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BARTER AND MONETARY EXCHANGE UNDER PRIVATE INFORMATION

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ABSTRACT

We analyze economies with private information concerning the quality of commodities. Without private information there is a nonmonetary equilibrium with only high quality commodities produced, and money cannot improve welfare. With private information there can be equilibria with bad quality commodities produced, and sometimes only nonmonetary equilibrium is degenerate. The use of money can lead to active (i.e., nondegenerate) equilibria when no active nonmonetary equilibrium exists. Even when active nonmonetary equilibria exist, with private information money can increase welfare via its incentive effects: in monetary equilibrium, agents may adopt trading strategies that discourage production of low quality output.

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> To barter is to exchange goods for other goods rather than money. This was common in early days. Presumably, however, the deal was not always fair. Barter is from the old French *barater* – to cheat! ("Word for Word," San Francisco Chronicle, 12/7/89).

1. Introduction

We present a model of production and exchange with private information, where the private information concerns the quality of commodities. Qualitative uncertainty and the impediments it presents to exchange have been important elements of the economics of information at least since the seminal contribution of Akerlof (1971). Inferior quality commodities are cheaper to produce, but they yield lower utility and hence are less desirable for consumption. In complex modern economies, however, obviously it is not always possible for a potential consumer to discern the quality of every commodity that needs to be purchased, and this can provide an incentive for sellers to produce low quality output and try to cheat buyers. The goal here is to analyze a model that captures these features, with emphasis on the impact of consumers' behavior on producers' incentives. We are especially interested in the role of *money*, an intrinsically worthless but universally recognizable object.

Many traditional discussions of money have emphasized its function as a medium of exchange and, in particular, its role in overcoming the double coincidence of wants problem with pure barter; see, for example, Jevons (1875). The focus is often on the intrinsic properties of objects that make them more or less natural media of exchange, including properties such as a relatively low storage or exchange cost, and a relatively high cost of producing the object privately, such as counterfeiting or digging precious metals out of the ground; see, for example, Menger (1892). Recently, some of these ideas have been formalized using search-theoretic, non-cooperative equilibrium models of the exchange process in Kiyotaki and Wright (1989, 1990, 1991). In these models, commodity money (a durable consumer good) and sometimes fiat money (a nonconsumable durable object) can arise endogenously as media of exchange, which leads to reductions in the search and transactions costs associated with direct barter.[1]

In addition to helping to solve the double coincidence problem, it has also been argued that money is important in mitigating frictions associated with moral hazard or adverse selection. These frictions can be impediments to exchange when agents have limited opportunities for enforcing contracts and there is private information concerning the quality of goods for sale or concerning the intentions of agents to honor private claims.[2] Alchian (1977) has gone so far as to argue that overcoming the double coincidence problem is a minor part of what money accomplishes, and that private information is the principal friction underlying the institution of monetary exchange:

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[1] For some related models of money as a medium of exchange, see the survey by Ostroy and Starr (1990).

[2] A general discussion of private information and monetary exchange is contained in Brunner and Meltzer (1971). Bernhardt and Engineer (1987) consider an adverse selection model in which money mitigates against a lemons problem in the exchange of goods. A related model is in King and Plosser (1986), where the use of a costly technology for producing a noncounterfeitable good from a counterfeitable one (e.g., gold coins from gold jewelry) can improve welfare. Freeman (1985) and Aiyagari (1989) consider models with private information concerning the quality of assets. Other models in which private information in credit markets expands the role for fiat currency include Smith (1986) and Williamson (1990). Townsend (1989) studies an economy where private information and spatial separation lead to the use of money as a record-keeping device.

> It is not the absence of double coincidence of wants, nor the costs of searching out the market of potential buyers and sellers of various goods nor of record keeping, but the costliness of information about the attributes of goods available for exchange that induces the use of money in an exchange economy (Alchian, 1977, p. 139).

We explore informational frictions here in a formal and, therefore, somewhat abstract economic model. The basic framework is a close relative of the fiat money economy in Kiyotaki and Wright (1990, 1991), except that we eliminate the double coincidence problem in order to focus on the property of recognizability. Agents have a choice between producing commodities that are of good or bad quality, the latter being cheaper to produce but yielding lower utility when consumed. No utility is derived from consuming one's own output, and agents therefore need to trade. They meet pairwise over time in random fashion, always carrying an inventory consisting of a good quality commodity, a bad quality commodity, or fiat currency, and trade when it is mutually agreeable. The model is constructed – again, so as to isolate the role of recognizability – so that pure barter is easy without private information. Therefore, even though there may be equilibria where fiat currency circulates, without private information money cannot improve welfare.

The key innovation in the paper is that in some trading opportunities one or both individuals may be unable to recognize the quality of the other's wares. This may or may not lead to some agents producing bad quality commodities, depending on relative production costs and the extent of the private information problem. For some parameter values, there can exist multiple equilibria. This illustrates an externality inherent in the exchange process which, to our knowledge, has not been discussed in the previous literature: the production of better quality output on average increases the cost of being turned down in an exchange opportunity, which discourages each individual from producing low quality commodities. The use of a generally recognizable fiat currency can be welfare improving in this context, not because it ameliorates Jevons' double coincidence problem, but because it can lead to agents adopting trading strategies that reduce the incentive to produce bad quality output.

More precisely, when the probability of recognizing the quality of a potential trading partner's inventory is one, the model has a unique active (nondegenerate) nonmonetary equilibrium, which is efficient in the sense that each agent produces a good quality commodity every period, and trades and consumes with every exchange opportunity. There may also exist monetary equilibria, but because there is no difficulty in finding a mutually acceptable barter transaction, the use of money does nothing to enhance exchange, and actually lowers welfare by reducing production. With private information, the efficient outcome with only good commodities being produced may or may not survive as an equilibrium, while other equilibria with production of bad commodities may emerge, depending on parameter values. For some parameter values the only nonmonetary equilibrium is the degenerate one with no production or consumption, while for other parameter values there can be multiple, Pareto-ranked, equilibria.

To illustrate the role of money, we first show that there are circumstances in which a monetary equilibrium exists when no active nonmonetary equilibrium exists. A universally recognizable fiat money can be a catalyst to exchange and, hence, production and consumption, when the private information problem is so severe that economic activity would otherwise shut down. Second, we demonstrate that there are circumstances in which active nonmonetary equilibria exist, but there also exist monetary equilibria that entail higher welfare. Money can increase welfare because it gives the seller of a good commodity the luxury of demanding payment in money, which in turn generates positive incentive effects by imposing discipline on the producers of bad quality output. For example, in some monetary equilibria no one ever trades a good commodity for a commodity of unrecognized quality. This effectively imposes a "cash-in-advance" constraint on producers of bad commodities, in that they must sell their output for fiat money before making another trade. Producers are more willing to bear the cost of high quality output in order to avoid this constraint.

The remainder of the paper is organized as follows. In Section 2 we present the basic framework and in Section 3 we analyze the complete information case as a benchmark. In Section 4 we consider nonmonetary equilibria with private information and in Section 5 we consider monetary equilibria with private information. In Section 6 we conclude.

2. The Basic Model

Time is discrete and continues forever. There is a continuum of homogeneous, infinite-lived agents, whose population is normalized to one. There are three "goods," a good quality commodity, a bad quality commodity, and money.[3] A good or bad commodity can be produced by any agent, with the cost in terms of disutility to producing one unit of the good commodity equal to $\gamma > 0$ and the cost to producing the bad commodity equal to 0. Money cannot be produced by any private individual. All objects (money and commodities) are indivisible, freely disposable, and storable at zero cost, but only one unit at a time. This implies that agents' inventories always

[3] To motivate the analysis of private information, it may help to imagine a large number of differentiated commodities. However, since consumers will derive the same utility from every commodity (holding quality fixed) here, we proceed as though there is a single consumption good.

consist of at most one unit of one object – either a good commodity, a bad commodity, or money. Consumption of either money or a bad commodity yields zero utility. Consumption of one unit of a good commodity yields utility u > 0 if it was produced by someone else, while consumption of one's own produce yields no utility.[4]

At the beginning of the initial period, a fraction M of the agents in the economy are chosen at random and endowed with one unit each of money, after which production takes place. In each succeeding period, agents meet pairwise and at random, and decide whether or not to trade in each meeting. Trade entails a one-for-one swap of inventories, and takes place if and only if it is mutually agreeable. There are no private credit arrangements, since agents who meet will meet again in the future with probability zero. Agents holding (good or bad) commodities are called *commodity traders*, while agents holding money are called *money traders*. Let p be the proportion of commodity traders holding commodities that are good and 1-p the proportion holding commodities that are bad.

Money is always identifiable, but in any meeting between an agent and a commodity trader, there is a probability θ that the former recognizes the quality of the latter's commodity.[5] This probability is independent across

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[4] The assumption that agents derive no utility from the consumption of their own output generates gains from trade in a very simple way, and is common in search-based models of the exchange process; see, for example, Diamond (1982, 1984) or Kiyotaki and Wright (1990, 1991).

[5] As suggested in the introduction, this is meant to capture the fact that, in modern economies with many commodities, it is typically not possible to identify the quality of everything one may have occasion to purchase. For example, one consumer may be well-informed concerning clothing but ignorant when it comes to electronics, while another is an expert in electronics but cannot tell Armani from K-Mart. Nevertheless, there are times when the

agents when two commodity traders meet. Traders do not know if other traders recognize their inventory, and do not know anything about other traders' histories. When two traders meet, they simply inspect each others' inventories and simultaneously announce whether or not they wish to trade. After a trade the agents separate, whereupon the quality of each object is revealed if it was previously unknown. Each trader then has the option of consuming the object, disposing of it, or storing it in inventory. If the object is consumed or disposed, then the agent can instantaneously produce a new commodity of either bad or good quality at the associated cost.

Future utility is discounted at the rate $r > 0$. Agents choose production, consumption, disposal, and trading strategies, in order to maximize the expected discounted utility of consumption net of production costs. In doing so, they take as given the strategies of others and the probabilities of meeting agents holding particular inventories. We look for stationary Nash equilibria, in which the relevant meeting probabilities are time-invariant and expectations are rational. The meeting probabilities are summarized by m, the probability of meeting a money trader, and p, the probability of meeting a trader with a good commodity conditional on meeting a commodity trader. Note that $m \leq M$, where M is the initial endowment of fiat money, and that $m = M$ if no agent disposes of money. For the most part, we confine attention to what we call *active* (or nondegenerate) equilibria, in which some good commodities are produced, traded and consumed, and utility is strictly positive.[6]

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former needs a stereo and the latter a suit of clothes.

[6] There will also exist degenerate equilibria in which utility is zero. For example, if no one ever accepts any trade, then no good commodities are produced; clearly, this is an inactive equilibrium implying zero utility.

3. Complete Information

As a benchmark, in this section we consider the case of $\theta = 1$, so that there is no private information concerning the quality of commodities. Let V_j denote the payoff or value function for an agent at the end of a period holding object j, where j = g, b or m denotes a good commodity, a bad commodity or money, respectively. Let $W = \max(V_g-\gamma, V_b)$ represent the value function for an agent with nothing in inventory and deciding which quality commodity to produce. By definition, in any active equilibrium at least some production of the good commodity must occur, which implies $p > 0$ and also $W = V_g-\gamma \geq V_b$. We first consider nonmonetary equilibria, in which money is never accepted in trade. In an active nonmonetary equilibrium, those initially endowed with fiat money dispose of it in the first period and, therefore, $m = 0$.

Some things are obvious. First, if a trader with a good commodity meets someone else with a good commodity, then each strictly prefers to trade if and only if $u + W > V_g$, or, equivalently, $u > \gamma$. Second, if a trader with a good commodity meets someone with a bad commodity, then the former strictly prefers not to trade, since $V_b < V_g$ by virtue of the fact that $V_g-\gamma \geq V_b$. Hence, traders with bad commodities cannot trade for good commodities, which implies that no bad commodities are produced and $p = 1$. This means that, if an active nonmonetary equilibrium exists, all agents produce good commodities in the initial period, and each period thereafter they meet, trade, consume and produce again. To verify that this is in fact an equilibrium, it only remains to check that an agent has no incentive to deviate from this strategy when everyone else is following it (that is, it is a fixed point of the best response correspondence).

We make extensive use of the fundamental principle of dynamic programming known as the *unimprovability criterion.*[7] To apply this, first note that the best response problem for a representative agent, given that others are following the above strategies and that $p = 1$ and $m = 0$, is described by the following equations (these and similar equations below are all special cases of some general results derived in Appendix B),

$$rV_g = u + W - V_g = u - \gamma \tag{3.1}$$

$$rV_b = W - V_b = V_g - \gamma - V_b. \tag{3.2}$$

Notice we have inserted the candidate strategy of always producing good quality output, $W = V_g - \gamma$. Equations (3.1) and (3.2) imply $V_g - \gamma \geq V_b$, and so the agent cannot improve his payoff by a one-time deviation of producing a bad commodity, if and only if $u \geq (1+r)\gamma$.

We conclude that producing only good commodities is a best response, and hence the unique active nonmonetary equilibrium, if and only if $u \geq (1+r)\gamma$. Let Z denote welfare, defined as the expected utility of the representative agent in the initial period *before* the goods are produced and the initial endowment of money is distributed. In this equilibrium, $Z = V_g - \gamma = Z^*$, where Z^* satisfies

$$rZ^* = u - (1+r)\gamma. \tag{3.3}$$

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[7] A policy Π is called *unimprovable* (in a single step) if the payoff from using Π cannot be increased by deviating to a different decision at a single date and then reverting back to Π for the rest of time. Obviously, a payoff maximizing policy is unimprovable; a more useful result is that an unimprovable policy is maximal (as long as the payoff function is bounded below). See Kreps (1990) for a very readable discussion.

Except for the borderline case where $u = (1+r)\gamma$, whenever this equilibrium exists we have $Z^* > 0$; hence, it Pareto dominates an inactive equilibrium.

We now consider monetary equilibria, in which fiat currency is accepted in at least some exchanges. In fact, in this economy, one can show that money is either accepted in all exchanges or in no exchanges (see below), and so we concentrate on the case of *pure* monetary equilibria in which money is universally accepted. Through an argument similar to that used above, for an active equilibrium we require $V_g - \gamma \geq V_b$. This implies that agents with good commodities will not trade for bad commodities. For money to be accepted in trade, we also require $V_m \geq V_g$, which implies that $V_m > V_g - \gamma$ and no one disposes of money. Therefore $m = M$. As no trader accepts bad commodities, $p = 1$ in this case, exactly as in the nonmonetary case. It only remains to check that producing good commodities and accepting money is a fixed point of the best response correspondence.

To this end, we note that the payoff functions for agents following this strategy satisfy

(3.4) $$rV_g = (1-M)(u-\gamma) + M(V_m - V_g)$$

(3.5) $$rV_b = V_g - \gamma - V_b$$

(3.6) $$rV_m = (1-M)(u - \gamma + V_g - V_m).$$

Manipulating these, we immediately see that $V_m = V_g$ (money has the same value as a good commodity). Hence, a one-step deviation of not accepting money does not improve the payoff. Further manipulation implies $V_g - \gamma \geq V_b$ if and only if

$$(3.7) \qquad M \le 1 - r\gamma/(u-\gamma) \equiv M_1.$$

For $M \le M_1$, producing a bad commodity also does not improve the payoff and, by the unimprovability criterion, accepting money and producing only good commodities is a best response.

We conclude that a monetary equilibrium exists if and only if $M \le M_1$. A monetary equilibrium exists for *some* $M > 0$ as long as $M_1 > 0$, which holds if and only if $u > (1+r)\gamma$. This is also the necessary and sufficient condition for the existence of the active nonmonetary equilibrium. Therefore, whenever a monetary equilibrium exists, so does the active nonmonetary equilibrium, and we claim that the latter is Pareto superior. In the monetary equilibrium expected utility is $Z = MV_m + (1-M)(V_g-\gamma)$, which simplifies to

$$(3.8) \qquad rZ = (1-M)[u-(1+r)\gamma].$$

Observe that Z is decreasing in M, and $M = 0$ implies $Z = Z^*$, where Z^* is welfare in the nonmonetary equilibrium. If we call the welfare-maximizing value of M the optimal quantity of money, then the optimal quantity of money is zero here (without private information).

We could also consider ex post welfare — that is, expected utility after the initial distribution of money. Since $rV_m = (1-M)(u-\gamma)$ in the monetary equilibrium and $rV_m = r(V_g-\gamma) = u - (1+r)\gamma$ in the nonmonetary equilibrium, those initially endowed with money are better off, ex post, in the monetary than in the nonmonetary equilibrium if and only if $M < r\gamma/(u-\gamma)$. Initial money holders are made better off by the fact that they do not have to produce in order to trade, although they are made worse off by the fact that *other* money traders do not produce, and lower production implies lower consumption in equilibrium. The former effect dominates when

$M < r\gamma/(u-\gamma)$. Agents not initially endowed with money are always worse off in the monetary equilibrium, because only the latter effect is relevant (*they* still have to produce). The result $Z < Z^*$ indicates that the average agent's utility, our ex ante welfare measure, is unambiguously lower in the monetary than in the nonmonetary equilibrium.[8]

It is helpful to compare the above results with those in Kiyotaki and Wright (1990), which is itself a simplified version of Kiyotaki and Wright (1991). The model in Kiyotaki and Wright (1990) is similar to the version of this model with $\theta = 1$, except that it includes a double coincidence problem. In particular, there is a number $x \in [0,1]$, such that x equals the probability that an agent selected at random will accept a given commodity, which also equals the probability that a commodity selected at random will be accepted by a given agent. Smaller values of x make barter more difficult; here we set $x = 1$ so that, at least when $\theta = 1$, barter is trivial. For $x \in (0,1)$ there are always three active equilibria: a nonmonetary equilibrium where money is never accepted, a pure monetary equilibrium where money is accepted with probability 1, and a mixed monetary equilibrium where money is accepted with probability x. When $x = 1$, the latter two equilibria coalesce, and there is no equilibrium where money is

[8] Following Aiyagari and Wallace (1991a,1991b), we could consider a slightly different set-up, in which agents cannot produce until they consume. This implies that those initially endowed with money never produce in a nonmonetary equilibrium, although they are still a part of the meeting technology (that is, agents with commodities still have a probability M of meeting a money trader every period). Under these assumptions agents initially endowed with money are always better off in the monetary equilibrium while everyone else is indifferent, and, therefore, ex ante welfare is higher. However, it remains the case that Z is a decreasing function of M in a monetary equilibrium, even under the Aiyagari - Wallace assumptions.

accepted with probability between zero and one.

We can also compare the welfare results for our model with $\theta = 1$ with those in Kiyotaki and Wright (1990). For arbitrary values of x, the generalized payoff functions in pure monetary equilibrium satisfy

$$rV_g = (1-M)x^2(u-\gamma) + Mx(V_m-V_g) \tag{3.9}$$

$$rV_m = (1-M)x(u-\gamma+V_g-V_m). \tag{3.10}$$

Welfare is given by $Z = Z_x$, where

$$rZ_x = (1-M)[Mx+(1-M)x^2](u-\gamma) - (1-M)\gamma r. \tag{3.11}$$

For small x, Z_x is increasing in M at M = 0. Thus, with a severe enough double coincidence problem, the optimal quantity of money is positive.[9] In this paper, we set x = 1 to remove the double coincidence friction entirely and focus on private information.

4. Private Information: Nonmonetary Equilibria

In this section we assume $\theta < 1$, so that in some meetings agents are not able to identify the quality of a commodity in a potential trading partner's inventory. We also restrict attention until the next section to nonmonetary equilibria (m = 0). In general, agents may wish to take

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[9] Actually, in Kiyotaki and Wright (1990) a slightly different welfare criterion is used, $W_x = MV_m + (1-M)V_g$, instead of $Z_x = MV_m + (1-M)(V_g-\gamma)$, with the interpretation that 1-M agents are endowed with commodities in the initial period and do not have to produce.

advantage of a lower cost to producing or storing bad commodities, with the hope of being able to trade them to an uninformed agent. Hence, we could have $p < 1$, although (by definition) we cannot have $p = 0$ in an active equilibrium.

As in the previous section, two commodity traders who meet and recognize each other's inventories as good quality will always want to trade. Clearly, an agent with a bad commodity will trade at every exchange opportunity, since at worst the agent gets another bad commodity in return. Hence, the only nontrivial decisions concern whether to produce good or bad quality output, and whether to accept or reject a commodity of unrecognized quality. Let Σ denote the probability with which an agent believes that other commodity traders will accept commodities of unrecognized quality. Note that $\Sigma > 0$ in any active equilibrium: If $\Sigma = 0$ then unrecognized commodities are never accepted, and so bad commodities are never produced; but then consumers are better off if they accept unrecognized commodities and this contradicts $\Sigma = 0$.

An agent's best response problem is now described by the following dynamic program

(4.1) $$rV_g = \theta p[\theta+(1-\theta)\Sigma](u+W-V_g)$$
$$+ (1-\theta) \max_{\sigma} \sigma\left\{p[\theta+(1-\theta)\Sigma](u+W-V_g) + (1-p)(W-V_g)\right\}$$

(4.2) $$rV_b = p(1-\theta)\Sigma(u+W-V_b).$$

Equation (4.1) sets the return to holding a good commodity equal to the probability the agent meets another agent with a commodity that is recognized as good quality, θp, multiplied by the probability the other agent is willing to trade, $\theta+(1-\theta)\Sigma$, multiplied by the gain from trading,

$u+W-V_g$, plus the probability the agent meets another *agent with a commodity* that cannot be recognized, $1-\theta$, multiplied by the gain from choosing the acceptance probability σ. Similarly, (4.2) sets the return to holding a bad commodity equal to the probability the agent meets a trader with a good commodity who is willing to accept something that cannot be recognized, $p(1-\theta)\Sigma$, multiplied by the gain from trading, $u+W-V_b$.

Let $\sigma = \sigma(\Sigma;p)$ denote the best response correspondence for a given p. Then a stationary Nash equilibrium is a value of p together with fixed point $\Sigma = \sigma(\Sigma;p)$, with the property that $V_g-\gamma > V_b$ implies $p = 1$, $V_g-\gamma < V_b$ implies $p = 0$, and $0 < p < 1$ implies $V_g-\gamma = V_b$. Potentially, there are three types of active equilibria. A type *a* equilibrium has $p = 1$, which implies $\Sigma = 1$; in this case, no bad commodities are ever produced and therefore traders always accept commodities even when they cannot recognize their quality. A type *b* equilibrium has $0 < p < 1$ and $\Sigma = 1$; in this case, some bad commodities are produced but traders always accept commodities even when they cannot recognize their quality. A type *c* equilibrium has $0 < p < 1$ and $0 < \Sigma < 1$; in this case, some bad commodities are produced and traders sometimes accept and sometimes reject commodities when they cannot recognize their quality. We consider each of these cases in turn.

First, consider a type *a* equilibrium, with $p = \Sigma = 1$. In such an equilibrium, if it exists, private information is not a problem in the sense that the outcome is the same as in the active nonmonetary equilibrium with $\theta = 1$. This could potentially be an equilibrium even if $\theta < 1$, as agents might be disciplined to produce good commodities by the possibility of having bad commodities rejected by informed agents. This requires $V_g-\gamma \geq V_b$. Using the unimprovability criterion once again, we insert $p = \Sigma = \sigma = 1$ into (4.1) and (4.2) and rearrange to find that $V_g-\gamma \geq V_b$ if and only if $\theta u \geq (1+r)\gamma$. We conclude that equilibrium *a* exists if and only if $\theta \geq \theta_1$, where we define

(4.3) $$\theta_1 = (1+r)\gamma/u.$$

Now consider a type *b* equilibrium, with $0 < p < 1$ and $\Sigma = 1$. If $V_g - \gamma = V_b$, then unimprovability implies it is a best response to produce a good commodity with any arbitrary probability.[10] If we substitute $\sigma = \Sigma = 1$ into (4.1) and (4.2) and solve for the value of p that implies $V_g - \gamma = V_b$, we find

(4.4) $$p = \frac{\gamma(1-\theta+r)}{\theta(u-\gamma)} = p_b.$$

Notice that $p_b > 0$, and $p_b < 1$ if and only if $\theta > \theta_1$ where θ_1 is defined in (4.3). We therefore need only check that $\sigma = 1$ is also a best response. If we insert $W = V_g - \gamma$ and $\Sigma = 1$ into (4.1), we see that $\sigma = 1$ is a best response if and only if $pu - \gamma \geq 0$. Using (4.4), this holds if and only if $\theta \leq \theta_2$, where

(4.5) $$\theta_2 = (1+r)u/(2u-\gamma).$$

Hence, equilibrium *b* exists if and only if $\theta_1 < \theta \leq \theta_2$.

Finally, consider a type *c* equilibrium, with $0 < p < 1$ and $0 < \sigma < 1$. Any $p \in [0,1]$ is a best response if $V_g - \gamma = V_b$, and any $\sigma \in [0,1]$ is a best response if agents are indifferent between accepting and rejecting a trade

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[10] By use of an appropriate law of large numbers for continuum economies (see Uhlig 1987), the proportion of agents with good commodities is equal to the probability with which the average agent produces a good commodity. Alternatively, we can simply impose that a fraction p of the agents always produce good commodities while the rest always produce bad commodities, since both are best responses, given $V_g - \gamma = V_b$. This yields the right proportions without appeal to a law of large numbers.

for an unrecognized commodity. Using (4.1), we see that the latter requires $p[\theta+(1-\theta)\Sigma](u-\gamma) = (1-p)\gamma$, or

$$(4.6) \qquad p = \frac{\gamma}{\gamma+[\theta+(1-\theta)\Sigma](u-\gamma)} = \pi(\Sigma).$$

Notice $0 < \pi(\Sigma) < 1$ for all $\Sigma \geq 0$. Using $p = \pi(\Sigma)$, we can solve for the value of Σ that yields $V_g-\gamma = V_b$,

$$(4.7) \qquad \Sigma_c = \frac{\theta(u-\gamma)+(1-\theta)\gamma-(1-\theta+r)[\gamma+\theta(u-\gamma)]}{(1-\theta)[\gamma+(1-\theta+r)(u-\gamma)]}.$$

Straightforward algebra implies $0 < \Sigma_c < 1$ and hence equilibrium *c* exists if and only if $\theta_3 < \theta < \theta_2$, where θ_2 is defined in (4.5) and θ_3 is defined by

$$(4.8) \qquad \theta_3 = .5r + \frac{.5}{u-\gamma}\sqrt{r^2(u-\gamma)^2 + 4r\gamma(u-\gamma)}\ .$$

The above analysis indicates that the set of equilibria depends on the value of θ relative to the three critical values, θ_1, θ_2 and θ_3, which themselves depend on the other parameters. In Figure 1, we graph θ_1, θ_2 and θ_3 as functions of r for given but arbitrary values of u and γ. Notice that there exist three values of r, with $0 < r_1 < r_2 < r_3$, such that the following is true. For $r \in (0,r_1)$ we have $\theta_3 < \theta_1 < \theta_2$; in this case, equilibrium *c* is the unique active equilibrium for $\theta_3 < \theta < \theta_1$, equilibria *a*, *b* and *c* coexist for $\theta_1 < \theta < \theta_2$, and equilibrium *a* is the unique active equilibrium for $\theta_2 < \theta < 1$. For $r \in (r_1,r_2)$ we have $\theta_1 < \theta_3 < \theta_2$; in this case, equilibria *a* and *b* exist for $\theta_1 < \theta < \theta_3$, all three coexist for $\theta_3 < \theta < \theta_2$, and equilibrium *a* is the unique active equilibrium for $\theta_2 < \theta < 1$. Finally, for $r \in (r_2,r_3)$ we have $\theta_1 < 1 < \theta_2 < \theta_3$; in this case, equilibria *a* and *b* coexist for $\theta_1 < \theta < 1$ while equilibrium *c* does not exist.

The following general observations can be made. First, assume $r < r_3$, which is equivalent to assuming that there exists an active equilibrium when $\theta = 1$, since $r_3 = (u-\gamma)/\gamma$. Then there always exist values of θ less than but close to 1 such that equilibrium *a* exists. Thus, a little private information can be introduced without creating a problem. Second, note that there are always values of θ close to 0 such that no active equilibrium exists. This means that enough private information can be introduced so that all economic activity shuts down. Third, in the case where $r < r_1$, notice that equilibrium *c* is the only active equilibrium for small θ. Equilibrium *c* has the greatest chance of surviving when the private information problem becomes severe, as $\Sigma < 1$ implies that the most discipline is imposed on the producers of bad commodities. A low probability of a bad commodity being accepted in trade by an uninformed trader reduces the incentive to produce bad commodities, which would otherwise be great when θ is small. Finally, notice that there always exist values of θ such that multiple equilibria coexist.

We now demonstrate that when multiple equilibria coexist they can be Pareto ranked. Let Z_j be welfare in equilibrium j, where j = *a*, *b*, or *c*. Then a little algebra implies Z_j can be written[11]

$$rZ_j = p_j[\theta+(1-\theta)\Sigma_j](u-\gamma) - (1-p_j)(1-\theta)\gamma - r\gamma. \tag{4.9}$$

[11] Notice that rZ_j is simply expected utility per period, which is the probability of meeting someone with a good commodity who is willing to trade, $p_j[\theta+(1-\theta)\Sigma_j]$, multiplied by the gain from trading, $u-\gamma$, minus the probability of meeting someone with a bad commodity that cannot be recognized, $(1-p_j)(1-\theta)$, multiplied by the loss from making the trade and having to produce again, γ, minus the capitalized value of the initial production cost, $r\gamma$.

As $p_a = \Sigma_a = 1$, welfare is greatest in equilibrium *a* and, of course, $Z_a = Z^*$, where Z^* is welfare in the active nonmonetary equilibrium with $\theta = 1$. Since $\Sigma_c < \Sigma_b = 1$, and since it is also possible to show that $p_c < p_b < 1$, it is immediate from (4.9) that $Z_c < Z_b < Z_a$. In Figure 2, we plot welfare in equilibria *a*, *b* and *c* as functions of θ, for all values of θ for which the equilibria exist. Three cases are shown, corresponding to values of r in each of the three intervals defined by Figure 1.

The multiplicity of Pareto-ranked equilibria is due to a *strategic complimentarity*, in the language of Cooper and John (1988), that works as follows. When more agents produce high quality output, the value of making a trade increases, and therefore so does the cost of "getting caught" with a bad commodity. When there are more high quality commodities in circulation, each individual is therefore more willing to bear the cost of producing high quality output. It is clear that this effect is due exclusively to the presence of private information, since in the economy with $\theta = 1$ there is a unique active nonmonetary equilibrium. That equilibrium could not be improved upon by the introduction of fiat currency. In the next section, we show in some circumstances that outcomes in the private information economy can be improved by using money.

5. Private Information: Monetary Equilibria

In this section, in the presence of private information we demonstrate that there can exist active monetary equilibria under circumstances in which the only nonmonetary equilibrium is degenerate, and that even if an active nonmonetary equilibrium exists, there may simultaneously exist a monetary equilibrium that entails higher welfare. We also demonstrate that several different monetary equilibria may coexist, interpret the mechanism by which the use of money mitigates the private information problem, and discuss some issues relating to the optimal quantity of money.

We restrict attention to equilibria with $0 < p < 1$. It is not difficult to show there exists a monetary equilibrium with $p = 1$ if and only if $\theta \geq \theta_4$, where $\theta_4 = (1+r)\gamma/[(1-M)u+M\gamma]$.[12] Since $\theta_4 > \theta_1$, whenever this equilibrium exists there also exists an active nonmonetary equilibrium with $p = 1$, and the latter implies a higher level of welfare. In other words, if money is to have a welfare enhancing role in this economy, it cannot *completely* alleviate the private information problem by driving out *all* bad commodities. In what follows, then, we examine monetary equilibria where $0 < p < 1$, implying $W = V_g - \gamma = V_b$. We also restrict attention to pure monetary equilibria, where money is universally accepted, which implies that $V_m \geq V_g$. In contrast to the situation with $\theta = 1$, when $\theta < 1$ there may well exist mixed monetary equilibria where money is sometimes but not universally accepted. However, it can be shown that if such an equilibrium exists, there also exists a nonmonetary equilibrium that implies a higher level of welfare (details are available upon request).

Denote the probabilities that good commodity traders and money traders accept commodities that they cannot recognize by Σ and Ω, respectively (bad commodity traders always accept). Then the best response problem of an agent is described by[13]

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[12] Alternatively, a monetary equilibrium with $p = 1$ exists if and only if $M < M_\theta = 1 - (1-\theta+r)\gamma/(u-\gamma)$. Note that this generalizes the necessary and sufficient condition for the existence of a monetary equilibrium when $\theta = 1$, given in (3.7). A monetary equilibrium exists for *some* $M > 0$ if and only if $M_\theta > 0$, which holds if and only if $\theta u < (1+r)\gamma$.

[13] Notice how we exploit the unimprovability criterion here. Since at each date the agent only needs to make a single decision (accept or reject money, produce a good or a bad commodity, etc.), we can demonstrate that a given strategy is a best response by showing that the agent's payoff cannot be

$$(5.1) \qquad rV_g = (1-M)\theta p[\theta+(1-\theta)\Sigma](u-\gamma) + M[\theta+(1-\theta)\Omega](V_m-V_g)$$

$$+ (1-M)(1-\theta) \max_{\sigma} \sigma\left\{p[\theta+(1-\theta)\Sigma](u-\gamma) - (1-p)\gamma\right\}$$

$$(5.2) \qquad rV_b = (1-M)p(1-\theta)\Sigma u + M(1-\theta)\Omega(V_m-V_b)$$

$$(5.3) \qquad rV_m = (1-M)\theta p(u-\gamma+V_g-V_m)$$

$$+ (1-M)(1-\theta) \max_{\omega} \omega\left\{p(u-\gamma+V_g-V_m) + (1-p)(-\gamma+V_g-V_m)\right\}.$$

These are natural extensions of the expressions in the previous sections. For example, (5.1) sets the return to holding a good commodity equal to the sum of three terms. The first term is the probability the agent meets a commodity trader with a good commodity and recognizes it, $(1-M)\theta p$, multiplied by the probability the other agent is willing to trade, $\theta+(1-\theta)\Sigma$, multiplied by $u-\gamma$. The second term is the probability the agent meets a money trader who is willing to trade, $M[\theta+(1-\theta)\Omega]$, multiplied by V_m-V_g. The final term is the probability the agent meets a commodity trader with an inventory that cannot be recognized, $(1-M)(1-\theta)$, multiplied by gain from choosing the acceptance probability σ.

Potentially, several different types of equilibria are possible, depending on whether Ω and Σ are elements of $\{0\}$, $\{1\}$, or Φ, where Φ denotes

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increased by deviating from this strategy at any single decision point. We do not have to also show that the agent's payoff cannot be improved by combinations of deviations (e.g., stop accepting money and simultaneously start producing only bad commodities, etc.)

the open interval $(0,1)$. The set of possibilities is shown in Table 1, where each case is given a label in terms of (Σ,Ω); for instance, equilibrium $(0,0)$ has $\Sigma = \Omega = 0$, equilibrium $(0,\Phi)$ has $\Sigma = 0$ and $0 < \Omega < 1$, and so on. There could never exist a $(0,0)$ equilibrium with $p < 1$, since someone has to accept commodities of unrecognized quality in order for them to be produced. Furthermore, it may be shown that whenever there exists an equilibrium with $\Sigma = 1$, there also exists a nonmonetary equilibrium which implies a higher level of welfare (details are available upon request). This leaves us with exactly five candidate equilibria that have the potential to Pareto dominate active nonmonetary equilibria: $(0,\Phi)$, $(0,1)$, $(\Phi,0)$, (Φ,Φ) and $(\Phi,1)$.

TABLE 1: Candidate monetary equilibria.

	$\Omega = 0$	$\Omega = \Phi$	$\Omega = 1$
$\Sigma = 0$	$(0,0)^*$	$(0,\Phi)$	$(0,1)$
$\Sigma = \Phi$	$(\Phi,0)$	(Φ,Φ)	$(\Phi,1)$
$\Sigma = 1$	$(1,0)^*$	$(1,\Phi)^*$	$(1,1)^*$

*Dominated by a nonmonetary equilibrium.

Because there are several qualitatively different types of equilibria and some of them are not amenable to simple closed form solutions, we do not attempt a complete analytical characterization, as we did for the nonmonetary equilibria in the previous section. Below we will describe the set of equilibria numerically for certain parameter values, and use the results to illustrate and interpret the potential welfare improving role of money. However, we start with a case in which analytical results are relatively tractable, the equilibrium with $(\Sigma,\Omega) = (0,\Phi)$. This case is important, because there is a region of parameter space (characterized by low values of θ) in which this equilibrium exists, no other monetary equilibrium exists, and the only nonmonetary equilibrium is degenerate.[14]

When $(\Sigma,\Omega) = (0,\Phi)$, the fact that $0 < \Omega < 1$ implies the final term in (5.3) vanishes. Then using $\Sigma = 0$, we can simplify (5.1)-(5.3) as follows:

(5.4) $$rV_g = (1-M)\theta^2 p(u-\gamma) + M[\theta+(1-\theta)\Omega](V_m-V_g)$$

(5.5) $$rV_b = M(1-\theta)\Omega(V_m-V_b)$$

(5.6) $$rV_m = (1-M)\theta p(u-\gamma+V_g-V_m).$$

To verify that $(0,\Phi)$ is an equilibrium, we need to find values for p and Ω in (0,1) with the following properties:

(5.7) $V_m-V_g \geq 0$ (so that accepting money is a best response);

(5.8) $V_g-\gamma = V_b$ (so that $0 < p < 1$ is a best response);

(5.9) $p\theta(u-\gamma)-(1-p)\gamma \leq 0$ (so that $\sigma = 0$ is a best response);

(5.10) $pu-\gamma+V_g-V_m = 0$ (so that $0 < \omega < 1$ is a best response).

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[14] At the point in parameter space where $u = 2\gamma$ and $r = 0.01$, the situation is as depicted in the first panel of Figure 2, with $\theta_3 = 0.11$, $\theta_1 = 0.505$ and $\theta_2 = 0.673$. As seen in the figure, for $\theta < \theta_3 = 0.11$ no active nonmonetary equilibrium exists. For $\theta = 0.10$, one can show numerically that no monetary equilibria other than $(0,\Phi)$ exist for any value of M, and equilibrium $(0,\Phi)$ exists at least for some M. An appropriate appeal to continuity guarantees that the situation is qualitatively similar in an open neighborhood of this point.

We show in the Appendix that all of these conditions are satisfied, and hence the $(0,\Phi)$ equilibrium exists, for all M in an interval $(\underline{M},\overline{M})$, where $0 < \underline{M} < \overline{M} < 1$, as long as r is not too large given u, γ and θ. As stated above, for a range of parameters with low values of θ, this is the unique active equilibrium: none of the other potential monetary equilibria in Table 1 exist, and the only nonmonetary equilibrium is the degenerate one that yields $Z = 0$. The $(0,\Phi)$ monetary equilibrium yields $Z > 0$ when it exists, and therefore Pareto dominates the only (inactive) nonmonetary equilibrium. This illustrates how the introduction of fiat currency can improve welfare by allowing the existence of an active monetary equilibrium when the private information problem is so severe that economic activity would otherwise shut down.

The private information problem is severe when θ is small, which makes the incentive to produce low quality output great. How is it that the use of fiat money can mitigate against the incentive problem under these circumstances? First, an agent wishing to sell a good commodity and confronted with an offer of an unrecognizable commodity has the luxury of turning down the offer and demanding either cash or a good that can be recognized. If the probability p is low, it is advantageous to incur the waiting cost and hold out for either money or something that can be recognized rather than taking a chance. Furthermore, the $(\Sigma,\Omega) = (0,\Phi)$ strategies impose the greatest amount of discipline on the producers of bad commodities. Since $\Sigma = 0$, bad commodity holders can never trade directly for a good commodity. They must trade first for money, which is possible since $\Omega > 0$ but not automatic since $\Omega < 1$, and then use the money to purchase a good commodity. This effectively subjects bad commodity traders to a cash-in-advance constraint, while good commodity holders can trade for cash whenever the opportunity presents itself, but may also barter directly whenever they meet a good commodity trader and they recognize each others' inventories.[15]

If the private information problem is not too severe, however, other equilibria may appear. For $u = 2\gamma$ and $r = .01$, when $\theta = 0.10$, $(0,\Phi)$ is the only active equilibrium that can exist. But when we increase θ to 0.20, all five of our candidate monetary equilibria exist for some values of M, and there also exists a unique active nonmonetary equilibrium, which is of type *c*. Figure 3 shows equilibrium welfare for a range of values of M for each of the equilibria when they exist. All of the monetary equilibria dominate the nonmonetary equilibrium for some values of M. The optimal quantity of money is the value of M that maximizes welfare across all equilibria, and the optimal monetary equilibrium is the one that yields the highest welfare at the optimal quantity of money. As seen in Figure 3, in this example the optimal monetary equilibrium is $(\Sigma,\Omega) = (0,\Phi)$, the one analyzed analytically above. For a range of parameters values that we examined, the optimal monetary equilibrium was either $(0,\Phi)$ or $(0,1)$, both of which impose discipline on the producers of bad commodities by effectively subjecting them to a cash-in-advance constraint.[16]

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[15] We emphasize that there is always *some* barter in any active equilibrium. If an agent has a good commodity, the probability of a direct exchange for another good commodity is bounded below by $(1-M)p\theta^2$.

[16] It might be thought that the more natural equilibria would involve $\Omega = 0$, which imposes discipline on the producers of bad commodities by forcing them to barter directly, since money traders never accept commodities they do not recognize. As seen in Figure 3, an equilibrium with $(\Sigma,\Omega) = (\Phi,0)$ exists and dominates the nonmonetary equilibrium for some values of M, but it can be dominated by other monetary equilibria. Direct barter is not particularly difficult in this environment and, therefore, forcing producers of bad commodities to barter does not impose a very effective discipline on them. It is more effective to force them to use money, since this requires

We computed the optimal quantity of money for a range of values for θ, with the other parameter values as described above. One perhaps surprising result is that the optimal quantity of money is not monotonic in θ. For low values of θ, the optimal monetary equilibrium is of type $(0,\Phi)$ and the optimal quantity of money falls with θ. For higher values of θ, the optimal monetary equilibrium is of type $(0,1)$ and the optimal quantity of money rises with θ. For sufficiently high values of θ the optimal quantity of money is zero since there will exist a type a nonmonetary equilibrium. Our calculations also reveal that p, the fraction of commodities that are high quality, is actually lower in the optimal monetary equilibrium than in the nonmonetary equilibrium, and that $(1-M)p$, the fraction of all traders holding high quality commodities, may be higher or lower. However, the probability of acquiring a good commodity each period was always highest in the optimal monetary equilibrium. Money need not raise the fraction of good commodities in order to improve welfare; it works by promoting useful exchange.

6. Conclusion

We have analyzed a model of production and exchange with private information, abstracting from the double coincidence problem in order to isolate the impact of informational frictions. With no private information, there is a nonmonetary equilibrium in which all agents produce good quality commodities, trade, and consume every period. In this case, there is no role for money in the sense that a monetary equilibrium may exist but it is Pareto dominated by the nonmonetary equilibrium. With a little private information, the complete information outcome can still be supported as an

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making two trades rather than one in order to acquire a consumption good.

equilibrium, but as the private information problem becomes severe other equilibria emerge with production of bad commodities. For some parameter values there exist multiple Pareto-ranked nonmonetary equilibria. The economy could conceivably end up in a dominated equilibrium, due to a type of coordination failure: if other traders produced fewer bad commodities then it would be in each individual's self interest to do the same, so as to reduce the chance of being turned down in an exchange opportunity.

The introduction of fiat money can lead to active equilibria when the only nonmonetary equilibrium is degenerate, as is the case when the private information problem becomes severe. Even when active nonmonetary equilibria exist, the introduction of fiat money can increase welfare. However, money never completely alleviates the private information problem, since welfare is lower in the optimal monetary equilibrium than in the nonmonetary equilibrium with full information. Furthermore, any monetary equilibrium that dominates an active nonmonetary equilibrium has the property that some bad quality commodities are produced and traded. Hence, money does not drive out all of the bad commodities. What the presence of fiat currency does is to enlarge the strategy space, and this leads to the possibility of agents adopting trading strategies that ultimately increase the probability of acquiring high quality output. For example, money gives good commodity traders the luxury of demanding payment in cash, which in turn generates positive incentive effects on producers.

We close by pointing out a fundamentally important property of fiat money illustrated by this class of models. Since bad commodities are similar to money in that they are perfectly durable objects with zero consumption value and can be produced at zero cost, in our model, one might conjecture that bad commodities could serve as a medium of exchange. But there is one important difference between fiat money and bad commodities: the latter can be produced privately. If bad commodities are to be accepted in exchange by holders of good commodities, we require $V_b \geq V_g$. But if any good commodities are to be produced at all, we require $V_g - \gamma \geq V_b$. Hence, if bad commodities are accepted as a medium of exchange, no one will ever produce good quality output. Either bad commodities will have to stop being media of exchange or the economy will be stuck in an inactive equilibrium.

The important characteristics of money clearly include durability and recognizability; but it is also important that money cannot be produced privately at zero (or very low) cost. However, there could be equilibria with privately produced money if the cost of producing it were sufficiently high. Suppose there is some intrinsically worthless and perfectly recognizable and durable object – say, a precious metal – that can be produced privately at a cost per unit that exceeds the cost of producing a good commodity. We could then look for equilibria where agents are indifferent between producing the precious metal and other commodities, and determine endogenously the quantity of this privately produced money in circulation. As Friedman (1960) has pointed out, however, it is socially preferable to adopt a fiat currency which can be produced essentially for free, as long as its private production (counterfeiting) can be controlled, since this arrangement avoids the initial production cost of private money. We leave further exploration of this topic to future work.[17]

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[17] Several other potentially interesting extensions suggest themselves. One can imagine allowing agents to invest in information, or allowing the economy to somehow choose the number and types of commodities it produces. In these or other ways, we could make θ endogenous in the model. One can also imagine deriving a role for specialized traders or middlemen. The effects on equilibrium and welfare of these and other generalizations are beyond the scope of the current project, but their analysis seems feasible.

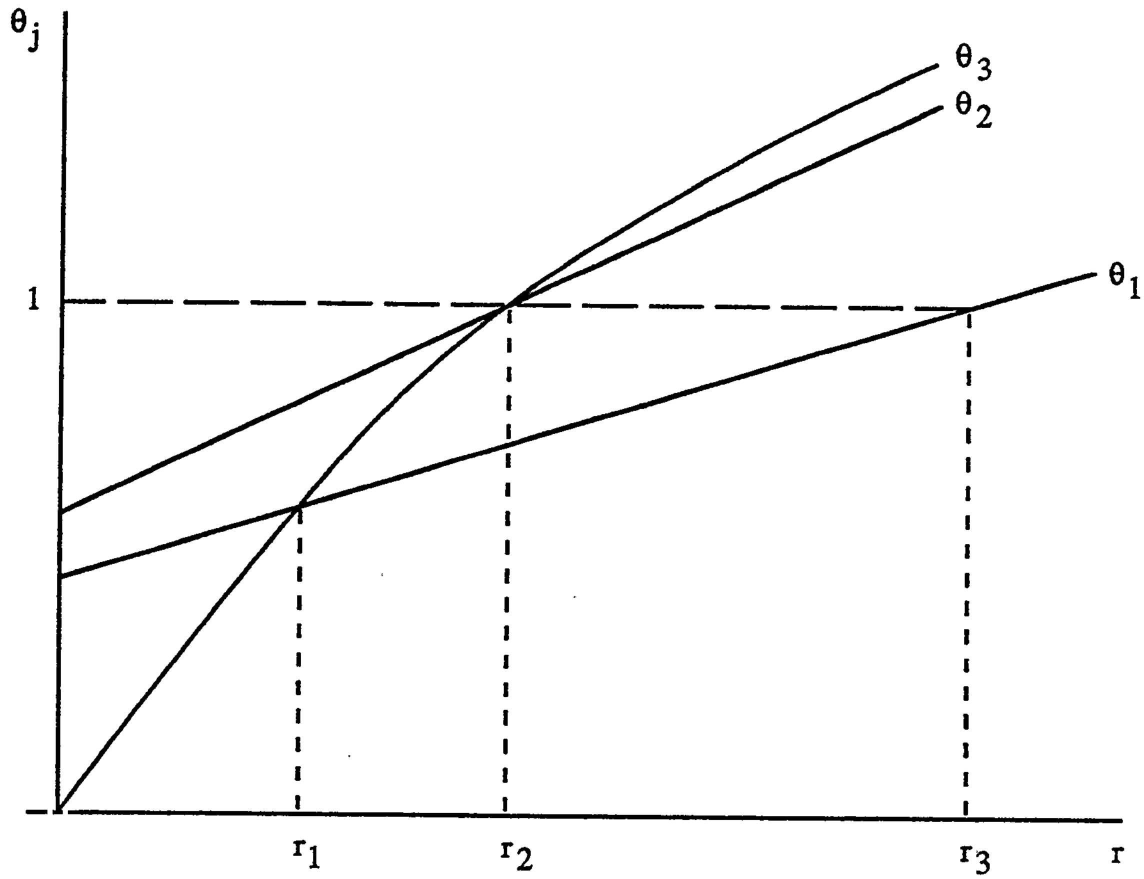


FIGURE 1

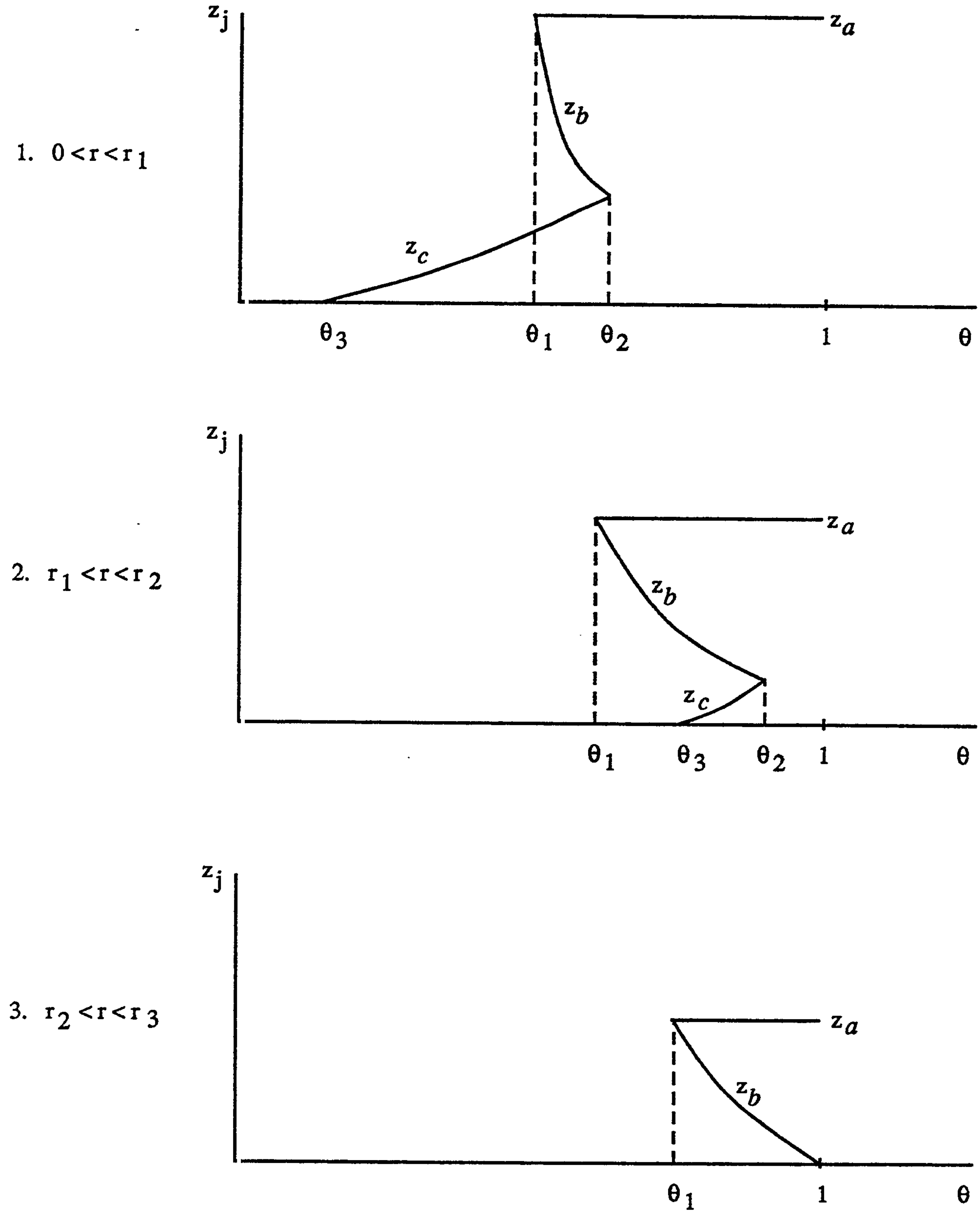
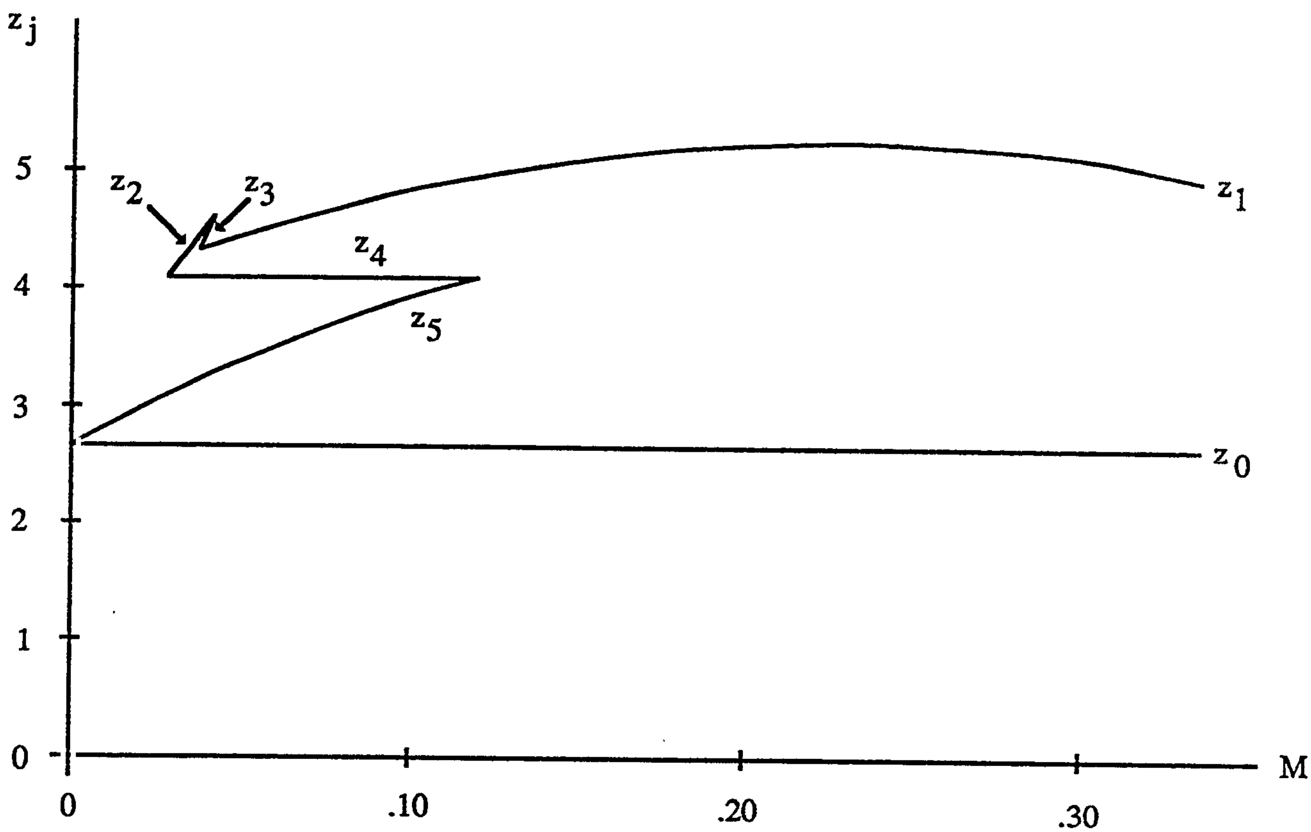

1. $0<r<r_1$
z_j
z_a
z_b
z_c
θ_3
θ_1
θ_2
1
θ
2. $r_1<r<r_2$
z_j
z_a
z_b
z_c
θ_1
θ_3
θ_2
1
θ
3. $r_2<r<r_3$
z_j
z_a
z_b
θ_1
1
θ

FIGURE 2



z_0 = welfare in nonmonetary equilibrium

z_1 = welfare in (0, Φ) equlilibrium

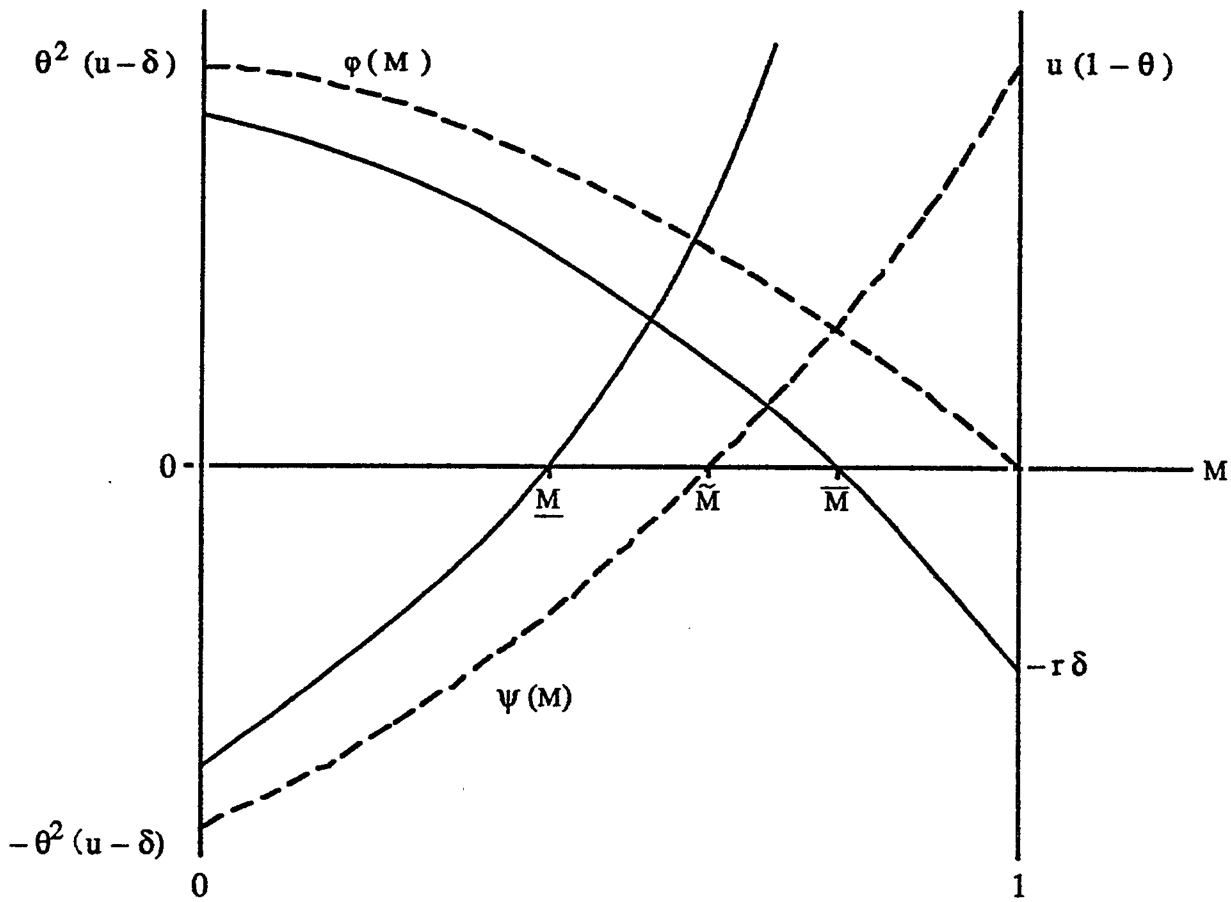
z_2 = welfare in (Φ, 1) equilibrium

z_3 = welfare in (0, 1) equilibrium

z_4 = welfare in (Φ, Φ) equilibrium

z_5 = welfare in (Φ, 0) equilibrium

FIGURE 3



$\theta^2\ (u - \delta)$
$\varphi(M)$
$u\ (1 - \theta)$
0
M
$\underline{M}$
$\tilde{M}$
$\overline{M}$
$-r\delta$
$\psi(M)$
$-\theta^2\ (u - \delta)$
0
1

FIGURE 4

Appendix A

Here we show that $(0,\Phi)$ is a monetary equilibrium for all M in a nondegenerate interval $(\underline{M},\overline{M})$, as long as r is not too big given u, γ and θ. First, note that (5.10) implies $V_m - V_g = pu-\gamma$. Using this and subtracting (5.4) from (5.6), we find

$$M(pu-\gamma)(1-\theta)\Omega = (1-M)\theta(1-\theta)p(u-\gamma) - [r+(1-M)\theta p+\theta M](pu-\gamma). \tag{A.1}$$

Now using (5.8) and subtracting (5.5) from (5.4), we find

$$M\gamma(1-\theta)\Omega = M\theta(pu-\gamma) + (1-M)\theta^2 p(u-\gamma) - r\gamma. \tag{A.2}$$

Solving (A.1) and (A.2), we find

$$p = \frac{\gamma}{\gamma+(u-\gamma)[\theta+(1-\theta)M]}, \tag{A.3}$$

$$\Omega = \frac{(1-M)\theta(u-\gamma)[\theta+(1-\theta)M]-rK}{(1-\theta)MK}, \tag{A.4}$$

where $K = \gamma + (u-\gamma)[\theta+(1-\theta)M]$.

By construction, equilibrium conditions (5.8) and (5.10) are satisfied (we used them to solve for p and Ω). Simple algebra implies that (5.7) and (5.9) hold for all parameter values. Clearly, (A.3) implies $0 < p < 1$. Therefore, all that remains to check in order to verify that $(0,\Phi)$ is an equilibrium is the condition $0 < \Omega < 1$. Equation (A.4) implies $\Omega > 0$ if and only if $\varphi(M) > 0$ and $\Omega < 1$ if and only if $\psi(M) > 0$, where

$$\varphi(M) = -\theta(1-\theta)(u-\gamma)M^2 + (1-2\theta-r)(1-\theta)(u-\gamma)M + C \tag{A.5}$$

$$\psi(M) = (1-\theta)(u-\gamma)M^2 + [(1-\theta)\gamma+(r-\theta r+\theta^2)(u-\gamma)]M - C \tag{A.6}$$

and $C = \theta(u-\gamma)(\theta-r) - r\gamma$. The functions φ and ψ are shown for the case $r = 0$ as the dashed curves in Figure 4, and both are positive if and only if M is in the nondegenerate interval $(\tilde{M},1)$. As r increases, the functions shift as indicated by the solid curves. We conclude that, as long as r is not too large given u, γ and θ, there will exist an equilibrium with $(\Sigma,\Omega) = (0,\Phi)$ if and only if $M\in(\underline{M},\overline{M})$, where $0 < \underline{M} < \overline{M} < 1$.

Appendix B

Here we derive equations (5.1)-(5.3), describing the best response problem for an agent in a monetary equilibrium with private information. Note that the best response problem in a nonmonetary equilibrium or without private information is a special case, derived by setting $M = 0$ or $\theta = 1$. To reduce notation, let $\lambda_1 = M[\theta+(1-\theta)\Omega]$ be the probability of meeting a money trader who is willing to trade, let $\lambda_2 = (1-M)\theta p[\theta+(1-\theta)\Sigma]$ be the probability of meeting a commodity trader with a good commodity that can be recognized who is willing to trade, let $\lambda_3 = (1-M)(1-\theta)$ be the probability of meeting a commodity trader with a commodity that cannot be recognized, and let $\Delta = [\theta+(1-\theta)\Sigma]$ be the probability that an agent with a good commodity is willing to trade.

Consider an agent with a good commodity. Bellman's equation of dynamic programming says that V_g is the discounted, maximized, expected value of the value function next period:

$$(B.1)\qquad V_g = \frac{1}{1+r}\max_{\sigma}\left\{\lambda_1 V_m + \lambda_2(u+W) + \lambda_3\sigma p\Delta(u+W) + \lambda_3\sigma(1-p)W \right.$$

$$\left. + [\lambda_3\sigma p(1-\Delta) + \lambda_3(1-\sigma) + (1-\lambda_1-\lambda_2-\lambda_3)]V_g\right\}.$$

With probability λ_1 money is acquired; with probability λ_2 a good commodity is acquired; with probability λ_3 an agent with an unrecognized commodity is encountered and, in this case, with probability $\sigma p\Delta$ a good commodity is acquired while with probability $\sigma(1-p)$ a bad commodity is acquired. If none of these events occur, nothing is acquired, and the agent continues with V_g.

If we multiply by 1+r and subtract V_g from both sides of (B.1), we arrive at

$$(B.2)\qquad rV_g = \lambda_1(V_m-V_g) + \lambda_2(u+W-V_g)$$

$$+ \lambda_3\max_{\sigma}\left\{\sigma p\Delta(u+W-V_g) + (1-p)(W-V_g)\right\}.$$

Upon substitution of $W = V_g-\gamma$ and the definitions of λ_j and Δ, it is seen that (B.2) is identical to (5.1). The derivations of (5.2) and (5.3) are very similar and are therefore not presented.

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