

Federal Reserve Bank of Minneapolis
Research Department Staff Report 169

April 1994

Two-Sided Search

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ABSTRACT

We integrate search theory into an equilibrium framework in a new way and argue that the result is a simple but powerful tool for understanding many issues related to bilateral matching. We assume for much of what we do that utility is less than perfectly transferable. This turns out to generate multiple equilibria that do not arise in the standard model, with transferable utility, unless one adds increasing returns. We also provide simple conditions for uniqueness that apply to models with or without transferable utility or increasing returns. Examples, applications, and extensions are discussed.

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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Abstract

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1 Introduction

The objectives of this paper are: to integrate the standard one-sided search problem from decision theory into an equilibrium framework in a new way; and to argue that the result is a simple but potentially powerful tool for understanding many issues related to markets with bilateral matching, including the labor market, the marriage market, and so on. Of course, there already exist equilibrium search models that have been applied successfully in a host of applications.¹ Our setup is different from existing models, and

¹A small but representative sample includes: Diamond (1981, 1982a) and Mortensen and Pissarides (1993a, 1993b), who focus on the labor market; Mortensen (1988), who looks at the marriage market; Kiyotaki and Wright (1991, 1993) and Trejos and Wright (1994), who study the money market; Diamond (1982b), Pissarides (1990), Howitt and McAfee (1987), and Howitt (1988), who look at a range of macroeconomic issues; and Rubinstein and Wolinsky (1985), who use the model as a foundation for Walrasian equilibrium. A related literature is concerned with generating a nondegenerate distribution of prices or wages endogenously; examples include Diamond (1971), Butters (1977), Burdett and Judd (1983), Albrecht and Axell (1984), and Burdett and Mortensen (1992).

generates some very different results. The best way to describe how things differ is to provide a brief overview of the environment and the results.

Consider a labor market where workers and employers are either unmatched or matched (in pairs). When unmatched, workers and employers search for partners. Upon contact each observes an expected payoff he will receive if they consummate the match and form a relationship. The payoffs differ across pairs, and include not only wages for the worker and profits for the employer, but utility generated by any number of other match-specific characteristics. If both are agreeable, an employment relationship ensues; if at least one of them decides not to enter into the relationship, they separate to look for other partners.

For much of what we do, it is assumed that utility is less than perfectly transferable between the parties. For instance, if a worker is enthusiastic about an employer but not vice-versa, there is a limit as to what he can do to convince the employer to hire him. If he offers to work for a reduced wage, say, we might want to restrict his offer to be a wage above some lower bound (such as a union-negotiated wage, a legislated minimum wage, or a zero wage). At what wage could typical economics professors get jobs as professional football players? Certainly negative, and probably more than they could afford even if they tapped their teaching and other income.

The point is that no matter how much a worker may want a certain job, there are limits on what he can do to convince the employer to hire him. Similarly, no matter how much an employer may want a certain worker, there are limits on how much he can pay.²

Once one accepts the notion that utility is not perfectly transferable, there is little loss in generality to focusing on an extreme version of the setup where utility is not transferable at all, and the terms of relationships are simply not subject to negotiation — that is, *what you see is what you get*. In this case it is clear that relationships cannot form unless *both* parties derive utility that

²The same observations are at least as valid for the marriage market. Suppose that if a particular male and female enter into a relationship, and equally divide household duties, the male obtains a payoff of 10 utils over and above the expected return to remaining single, while the female obtains -2 and therefore prefers to remain single. An apparent solution is for the male to perform more household duties in an attempt to transfer at least 2 utils to the female. But suppose the "exchange rate" is such that even if the male did enough additional housework to exhaust his 10 utils of surplus, the female's surplus was only -1. Then they simply will not match.

exceeds the reservation value of continued search for new partners. This is quite different from the situation in the standard model used in macro or labor economics, for example, in which relationships always form if there is a positive total surplus, and bargaining determines how this total surplus is split. Casual empiricism suggests that there are plenty of cases in the labor market, the marriage market, and other markets where our assumption of less than perfectly transferable utility seems more reasonable.³

In terms of results, the model with less than perfectly transferable utility can generate multiple equilibria, where the standard model predicts a unique outcome without auxiliary assumptions like increasing returns to scale in the meeting technology. This multiplicity results because of the following intuitively plausible considerations. If employers are very selective as to who they hire, workers will not get very many offers; in these circumstances workers cannot afford to reject many offers, which makes it easy for employers to hire and thereby rationalizes their selectivity. On the other hand, if employers are not very selective, workers will get more offers; in these circumstances workers are willing to only accept jobs they especially like, which makes it hard for employers to hire, and so on.

In the model with perfectly transferable utility, this type of multiplicity cannot arise. It can arise in our model even if workers and employers are completely symmetric, an observation that may be particularly relevant for marriage or mating markets. For instance, in many species females are very selective as to choice of mates, while males are not. But in other species it is the opposite, and in still others the two sexes behave similarly in this dimension. Does this mean males and females are fundamentally different, in ways that differ across species? Not necessarily, according to the model developed here. It could be the case that different species have, perhaps by some evolutionary process or perhaps by other means, simply settled on different equilibria.

The rest of the paper can be summarized as follows. In Section 2, we lay out the environment and the decision problems of individuals on both

³A model of the labor market with nontransferable utility is similar in this regard to search models of the role of money, in the sense that exchange requires a *double coincidence of wants*. An important difference, however, is that the labor market usually involves long-term relationships. Roth (forthcoming) provides an interesting description of several markets in the real world where matching is relevant, and the extent to which bargaining occurs.

sides of the market. In Section 3, we analyze steady state equilibria, in which several variables that are taken as given by individuals are determined by the market. We prove existence and discuss the possibility of multiple equilibria. However, we also provide some simple sufficient conditions – one version is that the distribution of payoffs across matches has a log-concavity property – that are sufficient to guarantee uniqueness.

It is important to point out that the model generates outcomes for any fixed wage, including wages set by unions or legislatures, for example. This is quite different from the standard market model, which only makes sense if the wage is set so that supply equals demand, unless one introduces some *ad hoc* rationing rule. Hence, the model can be used to discuss a variety of issues related to wage formation, some of which are mentioned in Section 4. In Section 5, we work through some examples.

In Section 6 we present some extensions, analyze dynamic equilibria, and consider versions of the model that allow either partially or completely transferable utility. We argue that the model with partially but not completely transferable utility yields results that are qualitatively similar to the model with nontransferable utility. The model with completely transferable utility, however, always has a unique nondegenerate equilibrium. Introducing increasing returns to scale in the meeting technology can generate multiple equilibria with completely transferable utility, as is well-known. However, we show that the same simple conditions which rule out multiplicity in the nontransferable utility model also rule out multiplicity in the transferable utility model, even with increasing returns.

2 The Basic Model

Consider an infinite horizon, stationary economy with two types of agents labelled $j = w, e$. We typically refer to these as *workers* and *employers*, since most of the discussion will be in terms of the labor market; but the model also applies to the marriage market, where the types are men and women, and to a variety of other markets, where the types are any sort of buyers and sellers. For simplicity, we assume that each worker wants to match with one employer and each employer wants to match with one worker.

All individuals are *ex ante* identical: there is no such thing as objectively better workers or employers. However, individuals have preferences over

partners that are idiosyncratic, in the sense that a worker may prefer one employer to another, or vice-versa. To make this concrete, let the flow utility, or payoff, to a worker in a relationship with a particular employer be z_w . For example, we could have $z_w = z_w(w, \varepsilon_w)$, where w is the wage, and ε_w is a vector of nonwage job characteristics comprised of everything from location of the job to the personality of the employer. Symmetrically, the payoff to an employer in a relationship with a particular worker is given by z_e . For example, we could have $z_e = z_e(p - w, \varepsilon_e)$, where p is the productivity of the match, and ε_e is a vector of other considerations the employer finds relevant. In this section, all we need to know are z_w and z_e , and it does not really matter what underlying model of preferences one has in mind.

Agents meet each other randomly over time, in a way that will be specified in more detail below. For type j agents, a random match yields payoff z_j , where z_j has cumulative distribution function F_j . Although not at all essential for the results, to simplify the notation we assume in much of what follows that F_j is differentiable and has its support contained in some interval $[\underline{z}, \bar{z}]$.

We assume for now that *what you see is what you get*, in the sense that utility is completely nontransferable. In this extreme version of the model, nothing can be done to change an agent's evaluation of a potential partner. Various interpretations of this in terms of fixed wages are explored below. However, as we argue in Section 6, the results are qualitatively similar if utility can be transferred via bilateral bargaining, but the wage is constrained to some set (say, it cannot be negative or greater than the total revenue generated by the relationship). What is important here is that it is the pair (z_e, z_w) that is relevant to the match, and not simply the sum $z_e + z_w$.

We begin by describing the decision problem of a typical worker, which is the textbook one-sided search problem (Mortensen 1985). On entering the market, the worker begins searching for an employer. Offers to form employment relationships arrive according to a Poisson process with parameter α_w ; the worker takes α_w as given, although we will ultimately need to determine its value endogenously as part of equilibrium. Each offer is characterized by an independent draw from F_w . While searching the worker also receives utility b_w per unit time, which could measure a variety of things, including unemployment compensation. Assume $F_w(b_w) < 1$, in order to rule out trivial situations where no relationships ever form.

Assume for now that no new offers arrive while a worker is employed, and

that z_w never changes after an offer is accepted (but see Section 6). In this case, a worker will never voluntarily leave a job he previously accepted. There is, however, the possibility that he is forced back on the market because a job ends, an event we think of as a permanent layoff. He expects layoffs to occur according to a Poisson process with parameter λ_w . Furthermore, according to an independent Poisson process with parameter δ_w the worker simply leaves the market for good – for example, he dies.

Let U_w denote the expected discounted lifetime utility of an unattached worker, and let $V_w(z)$ denote his expected discounted lifetime utility in a relationship characterized by utility flow z (the *value functions*). For now we consider only a stationary version of the model where nothing changes over time (dynamics are studied in Section 6). Then the value functions satisfy the following dynamic programming equations:

$$(r + \delta_w)U_w = b_w + \alpha_w \int_{\underline{z}}^{\bar{z}} \max[V_w(z) - U_w, 0] dF_w(z) \quad (1)$$

$$(r + \delta_w)V_w(z) = z + \lambda_w[U_w - V_w(z)], \quad (2)$$

where r is the subjective rate of time preference.⁴ For future reference, note that (2) implies that

$$V_w(z) = \frac{z + \lambda_w U_w}{r + \lambda_w + \delta_w}. \quad (3)$$

Therefore $V_w(z)$ is differentiable, and $V'_w(z) = 1/(r + \lambda_w + \delta_w)$.

Equations (1) and (2) describe the value functions in terms of flow utilities, b_w and z , plus expected capital gains, appropriately discounted by $r + \delta_w$. The capital gain in the case of an unattached worker is the arrival rate of offers times the value of the option to either accept or reject; the capital gain (loss, really) for an employed worker is the arrival rate of layoffs times the value of going back on the market and leaving the current relationship.

⁴Consider an unemployed worker. In discrete time where the length of a period is given by Δ , Bellman's equation is

$$U_w = \frac{1 - \delta_w \Delta}{1 + r \Delta} \{b_w \Delta + \alpha_w \Delta E \max[V_w(z), U_w] + (1 - \alpha_w \Delta)U_w + o(\Delta)\},$$

where $o(\Delta)$ captures the payoff in the event of more than one Poisson arrival in a period, and therefore satisfies $o(\Delta)/\Delta \rightarrow 0$ as $\Delta \rightarrow 0$. Manipulation of this expression yields (1). The derivation of (2) is similar.

The strategy that maximizes the value functions is to accept any offer above the *reservation utility* level R_w , defined by $V_w(R_w) = U_w$. By virtue of (3), $R_w = (r + \delta_w)U_w$. Combining this with (1), we have

$$\begin{aligned} R_w - b_w &= \alpha_w \int_{R_w}^{\bar{z}} [V_w(z) - V_w(R_w)] dF_w(z) \\ &= \alpha_w \int_{R_w}^{\bar{z}} [1 - F_w(z)] V_w'(R_w) dz, \end{aligned}$$

where the second equality results from integration by parts. Inserting V_w' into this expression yields

$$R_w - b_w = \frac{\alpha_w}{r + \lambda_w + \delta_w} \mu_w(R_w), \quad (4)$$

where μ_w is called the *surplus function*, defined by

$$\mu_w(R) = \int_R^{\bar{z}} [1 - F_w(z)] dz. \quad (5)$$

For future reference, note that $\mu_w'(R) = -[1 - F_w(R)] \leq 0$, and $\mu_w''(R) = F_w'(R) \geq 0$.

Employers in this model face a problem completely symmetric to that of workers. A similar analysis yields the employer reservation utility level, R_e , as the solution to

$$R_e - b_e = \frac{\alpha_e}{r + \lambda_e + \delta_e} \mu_e(R_e), \quad (6)$$

where b_e is his utility flow while he is unattached, α_e his arrival rate of offers (or employment applications), λ_e the rate at which workers leave and force him back onto the market, δ_e his death rate, and μ_e his surplus function. Equations (4) and (6) completely characterize the strategies that maximize the value functions, given the parameters. Of course, what is a parameter for one individual may depend, in equilibrium, on the behavior of others.

3 Equilibrium

In this section we put the two sides of the market together. Suppose there are constant and equal populations of each type, with measures normalized to unity. This happens if every type j agent who dies is replaced by a new type

j agent, who starts his life unattached.⁵ Also, we assume for simplicity that the payoffs z_e and z_w are drawn independently when a pair makes contact.

Suppose further that unattached workers and employers make contact according to a meeting technology that determines the total number of contacts per unit time, M , as a function of the number of unmatched workers u_w and the number of unmatched employers u_e . Since every relationship involves one worker and one employer, in this simple setup, $u_e = u_w \equiv u$ and $M = M(u, u)$. For now, we assume constant returns to scale in the meeting technology (but see Section 6). This implies $M = \beta u$, where $\beta \equiv M(1, 1)$. The contact rate for a type j individual equals the total number of contacts divided by the number of unattached agents of type j , which equals the constant β for both types.

However, every contact does not result in an offer. The offer arrival rate for an unattached employer is the contact rate times the probability that a worker finds him acceptable, whereas the offer arrival rate for an unattached worker is the contact rate times the probability that an employer finds him acceptable. Using the identity $1 - F_j(R) = -\mu'_j(R)$, we therefore have

$$\alpha_e = -\beta\mu'_w(R_w) \quad \text{and} \quad \alpha_w = -\beta\mu'_e(R_e). \quad (7)$$

Also, since no one ever terminates a relationship voluntarily, the only time an agent is forced back onto the market is when his partner dies (but see Section 6). Hence,

$$\lambda_e = \delta_w \quad \text{and} \quad \lambda_w = \delta_e. \quad (8)$$

Inserting (7) and (8) into (4) and (6), we arrive at

$$\frac{R_e - b_e}{\mu_w(R_e)} = -\pi\mu'_w(R_w), \quad (9)$$

$$\frac{R_w - b_w}{\mu_e(R_w)} = -\pi\mu'_e(R_e), \quad (10)$$

where $\pi \equiv \beta/(r + \delta_w + \delta_e)$. Equation (9) defines the employer *reaction function* (or best response function), $R_e = \rho_e(R_w)$, which expresses his reservation

⁵Potentially interesting extensions beyond the scope of the present study are to allow different numbers of each type, and to endogenize the number of one or both types via entry (see Pissarides 1990, e.g., for a search model with entry by employers).

utility as a function of the reservation utility of workers. Similarly, (10) defines the worker reaction function, $R_w = \rho_w(R_e)$.

A *steady state equilibrium* is a pair (R_e^*, R_w^*) that satisfies $R_w^* = \rho_w(R_e^*)$ and $R_e^* = \rho_e(R_w^*)$. The first result is that a steady state equilibrium always exists.⁶

Proposition 1 *There always exists a steady state equilibrium (R_e^*, R_w^*) , with R_j^* strictly less than the upper bound of the support of F_j for both types.*

Proof: First, we claim that for all R_w such that $F_w(R_w) = 0$ we have $\rho_e(R_w) = \bar{R}_e$, where $\bar{R}_e < \bar{z}$. To see this, note that $\rho_e(R_w)$ is constant at some value \bar{R}_e for any such R_w , since then marginal changes in R_w do not affect the arrival rate α_e . Suppose $\bar{R}_e \geq \bar{z}$. Then an employer rejects offers with probability 1, which yields lifetime utility $b_e/(r + \delta_e)$. But $F_e(b_e) < 1$ by assumption, and therefore an employer can increase his utility by accepting some offers. Next we claim that for all R_w such that $F_w(R_w) = 1$ we have $\rho_e(R_w) = b_e$. This follows directly from (9). Finally, ρ_e is continuous by our assumptions on F_j . Symmetric properties hold for ρ_w . Hence, the reaction functions look like those shown in Figure 1, from which the result is clear. \square

Differentiation of (9) and (10) implies $\rho'_j \leq 0$ for both j , and hence the reaction functions could potentially cross more than once. Indeed, the following scenario seems plausible. Suppose employers are *selective*, in the sense that they choose a high value of R_e ; then workers receive infrequent offers, and their best response is to be *easy*, in the sense that they choose a low value of R_w . But this means employers receive lots of job applications, thereby rationalizing their high value of R_e . Now suppose employers are *easy*; then workers receive lots of job offers, and their best response is to be *selective*. But this means employers receive few job applications, thereby rationalizing their low value of R_e .

We provide explicit examples with multiple equilibria in Section 5. Nevertheless, it is desirable in some contexts to have conditions that deliver a unique equilibrium. To this end, define a function φ to be *log-concave*, abbreviated LC, if $\log(\varphi)$ is concave. If φ is twice differentiable, then it is LC

⁶The proof uses the assumption that F_j is continuous, but this is not really necessary. More generally, equilibria will always exist if we allow agents to use mixed strategies, whereby they choose a probability of accepting any offer.

if and only if

$$\varphi\varphi'' - (\varphi')^2 \leq 0. \quad (11)$$

Moreover, one can show that if the density function F' is LC, then so is the distribution function F and the survivor function $1 - F$, and if the survivor function is LC, then so is the surplus function μ .⁷

Proposition 2 *Suppose the surplus function μ_j is LC (which is guaranteed if the survivor function is LC, which is guaranteed if the density function is LC), for both j ; then there is exactly one steady state equilibrium.*

Proof: No more than one equilibrium can exist if ρ_w is always steeper than ρ_e when they intersect. Calculating the slopes of the reaction functions from (9) and (10), we find that this is the case if and only if

$$1 + 2\pi\mu'_e\mu'_w + \pi^2 [(\mu'_w\mu'_e)^2 - \mu''_e\mu''_w\mu_e\mu_w] > 0.$$

Therefore, a sufficient condition for uniqueness is that the term in square brackets be nonnegative. This is true if μ_j is LC for both j , by (11). \square

In any equilibrium, one can calculate the implied path of unemployment, u (which is the same as the path of vacancies, given equal numbers of both types). The flow into unemployment is given by $(1 - u)(\delta_e + \delta_w)$, while the flow out is given by uH , where

$$H = \beta\mu'_e(R_e)\mu'_w(R_w)$$

is the *hazard rate*. Then $\dot{u} = (1 - u)(\delta_e + \delta_w) - uH$, and for any initial condition, $u \rightarrow u^*$, where

$$u^* = \frac{\delta_e + \delta_w}{\delta_e + \delta_w + H}.$$

⁷This follows from a theorem of Prekopa (1973) that says LC is preserved under integration; see Dharmadhikari and Joag-dev (1988) for an exposition. Note that LC is a common condition and has found applications in search theory dating back to Burdett (1981). Many well-known probability distributions are LC, although some are not. Properties of LC distributions include these: all moments exist, they are unimodal, and they have a nondecreasing hazard.

The reservation utilities R_j do not depend on u , because with a constant returns meeting technology the contact rate does not depend on u .

Since there can be multiple equilibria in (R_w, R_e) space, there can be multiple equilibrium unemployment rates. This is true despite the fact that we did not assume increasing returns in the meeting technology, which has been the usual way to generate multiplicity in equilibrium search models since Diamond (1982b). Moreover, in the standard model with perfectly transferable utility, multiple nondegenerate equilibria *cannot* arise with constant returns (see below). It is therefore clear that the multiplicity here is different from the standard result and depends on utility being less than perfectly transferable.

It is also worth noting that there can be equilibria where $R_j \leq z$ for one type. For example, if $R_w^* \leq z$, then the reaction functions cross where $\rho_e(R_w^*)$ is horizontal. In this case, workers are *especially easy*, in the sense that they accept all offers.⁸ In such an equilibrium, a change in an exogenous parameter like b_w , induced by a change in unemployment compensation, say, will have no observable effects: it shifts ρ_w , but as long as $R_w^* = \rho_w(R_e^*)$ is still less than the lower bound of the support of F_w , this will not affect R_e^* , H , or u .

In order to study the effects of parameter changes further, assume for the rest of this section that there is a unique equilibrium where R_j^* is in the interior of the support of F_j for both types. At the unique equilibrium, ρ_w is steeper than ρ_e , as in Figure 1.⁹ Then an increase in b_w shifts ρ_w to the right, leading to an increase in R_w^* and a reduction in R_e^* . Indeed, the increase in R_w^* is *greater* than what one would predict from a single-sided search model where α_w does not change. Here the initial increase in workers' reservation utility makes it harder for employers to hire, and so they become less selective, which means workers get more offers, which further increases R_w^* .

One can also ask how unemployment depends on b_w . Since u is decreasing

⁸This observation may alleviate the misperception that search theories of unemployment require that workers reject job offers, something that does not happen a lot in the data.

⁹When multiple equilibria exist, in any equilibrium where ρ_w is flatter than ρ_w , changes in parameters will have completely counterintuitive effects; see the example in Section 5.

in H , we need only check

$$\frac{\partial H}{\partial b_w} = \beta [\mu'_e \mu''_w + \mu'_w \mu''_e \rho'_e] \frac{\partial R_w}{\partial b_w},$$

where ρ'_e is the slope of the employer reaction function. If $\rho'_e = 0$, H unambiguously falls with b_w . If $\rho'_e < 0$, H falls by less, and could actually rise – e.g., more generous unemployment insurance benefits could actually reduce unemployment. Inserting ρ'_e and simplifying, we have

$$\frac{\partial H}{\partial b_w} = K [\mu'_e + \pi \mu'_w (\mu_e'^2 - \mu'_e \mu_e'')] \frac{\partial R_w}{\partial b_w},$$

where $K > 0$. This is ambiguous, in general, even if we restrict attention to the case of a unique equilibrium. If μ_e is LC, however, then the term in parentheses is positive, which implies that $\partial H / \partial b_w < 0$ and $\partial u / \partial b_w > 0$.

At this point we consider a symmetric version of the model, where $b_j = b$ and $F_j(z) = F(z)$, which implies that $\mu_j(R) = \mu(R)$, for both j . Then the reaction functions are the same, $\rho_j = \rho$, and it is natural to look for symmetric equilibria, where $R_e^* = R_w^* = R^*$. Such an equilibrium is characterized by the intersection of ρ with the 45° line in (R_w, R_e) space. It is straightforward to verify that there always exists a symmetric equilibrium, that there cannot be more than one symmetric equilibrium.¹⁰

In the symmetric equilibrium, an increase in b (that is, an increase in b_j for both types) unambiguously increases R^* and u . An increase in the contact rate β also increases R^* , but has an ambiguous effect on u , because the increase in β raises H directly and then lowers it through the indirect effect on R^* . However, one can show that if μ is LC then the net effect of an increase in β is an increase in H and a decrease in u . Intuitively, LC guarantees that an increase in the offer arrival rate does not increase the reservation utility value too much, which guarantees that the net effect on H is positive.

We summarize the effects of these parameter changes as follows.

¹⁰Even if b_j and F_j are symmetric, there can still exist nonsymmetric equilibria (see below). What we are claiming here is that there is a unique symmetric equilibrium. Of course, by Proposition 2, if μ is LC then there exists a unique equilibrium and it is the symmetric one.

Proposition 3 *Assume there is a unique steady state equilibrium. Then an increase in b_w increases R_w^* and decreases R_e^* , and increases u if μ_e is LC. The change in R_w^* is greater than predicted by a one-sided search model, while the change in u is less. In the symmetric equilibrium of the symmetric model, an increase in b increases R^* and u , while an increase in β always increases R^* and reduces u if μ is LC.*

4 Wage Determination

So far it has been argued that, when an unemployed worker and employer meet, two random variables are realized, $z_w = z_w(w, \varepsilon_w)$ and $z_e = z_e(p - w, \varepsilon_e)$, where p is the revenue flow if a relationship ensues, w is the wage rate for the type of labor services to be supplied, and ε_w and ε_e are the two idiosyncratic utility flows unique to the match. In Section 6 we consider bilateral bargaining between the parties when they are deciding whether or not to form a relationship, after they have met. Here we assume that the wage is not negotiable after the parties meet; nonetheless, we can still discuss how the wage is determined *ex ante*.

Given that an individual cannot transfer utility to the other party, by varying w or otherwise, Proposition 1 shows that there exists at least one steady state equilibrium for *any* w . This differs from the standard competitive model of the labor market, where the theory only makes sense when the wage is set so that supply equals demand, unless we impose some *ad hoc* rationing rule. Here, rationing is done by the search and matching process, which rather than being an afterthought is the very heart of the model.

Of course, when w changes, the equilibrium changes. For example, suppose the labor market under consideration is fully unionized. Before the search process begins, or perhaps during the process, representatives of the workers and employers meet to negotiate the wage to be paid to all employees. The results presented so far specify the possible equilibria as a function of w . Therefore, the negotiators can calculate the consequences in terms of steady state unemployment; steady state utility for an employed worker, an unemployed worker, or an average worker; and so on, for any suggested wage. In this case, the above results describe the menu over which the representatives negotiate. Further discussion is contained in the context of an example in Section 5.

Another approach is to regard the fixed wage w as the symmetric equilibrium of a wage-posting game between firms. Thus, each employer chooses his own wage, to which he commits, taking as given the wages posted by all other employers. The advantage of a low wage is that profits are higher once a relationship begins; the disadvantage is that workers are less likely to accept the job, and so hiring takes longer. One can show that there exists a unique symmetric Nash equilibrium in wages under the LC assumption; see Masters (in progress).

On any of these interpretations, the bottom line is a two-sided search problem in which it is the pair (z_e, z_w) that matters. Later, we also consider versions of the model where the wage is negotiable *ex post*. Then, in the extreme case where there are no constraints on bargaining whatsoever, it is only the sum $z = z_e + z_w$ that matters.

5 Examples

In this section we study some examples with particular distribution functions.

5.1 The Uniform Case

Consider the case where a worker's utility flow from a randomly selected employer is given by $z_w = w + \varepsilon_w$, where w indicates a (fixed) wage rate offered by all employers and ε_w is uniformly distributed on $[0, 1]$. Also, an employer's utility flow from a randomly selected worker is given by $z_e = p - w + \varepsilon_e$, where p is a (fixed) revenue flow and ε_e is uniformly distributed on $[0, 1]$. Assume that $b_w < w + 1$ and $b_e < p - w + 1$.

It is useful in this case to translate variables by defining

$$k_w \equiv R_w - w \quad \text{and} \quad k_e \equiv R_e - p + w.$$

Thus, k_j is the reservation utility level in terms of nonmonetary considerations, and agent j accepts any offer such that $\varepsilon_j \geq k_j$. Then equilibrium conditions (9) and (10) can be written as

$$\frac{2(k_e + p - w - b_e)}{(1 - k_e)^2} = \pi(1 - k_w) \tag{12}$$

$$\frac{2(k_w + w - b_w)}{(1 - k_w)^2} = \pi(1 - k_e). \quad (13)$$

It is easy to check that there is a unique solution to (12) and (13) (which is not surprising, given that the uniform distribution is LC).

Because these equations are so simple, we can say a lot about certain issues. For instance, consider the model where representatives of a trade union and an industry meet to negotiate a wage that must be paid to any worker who is employed. Let (k_e, k_w) be the equilibrium reservation utilities, given w , and assume $k_j \in (0, 1)$. Associated with this are the hazard $H = \beta(1 - k_w)(1 - k_e)$ and the implied unemployment rate u .

It is straightforward to show that if we increase w then workers become less selective and employers become more selective; in fact, $-\partial k_w / \partial w = \partial k_e / \partial w > 0$. The induced change in unemployment can be determined from the change in the hazard,

$$\frac{\partial H}{\partial w} = -\beta(1 - k_e) \frac{\partial k_w}{\partial w} - \beta(1 - k_w) \frac{\partial k_e}{\partial w} = \beta(k_e - k_w) \frac{\partial k_w}{\partial w}.$$

Notice that $\partial H / \partial w > 0$ if and only if $k_w > k_e$. However, from (12) and (13) it can be seen that $k_w > k_e$ if and only if $w < \hat{w} \equiv \frac{1}{2}(p + b_w - b_e)$. We conclude that H is increasing and u decreasing in w if and only if $w < \hat{w}$.

What is interesting is that higher union-negotiated wages lead to greater unemployment only after w exceeds the threshold \hat{w} . For low wages, increasing w reduces unemployment, because workers become less selective in terms of nonwage job considerations. At the same time, increasing w means employers become more selective, but the worker effect dominates as long as $w < \hat{w}$. Note that this result depends on the assumption that utility is not perfectly transferable; in the standard model, with perfectly transferable utility, changes in w can simply be undone by bilateral bargaining.¹¹

5.2 The Pareto Case

Now suppose that both workers and employers face a Pareto distribution, which is worth considering because its density is not LC. The Pareto distribution with parameters $x > 0$ and $\gamma > 1$ is given by $F(z) = 1 - (x/z)^\gamma$ for

¹¹A similar analysis could be used to argue that, up to some threshold, increases in the minimum wage reduce unemployment in a model without perfectly transferable utility.

any $z \geq x$, and $F(z) = 0$ for any $z < x$. Its surplus function is

$$\mu(R) = \frac{x^\gamma R^{1-\gamma}}{\gamma - 1}$$

for $R \geq x$.

Let F_e be Pareto with parameters x_e and γ_e , and let F_w be Pareto with parameters x_w and γ_w (the fact that the supports are unbounded gives us no trouble). After simplification, the employer reaction function can be written implicitly as:

$$\frac{R_e - b_e}{R_e^{1-\gamma_e}} = \begin{cases} \left(\frac{\pi x_e^{\gamma_e} x_w^{\gamma_w}}{\gamma_e - 1} \right) R_w^{-\gamma_w} & \text{if } R_w \geq x_w \\ \left(\frac{\pi x_e^{\gamma_e}}{\gamma_e - 1} \right) & \text{if } R_w < x_w. \end{cases}$$

A symmetric expression describes the worker reaction function. Also, assume for the moment that $b_e = 0 < b_w$.

Then there are several possible outcomes, depending on parameter values. One possibility is that there is a unique equilibrium with $R_w < x_w$ and $R_e > x_e$, where workers accept every offer but employers are selective. This is depicted in Figure 2. Another possibility is the reverse situation – a unique equilibrium with $R_w > x_w$ and $R_e < x_e$, where workers are selective and employers easy. The final possibility is that there exist exactly three equilibria, the two described above plus an equilibrium where $R_j > x_j$ for both j . This is depicted in Figure 3.

Notice that whenever the interior equilibrium exists (i.e., the one with $R_j > x_j$ for both j), so do the other two equilibria. Also, notice that the interior equilibrium has some weird comparative static properties; for example, if we increase b_w then ρ_w shifts out, but the net effect in equilibrium is that R_w^* falls. Finally, consider the symmetric case, where $\gamma^j = \gamma$ and $x_j = x$ for both j . If we also relax the assumption $b_w > 0$, and set $b_w = b_e = 0$, then the two reaction functions are coincident as long as $R_j \geq x$ for both j . Hence, there is a continuum of equilibria. In this strange case, over some region, any time employers become more selective workers respond by becoming easier by exactly enough to rationalize the employers' increase in selectivity.

5.3 A Discrete Example

Suppose that $z = (z_1, z_2)$ with probability $(1 - \theta, \theta)$ for both types, where $0 < z_1 < z_2 < 1$. Also, assume for simplicity that $b_j = 0$, and set $\beta = 1$. Then routine algebra implies that (6), which gives the reservation utility level for an employer as a function of α_e , can be reduced to

$$R_e = \begin{cases} \frac{\alpha_e[\theta z_2 + (1-\theta)z_1]}{r + \delta_e + \delta_w + \alpha_e} & \text{if } \alpha_e < \bar{\alpha} \\ \frac{\alpha_e \theta z_2}{r + \delta_e + \delta_w + \theta \alpha_e} & \text{if } \alpha_e \geq \bar{\alpha}, \end{cases} \quad (14)$$

where $\bar{\alpha} \equiv (r + \delta_e + \delta_w)z_1 / \theta(z_2 - z_1)$. A symmetric condition holds for workers.

We claim that for some parameter values there exist an equilibrium with selective employers who only accept z_2 and easy workers who accept either z_1 or z_2 , and another equilibrium with selective workers and easy employers. To verify this, choose z_1 and z_2 so that $\theta < \bar{\alpha} < 1$ (this can always be done). If $R_w \in (0, z_1)$ then workers accept all offers, $\alpha_e = 1 > \bar{\alpha}$, and from (14) one can check that $R_e \in (z_1, z_2)$. On the other hand, if $R_w \in (z_1, z_2)$ then workers are selective, $\alpha_e = \theta < \bar{\alpha}$, and from (14) one can check that $R_e \in (0, z_1)$. The situation is depicted in Figure 4.

6 Extensions

6.1 Voluntary Separations

Up to now, no individual ever terminates a relationship voluntarily. Several generalizations of the model change this result, including on-the-job search, as in Burdett (1978), or learning about the match, as in Jovanovic (1979). For the sake of illustration, we consider an alternative in which individuals sometimes *change their minds*. That is, according to a Poisson process with parameter γ_j , an attached type j agent reevaluates the relationship by drawing a new z , at which point he may either stay in the relationship or go back on the market. To simplify matters, assume the new z is independent of the current value.¹²

¹²It would not be difficult to recast this as a model with learning, where agents upon contact see only a signal of the true z_j .

Then (1) still holds, but (2) must be amended to

$$(r + \delta_j)V_j(z) = z + \lambda_j[U_j - V_j(z)] + \gamma_j E \max[V_j(z') - V_j(z), U_j - V_j(z)].$$

A strategy that maximizes the value functions is to accept or stay in any relationship with $z \geq R_j$ and reject or leave any relationship with $z < R_j$, where the reservation utility level satisfies the following generalization of (4):

$$R_j - b_j = \frac{\alpha_j - \gamma_j}{r + \delta_j + \lambda_j + \gamma_j} \mu_j(R_j).$$

Notice that the same reservation value is used for new and incumbent partners (this would not be true if we changed the model, by introducing a fixed cost of separations, say).

Equilibrium requires the same conditions on offer arrival rates (7); but now (8) must be generalized to

$$\lambda_e = \delta_w + \gamma_w F_w(R_w) \quad \text{and} \quad \lambda_w = \delta_e + \gamma_e F_e(R_e),$$

since individuals are forced back on the market not only when their partners die, but also when their partners change their minds and terminate the relationships. Hence, the employer reaction function satisfies the following generalization of (9):

$$\frac{R_e - b_e}{\mu_e(R_e)} = - \frac{\beta \mu'_w(R_w) + \gamma_e}{r + \delta_e + \delta_w + \gamma_e + \gamma_w F_w(R_w)}.$$

A similar equation holds for workers.

Consider the symmetric case, in which a symmetric equilibrium R^* occurs at an intersection of the common reaction function ρ with the 45° line in (R_w, R_e) space. Notice that $R^* > b$ if and only if $\gamma < -\beta \mu'(b) = \beta[1 - F(b)]$. One can also show that $\rho' < 0$ as long as $\gamma < \beta[1 - F(b)]$. Therefore, given this condition there exists a unique symmetric equilibrium with $R^* > b$.

Agents in this generalized model undergo several different types of transitions. For example, a worker can lose his job by exiting the market, by quitting voluntarily to go back on the market, or by getting laid off either because his employer leaves the market or because his employer reevaluates the relationship. The model does not yet generate job-to-job transitions, but we could introduce these with on-the-job search.

6.2 Dynamics

The reaction functions were derived above under the assumption that R_j did not change over time. If agents are myopic, we can trace out the dynamics, starting from any $R_w(0)$, say, by setting $R_e(t) = \rho_e[R_w(t)]$ and $R_w(t+1) = \rho_w[R_e(t)]$. In the case of a unique equilibrium, it will be stable under these dynamics, in the sense that $[R_w(t), R_e(t)] \rightarrow (R_w^*, R_e^*)$. In the case of multiple equilibria, they alternate between stable and unstable, with the one with the lowest value of R_w stable.

If agents are forward-looking, on the other hand, then (1) and (2) must be generalized to take the dynamics into account. This implies

$$(r + \delta_j)U_j = b_j + \alpha_j E \max[V_j(z) - U_j, 0] + \dot{U}_j \quad (15)$$

$$(r + \delta_j)V_j(z) = z + \lambda_j[U_j - V_j(z)] + \dot{V}_j, \quad (16)$$

where a "dot" denotes a time derivative.¹³ A strategy that maximizes the value functions is to accept $z \geq R_j$, where $V_j(R_j) = U_j$, at each point in time. The reason that R_j may vary over time is that the offer arrival rate may vary over time, and the arrival rates vary if the reservation values vary.

Differentiation of the identity $V_j(R_j) = U_j$ with respect to time implies that

$$V_j'(R_j)\dot{R}_j = \dot{U}_j - \dot{V}_j(R_j).$$

It can be shown that $\dot{V}_j(z)$ actually does not depend on z ; intuitively, the only reason $V_j(z)$ changes is that attached agents face changing arrival rates when they are forced back on the market after their partner dies, in which case the previous value of z is irrelevant. Hence, (16) implies that $V_j'(z) = 1/(r + \delta_j + \lambda_j)$. Now if we subtract (15) and (16) and evaluate the result at $z = R_j$, we have

$$\begin{aligned} 0 &= R_j - b_j - \alpha_j \int_{R_j}^{\bar{z}} [V_j(z) - U_j] dF_j(z) - [\dot{U}_j - \dot{V}_j(R_j)] \\ &= R_j - b_j - \alpha_j \int_{R_j}^{\bar{z}} [1 - F_j(z)] V_j'(z) dz - V_j'(R_j)\dot{R}_j. \end{aligned}$$

¹³These equations are derived assuming that agents cannot leave a relationship into which they voluntarily entered – a sort of *no divorce* restriction.

Inserting $V_j'(z)$ and rearranging yields

$$\dot{R}_j = (r + \delta_j + \lambda_j)(R_j - b_j) - \alpha_j \mu_j(R_j). \quad (17)$$

By virtue of the equilibrium conditions (7) and (8), the model reduces to the following dynamical system:

$$\begin{bmatrix} \dot{R}_w \\ \dot{R}_e \end{bmatrix} = \begin{bmatrix} (r + \delta_e + \delta_w)(R_w - b_w) + \beta \mu_w(R_w) \mu_e'(R_e) \\ (r + \delta_e + \delta_w)(R_e - b_e) - \beta \mu_e(R_e) \mu_w'(R_w) \end{bmatrix}. \quad (18)$$

A *rational expectations equilibrium* is any bounded solution to (18). Boundedness is a necessary condition, in addition to (17), for a path R_j to maximize the value functions. There are no initial conditions, because R_j is free to take on any value at $t = 0$.

The locus of points such that $\dot{R}_j = 0$ is simply the reaction function of agent j from the stationary model analyzed above. Thus, the steady states of (18) are the equilibria of the stationary model studied above, and a unique steady state always exists under LC. When the steady state is unique, it is a source, and the only bounded solution to (18) is the orbit that starts at (R_w^*, R_e^*) and stays there. Hence, this is the unique rational expectations equilibrium.

When there are multiple steady states, as shown in Figure 5, the dynamics are more interesting. The steady states alternate between sources and saddle points, with the one with the lowest R_w a source. Let \underline{R}_w and \bar{R}_w denote steady state values of R_w in two sources on either side of a saddle point. Then for any initial $R_w \in (\underline{R}_w, \bar{R}_w)$, if we choose R_e on the saddle path the implied orbit constitutes a dynamic rational expectations equilibrium. In this equilibrium, R_j changes over time in anticipation of changes in arrival rates, and anticipations of changes in arrival rates are rationalized by the changes in the reservation values.¹⁴

¹⁴This suggests the possibility of equilibria in which extrinsic uncertainty (sunspots or animal spirits) could matter, where we randomly jump between a state in which workers are selective and employers are easy and a state in which the opposite is true. See Howitt and McAfee (1992) and Wright (forthcoming) for analyses of extrinsic uncertainty in search models. It seems feasible (although not necessarily trivial) to apply similar methods here.

6.3 General Matching Technologies

Even though offer rates are endogenous, the underlying contact rate β has so far been assumed to be constant, corresponding to the assumption of constant returns to scale in the meeting technology. Some authors have suggested that this technology should display increasing returns, although the empirical evidence is mixed; see Diamond and Blanchard (1990) and Coles and Smith (1992). Increasing returns implies $\beta = B(u)$, where $B(0) = 0$ and $B'(u) > 0$. It has been well-known since Diamond (1982b) that increasing returns can generate multiple nondegenerate equilibria in search models.

For any candidate unemployment rate u_0 , set $\beta = B(u_0)$ and find the implied equilibrium (R_e, R_w) . Assume that this is unique, in order to focus on the multiplicity generated by increasing returns rather than the multiplicity analyzed earlier. This generates the hazard rate $H = \beta\mu_e(R_e)\mu_w(R_w)$ and the actual steady state unemployment rate $u_1 = T(u_0)$. An equilibrium is a fixed point, $u = T(u)$. To illustrate the essentials, consider the symmetric version of the model in which $F_j = F$ and $b_j = b$. In any symmetric equilibrium, the reservation utility level used by both types is R , and the hazard is $H = \beta\mu(R)^2$.

In general, it is certainly possible to have multiple nonzero solutions to $u = T(u)$. But the following result says this cannot happen under LC.

Proposition 4 *If μ is LC, then there is a unique symmetric equilibrium even with $B' > 0$.*

Proof: If μ is LC, we know from Proposition 3 that an increase in β increases R by less than the amount required to increase u . This implies that $T(u)$ is a decreasing function. Hence, there cannot exist multiple nonzero solutions to $u = T(u)$. \square

6.4 Transferable Utility

We now drop the assumption that utility is not transferable and assume it is perfectly transferable. The total payoff available in a match is $z = z_w + z_e$, a random draw from the distribution function G induced by the underlying distributions F_j . Let \hat{z} be the upper bound of G . Let $Z_w(z)$ and $Z_e(z) = z - Z_w(z)$ denote the outcome of bargaining over how to split this total utility between the agents; we will be more explicit about this in a moment. The key

point for now is that $V_j[Z_j(z)] \geq U_j$ is necessary for j to enter a relationship rather than stay on the market.

The usual techniques imply that

$$(r + \delta_j)U_j = b_j + \alpha_j E \max[V_j(Z_j) - U_j, 0]$$

$$(r + \delta_j)V_j(Z_j) = Z_j + \lambda_j[U_j - V_j(Z_j)].$$

A strategy that maximizes the value functions is to enter into a relationship with total payoff z if and only if $Z_j(z) \geq R_j$, where $V_j(R_j) = U_j$, or $R_j = (r + \delta_j)U_j$. To progress further, we need to say more about bargaining. Consider the Nash solution,¹⁵

$$\begin{aligned} Z_w &= \arg \max [V_w(Z_w) - U_w][V_e(z - Z_w) - U_e] \\ &= \arg \max [Z_w - R_w][z - Z_w - R_e]. \end{aligned}$$

This implies that

$$Z_w = \frac{1}{2}(z + R_w - R_e) \quad \text{and} \quad Z_e = \frac{1}{2}(z + R_e - R_w).$$

Note that $Z_w \geq R_w$ if and only if $Z_e \geq R_e$ if and only if $z \geq R \equiv R_w + R_e$. Hence, relationships will be consummated by mutual consent if and only if the total available utility exceeds R . Since $R_e = (r + \delta_e)U_e$, the equilibrium conditions allow us to express R_e as

$$R_e = b_e + \beta \int_R^{\hat{z}} [V_e(Z_e) - U_e] dG(z).$$

We also know that

$$V_e(Z_e) - U_e = \frac{Z_e - R_e}{r + \delta_e + \delta_w} = \frac{z - R}{2(r + \delta_e + \delta_w)}.$$

Hence, we have

¹⁵This corresponds to the equilibrium of a Rubinstein (1982) sequential bargaining game, under the assumption that agents continue to meet potential trading partners during the period of delay after an offer is rejected, in the limit as this period of delay becomes small. See Binmore et al. (1986) for an extended discussion.

$$R_e - b_e = \frac{\pi}{2} \int_R^{\hat{z}} (z - R) dG(z).$$

A similar condition holds for worker reservation utility. Adding these conditions yields

$$R - b_e - b_w = \pi \int_R^{\hat{z}} (z - R) dG(z). \quad (19)$$

At this stage, we can either look for values of R^* satisfying (19) and then solve for R_w^* and R_e^* , or define the reaction functions $R_e = \rho_e(R_w)$ and $R_w = \rho_w(R_e)$ and look for (R_w^*, R_e^*) directly. It is not difficult to show that there exist unique equilibrium values for (R_w^*, R_e^*) , R^* , the hazard $H = \beta[1 - G(R^*)]$, and unemployment u . The significance of this result is that it implies the multiplicity found earlier can be definitely attributed to the assumption of less than perfectly transferable utility.

Proposition 5 *In the model with perfectly transferable utility there is a unique equilibrium.*

Proof: Simply note that there is a unique R^* that satisfies (19). \square

Now consider introducing an increasing returns to scale meeting technology into the model with transferable utility, so that $\beta = B(u)$ with $B' > 0$. It is generally understood that a model like this can generate multiple equilibria. However, the following result indicates that LC is sufficient for uniqueness in this model, just as it is in the model with nontransferable utility.

Proposition 6 *In the model with perfectly transferable utility and increasing returns, there always exists an equilibrium; there may exist multiple equilibria, but not if μ is LC.*

Proof: The steady state and reservation utility conditions can be written $S(u, R) = T(u, R) = 0$, where

$$S(u, R) = (\delta_e + \delta_w)(1 - u) - uB(u)[1 - G(R)]$$

$$T(u, R) = R - b_e - b_w - \frac{B(u)}{r + \delta_e + \delta_w} \int_R^{\hat{z}} (z - R) dG(z).$$

It is easy to show that the $S = 0$ and $T = 0$ curves intersect at least once in (u, R) space. Moreover, they intersect no more than once if the former is steeper than the latter at their intersection. A little algebra implies that this is the case if and only if

$$(\delta_e + \delta_w - B'\mu')(r + \delta_e + \delta_w) - (\delta_e + \delta_w)B\mu' + BB'[(\mu')^2 - \mu\mu''] > 0.$$

Once again, this is true if μ is LC. \square

6.5 Partially Transferable Utility

In this last extension, we consider the case where the agents can transfer part of the utility generated by the match, by changing the wage, say; but they may not be able to transfer utility perfectly, because the wage cannot be adjusted beyond some level. For example, suppose we allow w to take on any value in some interval $[\underline{w}, \bar{w}]$. To fix ideas, let $\underline{w} = 0$ and $\bar{w} = p$, so that employers cannot pay negative wages and cannot pay more than total revenue; but other assumptions or interpretations entail the same basic message.

When a worker and employer make contact, $(\varepsilon_w, \varepsilon_e)$ is revealed, and then the pair bargain over the wage w . The payoffs are $z_e = \varepsilon_w + w$ and $z_e = \varepsilon_e + p - w$, the threat points are R_w and R_e , and the constraint is $0 \leq w \leq p$. We can be agnostic about the bargaining solution and simply write $w = w(\varepsilon_w, \varepsilon_e, R_w, R_e, p)$, since we are only interested here in showing that sometimes there will be no agreement that satisfies the constraints and is acceptable to both parties, even though $\varepsilon_w + \varepsilon_e + p > R_w + R_e$.

Figure 6 shows a case where $\varepsilon_w > R_e$ and $\varepsilon_e < R_e$. If utility is perfectly transferable, with no constraints, then the bargaining frontier is the 45° line, and the segment of this line that satisfies $z_w \geq R_w$ and $z_e \geq R_e$ is the set of acceptable agreements on the frontier. As shown in the figure, this set is nonempty, and so the worker could get the job if he were allowed to transfer enough utility to the employer. But, as shown, this would imply a negative wage. If we constrain $w \geq 0$, then the most the employer could get from the relationship is $\varepsilon_e + p < R_e$, which is not enough to hire this worker.

Clearly, as long as there is some degree beyond which utility cannot be transferred, situations like this can arise, and the model will be qualitatively similar to the version analyzed above where utility is completely nontransferable. The distribution of z_j will change to take into account the bargaining

over w that occurs whenever there is a nonempty intersection of the set of acceptable offers and the constraint set, but all of our results survive if we reinterpret F ; as the new distribution induced by the constrained bargaining solution.

7 Conclusion

The paper has presented an equilibrium search model with less than perfectly transferable utility. This framework seems a natural extension of the typical decision-theoretic model, in the sense that every agent solves a textbook one-sided search problem. However, in equilibrium, offer arrival rates for agents on one side of the market are endogenously determined by the reservation utilities of agents on the other side, and layoff rates for one side are endogenously determined by the rates at which agents on the other side leave the market, as well as the rates at which they revise their beliefs or change their minds about partners.

We demonstrated that there always exists a steady state equilibrium, and we showed that there could exist multiple steady state equilibria. This multiplicity is different from that discussed in the previous literature; rather than increasing returns in the meeting technology, it depends on positive feedback in search strategies. There can also exist multiple dynamic equilibria. These results depend on there being less than perfectly transferable utility – with perfectly transferable utility, multiple nondegenerate equilibria cannot arise without increasing returns.

Additionally, we showed that the simple condition known as *log-concavity* rules out multiplicity, whether or not one allows transferable utility or increasing returns. We also worked through some examples, and discussed several applications and extensions. Much more can be done. The goals here were to present the basic model, provide some key results and techniques, and suggest topics for future research.

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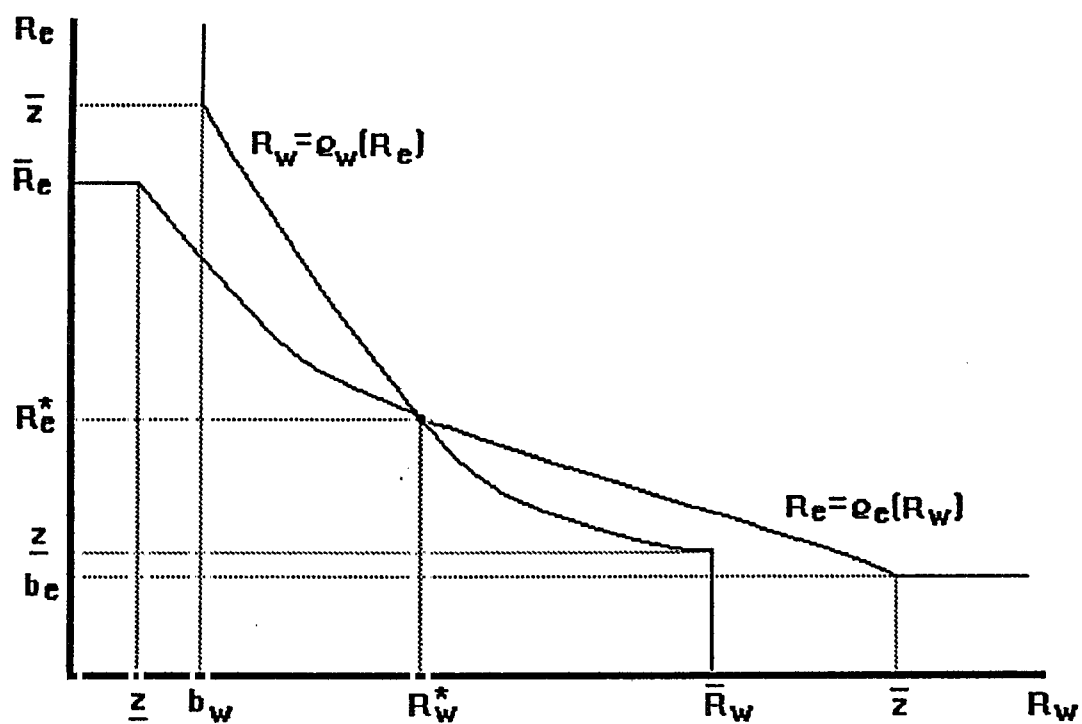


Figure 1: Existence of Equilibrium

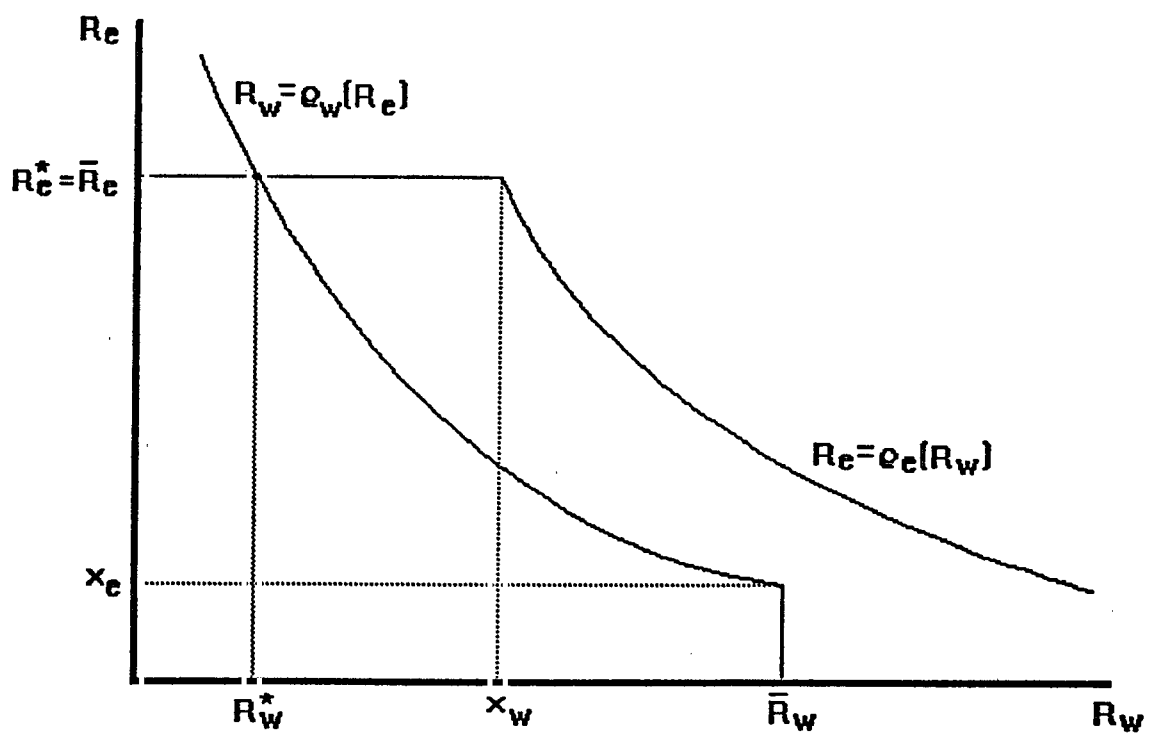


Figure 2: Pareto Distribution, Unique Equilibrium

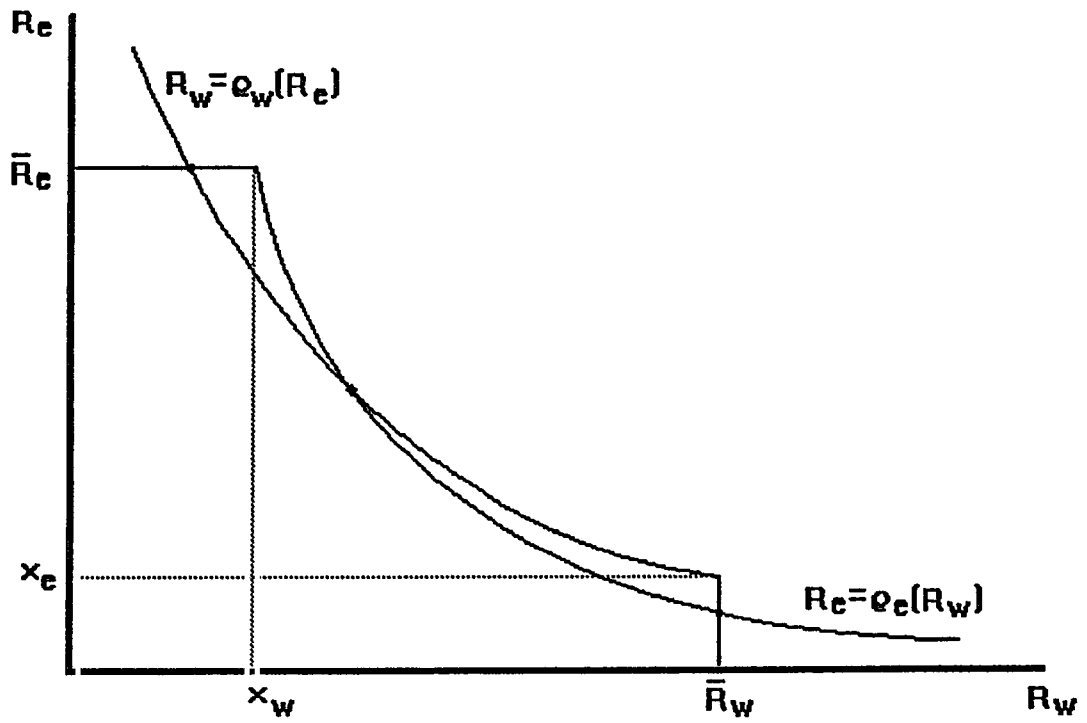


Figure 3: Pareto Distribution, Three Equilibria

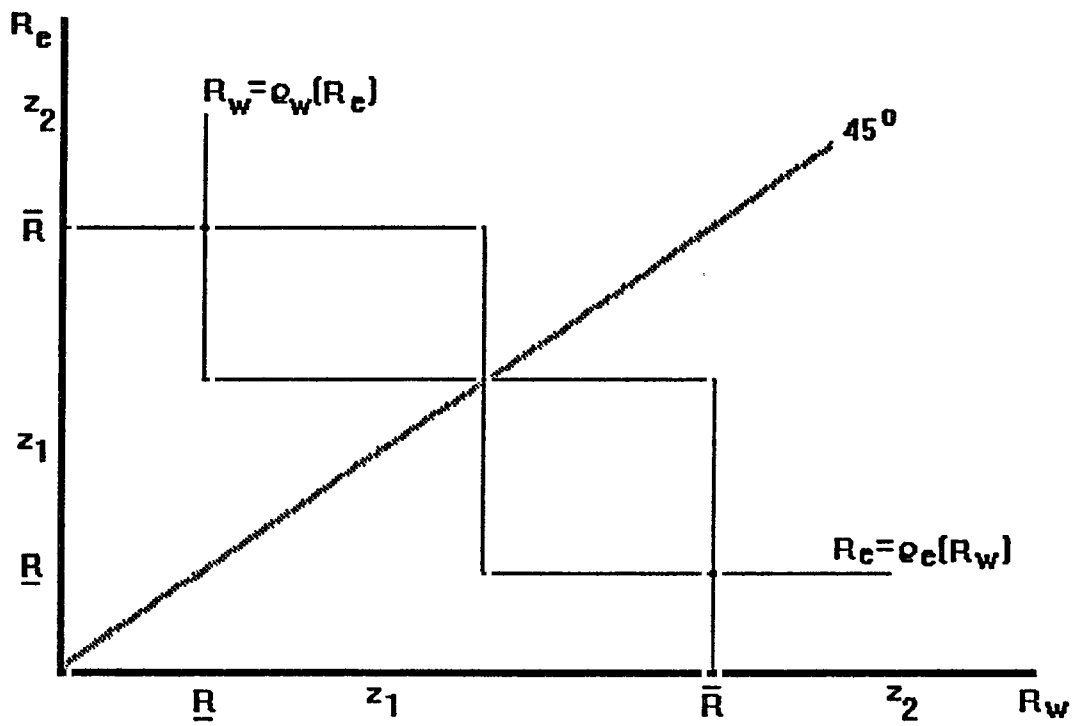


Figure 4: Discrete Distribution, Three Equilibria

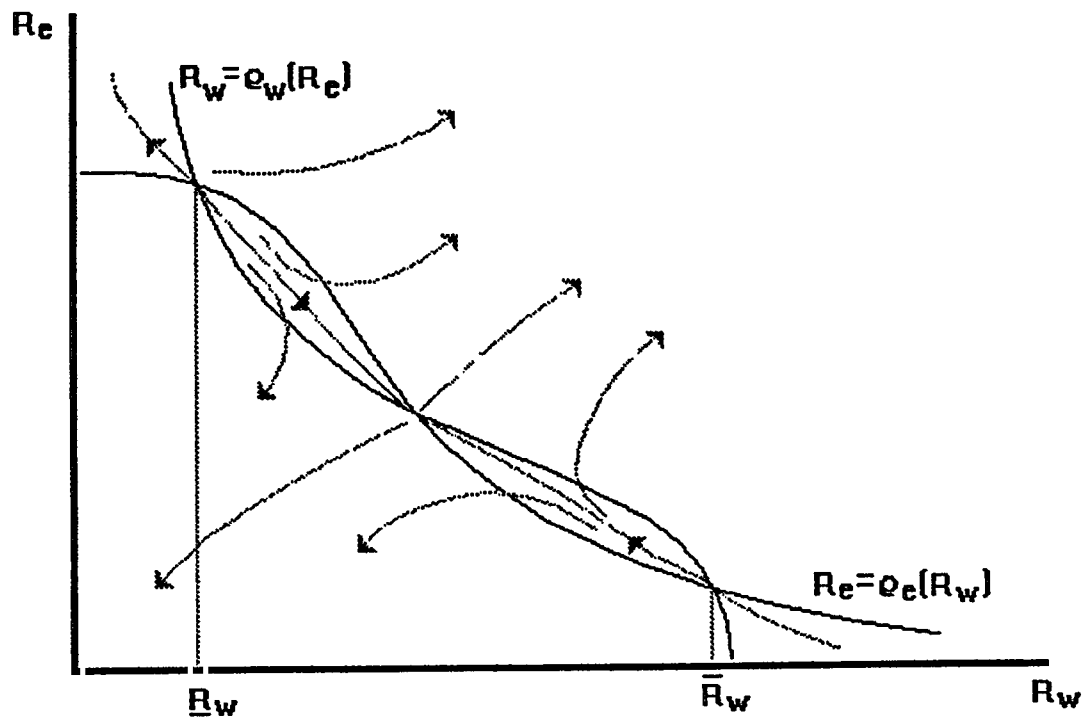


Figure 5: Dynamic Equilibria

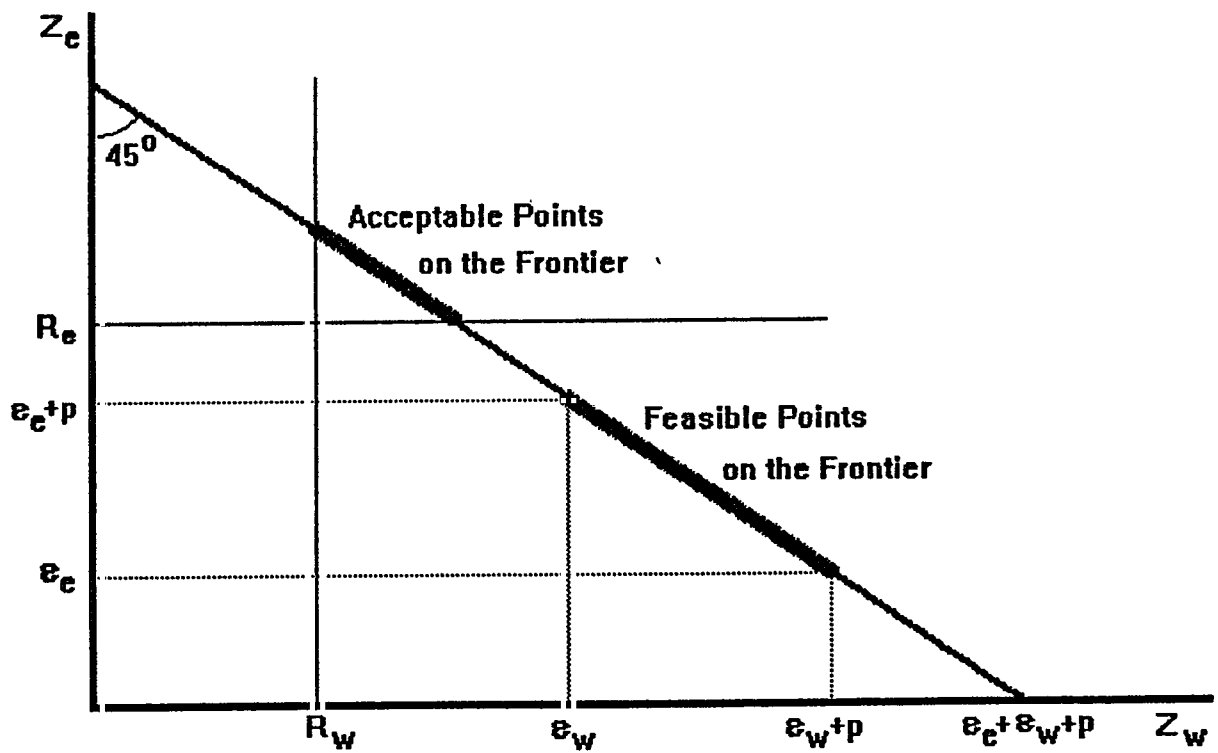


Figure 6: Bargaining with Partially Transferable Utility