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Seasonality and Equilibrium Business Cycle Theories

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ABSTRACT

We consider a dynamic, stochastic equilibrium business cycle model which is augmented to reflect seasonal shifts in preferences, technology, and government purchases. Our estimated parameterization implies implausibly large seasonal variation in the state of technology: rising at an annual rate of 24% in the fourth quarter and falling at an annual rate of 28% in the first quarter. Furthermore, our findings indicate that variation in the state of technology of this magnitude is required if the model is to explain the main features of the seasonal cycle.

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1. Introduction

Real business cycle models start from the assumption that variation in the state of technology is the single most important source of economic fluctuations. Kydland and Prescott (1991) conclude that variations in the rate of technological change account for 70% of postwar cyclical variation. This literature demonstrates that simple equilibrium models of optimizing agents can explain many of the business cycle features of U.S. data. While this literature focuses primarily on agents' responses to unanticipated and persistent technology shocks, perhaps the sharpest prediction of models of optimizing agents arise when the source of the impulse is anticipated.

Seasonal fluctuations provide valuable information about the response of the economy to anticipated events. Barsky and Miron (1989) find that over 90% of the total variation in many quantity variables can be explained by seasonal dummies. This evidence suggests that seasonal fluctuations contain a large anticipated component. By conditioning on this fact important new insights can be gained about the properties of models of optimizing agents.

In this paper we address two issues. First, we ask how large is the seasonal variation in the state of technology? Following the practice in the real business cycle literature, we assume that the aggregate production technology is correctly specified and its arguments are correctly measured. Under these assumptions variation in Solow's residual represents variation in the state of technology. The answer to this question is important because it is difficult to think of events that would lead the state of technology to vary dramatically over the course of the calendar year. Thus, large measured seasonal variation in Solow's residual would be evidence that the aggregate production technology is misspecified or that one of its inputs was incorrectly measured. Such evidence would call into question Prescott's (1986) identification of technology shocks and his estimate of its variance. Alternatively, such evidence could be viewed as support for arguments by Burnside, Eichenbaum, and Rebelo (1993) and Rotemberg and Summers (1990) who suggest that labor hoarding is important or arguments by Hall (1990) that technology exhibits increasing returns to scale. Second, we ask how much seasonal variation in technology is required for a simple real business cycle model to explain the stylized facts of the seasonal cycle? The second question is also important because seasonal cycles may be driven primarily by seasonal shifts in preferences like Christmas and government purchases and thus it may be possible for an equilibrium business cycle model to explain the seasonal cycle with only a small amount of seasonal variation in technology. On the other hand, if large seasonal variation in technology is required this represents additional evidence against the specification of the production technology commonly assumed by RBC theorists.

We consider a dynamic, stochastic equilibrium business cycle model which includes deterministic preference and technology shifters.[1] Our seasonal specification is relatively parsimonious: we use a four-state technology shifter, a two-state preference shifter, and a four-state government spending shifter to explain the seasonal fluctuations in output, hours, consumption, investment, real rates, government purchases, productivity, and the capital stock. A priori reasoning suggests that the important shifters would include a Christmas preference shifter and a winter weather shifter. Our econometric strategy uses the data to quantify the role of these and other seasonal shifters. Preferences exhibit nontime-separabilities in consumption goods and leisure, as in Braun (1989), Eichenbaum, Hansen, and Singleton (1988), and Kydland and Prescott (1982). We compute a perfect foresight seasonal equilibrium path for this economy and a log-linear approximation to the stochastic equilibrium around this equilibrium path. As in the exact linear quadratic analyses of Todd (1990), Hansen and Sargent (1990), and Ghysels (1990), the resulting decision rules are state-dependent with means and autoregressive coefficients that vary with the season. These periodic decision rules allow for the possibility that seasonal impulses may affect the properties of the model at business cycle frequencies.

This first question we posed above is investigated by estimating the model's structural parameters on postwar U.S. data using a Generalized Method of Moments (GMM) estimator. The parameter estimates indicate an important role for nontime-separable preferences. The technology seasonal estimates indicate a strong seasonal pattern, rising by an average of 24% in the fourth quarter and falling by 28% in the first quarter (at annualized rates of growth). We argue that variation of this magnitude is too large to represent true technological variation.

Does the model *require* this degree of technological variation to explain the observed seasonal patterns? To address this question we condition on the parameter estimates, and investigate the predicted seasonal patterns, and also the business cycle properties, of the equilibrium model. The model replicates many of the business cycle facts and seasonal patterns in output, consumption, capital, average labor productivity, government purchases, and the real interest rate. The fact that these successes arise in a parameterization with such dramatic variation in technology provides indirect evidence that this variation is important. In order to provide more direct evidence on the role of seasonal variation in technology we examine the properties of the model for the special case where technology is assumed to be the only seasonal shifter. This analysis suggests that the measured seasonal variation in technology is crucial for explaining the seasonal variation in output, investment, and labor productivity.

Finally, our research is related to work by Chatterjee and Ravikumar (1992), but the methods differ. In a stationary equilibrium economy with seasonal perturbations, they estimate a larger number of preference shifters in order to fit the seasonal patterns of the aggregate quantity data almost exactly with time-separable preferences. They calibrate their model using structural parameters from King, Plosser, and Rebelo (1988). Finally, their solution procedure assumes an orthogonal, spectral decomposition between seasonal and nonseasonal frequencies.

2. An equilibrium business cycle economy with seasonality

This section presents a one-sector, real business cycle economy which is subject to seasonal variation in technology, preferences, and government purchases. The basic model is similar to the models considered by Christiano and Eichenbaum (1992) and Braun (1989). However, in contrast to these papers, preferences are assumed to be nontime-separable. There is considerable empirical support for nontime-separable preferences as in Eichenbaum and Hansen (1990), Braun (1989), Eichenbaum, Hansen, and Singleton (1988), and Kydland and Prescott (1982).

2.1. The economy with growth and seasonality

Consider an economy composed of a large number of identical, infinitely-lived households each of which seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \{\tau_t \log c_t^* + \gamma_2 \log l_t^*\}, \quad \gamma_2 > 0 \tag{1}$$

where c* and l* represent consumption and leisure services, respectively. Consumption services are related to private consumption (cp) and public consumption (g) as follows:

$$c_t^* = cp_t + \gamma_1 g_t + a(cp_{t-1} + \gamma_1 g_{t-1}), \quad 0 \leq \gamma_1 < 1, \ |a| < 1 \tag{2}$$

where γ_1 governs the substitutability of public goods for private consumption goods. The parameter **a** governs the character and degree of nonseparability: if **a** is negative (positive), consumption goods are complements (substitutes) across adjacent time periods. The complementarity case can also be interpreted as habit-persistence in preferences. The variable τ_t is a deterministic preference seasonal which follows:

$$\tau_t = \tau_1 Q_{1t} + \tau_2 Q_{2t} + \tau_3 Q_{3t} + \tau_4 Q_{4t}, \quad \tau_j > 0 \text{ for all j} \tag{3}$$

and the variable Q_{jt} is a dummy variable taking on the value of one when period t corresponds to season j, and zero otherwise; consequently, τ_j is the preference seasonal in season j.[2] Leisure (l) is time not devoted to labor (n), leading to the time allocation constraint that $n_t + l_t = T$, where T is the maximum number of hours available per period. Preferences are defined over leisure services l_t^*:

$$l_t^* = l_t + \mathbf{b} l_{t-1}, \quad |b| < 1. \tag{4}$$

The parameter **b** governs the character and degree of nonseparability: if **b** is negative (positive), then leisure choices are complements (substitutes) across adjacent time periods. Finally, the operator E_t is the mathematical expectations operator conditional on all information known at time t.

Each household has access to a production function of the form:

$$y_t = (k_t^d)^\theta (z_t n_t^d)^{1-\theta} \tag{5}$$

where y is output and k and n are the quantities of capital and labor input demanded by the household. The household's output can be consumed (privately or publicly) or stored in the form of additional capital next period. Each period, the existing capital stock depreciates at the geometric rate δ. The variable z_t is a labor-augmenting technology shock which includes deterministic seasonal components:

$$z_t = z_{t-1}\exp(\lambda_t) \tag{6}$$

$$\lambda_t = \lambda_1 Q_{1t} + \lambda_2 Q_{2t} + \lambda_3 Q_{3t} + \lambda_4 Q_{4t} + \epsilon_t$$

where ϵ_t is a purely indeterministic, white noise random variable. Notice that log z_t is a random walk with seasonal drift: when the seasonal growth rates λ_j do not sum to zero, this economy experiences growth.[3]

The economy possesses competitive markets in labor and capital services: suppliers of labor services receive a wage w_t, suppliers of capital services receive the rental rate r_t. Finally, the government taxes each household in a lump-sum fashion, TL_t. This leads to the household's period budget constraint:

$$cp_t + k_{t+1} = y_t + (1-\delta)k_t - w_t(n_t^d - n_t) - r_t(k_t^d - k_t) - TL_t \tag{7}$$

where n_t and k_t represent labor and capital supplied by the household.

The government chooses a stochastic process for g_t which is uncontrollable from the household's perspective. The stochastic process adopted here resembles specifications used by Christiano and Eichenbaum (1992) and Braun (1989). Government purchases are assumed to contain a permanent and a transitory component. The permanent component is related to the technology shock z_t; the transitory component is an autoregressive process of order one with a seasonal mean. The stochastic process for g_t is

$$\log \frac{g_t}{z_t} - \log \tilde{g}_{jt} = \rho \left[\log \frac{g_{t-1}}{z_{t-1}} - \log \tilde{g}_{j-1,t-1} \right] + u_t, \quad 0 < \rho < 1 \quad (8)$$

where $\tilde{g}_{jt}$ is the seasonal mean of transitory government purchases when period t corresponds to season j; and u_t is an indeterministic, white noise random variable. In this Ricardian environment, we assume without loss of generality that the government's budget constraint is $g_t = TL_t$. The economy-wide resource constraint (in per capita terms) is

$$cp_t + k_{t+1} + g_t = y_t + (1-\delta)k_t. \quad (9)$$

As in King, Plosser, and Rebelo (1988), an empirical analysis of this economy is facilitated by rescaling the economy in a way which induces a stationary environment. For the posited unit root process governing technology, this transformation involves deflating capital, output, government purchases, investment, consumption, real wages, and transfers by the current value of the technology shock. Then, the competitive allocations can be calculated by solving the social planner's problem. These details are described in a technical appendix, which is available on request.

2.2. *Characterizing the seasonal equilibrium*

In section 4 we describe a method for approximating the stochastic equilibrium of the transformed economy. The solution method involves taking a Taylor approximation about a particular perfect foresight equilibrium path. In this subsection we characterize this perfect foresight path.

A perfect foresight path is a solution to the social planner's problem with the uncertainty removed. In general, different initial conditions will give rise to different paths. We, however, restrict attention to a particular perfect foresight path. This path has the characteristic that the value of a variable x in quarter j is always equal to its realization four quarters ago. For our economy with (quarterly) seasonals, the perfect foresight seasonal path for the transformed variables $\{\tilde{cp}_j, \tilde{k}_j, n_j; j = 1,2,3,4\}$ can be deduced from the two Euler equations for consumption-labor and consumption-savings decisions, the resource constraint, and the definitions of consumption and leisure services. These seasonal restrictions (without uncertainty) are

$$\frac{\gamma_2}{l_j^*} + b\beta \frac{\gamma_2}{l_{j+1}^*} = \frac{\tau_j}{\tilde{c}_j^*} (1-\theta)\tilde{k}_j^{\theta} n_j^{-\theta} e^{-\theta\lambda_j} \tag{10}$$

$$\frac{\tau_j}{\tilde{c}_j^*} = \beta\left\{ \frac{\tau_{j+1}}{\tilde{c}_{j+1}^*} \left(\theta\tilde{k}_{j+1}^{\theta-1} e^{-\theta\lambda_{j+1}} + (1-\delta)e^{-\lambda_{j+1}}\right)\right\} \tag{11}$$

$$\tilde{cp}_j + \tilde{k}_{j+1} + \tilde{g}_j = \tilde{k}_j^{\theta} n_j^{1-\theta} e^{-\theta\lambda_j} + (1-\delta)\tilde{k}_j e^{-\lambda_j} \tag{12}$$

where l_j^* and $\tilde{c}_j^*$ are defined for the transformed analogues of eqs. (4) and (2) and the seasonal index j runs from 1–4. We adopt a wrap-around dating convention that when $j = 5$, this represents the first quarter ($j = 1$). This leads to the following definition.

Definition. A sequence $\{\tilde{cp}_i, \tilde{k}_i, n_i;\ i = 1,2,3,4\}$ *which satisfies eqs.* (10)–(12) *is a perfect foresight seasonal equilibrium.*

In practice, we calculate the perfect foresight seasonal equilibrium by substituting l_j^* and $\tilde{c}_j^*$ into eqs. (10)–(12) and then solve these 12 equations in 12 unknowns using the Gauss procedure "nlsys." For all parameterizations of the model which we consider below, we found no evidence of nonuniqueness: different starting values always converged to the same seasonal equilibrium. Furthermore, in the course of solving for the stochastic equilibrium (in section 4), we calculate roots for the log-linear system which exhibit the proper characteristics to ensure local stability of the perfect foresight seasonal equilibrium. That is, in the state space representation, the fundamental matrix had equal numbers of roots inside and outside the unit circle.[4]

Given the perfect foresight path for $\{\tilde{cp}_i, \tilde{k}_i, n_i;\ i = 1,2,3,4\}$ and the seasonals $\{\lambda_j, \tau_j, g_j;\ j = 1,2,3,4\}$, seasonal paths for $\{\tilde{y}_j, \tilde{i}_j, \tilde{w}_j, r_j\}$ can be computed from the production function, the law of motion for capital and the marginal products of labor and capital. Before the model's predictions can be compared with the Barsky-Miron seasonal results, the following tasks must still be completed: the data must be precisely defined, the model's parameters must be estimated and the stochastic equilibrium must be calculated.

3. Estimation of the structural parameters

In this section we describe the data and report the estimation results. The data set employed in this study is the Barsky and Miron (1989) data: U.S. quarterly data which has not been adjusted for seasonality. For the empirical analysis to conform to the theoretical constructs of our model, however, we redefine some of the variables as follows (and convert to per capita values). Output (y) is Gross National

Product per capita. Private consumption (cp) is nondurables plus services consumption expenditures per capita. Investment (i) is the sum of Business Fixed Investment plus Durable consumption expenditures, per capita. Government (g) is Federal, State, and Local government purchases, per capita. The capital stock is computed using the flow investment expenditures, a quarterly depreciation rate of 2.5%, and an initial capital stock value for 1950. Labor hours are computed as the product of total nonagricultural employment times average hours per week of nonagricultural production workers times 13 weeks per quarter (per capita). Average labor productivity and the capital rental rate are constructed from the output, labor, and capital data. The data is converted to per capita values by using the civilian population, 16 years and older.

Given seasonally unadjusted time series data for the U.S economy, the Euler equation methods of Hansen and Singleton (1982) can be used to estimate the model's structural parameters and test the overidentifying restrictions implied by the model and choice of instruments. The parameter vector to be estimated is

$$\Psi = (\theta, a, b, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \tau_1, \tau_4, d_1, d_2, d_3, d_4, \rho, \sigma_u, \sigma_\epsilon)$$

where the d_i seasonals are related to the log $\tilde{g}_i$ seasonals by the relationship $d_i = \log \tilde{g}_i - \rho \log \tilde{g}_{i-1}$.

In estimating the model we restrict attention to preferences that vary only in the fourth quarter in response to Christmas. Thus in quarters one through three τ takes on the value τ_1 and in quarter four τ takes on the value τ_4. This assumption, is plausible on a priori grounds and resolves an identification problem. With four preference shifters the nontime-separabilities in consumption interact with the preference shifters to produce two observationally equivalent ways of matching the seasonal properties of consumption in the data. One parameterization implies implausibly large seasonal variation in preferences while the other shows little seasonal variation in quarters 1–3. Conditioning on a single fourth quarter shifter resolves this issue in a fashion that lines up with our a priori beliefs: Christmas is the most significant demand shift of the calendar year.

Notice next that Ψ does not exhaust the entire list of structural parameters. Some of the structural parameters are formally not identified, while others are difficult to identify in the data. Consequently, the parameters β, γ_1, γ_2, δ, and T are set a priori, in accordance with previous studies. We follow Christiano and Eichenbaum (1992) and Braun (1989) and set β to $1.03^{-0.25}$ and set γ_1 to 0.4 which is the number reported by Aschauer (1985).[5] The depreciation rate δ was chosen to be 2.5% per quarter, as in King, Plosser, and Rebelo (1988) and Kydland and Prescott (1982). The utility weight γ_2 on leisure which is formally not identified was normalized to be one. Finally, the total time endowment for the household is chosen to be 1,369 hours per quarter.

The moment equations used in estimation consist of two stochastic Euler equations, the production function, the transitory government spending autoregression, and two variance estimates. The two Euler equations arise from the household's time t decision for k_{t+1} and n_t yield the conditions:

$$\beta E_t\left\{\left[\frac{\tau_{t+1}}{c^*_{t+1}} + \beta a\,\frac{\tau_{t+2}}{c^*_{t+2}}\right]\left[\theta\,\frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right] - \left[\frac{\tau_t}{c^*_t} + \beta a\,\frac{\tau_{t+1}}{c^*_{t+1}}\right]\right\} = 0 \tag{13}$$

$$E_t\left\{\left[\frac{\tau_t}{c_t^*} + \beta a\,\frac{\tau_{t+1}}{c_{t+1}^*}\right](1-\theta)\,\frac{y_t}{n_t} - \frac{\gamma_2}{l_t^*} - l_t^* - \beta b\,\frac{\gamma_2}{l_{t+1}^*}\right\} = 0 \qquad (14)$$

where c* and l* can be constructed from eqs. (2) and (4) given values of γ_1, a, and b. All of the equations are estimated simultaneously with the cross-equation restrictions imposed.[6]

Table 1 presents two sets of parameter estimates of Ψ. The sample period covers 1964:1–1985:4. Column one contains the nontime-separable estimates (NTS). At the bottom of the column we report a test of the overidentifying restrictions.[7] This statistic is asymptotically distributed χ^2 with 15 degrees of freedom. This statistic uncovers little evidence against the overidentifying restrictions.[8] The NTS estimates display habit-persistence (or adjacent complementarity) in preferences for leisure hours and local durability (or adjacent substitutability) in preferences for consumption goods: that is, **b** is estimated to be negative and **a** is estimated to be positive. Furthermore, both **a** and **b** are significantly different from zero. Habit-persistence in leisure is consistent with previous empirical analyses using these preference specifications on seasonally adjusted aggregate data. [See, for example, Braun (1989) and Eichenbaum, Hansen, and Singleton (1988).] A number of researchers have estimated consumption preferences which are consistent with a > 0 [for example, Gallant and Tauchen (1989) and Eichenbaum, Hansen, and Singleton (1988)]. On the other hand, using quarterly consumption data, Braun (1989) finds evidence of habit-persistence and Constantinides (1990) shows that negative values of **a** can help explain the equity premium puzzle.

Turning to the rest of the parameterization the capital share parameter θ is estimated to be 0.28 with a small standard error. The seasonal patterns in transitory government spending exhibit low fourth quarter spending and high first quarter spending. The government spending autoregressive coefficient ρ is estimated to be approximately 0.88. Finally, the estimated standard deviations of ϵ_t and u_t are respectively 0.0195 and 0.0191.

The NTS parameterization displays a large degree of seasonal variation in technology. The fourth quarter growth in total factor productivity is estimated to be 6% (or 24% on an annualized basis). The first quarter experiences technical regress (on average), growing at a rate of -7% in one quarter (-28% annualized). If the technological specification (5) is correct, and the factor inputs and output are properly measured, then these seasonals represent true seasonal variation in aggregate technology. Does this seem reasonable? At first blush, the downturn in the first quarter may seem plausible. For instance, Barro (1990) suggests that weather conditions and seasonality in construction probably account for some of the seasonal patterns in aggregate technological growth. But Evans (1989) has measured aggregate seasonal Solow residuals making allowances for agriculture and construction, and the negative first quarter growth remains large. Also, Beaulieu and Miron (1992) find that output falls in the first quarter in Argentina and Australia, again casting doubt on the weather explanation since these countries are in the Southern Hemisphere. Finally, the dramatic rise in the fourth quarter is difficult to rationalize with explanations based on weather. Considering the rise in fourth quarter consumption demand ($\hat{\tau}_4 > \hat{\tau}_1$), the estimated fourth quarter shift in technology could plausibly be due to unobserved variations in labor effort or productive externalities.

The hypothesis of exogenous shifts in technology of the magnitude estimated here, however, seems implausible.

Finally, we report estimates in column two of table 1 for a time separable parameterization; **a** and **b** are constrained to be zero. This estimation produced a criterion value of 888 when using the converged weighting matrix from the NTS estimation. Following Eichenbaum, Hansen, and Singleton (1988) the difference between this criterion value and the criterion value from the nontime-separable estimates is asymptotically distributed χ^2 with two degrees of freedom. Thus, the additional restrictions imposed by time-separable preferences are sharply rejected at conventional significance levels. In other respects this parameterization bears many resemblances to the NTS estimates.

4. Evaluation of the stochastic model

4.1. Solving the stochastic model

To solve the model, we linearize the equations which characterize the solution of the stationary social planner's problem about the perfect foresight seasonal equilibrium path calculated in section 2. The linearized system can be reduced to 12 difference equations which are stochastic counterparts to eqs. (10)–(12). In a technical appendix (Braun and Evans 1993) we display the linearized system and describe how these 12 difference equations are mapped into a state space representation which can be solved using methods described in King, Plosser, and Rebelo (1990). The state space representation essentially has the same structure as Todd's (1990) time-invariant linear-quadratic representation or Hansen and Sargent's (1990) time-varying strictly periodic equilibrium. The model's solution is a series of 12 equations that describe the optimal decision rule for capital, hours, and private consumption, one equation for each season:

$$K_{t+1} = AS_t \tag{15}$$

where

$$K_{t+1} = [k^1_{t+1} k^2_{t+1} k^3_{t+1} k^4_{t+1} cp^1_t cp^2_t cp^3_t cp^4_t n^1_t n^2_t n^3_t n^4_t]'$$

and

$$S_t = [K'_t g^1_t g^2_t g^3_t g^4_t \lambda^1_t \lambda^2_t \lambda^3_t \lambda^4_t]'$$

where the superscripts denote the seasons.

Given these log-linear decision rules for capital, private consumption and hours, it is straightforward to generate time series for the model economy. First, a sequence of normal variables is drawn to mimic the empirical covariance structure of the forcing processes ϵ_t and u_t. Once ϵ_t and u_t have been constructed it is straightforward to calculate λ_t and g_t. Then given an initial K_0 we can construct a sequence of realizations for the capital stock, hours, and private consumption using the following method. If this is the jth quarter then use the jth, j + 4th, and j + 8th row of matrix A along with the current states: k^j_t, cp^{j-1}_{t-1}, n^{j-1}_{t-1}, λ^j_t, and g^j_t to determine the current decisions for next period's capital, and today's consumption and hours. Given the values of next period's stock of capital, today's consumption and today's work effort, it is straightforward to determine the current choices of output, investment, real wages, and the real interest rate using the production technology, investment identity, and marginal product pricing relations.[9]

With the simulated time series in hand, a variety of descriptive statistics are easy to compute. While the equilibrium model developed in this paper imposes restrictions across the entire spectrum, researchers often decompose time series to focus attention on a specific set of moments. Examples of such decompositions include first differencing to remove low frequency moments and seasonal adjustment to remove particular high frequency moments. Our objectives lead us to compute the model's predictions for first and second moments of the data. To facilitate comparisons with the literature, we adopt Barsky and Miron's decomposition of the stationary, stochastic processes into "deterministic seasonal" and "indeterministic" components. Specifically, after inducing stationarity by log-first differencing, we regress each series on four seasonal dummies: the coefficient estimates for the dummy variables define the seasonal patterns emphasized by Barsky and Miron (1989). We also adopt the convention of referring to moments calculated using the indeterministic residuals from these regressions as relating to cyclical or business cycle phenomena emphasized by Prescott (1986). Obviously, the properties of the "seasonal cycle" will vary depending on the particular decomposition used.

4.2. Comparing the model's seasonal predictions with the data

The first set of columns in table 2 present the seasonal patterns for the data set using the log first-difference filter. The seasonal means are reported in terms of percentage deviations from average growth rates for the sample period 1964:I to 1985:IV. The real interest rate, which is measured by the rental rate on capital, is reported in terms of annualized rates of return. The table also includes R-square statistics for each variable which describe the percentage of the total variation in the particular time series that is attributable to the deterministic seasonal. Finally, we report standard errors for each estimate that are based on the Newey and West (1987) covariance estimator with 12 autocorrelations.

The second and third sets of columns in table 2 contain simulation results for respectively the nontime-separable and time-separable preference specifications. For each specification, columns 1–4 label the average seasonal means for 500 draws. The fifth column contains the average R-square of the regressions.

Seasonal Predictions of the NTS Model

To facilitate comparison of the NTS results with the data, seasonal growth rates are also presented graphically in fig. 1 as well as in table 2. Examination of fig. 1 reveals that the NTS specification mimics the overall seasonal patterns in output, consumption, government purchases, average productivity, and capital. For these variables the model reproduces the sequential pattern of seasonal movements in the data and in most cases the magnitudes.[10] The model is less successful with respect to investment, the rental rate on capital, and hours. For the rental rate the model consistently overstates the magnitudes, although the sequential pattern is correct. In the case of investment, the model captures the sequential pattern of seasons found in the data, but understates the second quarter rise and overstates the fourth quarter rise in investment growth. The model's predictions for hours are also at odds with the data. The magnitudes are off in three out of four quarters and the model predicts a counterfactual rise in fourth quarter employment. Nevertheless, conditional on the technology seasonal estimates, the NTS parameterization captures many of the seasonal patterns found in the data.

Seasonal Predictions of the TS Model

Consistent with the econometric evidence reported in section 3 comparisons of the predictions of the TS model with the data in table 2 reveal several significant shortcomings relative to the NTS model. The TS model sharply overstates the seasonal means in output, hours, and investment. The second and fourth quarter consumption means are also a poor match. The predicted R-squares for these variables also exceeds the respective number in the data in each instance. Overall, the TS specification does not capture the seasonal properties of the data as well as the NTS specification.

4.3. The contribution of technology seasonals

To assess the role of the estimated technology seasonals for the model's predictions, we have considered versions of the NTS model which alternately possess only one seasonal shifter at a time. This leads to the following three cases: (1) a technology shock only case, (2) a consumption preference shock only case, and (3) a government purchases shock only case.[11]

The estimated seasonal variation in technology is *crucial* for the model to be able to predict the seasonal patterns in output, labor hours, labor productivity, and the rental rate on capital. In fig. 2 for output, labor hours, and labor productivity, the predictions of the basic NTS model and the "technology shock only" parameterization are virtually identical. In this model, seasonal shifts in consumption preferences and government purchases do not lead to significant variations in output, labor, productivity, or the capital rental rate. So not only are the estimated technology seasonals implausibly large, they are implausibly important.

As a matter of economic theory, it is interesting to consider some of the reasons for the seasonal patterns observed in these cases. In the technology shock only case, private consumption purchases are essentially smooth. Since the technology seasonal is an anticipated event, there is no wealth effect. So one source of consumption variability, unanticipated wealth effects, is absent.[12] In the preference seasonal only case, consumption follows the same pattern as the shift in preferences. Agents satisfy their transient increased desire to consume by drawing down their savings, increasing consumption, and decreasing investment. Interestingly, since investment is proseasonal in the data but driven in different directions by demand and supply shifts, investment may offer useful identifying information about alternative models. This is an interesting subject for future research.

In summary, without the implausibly large seasonal variation in technology that we estimate from the data, our equilibrium model is unable to generate the stylized facts of seasonal fluctuations in the postwar U.S.

4.4. Cyclical predictions of the stochastic model

Table 3 contains results relating to relative variability and cross-correlations with output for both parameterizations and the data under two different filters. The heading "one-quarter growth rate" corresponds to moments calculated using data that has been log first-differenced and regressed on four seasonal dummies. The heading "HP filter" corresponds to data that has been rendered stationary using the Hodrick and Prescott (1980) filter and then regressed on four dummies. For each filter we report moments for U.S data running from 1964:I–1985:IV in the first column. The second column contains standard errors for the data's moments reported in column one. The standard errors were calculated using a Newey-West covariance estimator with 12 lags. The third and fourth columns contain results from simulating the model using the NTS and TS parameterizations. In each case the reported statistics are sample averages based on 500 draws of length 88.

Consider the predictions of the NTS and TS parameterizations. In many cases both models' predictions lie outside a two standard deviation band around the data. The largest differences between the models occur under the one-quarter growth filter. With respect to relative variabilities, the NTS specification fails to capture the relative variabilities of average productivity, employment and government expenditures. The TS specification fails to capture the relative variabilities of consumption, investment, and average productivity. In addition, the TS specification overstates the variance of output by 24%; the NTS specification overstates the output variance by only 9%. Interestingly, the differences are much narrower under the HP-filter.

Turning next to cross-correlations we see that both specifications predict positive comovements of consumption, investment, government purchases, labor hours, and productivity with output after adjusting for deterministic seasonality. Comparing these predicted correlations with the data's correlations, however, reveals significant failures of the NTS and TS specifications. For example, both specifications fail to capture the contemporaneous correlations of hours and the rental rate with output. Thus, while the patterns of comovement are correctly predicted by both specifications, the exact match is not completely satisfactory.

To shed further light on the model's implications for second moments fig. 3 displays estimates of the spectra of output, consumption, and labor hours under the two filters.[13] Four observations are in order. First, most of the data's spectra resemble the output and hours spectra—that is, they fail to display significant power at seasonal frequencies after filtering with seasonal dummies. Consumption, on the other hand, is the one time series that does continue to exhibit a "hump" in the spectrum around $\pi/2$ under the log first-difference filter and the Hodrick-Prescott filter. Neither the NTS nor the TS model captures this feature of the data. Including a stochastic component in the preference shifter is worthy of future research. Second, under the one-quarter growth rate filter, the data displays an increase in spectral power moving from medium frequencies typically associated with business cycles to higher frequencies. The NTS model captures this phenomenon but the TS model does not. This difference can be attributed to the fact that consumption preferences display local durability which acts to increase variability in consumption between adjacent periods. Third, under the one-quarter growth rate filter, data on hours produces a spectrum which decreases in power from low to high frequencies. The NTS specification captures this feature of the data while the TS specification does not. This difference can be explained by the estimated habit persistence in leisure preferences which places a penalty on large quarterly variations in leisure. Fourth, under the Hodrick-Prescott filter the differences between the NTS and TS spectra noted above vanish.[14]

How do these cyclical results compare with the performance of standard business cycle models that ignore seasonality? With regard to spillover effects, if seasonal taste and technology shifters induce important spillover effects at business cycle frequencies, then our cyclical results should be different from the nonseasonal literature. First, as in the RBC literature, our model predicts that consumption is less variable than output and output is less variable than investment. This is especially true under the Hodrick-Prescott filter, which has become a standard detrending method in the literature. Second, hours are not as variable as in the data; again, this is true in the literature and is highlighted by the Hodrick-Prescott filter more than the one-quarter growth rate filter. Third, our model predicts patterns of positive comovement between aggregate output and other aggregate quantity variables as found in the data. Fourth, as in Christiano and Eichenbaum (1992) and Braun (1989), our model overstates the variability of output, this is attributable to our estimation of the parameterizations.[15] Thus, the cyclical properties of our model share many of the broad characteristics of the RBC literature's models. There are some differences; for example, the variability of consumption relative to output under the one-quarter growth rate is a large 0.73 in the data and 0.77 under the NTS parameterization. Nevertheless, the numerous similarities indicate that this model has no critical spillover effects from the seasonal shifters to the cyclical frequencies under our solution method.

Unlike the RBC literature, however, we find that nontime-separabilities in consumption preferences improve the model's ability to capture the first and second moment properties of the data. The nontime-separable specification outperforms the time-separable specification with regard to the Euler equation estimation, the predictions of the seasonal patterns, the relative variability of consumption growth (where substitutability in preferences is most likely to be important), and the variance of output growth. Furthermore, the spectra described above reveal that nontime-separabilities do affect the spectral power at seasonal frequencies. Relative to the time-separable case, the NTS specification displays more spectral power at high frequencies in the case of consumption and less power in the case of hours, in both cases improving the model's fit.

5. Conclusions

In this paper we have measured the seasonal pattern in the Solow residual and found that it implies an implausibly large amount of variation in the state of technology over the calendar year. In addition, we have found that real business cycle models require this much seasonal variation in technology if they are to explain the main features of the seasonal cycle. Without seasonal variation in technology in the class of models considered here, seasonal shifts in consumption preferences and government purchases produce negligible responses in output and a pattern of investment that is opposite to what we observe in the data. Since most economists agree that Christmas demand has a significant influence on fourth quarter output, the theory's inability to generate output shifts in response to a large fourth quarter preference shift indicates misspecification. The theory's reliance on large technology seasonals also indicates misspecification. The evidence presented here offers independent support to the contention that the production technology commonly used in RBC analyses is either misspecified or subject to important sources of measurement error. In other work Evans (1992) and Eichenbaum (1991) have arrived at similar conclusions.

Appendix: Seasonal Unit Roots

Recent research by Beaulieu and Miron (1993); Hylleberg, Engle, Granger, and Yoo (1990), henceforth HEGY; and Hylleberg, Jorgensen, and Sorensen (1991) has found evidence of unit roots at seasonal frequencies. Our specification of preference and technology shifters rules out this possibility. In order to investigate the plausibility of our assumptions we pretested the consumption and Solow residual data for seasonal unit roots. These two data series provide the most information about the seasonal processes preference and technology shifters are likely to follow. Table 4 reports the results of the HEGY tests for these two data series. These results are based upon the basic HEGY regression augmented by four lags of the dependent variable and an intercept term.[16] The null hypotheses of the tests are that seasonal unit roots exist at certain seasonal frequencies. These tests find no evidence against the null hypotheses of unit roots at the zero, semi-annual, and annual frequencies. If the HEGY regressions are augmented to include seasonal dummies and a trend term, or fewer lags in the autoregressive polynomial, the results are very similar.

These results cast some doubt on our modeling assumptions. However, unit root tests are known to have low power against particular stationary alternatives [see, for example, Christiano and Eichenbaum (1990)]. With respect to seasonal unit roots, Ghysels, Lee, and Noh (1993) and Hylleberg (1993) have investigated the power properties of HEGY tests for a number of interesting data generating processes. Their results suggest that the power of the HEGY tests is generally quite high. Still, it is well known that the power of any test depends on the alternative being considered.

Given the low computational cost of monte carlo simulations we decided to do a power calculation using an alternative that imposes restrictions from our model and information from the data on consumption. Under the null of our model the log first-difference of consumption is stationary with a deterministic seasonal component:

$$(1-L)c_t = \beta' d_t + B(L)\epsilon_t$$

where $B(L)^{-1} = 1 - a(L)$, so an autoregressive representation is possible. To parameterize the data generating mechanism, we assumed that a(L) is of order four and estimated this process on our consumption data. We generated 10,000 samples of length equal to our data's sample length (144 observations). For each sample we conducted a HEGY test using 5% critical values taken from HEGY (1990). The estimated power of the HEGY test, the probability of rejecting the null hypothesis conditional on the null hypothesis being false, is also reported in table 4. For this data generating mechanism which is empirically close to our model specification, the HEGY test has very little power to reject the false null hypothesis of seasonal unit roots. We conclude from this example that pretesting our data does not provide sufficient evidence to dissuade us from our preferred specification of preference and technology shifters. For a more detailed discussion of these and other specification tests for our data, a technical appendix is available on request.

Footnotes

[1]An alternative view of seasonality is that it is best characterized as a nonstationary process with unit roots at some (or all) seasonal frequencies [see Hylleberg, Engle, Granger, and Yoo (1990)]. We state our modeling assumptions in section 3 and defend them in the appendix.

[2]See Hansen and Sargent (1990) and Osborn (1988) for alternative specifications of seasonal preferences.

[3]Work by Beaulieu and Miron (1993); Ghysels, Lee, and Siklos (1992); and Hylleberg, Engle, Granger, and Yoo (1990) suggests we may want to allow for seasonal unit roots in either preferences or technology. This issue is investigated in the appendix.

[4]Chatterjee and Ravikumar (1989) provide a more formal characterization of existence and uniqueness in a simple seasonal model with inelastic labor supply and 100% depreciation of capital.

[5]Christiano and Eichenbaum (1992) and Braun (1989) report a range of values for γ_1 and find that small values of γ_1 help explain the correlation between hours and average productivity.

[6]The instruments for our moment conditions were selected as follows: for eq. (13), four seasonal dummies, and the time t growth rates of private consumption, leisure, output-capital ratio, and output-labor ratio; for eq. (14), four seasonal dummies and the time t and $t-1$ growth rates of private consumption, leisure, output-capital ratio, and output-labor ratio; for the production function, four seasonal dummies; for the transitory government spending autoregression, four seasonal dummies and the logarithm of g_{t-1}/z_{t-1}; and for the variance estimators, only unity.

A total of 31 instruments are used to estimate the 18 parameters of the nontime-separable specification, yielding 15 overidentifying restrictions. For a time-separable specification in which the parameters **a** and **b** are set to zero a priori, only 16 parameters are estimated, yielding 17 overidentifying restrictions.

[7]The moment conditions used to identify the technology and government seasonals serve to just identity them. Consequently the test offers no information about these restrictions.

[8]It could be argued that the degrees of freedom associated with this statistic are inflated by the use of four seasonal dummies in the two Euler equations.

[9]For more details, see the technical appendix.

[10]A cursory inspection of the graphs might suggest that the fit for capital is not particularly good. Notice however, that the vertical axis is in tenths of a percent.

[11]In each case we set the remaining two seasonal shifters equal to the mean of their estimated values. Notice that for the "technology only" case, government purchases varies seasonally. This variation is inherited from the seasonal variation in the permanent component of government purchases. Our specification for government purchases embodies the assumption that g_t and z_t cointegrate. This assumption is necessary for the economy's perfect-foresight equilibrium path to exhibit balanced growth.

[12]For this case, consumption is slightly counterseasonal due to the substitutability of government purchases for private consumption ($\gamma_1 = 0.4$). If $\gamma_1 = 0$, consumption is slightly proseasonal.

[13]The models' spectra are estimated from a single long draw of the NTS and TS simulations. The data's spectra are estimated over the sample period 1964–85.

[14]Cogley and Nason (1991) argue that this lack of discriminatory power under the HP filter is not unusual for data-generating processes with unit roots.

[15]Reducing our estimate of the technology shock variance would reduce the implied variability of output in the model. Assuming that the Solow residual series is contaminated by measurement errors which are orthogonal to the true technology shock would imply that our estimate is too large. Indeed, Prescott (1986) uses this assumption to reach a substantially lower variance estimate. However, the presence of labor hoarding or external increasing returns would invalidate the orthogonality assumption. We leave for future research an assessment of these issues (for example, see Braun and Evans 1991).

[16]The sample period for our consumption data runs from 1950–85, while the Solow residual data begins in 1964, which is the year our labor input time-series starts. Tests for consumption based on the shorter 1964–85 sample produced stronger evidence against the seasonal unit root hypothesis.

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Table 1:

GMM Estimates of the Structural Parameters

1964, QI - 1985, QIV

	Nontime–Separable		Time–Separable	
	Estimate	Standard Error	Estimate	Standard Error
θ	.2803	.0195	.2751	.0055
a	.3402	.0184	—	—
b	−.4956	.0154	—	—
λ_1	−.0724	.0031	−.0710	.0027
λ_2	.0385	.0026	.0375	.0022
λ_3	−.0193	.0019	−.0190	.0015
λ_4	.0600	.0024	.0596	.0019
τ_1	.1863	.0011	.1819	.0013
τ_4	.1929	.0011	.1929	.0012
d_1	.6048	.1521	.6115	.1747
d_2	.5712	.1518	.5807	.1743
d_3	.6072	.1523	.6162	.1747
d_4	.5382	.1520	.5469	.1748
ρ	.8779	.0319	.8756	.0368
σ_u	.0195	.0385	.0198	.0013
σ_ϵ	.0191	.0193	.0194	.0014
J-statistics	20.7		888.2	
P-value	(.146)		(.000)	
	$\chi^2(15)$		$\chi^2(17)$	

Table 2:

Seasonal Growth Rates[1]

X-Variables	Data[2] Winter	Spring	Summer	Fall	R^2	Nontime-Separable Estimates[3] Winter	Spring	Summer	Fall	R^2	Time-Separable Estimates[4] Winter	Spring	Summer	Fall	R^2
Output	−7.72	4.26	−1.04	4.16	.905	−7.48	3.35	−1.58	5.71	.892	−12.60	6.13	−3.49	9.91	.951
	(.59)	(.42)	(.22)	(.35)											
Consumption	−7.30	2.59	.04	4.34	.936	−7.08	2.38	−1.27	5.97	.927	−6.25	−.37	−.22	6.86	.959
	(.48)	(.40)	(.09)	(.26)											
Investment	−14.64	11.79	−2.15	4.33	.903	−11.39	6.62	−4.71	9.48	.842	−38.10	28.09	−16.50	26.49	.962
	(.68)	(1.08)	(.34)	(.58)											
Government	−4.89	2.86	.49	1.31	.696	−4.68	2.70	.65	1.33	.517	−4.72	2.70	.72	1.30	.513
	(.37)	(.79)	(.46)	(.32)											
Capital[5]	.16	−.26	.09	.02	.214	.041	−.082	.17	−.13	.163	.21	−.23	.51	−.50	.618
	(.10)	(.08)	(.10)	(.10)											
Labor Hours	−3.26	2.31	.73	.07	.842	−3.05	1.04	−.13	2.15	.921	−10.25	5.09	−2.83	7.99	.986
	(.16)	(.21)	(.20)	(.17)											
Average Production	−4.47	1.95	−1.78	4.09	.846	−4.42	2.31	−1.45	3.56	.868	−2.31	1.04	−.66	1.92	.655
	(.52)	(.40)	(.30)	(.40)											
Rental Rate on Capital	4.02	3.91	4.49	3.41	.084	3.75	3.52	4.35	3.27	.299	3.87	3.35	4.81	2.96	.537
	(.48)	(.44)	(.49)	(.42)											

[1]All variables except the rental rate are deviations from average growth rates as in Barsky and Miron (1989). They are calculated by constructing the log growth rate for each variable ($\log x_t - \log x_{t-1}$) and then removing its mean. The rental rate is an annualized return.

[2]The seasonal growth rates for the data were estimated by regressing each variable on four seasonal dummies. The standard errors are calculated using a Newey-West covariance estimator.

[3]The nontime-separable results are sample averages regression results based on simulations with 500 replications of draws of length 88.

[4]The time separable results are sample averages of regression results based on simulations with 500 replications of draws of length 88.

[5]Capital in contrast to the other variables is calculated using $\log k_{t+1} - \log k_t$.

Table 3:

Selected Second Moment Properties, Various Filters[1]

	One-Quarter Growth				HP Filter				Unadjusted One-Quarter Rates			
X-Variables	Data	Standard Error	NTS	TS	Data	Standard Error	NTS	TS	Data	Standard Error	NTS	TS
1. Relative Volatility (Standard Deviation of x)/(Standard Deviation of Output)												
Output[2]	.016	.002	.018	.020	.026	.002	.024	.026	.051	.003	.054	.090
Consumption	.73	.054	.77	.48	.60	.036	.56	.51	.88	.033	.93	.53
Investment	1.94	.26	2.09	2.81	2.42	.12	2.38	2.58	1.96	.069	1.72	3.22
Government	1.21	.19	1.56	1.39	.86	.24	1.41	1.35	.66	.063	.74	.44
Capital[3]	.18	.026	.17	.16	.32	.060	.24	.23	.07	.009	.060	.056
Labor Hours	.52	.065	.32	.42	.73	.033	.38	.41	.42	.025	.38	.80
Average Production	.89	.050	.70	.59	.75	.11	.64	.61	.70	.024	.63	.23
Real Rate[3]	.36	.073	.37	.33	—	—	—	—	.28	.086	.14	.11
2. Contemporaneous Correlation: X With Output												
Consumption	.72	.075	.83	.87	.87	.030	.85	.86	.96	.010	.98	.89
Investment	.72	.059	.86	.92	.93	.024	.96	.97	.94	.010	.97	.98
Government	.38	.081	.73	.74	.30	.11	.74	.75	.83	.030	.77	.72
Capital[2]	.26	.11	.25	.37	.28	.070	.29	.36	.50	.060	.42	.80
Labor Hours	.46	.085	.95	.98	.67	.10	.97	.96	.81	.020	.98	.99
Average Production	.82	.040	.99	.99	.59	.080	.99	.98	.93	.010	.99	.91
Real Rate	.25	.098	−.09	−.08	—	—	—	—	−.09	.095	−.30	−.46

[1]The sample period of the data is 1964:II–1985:IV. The standard errors are for the data's moments; and 12 lags are employed in the Newey-West estimator. The stochastic models were simulated 500 times using draws of length 88.

[2]The "output" rows reports the standard deviation of output.

[3]The capital stock refers to k_{t+1}, whereas the other variables are x_t.

[4]The real rate is not filtered, for comparability with Barsky and Miron (1989).

Table 4:

HEGY Test Results

	HEGY Tests			Power Results for Consumption	
	Consumption	Solow Residual	5% Critical Value	No Trend nor Seasonal Dummies	Trend and Seasonal Dummies
Unit root at frequency 0 (π_1)	-1.17	-1.52	-2.89	N/A	N/A
Unit root at semi-annual frequency (π_2)	-0.92	-1.34	-1.91	.002	.027
Complex root at frequencies 1/4 and 3/4 (π_3)	-1.70	-1.61	-1.88	.002	.315
Unit root at annual frequency ($\pi_3 \cap \pi_4$)	1.84	1.56	3.00	.001	.657

HEGY Tests, columns (2), (3), and (4): The consumption regression is estimated over the sample period 1950–85; the solow residual regression is estimated over the sample period 1964–85. An intercept is included in the regression, but no seasonal dummies nor a trend. Critical values are taken from HEGY (1990, pp. 226–27), T = 136 observations: The critical values are only slightly higher for T = 100 and T = 48 observations. Power Results: The numbers in the table refer to the probability of rejecting the null hypothesis when the null hypothesis is false. N/A = not applicable, test not conducted.

Figure 1

This figure plots the seasonals presented in Table 2, for the data and the nontime-separable specification.

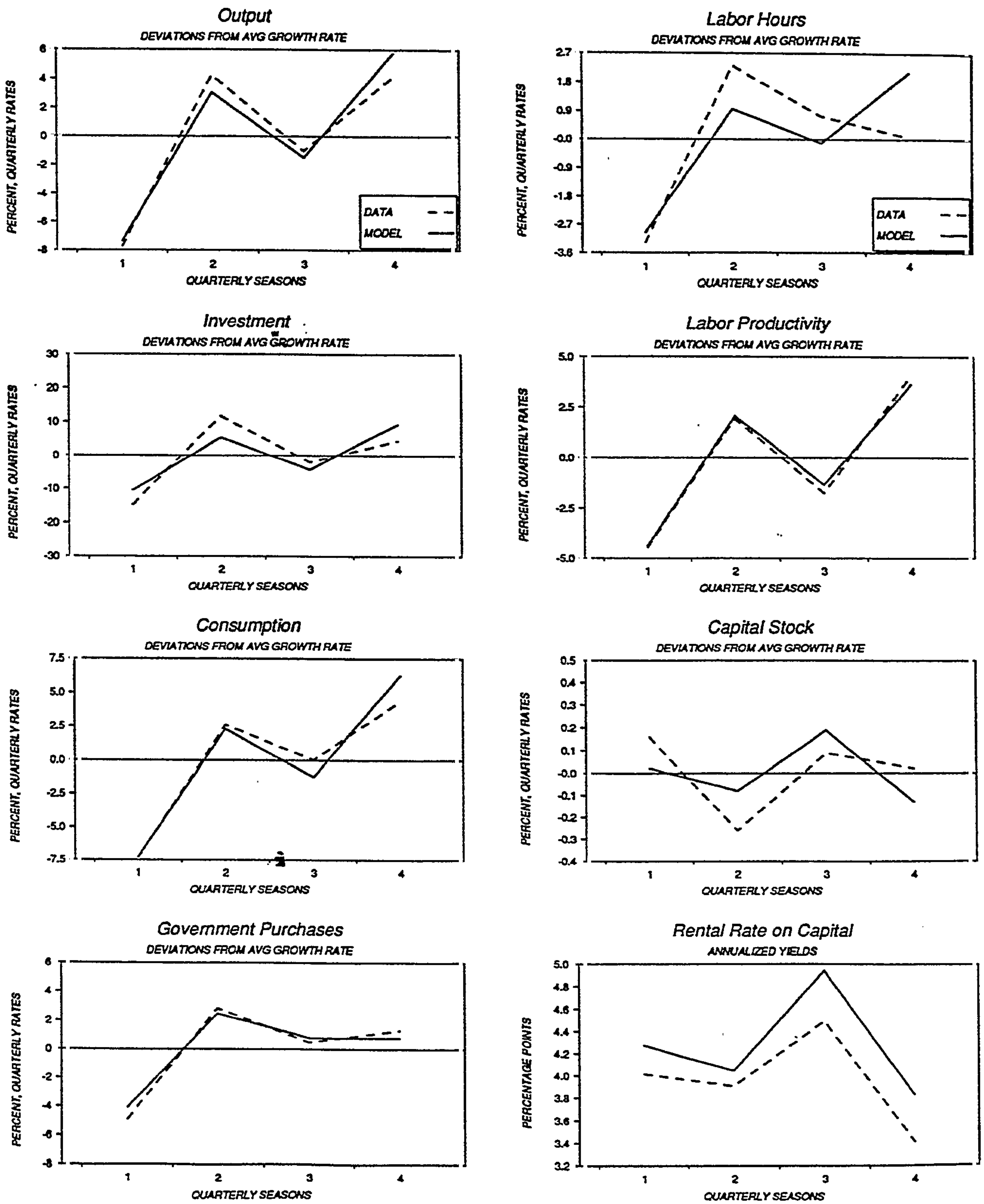

Figure 2

This figure reports seasonals for the nontime-separable model and three special cases. TECH only allows seasonal variation in technology, PREF only allows seasonal variation in preferences, and GOVT only allows seasonal variation in government purchases.

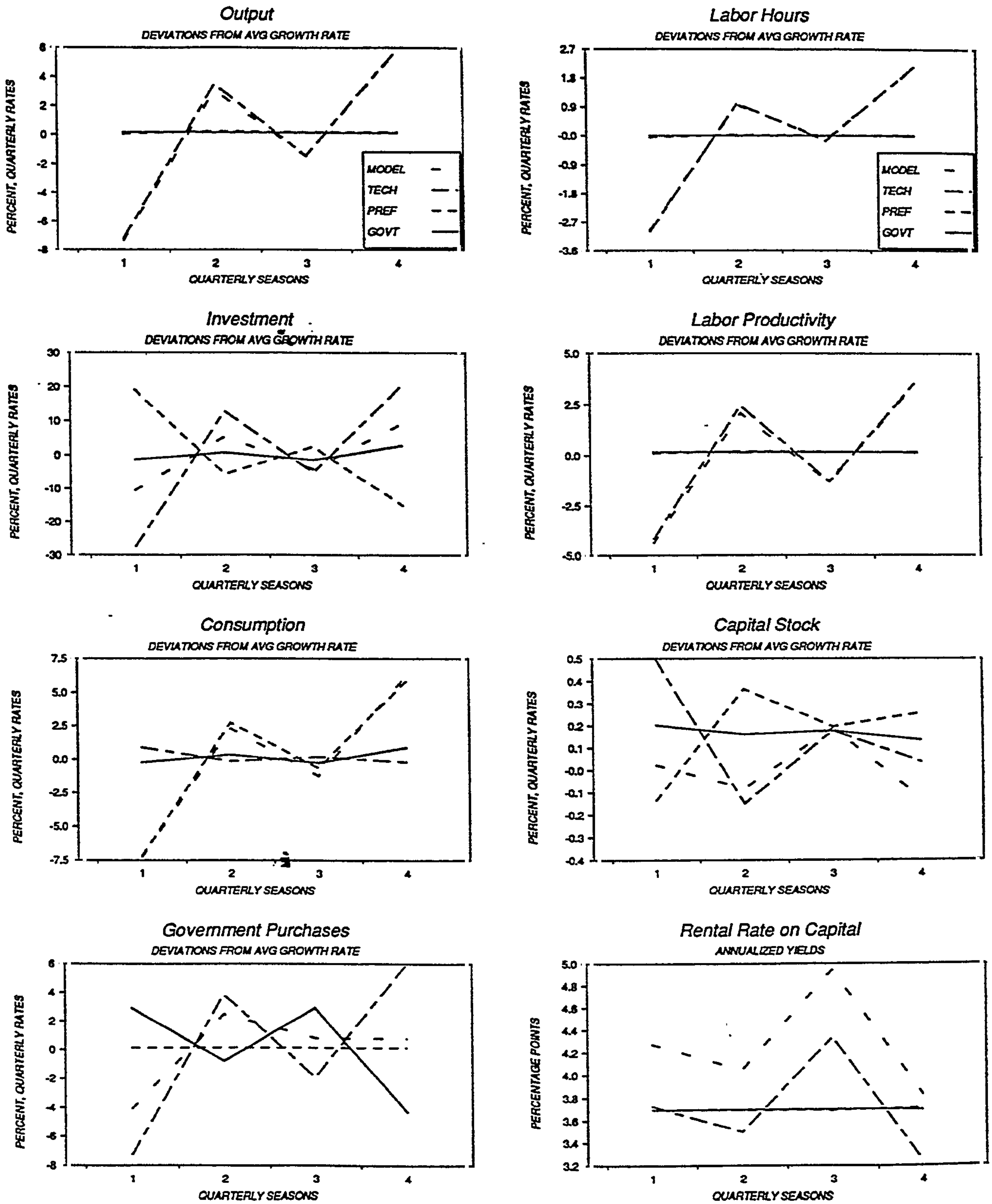

Figure 3

This figure reports spectral densities for output, consumption (Cons.), and labor hours (Labor) for the data, the nontime-separable specification (NTS), and the time-separable specification (TS). Results are reported for seasonally adjusted variables expressed in log-first differences (Differenced) or the Hodrick-Prescott filter (HP).