Federal Reserve Bank of Minneapolis
Research Department Staff Report 179

August 1994

The Role of Institutions in Reputation Models of Sovereign Debt*

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ABSTRACT

A standard explanation for why sovereign governments repay their debts is that they must maintain a good reputation to easily borrow more. We show that the ability of reputation to support debt depends critically on the assumptions made about institutions. At one extreme, we assume that bankers can default on payments they owe to governments. At the other, we assume that bankers are committed to honoring contracts made with governments. We show that if bankers can default, then a government gets *enduring benefits* from maintaining a good relationship with bankers and its reputation can support a large amount of borrowing. If, however, bankers must honor their contracts, then a government gets only *transient benefits* from maintaining a good relationship and its reputation can support zero borrowing.

A question central to international capital markets is this: Why do sovereign governments ever choose to repay their debt? The traditional answer is that governments choose to repay because they know that if they default, lenders will be less willing to lend to them in the future. (See, for example, Eaton and Gersovitz 1981; Manuelli 1986; Grossman and Van Huyck 1988; Cole, Dow, and English 1989; and Atkeson 1991.[1]) In a provocative paper, Bulow and Rogoff (1989b) have challenged this traditional explanation. They show that "under fairly general conditions, lending to small countries must be supported by the direct sanctions available to creditors, and cannot be supported by a country's 'reputation for repayment'" (1989b, p. 43). A key reason for the difference between this result and the results in the rest of the literature is that Bulow and Rogoff assume that legal institutions are such that a government can always safely invest abroad regardless of its past behavior. The rest of the literature, either explicitly or implicitly, assumes that legal institutions are such that if a country defaults it can be completely shut off from world capital markets.[2] We develop a model that highlights how different assumptions about legal institutions lead to different results about the ability of reputation to support debt.

The model that we consider consists of two countries. One country has a large number of identical one-period-lived agents called *bankers,* and the other has only a single, long-lived agent called the *government.* At one extreme, we assume that institutions in both countries are such that no agents can commit to repaying their debts. At the other, we assume that institutions are such that commitment is one-sided: the bankers can commit to honoring their contracts, but the government cannot. For brevity, we refer to the first institutional setup as one *without Swiss bankers* and the second as one *with Swiss bankers.* Specifically, *Swiss bankers* are agents who may or may not trust anyone but who are themselves completely trustworthy. These agents will let anyone save with them at the world rate of interest and can commit to not confiscating the savings.

We begin with a complete information model and show that without Swiss bankers, trigger strategies can support large amounts of debt but that with Swiss bankers, no equilibrium has any debt. The intuition behind this stark difference is the following. In the model, in order to achieve a good outcome, the government must either safely save or borrow repeatedly over time. Under either institutional setup, maintaining a good relationship with the bankers allows the government to borrow funds repeatedly. In the economy with no Swiss bankers, the government cannot safely save since confiscating savings is optimal for bankers. In such an economy, if the relationship with the bankers turns sour (resulting in the bankers refusing to lend), then the government can neither safely save nor borrow and is stuck in autarky forever. In contrast, in an economy with Swiss bankers, if the government defaults, it may never be allowed to borrow again, but its ability to safely save with the Swiss bankers undercuts the force of any borrowing restrictions. In this setup, if there were positive debt, a government could do better by defaulting when the debt is relatively large and then saving with the Swiss bankers than it could by repaying and maintaining a good relationship with the bankers.

To investigate the robustness of our earlier results, we extend the model with complete information to a Bayesian model with a small amount of uncertainty about the government's preferences. We find that our results are robust: Without Swiss bankers, a large amount of borrowing is possible while with them, effectively, none is.

In terms of the literature on debt and default, our Bayesian model builds on the seminal work of Jaffee and Russell (1976). Within the international literature, it is related to the work of Kahn (1989), Kletzer and Wright (1990) and, especially, Eaton (1992). For a survey of this area, see the work of Eaton and Fernandez (forthcoming).

Our Bayesian model also has some features that are quite similar to the reputation models familiar from the industrial organization literature, such as the chain store model. (For more

information on the chain store model, see Kreps and Wilson 1982 or Milgrom and Roberts 1982.) First, both our model and the chain store model have a single, long-lived player and a sequence of short-lived players. Second, the way that reputation spills over across bankers in the models with and without Swiss bankers is very similar to the way that it spills over across entrants in the chain store game.

The difference between the models is the ability of reputation spillover to support good outcomes. The chain store model is a repeated game, and consequently, the relationship between the long-lived player and the short-lived players has enduring benefits. Because of this feature, reputation spillover can support good outcomes. In contrast, our model is a dynamic game, and the ability of this reputation spillover to support good outcomes is more subtle. In the Bayesian model without Swiss bankers, the inability of governments to save makes the model function similarly to a repeated game. In sharp contrast, however, in the Bayesian model with Swiss bankers, this same reputation spillover can, effectively, support only autarky. The key to understanding these results is to examine not the spillover, per se, but the type of relationship: if it has enduring benefits, then the spillover can support good outcomes; if it has only transient benefits, then the spillover cannot.

The point of our paper can be summarized as follows. Standard models of debt produce dynamic games with physical state variables instead of the repeated games common in the industrial organization literature. In these dynamic games, the role that reputation spillover can play is subtle, and it depends critically on the nature of legal institutions as well as other features of the environment. In this paper, we clarify the nature of these interactions.

1. An economy with complete information

We begin with a complete information economy that consists of two countries. One country has a number of risk neutral bankers. The other country is represented by the government, which

has access to a country-specific investment project and needs to borrow resources to fund it. We will show that whether or not borrowing can occur in an equilibrium depends critically on the nature of the institutions in the countries. In particular, we will show that if institutions are such that neither country can commit to repaying debts, then positive borrowing can be supported through standard trigger strategies. If, however, commitment is one-sided (so that bankers can commit to repaying debts, but the government cannot), then no borrowing can occur in equilibrium.

Specifically, in each period t, t = 0, ..., ∞, the economy has a consumption-capital good, which is perishable and cannot be stored during a period. Bankers are risk neutral, live for one period, have a discount factor β, and are endowed with a large amount of the consumption-capital good in each period. Without loss of generality, we suppose that each period has two bankers who are denoted j = 1, 2. The government is infinitely lived, risk neutral, and discounts the future at rate β. In each period t, an investment of x_t units in period t produces output of $A_t x_t$ units in period t + 1. Here A_t is a deterministically fluctuating productivity parameter that specifies the project's gross return. For simplicity, we assume

$$A_t = \begin{cases} A, & \text{if t is even} \\ 0, & \text{if t is odd} \end{cases}.$$ (1)

(Letting productivity fluctuate is an easy way of giving the government an incentive to borrow. This simple pattern of fluctuations makes the resulting borrowing pattern simple but is otherwise inessential.)

The project has a maximal size of 1, so $x_t \leq 1$. Throughout the paper, we will assume the discount factor satisfies

$$\beta A > 1$$ (2)

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as well as $\beta < 1$. The government is endowed with $x_0 = 0$ units of the consumption-capital good at the beginning of period 0.

A precise description of the timing of events in the model is as follows. In each period t, the government starts with new output, $A_t x_t$, and the value of debt either owed or saved, $R_t b_t$. If $b_t > 0$, then the government decides whether to pay old loans subject to the constraint

$$z_t R_t b_t \leq A_t x_t, \tag{3}$$

where $z_t = 1$ corresponds to repayment by the government and $z_t = 0$, to default. Each banker, having seen the default decision as well as the past actions of all agents, offers the government a new loan contract. Each such contract s_{t+1} is a pair (R_{t+1}, b_{t+1}) that specifies a gross interest rate R_{t+1} and a loan amount. Let S_{t+1} denote the set of loan contracts offered. In the economy with Swiss bankers, we embed the savings possibilities in S_{t+1} by having S_{t+1} include savings contracts that specify $R_{t+1} = \rho$ for any $b_{t+1} < 0$. The government then chooses some specific contract s_{t+1} and decides how much to consume, c_t, and invest, x_{t+1}, subject to a constraint on the maximal size of the project

$$x_t \leq 1 \tag{4}$$

and the budget constraint

$$c_t + x_{t+1} - b_{t+1} = A_t x_t - z_t R_t b_t. \tag{5}$$

To build intuition, let us begin by examining an economy in which institutions are such that agents in both countries can commit to repaying their loans. Competition among bankers ensures that in each period t the equilibrium gross rate of interest on loans is $R_t = \rho$, where $\rho \equiv 1/\beta$. From (2), the return on the project A is greater than ρ, and hence with such an interest rate in each odd period, the government optimally borrows to fully fund the project. Thus in each odd period, starting with period 0, the government borrows 1, invests it, and consumes 0. In the next even

period, the project yields A units of output, out of which the government repays the banker ρ, consumes the rest $A - \rho$, and borrows 0. The discounted value of utility under commitment is thus

$$(A-\rho) + \beta^2(A-\rho) + \beta^4(A-\rho) + \ldots = \frac{A - \rho}{1 - \beta^2}. \tag{6}$$

Of course, given that the government has linear preferences and that the discount factor of the government β satisfies $\beta = 1/\rho$, the timing of consumption by the government can be structured in a variety of ways to yield the same discounted value of utility.

1.1 Without Swiss bankers

Now consider an institutional setup in which agents in neither country can commit to paying their loans, that is, one without Swiss bankers. In such an economy, since the bankers are one-period-lived, they will always confiscate all savings that the government makes with them. Thus the economy is equivalent to one in which the government cannot save, so from now on, we will simply impose this constraint.

We set up and define equilibrium as follows. The history $h_t = \{[z_0, S_1, s_1, x_1, c_0], \ldots, [z_{t-1}, S_t, s_t, x_t, c_{t-1}]\}$ records past actions for the government and the bankers up to period t. A *strategy for the government* at t is a default decision $z_t(h_t)$ made at the beginning of the period together with loan contract, investment, and consumption decisions denoted $s_{t+1}(h_t, z_t, S_{t+1})$, $x_{t+1}(h_t, z_t, S_{t+1})$, and $c_t(h_t, z_t, S_{t+1})$ made after both the default decision z_t and the offer of the new set of loan contracts S_{t+1}. A strategy for each banker j = 1, 2 at t is a new loan contract $s_{t+1}^j(h_t, z_t)$. We let $S_{t+1}(h_t, z_t)$ denote the set of such loan contracts.

A *perfect equilibrium* is a set of strategies for the government and the bankers for each period t that satisfy the following conditions:

(i) For each history h_t and (h_t,z_t,S_{t+1}), given bankers' strategies from t onward and given the government's strategies from t + 1 onward, the government's strategy at t maximizes its payoff over the set of strategies that satisfy (3)–(5) and $s_{t+1}(h_t,z_t,S_{t+1}) \in S_{t+1}(h_t,z_t)$.

(ii) For each banker j, for each history (h_t,z_t), given the other banker's strategy and given the government's strategies, the contract offered $s^j_{t+1}(h_t,z_t)$ maximizes its payoffs.

When interpreting this definition, note that we impose perfection by requiring conditions (i) and (ii) to hold for all histories, including those that do not occur in equilibrium. Note that in condition (i), we require that strategies be optimal only for a one-shot deviation. As Abreu (1988) shows, this is equivalent to requiring that these strategies be optimal for all possible deviations.

One equilibrium for this economy is the *autarky equilibrium* in which the government defaults on all debts. In this equilibrium, no loans are made and the government consumes 0 in each period. We can use trigger strategies that specify reversion to this equilibrium to support better outcomes in which the government does not default. In particular, let the strategies specify that the government begins by playing the full commitment allocation. In such an allocation, in odd periods the government borrows and invests 1, ρ, and consumes 0, while in even periods it repays ρ and borrows and consumes 0. Let the bankers begin by offering the following loan contracts. In odd periods the bankers let the government borrow 1 at an interest rate ρ. Thus the contract offered in odd periods specifies $R = \rho$ and $b = 1$. In even periods the government cannot save or borrow any amount and thus $S = \phi$.

If either the government or the bankers deviate from these allocations, then both revert to the autarky allocations. We claim that with sufficiently little discounting by the government, such strategies support the full commitment allocations. More precisely, we prove the following:

PROPOSITION 1. In the economy without Swiss bankers, there exists a $\underline{\beta}$ such that for all $\beta \in (\underline{\beta},1)$, the full commitment allocations are perfect equilibrium outcomes.

Proof. Consider histories in which no deviations have occurred up until some odd period t. For such histories, the government's strategies specify to repay new loans, and thus bankers find lending to be optimal. Competition among the bankers ensures that a loan of size 1 with an interest rate of ρ is offered. For such histories, the government finds that borrowing 1 to fully fund the project is optimal. Consider the government's repayment decision in period t + 1. If it repays the loan of 1 and continues with the full commitment allocations, it will consume $A - \rho$ at t + 1, t + 3, and so on. If it defaults, it will receive A units in period t + 1. After period t + 1, however, it cannot borrow from bankers, and if it ever saves with them, the bankers will confiscate the government's savings. Hence, if it defaults, the government will receive A at period t + 1 and 0 thereafter. Thus a government will repay if

$$\frac{A - \rho}{1 - \beta^2} \geq A. \tag{7}$$

Thus for all $\beta \geq \underline{\beta} = (\rho/A)^{1/2}$, the government will repay. Notice that with $\beta = 1/\rho$, this condition is equivalent to $\beta^3 A \geq 1$.

Now for histories for which deviations have occurred, the autarky strategies are optimal. Thus the trigger strategies constitute a perfect equilibrium if $\beta \geq \underline{\beta}$. $\square$

1.2 With Swiss bankers

Next, consider the institutional setup in which bankers can commit to repaying their debts but the government cannot, that is, one with Swiss bankers. The formal definition of an equilibrium is still the same as before, except that the bankers always offer a full menu of savings contracts of the form

$$\{(R,b)\,|\,R = \rho,\, b \leq 0\}, \tag{8}$$

in addition to the borrowing contract. Under such an institutional assumption, the full commitment allocations cannot be supported as equilibrium allocations, regardless of the discount factor.

To see this, consider the full commitment allocation and consider the decision to repay in some even period t. If the government repays at t, it gets $A - \rho$ at t, $A - \rho$ at t + 2, and so on. Consider the following deviation. Suppose instead that it defaults at t. After defaulting, it has A units of output, out of which it consumes $A - 1/\rho$ units and saves $1/\rho$ units with a Swiss banker. In period t + 1, an odd one, the Swiss banker safely returns 1 to the government and the government fully funds the project. In period t + 2, the project yields A, the government consumes $A - 1/\rho$ and saves $1/\rho$ with the Swiss banker, and so on. This deviation yields $A - 1/\rho$ in all even periods, while if the government continues with the full commitment allocations, it yields only $A - \rho$ in even periods. Since $\rho > 1$, the derivation is strictly preferred for all discount factors $\beta \in (0,1)$. Thus in the economy with Swiss bankers, the full commitment allocations cannot be supported as equilibrium allocations.

The intuition is simply that once the government has 1 unit on hand, it has no need to borrow any more, and thus the value of maintaining a good relationship with the bankers is 0. Moreover, if it breaks this relationship by defaulting, it saves the funds it owed, and thus defaulting dominates maintaining the good relationship. More generally, in the spirit of Bulow and Rogoff, we can prove the following:

PROPOSITION 2. In the economy with Swiss bankers, the unique equilibrium allocations are the autarky allocations.

Proof. The proof is by contradiction. Competition among bankers guarantee that they break even on any loan, so

$$(R_t z_t - \rho) b_t = 0, \tag{9}$$

which means that the government earns the market rate on both loans and savings. Therefore, if any loans are made, the gross interest rate is ρ; that is, if $z_t = 1$ and $b_t \neq 0$, then $R_t = \rho$. If $z_t = 0$, then no loans are made, so $b_t = 0$. Clearly, b_t cannot be greater than or equal to $1/\rho$ in any equilibrium. If it were, then the government would certainly prefer to deviate by defaulting on the amount owed, ρb_t, and then consuming $\rho b_t - 1/\rho$ in extra consumption at date t and saving $1/\rho$. In all future odd periods, it would use the payoff from its savings to fully fund the project. In all future even periods, it would consume $A - 1/\rho$ and save $1/\rho$. Since b_t is bounded in equilibrium, then

$$\lim_{t \to \infty} \beta^t b_t = 0. \tag{10}$$

Next, we show that b_t cannot be any strictly positive number between 0 and 1. By way of contradiction, suppose that at some date, say, date v, $b_v > 0$. Let

$$\beta^r b_r = \max_t \beta^t b_t. \tag{11}$$

Thus r is the date at which the present value of borrowing is the largest. Clearly, r is finite since $b_t \leq 1$ for all t. If multiple dates satisfy (11), then let r be the largest such date. Consider, for now, the government deviating at date r by defaulting at r and then saving at rate ρ the funds it would have been repaying the bankers and using those funds to self-finance the original consumption levels and investment. Specifically, new debt, consumption, and investment levels $\hat{b}_t$, $\hat{c}_t$, and $\hat{x}_t$ satisfy, for t > r,

$$\beta^t \hat{b}_t = \beta^t b_t - \beta^r b_r, \tag{12}$$

$$\hat{c}_t = c_t, \quad \hat{x}_t = x_t.$$

Notice that (12) simply states that the present value of the new debt sequence equals the present value of the original debt sequence minus the present value of the defaulted-on debt. Of course, we can also write this in time t units as

$$\hat{b}_t = b_t - \rho^{t-r} b_r, \quad \text{for } t \geq r, \tag{13}$$

so that the new debt sequence equals the original one minus the rolled-forward value of the defaulted-on debt.

To show this deviation is feasible, we must show that the new debt sequence, $\hat{b}_t$, is nonpositive and that at the original consumption and investment allocations, the following hold:

$$c_t + x_{t+1} - \hat{b}_{t+1} - A_t x_t + \rho \hat{b}_t = 0 \tag{14}$$

$$\rho \hat{b}_t \leq A_t x_t. \tag{15}$$

Clearly, $\hat{b}_t$ is nonpositive from the definition of date r. And $\hat{b}_t < b_t$, so (15) holds. To see that (14) holds, note that from (13)

$$-\hat{b}_{t+1} + \rho \hat{b}_t = -(b_{t+1} - \rho^{t+1-r} b_t) + \rho(b_t - \rho^{t-r} b_r) = -b_{t+1} + \rho b_t.$$

So (14) holds, since the budget constraint held at the old allocations. Thus this deviation, which makes the government as well off as the original allocation, is feasible.

To show that the agent can be made strictly better off, note that under our deviation

$$\lim_{t \to \infty} \beta^t \hat{b}_t = \lim_{t \to \infty} \beta^t b_t - \beta^r b_r = -\beta^r b_r.$$

Clearly, at some sufficiently large date, consumption can be increased while the rest of allocation is unaffected. □

The intuition behind this proposition is similar to the intuition behind why the full commitment allocations are not supportable as equilibrium allocations. Consider any equilibrium and

consider the period in which the present value of debt owed by the government is maximal. Since this value of debt is the largest it will ever be, then in each period after this, the government is paying back the bankers on net. If the government instead defaults and saves with the bankers, it can finance its original investment pattern and increase its consumption.

The intuition for why the economies with and without Swiss bankers function so completely differently is as follows. In setups without Swiss bankers, if the relationships with bankers turn sour, the government can neither borrow nor save and is thereafter stuck in autarky. For a sufficiently patient government, the present value of this loss is enormous, and it gives the government a big incentive to maintain a good relationship with bankers by repaying. Bankers understand this incentive, and they lend the full commitment amount to the government. In setups with Swiss bankers, the equilibrium is quite different. Even if the relationships with bankers turn sour, the government can always safely save with Swiss bankers and earn the market rate of return. This savings option undercuts the force of any borrowing restrictions. Indeed, in any conjectured equilibrium with positive debt, at the date where the debt has the highest present value, the value of maintaining a good relationship with bankers is 0: If the government defaults, it can use the savings option to finance the future investment projects just as well as it could have by borrowing. Moreover, by defaulting, it saves the cost of paying back the bankers. Bankers understand this lack of incentive by the government to pay them back, and they refuse to lend to the government in the first place.

2. An economy with incomplete information

We now consider a reputation model with two types of governments. We will show that, just like the economy with complete information, the nature of institutions is critical in determining the borrowing pattern of the equilibrium. Indeed, with no Swiss bankers, long-enough time horizons,

and high-enough discount factors, a borrowing pattern that is essentially the full commitment pattern can be supported. We then show that with Swiss bankers, anything close to the full commitment level of borrowing is impossible to support. Moreover, there is a sense in which the level of borrowing is effectively 0. This institutional setup, then, determines the borrowing possibilities in a way that is similar to the way it did in section 1.

Both of the economies we consider have the following two types of governments: a normal government and an honest government, denoted $i = n, h$. The *normal* government is risk neutral and discounts the future at rate β. The *honest* government evaluates consumption streams the same way the normal government does, but the honest government also assigns a large disutility to breaking any contract it has signed. In particular, we can write the preferences of the honest government as

$$\sum_{t=0}^{T} \beta^t c_t - (1-z_t)M, \qquad (16)$$

where M is some large, positive number. (Recall that $z_t = 1$ corresponds to repayment and $z_t = 0$, to default.) The type of the government is private information. Bankers hold subjective beliefs about what type the government is, and they update these beliefs after seeing the actions of the government. (This model of different types of borrowers formalizes in a game theory context some of the ideas in the early model of Jaffee and Russell 1976.)

We find that in the incomplete information context, an equilibrium in pure strategies for the lenders may not exist [essentially for reasons similar to those on the debt model of Jaffee and Russell (1976)]. The existence of an equilibrium can be ensured in two easy ways: Allow mixed strategies for bankers or change the timing slightly. We choose the latter option here. Specifically, we assume that bankers first choose the interest rates they will offer and then the loan sizes. This timing allows bankers to condition their loan sizes on their competitor's interest rate offer.

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The notation for the strategies of the normal and honest governments are the same as the strategy for the government in the incomplete information game, except that now a probability of repayment exists, denoted $\sigma_t^i(h_t)$, $i = n, h$. Given the new timing assumption, the notation for the strategies of the bankers is similar to that for the complete information game, except that now for each history h_t, bankers also have a common belief $p_t(h_t)$ that gives the probability of the government being honest, conditional on the history.

A *perfect Bayesian equilibrium* is a set of strategies of the normal and honest governments and a set of strategies and beliefs of the bankers that satisfy the following conditions:

(i) For each history, given the strategies and beliefs of the bankers from t onward and given the strategies of the governments from t + 1 onward, the strategies of the governments at t maximize their payoffs among the set of strategies that satisfy (3)–(5) and $s_{t+1}^i(h_t,z_t,S_{t+1}) \in S_{t+1}(h_t)$ for $i = h, n$.

(ii) For each history, given the strategies of the governments, the strategy of the other banker at t, and banker j's beliefs, banker j's strategy maximizes its payoffs.

(iii) Bankers' beliefs are updated according to Bayes' rule so that if $z_t = 1$, then

$$p_{t+1}(h_{t+1},z_{t+1}) = \frac{p_t(h_t,z_t)}{p_t(h_t,z_t) + (1 - p_t(h_t,z_t))\sigma_t^n(h_t)}.$$

Consider the setup without Swiss bankers, in which agents in neither country can commit to repaying their debts. Notice that the government has no way to transfer resources across odd periods, and thus the model is essentially reduced to a repeated game. It works almost exactly like the repeated game models of Kreps-Wilson (1982) and Milgrom-Roberts (1982) on the chain store paradox. In particular, for any fixed prior—no matter how small—our assumption (2) on the discount factor implies that as the time horizon gets longer, the discounted value of utility converges

to the value under full commitment and the initial level of borrowing converges to the level under full commitment. (See the working paper version of this paper for details.)

More interesting is the setup with Swiss bankers who can commit to repaying debts. For convenience later, we will define the *break-even interest rate* to be the rate $\bar{R}_t$ on a loan such that if both types of governments take the loan, then the expected profits to bankers are 0. Thus

$$-1 + \beta \bar{R}_t \big[p_t(h_t) \sigma_t^h(h_t) + (1 - p_t(h_t)) \sigma_t^n(h_t) \big] = 0,$$

or dropping dependence on histories and solving for $\bar{R}_t$ gives

$$\bar{R}_t = \frac{\rho}{p_t \sigma_t^h + (1-p_t)\sigma_t^n}.$$

We construct an equilibrium starting in the last period T.

Period T. By convention, T is odd (so $A_T = A$). The strategy of the honest government is as follows: If it has enough output to repay its loans, it will do so and consume the remainder. If not, it will default and consume all it has. Of course, in equilibrium, the honest government will never choose to borrow so much that it is forced to default, so from now on, we ignore this possibility.

Clearly, the normal government will default if it owes the bankers any positive amount, and it will consume the output realized from last period's investment x_T^n and any savings it may have. Thus $\sigma_T^n(h_T) = 0$, if $R_T b_T > 0$, and 1 otherwise, while $c_T^n(h_T) = Ax_T + \max\{0, -R_T b_T\}$. This yields the value for the normal government

$$V_T^n(h_T) = Ax_T + \max\{0, -R_T b_T\}.$$

Period T − 1. This is an investment period (since $A_{T-1} = 0$ and $A_T = A$). Consider the choice of a new loan (R_T, b_T) by an honest government with current debt (or savings) $R_{T-1} b_{T-1}$.

If it takes such a loan and $b_T - R_{T-1}b_{T-1} < 1$, then it would invest all its funds and in the next period consume $A(b_T - R_{T-1}b_{T-1}) - R_Tb_T$. If it takes such a loan and $b_T - R_{T-1}b_{T-1} > 1$, then it would invest 1 in the project, roll over the rest, and in the next period consume $A - R_Tb_T + \rho(b_T - R_{T-1}b_{T-1} - 1)$. Faced with multiple loans, it chooses the one that maximizes its consumption. The normal government intends to default in the next period and therefore chooses the contract with the larger loan amount. We assume that among contracts with the same loan size, it chooses the one with the smaller interest rate.

The behavior of the bankers is rather delicate. To appreciate this delicateness, suppose we had assumed that bankers choose both interest rate and loan sizes simultaneously. Suppose the first banker offers a contract (R_1, b_1) that attracts both the normal and the honest government. If the contract makes strictly positive profits for the first banker, then the second banker will simply undercut it a little. Now suppose it makes 0 profits when it attracts both types of governments. The second banker can easily structure a contract with a (substantially) lower interest rate and a (slightly) lower loan size that attracts only the honest government away from the first contract. Such a defection by the honest government makes the first contract unprofitable, and thus the original contract is not an equilibrium one. If we continue with this logic, we can easily see that no equilibrium exists in pure strategies.

Our timing assumption resolves this nonexistence problem by allowing bankers to condition the loan sizes on interest rate offers. To see this, suppose the prior satisfies $p_T \geq \rho/A$ and $\bar{R}_{T-1}b_{T-1} = 0$. Suppose the first banker offers an interest rate, say, $\bar{R}_T$, that will just break even if the subsequent loan attracts both types of governments. Let the first banker offer the following loan size depending on how the other banker's rate R_T compares to the break-even rate $\bar{R}_T$, namely,

$$b_T(R_T) \ = \ \begin{cases} 1, & \text{if } R_T > \bar{R}_T \\ b_T, & \text{if } R_T = \bar{R}_T \\ 0, & \text{if } R_T < \bar{R}_T \end{cases}.$$

Now suppose the second banker tries a strategy similar to the one above: it offers a lower rate, $R_T < \bar{R}_T$ and hopes to offer a loan size such that it only attracts the honest government. Here the first banker simply responds by offering a loan size of 0. If the second banker offers any positive loan size, it attracts the normal government, as well as the honest one, and will lose money; thus it also offers 0.

Likewise, suppose the second banker offers a higher rate $R_T > \bar{R}_T$ and hopes to offer a loan size larger than that offered by the first banker that will attract both types and make a positive profit. Here the first banker responds by offering 1 whenever $R_T > \bar{R}_T$. The second banker's best response is to offer $b \le 1$, and thus he attracts neither type. Thus for $p_T \ge \rho/A$, a whole continuum of other equilibria exists in which both bankers offer an interest rate that breaks even if both types are attracted and then offer any loan size from 0 up to 1. More generally, if $R_{T-1}b_{T-1} \le 0$, then the same argument shows that any break-even interest rate together with a loan size between 0 and b_T that satisfies $b_T - R_{T-1}b_{T-1} = 1$ (so the government has total funds of 1) is an equilibrium.

Suppose, next, the prior satisfies $p_T < \rho/A$. The rate the banker must charge to break even on a loan taken by both types of governments is $R_T = \rho/p_T > A$. But the honest government will not take such a loan, and thus bankers offer no loans, since only the normal government will take them.

If we combine the strategies of the bankers and the governments, we have five regions of the equilibrium that depend on the prior p_{T-1} and the outstanding debt (or savings) $R_{T-1}b_{T-1}$:

- Region 1. $R_{T-1}b_{T-1} \geq -1$. The honest government will not borrow since it can fully fund the project with funds on hand. Bankers know that any loans they make will be defaulted on with probability 1, and so they offer none. Both types of government invest 1 and consume the remainder, yielding a payoff $-R_{T-1}b_{T-1} - 1 + \beta A$. Since no opportunity to default exists, the new prior is simply the old one, $p_T = p_{T-1}$.

- Region 2. $R_{T-1}b_{T-1} > -1$, and $p_{T-1} \geq \rho/A$. A continuum of equilibria exists. In all of them, the interest rate offered on loans is the break-even rate, $\bar{R}_{T-1} = \rho/p_{T-1}$. The equilibria are indexed by the larger of the two loans made by the bankers. The loan size varies from 0 up to the amount that gives the government total funds of 1, namely, $b_T = 1 + \min(0, \bar{R}_{T-1}b_{T-1})$. In each such equilibrium, both the honest and normal government take the loan with the higher debt level and invest these funds plus any savings they have. In the next period T, the honest government repays and the normal government defaults.

- Region 3. $R_{T-1}b_{T-1} > -1$, and $p_{T-1} < \rho/A$. Bankers make no loans for the reasons discussed above. The government invests all of its savings, $\max(0, -R_{T-1}b_{T-1})$, and gets a payoff of $\beta A \max(0, -R_{T-1}b_{T-1})$. The prior is updated as $p_T = p_{T-1}$.

Period T − 2. This is a repayment period in which output is realized ($A_{T-2} = A$). Some decisions are straightforward. Investment is 0 since the project is not productive in the next period. Bankers offer only savings contracts at rate $R_{T-2} = \rho$. The honest government will repay its loans if it has the resources to do so; otherwise it defaults. After its repayment decision, the honest government saves everything. The normal government also saves everything. The interesting decision, however, is the repayment decision of the normal government.

Trivial, out-of-equilibrium regions exist in which the government saved when it should have invested and in which it has more debt than it can repay, given its resources. We describe the five nontrivial regions here, each of which has $0 \leq R_{T-2}b_{T-2} \leq Ax_{T-2}$.

- Region 1. $Ax_{T-2} \geq 1/\rho$. The normal government has enough resources so that if it defaults on its debt and saves $1/\rho$, it will have 1 at $T - 1$ and can fully fund the project then. It does so, consuming $Ax_{T-2} - 1/\rho$ at $T - 2$ and A at T. The honest government repays its loan, saves up to $1/\rho$, and consumes any remaining resources. Banks update their priors, setting $p_{T-1} = 1$ if repayment is made.

- Region 2. $Ax_{T-2} < 1/\rho$, and $p_{T-2} \geq \rho/A$. Here the prior is high enough so that if $p_{T-1} = p_{T-2}$, then the government will be in Region 2 at $T - 1$, which has a continuum of equilibria. We select the continuation equilibrium with the maximal loan size, 1, at $T - 1$. If the normal government defaults at $T - 2$, it would save Ax_{T-2} and invest ρAx_{T-2} at $T - 1$, giving it a payoff of $\beta^2(\rho A)Ax_{T-2}$. If it repays, borrows 1 at $T - 1$, and defaults at T, its payoff is higher, namely, $\beta^2 A$. Thus it repays with probability 1. The honest government repays, saves the remainder, and at $T - 1$ borrows enough so that its total funds are 1. Priors are updated by $p_{T-1} = p_{T-2}$.

We next consider three regions with $p_{T-2} < \rho/A$. Notice that the prior is such that if $p_{T-1} = p_{T-2}$, then no loans will be made at $T - 1$ and the normal government will prefer to default at $T - 2$ since there are no benefits to repaying. More interesting is a situation in which, if the normal government mixes and actually ends up repaying, the prior at $T - 1$ will be pushed up to be $p_{T-1} \geq \rho/A$. If so, then the anticipation of new loans at $T - 1$ may give the government an incentive to repay at $T - 2$. Recall that the region at $T - 1$ with $p_{T-1} \geq \rho/A$ is the one with the continuum of equilibria. As we will see, for an equilibrium to exist, we must delicately specify the continuation equilibria. We begin with the most delicate region at $T - 2$.

- Region 3. $Ax_{T-2} < 1/\rho$, and $(\rho/A)^2 \leq p_{T-2} \leq \rho/A$. The equilibrium here depends on how we select the continuation equilibrium from the continuum of equilibria in region 2 of period

T − 1. To get a feel for how this works, suppose we assume that the loan at T − 1 depends only on the "natural" state variables at T − 1: savings $R_{T-1}b_{T-1}$ and the prior p_{T-1}. Now suppose the prior updating rule is such that if the government repays at T − 2, its prior is pushed up to be $p_{T-1} \geq \rho/A$. Also suppose it can borrow, say, b_T, such that its total funds are 1. Then if it repays at T − 2, borrows the maximum at T − 1, invests it, and then defaults at T, its payoff is $\beta^2 A$. If it instead defaults at T − 2, saves its funds, and then invests them at T − 1, it gets a smaller amount $\beta^2(\rho A)(Ax) < \beta^2 A$. Thus the government strictly prefers to repay, and so our conjectured updating rule is not consistent with the resulting strategies. Suppose we instead conjecture that the updating rule specifies that if the government repays, the prior is not pushed up to be above ρ/A. If the normal government repays, saves all its funds, and invests them at T − 1, it gets a payoff of $\beta^2(\rho A)(Ax_{T-2}-R_{T-2}b_{T-2})$. If it defaults, saves its funds, and invests them, it gets a larger amount, namely, $\beta^2(\rho A)(Ax_{T-2})$. But then the government prefers to default with probability 1, and the prior updating rule is again inconsistent with the resulting strategies. Thus no equilibrium of this type exists.

We can easily show that no matter what level of debt we choose for the equilibria at T − 1, if it only depends on the natural state variables, then no equilibria at T − 2 exists. A Markov-type equilibrium does exist only if we allow the equilibria chosen at T − 1 to depend on extra information, namely, the value of inherited debt, $R_{T-2}b_{T-2}$. (For an early example of the nonexistence of a Markov-type equilibrium, see Peleg and Yaari 1972; for a discussion of how to ensure the existence of a Markov-type equilibrium by extending the state space, see Fudenberg and Tirole 1991.)

Specifically, let the maximum debt offered at T − 1 in region 2 be $b_T = \rho R_{T-2}b_{T-2}$. Now suppose that at T − 2 the updating rule is such that if the government repays, the prior at T − 1 is, say, equal to ρ/A. Then if the government repays, saves $Ax_{T-2} - R_{T-2}b_{T-2}$, and borrows the

maximum amount at $T - 1$, it gets a payoff $\beta^2(\rho A)(Ax_{T-2})$. If it instead defaults, saves its funds Ax_{T-2}, and then invests them, its payoff is the same. To solve for the mixing probability σ^n_{T-2}, recall Bayes' rule

$$p_{T-1} = \frac{p_{T-2}}{p_{T-2} + \sigma^n_{T-2}(1-p_{T-2})}.$$

So if the updated prior is $p_{T-1} = \rho/A$, then the mixing probability of the normal government σ^n_{T-2} solves

$$\frac{\rho}{A} = \frac{p_{T-2}}{p_{T-2} + \sigma^n_{T-2}(1-p_{T-2})}.$$

This is an equilibrium for this region. Others exist in which the prior is pushed up strictly above ρ/A, but the behavior is similar.

- Region 4. $Ax_{T-2} > 1/\rho$, and $(\rho/A)^2 \leq p_{T-2} \leq \rho/A$. Here the normal government strictly prefers to default, and no loans are made.

- Region 5. $p_{T-2} < (\rho/A)^2$. Recall that the normal government's only incentive to repay is its anticipation of being able to borrow at $T - 1$. For this to be the case, the government must mix with a low-enough probability so that $p_{T-1} \geq \rho/A$, with the resulting break-even rate R_{T-1} being greater than A. With a prior this low, no such mixing probability exists and no loans are made.

Period $T - 3$. This is an investment period (since $A_{T-3} = 0$ and $A_{T-2} = A$). Trivial, out-of-equilibrium regions exist in which the governments borrowed, but we will ignore them. Four nontrivial regions exist with $R_{T-3}b_{T-3} < 0$ that depend on the values of $(R_{T-3}b_{T-3}, p_{T-3})$.

- Region 1. $R_{T-3}b_{T-3} \leq -1$. Governments have more than enough resources to fully fund the project and so do not borrow. They invest 1 unit and consume the rest.

- Region 2. $R_{T-3}b_{T-3} > -1$, and $p_{T-3} \geq \rho/A$. As in region 2 of period $T - 1$, a continuum of equilibria exists. The loan size varies from 0 up to the amount that gives the government total funds of 1. The normal government borrows, and the interest rate $R_{T-3} = \rho/p_{T-3}$. In terms of developing a value function, we concentrate on the equilibrium with the largest loan size.

- Region 3. $R_{T-3}b_{T-3} > -1$, and $(\rho/A)^2 \leq p_{T-3} \leq \rho/A$. The equilibrium loan size is such that the government has total funds of $1/\rho A$. The interest rate is consistent with the mixing probability on repayments in period $T - 2$ in region 3.

- Region 4. $R_{T-3}b_{T-3} > -1$, and $p_{T-3} < (\rho/A)^2$. The equilibrium loan size is 0, and governments self-finance the project. The value function in this period is summarized by

$$V_{T-3}^n(b_{T-3}) = \begin{cases} 1 - R_{T-2}b_{T-2} + \beta(A-1/\rho) + \beta^3 A, & R_{T-3}b_{T-3} \geq -1 \\ \beta(A-1/\rho) + \beta^3 A, & R_{T-3}b_{T-3} > -1 \ \& \ p_{T-3} \geq \rho/A \\ \beta^3 A, & R_{T-3}b_{T-3} > -1 \ \& \ (\rho/A)^2 \leq p_{T-3} \leq \rho/A. \\ \beta(-AR_{T-3}b_{T-3}-1/\rho) + \beta^3 A, & R_{T-3}b_{T-3} < -1/\rho A, \ p_{T-3} < (\rho/A) \\ -\beta^3(\rho A^2 R_{T-3}b_{T-3}), & R_{T-3}b_{T-3} > -1/\rho A, \ p_{T-3} \leq (\rho/A)^2 \end{cases}$$

Period 0. The analysis continues in a similar fashion all the way back to period 0. An analysis of all the regions in period 0 is very tedious. In period 0 for the lending regions, however, we can easily determine that if the initial prior p_0 and the initial capital stock x_0 satisfy $(\rho/A)^{k+1} \leq p_0 \leq (\rho/A)^k$ and $x_0 < (1/\rho A)^k$ for some integer k, then if $2k+1 < T$, the value function is

$$V_0(x_0,p_0) = \beta^{2k+1}(A-1/\rho) + \beta^{2k+3}(A-1/\rho) + \ldots + \beta^{T-2}(A-1/\rho) + \beta^T A. \tag{17}$$

If $2k+1 = T$, the value function is $\beta^T A$, and if $2k+1 > T$, the value function is zero. The initial borrowing level is $b_0 = 1/(A\rho)^k$ if $2k+1 \leq T$ and 0 if $2k+1 > T$.

The borrowing patterns and utility levels in the private information economies are quite similar to those in the complete information economies of section 1. As we have mentioned, in the economy without Swiss bankers, for any initial prior, the time horizon is long enough so that the utility level and initial borrowing level are arbitrarily close to the full commitment levels. A more precise way to state this result is the following: Consider a sequence of economies without Swiss bankers in which the prior shrinks to 0 as the horizon length grows to infinity. If the prior shrinks at a slow-enough rate, then the discounted value of utility and the initial borrowing level converge to those obtained under full commitment. In contrast, in any sequence of economies with Swiss bankers, positive discounting, and the horizon length converging to infinity, if the prior shrinks to 0, then the discounted value of utility and the initial borrowing level both converge to 0.

More precisely, we prove the following:

PROPOSITION 3. Consider a sequence of economies indexed by the horizon length T with $\beta < 1$ in which the initial prior $p_0(T)$ converges to 0 as T converges to infinity. With Swiss bankers, the equilibrium discounted value of utility converges to the autarky level of 0 and the initial borrowing level converges to the autarky level of 0.

Proof. Given a sequence $p_0(T)$ that converges to 0, for each T, define k(T) to be that integer such that

$$(\rho/A)^{k(T)+1} \leq p_0(T) < (\rho/A)^{k(T)}.$$

Note that if k(T) converges to infinity as T does, then we are done. To see this, recall that with Swiss bankers, the discounted value of utility has 0 terms for the first k periods and that the initial borrowing level is $b_0 = 1/(\rho A)^k$ and $\rho A > 1$. Since $\rho/A < 1$ as $p_0(T)$ converges to 0, k(T) converges to infinity. □

The intuition for why the economies with and without Swiss bankers function so differently is similar to the intuition given at the end of section 1. In the setup without Swiss bankers, if the government defaults, its bad reputation spills over to each of the subsequent bankers, and the government is stuck in autarky until the end of the game. For long time horizons, this has an enormous cost and the government has a big incentive to repay. Bankers realize this incentive is there, and even if the government's prior probability of being honest is quite low, they are willing to lend it nearly the full commitment amount.

In contrast, in the economy with Swiss bankers, if the government borrows more than a trivial amount in the early periods, it can do quite well by defaulting, safely saving with the Swiss bankers, and gradually increasing the level at which it runs the project. Thus, as before, adding the Swiss bankers introduces the option of safely saving and undercuts the borrowing restrictions. Bankers realize that the government has little incentive to repay, and they are willing to lend it only a trivial amount. If, over time, the government mixes over defaulting and repaying, and the realizations are such that it keeps repaying, the bankers become more and more convinced that they are dealing with an honest government and are willing to lend more funds. But this takes so long that in present-value terms, the government is not much better off than it is under autarky. Indeed, for smaller and smaller priors, the government's payoff eventually converges to the autarky payoff, even if the horizon is simultaneously increasing. This result clearly shows that while the economy without Swiss bankers functions similarly to the class of economies considered in the chain store literature, the economy with Swiss bankers functions completely differently.

3. Conclusion

In this paper we have shown how the ability of reputation to support debt depends crucially on the assumptions made about government's ability to safely save. Here we have interpreted this assumption to mean that a legal institution exists which allows foreign bankers to commit to repaying their loans. Of course, in this simple environment, we could instead interpret this assumption as one on technology. Indeed, if the government has some way to safely save at ρ by investing in its own country, then we get the same result as we do in a setup with Swiss bankers. However, suppose we consider a more general setup in which domestic consumers are risk averse and all domestic technologies have stochastically fluctuating productivities with a country-specific component. Then in order to diversify away some of the country-specific risk, the country needs more than just access to the domestic savings technologies: it needs to be able to safely invest in world capital markets. Thus in more elaborate models, the institutional interpretation of the assumption is more compelling then the technology interpretation.

One question that comes to mind is this: Can reputation support debt under the Swiss banker assumption? In our paper, Cole and Kehoe 1992, we argue that if misbehavior in the debt relationship spills over to tarnish the reputation in another enduring benefit relationship, then reputation can indeed support debt.

Footnotes

[1]For some interesting nonreputation-based models of debt and default, see Calvo 1988, Bulow and Rogoff 1989a, and Fernandez and Rosenthal 1990.

[2]Across countries and time, a whole myriad of legal institutions and codes have evolved. Moreover, even within a particular legal structure, substantial uncertainty seems to exist among economists about the effective regulations and sanctions that that structure entails in practice. (See Alexander 1987, Bradlow and Jourdin 1984, and Bulow and Rogoff 1989a for a discussion of the various legal institutions and for case studies of how the institutions actually function.) In terms of modeling the legal structure in a model of sovereign borrowing, we cannot clearly identify from the data which of the different assumptions used in the literature is the most appropriate. Indeed, maybe no single set of assumptions on legal institutions can well approximate the variety seen in practice across countries and time.

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