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Pattern Bargaining

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ABSTRACT

Many unions in the United States have for several years engaged in what is known as pattern bargaining—a union determines a sequence for negotiations with firms within an industry where the agreement with the first firm becomes the take-it-or-leave-it offer by the union for all subsequent negotiations. In this paper, we show that pattern bargaining is preferred by a union to both simultaneous industrywide negotiations and sequential negotiations without a pattern. In recent years, unions have increasingly moved away from patterns that equalized wage rates across firms when these patterns did not equalize interfirm labor costs. Allowing for interfirm productivity differentials within an industry, we show that for small interfirm productivity differentials, the union most prefers a pattern in wages, but for a sufficiently wide differential, the union prefers a pattern in labor costs.

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1 Introduction

In many industries in the United States, a large part of the workforce is represented by a nationally organized union. The United Auto Workers (UAW) and the International Brotherhood of Teamsters are two examples. At the time of contract renegotiation, many unions engage in a process which is referred to as *pattern bargaining*. Annual surveys over the past decade indicate that approximately 25 percent of all manufacturing and nonmanufacturing employers in the country who participated in collective bargaining agreements intended to bargain a pattern contract within the next year.[1] These surveys include a broad range of industries and firms. Many of the industries are highly unconcentrated, such as agriculture. In automobile assembly, steel, petroleum, and several other major industries, pattern bargaining has determined compensation for unionized workers for the past several decades.

Three features characterize pattern bargaining. First, the union negotiates with firms sequentially. Second, the union chooses the order with which it negotiates with firms. Third, the agreement reached with the first firm in the sequence (this firm is often referred to as the *target*) sets the pattern for all subsequent negotiations. In the strictest interpretation of pattern bargaining, the agreement with the target exactly defines the take-it-or-leave-it offer that the union makes to all firms with which it subsequently negotiates.

In recent years, pattern bargaining has loosened.[2] Negotiations are still sequential, but unions have not used the agreement with the target firm as a take-it-or-leave-it offer to subsequent firms. Rather, unions have become increasingly attentive to the fact that equalization of wages among firms may not be in their best interest when there is substantial interfirm heterogeneity in production technology and/or age of the workforce.[3]

[1] See Bureau of National Affairs (1996, p. 3).

[2] Voos (1994, p. 6) notes that in recent years, there has been "a loosening of bargaining patterns, an increased tendency of collective agreements to be tailored to a particular company or particular operation's economic situation...." With regard to the trucking industry and the International Brotherhood of Teamsters in the late 1970s, Levinson (1980, p. 145) notes, "there is widespread recognition among leading union representatives that some relief must be provided, particularly to the short haul carriers, if they were to continue to operate under union conditions." Begin and Beal (1989, p. 374) note several reasons for deviation from a strict wage pattern, including "competitors have lower non-wage costs."

[3] Voos (1994, p. 20) offers the following observation. "Nonetheless, it seems to me that a more sophisticated and subtle type of pattern bargaining has emerged in steel from union attempts to equalize the employee cost burden across companies. That is, because

In this paper, we will consider two kinds of pattern bargaining—*pattern in wages* and *pattern in labor costs*.[4] Both involve sequential negotiations. With the former, the union holds all firms to the terms of the wage agreement with the target firm. With the latter, the union adjusts the wage paid by each firm in order to equalize the labor costs across firms—the target determines costs for all firms.

Our analysis will provide plausible explanations for the following observed phenomena.

1. Unions generally negotiate contracts via pattern bargaining rather than engaging in simultaneous negotiations with all firms in an industry or negotiating sequentially without a pattern.
2. In the 1950s, 1960s, and 1970s, a pattern in wages was very common in union labor negotiations within many industries.[5] In later time periods, there was a movement away from a pattern in wages and toward, in many cases, a pattern in labor costs. The movement away from a pattern in wages occurred at different times for different industries. In meatpacking, it was in the mid 1970s,[6] for steel[7] and automobiles it was in the late 1970s and early 1980s,[8] while for aerospace it was in the late 1980s.[9]

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benefit costs have become a major percentage of total compensation, and companies differ strikingly in the age composition of their work forces (and hence, in the cost of providing pensions and health insurance) the union discovered that equalizing wage rates and benefit provisions no longer allowed it to equalize labor costs and thereby remove labor from competition. The Steelworkers have not dropped the elimination of competition based on labor costs as a goal. Instead, they are now using pattern bargaining of a more subtle form to achieve this end."

[4]In some studies, pattern bargaining has been exclusively characterized (partially) by equality of wages and benefits across firms within an industry (see, e.g., Ready (1990) and Cappelli (1990)) rather than allowing for a broader notion of pattern bargaining.

[5]Pattern bargaining in wages was commonplace in industries such as automobile assembly, steel, meatpacking, trucking, and aerospace, to name a few.

[6]See Craypo (1994, p. 70).

[7]See Arthur and Konzelmann-Smith (1994, p. 159).

[8]See Katz and McDuffie (1994, pp. 201-202). Also, see Budd (1992) for an empirical analysis of pattern bargaining in the automobile assembly industry.

[9]"The [1989] settlements at the other companies, which were not in as good financial shape as Boeing due to the decline in military expenditures, deviated from the exact terms and the overall value of the Boeing settlement." (Erickson (1994, p. 121)) Aerospace production for the military, which involves low volume runs with large amounts of pre-

3. Whenever pattern bargaining in wages was commonplace, the target generally has been a leading firm in the industry.

In our model, two firms with constant returns to scale production technologies compete in the product market as Cournot duopolists. We allow for the possibility that the two firms differ in terms of their productive efficiency.[10] We also allow for the possibility that their products are not perfect substitutes. There is a single industrywide labor union. The wage rates paid by the two firms are determined in bargaining between the union and the firms. To characterize the outcome of the negotiations, the Nash bargaining solution is employed. Our analytic framework is similar to that of Horn and Wolinsky (1988) and Dobson (1994).[11] These authors provide comparisons of simultaneous to sequential bargaining, but neither examines bargaining when the union negotiates sequentially and commits to uniformity (either in wages or costs) in the contracts across firms.[12]

We consider four distinct bargaining environments. In the *simultaneous bargaining* environment, the union negotiates with both firms at the same time. In the *sequential bargaining* environment, the union negotiates with one of the firms first and then negotiates with the second firm. In the third environment, which we call *pattern bargaining in wages*, bargaining is also sequential, but the wage rate negotiated at the first firm becomes a take-it-or-leave-it offer to the second firm. In the fourth bargaining environment, which we call *pattern bargaining in costs*, the outcome of the first negotiation also establishes a pattern, but now the second firm confronts a take-it-or-leave-it wage rate which equalizes the marginal cost of production between the two firms.

production research and development, is particularly ill suited to the commercial aircraft world. In other words, with the military decline in the late 1980s, Boeing held an advantage over other aerospace firms in terms of its productive efficiency.

[10] The model can be reinterpreted as one where the two firms are endowed with the same production technology, but they have access to different workforces, and one workforce is relatively more costly than the other (e.g., it is an older workforce with higher health care and pension costs). This alternative interpretation of the model is discussed further in Section 5.1.

[11] See Davidson (1988) for a noncooperative analysis of union bargaining in an oligopolistic setting.

[12] Note that Dobson (1994) uses the term pattern bargaining as a synonym for sequential negotiations. As we note above, the fact that the union negotiates with firms sequentially is only part of our definition of pattern bargaining.

Our analysis provides explanations for all of the observed phenomena we enumerate above. Of the numerous results produced from our analysis, three constitute our central findings.

1. Pattern bargaining (either in wages or costs) dominates all other bargaining options for the union. (See observed phenomenon #1.)

2. For a given substitutability of the products within an industry, if the differential in productive efficiency between the two firms is small, then the union's payoff from negotiating a pattern in wages exceeds its payoff from negotiating a pattern in costs; if the differential in productive efficiency between the two firms is sufficiently large, then the union's payoff from negotiating a pattern in costs exceeds its payoff from negotiating a pattern in wages. (See observed phenomenon #2.)

3. Under pattern bargaining in wages, the target firm chosen by the union is the relatively more efficient firm. (See observed phenomenon #3.)

To understand these results, we begin by supposing that the firms are equally productive. With pattern bargaining in wages (same as costs with equal productivity), a dollar increase in the wage at the target firm results in a dollar increase in the wage at the other firm. In no other bargaining game that we consider does a change in the wage rate paid by one firm have such a strong external effect on the wage rate paid by the other firm. Intuitively, higher wages at both firms are good for the union. Furthermore, it is important to realize that there are two interconnected parts of each game—the firms are competing in the product market, and the union is negotiating with each firm. Firms are more willing to agree to pay high wages when the other firm will also pay high wages. With pattern bargaining, the wage rates paid by the two firms are identical, by definition. Now suppose the firms differ in their productive efficiency. The more efficient firm is always capable of paying a higher wage than the less efficient firm, and the union wants to take advantage of this fact. However, the union's payoff decreases as the industry moves away from oligopoly and toward monopoly.[13] With pattern

[13] A major motivation offered by unions and employers alike for the use of pattern bargaining is that it equalizes labor costs between firms and, therefore, eliminates competition over the cost of labor (see, e.g., Begin and Beal (1989, p. 374)). Crane (1990, p. 106) notes that "...the UAW established pattern bargaining to provide uniform wages and benefits throughout the industry....Pattern bargaining was designed to bring stability and standardization of settlements to bargaining."

bargaining in wages, the union must weigh the trade-off between obtaining a uniform higher wage at both firms through negotiation with the efficient firm versus enhancing the asymmetry of the two firms in the industry by not offering a wage concession to the less efficient firm. When the firms are close in terms of productive efficiency, the first effect dominates, whereas if they are very different, the second effect dominates. When the second effect dominates, the union prefers a pattern in costs.[14]

The paper proceeds as follows. In Section 2, we pose the model. In Section 3, we describe the bargaining environments. Section 4 contains our results. Extensions and robustness of our findings are discussed in Section 5. Concluding remarks and topics for further research are discussed in Section 6.

2 The Model

We consider an industry where two firms, 1 and 2, produce related products. Each firm is endowed with a constant returns to scale technology that uses a single homogeneous input, labor. Firm 1 uses labor inputs l_1 to produce output $x_1 = l_1$ of good 1. Firm 2 uses labor inputs l_2 to produce output $x_2 = tl_2$ of good 2, where $t \in (0, 1]$ is a parameter that measures the relative inefficiency of firm 2's technology. The smaller t, the more inefficient firm 2 is compared to firm 1.

The demand for product i is

$$p_i(x_i, x_j) = a - cx_j - x_i, \tag{1}$$

$i \neq j = 1, 2$, where $a > 0$ and $c \in (0, 1]$ is a parameter that measures the degree of substitutability between the products. If $c = 1$, the two products are perfect substitutes. We assume that firms 1 and 2 compete in the product market by setting quantities.[15]

[14] In practice, information asymmetries may make a pattern in labor costs very difficult to implement. If a pattern in costs is not a feasible option for the union, then if one firm is much more efficient than the other, the union would prefer sequential negotiations to a pattern in wages.

[15] The parameter c measures a relatively time-invariant characteristic of an industry while t may change within an industry through time. In terms of actual industries, it is reasonable to characterize steel as having a higher c than automobile assembly. With regard to intra-industry changes in t, it is reasonable to argue that steel had a much wider

All workers are assumed to be organized into an industrywide union. The wage rate paid by each firm is determined by bargaining between the union and the firms. Given the wage rates w_1 and w_2 paid by the two firms, respectively, the firms interact in the product market by simultaneously choosing the quantities they will produce, and hence the amounts of labor inputs they will hire. Note that given the wage rates w_1 and w_2, the marginal costs of production for the two firms are equal to w_1 and w_2/t, respectively.

Given w_1 and w_2 and given the demand and cost functions assumed above, the Cournot-Nash equilibrium outputs are equal to

$$x_1(w_1, w_2) = l_1(w_1, w_2) = \frac{a(2-c) + c\frac{w_2}{t} - 2w_1}{4-c^2} \geq 0 \tag{2}$$

and

$$x_2(w_1, w_2) = tl_2(w_1, w_2) = \frac{a(2-c) + cw_1 - 2\frac{w_2}{t}}{4-c^2} \geq 0, \tag{3}$$

and the equilibrium profits of the two firms are

$$\pi_1(w_1, w_2) = [x_1(w_1, w_2)]^2 \tag{4}$$

and

$$\pi_2(w_1, w_2) = [x_2(w_1, w_2)]^2, \tag{5}$$

respectively.

Note that if firm 1 were to operate in the product market as a monopoly, given the wage rate it has to pay, its output and profit levels would be equal to

$$x_1^m(w_1) = l_1^m(w_1) = \frac{a - w_1}{2} \tag{6}$$

and

$$\pi_1^m(w_1) = [x_1^m(w_1)]^2, \tag{7}$$

respectively. Similarly, if firm 2 were to operate as a monopoly, its output and profit levels as functions of w_2 would be given by

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dispersion in terms of interfirm production technology in the late 1970s and early 1980s than it did in the late 1950s and the 1960s (see Arthur and Konzelmann-Smith (1994, p. 164)).

$$x_2^m(w_2) = tl_2^m(w_2) = \frac{a - \frac{w_2}{t}}{2} \tag{8}$$

and

$$\pi_2^m(w_2) = [x_2^m(w_2)]^2, \tag{9}$$

respectively.

We assume that the objective of the union is to maximize the wage bill

$$\pi_u(w_1, w_2) = w_1 l_1(w_1, w_2) + w_2 l_2(w_1, w_2); \tag{10}$$

that the wage rates w_1 and w_2 are negotiated between the union and the firms; that employment is decided by the firms after w_1 and w_2 are determined and is not subject to bargaining; and that the equilibrium in the product market is common knowledge.[16]

Negotiations between the union and each of the two firms may be conducted either simultaneously (i.e., both firms independently bargain with the union over their own wage rate at the same time), or sequentially (i.e., the union negotiates first with one firm and then with the other firm). Furthermore, when negotiating with the two firms in sequence, the union may commit to bargain with the first firm over a common wage for the entire industry. Alternatively, the outcome of the first negotiation may be understood to set the basis for the determination of the wage rate paid by the other firm, so as to equalize the marginal costs of production of the two firms.

These four bargaining environments represent the four basic institutions we focus on here. Each environment defines a game (two if we consider that when the negotiations are sequential, the order in which the contracts are negotiated must also be specified). Before we turn our attention to the description and the solution of each of these games, a few general remarks are in order.

For simplicity, we model the negotiation between the union and a firm over a wage rate as a Nash bargaining problem, and we characterize its equilibrium using Nash's solution. When the union and a firm bargain, they take into account that the wage rate paid by the other firm is determined

[16]The assumption that all workers are represented by an industrywide union whose objective is to maximize the wage bill is fairly common in the literature. Since the wage rates paid by the two firms may be different, it is implicitly assumed that the union provides insurance to its members by equalizing their earnings.

in bargaining between that firm and the union and that the two bargaining problems are interdependent. In particular, if we let w_j^* denote the equilibrium wage rate paid by firm j, the bargaining problem between the union and firm i over the wage rate w_i, $i \neq j = 1, 2$, is indexed by $(S_i, \mathbf{d}_i)$, where $S_i = \{(\pi_i(w_i, w_j^*), \pi_u(w_i, w_j^*)) : w_i \geq 0\}$ is the set of feasible payoff vectors that may be agreed upon, and $\mathbf{d}_i = (d_i, d_u)_i \in S_i$ represents the disagreement point. The Nash solution to this problem is given by

$$w_i^* = \arg\max_{w_i} [\pi_i(w_i, w_j^*) - d_i][\pi_u(w_i, w_j^*) - d_u]. \tag{11}$$

Following Binmore, Rubinstein and Wolinsky (1986), we interpret the static Nash bargaining game as the reduced form of a suitably specified dynamic bargaining game of the type that is studied by Rubinstein (1982). This implies that the disagreement point should correspond to the streams of payoffs that accrue to the negotiating parties when they are in a state of disagreement. Hence, we assume that when a firm and the union cannot agree, the firm earns zero profits, and the payoff to the union is equal to the wage bill that attains when the other firm operates in the product market as a monopoly.

3 Bargaining Environments

In this section, we describe four bargaining environments, and we characterize the equilibria they induce. The derivation of the equilibria is presented in the Appendix.

Environment A: Simultaneous Bargaining.

This environment corresponds to the case in which the union bargains with the two firms simultaneously. The equilibrium wage rates are the solutions to the following problem:

$$\begin{cases} w_1^{A*} = \arg\max_{w_1} \pi_1(w_1, w_2^{A*})[\pi_u(w_1, w_2^{A*}) - w_2^{A*} l_2^m(w_2^{A*})], \\ w_2^{A*} = \arg\max_{w_2} \pi_2(w_1^{A*}, w_2)[\pi_u(w_1^{A*}, w_2) - w_1^{A*} l_1^m(w_1^{A*})]. \end{cases} \tag{12}$$

As we note above, we assume here that in the event that firm i and the union cannot agree, firm i earns zero profits, and the payoff to the union is equal to the wage bill that results when firm j operates in the product market as a monopoly, given its anticipated equilibrium wage rate w_j^{A*}, $i \neq j = 1, 2$. [17]

Environment B: Sequential Bargaining.

This environment corresponds to the case in which the union bargains with one firm first, and only after an agreement is reached in the first negotiation, it bargains with the other firm. Since the order in which the contracts are negotiated is important, we specify two games depending on the identity of the firm that engages in the first negotiation with the union.

B_1: Firm 1 negotiates first.

Since the negotiation between the union and firm 2 follows the one with firm 1, for any given outcome of the first negotiation, $\overline{w}_1$, the outcome of the second negotiation is given by

$$w_2^{B_1*}(\overline{w}_1) = \arg\max_{w_2} \pi_2(\overline{w}_1, w_2)[\pi_u(\overline{w}_1, w_2) - \overline{w}_1 l_1^m(\overline{w}_1)], \tag{13}$$

whereas in the event that firm 1 and the union cannot agree, the solution of the bargaining problem between the union and firm 2 is equal to

$$w_2^{m*} = \arg\max_{w_2} \pi_2^m(w_2)[w_2 l_2^m(w_2)]. \tag{14}$$

Plugging these results into the bargaining problem faced by the union and firm 1, we obtain that the equilibrium wage rates are the solutions to the following problem:

$$\begin{cases} w_1^{B_1*} = \arg\max_{w_1} \pi_1(w_1, w_2^{B_1*}(w_1))[\pi_u(w_1, w_2^{B_1*}(w_1)) - w_2^{m*} l_2^m(w_2^{m*})], \\ w_2^{B_1*} = w_2^{B_1*}(w_1^{B_1*}). \end{cases} \tag{15}$$

B_2: Firm 2 negotiates first.

In this case, since the negotiation between the union and firm 2 precedes the one with firm 1, for any given outcome of the first negotiation, $\overline{w}_2$, the outcome of the second negotiation is given by

$$w_1^{B_2*}(\overline{w}_2) = \arg\max_{w_1} \pi_1(w_1, \overline{w}_2)[\pi_u(w_1, \overline{w}_2) - \overline{w}_2 l_2^m(\overline{w}_2)], \tag{16}$$

[17]Note that even if one of the firms were to fail to agree with the union (an event that is never observed in equilibrium), the efficiency of the Nash solution implies that the wage agreement between the other firm and the union would not be renegotiated.

whereas in the event that firm 2 and the union cannot agree, the solution of the bargaining problem between the union and firm 1 is equal to

$$w_1^{m*} = \arg\max_{w_1} \pi_1^m(w_1)[w_1 l_1^m(w_1)]. \tag{17}$$

Plugging these results into the bargaining problem faced by the union and firm 2, we obtain that the equilibrium wage rates are the solutions to the following problem:

$$\begin{cases} w_1^{B_2*} = w_1^{B_2*}(w_2^{B_2*}), \\ w_2^{B_2*} = \arg\max_{w_2} \pi_2(w_1^{B_2*}(w_2), w_2)[\pi_u(w_1^{B_2*}(w_2), w_2) - w_1^{m*} l_1^m(w_1^{m*})]. \end{cases} \tag{18}$$

Environment C: Pattern Bargaining in Wages.

This environment corresponds to the case in which the union selects a target firm to negotiate a common wage for the entire industry, and all parties understand that the union's commitment is binding.[18] As before, we distinguish between two cases that are indexed by the identity of the target firm.

C_1: Firm 1 is the target.

In this environment, the equilibrium wage rates are the solutions to the following problem:

$$\begin{cases} w_1^{C_1*} = \arg\max_w \pi_1(w, w)[\pi_u(w, w) - w_2^{m*} l_2^m(w_2^{m*})], \\ w_2^{C_1*} = w_1^{C_1*}. \end{cases} \tag{19}$$

C_2: Firm 2 is the target.

In this environment, the equilibrium wage rates are the solutions to the following problem:

$$\begin{cases} w_1^{C_2*} = w_2^{C_2*}, \\ w_2^{C_2*} = \arg\max_w \pi_2(w, w)[\pi_u(w, w) - w_1^{m*} l_1^m(w_1^{m*})]. \end{cases} \tag{20}$$

[18] It is implicitly assumed that once the negotiation with the target firm is concluded, the union will face the other firm with a take-it-or-leave-it offer identical to the wage agreement reached with the target firm. Alternatively, we could assume that before the negotiation with the target firm begins, the other firm commits to accept the wage agreement resulting from that negotiation. In Section 4, we show that the two models are equivalent and, in equilibrium, it is never mutually beneficial for the union and the non-target firm to break the commitment and deviate from the pattern.

Environment D: Pattern Bargaining in Costs.

The last environment we consider here is similar to the previous one. In this case, however, the union and the target firm bargain to set a uniform marginal cost of production for the two firms in the industry. In particular, all parties understand that the wage agreement between the union and the target firm will be used to determine the wage rate paid by the other firm, so as to equalize the production costs of the two firms.[19]

D_1: Firm 1 is the target.

When firm 1 is the target, the equilibrium wage rates are the solutions to the following problem:

$$\begin{cases} w_1^{D_1*} = \arg\max_w \pi_1(w, tw)[\pi_u(w, tw) - w_2^{m*} l_2^m(w_2^{m*})], \\ w_2^{D_1*} = tw_1^{D_1*}. \end{cases} \tag{21}$$

D_2: Firm 2 is the target.

When firm 2 is the target, the equilibrium wage rates are the solutions to the following problem:

$$\begin{cases} w_1^{D_2*} = \frac{w_2^{D_2*}}{t}, \\ w_2^{D_2*} = \arg\max_w \pi_2(\frac{w}{t}, w)[\pi_u(\frac{w}{t}, w) - w_1^{m*} l_1^m(w_1^{m*})]. \end{cases} \tag{22}$$

For each game $g = A, B_1, B_2, C_1, C_2, D_1, D_2$, given the equilibrium wage rates w_1^{g*} and w_2^{g*}, we let $\pi_u^{g*} \equiv \pi_u(w_1^{g*}, w_2^{g*})$, $\pi_1^{g*} \equiv \pi_1(w_1^{g*}, w_2^{g*})$, and $\pi_2^{g*} \equiv \pi_2(w_1^{g*}, w_2^{g*})$ denote the equilibrium payoffs to the union, firm 1, and firm 2, respectively. The equilibrium wage rates and payoffs for all the games are reported in Tables 1 and 2 in the Appendix, respectively. Note that the equilibrium payoffs depend on the parameters of the model a, c, and t.

4 Results

We begin this section by comparing the equilibrium outcomes of the different games with respect to the payoffs they yield to the parties for the case in which firm 1 and firm 2 are endowed with the same production technology, i.e., $t = 1$. In Section 4.2, we consider the case in which the two firms are

[19]Note that if $t = 1$, environments C and D coincide.

heterogeneous with respect to their production efficiency and establish the main results of the paper.

Since the proofs of all the results presented in this section simply involve tedious comparisons of the equilibrium outcomes reported in Tables 1 and 2 in the Appendix, we present the formal arguments in the Appendix, and we focus here on the intuition underlying the results.

4.1 Homogeneous Firms ($t = 1$)

As noted in Section 3, when the two firms have access to the same production technology, pattern bargaining in costs reduces to pattern bargaining in wages. Also, the identity of the target firm becomes irrelevant, and the outcomes of the two sequential bargaining games where either firm negotiates first are symmetric with respect to the order in which the contracts are negotiated. These considerations imply that when the two firms are homogeneous with respect to their production efficiency, there are only three games that need to be considered: simultaneous, sequential, and pattern bargaining. When we compare the outcomes of these games with respect to the equilibrium payoffs to the parties, we obtain the following.

Proposition 1 (i) *For any a and c, the equilibrium wage bill is higher under pattern bargaining than under sequential bargaining, and the equilibrium wage bill under sequential bargaining is higher than under simultaneous bargaining.* (ii) *For any a and c, the profits of either firm are the same under simultaneous bargaining as under sequential bargaining when the firm negotiates last; this level of profits is larger than the one that attains under sequential bargaining when the firm negotiates first, and this level of profits is, in turn, larger than either firm's profits under pattern bargaining.*

When firms are homogeneous with respect to their production efficiency, sequential bargaining dominates simultaneous bargaining from the point of view of the union. This result confirms previous findings by Dobson (1994) and Horn and Wolinsky (1988). Our analysis, however, indicates that in this case, pattern bargaining is the most preferred alternative by the union.

The intuition for these results is as follows. An increase in a firm's wage rate is detrimental for the firm, while an increase in the wage rate paid by its competitor is beneficial. The bargaining environments we consider here differ with respect to the way the outcome of one negotiation affects the equilibrium outcome of the other negotiation. In particular, if a firm agrees to pay a higher wage rate to the union, then this has a positive effect on its competitor's equilibrium wage rate—this effect is higher under pattern bargaining than under sequential bargaining when the firm negotiates first. This external effect is instead zero either under simultaneous bargaining or under sequential bargaining when the firm negotiates last.[20] The firms want to avoid taking any action which harms them in the product market relative to their competitor. An increase in its own factor price harms a firm no matter what is the factor price paid the other firm, but it is increasingly harmful the smaller the wage increase it induces for the rival firm. This implies that the returns to the union from adopting an aggressive bargaining strategy are highest under pattern bargaining where firms are more likely to agree to a wage increase.

4.2 Heterogeneous Firms ($t < 1$)

We begin the presentation of the general case in which firms are heterogeneous with respect to their production efficiency by noting that under pattern bargaining in costs, the game where firm 1 is the target has the same equilibrium outcome as the game where firm 2 is the target.[21] In the remainder of the paper, we refer to pattern bargaining in costs without specifying the identity of the target.

We restrict attention to equilibria in which, in any bargaining environment, both firms produce positive levels of output. For a given substitutability of the products within the industry, this restriction implies a lower bound on the parameter that measures the heterogeneity in production efficiency between the two firms. We let $\Omega = \{(c,t) : 0 < c \leq 1, c/2 < t < 1\}$ denote the set of admissible parameter values for c and t.[22]

The following lemma establishes a useful characterization that applies to

[20]In the Appendix, we show that for $t = 1$, $w_2^{B_1*}(w_1) = (4cw_1 - a(c-2))/8$, which implies that $\partial w_2^{B_1*}(\cdot)/\partial w_1 = c/2 > 0$. Under pattern bargaining, $w_2^{C_1*}(w_1) = w_1$, which implies that $\partial w_2^{C_1*}(\cdot)/\partial w_1 = 1 > c/2$ for any $c \in (0,1]$. Under simultaneous bargaining or in the last negotiation under sequential bargaining, however, in equilibrium, the outcome of a negotiation does not affect the wage rate paid by the other firm.

[21]This follows immediately from the fact that when $w_1 = w_2/t$, problems (21) and (22) coincide.

[22]For a given c, if $t \leq c/2$, under pattern bargaining in wages, the relatively less efficient firm does not operate in equilibrium. The intuition for this result comes from the fact that when one firm is much more efficient than the other firm, the restriction that the two firms pay the same wage rate in equilibrium drives the inefficient firm out of the market.

all pairwise comparisons of individual equilibrium payoffs between the games described in Section 3.

Lemma 1 *For any pair of games $g \neq h = A, B_1, B_2, C_1, C_2, D$, and for any player $j = u, 1, 2$, the sign of the difference $(\pi_j^{g*} - \pi_j^{h*})$ is independent of a. Furthermore, either* (i) *The difference $(\pi_j^{g*} - \pi_j^{h*})$ is always positive or always negative for any $(c, t) \in \Omega$; or* (ii) *The equation $\pi_j^{g*} - \pi_j^{h*} = 0$ implicitly defines a threshold for t, a function $\tau_j^{gh}(c)$, which is increasing in c.*

Note that when $\tau_j^{gh}(c)$ exists, it partitions the parameter space Ω into two regions such that in one region $\pi_j^{g*} > \pi_j^{h*}$, whereas in the other region $\pi_j^{g*} < \pi_j^{h*}$, $g \neq h = A, B_1, B_2, C_1, C_2, D$, $j = u, 1, 2$.

The following proposition exploits Lemma 1 to characterize the way each market participant ranks the bargaining games we consider here.

Proposition 2 *The results of equilibrium payoff comparisons for the union, firm 1, and firm 2, respectively, for each pair of games are summarized in the following tables:*

Union	π_u^{A*}	$\pi_u^{B_1*}$	$\pi_u^{B_2*}$	$\pi_u^{C_1*}$	$\pi_u^{C_2*}$	π_u^{D*}
π_u^{A*}						
$\pi_u^{B_1*}$	$>$					
$\pi_u^{B_2*}$	$>$	$=$				
$\pi_u^{C_1*}$	$\gtrless$	$\gtrless$	$\gtrless$			
$\pi_u^{C_2*}$	$\gtrless$	$\gtrless$	$\gtrless$	$<$		
π_u^{D*}	$>$	$>$	$>$	$\lessgtr$	$>$	

,

Firm 1	π_1^{A*}	$\pi_1^{B_1*}$	$\pi_1^{B_2*}$	$\pi_1^{C_1*}$	$\pi_1^{C_2*}$	π_1^{D*}
π_1^{A*}						
$\pi_1^{B_1*}$	$<$					
$\pi_1^{B_2*}$	$=$	$>$				
$\pi_1^{C_1*}$	$\lessgtr$	$\lessgtr$	$\lessgtr$			
$\pi_1^{C_2*}$	$\lessgtr$	$\lessgtr$	$\lessgtr$	$>$		
π_1^{D*}	$<$	$<$	$<$	$<$	$<$	

,

and

Firm 2	π_2^{A*}	$\pi_2^{B_1*}$	$\pi_2^{B_2*}$	$\pi_2^{C_1*}$	$\pi_2^{C_2*}$	π_2^{D*}
π_2^{A*}						
$\pi_2^{B_1*}$	$=$					
$\pi_2^{B_2*}$	$<$	$<$				
$\pi_2^{C_1*}$	$<$	$<$	$<$			
$\pi_2^{C_2*}$	$<$	$<$	$<$	$>$		
π_2^{D*}	$<$	$<$	$<$	$>$	$>$	

,

where each cell in a table displays the binary relation between the row payoff and the column payoff, and two entries in the same cell indicate that such relation is different in different regions of the parameter space Ω. In such cases, the top entry refers to the binary relation that holds for all (c,t) combinations that lie above $\tau_j^{gh}(c)$, and the bottom entry refers to the binary relation that holds for all (c,t) combinations that lie below $\tau_j^{gh}(c)$, where $g \neq h = A, B_1, B_2, C_1, C_2, D$, and $j = u, 1, 2$.

By combining Proposition 2 and Lemma 1, we obtain the following:

Corollary 1 (i) *For any a, and for any $(c,t) \in \Omega$, the wage bill is highest either under pattern bargaining in costs or under pattern bargaining in wages when firm 1 is the target.* (ii) *For any a, given a $c \in (0,1]$, there exists a critical level of t, $\tau_u^{C_1D}(c)$, such that for $\tau_u^{C_1D}(c) < t < 1$, pattern bargaining in wages with firm 1 as the target is best for the union, whereas for $c/2 < t < \tau_j^{gh}(c)$, pattern bargaining in costs yields the highest payoff to the union.*

We first offer some intuition for these results before linking them to the observed phenomena associated with collective bargaining.

The intuition for the results contained in Corollary 1 is similar to the one we provide above for the case in which the two firms are endowed with identical production technologies. The more efficient firm is always capable of paying a higher wage than the less efficient firm, and the union wants to take advantage of this fact. Each firm takes into account the externality generated by the outcome of its union negotiation on the other firm's equilibrium wage rate, which affects its willingness to agree to pay a higher wage rate to the union. Like in the case in which the two firms are homogeneous with respect to their production efficiency, this externality is strongest under pattern bargaining in wages.[23] When the two firms are heterogeneous with

[23]For $(c,t) \in \Omega$, note for example that $\partial w_2^{A*}(\cdot)/\partial w_1 = \partial w_2^{B_2*}(\cdot)/\partial w_1 = 0 < \partial w_2^{B_1*}(\cdot)/\partial w_1 = ct/2 < \partial w_2^{D*}(\cdot)/\partial w_1 = t < \partial w_2^{C_1*}(\cdot)/\partial w_1 = 1$.

respect to their production efficiency, however, the union does not necessarily prefer a uniform high wage rate in the industry. Maintaining a balanced duopoly is an important goal for the union—the union's payoff decreases as the industry moves away from a balanced duopoly and toward a monopoly. As the heterogeneity in the production efficiency of the two firms increases, it may be in the union's best interest to equalize the production costs of the two firms rather than achieve a higher uniform wage rate in the industry. Note that if equalization of production costs is not possible, for whatever reason, then the union would prefer sequential negotiation to a pattern in wages for a sufficiently large interfirm efficiency differential.[24]

The results in this section provide an understanding for many of the observed phenomena associated with collective bargaining. First, as noted in the introduction, pattern bargaining has dominated collective bargaining in the United States since World War II.

Second, under pattern bargaining in wages the target usually has been a leading firm in the industry. In aerospace, the target usually has been Boeing. Boeing has been recognized as the most efficient producer in the industry with the largest commercial sales revenues and a relatively small part of its revenues coming from military contracts. In the steel industry, there has been a consortium of three producers—U.S. Steel, Bethlehem, and Republic—which negotiated in unison as the target in the industry. In trucking, for a period of 15 years, the National Master Freight Agreement, which was binding on the largest of the trucking firms in the country, set the pattern for the industry. Unlike so many other industries, there was no clear target in the automobile assembly industry. From 1955 to 1996, the UAW has selected Ford as target seven times, GM five times, Chrylser twice. It is, however, the case that in each of these negotiations, the weakest firm was never chosen as the target.

Third, as noted in the introduction, many industries in the past 20 years moved away from a pattern in wages. In the steel industry, firms with older plants where the workforce was also older received concessions in the early 1980s relative to the contracts at the major producers. Arthur and Konzelmann-Smith (1994, p. 164) offer the following observation regarding the steel industry at the present time.

> "Instead of one dominant industrial relations pattern for the industry, there appears to be a series of patterns emerging for plants

[24] A specific context is discussed in Section 5.2.

> whose products compete in different segments of the industry's market with different levels of product market competition and technology change."

In our discussion so far, we have been implicitly assuming that the union's and the firms' commitment in pattern bargaining, either in wages or in costs, is credible. We conclude this section by showing that this is, indeed, the case, since once an agreement is reached with the first firm in a pattern negotiation, it is never mutually advantageous for the union and the second firm to break the commitment and deviate from the pattern.

Proposition 3 *For any a, and for any $(c,t) \in \Omega$, the equilibrium outcomes of pattern bargaining in wages and pattern bargaining in costs are Pareto efficient.*

5 Discussion

5.1 A Reinterpretation/Extension of the Model

The model presented in Section 2 assumes that the two firms are heterogeneous with respect to their production efficiency but have access to a homogeneous labor force. Alternatively, we could have posed the model by assuming that the two firms are endowed with the same production technology but have heterogeneous workforces. In this alternative formulation, $x_i = l_i$ for $i = 1, 2$, the compensation rate paid by firm 1 is w_1, the compensation rate paid by firm 2 is w_2/t, and the payoff to the union is $\pi_u(w_1, w_2) = w_1 l_1(w_1, w_2) + \frac{w_2}{t} l_2(w_1, w_2)$. This model corresponds to an environment where firm 2 has a relatively more costly workforce (e.g., an older workforce with higher health and pension costs) than firm 1, and the parameter $t \in (0, 1]$ measures the relative labor costs differential between the two firms. This alternative model is equivalent to the model of Section 2, and all the results presented in the previous section apply to this model as well. So, our analysis provides a credible explanation for the emergence of a pattern in costs in recent years, whether one believes the underlying cause is widening interfirm differentials in productive efficiency or interfirm differentials in the underlying costliness of workers.[25]

[25] In 1991, benefits were 34 percent of total compensation for unionized workers in the private sector. Approximately one-third of these costs were for medical benefits and one-

5.2 Concessionary Bargaining

As noted in the introduction, pattern bargaining (either in wages or in labor costs) has been a pervasive phenomenon in union labor negotiations within many industries since World War II. However, in the automobile assembly industry and other industries like, e.g., trucking and paper, there was a transition period in the early 1980s—at the time the movement away from pattern in wages began—characterized by what the labor and industrial relations literature typically refers to as *concessionary bargaining* (see, e.g., Begin and Beal (1989, p. 373)). Concessionary bargaining is characterized by individual firms attempting to negotiate better terms with a union than its competitors were able to negotiate. Concessionary bargaining corresponds to our sequential bargaining environment.

Under the maintained assumption that the union has control over the way the negotiations with the firms are to be conducted, our model predicts that sequential bargaining could only be observed if pattern bargaining in costs were not a viable option. In fact, Proposition 2 implies that for a given substitutability of the products within an industry, if the firms are sufficiently heterogeneous with respect to their production efficiency, then the union would prefer to negotiate with the firms sequentially with no commitment than to negotiate a pattern in wages with either firm as the target. In such circumstances, however, the union's most preferred option would be to negotiate a pattern in costs.

In reality, unlike a pattern in wages, a pattern in costs can be very difficult to implement on a practical level. There may be many things that are unknown to the union that are relevant to the equalization of labor costs among firms—information that is irrelevant for equalizing wages across firms. This may be especially true in times of rapid and dramatic changes like the ones that occurred in the early 1980s in the automobile assembly industry. Our model assumes that the firms and the union have access to the same information, and it is, therefore, incapable of addressing these issues. These considerations, however, suggest that pattern bargaining in costs may not always be a feasible option, in which case, sequential bargaining may actually be the union's most preferred alternative.

sixth were for pensions. In 1965, benefit costs were only 18 percent of total compensation for all workers. (see Bureau of National Affairs (1992, pp. 96 and 117); also see Bureau of Labor Statistics (1992, p. 3)).

5.3 Additional Bargaining Environments

Since the bargaining environments we consider in our analysis do not exhaust the set of institutions that may govern the negotiations between a union and the firms in a duopoly, we consider others as well. For example, when the union bargains with the firms in sequence, the outcome of the first negotiation may be used as either a wage or a cost floor (or as a ceiling) in the subsequent negotiation. Alternatively, the union may commit never to give the firm negotiating first a deal worse than the one obtained by the other firm in the subsequent negotiation. To understand why these alternative institutions are not observed in the real world we characterize the equilibrium outcomes they induce and compare them to the outcomes of the four basic environments described in Section 3. What we find is that each of these alternative bargaining environments is either equivalent to one of our four basic environments, or it is strictly dominated by at least one of the basic environments from the point of view of the union.[26]

5.4 Interindustry Pattern Bargaining

The model we present in Section 2 refers to a single industry in which two firms that produce related products compete as Cournot duopolists. Alternatively, we could consider an environment where two firms operate each as a monopoly in two unrelated industries, and all workers in the two industries are organized into a single union. In the context of our model, this corresponds to the case in which $c = 0$. Since Corollary 1 applies to this case as well,[27] the results of our analysis are consistent with the observation that for unions whose membership extends beyond a single industry, like the United Automobile Workers who represent a large portion of workers in the farm/construction machinery industry, agreements in one industry (auto assembly) often set the pattern for negotiations in others (farm/construction machinery).

[26] The analysis of these alternative bargaining environments is available from the authors upon request.

[27] The proof is contained in the Appendix.

6 Concluding Remarks

Our main finding is that pattern bargaining produces a higher payoff for a union than anything else we consider here, including sequential and simultaneous bargaining. This provides an explanation for much of collective bargaining in the United States since World War II.

We mention three of several areas for future investigation. First, industry-wide demand shocks may clearly have an effect on bargaining, particularly by affecting the relative bargaining power of the union and the firms. Changes in our demand intercept have no qualitative effects on our results. Furthermore, bargaining power is held constant in our analysis.

Second, in our model, we always assume that in the event that the union and a firm disagree, the firm is unable to produce and hence earns zero profits. However, 77 percent of firms in a recent survey indicate that if struck, they would consider the use of replacement workers.[28] The willingness or ability to use replacement workers seems to have changed over time, perhaps as a result of the general decline of unionization in the country (from 34 percent of the workforce in 1954 to 11 percent in recent years).

Third, in our analysis we assume that the union controls the way negotiations are to be conducted. Casual empiricism indicates that this may be a good approximation for what we observe in many U.S. industries. This assumption, however, may not be appropriate in many other situations. In future work, we will incorporate within the model a political mechanism that selects the bargaining environment in which collective negotiations are to be conducted.

[28] See Bureau of National Affairs (1992).

A Appendix

We begin this section by deriving the equilibrium wage rates for each game we describe in Section 3. In section A.2, we then compare the equilibrium payoffs across games.

A.1 Derivation of the Equilibrium Wage Rates

Environment A: Simultaneous Bargaining

From the first order conditions for problem (12), we obtain the following system of equations:

$$\begin{cases} w_1^{A*} = \frac{ta(2-c)+4cw_2^{A*}}{8t}, \\ w_2^{A*} = \frac{ta(2-c)+4tcw_1^{A*}}{8}. \end{cases}$$

Solving for the equilibrium wage rates yields

$$\begin{cases} w_1^{A*} = a\frac{1}{4}, \\ w_2^{A*} = ta\frac{1}{4}. \end{cases}$$

Note that for any a, c, and t,

$$w_1^{A*} - \frac{w_2^{A*}}{t} = 0.$$

Hence, in equilibrium, the marginal costs of production of the two firms are equal, which implies that they produce the same quantity. Plugging the equilibrium wage rates into (2) and (3), we obtain an expression for the equilibrium output level as a function of the parameters of the model as

$$x_1^{A*} = x_2^{A*} = a\frac{3}{4(c+2)}.$$

This level of output is positive for any a, c, and t.

Environment B: Sequential Bargaining

B_1*: Firm* 1 *negotiates first*

For any given outcome of the first negotiation, $\overline{w}_1$, solving the bargaining problem between firm 2 and the union (equation (13)) yields

$$w_2^{B_1*}(\overline{w}_1) = \frac{ta(2-c)+4tc\overline{w}_1}{8}.$$

Plugging this result into (15) and solving for the equilibrium wage rates yields

$$\begin{cases} w_1^{B_1*} = a\frac{20+7c-\sqrt{144-15c^2+24c}}{16(c+2)}, \\ w_2^{B_1*} = ta\frac{16+3c^2+20c-c\sqrt{144-15c^2+24c}}{32(c+2)}. \end{cases}$$

Note that

$$w_1^{B_1*} - \frac{w_2^{B_1*}}{t} = \frac{24-3c^2-6c-(2-c)\sqrt{144-15c^2+24c}}{32(c+2)} > 0,$$

for any a, c, and t. Hence, in equilibrium, firm 1's marginal cost of production is higher than firm 2's cost. The equilibrium output levels are equal to

$$\begin{cases} x_1^{B_1*} = a\frac{12-3c+\sqrt{144-15c^2+24c}}{32(c+2)}, \\ x_2^{B_1*} = a\frac{3}{4(c+2)}. \end{cases}$$

These output levels are positive for any a, c, and t.

B_2: Firm 2 negotiates first

For any given outcome of the first negotiation, $\overline{w}_2$, solving the bargaining problem between firm 1 and the union (equation (16)) yields

$$w_1^{B_2*}(\overline{w}_2) = \frac{ta(2-c)+4c\overline{w}_2}{8t}.$$

Plugging this result into (18) and solving for the equilibrium wage rates yields

$$\begin{cases} w_1^{B_2*} = a\frac{16+3c^2+20c-c\sqrt{144-15c^2+24c}}{32(c+2)}, \\ w_2^{B_2*} = ta\frac{20+7c-\sqrt{144-15c^2+24c}}{16(c+2)}. \end{cases}$$

Note that

$$w_1^{B_2*} - \frac{w_2^{B_2*}}{t} = -\frac{24-3c^2-6c-(2-c)\sqrt{144-15c^2+24c}}{32(c+2)} < 0,$$

for any a, c, and t. Hence, in equilibrium, firm 2's marginal cost of production is higher than firm 1's cost. The equilibrium output levels are equal to

$$\begin{cases} x_1^{B_2*} = a\frac{3}{4(c+2)}, \\ x_2^{B_2*} = a\frac{12-3c+\sqrt{144-15c^2+24c}}{32(c+2)}. \end{cases}$$

These output levels are positive for any a, c, and t.

Environment C: Pattern Bargaining in Wages

In this bargaining environment, we have to distinguish between two cases depending on the values of the parameters of the model. In particular, for any a and for a given degree of substitutability of the products, if firm 1's technology is sufficiently more efficient than firm 2's technology, then the restriction that the two firms pay the same equilibrium wage rate drives firm 2 out of the market. This situation arises when $t \leq c/2$. Since we focus on duopolistic markets where both firms operate in equilibrium, we restrict the parameter t to be in the interval $(c/2, 1]$.

C_1*: Firm* 1 *is the target*

Solving (19) we obtain that for any a and c, and for any $t \in (c/2, 1]$, the equilibrium wage rates are equal to

$$w_1^{C_1*} = w_2^{C_1*} = at\frac{1}{16\,(2t-c)\,(t^2-tc+1)} \cdot \left(\begin{array}{c} (2-c)\,(4-3c-7tc+6t+10t^2) - \sqrt{2-c} \cdot \\ \sqrt{\left(\begin{array}{c} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{array} \right)} \end{array} \right).$$

Note that, by construction, the equilibrium marginal cost of production is higher for firm 1 than for firm 2. Given the equilibrium wage rates, using (2) and (3) we obtain that the equilibrium output levels of the two firms are equal to

$$x_1^{C_1*} = a\frac{1}{16\,(4-c^2)\,(t^2-tc+1)} \cdot \left(\begin{array}{c} 3\,(2-c)\,(4+c-3tc-2t+2t^2) + \sqrt{2-c} \cdot \\ \sqrt{\left(\begin{array}{c} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{array} \right)} \end{array} \right)$$

and

$$
\begin{aligned}
x_2^{C_1*} &= a\frac{1}{16\left(4-c^2\right)\left(2t-c\right)\left(t^2-tc+1\right)}\cdot \\
&\left(\begin{array}{c} (2-c)\left(\begin{array}{c} -8+13c^2t-7c^2t^2-10c+18tc- \\ 42ct^2+10ct^3-20t^2+20t+32t^3 \end{array}\right)+(2-tc)\cdot \\ \hline \sqrt{2-c}\sqrt{\left(\begin{array}{c} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{array}\right)} \end{array}\right),
\end{aligned}
$$

respectively. These output levels are positive for any a and c and for any $t \in (c/2, 1]$.

C_2: Firm 2 is the target

Solving (20) we obtain that for any a and c, and for any $t \in (c/2, 1]$, the equilibrium wage rates are equal to

$$
\begin{aligned}
w_1^{C_2*} &= w_2^{C_2*} = at\frac{1}{16\left(2-tc\right)\left(t^2-tc+1\right)}\cdot \\
&\left(\begin{array}{c} (2-c)\left(10-7tc-3ct^2+6t+4t^2\right)-\sqrt{2-c}\cdot \\ \hline \sqrt{\left(\begin{array}{c} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{array}\right)} \end{array}\right).
\end{aligned}
$$

Note that, by construction, the equilibrium marginal cost of production is higher for firm 1 than for firm 2. Given the equilibrium wage rates, using (2) and (3) we obtain that the equilibrium output levels of the two firms are equal to

$$
\begin{aligned}
x_1^{C_2*} &= a\frac{1}{16\left(4-c^2\right)\left(2-tc\right)\left(t^2-tc+1\right)}\cdot \\
&\left(\begin{array}{c} (2-c)\left(\begin{array}{c} 32-7c^2t+13c^2t^2+10c-42tc+ \\ 18ct^2-10ct^3+20t^2-20t-8t^3 \end{array}\right)+(2t-c)\cdot \\ \hline \sqrt{2-c}\sqrt{\left(\begin{array}{c} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{array}\right)} \end{array}\right)
\end{aligned}
$$

and

$$x_2^{C_2*} = a\frac{1}{16\left(4-c^2\right)\left(t^2-tc+1\right)} \cdot \left(\frac{3\left(2-c\right)\left(2-3tc+ct^2-2t+4t^2\right)+\sqrt{2-c}\,\cdot}{\sqrt{\left(\begin{array}{c} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{array}\right)}} \right),$$

respectively. These output levels are positive for any a and c and for any $t \in (c/2, 1]$.

Environment D: Pattern Bargaining in costs

When we restrict the wage rates paid by the two firms so that $w_1 = w_2/t$, the two firms become symmetric—that is, they always produce the same quantity—and problems (21) and (22) coincide. Hence, in this bargaining environment, the equilibrium outcome is independent of the identity of the target firm. Solving the bargaining problem between the union and the target firm, we obtain that the equilibrium wage rates are equal to

$$\begin{cases} w_1^{D*} = a\frac{10-\sqrt{24-6c}}{16}, \\ w_2^{D*} = ta\frac{10-\sqrt{24-6c}}{16}. \end{cases}$$

The equilibrium output levels of the two firms are the same and are equal to

$$x_1^{D*} = x_2^{D*} = a\frac{6+\sqrt{24-6c}}{16(c+2)}.$$

This level of output is positive for any a, c, and t.

The equilibrium wage rates of the six games we consider here are summarized in Table 1. For each game, plugging the equilibrium wage rates into (4), (5), and (10), we obtain the equilibrium payoffs for firm 1, firm 2, and the union, respectively. The equilibrium payoffs for all the games are reported in Table 2

Table 1: Equilibrium Wage Rates

game	wage rates
A	$w_1^{A*} = \frac{a}{4}$ $w_2^{A*} = \frac{ta}{4}$
B_1	$w_1^{B_1*} = a\frac{20+7c-\sqrt{144-15c^2+24c}}{16(c+2)}$ $w_2^{B_1*} = ta\frac{16+3c^2+20c-c\sqrt{144-15c^2+24c}}{32(c+2)}$
B_2	$w_1^{B_2*} = a\frac{16+3c^2+20c-c\sqrt{144-15c^2+24c}}{32(c+2)}$ $w_2^{B_2*} = ta\frac{20+7c-\sqrt{144-15c^2+24c}}{16(c+2)}$
C_1	$w_1^{C_1*} = w_2^{C_1*} = at\frac{1}{16(2t-c)(t^2-tc+1)} \cdot \left(\begin{array}{c} (2-c)\left(4-3c-7tc+6t+10t^2\right) - \sqrt{2-c} \cdot \\ \sqrt{\left(\begin{array}{c} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{array} \right)} \end{array} \right)$
C_2	$w_1^{C_2*} = w_2^{C_2*} = at\frac{1}{16(2-tc)(t^2-tc+1)} \cdot \left(\begin{array}{c} (2-c)\left(10-7tc-3ct^2+6t+4t^2\right) - \sqrt{2-c} \cdot \\ \sqrt{\left(\begin{array}{c} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{array} \right)} \end{array} \right)$
D	$w_1^{D*} = a\frac{10-\sqrt{24-6c}}{16}$ $w_2^{D*} = ta\frac{10-\sqrt{24-6c}}{16}$

Table 2: Equilibrium Payoffs

game	union's payoff
A	$\pi_u^{A*} = a^2 \frac{3}{8(c+2)}$
B_1	$\pi_u^{B_1*} = a^2 \frac{144+15c^2+120c+(4-c)\sqrt{144-15c^2+24c}}{256(c+2)^2}$
B_2	$\pi_u^{B_2*} = a^2 \frac{144+15c^2+120c+(4-c)\sqrt{144-15c^2+24c}}{256(c+2)^2}$
C_1	$\pi_u^{C_1*} = a^2 \frac{1}{128(4-c^2)(2t-c)^2(t^2-tc+1)} \cdot \left((2-c)\left(-4-tc-5c+10t+6t^2\right) + \sqrt{2-c} \cdot \sqrt{\left(\begin{array}{c} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{array} \right)} \right) \cdot \left((2-c)\left(4-3c-7tc+6t+10t^2\right) - \sqrt{2-c} \cdot \sqrt{\left(\begin{array}{c} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{array} \right)} \right)$
C_2	$\pi_u^{C_2*} = a^2 \frac{1}{128(4-c^2)(2-tc)^2(t^2-tc+1)} \cdot \left((2-c)\left(6-tc-5ct^2+10t-4t^2\right) + \sqrt{2-c} \cdot \sqrt{\left(\begin{array}{c} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{array} \right)} \right) \cdot \left((2-c)\left(10-7tc-3ct^2+6t+4t^2\right) - \sqrt{2-c} \cdot \sqrt{\left(\begin{array}{c} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{array} \right)} \right)$
D	$\pi_u^{D*} = a^2 \frac{\left(6+\sqrt{(24-6c)}\right)\left(10-\sqrt{(24-6c)}\right)}{128(c+2)}$

Table 2 (continued)

game	firm 1's payoff
A	$\pi_1^{A*} = a^2 \left(\frac{3}{4(c+2)}\right)^2$
B_1	$\pi_1^{B_1*} = a^2 \left(\frac{12-3c+\sqrt{144-15c^2+24c}}{32(c+2)}\right)^2$
B_2	$\pi_1^{B_2*} = a^2 \left(\frac{3}{4(c+2)}\right)^2$
C_1	$\pi_1^{C_1*} = a^2 \left(\frac{1}{16(4-c^2)(t^2-tc+1)}\right)^2 \cdot \left(\begin{array}{c} 3\,(2-c)\,(4+c-3tc-2t+2t^2)+\sqrt{2-c}\,\cdot \\ \sqrt{\left(\begin{array}{c} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{array} \right)} \end{array} \right)^2$
C_2	$\pi_1^{C_2*} = a^2 \left(\frac{1}{16(4-c^2)(2-tc)(t^2-tc+1)}\right)^2 \cdot \left(\begin{array}{c} (2-c)\left(\begin{array}{c} 32-7c^2t+13c^2t^2+10c-42tc+ \\ 18ct^2-10ct^3+20t^2-20t-8t^3 \end{array} \right)+(2t-c)\,\cdot \\ \sqrt{2-c}\sqrt{\left(\begin{array}{c} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{array} \right)} \end{array} \right)^2$
D	$\pi_1^{D*} = a^2 \left(\frac{6+\sqrt{24-6c}}{16(c+2)}\right)^2$

Table 2 (continued)

game	firm 2's payoff
A	$\pi_2^{A*} = a^2 \left(\frac{3}{4(c+2)}\right)^2$
B_1	$\pi_2^{B_1*} = a^2 \left(\frac{3}{4(c+2)}\right)^2$
B_2	$\pi_2^{B_2*} = a^2 \left(\frac{12-3c+\sqrt{144-15c^2+24c}}{32(c+2)}\right)^2$
C_1	$\pi_2^{C_1*} = a^2 \left(\frac{1}{16(4-c^2)(2t-c)(t^2-tc+1)}\right)^2 \cdot \left(\begin{array}{c} (2-c)\begin{pmatrix} -8+13c^2t-7c^2t^2-10c+18tc- \\ 42ct^2+10ct^3-20t^2+20t+32t^3 \end{pmatrix} + (2-tc)\cdot \\ \sqrt{2-c}\sqrt{\begin{pmatrix} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{pmatrix}} \end{array} \right)^2$
C_2	$\pi_2^{C_2*} = a^2 \left(\frac{1}{16(4-c^2)(t^2-tc+1)}\right)^2 \cdot \left(\begin{array}{c} 3(2-c)(2-3tc+ct^2-2t+4t^2)+\sqrt{2-c}\cdot \\ \sqrt{\begin{pmatrix} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{pmatrix}} \end{array} \right)^2$
D	$\pi_2^{D*} = a^2 \left(\frac{6+\sqrt{24-6c}}{16(c+2)}\right)^2$

A.2 Payoff Comparisons

This section contains the proofs of all the results presented in the paper. It is divided into three parts that correspond to the case in which the two firms are endowed with the same production technology (i.e., $t = 1$), the case in which the two firms are heterogeneous with respect to their production efficiency (i.e., $t < 1$), and the case of two unrelated monopolies (i.e., $c = 0$), respectively.

Before we analyze each of the three cases in detail, we establish the following general result.

Lemma A1 *For any a, c, and t, the following equivalences hold:*
(i) $\pi_u^{B_1*} = \pi_u^{B_2*}$.
(ii) $\pi_1^{B_2*} = \pi_2^{B_1*} = \pi_1^{A*} = \pi_2^{A*}$.
(iii) $\pi_1^{B_1*} = \pi_2^{B_2*}$.
(iv) $\pi_1^{D*} = \pi_2^{D*}$.

Proof of Lemma A1. Equivalences (i)–(iv) follow directly from the results we report in Table 2. ■

A.2.1 Homogeneous Firms

Proof of Proposition 1. (i) Using the equilibrium payoffs to the union as displayed in Table 2, note that when $t = 1$, the following inequalities hold for any $a > 0$ and for any $c \in (0, 1]$:

$$\pi_u^{A*} - \pi_u^{B_1*} = a^2 \frac{48 - 24c - 15c^2 - (4-c)\sqrt{144 - 15c^2 + 24c}}{256(c+2)^2} < 0,$$

$$\pi_u^{A*} - \pi_u^{D*} = a^2 \frac{6 - 3c - 2\sqrt{24 - 6c}}{64(c+2)} < 0,$$

and

$$\pi_u^{B_1*} - \pi_u^{D*} = a^2 \frac{3c^2 + 24c + (4-c)\sqrt{144 - 15c^2 + 24c} - (16 + 8c)\sqrt{24 - 6c}}{256(c+2)^2} < 0.$$

Since when $t = 1$ the bargaining environments C and D coincide, this part of the theorem is established by combining these inequalities with Lemma A1.
(ii) Using the equilibrium payoffs to firm 1 as displayed in Table 2, note that when $t = 1$, the following inequalities hold for any $a > 0$ and for any

$c \in (0,1]$:

$$\pi_1^{A*} - \pi_1^{B_1*} = 3a^2 \frac{48 + c^2 + 8c - (4-c)\sqrt{144 - 15c^2 + 24c}}{512\,(c+2)^2} > 0,$$

$$\pi_1^{A*} - \pi_1^{D*} = 3a^2 \frac{14 + c - 2\sqrt{24 - 6c}}{128\,(c+2)^2} > 0,$$

and

$$\pi_1^{B_1*} - \pi_1^{D*} = 3a^2 \frac{8 + 8\sqrt{24 - 6c} - c^2 - 4c + (4-c)\sqrt{144 - 15c^2 + 24c}}{515\,(c+2)^2} > 0.$$

Since when $t = 1$ the bargaining environments C and D coincide, this part of the theorem is established by combining these inequalities with Lemma A1. ■

A.2.2 Heterogeneous Firms

Proof of Lemma 1. For any game $g = A, B_1, B_2, C_1, C_2, D$ and for any player $j = u, 1, 2$, the results we report in Table 2 imply that $\pi_j^{g*} = a^2 f_j^g(c,t)$, where $f_j^g : \Omega \rightarrow R^+$ is a continuous and differentiable function. Hence, for any pair of games $g \neq h = A, B_1, B_2, C_1, C_2, D$ and for any player $j = u, 1, 2$, the sign of the difference $(\pi_j^{g*} - \pi_j^{h*})$ is independent of a. The proof of the second part of the lemma is contained in the proof of Proposition 2. ■

Proof of Proposition 2. Since for any player $j = u, 1, 2$, π_j^{A*}, $\pi_j^{B_1*}$, $\pi_j^{B_2*}$, and π_j^{D*} do not depend on t, Proposition 1 and Lemma A1 imply that for any $a > 0$ and for any $(c,t) \in \Omega$, the following relations hold:

$$\pi_u^{A*} < \pi_u^{B_1*} = \pi_u^{B_2*} < \pi_u^{D*},$$

$$\pi_1^{D*} < \pi_1^{B_1*} < \pi_1^{B_2*} = \pi_1^{A*},$$

and

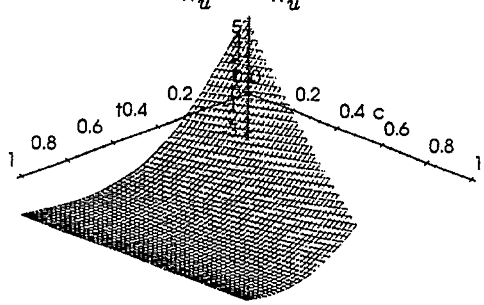
$$\pi_2^{D*} < \pi_2^{B_2*} < \pi_2^{B_1*} = \pi_2^{A*}.$$

The characterization of the remaining payoff comparisons for the union, firm 1, and firm 2, respectively, is substantially more involved, and it entails algebraic manipulations of the expressions reported in Table 2. Since none of the derivations provides useful intuition for the results and since all equilibrium payoffs are continuous and differentiable functions of the parameters of the derivations provides useful intuition for the results and since all equilibrium payoffs are continuous and differentiable functions of the parameters of the model, we present here a graphical characterization of these payoff comparisons. The Maple V file containing all the analytic derivations is available from the authors upon request.

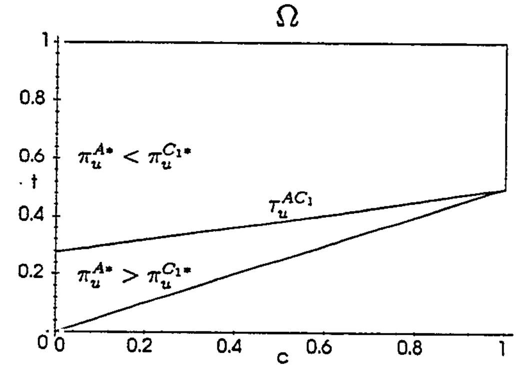
For any player $j = u, 1, 2$, for any relevant pair of games $g \neq h = A, B_1, B_2, C_1, C_2, D$, we present two pictures. The first picture displays the graph of the difference $(\pi_j^{g*} - \pi_j^{h*})$ as a function of $(c, t) \in \Omega$. Without loss of generality, to generate these graphs we set $a = 10$. The second picture summarizes the relation between π_j^{g*} and π_j^{h*} in the parameter space Ω.

Union:

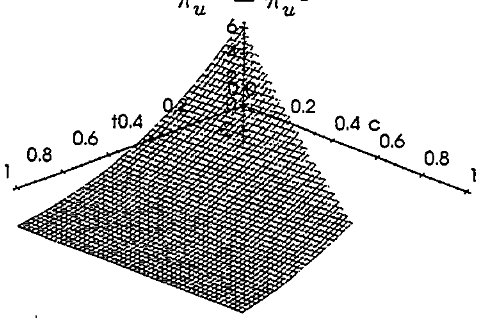
$\pi_u^{A*} - \pi_u^{C_1*}$



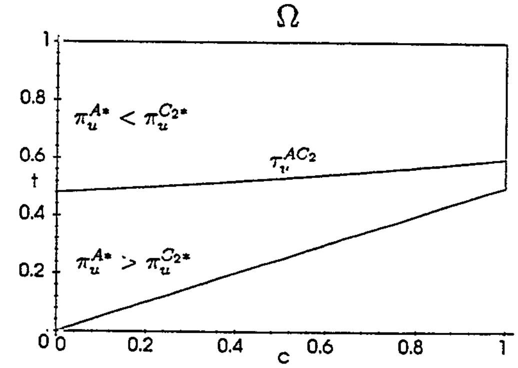
Ω

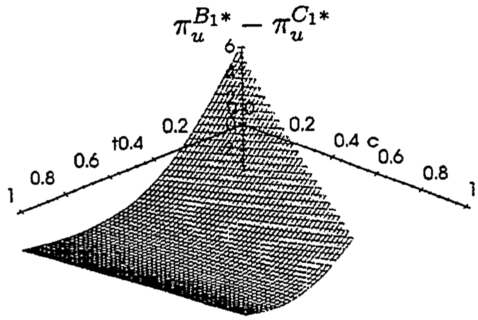


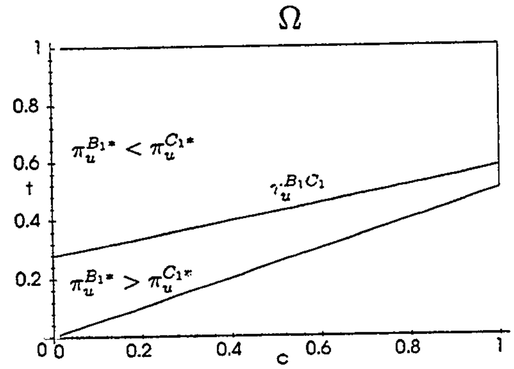
$\pi_u^{A*} - \pi_u^{C_2*}$

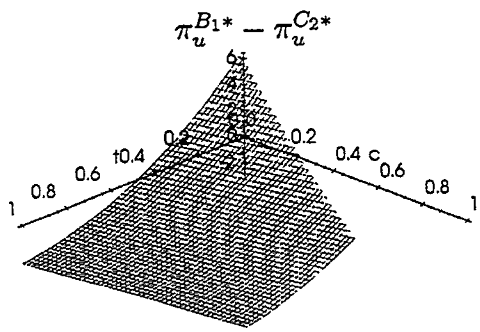


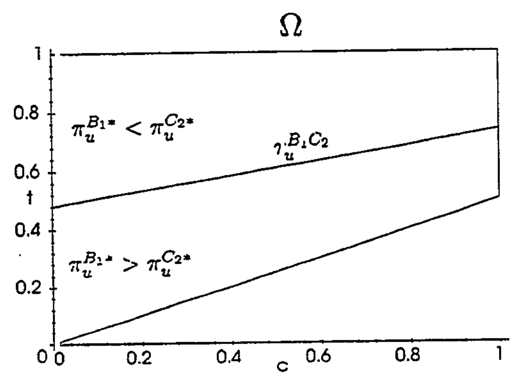
Ω

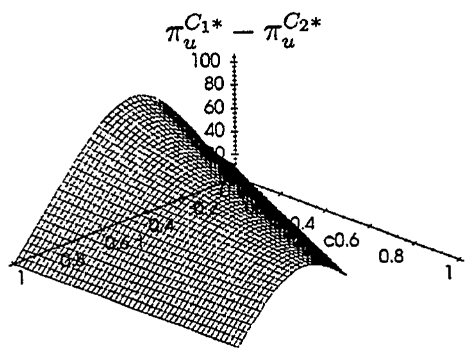


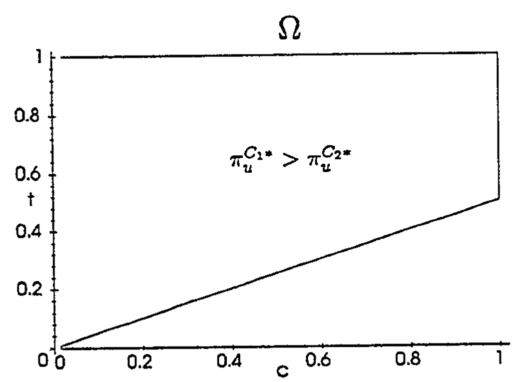

$\pi_u^{B_1*} - \pi_u^{C_1*}$
t
c

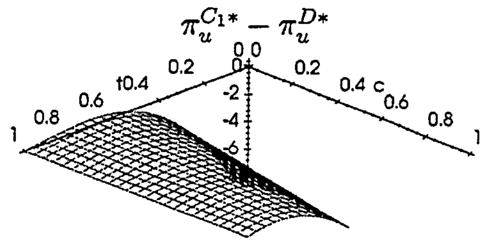

Ω
$\pi_u^{B_1*} < \pi_u^{C_1*}$
$\gamma_u^{B_1C_1}$
$\pi_u^{B_1*} > \pi_u^{C_1*}$
t
c

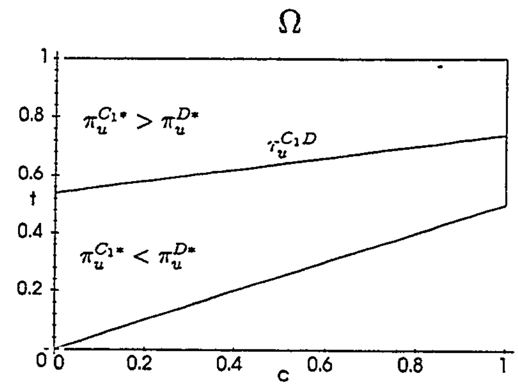

$\pi_u^{B_1*} - \pi_u^{C_2*}$
t
c

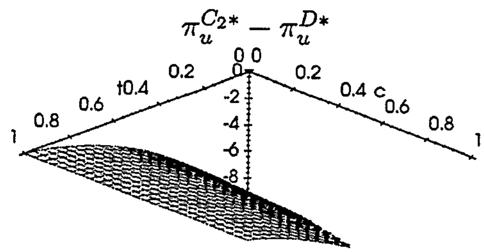

Ω
$\pi_u^{B_1*} < \pi_u^{C_2*}$
$\gamma_u^{B_1C_2}$
$\pi_u^{B_1*} > \pi_u^{C_2*}$
t
c

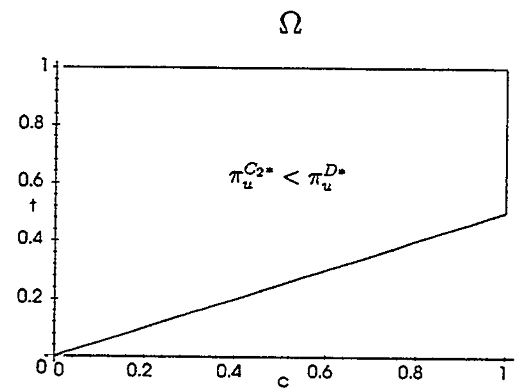

$\pi_u^{C_1*} - \pi_u^{C_2*}$
c


Ω
$\pi_u^{C_1*} > \pi_u^{C_2*}$
t
c

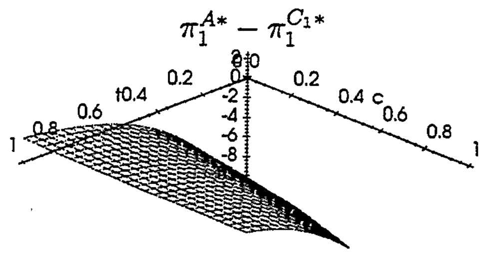

$\pi_u^{C_1*} - \pi_u^{D*}$
t
c

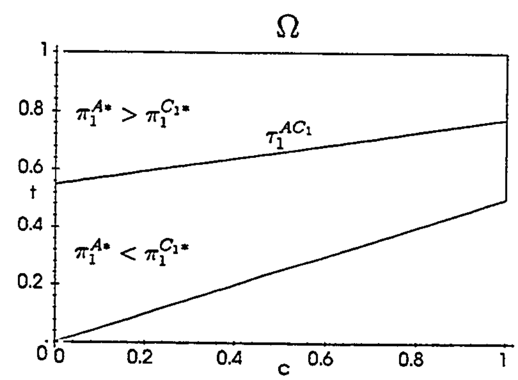

Ω
$\pi_u^{C_1*} > \pi_u^{D*}$
$\tau_u^{C_1 D}$
$\pi_u^{C_1*} < \pi_u^{D*}$
t
c

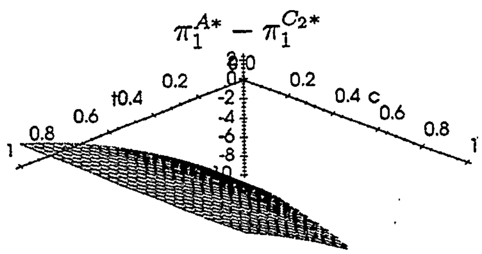

$\pi_u^{C_2*} - \pi_u^{D*}$
t
c


Ω
$\pi_u^{C_2*} < \pi_u^{D*}$
t
c

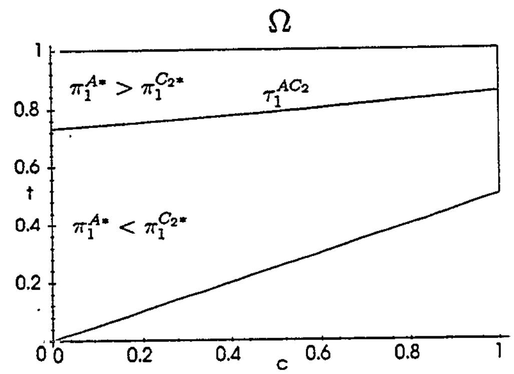
Firm 1:


$\pi_1^{A*} - \pi_1^{C_1*}$
t
c

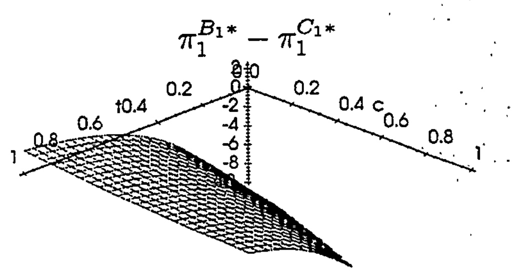

Ω
$\pi_1^{A*} > \pi_1^{C_1*}$
$\tau_1^{AC_1}$
$\pi_1^{A*} < \pi_1^{C_1*}$
t
c



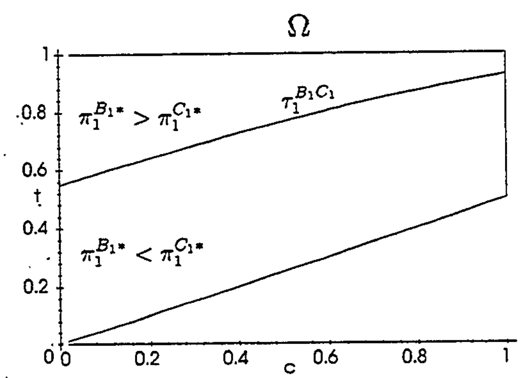
$\pi_1^{A*} - \pi_1^{C_2*}$



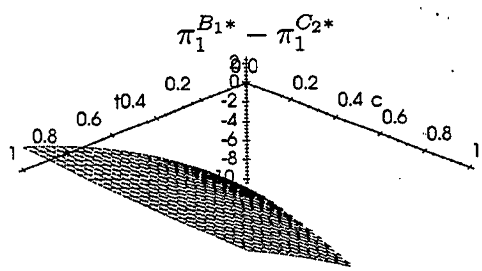
Ω
$\pi_1^{A*} > \pi_1^{C_2*}$
$\tau_1^{AC_2}$
$\pi_1^{A*} < \pi_1^{C_2*}$
t
c



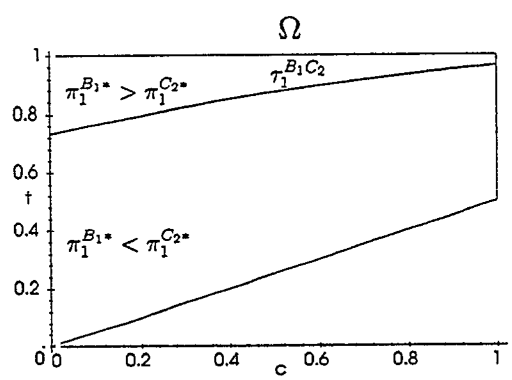
$\pi_1^{B_1*} - \pi_1^{C_1*}$



Ω
$\pi_1^{B_1*} > \pi_1^{C_1*}$
$\tau_1^{B_1C_1}$
$\pi_1^{B_1*} < \pi_1^{C_1*}$
t
c

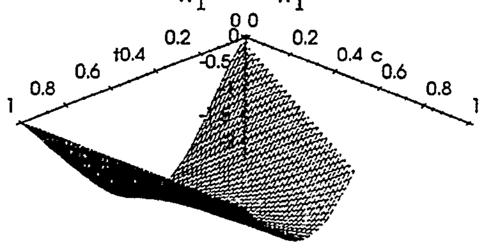


$\pi_1^{B_1*} - \pi_1^{C_2*}$

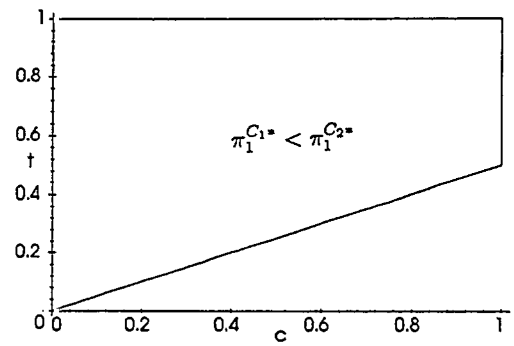


Ω
$\pi_1^{B_1*} > \pi_1^{C_2*}$
$\tau_1^{B_1C_2}$
$\pi_1^{B_1*} < \pi_1^{C_2*}$
t
c

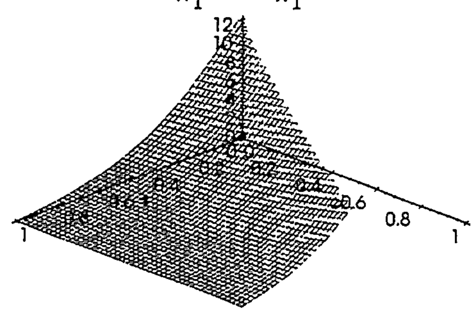
$\pi_1^{C_1*} - \pi_1^{C_2*}$



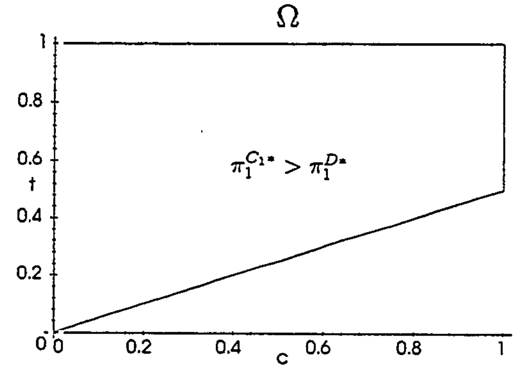
Ω



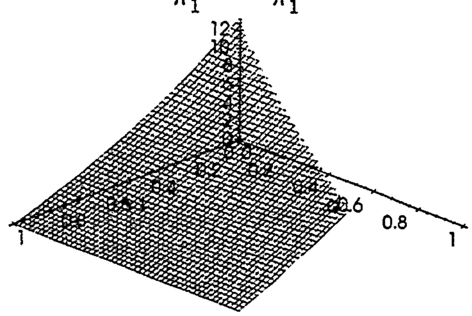
$\pi_1^{C_1*} - \pi_1^{D*}$



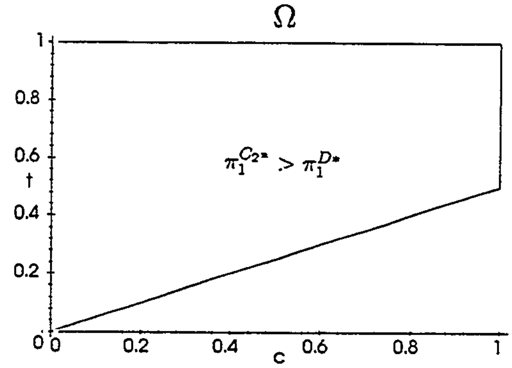
Ω



$\pi_1^{C_2*} - \pi_1^{D*}$

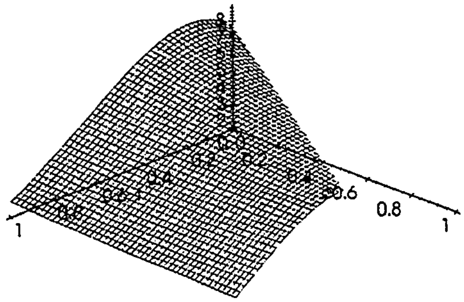


Ω

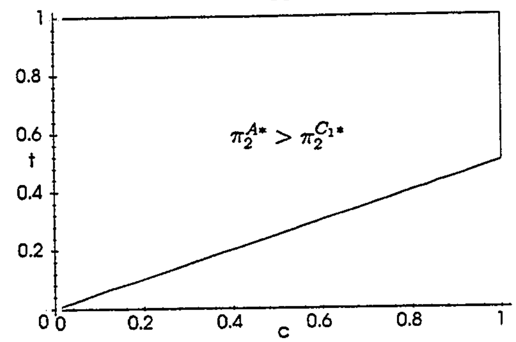


Firm 2:

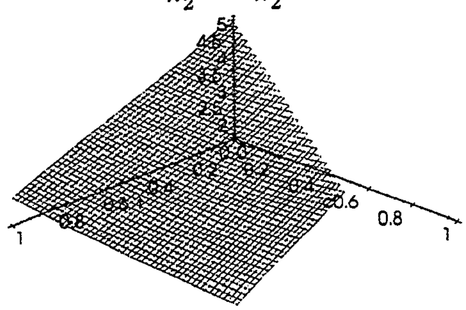
$\pi_2^{A*} - \pi_2^{C_1*}$



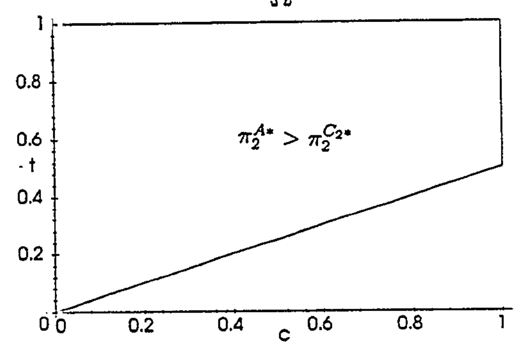
Ω



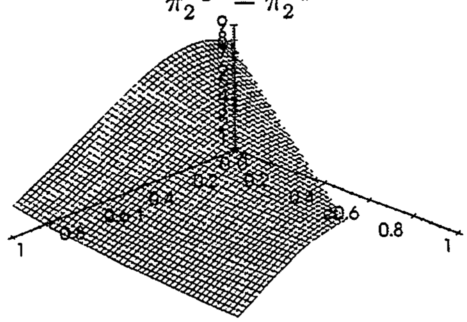
$\pi_2^{A*} - \pi_2^{C_2*}$



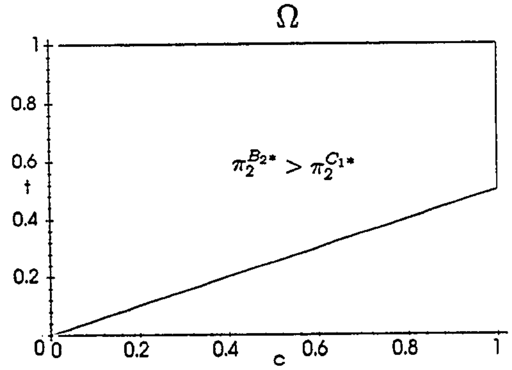
Ω



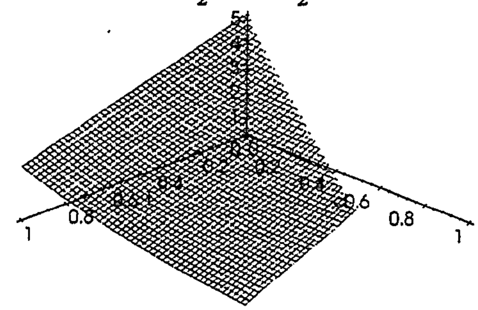
$\pi_2^{B_2*} - \pi_2^{C_1*}$



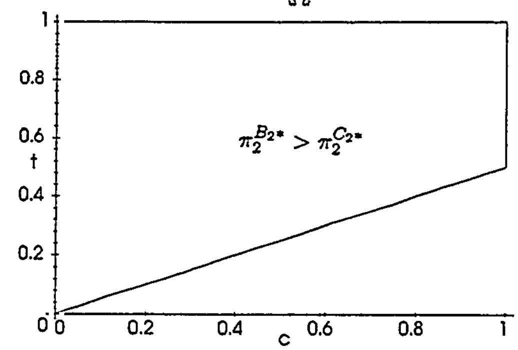
Ω



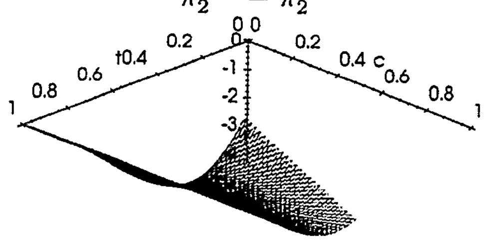
$\pi_2^{B_2*} - \pi_2^{C_2*}$



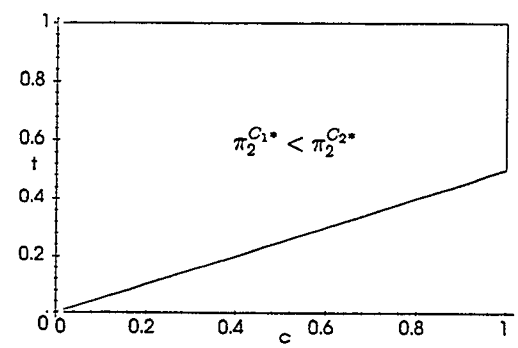
Ω



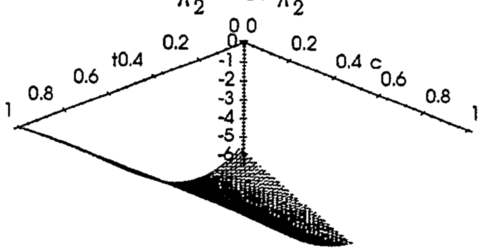
$\pi_2^{C_1*} - \pi_2^{C_2*}$



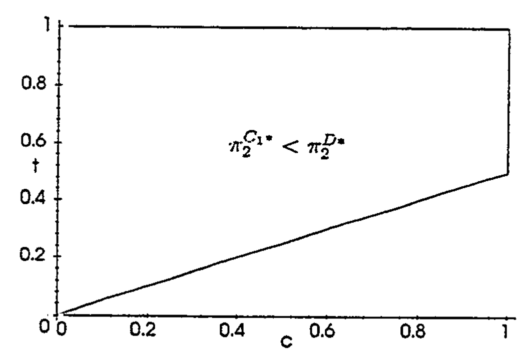
Ω

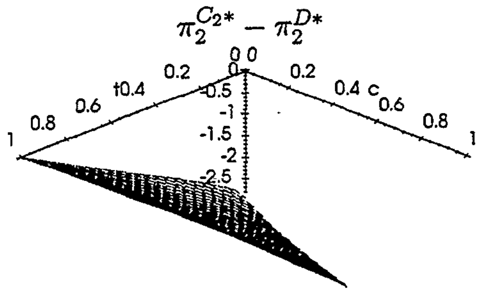


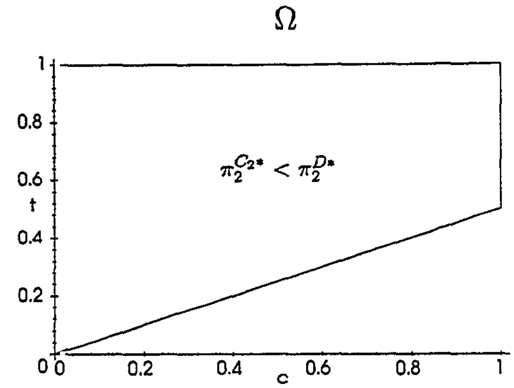
$\pi_2^{C_1*} - \pi_2^{D*}$



Ω







The proof of the theorem is completed by combining these binary relations with the equivalence results contained in Lemma A1. ■

Proof of Proposition 3. Since the Nash bargaining solution is Pareto efficient, to prove the theorem, it is sufficient to show that both under pattern bargaining in wages and under pattern bargaining in costs, given the (efficient) outcome of the first negotiation, the union and the other firm cannot achieve a mutually beneficial agreement by reneging on their commitment. We show that this is indeed the case by showing that given the equilibrium wage rate paid by the firm that engaged in the first negotiation, the derivatives of the payoff functions of the union and the other firm, respectively, with respect to the wage rate paid by that firm, evaluated at the equilibrium wage rate have the opposite sign.

Environment C: Pattern Bargaining in Wages.

C_1*: Firm* 1 *is the target.*

$$\frac{\partial}{\partial w_2}\pi_u(w_1^{C_1*}, w_2)\mid_{w_2=w_2^{C_1*}} = \frac{a}{8t\left(4-c^2\right)\left(2t-c\right)\left(t^2-tc+1\right)}S_u$$

and

$$\frac{\partial}{\partial w_2}\pi_2(w_1^{C_1*}, w_2)\mid_{w_2=w_2^{C_1*}} = \frac{a}{4t\left(4-c^2\right)^2\left(2t-c\right)\left(t^2-tc+1\right)}S_2,$$

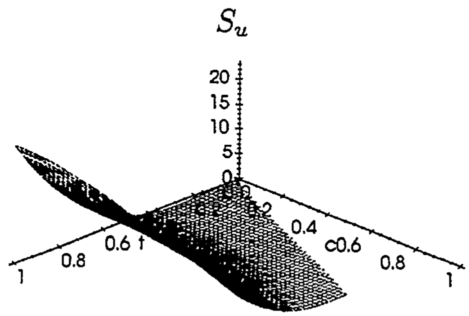
where

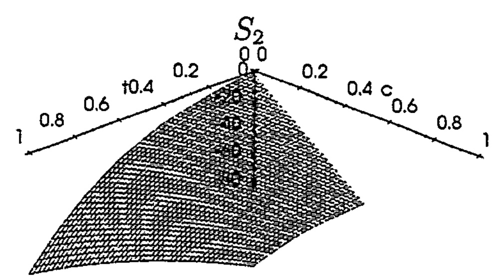
$$S_u = \left(\frac{(2-c)\begin{pmatrix} -8-7c^2t^2+5c^2t+10ct^3+18tc- \\ 18ct^2-2c+16t^3+4t-20t^2 \end{pmatrix} + (2-ct)\cdot}{\sqrt{2-c}\sqrt{\begin{pmatrix} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{pmatrix}}} \right)$$

and

$$S_2 = \left(\frac{(2-c)\begin{pmatrix} 8+7c^2t^2-13c^2t+10c-10ct^3- \\ 18tc+42ct^2-20t-32t^3+20t^2 \end{pmatrix} - (2-ct)\cdot}{\sqrt{2-c}\sqrt{\begin{pmatrix} 32+6c^4t-47c^3t^2+2c^3t-15c^3+104c^2t+ \\ 22c^2t^2-2c^2+92c^2t^3-56tc-48ct^3- \\ 60ct^4-172ct^2+56t^2-32t+112t^3+24t^4 \end{pmatrix}}} \right).$$

To evaluate the sign of the two derivatives, note that the first term in both expressions is positive. Hence, the sign of each derivative is determined by the sign of S_u and S_2, respectively. The graphs of S_u and S_2 in the parameter space Ω are depicted in the following pictures.





Hence, $\frac{\partial}{\partial w_2}\pi_u(w_1^{C_1*}, w_2)\mid_{w_2=w_2^{C_1*}} > 0$ and $\frac{\partial}{\partial w_2}\pi_2(w_1^{C_1*}, w_2)\mid_{w_2=w_2^{C_1*}} < 0$ for any $a > 0$ and for any $(c,t) \in \Omega$.

C_2: Firm 2 is the target.

$$\frac{\partial}{\partial w_1}\pi_u(w_1, w_2^{C_2*})\mid_{w_1=w_1^{C_2*}} = \frac{a}{8\,(4-c^2)\,(2-tc)\,(t^2-tc+1)} S_u'$$

and

$$\frac{\partial}{\partial w_1}\pi_1(w_1, w_2^{C_2*}) \mid_{w_1=w_1^{C_2*}}= \frac{a}{4\left(4-c\right)^2\left(2-tc\right)\left(t^2-tc+1\right)}S_1,$$

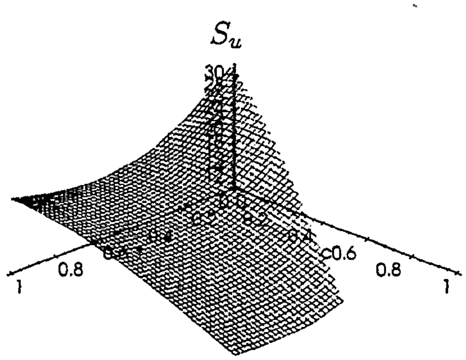
where

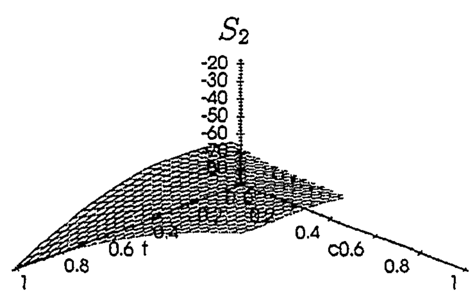
$$S_u' = \left(\begin{array}{c} (2-c)\begin{pmatrix} 16+5c^2t^2-7c^2t+10c-2ct^3+ \\ 18ct^2-18tc-8t^3+4t^2-20t \end{pmatrix} + (2t-c)\cdot \\ \sqrt{2-c}\sqrt{\begin{pmatrix} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{pmatrix}} \end{array} \right)$$

and

$$S_1 = \left(\begin{array}{c} (2-c)\begin{pmatrix} -32-13c^2t^2+7c^2t+10ct^3-10c- \\ 18ct^2+42tc+8t^3-20t^2+20t \end{pmatrix} - (2t-c)\cdot \\ \sqrt{2-c}\sqrt{\begin{pmatrix} 24+6t^3c^4+2t^3c^3-47c^3t^2-15t^4c^3+92c^2t+ \\ 22c^2t^2-2t^4c^2+104c^2t^3-172ct^2-56ct^3- \\ 48tc-60c+56t^2+112t-32t^3+32t^4 \end{pmatrix}} \end{array} \right).$$

To evaluate the sign of the two derivatives, note that the first term in both expressions is positive. Hence, the sign of each derivative is determined by the sign of S_u' and S_1, respectively. The graphs of S_u' and S_1 in the parameter space Ω are depicted in the following pictures.





Hence, $\frac{\partial}{\partial w_2}\pi_u(w_1, w_2^{C_2*}) \mid_{w_1=w_1^{C_2*}}> 0$ and $\frac{\partial}{\partial w_1}\pi_1(w_1, w_2^{C_2*}) \mid_{w_1=w_1^{C_2*}}< 0$ for any $a > 0$ and for any $(c, t) \in \Omega$.

Environment D: Pattern Bargaining in Costs.

$$\frac{\partial}{\partial w_2}\pi_u(w_1^{D*}, w_2) \mid_{w_2 = w_2^{D*}} = a\frac{-2+\sqrt{24-6c}}{8t\,(c+2)} > 0$$

and

$$\frac{\partial}{\partial w_2}\pi_2(w_1^{D*}, w_2) \mid_{w_2 = w_2^{D*}} = a\frac{-6-\sqrt{24-6c}}{4t\,(c+2)\,(4-c^2)} < 0$$

for any a, c, and t. ■

A.2.3 Unrelated Monopolies

Looking at the equilibrium payoffs we report in Table 2, we see that there are no discontinuities at $c = 0$. Thus, Corollary 1 readily extends to the case of two unrelated monopolies. In addition, note that when $c = 0$, for any $a > 0$ and for any $t \in (0, 1]$, we have that $\pi_u^{A*} = \pi_u^{B_1*} = \pi_u^{B_2*}$ and $\pi_1^{A*} = \pi_2^{A*} = \pi_1^{B_1*} = \pi_1^{B_2*} = \pi_2^{B_1*} = \pi_2^{B_2*}$.

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