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The CAPM Is Alive and Well

Ravi Jagannathan*

Federal Reserve Bank of Minneapolis
and University of Minnesota

Zhenyu Wang*

University of Minnesota

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Abstract

In empirical studies of the CAPM, it is commonly assumed that, (a) the return to the value-weighted portfolio of all stocks is a reasonable proxy for the return on the market portfolio of all assets in the economy, and (b) betas of assets remain constant over time. Under these assumptions, Fama and French (1992) find that the relation between average return and beta is flat. We argue that these two auxiliary assumptions are not reasonable. We demonstrate that when these assumptions are relaxed, the empirical support for the CAPM is very strong. When human capital is also included in measuring wealth, the CAPM is able to explain 28% of the cross sectional variation in average returns in the 100 portfolios studied by Fama and French. When, in addition, betas are allowed to vary over the business cycle, the CAPM is able to explain 57%. More important, relative size does not explain what is left unexplained after taking sampling errors into account.

1 Introduction

A substantial part of the research effort in finance is directed toward improving our understanding of how investors value risky cash flows. It is generally agreed that investors demand a higher expected return for investing in riskier projects, or securities. However, we still do not fully understand how investors assess the risk of a project's cash flow and how they determine what risk premium to demand.

Several capital asset pricing models have been suggested in the literature that describe how investors assess risk and value risky cash flows. Among them, the Sharpe-Lintner-Mossin-Black model (CAPM)[1] is the one that financial managers most often use for assessing the risk of the cash flow from a project and arriving at the appropriate discount rate to use in valuing the project. According to the CAPM, (a) the risk of a project is measured by the beta of the cash flows with respect to the return on the market portfolio of all assets in the economy, and (b) the relation between required expected return and beta is linear.

Over the past two decades a number of studies have empirically examined the validity of the CAPM. The results reported in these studies support the view that it is possible to construct a set of portfolios such that the CAPM[2] has little ability to explain the cross sectional variation in average returns among them. In particular, portfolios containing stocks with relatively small capitalization appear to earn positive excess returns on average than those predicted by the CAPM[3]. In spite of the lack of empirical support, the CAPM is still the preferred model for classroom use in MBA and other managerial

[1]See: Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972).

[2]See Banz (1981), Basu (1983), Reinganum (1981), Chan, Chen and Hsieh (1985), Bhandari (1988), Gibbons (1982) and Shanken (1985).

[3]Hansen and Jagannathan (1991) find that this is true even after controlling for systematic risk using a variety of other measures.

finance courses. In a way it reminds us of cartoon characters like Wiley Cayote who have the ability to come back to original shape after being blown to pieces or hammered out of shape. Maybe, the CAPM survives because: (a) empirical support for other asset pricing models is no better[4], (b) the theory behind the CAPM has an intuitive appeal that is hard to beat using the other models, and (c) the economic importance of the empirical evidence against the CAPM reported in empirical studies is ambiguous.

In their widely cited study, Fama and French (1992) present evidence suggesting that the statistical rejections of the CAPM that have been reported in the literature may also be economically important. They examined the CAPM using return data on a large collection of assets and found that the "relation between market β and average return is flat"[5]. The CAPM is widely viewed as one of the two or three major contributions of academic finance to practicing managers during the postwar era. As Fama and French point out, the robustness of the size effect and the absence of a relation between β and average return is so contrary to the CAPM, that it shakes the foundation on which MBA and other managerial course material in finance is built.

While the evidence presented by Fama and French (1992) against the CAPM is impressive, it is still not sufficient to conclude that the CAPM does not provide a useful framework for explaining the cross section of expected returns, for the following reasons.

First, in order to implement the CAPM, for practical purposes, it is commonly assumed that the return on the value-weighted portfolio of all stocks listed in the New York and AMEX stock exchanges (as well as those traded on NASDQ) is a reasonable proxy for the return on the market portfolio of all assets. Hence one possible interpretation of the evidence is that the par-

[4]See Hansen and Singleton (1982), Lehmann and Modest (1988), Connor and Korajczyk (1988a and 1988b) and Hansen and Jagannathan (1991).

[5]Also see Jegadeesh (1992) who obtains results similar to Fama and French.

ticular proxy Fama and French use for the return on the market portfolio of all assets is a major cause for the unsatisfactory performance of the CAPM.

Second, the CAPM is a static (two period) model, whereas the time-series data is from the real world which is inherently dynamic. It is therefore necessary to make some auxiliary assumptions in assessing the empirical support for the CAPM. Fama and French (1992) assume that the betas of the portfolios they study remain constant over time. This may not be a reasonable assumption since betas of firms may vary over the business cycle. For example, during recessions financial leverage of firms in relatively poor shape may increase sharply relative to other firms, causing their stock betas to rise. Also, to the extent business cycles are induced by technology or taste shocks, relative share of different sectors in the economy will fluctuate and this will induce fluctuations in the betas of firms in these sectors.

We find that when human capital is also included in measuring wealth, the CAPM is able to explain 28% of the cross sectional variation in average returns on the portfolios used in the Fama-French study. This is a substantial improvement when compared to the 1.35% explained when a traditional market portfolio of NYSE-AMEX stocks alone is used as a proxy for aggregate wealth. When betas are allowed to vary over the business cycle, the CAPM is able to explain 57 % of the cross sectional variation in average returns. Size has little ability to explain what is left unexplained. These findings suggest that it is rather premature to discard the CAPM.

The rest of the paper is organized as follows. In Section 2 we replicate the results reported by by Fama and French (1992) using our set of data. In Section 3 we develop and examine an empirical specification of CAPM that incorporates human capital and allows for time variations in betas. In Section 4 we provide some additional diagnostics. We conclude in Section 5.

2 Robustness of the Fama-French Findings

2.1 Data

Fama and French (1992) find that while the relation between beta and average return is flat, the relation between average return and size is negative and statistically significant. While the data set we use is similar to the one used by Fama and French, there are some significant differences. Hence in order to show meaningful connection between our analysis and that of Fama and French, it is necessary to replicate their work using our data set.

While Fama and French use returns to common stocks of non-financial corporations listed in NYSE, AMEX (1962-90) and NASDAQ (1973-90) that are covered by CRSP as well as COMPUSTAT in their study, we study return to stocks of non-financial firms listed in NYSE and AMEX (1962-90) covered by CRSP alone.

NASDAQ stocks are not included because we do not have monthly data for NASDAQ stocks available to us at the University of Minnesota. This should not be an issue since Fama and French report that their results do not depend on the inclusion of NASDAQ stocks.

It is well known that firms in COMPUSTAT may have some forward looking bias[6], since stocks move in and out of the COMPUSTAT list depending on their past performance. Kothari, Shanken and Sloan (1992) provide indirect evidence for the existence of such a bias — they point out that the annual returns are about 10 percentage points more for small firms in COMPUSTAT when compared to small firms that are only in CRSP. Fama and French themselves are aware of this problem, since in their follow-up paper (Fama and French (1993)), they omit the first two years of data as, according to them, COMPUSTAT claims that they rarely add more than 2 years

[6]See Chari, Jagannathan and Ofer (1986).

of data when they add a firm to their list. However, it is not clear whether this completely eliminates the bias in COMPUSTAT tape. In view of this, we do not examine the relation between book to market equity and the cross section of returns[7]. Hence, we are not constrained to limit our attentions to stocks that are in CRSP as well as COMPUSTAT.

We form 100 portfolios of NYSE and AMEX stocks in the same way as Fama and French (1992) did. For every calendar year, starting from 1963, we first sort firms into 10 size deciles based on their market value at the end of June. For each size category, we then estimate the beta of each firm using 24 to 60 months of past return data and the CRSP value-weighted index as the market index proxy. Firms that have less than 24 continuous monthly return observations are omitted. Following Fama and French, we denote this beta as "pre-ranking" β estimates or "pre-beta" estimates for short. We then sort firms within each size decile into 10 pre-beta deciles. This gives us 100 portfolios and we compute the return on each of these portfolios for the next 12 calendar months, where portfolio returns are calculated by equally weighting the returns on stocks in the portfolio. We repeat this procedure for each calendar year. This gives a time series of monthly returns (July 1963 – December 1990, i.e., 330 observations) for each of the 100 size-prebeta portfolios.

The Fama and French sorting procedure produces an impressive dispersion in the characteristics of interest. Time series averages of portfolio returns are given in Table 1. The rates of return range from a low of 0.61% to a high of 1.72% per month. We calculate the beta of a portfolio by regressing the portfolio return on the CRSP value-weighted index. Betas of the portfolios are presented in Table 2. They range from a low of 0.57 to a high of 1.70.

[7]Jagannathan, Krueger and McGrattan (1993) study the selection bias in the COMPUSTAT tape by comparing the ability of book to market ratio to explain the cross section of stock returns when attention is limited to the list of firms that appeared in the 1975 COMPUSTAT tape. However, their analysis is incomplete.

We calculate the size of a portfolio as the equally weighted average of the logarithm of market value of stocks (in million dollars). Time series averages of portfolio size are presented in Table 3. They range from a low of 2.34 to a high of 7.81. Properties of these three characteristics of the portfolios are very similar to that of the portfolios formed by Fama and French.

2.2 Fama-MacBeth Regression

The CAPM relation between expected return and beta is given by

$$\mathrm{E}[R_{it}] = \gamma_0 + \gamma_1 \beta_i \tag{1}$$

where R_{it} is the month t return on portfolio i and

$$\beta_i = \mathrm{Cov}(R_{it}, R_{mt})/\mathrm{Var}[R_{mt}]$$

where R_{mt} is the return on the proxy for the market index portfolio at month t. Let $R_{\mathrm{vw}t}$ be the value-weighted portfolio of all stocks traded on NYSE and AMEX. Under the assumptions that $R_{\mathrm{vw}t}$ is a good proxy and betas are constant over time, the empirical specification of CAPM becomes

$$\mathrm{E}[R_{it}] = \gamma_0 + \gamma_1 \beta_i^{\mathrm{vw}} \tag{2}$$

where

$$\beta_i^{\mathrm{vw}} = \mathrm{Cov}(R_{it}, R_{\mathrm{vw}t})/\mathrm{Var}[R_{\mathrm{vw}t}].$$

To test this specification of the CAPM, we can consider the following two regressions across portfolios for each month t

$$R_{it} = \gamma_{0t} + \gamma_{\mathrm{vw}t}\beta_i^{\mathrm{vw}} + \epsilon_{it} \tag{3}$$

$$R_{it} = \gamma_{0t} + \gamma_{\mathrm{vw}t}\beta_i^{\mathrm{vw}} + \gamma_{\mathrm{size}t}\log(\mathrm{ME}_{it}) + \eta_{it} \tag{4}$$

where $\log(\mathrm{ME}_{it})$ denotes the size for portfolio i at month t. Fama and French (1992) assign the beta of a portfolio to individual stocks in that portfolio, and estimate γ_{vwt} and γ_{sizet} for each month t using the cross section of returns on individual stocks. The estimates obtained using this procedure will be almost the same as the one obtained by using the cross section of returns on the 100 portfolios[8]. Let $\bar{\gamma}_0$, $\bar{\gamma}_{vw}$ and $\bar{\gamma}_{size}$ denote the time series averages of the γ's. We can then examine whether the risk premium γ_{vw} in equation (2) is positive by testing whether $\mathrm{E}[\bar{\gamma}_{vw}]$ is zero against the alternative that it is positive. Following Fama and French we also examine if the empirical CAPM specification given in (2) is correct by testing whether $\mathrm{E}[\bar{\gamma}_{size}] = 0$. The assumption is that if one can reject the hypothesis that $\mathrm{E}[\bar{\gamma}_{size}] = 0$, then (2) is misspecified[9].

Table 4 gives the results reported by Fama and French (1992). $\bar{\gamma}_{vw}$ is 0.15 with an associated t-statistic of 0.46. When $\log(\mathrm{ME}_{it})$ is also included in the regression, $\bar{\gamma}_{vw} = -0.37$ ($t = -1.21$) and $\bar{\gamma}_{size} = -0.17$ ($t = -3.41$). Fama and French interpret this as evidence that the relation between beta and average return is flat. Further, the univariate relation between market value and the cross section of average returns is statistically significant – which is taken as further evidence against the CAPM.

Fama and French interpret the "flat" relation between beta and average return as stong evidence against the CAPM. When models are viewed as approximations to reality, a natural question that arises is: "how bad is the model's performance?". A flat relation between beta and average return, should be viewed as extremely poor performance indeed, subject to certain caveats to be made clear shortly.

The CAPM implies that the market portfolio is mean variance efficient. One implication is that individuals need only invest in the market portfolio

[8]The CAPM is a pricing model for both portfolios and stocks. Since we use portfolio betas, it is more natural to run regression across portfolios instead of stocks. If the numbers of stocks in portfolios are the same, it can be shown that estimates obtained using portfolios will be identical to those obtained using stocks.

[9]Berk (1992) provides theoretical justification for this test.

and the riskless asset in suitable combinations. From the perspective of such an individual who considers investing in a market proxy portfolio and the riskless asset, the measure of how bad the market proxy is, "how inefficient is the market proxy in the mean variance space?" Roll and Ross (1992) correctly point out that even when the proxy used for the market portfolio is very close to the mean-variance efficient frontier, the relation between beta computed using the proxy and the expected return on the assets can be "flat". Hence, an individual investing in the market proxy portfolio may not be significantly worse off when compared to investing in the true market portfolio, even when such a proxy has no ability to explain the cross section of expected returns. Hence, some one who is interested in explaining the cross section of expected returns may well end up picking a very poor proxy if he/she uses closeness of the return on the proxy to the mean variance frontier as the only criterion in selecting from among various candidates. This is because closeness in the mean-variance space need not necessarily imply that expected returns assigned by model using the proxy will be close to the true expected returns, as Green (1986) pointed out.

In view of this, the observation by Roll and Ross that a proxy that is close to the mean variance frontier may perform poorly in explaining the cross section of expected returns cannot be interpreted as a criticism of the conclusions reached by Fama and French that the CAPM performs very poorly when the value weighted index of stocks is used as the market proxy.

Figure 1 gives a plot of the 100 portfolios in our sample in the standard deviation-mean plane. The figure also gives the ex post efficient frontier of returns with and without positivity constraints on portfolio weights. (The frontier with positivity constraints is represented by the dotted line and that without positivity constraints is represented by the solid line in Figure 1.) As can be seen from the figure, the value weighted index portfolio is well within the frontier even when portfolio weights are constrained to be positive.

Table 4 also gives the corresponding results for our data set (NYSE & AMEX). Although our data set is different from the one used by Fama and French, the coefficient for beta is not statistically different from zero no matter whether or not size is included in the regression, and the coefficient for size remains very significant (although the t-ratio is smaller than that reported by Fama and French).

We need an r-square like goodness of fit measure for comparing the performance of different empirical specifications of the CAPM in explaining the cross sectional variation in average returns across portfolios. The Fama-MacBeth regressions are cross sectional, and there is one such regression for each month. Hence, in order to compute an r-square like statistic, we follow the following procedure. Notice that the fitted residuals for each month in equation (3) have an average of zero for each month. Let $\bar{R}_i$ and $\bar{e}_i$ denote the time series average of the returns and residuals for portfolio i. Let $\mathrm{Var}_c[\bar{R}_i]$ and $\mathrm{Var}_c[\bar{e}_i]$ denote the cross sectional variance of $\bar{R}_i$ and $\bar{e}_i$. We define r_{I}^2 as $\left(\mathrm{Var}_c[\bar{R}_i] - \mathrm{Var}_c[\bar{e}_i]\right)/\mathrm{Var}_c[\bar{R}_i]$. Note that

$$\mathrm{Var}_c(\bar{R}_i) = \mathrm{Var}_c(\bar{\gamma}_0 + \bar{\gamma}_{\mathrm{vw}}\beta_i^{\mathrm{vw}}) + \mathrm{Var}_c(\bar{e}_i)$$

Hence r_{I}^2 does indeed measure the ability of the model to explain the cross sectional variation in average returns. Since $\log(\mathrm{ME}_{it})$ varies over time and thus the above decomposition of the variance will not hold generally, this r_{I}^2 measure is not defined when size is included in the regression. In Table 4, r_{I}^2 for β^{vw} alone is 1.35%. This confirms and supplements the results reported by Fama and French (1992) that β^{vw} does not explain the cross-sectional variation of average stock returns[10].

It is necessary to use caution in using r-square to compare the performance of different specifications of an asset pricing model. To see why,

[10]However, Fama and French do not report this type of a goodness of fit measure in their study.

consider a hypothetical economy where the econometrician has observations on four assets. The betas with respect to a proxy market portfolio for the four assets are 0.5, 0.5, 2 and 2. The corresponding expected rates returns are 12%, 8%, 24% and 20%. There are no measurement errors involved here. It can be verified that in this case, the expected rate of return R_{it} is given by

$$\mathrm{E}[R_i] = 6 + 8\beta_i + \epsilon_i \qquad i = 1, 2, 3, 4$$

and that the r-square of the cross sectional regression is 95%.

Now consider forming four other portfolios (by an invertible linear transformation) of the four given assets as follows. Let $z = R_3 - R_4$ denote the payoff on the zero investment portfolio constructed by going long one dollar on the third asset and going short one dollar on the fourth asset. The beta of the payoff, z, is 0 by construction. Define the return on the four new portfolios by: $R_1^* = R_1 + 3z$; $R_2^* = R_2 + 3z$; $R_3^* = R_3$; and $R_4^* = R_4$. Notice that the original set of four assets can be constructed as portfolios of these four portfolios. The betas of the four portfolios defined this way are, 0.5, 0.5, 2, and 2 respectively. The expected returns on these portfolios are, 24%, 20%, 24% and 20% respectively. Clearly, when these four portfolios are used, the relation between expected return and beta is flat (i.e., the r-square is 0%)[11].

This shortcoming is not an issue for the way we use r-square to compare the performance of different competing specifications of the CAPM, since we use the same set of portfolios across all the specifications, and use OLS to estimate model parameters in every case.

As noted by Fama and French (1992), although beta has very little ability to explain the cross section of average returns on the 100 portfolios, it does explain the cross sectional variability of returns in any given month. To see this more clearly, it is useful to consider the average r-square of the

[11] This example is motivated by the results in Kandel and Stambaugh (1993).

cross sectional regressions, r^2_{II}, defined below. For each month t, we calculate the conventional sum of squared total variance SST_t and sum of squared residual SSR_t. Let $\text{SST} = \sum_t \text{SST}_t$ and $\text{SSR} = \sum_t \text{SSR}_t$. Define r^2_{II} as $(\text{SST} - \text{SSR})/\text{SST}$. As can be seen from Table 4, r^2_{II} for β^{vw} alone is 26.92%, which is substantially higher than $r^2_{\text{I}}(= 1.35\%)$. Clearly, β^{vw} explains a substantial part of the cross sectional variability in returns in any given month. However, it does not explain the cross sectional variability in average returns. This is consistent with Chan and Lakonishok (1992). Including log(ME) increases r^2_{II} to 43.69%, i.e., size does measure systematic risk not captured by β_i^{vw} alone.

The market value of equity in portfolios ranges from a low of \$0.7 billion to a high of \$155 billion (Table 5). Firms in the 40 largest size portfolios account for more than 90% of the average market capitalization. However, those 40 portfolios contain only about 30% of the firms (see Table 6). Nearly 30% of the firms are in the 10 small-size portfolios. This would cast some doubt about the economic significance of the lack of support for the CAPM reported by Fama and French if the conclusions critically depend on the return characteristics of 70% of securities that constitute only 10% of the total market capitalization of stocks included in the sample. We therefore omit the firms traded on AMEX – but the results do not materially change (see Table 4), confirming the conclusions reached by Fama and French.

2.3 Discussion

Like Fama and French, with the empirical specification (2), we find zero risk premium for beta and a strong size effect. One possible conclusion is that the CAPM does not provide a useful framework for explaining the cross section of expected returns. However, other explanations are possible.

We (as well as Fama and French) use a time series of 330 monthly observations on returns in our study. This corresponds to only 28 years of data and there are only about 5 business cycles during this period[12]. There may be too few significant observations to measure expected returns accurately using realized average returns. Use of monthly returns over a relatively short calendar period may lead to inappropriate specification of statistical tests. One way to examine the robustness to this type of specification error would be to use annual data over a fairly long time period instead of using a large number of observations sampled more frequently over a relatively short time period. But we will not examine this issue since it has received attention in other studies. (See Jagannathan and Wang (1992), Amihud, Christensen and Mendelson (1992) and Kothari, Shanken and Sloan (1992).)

The empirical specification (2) is obtained from the CAPM with the auxiliary assumptions that R_{vwt} is a good proxy and betas are constant over time. Thus the above test is actually a joint test of the CAPM and the two auxiliary assumptions. As we have pointed out in Section 1, use of the value-weighted portfolio of all stocks listed in NYSE and AMEX (as well as those traded on NASDAQ) as a proxy for the market portfolio may be one of the reason for the unsatisfactory performance of the CAPM. The rather unrealistic assumption that betas of the portfolios remain constant over time may be another reason. To investigate this issue, we will relax these two assumptions in a reasonable way and examine whether the results will change significantly.

[12] "The Recession in Historical Context", Economic Report of the President, January 1993.

3 Econometric Specifications

3.1 Human Capital and Aggregate Wealth

To appreciate the need for examining other proxies for systematic risk before giving up on the CAPM, note that the CAPM cannot be tested because the market portfolio is not observable. In fact this observation by Roll (1977) is what lead to the wide attention paid to the alternative model proposed by Ross (1976) – the Arbitrage Pricing Theory (APT)[13]. The monthly per capita income in the U.S. from dividends during the period 1959:1 – 1992:12 was less than 3% of the monthly personal income from all sources, whereas income from salaries and wages was about 63% during the same period. While these income flows ignore capital gains, these proportions have remained relatively steady over time during this period[14]. This suggests that common stocks of corporations constitute about a thirtieth of national income, and probably, national wealth as well. Another way to see this is as follows. As Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) point out, almost two-thirds of non-government tangible assets are owned by the household sector, and only one-third is owned by the corporate sector (p. 536, op. cit.). Approximately a third of the corporate assets are financed by equity (see Table 2, op. cit.) Hence, it does appear that the return to stocks alone is unlikely to measure the return to the wealth portfolio sufficiently accurately[15].

Even when stocks constitute only a small fraction of total wealth, the

[13]In our view, the main contribution of the APT is in the observation that, to a first order approximation it may be reasonable to assume that the return on the aggregate wealth portfolio is only affected by a few factors. This assumption makes the CAPM implementable.

[14]Federal Reserve Bulletin, Table 2.17.

[15]This was pointed out by Ravi Jagannathan during his lecture at *The Berkeley Program in Finance: Are Betas Irrelevant? Evidence and Implications for Asset Management*, September 13–15, 1992 at Santa Barbara, California.

stock index portfolio return could well be an excellent proxy for the return on the wealth portfolio if the two returns are perfectly correlated[16] In what follows, we therefore present some evidence that supports the view that the return on the value-weighted stock index portfolio is unlikely to be highly correlated with the return on the unobserved wealth portfolio return.

Suppose the return to the wealth portfolio is affected by two pervasive forces, which we will call factors, following Connor (1984). Suppose further that one of these factors has a relatively small effect on the return to stocks, whereas the second factor has a relatively larger effect on stock returns. If this were indeed the case, then stock index return need not be highly correlated with the return on the wealth portfolio. If we observe these shocks separately, we can then reconstruct the return to the wealth portfolio and measure the betas correctly, using methods analogous to Chen, Roll and Ross (1986). Even if we do not observe these shocks directly, we can test for their presence by examining if stock returns respond in a different way to small and large unanticipated returns to the value-weighted index of stocks[17].

To this end, consider the relation given in equation (5) below:

$$R_{it} = b_{i1} + b_{i2}R_{\mathrm{vw}t} + b_{i3}|R_{\mathrm{vw}t}|R_{\mathrm{vw}t} + \epsilon_{it}. \tag{5}$$

The estimated values of b_{i3} and the associated t-values are given in Table 7 and Table 8. First notice that for portfolios in a given size decile, b_{i3} decreases with beta (i.e., as we move along horizontally across different columns in a row). Hence, for any given size decile, larger beta stocks have smaller sensitivity to the second factor and larger sensitivity to the first factor. The associated t-statistics exibit a similar but stronger pattern. This supports our

[16]See Shanken (1987) and Kandel and Stambaugh (1993) who show how the correlation between the market index proxy return and the unobserved wealth return is related to the mean variance efficiency of the market index proxy portfolio.

[17]Bansal and Viswanathan (1993) use a related non-linear factor model to empirically study asset prices.

view that it is necessary to consider other proxies for the return to aggregate wealth.

Apparently, the observation that stocks only form a small part of the total wealth is what motivated Stambaugh (1982) to examine the sensitivity of tests of the CAPM to different proxies for the market portfolio. In his seminal comparative study of the various market proxies, he found that "even when stocks represent only 10% of the portfolio's value, inferences about the CAPM are virtually identical to those obtained with a stocks-only portfolio". However, he did not consider return to human capital in his otherwise extensive study.

The commonly held view appears to be that human capital is not tradable and hence should be treated differently than other capital (see Mayers (1973)[18]). This view is not completely justified. Note that mortgage loans constitute about a third of all outstanding loans. At the end of 1986 the total market value of equities held by households category was 0.80 GNP, whereas the outstanding stock of mortgages (0.60 GNP), consumer credit (0.16 GNP) and bank loans to the household sector (0.04 GNP) also summed up to 0.80 GNP (Table 4, Diaz-Gimenez et al. (1992)). Since mortgage loans, consumer credit and bank loans to the household sector (although they may be collaterized) can be viewed as borrowings against future income, it does not appear inappropriate to view human capital just like any other form of physical capital, cash flows from which are traded through issuance of financial assets.

There is however an important difference between physical assets and financial assets. Friction in the market for physical assets can indeed be substantial, whereas friction in the financial market (for paper claims to these physical assets) is typically negligible in comparison. Hence it is no surprise

[18]Mayers was the first to point out the importance of including return to human capital in measuring the return to the aggregate wealth portfolio.

that financial markets are very liquid, unlike markets for physical assets, cash flows from which back the financial assets. The real difficulty in valuing human capital lies in the fact that the entire cash flow to human capital is not promised away through financial claims. Unlike stocks which are the residual claimants in firms, cash flow from labor income promised to mortgages comes from the top. Hence factors that affect return to human capital cannot be identified by examining returns to financial assets like mortgages, sufficiently precisely. We therefore follow a different strategy in measuring the return to human capital, and assume that the return on the wealth portfolio is an exact linear function of the return to the stock index portfolio and the growth in per capita income.

While the use of the growth rate in per capita income is rather ad hoc, we can provide some rationale for using it. For example, suppose, to a first order approximation, the expected return to human capital is a constant r, and that per capita labor income follows an autoregressive process of the form $L_t = (1+g)L_{t-1} + \epsilon_t$. In such a case, the realized capital gains part of the return to human capital (not correcting for additional investment in investment in human capital made during this period) will be the realized growth rate in per capita labor income. To see this, note that, under these assumptions, wealth due to human capital is $W_t = L_t/(r-g)$ where L_t is date t labor income, r is the discount rate, and g is the growth rate in labor income. The change in this wealth from date $t-1$ to t is then given by $R_{\text{labor}t} = L_t/L_{t-1} - 1$, i.e., the growth rate in per capita labor income from date $t-1$ to t. Fama and Schwert (1977) and Campbell (1993b) arrived at similar measures based on different lines of reasoning. When the growth in labor income follows a logarithmic autoregressive process, and the discount rate applied to labor income to value the endowment of human capital is a constant through time, it can be shown that the growth rate in human capital can be approximated by a linear function of current and past growth rates in labor income.

Motivated by these observations we make the ad hoc assumption that the return on the aggregate wealth portfolio is a linear function of the return to stocks and the growth rate in per capita labor income. Since there are other components to wealth, viz., investment in durable goods and housing, which we do not measure, we do not impose the restriction that the return to wealth is a weighted average of the return to stocks and return to human capital.

Labor income from salaries and wages is taken from the monthly seasonally adjusted personal income numbers reported in Table 2.17 of the Federal Reserve Bulletin. The monthly income numbers for a given month are typically announced during the last week of the following month. Further, labor income is measured with substantial error, unlike financial market data. Hence, (a) we use a 2-month moving average of labor income in order to minimize the role of measurement errors, and (b) we use the growth rate of the moving average of reported per capita labor income that becomes known at the end of month t as a measure of return to human capital for that month. This is because labor income does not fluctuate much at the individual level from month to month. A substantial part of the fluctuation in total wages paid during a month comes from variations in the number of persons employed. Our view is that when a person finds out that Firm X reduced its work force, he/she will revise his/her expected future income from wages and salaries downward and this will lead to a lower realized return on human capital for that month.

Formally, we assume that the return to the wealth portfolio R_{mt} is given by

$$R_{mt} = \phi_0 + \phi_{\text{vw}} R_{\text{vw}t} + \phi_{\text{labor}} R_{\text{labor}t}. \tag{6}$$

Hence

$$\beta_i = h_{\text{vw}}\beta_i^{\text{vw}} + h_{\text{labor}}\beta_i^{\text{labor}} \tag{7}$$

where

$$\begin{aligned}
\beta_i &= \text{Cov}(R_i, R_m)/\text{Var}(R_m) \\
\beta_i^{\text{vw}} &= \text{Cov}(R_i, R_{\text{vw}})/\text{Var}(R_{\text{vw}}) \\
\beta_i^{\text{labor}} &= \text{Cov}(R_i, R_{\text{labor}})/\text{Var}(R_{\text{labor}}) \\
h_{\text{vw}} &= \phi_{\text{vw}}\text{Var}(R_{\text{vw}})/\text{Var}(R_m) \\
h_{\text{labor}} &= \phi_{\text{labor}}\text{Var}(R_{\text{labor}})/\text{Var}(R_m).
\end{aligned}$$

Substituting equation (7) into (1), we get

$$\text{E}[R_i] = a_0 + a_{\text{vw}}\beta_i^{\text{vw}} + a_{\text{labor}}\beta_i^{\text{labor}} \tag{8}$$

for some constants a_0, a_{vw} and a_{labor}. It is tempting to conclude that equation (8) resembles the linear factor models studied in the finance literature, when stock index return and the growth rate in per capita labor income are used as factors. Note however that Fama and Schwert (1977) used this specification of the CAPM even before linear factor models received attention in the empirical finance literature. The betas we use, β_i^{vw} and β_i^{labor} for each portfolio i, are obtained from separate univariate regressions. We prefer this to using betas obtained from multiple regressions, since the two right side variables in equation (6) will in general be correlated. To estimate the betas from multiple regression, we therefore need to estimate the covariance between the two right side variables. Our approach of working with univariate betas, may be preferable in finite samples.

3.2 Beta Variations Over the Business Cycle

The need to take time variations in betas into account is demonstrated by the commercial success of firms like BARRA which provide beta estimates for risk management and valuation purposes using elaborate time series models. Further, several empirical studies of beta pricing models reported in the literature find that estimated betas exhibit statistically significant variability over time (see: Harvey (1989), Ferson and Harvey (1991, 1993) and Ferson and Korajczyk (1993)).

To understand the intuition as to why ignoring such time variations in betas can lead to an incorrect inference regarding the empirical support for the CAPM, consider the following hypothetical world in which the conditional version of the CAPM holds. Suppose that the econometrician considers only two stocks and that there are only two possible types of dates in the world – wealthy and poor. The betas of the low beta stock in the two date-types are, respectively, 0.5 and 1.25 (corresponding to an average beta of 0.875). The corresponding betas of the high beta stock are 1.5 and 0.75 (corresponding to an average beta of 1.125). Suppose the expected risk premium on the market is 10% in the wealthy date and 20% in the poor date. Then the expected risk premium on the first stock will be 5% in the wealthy date and 25% in the poor date. The expected risk premium on the second stock will be 15% in both dates. Hence an econometrician who ignores the fact that betas and risk premiums vary over time will mistakenly conclude that the CAPM does not hold since both stocks earn an average risk premium of 15% but their betas differ. While the numbers we use in this example are rather extreme and unrealistic, they do illustrates the pitfalls involved in any empirical study of the CAPM that ignores time variation in betas.

Following Ferson (1985) and Ferson and Harvey (1991, 1993) we put the CAPM model into a multi-period scenario, giving the following conditional version of the CAPM

$$\mathrm{E}_{t-1}[R_{it}] = \gamma_{0t-1} + \gamma_{1t-1}\beta_{it-1} \tag{9}$$

where

$$\begin{aligned} \gamma_{0t-1} &= \mathrm{E}_{t-1}[R_{0t}] \\ \gamma_{1t-1} &= \mathrm{E}_{t-1}[R_{mt} - R_{0t}] \\ \beta_{it-1} &= \mathrm{Cov}_{t-1}(R_{it}, R_{mt})/\mathrm{Var}_{t-1}(R_{mt}) \end{aligned}$$

and R_{0t} is the return such that $\mathrm{Cov}_{t-1}(R_{0t}, R_{mt}) = 0$ (i.e., the return on the orthogonal portfolio). We use E_{t-1}, Cov_{t-1} and Var_{t-1} to denote the mean, covariance, and variance conditional on the information available at $t-1$.

However, notice that β_{it-1} is not observable. Fama and French (1992) rely on the assumptions made by Chan and Chen (1988) that lead to the unconditional model given in (1). As we have argued, this assumption is not realistic since conditional betas are likely to vary over the business cycle along with risk premium. For example, Keim and Stambaugh (1986) showed that the risk premium γ_{1t-1} varies over the business cycle, and Harvey (1989) and Ferson and Harvey (1991) showed that β_{it-1} also varies over the business cycle. In order to examine (9), take the unconditional expectation of both sides to get

$$\mathrm{E}[R_{it}] = \gamma_0 + \gamma_1 \tilde{\beta}_i + \mathrm{Cov}(\gamma_{1t-1}, \beta_{it-1})$$

where $\gamma_0 = \mathrm{E}[\gamma_{0t-1}]$, $\gamma_1 = \mathrm{E}[\gamma_{1t-1}]$, and $\tilde{\beta}_i = \mathrm{E}[\beta_{it-1}]$. Notice that the last term depends only on that part of β_{it-1} that is in the linear span of γ_{1t-1}. Hence project β_{it-1} on a constant and γ_{1t-1} to get

$$\beta_{it-1} = \tilde{\beta}_i + \vartheta_i(\gamma_{1t-1} - E[\gamma_{1t-1}]) + \eta_{it-1}.$$

The first term is the mean of β_{it-1}. The second term is that part of β_{it-1} that is correlated with γ_{1t-1} and the last term is asset specific, zero on average, and uncorrelated with γ_{1t-1}. We assume that this term is not affected by what happens to the economy as a whole, i.e., η_{it-1} is orthogonal to all other variables in the economy.

We still do not have an operational specification since γ_{1-1} is unobservable. Since Keim and Stambaugh (1986) showed that the risk premium γ_{1t-1} varies over the business cycle, we assume that γ_{1t-1} is a linear function of the variable that best predicts business cycles. Stock and Watson (1989) found that (i) the spread between six-month commercial paper and six-month Treasury bill rates and (ii) the spread between ten-year and one-year Treasury bond rate, outperformed nearly every other variable as forecasters of the business cycle. Bernanke (1990) ran a horse race between a number of interest rate variables suggested in the literature. He found that the best single variable is the spread between the commercial paper rate and Treasury bill rate first used by Stock and Watson. We choose the spread between BAA rated and AAA rated bonds as the variable that is by assumption perfecly correlated with γ_{1t-1}. It is similar to the spread between commercial paper rate and the Treasury bill rate, but has been extensively used in earlier studies of asset pricing models. Let $Prem_{t-1}$ equal the date $t-1$ yield on low grade bonds minus the date $t-1$ yield on high grade bonds (measured as deviation from its time series mean) and assume that it is propotional to γ_{1t-1}. We thus have

$$\beta_{it-1} = \tilde{\beta}_i + \theta_i Prem_{t-1} + \eta_{it-1}. \tag{10}$$

Using (9) and (10), we can write the the return generating process as

$$R_{it} = \gamma_{0t-1} + \gamma_{1t-1}\tilde{\beta}_i + \gamma_{1t-1}Prem_{t-1}\theta_i + \gamma_{1t-1}\eta_{it-1} + \epsilon_{it}. \tag{11}$$

Taking unconditional expectation of both sides, we obtain

$$\begin{aligned} \mathrm{E}[R_{it}] &= E[\gamma_{0t-1}] + \mathrm{E}[\gamma_{1t-1}]\tilde{\beta}_i + \mathrm{E}[\gamma_{1t-1}Prem_{t-1}]\theta_i \\ &= a_0 + \begin{pmatrix} a_1 & a_2 \end{pmatrix} \begin{pmatrix} \tilde{\beta}_i \\ \theta_i \end{pmatrix} \end{aligned} \tag{12}$$

for some constants a_0, a_1 and a_2. Hence the unconditional expected return on any asset i is a linear function of $\tilde{\beta}_i$ and θ_i. To relate $\tilde{\beta}_i$ and θ_i to the betas estimated by univariate regressions on the true market index return and *Prem*, note that from (11) we have

$$\begin{aligned}
\mathrm{Var}(R_{mt})\beta_i &= \mathrm{Cov}(R_{it}, R_{mt}) \\
&= \mathrm{Cov}(\gamma_{0t-1}, R_{mt}) + \tilde{\beta}_i \mathrm{Cov}(\gamma_{1t-1}, R_{mt}) \\
&\qquad + \theta_i \mathrm{Cov}(\gamma_{1t-1} Prem_{t-1}, R_{mt}) \\
\mathrm{Var}(Prem_{t-1})\beta_i^{\text{prem}} &= \mathrm{Cov}(R_{it}, Prem_{t-1}) \\
&= \mathrm{Cov}(\gamma_{0t-1}, Prem_{t-1}) + \tilde{\beta}_i \mathrm{Cov}(\gamma_{1t-1}, Prem_{t-1}) \\
&\qquad + \theta_i \mathrm{Cov}(\gamma_{1t-1} Prem_{t-1}, Prem_{t-1}).
\end{aligned}$$

Thus, we can write the above equations as

$$\begin{pmatrix} \beta_i \\ \beta_i^{\text{prem}} \end{pmatrix} = \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \end{pmatrix} + \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} \begin{pmatrix} \tilde{\beta}_i \\ \theta_i \end{pmatrix}$$

for some constants λ_{ij} ($i = 1, 2$ and $j = 0, 1, 2$). Solving for $\tilde{\beta}_i$ and θ_i, we get

$$\begin{pmatrix} \tilde{\beta}_i \\ \theta_i \end{pmatrix} = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \beta_i \\ \beta_i^{\text{prem}} \end{pmatrix} - \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \end{pmatrix} \right\}. \tag{13}$$

Substituting (13) into (12), we obtain

$$\begin{aligned}
\mathrm{E}[R_{it}] &= a_0 - \begin{pmatrix} a_1 & a_2 \end{pmatrix} \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}^{-1} \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \end{pmatrix} \\
&\qquad + \begin{pmatrix} a_1 & a_2 \end{pmatrix} \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}^{-1} \begin{pmatrix} \beta_i \\ \beta_i^{\text{prem}} \end{pmatrix}
\end{aligned} \tag{14}$$

which can be written as

$$\mathrm{E}[R_{it}] = b_0 + b_1 \beta_i + b_2 \beta_i^{\text{prem}}. \tag{15}$$

Equation (15) gives an unconditional specification of the CAPM when betas vary over time. Hence expected returns are linear in β_i and β_i^{prem}.

3.3 Statistical Test of CAPM Specifications

Combining (7) and (15), we have the following linear specification of CAPM

$$\mathrm{E}[R_{it}] = c_0 + c_1\beta_i^{\mathrm{vw}} + c_2\beta_i^{\mathrm{labor}} + c_3\beta_i^{\mathrm{prem}} \tag{16}$$

for some constants c_0, c_1, c_2 and c_3. When $c_2 = c_3 = 0$, We get the conventional specification of the CAPM studied in the literature. We will empirically examine how the specification in (16) performs.

While Fama-MacMeth regression used by Fama and French (1992) is a useful diagnostic tool, it does not impose the restriction that the true relation between expected return and betas is linear in the cross section. While several procedures have been suggested in the literature for testing linear beta pricing models, they are not directly applicable to our representation of the conditional CAPM given in (16). This is because the betas in (16) do not correspond to betas estimable by multiple regression as is assumed in these tests. We will therefore use the Generalized Method of Moments of Hansen (1982) to test whether (16) describes cross sectional variations in expected returns. For this purpose, we will rewrite (16) as:

$$\mathrm{E}[d_t r_{it}] = 1 \tag{17}$$

where

$$d_t = \delta_0 + \delta_{\mathrm{vw}} r_{\mathrm{vw}t} + \delta_{\mathrm{labor}} r_{\mathrm{labor}} + \delta_{\mathrm{prem}} Prem_{t-1} \tag{18}$$

and $r_{it} = 1 + R_{it}$, $r_{\mathrm{vw}t} = 1 + R_{\mathrm{vw}t}$, and $r_{\mathrm{labor}} = 1 + R_{\mathrm{labor}t}$ are gross rates of returns. (See Appendix A.1 for detailed derivations.) Following Hansen and Jagannathan (1991), we will refer to d_t as the stochastic discount factor. In our empirical work we will compare the relative performance of the following three versions of (18), i.e., we will consider the following three candidate stochastic discount factors:

$$\text{(A)} \qquad d_t = \delta_0 + \delta_{\mathrm{vw}} r_{\mathrm{vw}t}$$

$$
\begin{aligned}
\text{(B)} \qquad d_t &= \delta_0 + \delta_{\text{vw}} r_{\text{vw}t} + \delta_{\text{labor}} r_{\text{labor}t} &(19)\\
\text{(C)} \qquad d_t &= \delta_0 + \delta_{\text{vw}} r_{\text{vw}t} + \delta_{\text{labor}} r_{\text{labor}t} + \delta_{\text{prem}} Prem_{t-1}.
\end{aligned}
$$

Let $w_{it} = d_t r_{it} - 1$ for each portfolio i, or simply $\mathbf{w}_t = d_t \mathbf{r}_t - \mathbf{1}$ where $\mathbf{w}_t$, $\mathbf{r}$ and $\mathbf{1}$ represent $N(=100)$ dimensional vectors. Then we can write (17) as $\mathrm{E}[\mathbf{w}_t(\boldsymbol{\delta})] = 0$ where $\boldsymbol{\delta}$ is the vector of parameters in discount factor d_t in (19). Consider the time series average of the N dimensional vector $\mathbf{w}_t(\boldsymbol{\delta})$ given by $\bar{\mathbf{w}} = \sum_t^T \mathbf{w}_t(\boldsymbol{\delta})/T$. Since $\mathrm{E}[\mathbf{w}_t]$ is the vector of pricing errors associated with a given candidate stochastic discount factor d_t, we will refer to $\bar{\mathbf{w}}$ as the sample pricing error — which differs from 0 due to sampling error as well as model specification error. If d_t is a valid stochastic discount factor, i.e., if the particular specification of the CAPM is right, then $\mathrm{E}[\bar{\mathbf{w}}] = 0$.

A natural way to test whether $\mathrm{E}[\bar{\mathbf{w}}] = 0$ when the parameter vector $\boldsymbol{\delta}$ is known is to examine the Wald test of statistic given by the quadratic form

$$
Wald = T\bar{\mathbf{w}}'\mathbf{S}_{\bar{\mathbf{w}}}^{-1}\bar{\mathbf{w}} \tag{20}
$$

where

$$
S_{\bar{\mathbf{w}}} = \mathrm{E}[\bar{\mathbf{w}}\bar{\mathbf{w}}'].
$$

When $\mathrm{E}[\bar{\mathbf{w}}] = 0$, *Wald* is asymptotically distributed as a χ^2 with N degrees of freedom. This is true even when $\mathbf{S}_{\bar{\mathbf{w}}}$ is replaced by any consistent estimate $\mathbf{S}_{\bar{\mathbf{w}}T}$. When the parameter vector $\boldsymbol{\delta}$ is not known and has to be estimated, Hansen (1982) suggests choosing $\boldsymbol{\delta}$ to minimize the quadratic form *Wald*. Since $S_{\bar{\mathbf{w}}}$ is not known and must be estimated, first choose any positive definite matrix $\mathbf{S}$ in place of $\mathbf{S}_{\bar{\mathbf{w}}}$. Under suitable regularity conditions, Hansen (1982) shows that the resulting estimator $\boldsymbol{\delta}^1$ of $\boldsymbol{\delta}$ will be consistent. Use this estimator of $\boldsymbol{\delta}$ to construct $\mathbf{w}_t(\boldsymbol{\delta}^1)$, and arrive at a consistent estimator $\mathbf{S}_{\bar{\mathbf{w}}T}$ of $\mathbf{S}_{\bar{\mathbf{w}}}$. In the second stage choose that $\boldsymbol{\delta}_T$ as the estimator of $\boldsymbol{\delta}$ where $\boldsymbol{\delta}_T$ minimizes

$$
T\bar{\mathbf{w}}'\mathbf{S}_{\bar{\mathbf{w}}T}^{-1}\bar{\mathbf{w}}.
$$

The minimized value in the second stage is asymptotically distributed as a χ^2 with $N - K$ degrees of freedom, where K is the dimension of $\boldsymbol{\delta}$[19].

While the GMM method is intuitively appealing, there are drawbacks. First, the consistent estimate $\mathbf{S}_{\bar{\mathbf{w}}T}$ of the weighting matrix $\mathbf{S}_{\bar{\mathbf{w}}}$ will depend on estimated values of the unknown model parameters. While the use of consistent estimator $\mathbf{S}_{\bar{\mathbf{w}}T}$ for the wighting matrix $\mathbf{S}_{\bar{\mathbf{w}}}$ can be justified using large sample theory, use of a weighting matrix that does not depend on estimated model parameters is likely to have better properties in small samples. Second, keep in mind that our objective is to compare the empirical performance of the three candidates for the stochastic discount factor given in (19). The minimized value of the GMM criterion is a quadratic form involving the average of pricing error (i.e., the length of pricing error is measured as the square root of the quadratic form) and should not be different from zero after allowing for sampling error if our model holds. Note however that the weighting matrix used in measuring "length" depends on the particular stochastic discount factor used (i.e., the version of the model examined). Hence it is not appropriate to compare the minimized value of the criterion function across the three specifications[20]. In view of this, Hansen and Jagannathan (1992) suggest using the weighting matrix $\mathbf{S}_{\mathbf{r}} = \mathrm{E}[\mathbf{r}\mathbf{r}']$, which does not depend on the asset pricing model under consideration and consistent estimates of which can be obtained without first estimating model parameters.

Hansen and Jagannthan motivate the use of $\mathbf{S}_{\mathbf{r}} = \mathrm{E}[\mathbf{r}\mathbf{r}']$ as follows. If there is only one asset, then it is relatively straight-forward to compare the performance of different candidates for the stochastic discount factor. All we

[19]See Hansen (1982) for a set of sufficient conditions that the stochastic process $\mathbf{w}_t$ has to satisfy for this result to hold, MacKinlay and Richardson (1991) for a comparison of the GMM and classical methods.

[20]This difficulty is similar to the one that arises when using t-statistics to compare different models. The t-statistic could be small either because the numerator is close to zero or because the denominator is very large.

have to do is to compare the pricing error – i.e., the difference between the market price of an asset and the hypothetical price assigned to it by the given stochastic discount factor. When there are many assets (100 in our study) it is rather difficult to compare the pricing errors across the different candidate stochastic discount factors for the CAPM. In view of this, Hansen and Jagannathan suggest examining the pricing error on the portfolio that is most mispriced by a given model. There is a practical problem in implementing this simple idea.

Suppose there are at least two assets which do not have the same pricing error for a given candidate stochastic discount factor. Let r_{1t} and r_{2t} denote the corresponding gross returns. The date $t-1$ prices of these payoffs are both 1, i.e., by investing one dollar at date $t-1$ in portfolio i, the investor gets the payoff r_{it} at date t. A given asset pricing model may not assign a price of 1 at date $t-1$ to the payoff r_{it}. Suppose the pricing error is ψ_i , i.e., the model assigns a price of $1+\psi_i$. Consider forming a zero investment portfolio by going long one dollar in security 1 and short one dollar in security 2. The pricing error on this zero investment portfolio is $\psi_1 - \psi_2$. So long as this is not zero, the pricing error on any portfolio of the two assets with a price of one dollar can be made arbitrarily large by adding a scale multiple of this zero investment portfolio which is mispriced. The same problem arises if instead of examining the pricing error we examine the difference between the expected return on a portfolio and the expected return assigned by a particular asset pricing model to that portfolio. To overcome this problem, it is necessary to examine the pricing error on portfolios that have the same "size". Hansen and Jagannathan suggest using the second moment of the payoff as a measure of "size", i.e., examine the portfolio which has the maximum pricing error among all portfolio payoffs which have unit second moment.

Following Ross (1976), Hansen and Richard (1983) and Hansen and Jagannathan (1992), in our empirical work we will work with the representation of the capital asset pricing model given in equation (17), i.e., $\mathrm{E}[d_t r_{it}] = 1$. As Ross (1976) shows, so long as financial markets satisfy the law of one price, there will be at least one d_t that satisfies (17). We will denote the set of all valid stochastic discount factors that satisfy (17) by $\mathcal{M}$. As Hansen and Richard (1983) pointed out, an asset pricing model specifies a candidate for the stochastic discount factor. If the model is correctly specified, then the candidate given by a model will satisfy (17), i.e., it will be in the set $\mathcal{M}$ and $\mathrm{E}[\bar{\mathbf{w}}] = 0$.

Consider a portfolio of the N primitive assets defined by the portfolio weights $\mathbf{x}$. The date t payoff on this portfolio is given by $\mathbf{x}'\mathbf{r}_t$. It has a price of $\mathbf{x}'\mathbf{1}$ at the beginning of each date. The pricing error on this portfolio is $\mathbf{x}'\mathrm{E}[\mathbf{w}_t]$, and its sample analog is $\mathbf{x}'\bar{\mathbf{w}}$. Notice that the pricing error as well as its sample analog depend on the K dimensional vector of parameters $\boldsymbol{\delta}$. The second moment of this portfolio payoff is $\mathrm{E}[\mathbf{x}'\mathbf{r}_t]^2$, i.e., the norm of this portfolio is $\sqrt{\mathrm{E}[\mathbf{x}'\mathbf{r}_t]^2}$. For a given vector of parameters $\boldsymbol{\delta}$, Hansen and Jagannathan (1992) show that the maximum pricing error per unit norm on any portfolio of this N assets is given by:

$$Dist \equiv \sqrt{\mathrm{E}[\mathbf{w}_t]'\mathbf{S}_{\mathbf{r}}^{-1}\mathrm{E}[\mathbf{w}_t]}. \tag{21}$$

Dist is also the least squares distance between the given candidate stochastic discount factor and the nearest point to it in the set $\mathcal{M}$.

Since the parameters $\boldsymbol{\delta}$ describing a particular asset pricing model are unknown, a natural way is to choose that value for $\boldsymbol{\delta}$ that minimizes *Dist* given in (21). We can then assess the specification error of a given stochastic discount factor by examining the maximum pricing error *Dist* associated with it, as suggested by Hansen and Jagannathan (1992).

We will therefore estimate $\boldsymbol{\delta}$ by minimizing the sample analog of (21), i.e.,

$$\boldsymbol{\delta}_T = \operatorname*{Arg\,Min}_{\delta} \bar{\mathbf{w}}'\mathbf{S}_{\mathbf{r}T}^{-1}\bar{\mathbf{w}}$$

where $\mathbf{S}_{\mathbf{r}T}$ is the sample analog of $\mathbf{S}_{\mathbf{r}}$. Let $Dist_T = \sqrt{\min_\delta \bar{\mathbf{w}}'\mathbf{S}_{\mathbf{r}T}^{-1}\bar{\mathbf{w}}}$. To empirically examine a particular candidate stochastic discount factor, we will assume that it is a valid one and hence the maximum pricing error $Dist$ is zero, and then test whether $Dist_T$ is different from zero after allowing for sampling error. For this purpose, we derive the asymptotic distribution of the minimized value of the general quadratic form $T \min_\delta \bar{\mathbf{w}}'\mathbf{G}_T^{-1}\bar{\mathbf{w}}$ under the null hypothesis that $\mathrm{E}[\mathbf{w}_t] = 0$, where $\mathbf{G}_T$ is a consistent estimate of a positive definite weighting matrix $\mathbf{G}$. (For details, see Appendix A.2.) The sampling distribution of the squared maximum pricing error $Dist_T$ follows directly by setting $\mathbf{G} = \mathbf{S}_{\mathbf{r}T}$. Note that Hansen's result is obtained by setting $\mathbf{G}_T = \mathbf{S}_{\mathbf{w}T}$, and is a special case of the result given in Appendix A.2. We will compare the performance of the different candidates for the stochastic discount factor by comparing the corresponding p-values of $Dist_T$.

Table 9 gives the estimate and p-values of the parameters for various stochastic discount factors. When r_{vw} is the only variable used to construct the discount factor, the data strongly reject the model. The p-value for $TDist^2$ is only 0.34% and that for δ_{vw} is 27.59%. This is consistent with the fact that β_i^{vw} explains very little of the cross section of average returns in the Fama-MacBeth regressions. The point estimate of δ_{vw} is positive and not statistically different from zero. This is consistent with the negative but not significant slope coefficient for β^{vw} in Fama-MacBeth regressions. When r_{labor} is also used to construct the discount factor, δ_{labor} is significant with a p-value of 2.46%. The p-value of δ_{vw} increases from 27.59% to 49.18%. This supports our argument that labor income is a major part of aggregate wealth. But the data still reject the model and the p-value of $TDist_T^2$ is 2.25%. However, when *Prem* is also used in constructing the discount factor, the data no longer reject the model as the p-value of $TDist_T^2$ jumps to 42.72%. Both δ_{labor} and δ_{prem} are very significant — p-value of δ_{labor} is 0.1% and p-value of δ_{prem} is 0.05%. Hence when we take human capital into account in measuring the return to the aggregate wealth portfolio and allow betas to vary over the business cycles, there is little evidence against the CAPM.

3.4 Assessing the Specifications by Regression

It is possible that our inability to reject the CAPM using the Hansen-Jagannathan test statistic as described in Section 3.3 may be due to lack of power. We will therefore follow Berk's (1992) suggestion and examine whether size has significant explanatory power in Fama-MacBeth cross-sectional regressions. For this purpose, first consider the following three cross-sectional regressions

$$\begin{aligned} R_{it} &= c_{0t} + c_{\text{vw}t}\beta_i^{\text{vw}} + \epsilon_{it} \\ R_{it} &= c_{0t} + c_{\text{vw}t}\beta_i^{\text{vw}} + c_{\text{labor}t}\beta_i^{\text{labor}} + \epsilon_{it} \\ R_{it} &= c_{0t} + c_{\text{vw}t}\beta_i^{\text{vw}} + c_{\text{labor}t}\beta_i^{\text{labor}} + c_{\text{prem}t}\beta_i^{\text{prem}} + \epsilon_{it}. \end{aligned}$$

Estimated values for β^{labor} and β^{prem} are given in Table 10 and Table 11. Since β^{vw}, β^{labor} and β^{prem} are correlated with each other in the cross section, we report betas orthogonalized in the following way. We regress β^{labor} on a constant and β^{vw} and report the residual as the "orthogonalized β_i^{labor}". We then regress β^{prem} on a constant, β^{vw} and β^{labor} and report the residual as the "orthogonalized Prem Beta". Notice that orthogonalized β_i^{labor} decreases as we move from left to right in any given size decile, while stock beta increases. The increase in stock beta alone suggests that the expected return on stocks should increase when we move from left to right in any given size decile. However, the decrease in β_i^{labor} suggests that this conclusion need not hold, so long as the risk premium on human capital is also positive. There is no particluar pattern to orthogonalized β_i^{prem}. This suggests that, to the extent the specification we suggest improves the ability of the CAPM to explain the cross section of average returns, the poor performance of the specification employed by Fama and French (1992) cannot be entirely due to inaccurate measurement of the return to the aggregate wealth portfolio.

The regression results are reported in Table 12. When β^{vw} is the only variable, the CAPM specification explains only 1.35% of the cross sectional variation of the average portfolio returns, as reported earlier. When β^{labor} is added to the CAPM specification, r_{I}^2 goes up to 28% and c_{labor} is statistically significant at conventional level (p-value = 4.33%). When β^{prem} is also added, r_{I}^2 goes up to 57%, while c_{labor} remains significant (with p-value unchanged), c_{prem} is very significant with an associated p-value of 0.11%. All these results are consistent with the results reported in Table 9, confirming our earlier conclusions in Section (3).

Finally, we add the variable $\log(\mathrm{ME}_{it})$ to the regression, i.e., consider

$$R_{it} = c_{0t} + c_{\mathrm{vw}t}\beta_i^{\mathrm{vw}} + c_{\mathrm{labor}t}\beta_i^{\mathrm{labor}} + c_{\mathrm{prem}t}\beta_i^{\mathrm{prem}} + c_{\mathrm{size}t}\log(\mathrm{ME}_{it}) + \epsilon_{it}$$

The result is shown in the last line of Table 12. First, there is no substantial increase in r_{I}^2. Second, c_{size} fails to be significant at the conventional 5% level (p-value = 9.28%). This suggests that, instead of viewing the size effect as evidence against the CAPM, we should view it as evidence against the use of stocks as proxy for aggregate wealth and the assumption that betas remain constant over time. Once these two additional ad hoc assumptions are relaxed, the size effect is greatly reduced.

In order to visually compare the performance of the different specifications, we plotted the fitted expected return computed using the estimated parameter values against the realized average return, for each of the four specifications. These are given in Figures 2, 3, 4 and 5. If the fitted expected returns and the realized average returns are the same, then all the points should lie on the 45 degree line through the origin. When stock beta alone is used the fitted expected returns are all about the same whereas the realized average returns vary substantially across the 100 portfolios. The performance substantially improves when β_i^{labor} is also used. It is even better when β_i^{prem} is used in addition. The distribution of the points around the 45 degree line suggests that the improved performance of the CAPM using the specification we suggest in this paper is not due to a few outliers. The distribution of the points around the 45 degree line does not significantly change when we add log(ME) as an additional explanatory variable – confirming our earlier findings.

4 Some Additional Investigations

4.1 Time Variations in Betas

While the results reported in the previous sections are encouraging, it is still possible that β^{prem} could be capturing something other than time variations in β. This is because β^{prem} measures the co-movement of the expected return with *Prem* and hence the predictability of the portfolio return. The only model-free conclusion that is possible is that stocks that have relatively large predictable components in expected returns are the ones that cause mispricing from the perspective of the static CAPM which assumes that betas are constant. Since we did not provide any direct evidence that the cross sectional variations in the predictable component in returns is due to time varying betas, it is conceivable that we are documenting an entirely different phenomenon. While it is possible that perceived financial risk could be varying over time in a systematic manner and some firms could be more susceptible to this risk than others, it is also possible that certain types of securities could be temporarily mispriced and this mispricing may be caused by incorrect assessment of risk. Even though we cannot rule out such possibilities entirely, we will provide some evidence that *Prem* does capture time variations in beta.

The return generating process can always be written as

$$R_{it} = \varphi_{it-1} + \beta_{it-1}R_{mt} + \epsilon_{it} \tag{22}$$

where $\mathrm{E}[\epsilon_{it}] = 0$ and $\mathrm{Cov}(R_{mt}, \epsilon_{it}) = 0$. Substituting (10) for β_{it-1} we get

$$R_{it} = \varphi_{it-1} + \tilde{\beta}_i R_{mt} + \theta_i Prem_{t-1} R_{mt} + \eta_{it-1} R_{mt} + \epsilon_{it}. \tag{23}$$

Let $\varphi_i = \mathrm{E}[\varphi_{it-1}]$ and $v_{it-1} = \varphi_{it-1} - \varphi_i$. Substituting (6) for R_{mt} we get

$$\begin{aligned} R_{it} &= \varphi_i + \tilde{\beta}_i\phi_0 + \tilde{\beta}_i\phi_{\mathrm{vw}} R_{\mathrm{vw}t} + \tilde{\beta}_i\phi_{\mathrm{labor}} R_{\mathrm{labor}t} + \theta_i\phi_0 Prem_{t-1} \\ &\quad + \theta_i\phi_{\mathrm{vw}} Prem_{t-1} R_{\mathrm{vw}t} + \theta_i\phi_{\mathrm{labor}} Prem_{t-1} R_{\mathrm{labor}t} \\ &\quad + v_{it-1} + \eta_{it-1} R_{mt} + \epsilon_{it} \end{aligned}$$

which can be written as

$$\begin{aligned} R_{it} &= \mu_{0i} + \mu_{1i} R_{\mathrm{vw}t} + \mu_{2i} R_{\mathrm{labor}t} + \mu_{3i} Prem_{t-1} \\ &\quad + \mu_{4i} Prem_{t-1} R_{\mathrm{vw}t} + \mu_{5i} Prem_{t-1} R_{\mathrm{labor}t} + u_{it}. \end{aligned}$$

The error term u_{it} is on average zero. If it is orthogonal to the right side variables, we can then obtain consistent estimates of the parameters from the OLS regression. However, it is difficult to verify whether these orthogonality conditions hold. In particular, it is possible that $Prem_{t-1}$ is not orthogonal to v_{it-1}. Hence we examine whether θ_i is zero by testing the null hypothesis that μ_{4i} and μ_{5i} equal to zero.

Table 13 gives the F-test to the hypothesis, $\mu_{4i} = \mu_{5i} = 0$. Portfolios corresponding to F-values in the critical rejection region at 5% are marked with a * in the table. We can reject the null hypothesis for 33 of the 100 portfolios at the 5% level and for 51 portfolios at the 10 percent level. Notice that the largest F-value in the table is 7.07. If we suppose the null hypothesis is true for every portfolio, the probability that at least one of the F_i-values is greater than or equal to k is

$$p = P\left[\left(\bigcap_{i=1}^{n}\{F_i < k\}\right)^c\right] = P\left[\bigcup_{i=1}^{n}\{F_i < k\}^c\right] \leq \sum_{i=1}^{N} P[F_i \geq k] = NP[F_i \geq k].$$

When $k = 7.07$, the largest F-value in Table 13, we have $p = 100\times\ 0.000989 = 0.0989$. Hence, using the Bonferoni test described above, which is rather conservative, we can reject the null hypothesis that all the θ_i's are zero at 10% level. It suggests that the ability of β^{prem} to explain the cross section of average returns is likely to be at least in part due to temporal covariance between betas and *Prem.*

4.2 28 Years May Be Too Short a Time Period

While the results presented in Section 3 lend empirical support for the conditional CAPM, it should not be viewed as though this support is total. What we have documented is that the conditional version of the CAPM with some auxiliary assumptions explains a substantial part of the cross sectional variation in average returns in the particular sample analyzed. Any conclusions regarding the validity of the conditional CAPM should be tempered by the fact that the assumptions that are necessary to justify the applicability of the statistical methods we use may not be satisfied.

It is well recognized by macro economists (see Slutzky (1937)) that small but recurrent shocks can cause business cycle like behavior in macro economic models, i.e., cycles that exhibit a periodicity of approximately 4 to 5 years. However, this does not imply that business cycles in the US economy is induced by small and recurrent technology or taste shocks alone. For example, Hamilton (1983) finds that almost every post war recesssion is preceded by unanticipated shock to oil prices. Our view is that there are only about as many major information events as there are business cycles in a given period. Consider the major shock that affected the relative price of oil during the early 1970s. Suppose it takes 10 years for everyone to figure out its impact on the economy – and hence its effect on the operations of firms. It is conceivable that some large firms will become small at the end of the 10 years and some small firms will become big during the same period. In this case the naive econometrician who used the sample standard deviation of monthly returns on stocks of these firms to assess the sampling error associated with the average monthly return (as a measure of the monthly expected return), will seriously understate the sampling error. This effect is analogous to the well known statistically significant upward (or downward trend) prior to announcement dates found in event studies. In view of this, as Fischer Black points out[21], it is sometimes useful to use a bit of introspection instead of entirely relying on conventional data analysis. Expected returns cannot be measured precisely using data over a short time period. Using 28 years of data (as in the Fama-French study) probably involves only 5 major informational events.

It is also necessary to keep in mind that the CAPM assumes that economic agents live in an ideal world with no friction where everyone possesses the same information. While such simplifying assumptions provide important insights about the nature of the relation between asset prices and real economic activity, it is not at all clear that models based on such assumptions will be able to explain the observed empirical patterns in asset prices at small time intervals.

One particular difficulty is due to the presence of deterministic calendar month seasonalities in the data. For example, Keim (1983) pointed out that

[21]In his lecture at *The Berkeley Program in Finance: Are Betas Irrelevant? Evidence and Implications for Asset Management*, September 13–15, 1992 at Santa Barbara, California

most of the abnormal return to stocks occur during January. Ariel (1987) documents beginning and end of the month seasonal patterns in stock return data. Glosten, Jagannathan and Runkle (1993) document a October seasonal in the volatility of stock returns. If deterministic monthly seasonal patterns are present in the return data, then it would be one more reason to examine the cross section of annual holding period returns when empirically examining the CAPM.

To demonstrate that deterministic monthly seasonal patterns are present in our data set as well, we treat each calendar month as being different from other calendar months and examine the cross sectional relation between returns and other variables using the Fama-MacBeth regression approach.

Table 14 gives the results when beta alone is used as the explanatory variable. The relation between beta and average return is positive and statistically significant only in the month of January. It is negative and significant in June and October. The average r-square of the cross sectional regressions, r^2_{II}, varies from a low of 16% in December to a high of 38% in October.

The ability of beta to explain the cross section of average returns as measured by r^2_{I} also exhibits substantial variation across months — from a low of 2% in November and April to a high of 66% in October. The average value of r^2_{I} during months when the estimated relation between beta and average return is positive is 9% whereas it is 35% when the relation is negative. Hence, in this data set, the relation between average return and beta is stronger during down markets than in up markets.

The relation between beta and return during December is especially weak. This is consistent with the hypothesis that a large part of the trading during December may be motivated by tax considerations, as pointed out by Roll (1981) and Constantinides (1984).

Table 15 gives the results when beta and size are both used as explanatory variables. The coefficient for size is negative in January and positive in October and statistically significant in both months. However, the coefficient for beta loses significance only in January. It appears as though in January the return to small stocks stochastically dominates the return to large stocks, in this sample, whereas it is the other way around in October. While the size coefficient is statistically significant for 7 of the 12 months, it is positive for 2 of these 7 months, suggesting that the sampling theory based on which the t-statistics are computed may not be valid.

These results confirm the presence of deterministic seasonal patterns in our sample, suggesting the need to examine annual holding period returns using data over several years.

5 Conclusion

There are two major difficulties in examining the empirical support for the CAPM. First, the return to the aggregate wealth portfolio is not observable. Second, the CAPM is a static model while the real world is inherently dynamic. Hence it is necessary to make some auxiliary assumptions. In order to overcome these difficulties, it is generally assumed that (a) the return to stocks measures the return to the aggregate wealth portfolio and (b) betas of assets remain constant over time. Under these assumptions, Fama and French (1992) find that the relation between average return and beta is flat and that there is a strong size effect. We find that the CAPM, with these assumptions, is able to explain only about 1 percent of the cross sectional variation in average returns of the 100 size/beta portfolios constructed using the Fama-French sorting procedure.

We argue that assumptions (a) and (b) are not reasonable and demonstrate that the empirical support for the CAPM is surprisingly strong when these assumptions are relaxed. When human capital is also included in measuring wealth, the CAPM is able to explain 28% of the cross section of average returns. When, in addition, betas are allowed to vary over the business cycles, CAPM is able to explain 57%. More important, size does not explain the residual variation in average return after taking sampling errors into account.

While relaxing these restrictive assumptions considerably improves the empirical performance of the CAPM, we still advocate caution in interpreting this as strong support for the CAPM for the following reasons.

First, our modeling of the time variations in betas is rather ad hoc in nature. If one were to take the criticism that the real world is inherently dynamic seriously, then it may be necessary to explicitly model what is missing in a static model. In particular, in a dynamic world investors may care about hedging against a variety of risks that do not arise in a static economy. One possibility is to extend Merton's inter-temporal CAPM along the lines suggested by Campbell (1993a) for empirical analysis using time series data on returns and economic aggregates.

Second, we believe that relying on monthly return data for only 28 years to examine cross sectional variations in expected returns to some extent borders on naïveté. Our view is that there are only about as many major information events as there are business cycles in the given period. Using 28 years of data (as in the Fama-French study) probably involves only 5 major informational events. Also, a number of events occur at deterministic monthly and yearly frequencies and it may be reasonable to expect that such events may influence the behavior of asset prices at monthly frequencies. Since such events are outside the scope of asset pricing models we study, one natural strategy would be to examine the empirical performance of models using annual data over a sufficiently long period of time. Such an approach has its own set of shortcomings, most important of which is that the economy may not really be stationary.

A Appendix

A.1 Discount Factors Implied by the CAPM

Denote the variables r_{vwt}, r_{labort} and $Prem_{t-1}$ simply by F_{2t}, F_{3t} and F_{4t} and let

$$\beta_{ik} = \frac{\mathrm{E}[r_{it}(F_{kt} - \mathrm{E}[F_{kt}])]}{\mathrm{Var}[F_k]} \quad k = 2,\ 3,\ 4 \tag{24}$$

then the model in (16) can be written as

$$\mathrm{E}[r_{it}] = \gamma_1 + \sum_{k=2}^{K} \gamma_k \beta_{ik} \tag{25}$$

where $K = 4$. Substituting (24) into (25), we can have

$$\mathrm{E}\left[r_{it}\left(\frac{1}{\gamma_1} + \sum_{k=2}^{K} \frac{\gamma_k \mathrm{E}[F_{kt}]}{\mathrm{Var}[F_{kt}]\gamma_1} - \sum_{k=2}^{K} \frac{\gamma_k}{\mathrm{Var}[F_{kt}]\gamma_1} F_{kt}\right)\right] = 1$$

which can be written as

$$\mathrm{E}\left[r_{it}\left(\delta_1 + \sum_{k=2}^{K} \delta_k F_{kt}\right)\right] = 1. \tag{26}$$

Thus the stochastic discount factor for the model in (16) should be

$$d_t(\delta) = \sum_{k=1}^{K} \delta_k F_{kt}$$

with $F_{1t} \equiv 1$ and $\delta \equiv (\delta_1, \ldots, \delta_K)$. Then (26) becomes

$$\mathrm{E}[r_{it} d_t] = 1. \tag{27}$$

A.2 Distribution of Hansen-Jagannathan Distance

Let $w_t(\delta) \equiv (w_{1t}(\delta), \ldots, w_{Nt}(\delta))'$ where $w_{it}(\delta) = r_{it} d_t(\delta) - 1$, and $\bar{w}(\delta) = (\bar{w}_1(\delta), \ldots, \bar{w}_N(\delta))'$ where $\bar{w}_i = \sum_t r_{it} \sum_k \delta_k F_{kt}/T$. Let G be any $N \times N$

positive definite matrix and G_T be a consistent estimate of G and let

$$\delta_T \equiv \operatorname*{Arg\,Min}_{\delta} \bar{w}' G_T^{-1} \bar{w}. \tag{28}$$

The first order condition for minimizing the quadratic form in (28) is

$$\left[\frac{\partial \bar{w}'}{\partial \delta} G_T^{-1} \frac{\partial \bar{w}}{\partial \delta'}\right] \delta_T = \frac{\partial \bar{w}'}{\partial \delta} G_T^{-1} \mathbf{1}$$

which is a linear equation system for δ_T. So we can simply solve the linear system to obtain δ_T.

Since $\bar{w}(\delta)$ is a linear function in δ, we have

$$\bar{w}(\delta_T) = \bar{w}(\delta_0) + \frac{\partial \bar{w}}{\partial \delta'}(\delta_T - \delta_0). \tag{29}$$

Pre-multiplying $\partial \bar{w}'/\partial \delta G_T^{-1}$ to both sides, we have

$$\frac{\partial \bar{w}'}{\partial \delta} G_T^{-1} \bar{w}(\delta_T) = \frac{\partial \bar{w}'}{\partial \delta} G_T^{-1} \bar{w}(\delta_0) + \frac{\partial \bar{w}'}{\partial \delta} G_T^{-1} \frac{\partial \bar{w}}{\partial \delta'}(\delta_T - \delta_0).$$

Substituting the first order condition into the left side, we obtain

$$\delta_T - \delta_0 = -\left[\frac{\partial \bar{w}'}{\partial \delta} G_T^{-1} \frac{\partial \bar{w}}{\partial \delta'}\right]^{-1} \frac{\partial \bar{w}'}{\partial \delta} G_T^{-1} \bar{w}(\delta_0). \tag{30}$$

Let $S = \mathrm{E}[\bar{w}\bar{w}']$ and $D = \mathrm{E}[r_t F_t']$, where $F_t = (F_{1t}, \ldots, F_{Kt})'$. Since $\partial \bar{w}/\partial \delta' \xrightarrow{\mathrm{p}} D$ and (27) implies $\sqrt{T}\bar{w}(\delta_0) \xrightarrow{\mathrm{d}} N(0, S)$, we have

$$\sqrt{T}(\delta_T - \delta_0) \xrightarrow{d} N(0, C)$$

where

$$C = (D'G^{-1}D)^{-1} D'G^{-1}SG^{-1}D(D'G^{-1}D)^{-1}.$$

Substituting (30) into (29), we obtain

$$\sqrt{T}\bar{w}(\delta_T) = \left(I - \frac{\partial \bar{w}}{\partial \delta'}\left[\frac{\partial \bar{w}'}{\partial \delta} G_T^{-1} \frac{\partial \bar{w}}{\partial \delta'}\right]^{-1} \frac{\partial \bar{w}'}{\partial \delta} G_T^{-1}\right) \sqrt{T}\bar{w}(\delta_0)$$

which converges in distribution to

$$\lim_{T\to+\infty} \sqrt{T}\bar{w}(\delta_T) = (I - D[D'G^{-1}D]^{-1}D'G^{-1})N(0,S).$$

Then we have

$$\begin{aligned}
TDist_T(\delta_T) \quad &\xrightarrow{\mathrm{d}} \quad N(0,S)'(I - G^{-1}D[D'G^{-1}D]^{-1}D') \\
&\qquad\qquad G^{-1}(I - D[D'G^{-1}D]^{-1}D'G^{-1})N(0,S) \\
&= \quad N(0,S)'(G^{-1} - G^{-1}D[D'G^{-1}D]^{-1}D'G^{-1})N(0,S) \\
&= \quad N(0,I)'S^{\frac{1}{2}}G^{-\frac{1}{2}}(I - (G^{-\frac{1}{2}})'D[D'G^{-1}D]^{-1}D'G^{-\frac{1}{2}}) \\
&\qquad\qquad (G^{-\frac{1}{2}})'(S^{\frac{1}{2}})'N(0,I) \\
&= \quad N(0,I)'\Gamma N(0,I)
\end{aligned}$$

where

$$\Gamma = S^{\frac{1}{2}}G^{-\frac{1}{2}}(I - (G^{-\frac{1}{2}})'D[D'G^{-1}D]^{-1}D'G^{-\frac{1}{2}})(G^{-\frac{1}{2}})'(S^{\frac{1}{2}})' ,$$

which is symmetric and semi-positive definite. Here $G^{\frac{1}{2}}$ and $S^{\frac{1}{2}}$ are uper-triangle matrices from the Cholesky decomposition of G and S, i.e. $G = (G^{\frac{1}{2}})'G^{\frac{1}{2}}$ and $S = (S^{\frac{1}{2}})'S^{\frac{1}{2}}$. Since

$$I - (G^{-\frac{1}{2}})'D[D'G^{-1}D]^{-1}D'G^{-\frac{1}{2}}$$

is symmetric idempotent and its trace is $N-K$, we know that its rank is $N-K$ and thus the rank of Γ is also $N-K$. Then there is an orthogonal matrix H and a diagonal matrix $\Lambda = \mathrm{diag}\{\lambda_1, \ldots, \lambda_{N-K}, 0, \ldots, 0\}$, where λ_is are the $N-K$ positive eigen values of Γ, such that $\Gamma = H'\Lambda H$. Then the limiting distribution of $TDist_T(\delta_T)$, denoted by x, is

$$\begin{aligned}
\lim TDist_T(\delta_T) \overset{d}{=} x \quad &= \quad N(0,I)'H'\Lambda H N(0,I) \\
&= \quad N(0,I)'\Lambda N(0,I) \\
&= \quad \sum_{j=1}^{N-K} \lambda_j v_j
\end{aligned}$$

where $\{v_i\}_{i=1}^{N-K}$ are $N-K$ independent random variables, each of which has distribution of $\chi^2(1)$. The sum of these eigen values is the mean of x, i.e.,

$$\mathrm{E}[x] = \sum_{i=1}^{N-K} \lambda_i.$$

The random variable x does not have a simple distribution function. However, using the above expression, we can still conveniently compute the p-value to test the null hypothesis that the discount factors are specified correctly. Random sampling

$$\{v_{ij}\}_{i=1,\ldots,T^*;\ j=1,\ldots,N-K}$$

independently from $\chi^2(1)$ (on computer), we can obtain a set of independent samples, $\{x_i\}_{i=1}^{T^*}$, by letting

$$x_i = \sum_{j=1}^{N-K} \lambda_j v_{ij}\ .$$

Then, by Law of Large Numbers, we have, as $T^* \longrightarrow \infty$,

$$\frac{1}{T^*}\sum_{i=1}^{T^*} I(x_i \leq TDist_T(\delta_T)) \xrightarrow{p} \int_0^{TDist_T(\delta_T)} d\psi(x) = \mathrm{Prob}\{\ x \leq TDist_T(\delta_T)\ \}$$

where $\psi(x)$ is the probability distribution of x and p is the asymptotic p-value of $Td_T(\delta_T)$. Notice that, if the eigen values are all equal to 1, then x has χ^2 distribution with degree of freedom of $N-K$, which is the case of $G = S$.

B Tables and Figures

B.1 Tables

Table 1. Time Series Averages of Portfolio Returns (%)

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME-1	1.44	1.53	1.56	1.71	1.36	1.44	1.37	1.33	1.46	1.34
ME-2	1.13	1.22	1.09	1.19	1.38	1.37	1.37	1.30	1.15	0.95
ME-3	1.26	1.27	1.22	1.26	1.16	1.29	1.34	1.19	1.12	0.89
ME-4	1.37	1.47	1.40	1.28	1.01	1.39	1.11	1.33	1.07	0.95
ME-5	0.97	1.53	1.10	1.28	1.18	1.04	1.35	1.07	1.23	0.82
ME-6	1.07	1.36	1.34	1.12	1.25	1.27	0.84	0.94	0.92	0.77
ME-7	0.99	1.18	1.13	1.19	0.96	0.99	1.11	0.91	0.90	0.83
ME-8	0.95	1.19	1.02	1.39	1.18	1.24	0.94	1.02	0.88	1.08
ME-9	0.94	0.92	1.05	1.17	1.15	1.03	1.02	0.84	0.80	0.51
ME-10	1.06	0.97	1.02	0.94	0.83	0.93	0.82	0.83	0.61	0.72

Table 2. Beta to CRSP Value-Weighted Index

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME-1	0.90	0.99	1.01	1.13	1.17	1.21	1.20	1.31	1.44	1.54
ME-2	0.83	1.00	1.09	1.12	1.18	1.29	1.33	1.39	1.48	1.63
ME-3	0.78	0.93	1.09	1.11	1.18	1.27	1.29	1.40	1.42	1.70
ME-4	0.75	0.91	1.05	1.13	1.19	1.32	1.25	1.32	1.56	1.61
ME-5	0.57	0.78	1.10	1.10	1.12	1.20	1.25	1.43	1.45	1.54
ME-6	0.62	0.77	0.88	1.01	1.08	1.25	1.22	1.34	1.32	1.59
ME-7	0.64	0.84	1.01	1.07	1.16	1.21	1.26	1.26	1.31	1.54
ME-8	0.64	0.73	0.91	1.04	1.07	1.17	1.22	1.19	1.23	1.50
ME-9	0.62	0.78	0.88	0.96	1.04	1.05	1.13	1.17	1.22	1.34
ME-10	0.68	0.76	0.80	1.00	0.97	1.00	1.04	1.09	1.10	1.28

Table 3. Time Series Average of Portfolio Size (log of $million)

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME-1	2.48	2.50	2.49	2.48	2.48	2.50	2.46	2.46	2.46	2.34
ME-2	3.71	3.72	3.73	3.73	3.71	3.71	3.72	3.72	3.72	3.72
ME-3	4.21	4.21	4.21	4.21	4.21	4.23	4.21	4.22	4.21	4.20
ME-4	4.67	4.65	4.64	4.65	4.65	4.65	4.65	4.64	4.64	4.64
ME-5	5.07	5.09	5.07	5.08	5.08	5.07	5.07	5.07	5.07	5.05
ME-6	5.47	5.48	5.47	5.48	5.48	5.48	5.48	5.48	5.47	5.48
ME-7	5.91	5.92	5.93	5.92	5.92	5.89	5.91	5.90	5.92	5.90
ME-8	6.44	6.42	6.43	6.39	6.43	6.41	6.43	6.42	6.40	6.40
ME-9	6.98	6.98	7.00	6.98	6.96	6.97	6.95	6.96	6.95	6.97
ME-10	8.11	8.26	8.22	8.19	8.16	8.18	8.06	8.03	7.92	7.81

Table 4. Fama-MacBeth Regressions

	$\bar{\gamma}_{vw}$	$\|t\|$	$\bar{\gamma}_{size}$	$\|t\|$	r^2_{I}	r^2_{II}
Fama and French (1992)	0.15	(0.46)				
			-0.15	(2.58)		
	-0.37	(1.21)	-0.17	(3.41)		
NYSE and AMEX	-0.10	(0.28)			1.35	26.92
			-0.10	(1.91)		23.01
	-0.32	(0.95)	-0.12	(2.47)		43.69
NYSE only	-0.03	(0.08)			0.12	24.02
			-0.11	(1.89)		19.23
	-0.23	(0.67)	-0.12	(2.41)		37.70

Table 5. Time Series Average of Portfolio Market Value ($Billion)

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME-1	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.8	0.8	1.1
ME-2	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9	1.1
ME-3	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.2	1.1	1.4
ME-4	1.5	1.6	1.6	1.7	1.7	1.6	1.7	1.6	1.7	2.1
ME-5	2.2	2.4	2.3	2.4	2.6	2.3	2.5	2.4	2.4	2.6
ME-6	3.2	3.5	3.5	3.6	3.6	3.6	3.7	3.7	3.6	4.0
ME-7	5.0	5.7	5.6	5.3	5.5	5.1	5.4	5.5	5.8	6.0
ME-8	8.3	8.6	8.7	8.6	8.8	8.4	9.2	9.2	8.8	9.5
ME-9	15.0	15.2	15.3	15.3	15.1	15.5	14.9	15.2	14.9	16.2
ME-10	87.7	155.3	86.1	75.6	72.7	73.8	69.1	59.5	49.0	41.5

Table 6. Time Series Average of Number of Firms

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME-1	51	41	42	44	43	44	46	50	53	80
ME-2	17	16	15	17	16	17	17	18	19	23
ME-3	13	13	13	13	14	14	14	15	15	18
ME-4	12	12	13	13	13	13	13	13	13	16
ME-5	11	12	12	12	13	12	13	13	13	14
ME-6	11	12	12	12	12	12	12	12	12	13
ME-7	11	12	12	12	12	11	12	12	12	13
ME-8	11	11	11	12	12	11	12	12	12	12
ME-9	11	11	11	11	12	11	11	11	11	12
ME-10	11	11	11	12	12	11	11	11	11	12

Table 7. Estimates of coefficient of $|R_{vw}|R_{vw}$

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME- 1	2.50	1.52	1.08	0.23	1.29	0.72	0.83	-0.50	0.29	-0.15
ME- 2	1.62	2.59	1.62	1.22	0.57	-0.38	0.66	-0.21	-0.19	-0.33
ME- 3	2.68	0.24	0.95	-0.10	0.71	0.05	-0.43	0.22	-0.50	-0.55
ME- 4	0.99	-0.04	0.97	1.08	1.41	0.01	-0.94	-2.33	-0.83	-2.00
ME- 5	0.03	-0.22	0.92	0.25	0.46	-0.20	0.86	0.15	-0.68	-0.78
ME- 6	0.60	-0.63	0.09	-0.04	0.12	0.94	-1.14	-0.11	-0.40	-2.01
ME- 7	-0.10	0.30	0.70	1.11	0.43	0.54	1.06	-1.21	-0.40	0.36
ME- 8	-0.76	0.14	0.17	0.59	0.01	0.23	0.51	0.02	-1.00	-2.10
ME- 9	-0.17	0.62	0.76	0.02	-0.54	-0.33	1.69	-0.72	-1.29	-1.60
ME-10	0.12	0.79	-0.79	0.38	1.04	0.78	1.15	0.36	-1.07	-1.72

Table 8. t-value of coefficient of $|R_{vw}|R_{vw}$

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME- 1	2.40	1.45	0.94	0.19	1.03	0.58	0.63	-0.37	0.20	-0.09
ME- 2	1.98	2.74	1.82	1.33	0.53	-0.36	0.64	-0.20	-0.16	-0.25
ME- 3	3.42	0.29	1.03	-0.11	0.73	0.05	-0.43	0.22	-0.50	-0.47
ME- 4	1.42	-0.06	1.26	1.36	1.71	0.01	-1.01	-2.57	-0.71	-1.74
ME- 5	0.04	-0.34	1.50	0.36	0.59	-0.23	1.08	0.17	-0.69	-0.73
ME- 6	0.85	-1.05	0.16	-0.06	0.18	1.22	-1.43	-0.14	-0.40	-1.90
ME- 7	-0.14	0.44	1.19	1.90	0.66	0.85	1.61	-1.79	-0.53	0.38
ME- 8	-1.00	0.22	0.31	1.03	0.02	0.41	0.85	0.03	-1.32	-2.28
ME- 9	-0.24	1.10	1.50	0.04	-0.94	-0.61	2.83	-1.24	-2.22	-2.16
ME-10	0.18	1.43	-1.46	0.78	2.33	1.55	2.41	0.76	-2.00	-2.70

Table 9. Hansen-Jagannathan Test of the CAPM

$TDist_T^2$	p	δ_{vw}	p	δ_{labor}	p	δ_{prem}	p
141.51	(0.34)	1.55	(27.59)				
137.15	(2.25)	1.02	(49.18)	-77.53	(2.46)		
133.08	(42.73)	1.54	(31.38)	-138.09	(0.10)	-0.83	(0.05)

Table 10. Orthogonalized β_i^{labor}

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME- 1	1.27	1.39	1.07	1.09	1.37	1.09	2.12	1.33	0.76	0.99
ME- 2	0.90	0.76	-0.13	0.11	0.93	1.11	-0.08	-0.46	0.44	-0.69
ME- 3	0.33	0.23	-0.41	0.51	0.07	0.14	1.59	0.71	-0.40	0.59
ME- 4	0.04	0.64	0.34	0.17	-0.14	0.15	-0.74	-0.05	0.27	1.02
ME- 5	0.21	0.57	0.05	-0.10	-0.03	-0.51	-0.77	-0.56	0.59	0.97
ME- 6	0.02	0.04	0.21	0.04	-0.02	-1.25	0.03	-0.69	-0.21	-0.62
ME- 7	-0.42	-0.53	-0.21	-0.47	-0.21	-0.63	-0.99	-0.28	-0.59	-0.96
ME- 8	0.27	-0.28	-0.49	-0.45	-1.14	-0.31	-0.93	-0.88	0.53	-0.79
ME- 9	-0.18	-0.16	-0.69	-0.76	-0.04	-0.49	-0.76	-0.81	-0.64	-0.82
ME-10	0.53	-0.10	-0.32	-0.87	-0.44	-0.31	-0.41	-0.68	0.39	-1.16

Table 11. Orthogonalized β_i^{prem}

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME- 1	-0.02	0.30	0.21	0.09	0.44	0.25	0.18	0.26	0.47	0.24
ME- 2	-0.46	0.41	0.47	-0.23	0.02	0.33	0.40	0.66	0.34	-0.04
ME- 3	-0.04	0.10	0.41	0.28	-0.01	0.06	-0.49	-0.04	0.18	-0.38
ME- 4	-0.04	0.38	-0.07	-0.28	0.24	0.13	0.71	-0.28	-0.26	-0.18
ME- 5	-0.06	-0.12	0.18	0.08	0.06	0.16	0.11	0.14	-0.19	-0.78
ME- 6	0.21	0.38	0.38	-0.01	0.12	0.61	-0.30	0.01	-0.45	-0.49
ME- 7	-0.07	0.44	0.44	0.26	0.19	0.15	-0.08	0.06	0.31	0.05
ME- 8	-0.44	-0.33	-0.02	0.16	0.40	0.11	0.04	-0.34	-0.69	-0.38
ME- 9	-0.26	-0.34	0.13	0.08	-0.14	-0.14	0.14	-0.54	-0.04	-0.40
ME-10	-0.45	-0.07	-0.02	-0.26	-0.09	-0.64	-0.54	-0.34	-0.89	-0.26

Table 12. Results of Fama-MacBeth regressions

r^2_{I}	r^2_{II}	c_{vw}	p	c_{labor}	p	c_{prem}	p	c_{size}	p
1.35	26.92	-0.10	(78.00)						
28.28	37.35	-0.20	(56.02)	0.17	(4.33)				
56.92	41.66	-0.41	(22.76)	0.17	(4.21)	0.07	(0.11)		
	45.93	-0.40	(24.51)	0.07	(14.73)	0.21	(0.41)	-0.08	(9.28)

Table 13. F-values of portfolios

	β-1	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	β-10
ME-1	1.92	4.24*	4.35*	4.96*	3.14*	3.85*	2.92	4.68*	3.63*	2.76
ME-2	1.42	2.95	0.98	3.41*	7.34*	2.71	3.49*	2.51	3.48*	2.51
ME-3	5.20*	4.66*	0.78	2.17	2.77	2.00	2.48	5.15*	1.06	4.67*
ME-4	1.75	0.32	1.16	2.43	1.96	1.31	3.03*	4.87*	4.50*	0.81
ME-5	3.09*	3.43*	1.09	2.33	4.22*	0.53	1.02	1.22	2.52	3.13*
ME-6	1.73	0.66	0.64	4.34*	2.50	4.30*	3.53*	2.38	1.48	1.42
ME-7	1.71	1.15	1.76	3.13*	0.25	0.30	1.18	0.00	1.01	1.92
ME-8	3.36*	0.14	2.85	0.46	3.02	0.14	1.12	0.30	0.89	2.28
ME-9	3.19*	2.72	0.43	2.31	0.72	0.89	3.96*	0.81	0.11	0.79
ME-10	7.07*	3.75*	0.85	2.48	0.18	4.58*	6.74*	0.17	2.54	0.79

Table 14. Fama-MacBeth regressions in separate months

Month	r^2_{I}	r^2_{II}	$\bar{\gamma}_{vw}$	(t)
Jan	14.39	25.92	3.82	(2.36)
Feb	7.32	20.62	0.78	(0.75)
Mar	23.09	26.26	1.33	(1.17)
Apr	2.29	24.39	0.29	(0.27)
May	24.10	21.39	-0.88	(-0.87)
Jun	52.09	24.34	-2.12	(-2.12)
Jul	21.57	36.74	-0.93	(-0.64)
Aug	13.16	30.19	0.91	(0.75)
Sep	10.37	29.80	-0.71	(-0.60)
Oct	66.21	37.92	-4.34	(-2.89)
Nov	1.54	22.68	0.34	(0.30)
Dec	2.92	16.19	0.41	(0.46)

Table 15. Fama-MacBeth regressions in separate months

Month	r^2_{II}	$\bar{\gamma}_{\text{vw}}$	(t)	$\bar{\gamma}_{\text{size}}$	(t)
Jan	61.95	1.02	(0.70)	-1.47	(-6.67)
Feb	38.66	0.13	(0.13)	-0.33	(-2.13)
Mar	36.19	0.89	(0.85)	-0.22	(-1.91)
Apr	31.18	0.21	(0.20)	-0.04	(-0.42)
May	40.13	-0.88	(1.00)	-0.02	(-0.11)
Jun	32.98	-2.27	(-2.39)	-0.08	(-0.75)
Jul	46.03	-1.17	(-0.81)	-0.13	(-1.07)
Aug	44.50	1.40	(1.20)	0.26	(2.89)
Sep	38.03	-1.14	(-1.01)	-0.23	(-2.26)
Oct	53.40	-3.50	(-2.59)	0.41	(2.46)
Nov	35.41	0.77	(0.69)	0.21	(1.51)
Dec	35.13	0.75	(0.94)	0.15	(0.92)

B.2 Figures

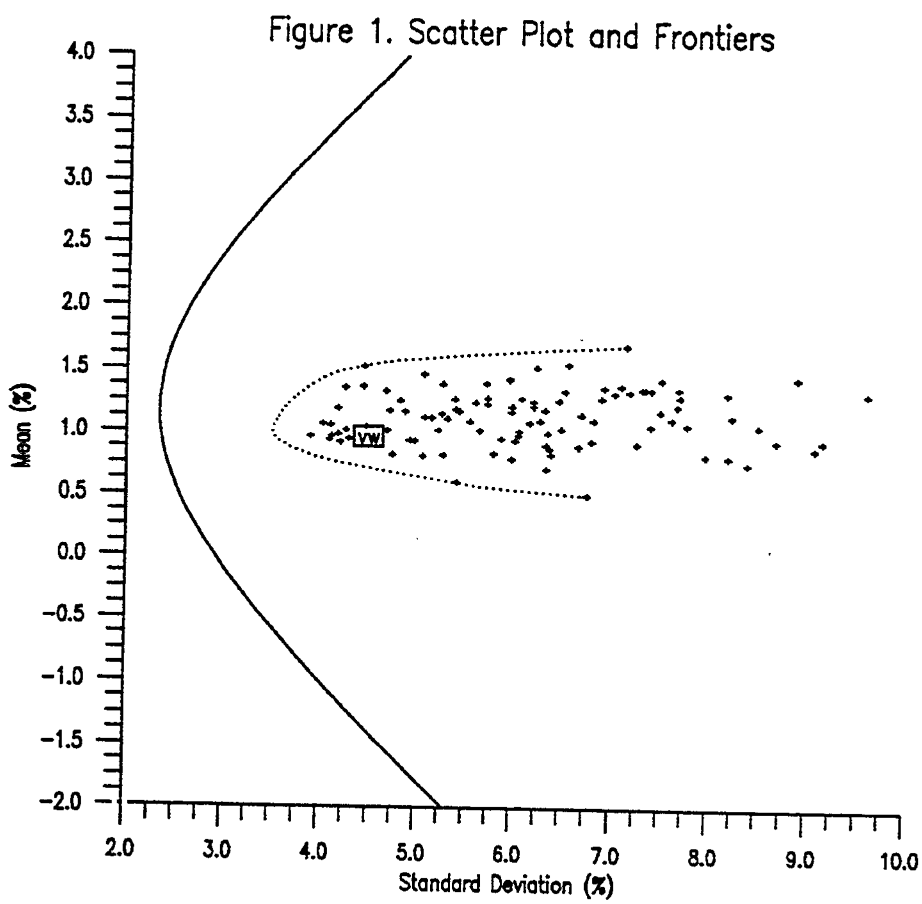


Figure 1. Scatter Plot and Frontiers
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Mean (%)
VW
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
Standard Deviation (%)

Figure 2.

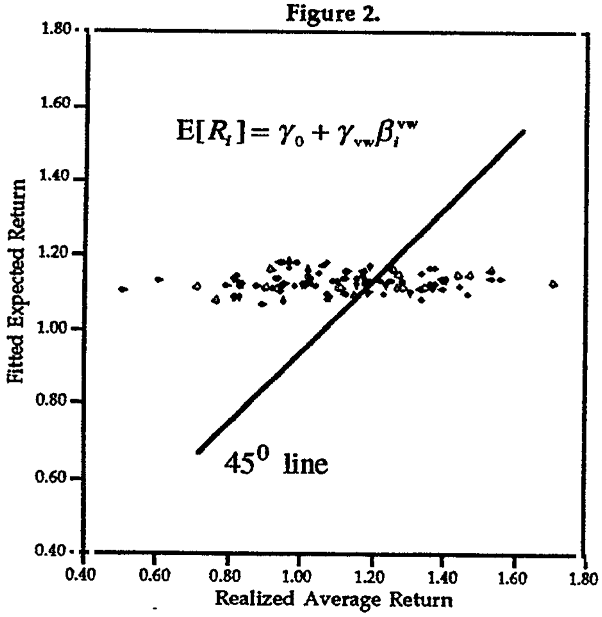


Figure 3.

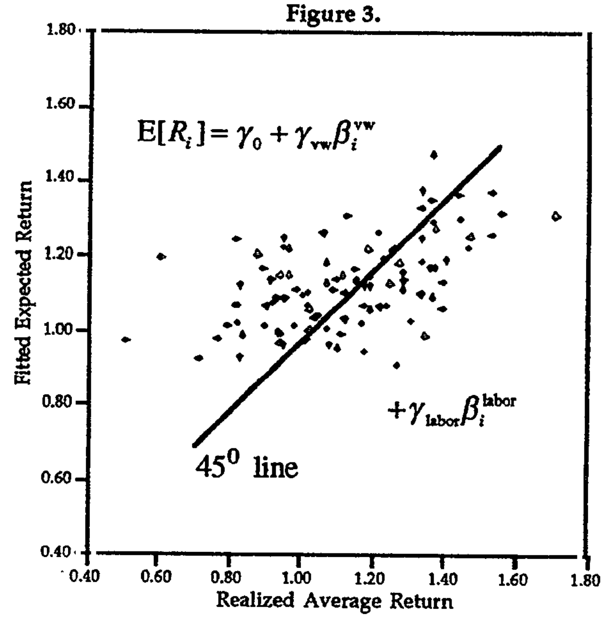


Figure 4.

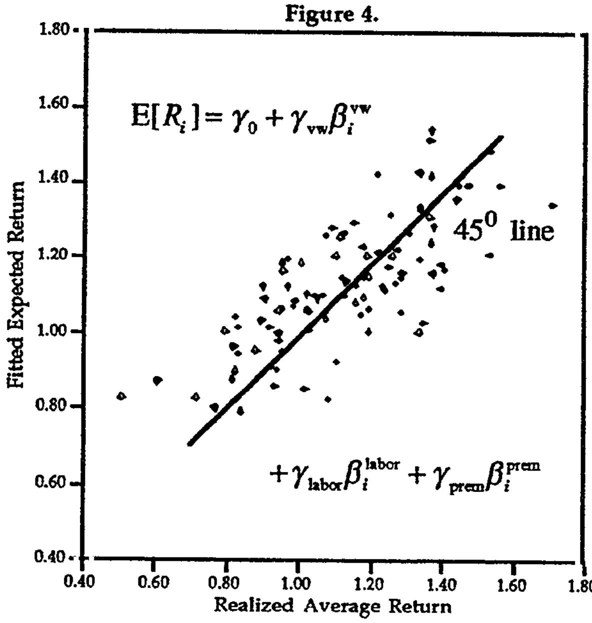
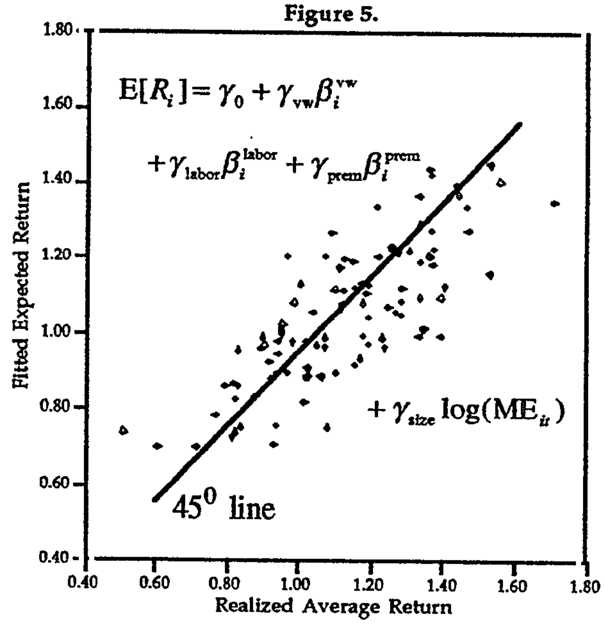


Figure 5.



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