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A Contribution to the Theory of Pork Barrel Spending

V. V. Chari*

University of Minnesota
and Federal Reserve Bank of Minneapolis

Harold L. Cole*

Federal Reserve Bank of Minneapolis

ABSTRACT

In this paper we present a formal model of vote trading within a legislature. The model captures the conventional wisdom that if projects with concentrated benefits are financed by universal taxation, then majority rule leads to excessive spending. This occurs because the proponent of a particular bill only needs to acquire the votes of half the legislature and hence internalizes the costs to only half the representatives. We show that Pareto superior allocations are difficult to sustain because of a free rider problem among the representatives. We show that alternative voting rules, such as unanimity, eliminate excessive spending on concentrated benefit projects but lead to underfunding of global public goods.

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Government programs that deliver benefits to small, well-organized groups and impose costs on the rest of the population are called *pork barrel* programs. Such programs frequently lead to inefficient outcomes, in the usual sense that the program's costs exceed its benefits. In a representative democracy like the United States, such programs require the assent of at least a majority of the legislators. Why do legislators consent to such pork barrel programs? Furthermore, if a particular inefficient program benefits a small group, why don't legislators agree to an alternative policy that leaves this group at least as well off and makes the rest of the people better off?

Our answer to the first question is that side payments (say, in the form of campaign contributions) can be used to induce a majority of legislators to vote for pork barrel programs.[1] Our answer to the second question is that the free rider problem inhibits cooperative action which might lead to Pareto improvements. The idea that the free rider problem leads to programs with concentrated benefits and diffuse costs to be adopted is very much a part of the conventional wisdom in political economy. (See, for example, Downs 1957, Buchanan and Tullock 1962, Olson 1965, Ferejohn 1974, and Ferejohn and Fiorina 1975.)

In this paper, we make three contributions to the conventional wisdom. First, we make explicit the modeling assumptions which lead to a free rider problem. Specifically, we show that private information about program benefits plays an important role in generating a free rider problem. Second, we show that pork barrel spending can be eliminated if the voting rules in the legislature require unanimity. Finally, we show that the allocation of global public goods is extremely inefficient with unanimity. The last two findings suggest that designers of voting

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[1]An extensive literature analyzes logrolling as a way to induce a majority of legislators to vote for pork barrel programs (see Weingast, Shepsle, and Johnsen 1981; Baron and Ferejohn 1989; Chari, Jones, and Marimon 1994; and the references therein).

systems must confront the tradeoff between systems that work well for allocating local public goods and those that work well for allocating global public goods.

We first consider a model in which a central government provides local public goods financed by uniform taxation. The amounts of these goods are determined by majority voting in a legislature. We allow legislators to make payments to other legislators contingent on how these other legislators vote. We show that in such an environment, an inefficiently high level of local public goods is provided. This inefficiency is reduced if a supermajority vote is required for passage, and the allocations are efficient if unanimous consent is required. We then allow legislators to make payments to bill proposers contingent on the nature of the bills that are proposed. We show that when we allow for such payments, there is a free rider problem, so that an inefficiently high level of expenditures results (provided unanimous consent is not required) if the legislature has a large number of members and if other legislators are uncertain about the proposer's preferences. Again, the inefficiency disappears with unanimous consent rules.

Since unanimous consent seems desirable for local public goods, we ask how such a rule works for global public goods. We show that such a rule can work very badly. This result is familiar from the literature on mechanism design in public good economies (Rob 1989, Mailath and Postlewaite 1990, and Chari and Jones 1991). The conflict between desirable voting rules for allocating global public goods and those for allocating local public goods means that a single voting rule cannot achieve desirable outcomes for both types of problems.

There is a large related literature on public choice, rent-seeking behavior, and political economy. While we do not survey this literature, several related papers deserve mention. Philipson and Levy (1992) and Philipson and Snyder (1992) have a model of vote trading with an intermediary. The results that they derive are somewhat similar in character to our complete information results. Weingast, Shepsle, and Johnsen (1981) offer an explanation for pork barrel projects based on differences in preferences between the legislators and the voters. They argue plausibly for such differences in preferences, but it is not clear why voters would elect such legislators.

Baron and Ferejohn (1989) study a model of redistributive politics under a variety of rules governing legislative behavior. They show that the proposer of a redistributive policy extracts a large amount from the other legislators. In their model, redistribution has no efficiency costs. Our model is, in some ways, an extension of their results to environments with efficiency losses from excessive redistribution. Our work is also closely related to that of Chari, Jones, and Marimon (1994). They extend Baron and Ferejohn's model to allow for logrolling. In contrast, in our model, we assume that votes are traded for money (or campaign contributions). The motivation for our assumption is the observation that money does play a role in intermediating vote trades and determining legislative outcomes.[2]

1. Local Public Goods and Vote Buying

In our model of local public goods, there are I districts, each of which has a single representative. Both districts and representatives are indexed by $i = 1, \ldots, I$. For notational convenience, we normalize the population of each district to unity. The government provides a local public good for each district. Let g_i denote the amount of the local public good provided in district i. The local public goods are financed by a uniform tax τ on the entire population, so that $\tau = (1/I)\Sigma_{i=1}^{I} g_i$. Each representative's preferences over the level of public expenditures and

[2]For example, the *New York Times*, March 21, 1994, p. A16, reports that leaders in Congress form leadership Political Action Committees (PACs) and use the proceeds to contribute to other candidates' campaigns. Senator Robert Dole is reported to have raised over $7 million; Congressman Newt Gingrich, over $5 million; and Congressman Richard Gephardt, over $2 million. Other examples are the political parties that funnel payments to candidates and the various interest groups that make campaign contributions to secure votes.

taxes in his or her district, as well as over the representative's own private consumption c_i, are given by

(1.1) $U_i(g_1,\ldots,g_I,c_i) = \theta_i f(g_i) - \tau + c_i$.

We assume that f is strictly concave and twice differentiable, $f(0) = 0$, $f'(0) > 1$, and $\lim_{g\to\infty} f'(g) = 0$. For now, θ_i is a given parameter known to all agents. In later sections, we will assume that θ_i is private information to representative i.

There are a variety of ways of modeling the bargaining process among the representatives which determines the provision of the local public goods. We consider an environment in which representatives can make side payments to influence each other's votes. These side payments can be interpreted as payments made by constituents in a district to other legislators.[3] (This interpretation is discussed at the end of this section.) We take as a given institutional feature that the spending outcomes are determined by plurality voting separately on each local public good. Specifically, representative i proposes a bill to spend g_i units in district i. If more than kI representatives vote for the bill, it passes, where $k \in [0.5,1]$ is fixed exogenously as part of the prevailing political constitution. If not, spending in that district is set at zero. Let K denote the smallest integer such that $K + 1 > kI$. For notational convenience, we will suppress spending decisions on all districts except district 1. This suppression is without loss of generality given that preferences are separable and linear in taxes.

Formally, we model the level of spending in each district as a two-stage game. In stage 1, representative 1 chooses a number $g \in \mathbf{R}_+$ and a vector $m \in \mathbf{R}_+^{I-1}$. An element of the vector, m_i, is interpreted as the amount of money that representative 1 has pledged to pay

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[3]An alternative interpretation of these side payments, suggested by Antonio Merlo, is that they are construction contracts for those components of the local public good which do not have to be produced on-site.

representative i if i votes for representative 1's proposal to spend g on local public goods in 1's district. In stage 2, each representative chooses a number $v_i \in \{0,1\}$. The number v_i is interpreted as representative i's vote on 1's spending proposal, where a 1 is interpreted as a "yes" vote and a zero as a "no" vote. If $(1/I)\Sigma_{i=1}^{I}v_i > k$, then the proposal passes, and the level of government spending in district i is g, and if $(1/I)\Sigma_{i=1}^{I}v_i \leq k$, then the proposal fails and the spending level in district i is zero.

The payoff to representative 1 from this game is given by

$$(1.2)\quad U_1(m,g,v) = \theta f(\delta(v)g) - \delta(v)g/I - \sum_{i=2}^{I} m_i v_i,$$

and the payoff to each of the other representatives is given by

$$(1.3)\quad U_i(m,g,v) = -\delta(v)g/I + m_i v_i, \text{ for } i = 2, \ldots, I,$$

where

$$(1.4)\quad \delta(v) = \begin{cases} 1 & \text{if } (1/I)\sum_{i=1}^{I} v_i > k \\ 0 & \text{otherwise} \end{cases}.$$

An equilibrium of the game is a vector $m \in \mathbf{R}_{+}^{I-1}$, a number $g \in \mathbf{R}_{+}$, and a set of functions $v_i(g,m) \rightarrow \{0,1\}$, for $i = 2, \ldots, I$, such that

(i) for all $g \in \mathbf{R}_{+}$ and $m \in \mathbf{R}_{+}^{I-1}$, v_1 satisfies

$$(1.5a)\quad \theta f(\delta(v)g) - \delta(v)g/I - \sum_{i=2}^{I} m_i v_i$$
$$\geq \theta f(\delta(v_{-1},\hat{v}_1)g) - \delta(v_{-1},\hat{v}_1)g/I - \sum_{i=2}^{I} m_i v_i \quad \text{for } \hat{v}_1 \neq v_1;$$

(ii) for $i = 2, \ldots, I$, v_i satisfies

$$(1.5b)\quad -\delta(v)g/I + m_i v_i \geq -\delta(v_{-i},\hat{v}_i)g/I + m_i\hat{v}_i \quad \text{for } \hat{v}_i \neq v_i; \text{ and}$$

(iii) given $v(\cdot)$, g and m satisfy

$$(1.6) \quad \theta f(\delta(v(g,m))g) - \delta(v(g,m))g/I - \sum_{i=2}^{I} m_i v_i(g,m)$$
$$\geq \theta f(\delta(v(\hat{g},\hat{m}))\hat{g}) - \delta(v(\hat{g},\hat{m}))\hat{g}/I - \sum_{i=2}^{I} \hat{m}_i v_i(\hat{g},\hat{m})$$

for all $\hat{g}$ and $\hat{m}$.

Condition (i) is a requirement that given g and the voting behavior of the other representatives, representative 1 weakly prefers to vote according to the equilibrium prescription. Condition (ii) is an analogous condition for representatives $i \neq 1$. Condition (iii) is a requirement that given the specification of voting behavior, g and m are chosen optimally by representative 1.

Next, we construct an equilibrium, called the *pivotal* equilibrium, of the model. In this equilibrium, the bill receives exactly $K + 1$ affirmative votes, so that every legislator who votes for the bill plays a pivotal role. Specifically, we assume that exactly K representatives are each paid an amount $m = g/I$, where g is the spending level proposed in representative 1's bill. This spending level is chosen to solve the following problem:

$$(P1) \quad \max_{g,m} \theta f(g) - g/I - Km$$

subject to

$$(1.7) \quad m \geq g/I.$$

This concave programming problem has a unique solution $(\bar{g},\bar{m})$ given by

$$(1.8) \quad \theta f'(\bar{g}) = \frac{K + 1}{I}$$

and

$$(1.9) \quad \bar{m} = \bar{g}/I.$$

Now for the equilibrium strategies. For representative 1, in the first stage, select K representatives at random and offer each of them $\bar{m} = \bar{g}/I$, while offering the rest zero, and propose a bill to spend $\bar{g}$ in district 1. The second-stage voting strategy of representative 1 is given by

(a) $v_1 = 1$ if $\theta f(g) - g/I \geq 0$ and $v_1 = 0$.

We describe the second-stage voting strategies of the other representatives in two parts. First, we let

(b) $v_i = 0$ if $m_i = 0$ and $v_i = 1$ if $m_i \geq g$.

Next, we specify their voting behavior when $0 < m_i < g/I$. Let L equal the number of representatives who have been promised payments of at least g/I. Recall that K is defined as the smallest integer such that $K + 1 > kI$. Set $v(\cdot)$ as follows:

(c) if $L \geq K$, set $v_i = 1$ if $0 < m_i < g/I$

(d) if $L < K$, set $v_i = 1$

for $K - L - 1$ representatives chosen randomly for whom $0 < m_i < g/I$, and set $v_i = 0$ for the rest.

The pivotal equilibrium strategies are then described by $\bar{g}$, $\bar{m}$, and the voting functions given above.

PROPOSITION 1. The pivotal strategies $\{g,m,v(\cdot)\}$ constructed above constitute an equilibrium.

Proof. The proof consists of verifying that $\{g,m,v(\cdot)\}$ satisfy (1.5) and (1.6). Since $v(\cdot)$ satisfies (a)–(d), (1.5) is satisfied. Given the specification of $v(\cdot)$, representative 1 does best by setting m_i to be at least as large as g/I on exactly K representatives. Since $\bar{g}$ and $\bar{m}$ solve (P1), it follows that (1.6) is satisfied. □

The equilibrium that we have just described is clearly not unique, even among the class of pivotal equilibria, since there are many ways in which the choice of whose vote to buy could be specified. However, as the following proposition makes clear, within this class of equilibria there is a unique level of local government spending.

PROPOSITION 2. There is a unique level of government spending in any pivotal equilibrium given by $g = \bar{g}$.

Proof. Since by the definition of a pivotal equilibrium, only K + 1 representatives vote "yes" on any proposal, a pivotal equilibrium must satisfy $m_i \geq g/I$ for exactly K representatives. Since (P1) gives the best allocation for representative 1 given this constraint, the proposition follows. □

The level of government spending is inefficiently high in the pivotal equilibrium to the extent that $k < 1$. The simplest way to see this is to consider the level that would be chosen by an egalitarian social planner. The planner would set the level of spending in district 1 so that $\theta f'(g) = 1$. In the pivotal equilibrium, $\bar{g}$ satisfies $\theta f'(\bar{g}) = (K+1)/I$. For large I, $(K+1)/I$ is approximately equal to k. If $k < 1$, it follows that government spending is inefficiently high in the pivotal equilibrium. The extent to which a representative internalizes the effect of his or her proposed spending on other districts depends on how close k is to 1. It is also interesting to compare the pivotal equilibrium outcome to that which would occur if side payments were prohibited; that is, $m_i = 0$ for all $i = 2, \ldots, I$. In the voting stage, $v(\cdot)$ must still satisfy (b), which implies that zero local spending is the unique outcome. Thus, the outcome under majority voting without vote buying is for spending on local public goods to be inefficiently low.[4]

There can also exist nonpivotal equilibria, since any representative who believes that his or her vote cannot affect the outcome of the proposal to spend g will strictly prefer to set $v_i = 1$ if $m_i > 0$. For example, suppose representative 1 offers $m_i = \epsilon > 0$ to each of the other representatives where ϵ is an arbitrarily small number. Then there is an equilibrium in which the proposal passes unanimously because no representatives believe that the outcome can be affected if they change their own vote. Other equilibria like this one can also be constructed.

We find such equilibria inherently implausible because they cease to exist under minor perturbations of the game. To demonstrate this implausibility, we consider two perturbations. In the first, each of the representatives 2, ..., I only observes payments made to him or her. In the second, representatives vote sequentially.

Consider the incomplete information variant. Since we now have a game with incomplete information, we must describe how representatives form beliefs about how others are likely to vote. That is, we need to describe how representatives form beliefs about how much other representatives have been promised for their votes by representative 1. In order to introduce these inferences, we now have to distinguish between the history of the game in the second stage, $h = (g,m)$, and the private history that a given representative—say, i—observes, $h_i = (g,m_i)$, $i = 2, \ldots, I$. We also have to specify each representative's beliefs about the history of the game h, based upon the representative's private history, h_i. Let $\mu_i(h|h_i)$ denote representative i's

[4]This is the outcome of median voter models. A similar result, that the level of expenditure is greater than that which would emerge from a simple median voter model, was derived by Romer and Rosenthal (1979) in a very different model in which they assumed that an expenditure maximizing bureaucracy had monopoly control over the proposal right. It is also worth noting that the prediction of our model is for a substantially lower level of government spending than would be implied by a "norm of universalism" in which, by convention, members were allowed to set spending levels in their own districts (Weingast 1979).

conditional probability distribution over h given h_i. Note that representative 1's private history is the same as the history of the game.

We now define a perfect Bayesian equilibrium of this model as a scalar g and a vector m and a set of pairs of functions $\{v_i(h_i),\mu_i(h|h_i)\}_{i=1}^{I}$ such that, for all g and m, v_i satisfies

$$\text{(1.5b}'\text{)} \qquad E_{\mu_i(h|h_i)}\{-\delta(v)g/I + m_i v_i\} \geq E_{\mu_i(h|h_i)}\{-\delta(v_{-i},\hat{v}_i)g/I + m_i\hat{v}_i\}, \quad i = 2, \ldots, I,$$

for all $\hat{v}_i \in \{0,1\}$, along with conditions (1.5a) and (1.6) from our complete information game, and that μ_i be consistent with the specification of the strategies. Note that since in our definition the strategies are pure, μ_i must put probability 1 on the equilibrium history, as long as the private histories are consistent with it. Thus, the only issue with respect to μ_i will be specifying out of equilibrium beliefs.

It is easy to see how one would specify beliefs so that the pivotal equilibrium outcomes are perfect Bayesian equilibrium outcomes. But the nonpivotal equilibria cannot be sustained in the incomplete information game. In the nonpivotal equilibria, representatives vote for the proposal even when they would be better off if it did not pass because they cannot affect the outcome by changing their vote. Suppose such outcomes are equilibrium outcomes in the incomplete information game. Now representative 1 gains by paying a positive amount to exactly K legislators since no representative observes payments to other representatives. Thus, each representative who receives a positive payment must believe that he or she is pivotal and should behave accordingly. This rules out nonpivotal equilibria with $m^* > 0$. The nonpivotal equilibrium with $\bar{m} = 0$ seems implausible since all the representatives are playing a weakly dominated strategy in the voting stage.

In the second perturbation, suppose legislators observe payments to each other but vote sequentially on representative 1's proposal. Now simple backward induction arguments imply that the pivotal equilibrium is the unique subgame perfect equilibrium. To see this, consider the problem faced by the last representative. This representative votes in his or her own best interest if the representative is pivotal. Recognizing this, the next-to-last representative behaves as in the pivotal equilibrium. By induction, all representatives behave as prescribed in the pivotal equilibrium.

For future use, note that the construction of the pivotal equilibrium does not require that the other representatives know θ. Thus, our results carry through to the situation where θ is private information to representative 1. It should also be clear that our results carry through unchanged if the legislature provides public goods to all the districts. In particular, if the legislature votes sequentially on each local public good, then simple backward induction arguments imply that the equilibrium outcome in each district is as described in the pivotal equilibrium.

Incorporating Constituents' Behavior

The model we have thus far described is of a legislature bargaining over spending that benefits particular legislators financed by equal taxes on all legislators. We now digress briefly to indicate how one might extend the model to capture some features of the behavior of the constituents whom the legislators represent.

Suppose that each district has N constituents and each district is represented by a legislator. Each constituent's preferences are represented by a utility function $f(g_i) - t + c_{ij}$, where g_i denotes local spending in district i, t denotes the level of uniform taxes, and c_{ij} denotes private consumption by constituent j in district i. We assume that constituents can coordinate their activities perfectly within a district, but not across districts. The constituents in a particular district jointly choose a level of side payments to legislators to influence the voting outcome. We assume, realistically, that side payments received by a legislator are retained for private consumption by that legislator and are not shared with the constituents. The side payment levels are chosen to maximize the sum of utilities of the constituents in a district, taking as given the side payments made by constituents in other districts. The voting game is, in all other respects, as described above.

The problem faced by constituents in district 1 is to choose side payments m to K legislators to solve the following problem:

$$(P1') \quad \max N\left[\theta f(g) - \frac{g}{IN}\right] - Km$$

subject to

$$m \geq \left[\frac{g}{IN}\right].$$

It should be clear that the solution to (P1′) is a pivotal equilibrium outcome and has the same *qualitative* features as the outcome in the voting game described above. Notice that no single district gains by prohibiting its legislator from receiving contributions. If a district prohibited its legislator from accepting contributions, payments would simply be made to other legislators, and the equilibrium outcome would be unaffected; the only loser would be the legislator from that district. Of course, there may be collective gains from prohibiting side payments to all legislators. Notice also that here the side payment required to induce a legislator to vote for a proposal is equal to that legislator's personal tax burden. That is, equilibrium side payments are small relative to a district's gains. This formulation does not address how constituents within a district coordinate their actions. Since side payments are small, relative to gains, it may be possible to solve the free rider problem within each district. (For more on this issue, see Chari and Jones 1991 and Chari, Jones, and Marimon 1994.)

2. Local Public Goods and Proposal Buying

We turn now to an analysis of local public good provision in environments where representatives can make side payments to influence the bills that can be proposed. One motivation for this analysis comes from our result that in plurality voting systems, vote buying results in inefficiently high levels of local public good expenditures. The representatives may then have an incentive to bribe the proposer to offer a bill with lower levels of local public goods. Since the level of public good expenditures is inefficiently high in the vote-buying equilibrium, bribes or side payments can be constructed which make everybody better off than in the vote-buying equilibrium. The problem, however, is that as long as all other representatives make such side payments, each individual representative's contribution has a negligible effect on the outcome. Thus, each representative has an incentive not to bribe the proposer to reduce expenditure. That is, there is potentially a free rider problem. The purpose of this section is to show that this free rider problem results in an outcome in which each bribe is close to zero and government expenditure is essentially the same as in the vote-buying equilibrium.

For simplicity, we again consider the determination of spending in one district only—say, district 1. Similar arguments apply to spending in other districts. We model the process for determining spending by a particular noncooperative game—though, as will become clear, our results generalize to a large class of games.

Our game has three stages. The first stage is the proposal-buying stage, and the last two are the same as in our vote-buying game.

In the first stage, representative 1 proposes to each of the other representatives a level of spending and makes a request for a side payment. Formally, representative 1 chooses a proposed spending level, γ, and requests for payments, n_i, $i = 2, \ldots, I$, from each of the other representatives. Each of the other representatives then agrees or disagrees with the request. We denote the decisions of the other representatives by y_i, $i = 2, \ldots, I$, where $y_i = 1$ signifies agreement and $y_i = 0$ signifies disagreement. The interpretation is that if representative i agrees to a proposal (γ, n_i) and if representative 1 proposes a bill to spend γ at the second stage, then representative i must then pay representative 1 n_i units of the consumption good. Note that representative i's payments are not contingent on the subsequent vote. We have made these payments not contingent in order to allow representative i to influence the outcome without having to commit to a voting decision.

The second and third stages of the proposal-buying game are exactly the same as in the vote-buying game. That is, representative 1 chooses a number g and a set of numbers m_i, $i = 2, \ldots, I$. Then the representatives choose a vote vector v_i, $i = 1, \ldots, I$. A strategy profile for this game consists of the vector of numbers (γ, n) denoting offers by representative 1; a set of functions $y(\gamma, n) = \{y_i(\gamma, n)\}_{i=2}^{I}$ mapping offers into $\{0,1\}$ denoting agreement decisions; functions $g(\gamma, n, y)$, $m(\gamma, n, y) = \{m_i(\gamma, n, y)\}_{i=2}^{I}$ denoting second-stage decisions by representative 1; and voting functions $v(\gamma, n, y, g, m)$ for the representatives. Let s denote the collection of strategies. The payoff to representative 1 is then given by

$$(2.1) \quad U_1(s) = \theta f(\delta g) - \frac{1}{I}\, \delta g - \sum_{i=2}^{I} m_i v_i + \sum_{i=2}^{I} n_i y_i \alpha_i,$$

where $\delta = 1$ if the vote is successful and $\delta = 0$ otherwise and $\alpha_i = 1$ if $g = \gamma_i$ and $\alpha_i = 0$ otherwise. The payoffs to the other representatives are given by

$$(2.2) \quad U_i(s) = -\frac{\delta g}{I} + m_i v_i - n_i y_i \alpha_i.$$

A (subgame perfect) equilibrium of this game is defined in the usual fashion. In the discussion of the vote-buying game, we argued that there are strong reasons to restrict attention to the pivotal equilibrium. We will, therefore, restrict attention to equilibria such that for every history of the game, at the second stage the continuation strategies constitute a pivotal equilibrium for the vote-buying game. Recall that in that game, representative 1 randomly picked K representatives and purchased their votes for $m_i = g/I$ each. The payoff of representative 1 is, therefore, given by

$$(2.3)\quad U_1(s) = \theta f(g) - \frac{(K+1)g}{I} + \sum_{i=2}^{I} n_i y_i \alpha_i.$$

The other representatives each receive a payment of g/I with probability K/(I−1) and with the complementary probability receive zero payment. The expected payoffs at the second stage for the other representatives are, then,

$$(2.4)\quad U_i(s) = -\frac{g}{I} + \frac{K}{I-1}\left[\frac{g}{I}\right] - n_i y_i \alpha_i.$$

An equilibrium (for the first stage) is a set of strategies for proposals, requests for payments, agreement decisions, and a spending decision on the local public good that satisfy the usual conditions. We restrict our analysis to symmetric subgame perfect equilibria. Since the offers (γ_i,n_i) and the agreement decisions $y_i(\gamma,n)$ are the same for all i, a symmetric equilibrium is characterized by three numbers (γ,n,g). We can without loss of generality examine only the cases in which $\gamma = g$ and $w(\gamma,n) = 1$, since the case where $n = 0$ subsumes the remainder.

It turns out that there are many equilibria for our game. We characterize the equilibrium set by developing necessary conditions for equilibrium outcomes. In particular, we develop conditions for a pair (g,n) to be part of a symmetric, subgame perfect equilibrium. Let g_v denote the equilibrium of the vote-trading game without proposal buying. Since representative 1 can always deviate to g_v, it follows that if (g,n) is a symmetric equilibrium outcome, it must satisfy

$$(2.5)\quad \theta f(g) - \frac{(K+1)g}{I} + (I-1)n \geq \theta f(g_v) - \frac{(K+1)g_v}{I}.$$

Next note that since g_v solves (P1), we have that $\theta f(g_v) - (K+1)g_v/I \geq \theta f(g) - (K+1)g/I$. Therefore, it follows that $n \geq 0$. Now consider the decision problem of one of the representatives—say, i—who has been asked to contribute n units of the consumption good given that all the other representatives agree. Denote the payoff to this representative if he or she agrees by $U_i(g,n)$:

$$(2.6)\quad U_i(g,n) = -n - \frac{g}{I} + \left[\frac{K}{I-1}\right]\frac{g}{I}.$$

The first term on the right side of (2.6) is the payment that must be made to representative 1 at the first stage, the second term is the taxes that must be paid to finance g, and the last term is the expected payment at the voting stage. To see that the last term is the expected payment, recall that the probability of being chosen to have one's vote bought at the voting stage is $K/(I-1)$ and the payment conditional upon being chosen is g/I. The representative receives no payment if he or she is not chosen at the voting stage.

Now assume that $n > 0$. Recall that n denotes payments from representatives 2, ..., I to representative 1. Consider representative i, i = 2, ..., I. This representative is better off by disagreeing if the outcome is unaffected by his or her actions. Suppose $\theta f(g) - (K+1)g/I + (I-2)n > \theta f(g_v) - (K+1)g_v/I$. At the voting stage, representative 1 now has an incentive to propose g_v even though representative i has disagreed. Subgame perfection implies that a necessary condition for (g,n) to be an equilibrium is given by

$$(2.7)\quad \theta f(g) - \frac{(K+1)g}{I} + (I-2)n \leq \theta f(g_v) - \frac{(K+1)g_v}{I}.$$

The last necessary condition is given by the requirement that each representative i, i = 2, ..., I, be better off agreeing than disagreeing. Representative i, i = 2, ..., I, could be better off by disagreeing if his or her payoff under g_v is greater than that payoff under (g,n). Thus, if (g,n) is to be part of a symmetric subgame perfect equilibrium, it must satisfy

$$(2.8) \quad -n - \left[1 - \frac{K}{I-1}\right] \frac{g}{I} \geq -\left[1 - \frac{K}{I-1}\right] \frac{g_v}{I}.$$

Conditions (2.5), (2.7), and (2.8) are also sufficient for (g,n) to be an equilibrium outcome. To show this, we must argue that none of the representatives wishes to deviate. Consider first the problem faced by representatives 2, ..., I. If any of these representatives disagrees, (2.7) implies that representative 1 will propose g_v at the voting stage. But under (2.8) each of representatives 2, ..., I is better off agreeing than reverting to g_v. Consider next the problem faced by representative 1. To establish sufficiency of (2.5), (2.7), and (2.8) we construct the equilibrium strategies as follows. If representative 1 offers $(\hat{g},\hat{n}) \neq (g,n)$, then all the other representatives disagree. If representative 1 offers (g,n), where (g,n) satisfies (2.5), (2.7), and (2.8), then all the representatives agree. The strategies constructed for representatives 2, ..., I are best responses in this case since no representative can alter the outcome by agreeing. Given these strategies, representative 1 cannot do better than to offer (g,n). Thus, (g,n) is a subgame perfect equilibrium outcome.

We summarize the above discussion with

PROPOSITION 3. Suppose an arbitrary pair of numbers (g,n) satisfies (2.5), (2.7), and (2.8) and $I \geq 4$. Then (g,n) is a (symmetric) subgame perfect equilibrium outcome of the proposal-buying game.

It should be clear that many pairs (g,n) will satisfy (2.5), (2.7), and (2.8). In Figure 1 we plot the iso-utility lines when (2.5), (2.7), and (2.8) hold with equality. The shaded area is the set of equilibrium outcomes. This set includes among others the efficient level of public good provision as well as $(g_v,0)$. The plethora of equilibria for the proposal-buying game is troublesome. The theory appears to provide no clear outcome when representatives are allowed to buy proposals. It turns out, however, that with a small amount of private information, the equilibrium outcomes are close to $(g_v,0)$ when the number of representatives is large.

Proposal Buying With Private Information

We now examine the proposal-buying outcomes when other representatives are uncertain about the benefits to representative 1 from the public good. We model this uncertainty by assuming that θ is a random variable drawn from a distribution $H(\theta)$ with strictly positive density $h(\theta)$ on its support $[\underline{\theta},\bar{\theta}]$. This uncertainty means that the other representatives are uncertain about g_v. We show that this uncertainty interacts with the free rider problem implicit in (2.8) and leads to a spending level of g_v.

A strategy profile for the proposal-buying game now consists of (measurable) functions $(\gamma(\theta),n_i(\theta))$ which map $[\underline{\theta},\bar{\theta}]$ into offers, agreement or disagreement decisions $y(\gamma,n)$, and a spending level $g(\gamma,n,w)$. To ensure that the other representatives have well-defined decision problems, we also need a probability distribution $\mu_i(\theta|\gamma,n)$ which describes representative i's beliefs about the representative 1's type. A perfect Bayesian equilibrium for this game is defined in the usual fashion.

Consider a sequence of economies indexed by I and the associated sequence of equilibrium outcome functions $g(\theta;I)$, $n(\theta;I)$. Let $g_v(\theta;I)$ denote the solution to (P1). Clearly, as I goes to infinity, $g_v(\theta;I)$ converges to $g_v(\theta)$, where $g_v(\theta)$ maximizes $\theta f(g) - kg$. We will show that $g(\theta;I)$ converges to $g_v(\theta)$ and $n(\theta;I)$ converges to zero.

We begin by establishing this result under strong assumptions on the functions g and n and then weaken these strong assumptions. Suppose that g and n are differentiable, and suppose by way of contradiction that for all I sufficiently large, $g(\theta;I) \neq g_v(\theta;I)$ for all θ in some interval $(\underline{\theta},\bar{\theta})$. Suppose also that the equilibrium is separating in this interval, so that $g(\theta;I) \neq g(\hat{\theta};I)$ for $\theta \neq \hat{\theta}$. Now (suppressing the index I) maximization by representative 1 requires that $g(\theta)$, $n(\theta)$ satisfy the following:

$$(2.9) \quad \theta \in \arg_{\hat{\theta}}\max\left\{\theta f(g(\hat{\theta})) - \frac{(K+1)g(\hat{\theta})}{I} + (I-1)n(\hat{\theta})\right\}.$$

Since g and n are differentiable, a necessary condition is that

$$(2.10) \quad \left[\theta f'(g(\theta)) - \frac{(K+1)}{I}\right] g'(\theta) + (I-1)n'(\theta) = 0.$$

Next we can use the same arguments as in the complete information environment to show that in a separating equilibrium, the analogues of conditions (2.5), (2.7), and (2.8) must continue to hold. That is, we have

$$(2.11) \quad \theta f(g(\theta)) - \frac{(K+1)g(\theta)}{I} + (I-1)n(\theta) \geq \theta f(g_v(\theta)) - \frac{(K+1)g_v(\theta)}{I},$$

$$(2.12) \quad \theta f(g(\theta)) - \frac{(K+1)g(\theta)}{I} + (I-2)n(\theta) \leq \theta f(g_v(\theta)) - \frac{(K+1)g_v(\theta)}{I},$$

$$(2.13) \quad n(\theta) \leq \left[\frac{g_v(\theta)}{I} - \frac{g(\theta)}{I}\right]\left[1 - \frac{K}{I-1}\right]$$

(where, again, we have suppressed the index I for notational convenience). Recall that $g_v(\theta;I)$ maximizes $\theta f(g) - (K+1)g/I$ and is, therefore, bounded. We assume that as I increases, $k = K/I$ remains constant. That is, the voting rule remains unchanged. From (2.13) it then follows that, since $g_v \geq g$, $n(\theta) \to 0$ and $(I-1)n(\theta)$ is uniformly bounded for all I. Next, note that the left sides of (2.11) and (2.12) differ only by $n(\theta)$, which goes to zero. Thus, the utility of representative 1 is arbitrarily close to utility under g_v for I sufficiently large. That is, we have that, asymptotically,

$$(2.14)\quad \theta f(g(\theta)) - \frac{(K+1)g(\theta)}{I} + (I-1)n(\theta) = \theta f(g_v(\theta)) - \frac{(K+1)g_v(\theta)}{I}.$$

Differentiating (2.14) with respect to θ, recalling that $g_v(\theta)$ maximizes the right side of (2.14), and using the envelope theorem, we have that

$$(2.15)\quad f(g(\theta)) + \left[\theta f'(g(\theta)) - \frac{(K+1)}{I}\right] g'(\theta) + (I-1)n'(\theta) = f(g_v(\theta)).$$

Comparing (2.10) to (2.15), one can see that both can hold if and only if $g_v(\theta) = g(\theta)$ for all $\theta \in [\underline{\theta},\bar{\theta}]$.

In the Appendix, we make the same argument more formally without assuming differentiability. First we show that for any sequence of separating equilibria, the convergence result holds. Then we show that all equilibria are asymptotically separating. The propositions we prove are:

PROPOSITION 4. Suppose that $g(\theta;I)$, $n(\theta;I)$ is a sequence of separating equilibrium outcomes. Then $g(\theta;I)$ converges almost surely to $g_v(\theta)$ and $(I-1)n(\theta;I)$ converges almost surely to zero.

PROPOSITION 5. Suppose $g(\theta,I)$, $n(\theta,I)$ is a sequence of (partially) pooling equilibrium outcomes; that is, there exists a sequence of measurable sets $A(I) \subset [\underline{\theta},\bar{\theta}]$ with the property that for any θ, $\theta' \in A(I)$, $g(\theta;I) = g(\theta';I)$ and $n(\theta;I) = n(\theta';I)$. Then the Lebesgue measure of $A(I)$ goes to zero as $I \rightarrow \infty$.

We have established that with even a small amount of uncertainty, government spending must be close to g_v if there are many representatives. This result reinforces the policy implication of the simple vote-buying game that unanimity is an efficient voting rule with regard to local public goods spending.

The intuition for this result is as follows. Since the asymptotic level of government expenditure in any district in the pure vote-buying game is bounded, the contribution that any representative is willing to make to reduce another district's spending level gets arbitrarily small as the number of districts gets large. This, along with the need for each representative to be pivotal with regard to his or her contribution, implies that the gain to a representative from agreeing to reduce his or her district's spending level relative to that which would prevail under pure vote buying is also becoming arbitrarily small. This implies one constraint on how $[g(\theta),n(\theta)]$ varies with θ. In addition, the need to preserve incentive compatibility induces another constraint on how $[g(\theta),n(\theta)]$ varies with θ. These two constraints are consistent if and only if $g(\theta) = g_v(\theta)$.

3. Global Public Goods

We are interested now in examining the implications of our basic political model for the determination of government spending upon a global public good. Our analysis thus far suggests that voting systems which require unanimity yield good outcomes. Drawing upon the work of Rob (1989), Mailath and Postlewaite (1990), and Chari and Jones (1991), we show quite generally that with private information, unanimous consent rules lead to undesirable outcomes. Specifically, we show that a large class of mechanisms [the interim efficient mechanisms in the Holmstrom and Myerson (1983) sense] lead to outcomes that are extremely inefficient in the full-information (or ex post) sense. The interim efficient mechanisms produce too little spending on global public goods. We will then show that our simple vote-buying game implies that higher levels of government spending will result once we relax the unanimity constraint.

Let G denote the level of per capita spending on the global public good. As before, the spending on this good is financed by a uniform tax on the entire population. Let representative i's preferences be given by

$$(3.1)\qquad \left[\frac{\theta_i}{I}\right] f(G) - G + c_i, \quad \text{for } i = 1, \ldots, I.$$

Note that we have scaled the benefits by 1/I. We will want to increase the number of representatives. When we do so, we keep the efficient level of government consumption under full information unchanged with this device. We assume that each θ_i is an identically, independently drawn random variable from a distribution $H(\theta)$ with support $[\underline{\theta},\bar{\theta}]$ and a strictly positive density $h(\theta)$. We assume that $\theta - [1 - H(\theta)]/h(\theta)$ is increasing in θ. This assumption is standard in the literature on mechanism design under private information. It is satisfied by a large class of distribution functions. We assume that $\underline{\theta}f'(0) < 1$. This assumption implies that the efficient level of government consumption is zero when θ is close to $\underline{\theta}$. That is, under full information, it is efficient to spend zero with positive probability.

Without side payments, the probability of any positive level of G passing by unanimous approval gets small as the number of representatives gets large, given our assumptions on H. To see this, let $\hat{\theta}$ be such that $(\hat{\theta}/I)f'(0) = 1$. Note that by assumption $\hat{\theta} > \underline{\theta}$ and that therefore $H(\hat{\theta}) > 0$. The probability that all of the I representatives will strictly prefer a positive level of G is $[1 - H(\hat{\theta})]^I$, which goes to zero as I gets large. We will show that even allowing for elaborate systems of side payments doesn't overcome this basic problem in large legislatures if the benefits from spending θ are private information.

Consider a game in which one of the representatives—say, representative 1—is chosen randomly. This representative then chooses some mechanism to allocate the public good.

Without loss of generality, we restrict attention to direct mechanisms in which all representatives reveal their own type θ_i. Let $\theta = (\theta_1,\ldots,\theta_I)$. A direct mechanism is then a set of functions $m_i(\theta)$ denoting payments to representative 1 and a government spending function $G(\theta)$.

Truth-telling is an equilibrium of the direct mechanism if for all θ_i, $\hat{\theta}_i$,

$$(3.2)\quad E_{-i}\left\{\frac{\theta_i}{I}f(G(\theta)) - \frac{G(\theta)}{I} - m_i(\theta)\right\} \geq E_{-i}\left\{\frac{\theta_i}{I}f(G(\hat{\theta}_i,\theta_{-i})) - \frac{G(\hat{\theta}_i,\theta_{-i})}{I} - m_i(\hat{\theta}_i,\theta_{-i})\right\}.$$

We restrict attention to truth-telling equilibria. We say that a truth-telling equilibrium of the mechanism satisfies unanimous consent if

$$(3.3)\quad E_{-i}\left\{\frac{\theta_i}{I}\, f(G(\theta)) - \frac{G(\theta)}{I} - m_i(\theta)\right\} \geq 0, \quad \text{for all i, for all } \theta_i.$$

Representative 1's expected payoffs are then given by

$$(3.4)\quad U_1(\theta_1) = E_{-i}\left\{\frac{\theta_1}{I}\, f(G(\theta)) - \frac{G(\theta)}{I} + \sum_{i=2}^{I} m_i(\theta)\right\}.$$

This representative's problem is to choose a mechanism to maximize (3.4) subject to (3.2) and (3.3). Let $G(\theta;I)$ denote spending levels under such a mechanism.

PROPOSITION 6. As $I \to \infty$, $G(\theta;I)$ converges to zero in probability.

Proof. See the Appendix.

This proposition includes as a special case our simple vote-buying game with unanimity, as well as unanimity voting games with much more elaborate schemes for side payments, such as the one we considered in the proposal-buying section. The result that spending converges to zero suggests that unanimous voting rules will perform poorly in global public good environments. In contrast, majority voting rules can perform much better.

The simplest way to establish that majority voting rules can perform better is to consider a specific game and show that equilibrium outcomes of this game with private information are approximately the same as those with full information. We will prove this result when I is large.

Consider the following game. One of the representatives is chosen at random. Without loss of generality, let this representative be from district 1. This representative offers a bill that specifies an appropriate level of spending G on the global public good. No side payments are allowed. If a majority of the legislators accept the proposal, it passes; and if a majority reject the proposal, it fails.

Consider first the full information version of this game. Representative 1 proposes the bill that maximizes his or her payoff subject to the constraint that a majority of the legislators vote for the bill. Any representative whose payoff is positive will vote for the bill. Without loss of generality, order the representatives so that $\theta_2 \geq \theta_3 \geq \ldots \geq \theta_I$. It then follows that

$$(3.5)\quad \frac{\theta_i f(G)}{I} - \frac{G}{I} \geq \frac{\theta_{i+1} f(G)}{I} - \frac{G}{I}.$$

If the right side of (3.5) is positive, then so is the left side. Thus, if representative i + 1 votes for the bill, so does representative i.

Representative 1's problem can now be written succinctly. For notational convenience, let I be odd. Then representative 1's problem is to

$$(3.6) \quad \max \ \theta_1 \frac{f(G)}{I} - \frac{G}{I}$$

subject to

$$(3.7) \quad \frac{\theta_{I-1}}{2} \frac{f(G)}{I} - \frac{G}{I} \geq 0.$$

Let $G^*(\theta;I)$ denote the solution to this problem.

Consider next the private information version of the game. Representative 1 now has a more difficult problem to solve. Since the types of the other representatives are not known to representative 1, proposals may sometimes fail. Let θ_m denote the median value of the vector of random variables $\theta_2, \ldots, \theta_I$. Define $\theta(G)$ as that value of θ such that $\theta f(G) - G = 0$. Then a bill proposing to spend G passes if $\theta(G) \leq \theta_m$. Let Prob(G;I) denote the probability that $\theta(G) \leq \theta_m$. Then the proposer's problem is to choose G to solve

$$(3.8) \quad \max \ \mathrm{Prob}(G;I)[\theta_1 f(G) - G].$$

Let $G(\theta_1;I)$ denote the solution to this problem. We will show that $G(\theta_1;I)$ converges in probability to $G^*(\theta;I)$. Let $\hat{G}(\theta_1)$ solve the problem $\max[\theta_1 f(G) - G]$. That is, $\hat{G}(\theta_1)$ is the unconstrained solution to the proposer's problem. Let $\bar{\theta}_m$ denote the median of the distribution $H(\theta)$.

Consider two cases. First, suppose that $\bar{\theta}_m f(\hat{G}(\theta_1)) - \hat{G}(\theta_1) \geq 0$. Then, for sufficiently large I, the unconstrained proposal passes with probability close to 1. The optimal proposal must, therefore, be close to the unconstrained proposal. Formally, we have

$$(3.9) \quad \begin{aligned} \theta_1 f(\hat{G}(\theta_1)) - \hat{G}(\theta_1) &\geq \theta_1 f(G(\theta_1)) - G(\theta_1) \\ &\geq \mathrm{Prob}(G(\theta_1);I)[\theta_1 f(G(\theta_1)) - G(\theta_1)] \end{aligned}$$

where the first inequality follows because $\hat{G}(\theta_1)$ maximizes $\theta_1 f(G(\theta_1)) - G(\theta_1)$ and the second inequality because $\text{Prob}(G;I) \leq 1$. We also have

$$\text{(3.10)}\quad \text{Prob}(G(\theta_1);I)[\theta_1 f(G(\theta_1)) - G(\theta_1)] \geq \text{Prob}(\hat{G}(\theta_1))[\theta_1 f(\hat{G}(\theta_1)) - \hat{G}(\theta_1)]$$

because the left side of (3.10) maximizes (3.8). Since $\text{Prob}(\hat{G}(\theta_1);I) \to 1$, the right side of (3.10) converges to the left side of (3.9). Since payoffs make the solution to (3.8) be between these numbers, it follows that $G(\theta_1;I) \to \hat{G}(\theta_1)$ in probability.

Second, suppose that $\bar{\theta}_m f(\hat{G}(\theta_1)) - \hat{G}(\theta_1) < 0$. Then the maximizing proposal under full information solves $\bar{\theta}_m f(\hat{G}(\theta_1)) - \hat{G}(\theta_1) = 0$. It should be clear that under private information any proposal to spend more than this amount will also fail with probability approaching 1 as $I \to \infty$. Thus, this proposal also maximizes the proposer's utility with private information.

4. Concluding Remarks

We have shown within a simple unicameral model of the legislature that representative democracies are prone to pork barrel spending. Such spending arises from institutional constraints like majority voting rules, along with a free rider problem which prevents Pareto superior proposals from being adopted. We have also shown that while unanimous voting rules lead to efficient allocations for local public goods, such rules are likely to do poorly for global public goods. The main distinction between our model and the standard median voter model is that we have allowed agents to explicitly take advantage of the nonanonymous voting rules of the legislature by enabling them to make side payments.

While our model has legislators making payments to each other, one interpretation is that the interest groups which a particular legislator represents make campaign contributions. We prefer this interpretation. The ability of these interest groups to influence legislators' decisions, combined with majority voting, allows interest groups to extract surplus by suitably compensating a majority of the legislature. In order to accomplish this goal, it is important that voting decisions be observable, as they are in legislatures. The uncompensated minority would like to be able to bribe interest groups to offer welfare enhancing proposals. But because the minority is not certain about the benefits to the interest groups, such groups have incentives to demand excessive bribes, leading each skeptical legislator in the minority to believe that the outcome will not be materially affected even if he or she does not contribute.

Given the simplicity of the model, a natural question to ask is, to what extent do our results generalize to a model that incorporates additional features of standard governmental arrangements? While we obviously cannot analyze every feature of a specific constitution, two particular deviations of our model from the U.S. Constitution seem worth discussing: presidential veto power and a bicameral legislature. Introducing a president and adding the requirement that it takes a two-thirds majority to override a presidential veto seem unlikely to affect any of our major results. In effect, this feature raises k from one-half to two-thirds. Even this effect might be substantially weakened if the president has a legislative agenda that requires cooperation by the representatives.[5]

With respect to a bicameral legislature, there are a number of different ways in which one could imagine altering the model. Since the representatives of a given district in each of the two houses would presumably have the same preferences with regard to local spending, one simple way to adjust the model is simply to imagine that there is implicitly only one representative per district who votes twice for a given spending proposal. If the two votes are simultaneous, and

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[5]Fitts and Inman (1992) argue that while the formal powers of the presidency, arising from the president's veto power and role as party leader, may not be strong, the president's informal powers, due for example to access to the media or control over the executive appointments and decision making, do give the president substantial leverage.

if the vote-buying contract is altered so that a payment is only forthcoming if both of the representatives from a district vote "yes," our results are unchanged. If, however, the two votes are sequential, then there is the possibility that information revealed in the first vote—in particular, whose vote has been acquired—could be exploited by a party in the losing minority of the first vote to induce a switch in one of the majority votes in the second vote. However, each member of the minority coalition would be subject to the sort of free rider problem discussed in the proposal buying and global public goods sections. We conjecture that this free rider problem would prevent our results from being substantially altered.

Finally, it is worth speculating on how alternative ways of modeling legislators' behavior would alter the results. In particular, would our results continue to hold in a model where legislators cannot make or receive side payments but can engage in logrolling? The results in Baron and Ferejohn (1989) suggest that such a model would yield the same kinds of inefficiencies as in our vote-buying model and that the free rider problem would prevent the legislature from adopting efficient policies as in our proposal-buying game. Our results obviously hinge critically on there being a free rider problem at the proposal-buying stage. Clearly, if coalitions can costlessly enforce agreements among themselves, there is no free rider problem. In practice, while large coalitions may sometimes be able to enforce collusive arrangements, such arrangements are often unstable because individual legislators have an incentive to defect. We think some useful insights can be provided by a model that assumes collusive arrangements must be self-enforcing.

Appendix

Proof of Proposition 4. The proof is by contradiction. Suppose not. Then, taking subsequences if necessary and dropping the subscript on subsequences, we have that there exist some interval $(\underline{\theta},\bar{\theta})$ and a number $\delta > 0$ such that for almost all θ in this interval, $g(\theta,I)$ converges to, say, $\hat{g}(\theta)$ and $(I-1)n(\theta,I)$ converges to, say, $\hat{N}(\theta)$, where $\hat{g}(\theta) < g_v(\theta) - \delta$.

We claim that $\hat{g}(\theta)$ and $\hat{N}(\theta)$ must satisfy the following:

$$\text{(A1)} \quad \theta f(\hat{g}(\theta)) - k\hat{g}(\theta) + \hat{N}(\theta) = \theta f(g_v(\theta)) - kg_v(\theta).$$

To prove this claim, note that from (2.13), $n(\theta)$ converges to zero. Now taking limits (of subsequences, if necessary) in (2.11) and using (2.12), we obtain (A1). Furthermore, it is also clear that this convergence is uniform in $\theta \in (\underline{\theta},\bar{\theta})$. We construct the rest of the argument assuming that we are at the limit. The details of the (ϵ,δ) arguments are available upon request.

Choose θ_1, $\theta_2 \in (\underline{\theta},\bar{\theta})$ with $\theta_2 > \theta_1$. Since $\hat{g}$, $\hat{N}$ is an equilibrium, we have that

$$\text{(A2)} \quad \theta_1 f(\hat{g}(\theta_1)) - k\hat{g}(\theta_1) + \hat{N}(\theta_1) \geq \theta_1 f(\hat{g}(\theta_2)) - k\hat{g}(\theta_2) + \hat{N}(\theta_2).$$

Using (A1) we have that

$$\text{(A3)} \quad \theta_1 f(g_v(\theta_1)) - kg_v(\theta_1) \geq \theta_1 f(\hat{g}(\theta_2)) - \theta_2 f(\hat{g}(\theta_2)) + \theta_2 f(g_v(\theta_2)) - kg_v(\theta_2).$$

Now recall that g_v maximizes $\theta f(g) - kg$. Let $U_v(\theta) = \theta f(g_v) - kg_v$. We then have $U_v(\theta_2) - U_v(\theta_1) = \int_{\theta_1}^{\theta_2} f(g_v(x))dx$. Using this result and rearranging (A3), we have that

$$\text{(A4)} \quad (\theta_2-\theta_1)f(\hat{g}(\theta_2)) \geq \int_{\theta_1}^{\theta_2} f(g_v(x))dx.$$

Now from the contradiction hypothesis, $g_v(\theta) - \hat{g}(\theta) \geq \delta$ for all $\theta \in (\underline{\theta},\bar{\theta})$. Thus, for θ_2 sufficiently close to θ_1, we have that $f(g_v(\theta_1)) - f(\hat{g}(\theta_2)) > 0$. The right side of (2.19) is at least as large as $(\theta_2-\theta_1)f(g_v(\theta_1))$ since g_v is increasing in θ. Thus, we have a contradiction. □

Proof of Proposition 5. Suppose A(I) satisfies the hypothesis in the proposition. Then, for all $\theta \in A(I)$, condition (2.11) must hold. Further, there must exist a subset of positive measure of the set $A'(I) \equiv \{\theta \in [\underline{\theta},\bar{\theta}]$ such that $g(\theta;I) = g(\theta';I)$ for $\theta' \in A(I)\}$ upon which (2.12) holds; otherwise one of the other representatives would gain by refusing to pay $n(\theta';I)$.

The proof proceeds by contradiction. Assume that there exists an $\epsilon > 0$ such that for any I, there exists an $I' \geq I$, and a pair $\theta_1, \theta_2 \in [\underline{\theta},\bar{\theta}]$, where $\theta_2 - \theta_1 > \epsilon$, such that $g(\theta_1;I') = g(\theta_2;I')$, $n(\theta_1;I') = n(\theta_2;I')$, and yet (2.11) and (2.12) hold. However, we have already established that for any θ such that (2.11) and (2.12) held, asymptotically (A1) holds. This implies that in the limit

$$\text{(A5)} \quad (\theta_2-\theta_1)f(\hat{g}(\theta_1)) = \theta_2 f(g_v(\theta_2)) - \theta_1 f(g_v(\theta_1)) - k[g_v(\theta_2) - g_v(\theta_1)].$$

However, recalling that $g_v(\theta)$ maximizes $\theta f(g_v) - kg_v$, we have that the right side of (A5) is at least as large as $(\theta_2-\theta_1)f(g_v(\theta_1))$, which implies that $\hat{g}(\theta_1) \geq g_v(\theta_1)$. But then since $\hat{N}(\theta_1)$ is nonnegative, it follows from (A1) that it must equal zero, which generates the contradiction since this implies that $\hat{g}(\theta_1) = g_v(\theta_1)$ and $\hat{g}(\theta_2) = g_v(\theta_2)$. But g_v is strictly increasing in θ. This contradicts the hypothesis that $\hat{g}(\theta_1) = \hat{g}(\theta_2)$. □

Proof of Proposition 6. We use standard results from the mechanism design literature to solve the problem of maximizing (3.4) subject to (3.2) and (3.3). Let $U_i(\theta_i,\hat{\theta}_i)$ denote the utility of representative i when the representative's type is θ_i and he or she reports $\hat{\theta}_i$. This is given by the right side of (3.2). Let $V_i = U_i(\theta_i,\theta_i)$. Let $q_i(\hat{\theta}) = E_{-i}f(G(\hat{\theta}_i,\theta_{-i}))$. Then it is straightforward, using the techniques in Myerson (1981), to establish that the incentive compatibility conditions are equivalent to the conditions that $q_i(\cdot)$ is increasing and that $V_i(\cdot)$ satisfies

$$\text{(A6)}\quad V_i(\theta_i) = V_i(\underline{\theta}) + \frac{1}{I}\int_{\underline{\theta}}^{\theta_i} q_i(x)dx.$$

An informal argument to establish (A6) is as follows. Incentive compatibility requires that θ maximize $U_i(\theta,\hat{\theta})$ over $\hat{\theta}$. Using the envelope theorem, we have that $V'(\theta)$ equals the derivative of U_i with respect to its first argument. Integrating this condition yields (A1). Notice that if q_i is increasing, $V_i(\theta)$ is increasing, and then (3.3) is satisfied if $V_i(\underline{\theta}) \geq 0$.

Next, using the definition of $V(\cdot)$ and (A6), we have that the expected payments from i are given by

$$\text{(A7)}\quad Em_i(\theta) = E\left\{\frac{\theta_i}{I} f(G(\theta)) - \frac{G(\theta)}{I} - \frac{1}{I}\int_{\underline{\theta}}^{\theta_i} q_i(x)dx - V_i(\underline{\theta})\right\}.$$

Integrating the expected value of the third term in (A7) by parts and substituting (A7) into (3.4) yields that the expected utility of 1 is given by

$$\text{(A8)}\quad EU_1(\theta) = E\left\{\frac{\theta_1}{I} f(G(\theta)) - \frac{G(\theta)}{I} + \sum_{i=2}^{I}\left\{\left[\theta_i - \frac{1-H(\theta_i)}{h(\theta_i)}\right]\frac{f(G(\theta))}{I} - \frac{G(\theta)}{I} - V_i(\underline{\theta})\right\}\right\}.$$

Consider the problem of maximizing (A8) subject to the constraints that $V_i(\underline{\theta}) \geq 0$. It is easy to show that the solution to this problem has $G(\theta)$ increasing in θ and has $V_i(\cdot)$ increasing in θ, so the solution to this problem is also the solution to the original problem of maximizing (3.4) subject to (3.2) and (3.3). Let $G(\theta{:}I)$ denote the solution to this problem. We will show that as I goes to infinity, $G(\theta;I)$ converges to zero in probability. The argument is by contradiction, so suppose not. Then, choosing subsequences if necessary, it follows that there exist δ and $\epsilon > 0$

such that Prob$\{G(\theta;I) \geq \delta\} \geq \epsilon$ for all sufficiently large I. Now, maximizing (A8), it follows that $G(\theta) = 0$ if

$$\text{(A9)} \quad \left[\frac{\theta_1}{I} + \frac{1}{I}\sum_{i=2}^{I}\left\{\theta_i - \frac{1 - H(\theta_i)}{h(\theta_i)}\right\}\right] f'(0) < 1$$

and $G(\theta) > 0$ if the inequality is reversed.

But making use of another integration by parts argument, one can show that the expected value of $[\theta - (1 - H(\theta)/h(\theta))] = 0$ (which is the lower end of the support of θ). Hence, the second term in the brackets in (A9) converges to zero. From Chebyshev's inequality, it follows that

$$\text{(A10)} \quad \text{Prob}\left[\frac{\theta_1}{I} + \frac{1}{I}\sum_{i=2}^{I}\left\{\theta_i - \frac{1 - H(\theta_i)}{h(\theta_i)}\right\} > \frac{1}{f'(0)}\right]$$

goes to zero as I goes to infinity. Thus, for large I, $G(\theta)$ converges to zero.

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FIGURE 1

Complete Information Equilibria for the Proposal-Buying Game