion below parallels that in Greenwood, Hercowitz and Huffman (1988).] Note that the solution for next period's aggregate stock, K'_j, is such that given an initial aggregate capital stock, K_i, and a value for the technology shock, ϵ_r, a unique value for $K'_j = K(K_i,\epsilon_r) \in \mathcal{K}$ is determined. Thus, the probability prob$[K'=K_j | K=K_i, \epsilon=\epsilon_r]$ will be equal to one for some $j \in \{1,\ldots,n\}$ and zero for the rest. Accordingly, the transition probability $p_{ir,js}$ of moving from the state characterized by capital stock k_i and shock ϵ_r to the one represented by k_j and ϵ_s can be expressed as $p_{ir,js} = \text{prob}[K'=k_j | k=k_i, \epsilon=\epsilon_r]\, \pi_{rs}$ for i, j = 1,...,n and r,s = 1,2. Next the $2n \times 2n$ transition matrix P with elements $p_{ir,js}$ is formed. The asymptotic joint distribution function for the capital stock and technology shock is a $1 \times 2n$ vector, ρ, which specifies a probability ρ_{ir} that the long–run capital stock/technology shock is (K_i,ϵ_r) for each i and r pair. The vector ρ solves the equation

$$\rho = \rho P,$$

subject to the constraints that $0 \leq \rho_{ir} \leq 1$ for all i, r and $\sum_{i=1}^{n} \sum_{r=1}^{2} \rho_{ir} = 1$.

Note that in general equilibrium all of the model's endogenous variables can be expressed as functions of the aggregate state of the world. Thus, one may write $X = X(K,\epsilon)$ for $X = C, K', H, L$, and Y. Consequently, the stationary moments for Y, CY and Y′Y can be written as

$$E[Y] = \sum_{r=1}^{2} \sum_{i=1}^{n} \rho_{ir}\, Y(K_i,\epsilon_r)$$

$$E[CY] = \sum_{r=1}^{2} \sum_{i=1}^{n} \rho_{ir}\, C(K_i, \epsilon_r) Y(K_i, \epsilon_r)$$

$$E[Y'Y] = \sum_{s=1}^{2} \sum_{j=1}^{n} \sum_{r=1}^{2} \sum_{i=1}^{n} p_{ir,js}\, \rho_{ir}\, Y'(K_j, \epsilon_s) Y(K_i, \epsilon_r).$$

Also, when conducting the simulations an evenly spaced grid of capital stock values is chosen for the set K such that further subdivision does not affect the value of the population moments under study. A grid of 120 point spanning the interval [0.175, 0.327] turns out to be sufficient for the benchmark economy under study. Similar discretization procedures are employed in Sargent (1980) and Danthine and Donaldson (1988). It is interesting to note that the aggregate law of motion governing capital accumulation, $K_{t+1} = K(K_t, \epsilon_t)$, turned out to be extremely close to being linear in form. For each of the two values of $\epsilon_t \in E$ a linear regression of the form $K_{t+1} = a + bK_t$ was fitted to the aggregate law of motion for capital. The R^2's for these regressions were .99997 and .99996, indicating an extremely tight fit. These linearization procedures, such as the one used in King, Plosser, and Rebelo (1988), would likely be very accurate for the model under study.

B. Welfare Measure

Finally, the welfare gain measure used in the analysis will be discussed. Let an asterisk attached to a variable denote its value in the benchmark economy. Then for the benchmark economy the agent's (unconditional) expected value of lifetime utility, E[V*], is given by

$$E[V^*] = \sum_{r=1}^{2} \sum_{i=1}^{n} \rho^*_{ir}\, U(C^*(K_i, \epsilon_r), L^*(K_i, \epsilon_r))/(1-\beta).$$

Similarly, the agent's expected level of lifetime utility under the new policy regime reads

$$E[V^{\sim}] = \sum_{r=1}^{2} \sum_{i=1}^{n} \rho^{\sim}_{ir}\, U(C^{\sim}(K_i,\epsilon_r), L^{\sim}(K_i,\epsilon_r))/(1-\beta),$$

where a tilde attached to a variable denotes its value under the new policy regime. Now the constant amount of consumption, $\Delta C^{\sim}$, that would have to be given to the agent in each state to make him as well off under the new policy regime as in the old one, is defined by the equation shown below.

$$\sum_{r=1}^{2} \sum_{i=1}^{n} \rho^{\sim}_{ir}\, U(C^{\sim}(K_i,\epsilon_r) + \Delta C^{\sim}, L^{\sim}(K_i,\epsilon_r))/(1-\beta) = E[V^*].$$

The welfare gain of switching from the benchmark economy to the new policy regime is then expressed as a percentage of expected output as follows:

$$\frac{\Delta C^{\sim}}{E[Y^{\sim}]} \times 100 = \frac{\Delta C^{\sim}}{\sum_{r=1}^{2} \sum_{i=1}^{n} \rho^{\sim}_{ir}\, Y^{\sim}(K_i,\epsilon_r)} \times 100.$$

FOOTNOTES

[1]Recent papers by Braun (1990), Cassou (1990), Gali (1990), and McGratten (1990) also simulate real business cycle models with distortional taxation and government policy. As in the current work, these papers examine the impact that fiscal policy has on macroaggregates—see Danthine and Donaldson (1985) for an early treatment of this issue. Braun (1990) and McGratten (1990) use estimation procedures to obtain values for the parameters that govern tastes, technology, and government policy in their models. Some welfare cost measures associated with distortional taxation are presented in Cassou (1990) and McGratten (1990), as is done here.

[2]The above definition for a competitive equilibrium presupposes that the initial condition $k_0 = K_0$ holds; that is, at the beginning of time the aggregate and household stocks of capital coincide.

[3]Specifically, given the current parameterization for tastes and technology, the steady–state analogues to equations (7), (8), and (9) are: (i) $(1-\lambda_i) = \beta[(1-\lambda_k)\alpha(L/KH)^{1-\alpha}H+(1-\lambda_i)(1-H^{\omega}/\omega)]$, (ii) $(1-\lambda_\ell)\alpha(L/KH)^{1-\alpha} = (1-\lambda_i)H^{\omega-1}$ and (iii) $(1-\lambda_i)(1-\alpha)(KH/L)^{\alpha}(1-\theta)(1-L) = \theta C$, where $C = (KH)^{\alpha}L^{1-\alpha} - H^{\omega}K/\omega$. The above two restrictions imply (iv) $L = 0.26$ and (v) $H^{\omega}/\omega = 0.10$. Given values for β, α, λ_k, λ_ℓ and λ_i this system of five equations can be thought of as determining a solution for the five unknowns K, H, L, θ, and ω.

[4]A detailed discussion of the construction of the welfare measure being used is contained in the Appendix.

[5]Some issues surrounding the transition from the old to the new stochastic steady–state have been ignored here. For instance, to calculate the immediate effect of a shift in the tax structure on the agent's expected

lifetime utility level, the initial state of the economy must be specified. The size of this effect will be different for different initial conditions. Similarly, the transitional dynamics of the system will depend upon the initial state of the economy. The comparison of the model's stochastic steady–states avoids the problem of having to specify the economy's initial position, but at the expense of abstracting away from some potentially interesting questions surrounding the transition from one steady–state to another.

[6]This stabilization scheme is similar to the one presented in Aschauer and Greenwood (1985).

[7]See footnote 2.

[8]The capital stock numbers have been normalized via a linear transformation to lie on the domain 1 to 120, the latter being the number of capital stock grid points in the simulated model's discrete space.

[9]This figure plots the fitted data points from a linear–quadratic regression that was estimated from the subsidy data obtained from the model. This procedure was employed to smooth out the approximation error associated with the computational procedure employed. The R^2 for this regression was .9829, indicating a very close fit. Hence, a quadratic function approximates well the state contingent subsidy schedule.

[10]See Huffman (1988) for an analysis of the accuracy of this approximation.

[11]Given the discrete nature of the state space such a strict criteria may never be fulfilled. In particular the algorithm may exhibit cycling behavior between the individual and aggregate laws of motion governing capital accumulation. Specifically, it can happen that for some points $(K_i, \epsilon_r) \in K \times E$, there exists a finite number T such that

$$k^t(K_i, K_i, \epsilon_r) = \begin{cases} K'_s & \text{for all odd } t \geq T \\ K'_{s\pm1} & \text{for all even } t \geq T \end{cases}$$

and

$$K^t(K_i, \epsilon_r) = \begin{cases} K'_s & \text{for all even } t \geq T \\ K'_{s\pm1} & \text{for all odd } t \geq T \end{cases}.$$

That is, given an initial state of world characterized by (K_i, ϵ_r), when the aggregate law of motion $K^t(K_i, \epsilon_r)$ dictates moving to K'_s, it is optimal for the individual to choose one of the adjacent points K'_{s+1} or K'_{s-1}, but when the aggregate capital stock moves to either K'_{s+1} or K'_{s-1}, the individual then picks K'_s. The algorithm should be terminated when such cycling is detected. The grid for the capital stock was chosen to be sufficiently fine so that the reported results were not sensitive to the point at which the algorithm was terminated.

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