Federal Reserve Bank of Minneapolis
Research Department Staff Report 247

May 1998

Comments on Gordon, Leeper, and Zha's "Trends in Velocity and Policy Expectations"

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ABSTRACT

I argue that low-frequency movements in U.S. base velocity are well explained by standard models of money demand. The model of Gordon, Leeper, and Zha is not standard because they assume a very high interest elasticity. The positive conclusion that they reach about the model's ability to mimic movements in velocity necessarily implies that predicted movements in interest rates are too smooth.

*This discussion was prepared for the 1997 Carnegie-Rochester Conference on Public Policy. I received very helpful comments from my colleagues at the Federal Reserve Bank of Minneapolis. I especially thank Hal Cole and Patrick Kehoe. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Gordon, Leeper, and Zha (hereafter GLZ) construct a model to explain low-frequency movements in U.S. base velocity over the period 1960–1997. In the data, trends in velocity are mirrored by trends in interest rates; hence, these movements in velocity are well explained by a standard money demand equation with an interest elasticity around one-third. GLZ find this explanation inadequate and propose their own model of money demand and an alternative methodology for explaining the observed trends in velocity. However, their model of velocity effectively boils down to a standard money demand equation with a very high interest elasticity of one. Thus, given observed movements in interest rates, their model necessarily predicts movements in velocity that are too large. Alternatively, given observed movements in velocity, their model necessarily predicts movements in interest rates that are too smooth.

GLZ argue that it is a mistake to use observed interest rates to uncover the predictions of standard money demand models for velocity movements because these models do not produce the high-frequency liquidity effects that are thought to be in the data. They argue that the right interest-rate series can be constructed by using the model to write the current interest rate as a function of expectations of future policy variables, like inflation, and then a Bayesian vector autoregression (BVAR) model to fill in the relevant expectations. I do not find this argument convincing, because GLZ's dispute with using interest rates has to do with a high-frequency issue, but the question of interest is a low-frequency one. In response to their argument, I show that when I abstract from these high-frequency movements by using longer-term interest rates, my original claim stands: Low-frequency movements in velocity are well explained by low-frequency movements in observed interest rates. Thus, I do not find the BVAR analysis helpful in answering the question addressed by GLZ.

In my comments, I elaborate on these points.

1 Is there a puzzle?

For many models of money, equilibrium behavior imposes a static relationship between the demand for real money balances and a measure of income and an interest rate. At the Carnegie-Rochester conference ten years ago, Lucas (1988) reviewed and replicated the results of Meltzer (1963), who estimated a money demand equation of the form

$$\ln m_t = a_0 + a_1 \ln y_t - a_2 \ln i_t + \varepsilon_t \tag{1}$$

where m_t is the stock of real balances, y_t is real income, i_t is an interest rate, ε_t is an error term, the subscript t indexes time, and a_0, a_1, and a_2 are parameters. With thirty additional years of data, Lucas (1988) confirmed the findings of Meltzer (1963). In particular, he found a stable money demand relation, an income elasticity (a_1) near one, and an interest elasticity (a_2) around one-third.[1]

Lucas's (1988) and Meltzer's (1963) results are relevant for GLZ, who are interested in trends in velocity. Assuming a unit income elasticity of money demand and a money demand equation of the form (1) implies that movements in velocity (y_t/m_t) are directly related to movements in interest rates (i.e., $\ln v_t = -a_0 + a_2 \ln i_t$, where $v_t = y_t/m_t$). Consider the data in Figure 1. I plot GLZ's measure of velocity (defined to be consumption plus investment expenditures divided by base money) over the period 1960–1997 along with 3-month Treasury bill rates. Both series are annualized, and both are logged, since the relationship in (1) suggests a linear relationship between the logarithm of velocity and the logarithm of interest rates.

As Figure 1 illustrates, over the period 1960–1980, there was a large increase in velocity. After 1980, velocity flattened out, and in the 1990s, it fell significantly. In 1995, velocity was at the same level it had been in 1975. Over this same period, there is a similar pattern for the 3-month Treasury bill rate. Rates went from a little over 2 percent in 1960 to about 15 percent in 1981. Rates then fell. By 1995, rates had fallen slightly below their 1975 levels. That is, trends in interest rates coincide with trends in base velocity. These facts suggest to me that there is no puzzle with regard to secular movements in base velocity. Furthermore, if I run a regression of the logarithm of velocity on the logarithm of the 3-month Treasury bill rate over the 1960–1997 period (using quarterly data), I find an interest elasticity equal to 0.28—very close to Lucas's (1988) estimate of one-third.

I should note here that typical measures of velocity use M1 rather than the monetary base. With M1 velocity, there is indeed a puzzle as far as standard money demand theory is concerned. As interest rates fell in the 1980s and 1990s, M1 velocity did not. This pattern has been attributed to technological changes in transactions such as the introduction of ATMs, the increased use of credit cards, and financial deregulation. These changes have the effect of increasing velocity.

2 Why GLZ miss this point

In their paper, GLZ develop a model that imposes a static relationship between velocity and interest rates. The logarithm of velocity in equilibrium is given by

$$\ln v_t = \ln\left(\psi\left(\frac{k_t}{y_t}, \frac{g_t}{y_t}\right)\right) + b_1 \ln\left(\frac{i_t}{1+i_t}\right) \tag{2}$$

where v_t is the ratio of consumption to real balances, ψ is a function that depends on both the ratio of capital to output k_t/y_t and the ratio of government spending to output g_t/y_t, i_t is an interest rate, t indexes time, and b_1 is a parameter. Note that the expression in (2) depends on the capital-output ratio and government share. However, $\psi(k/y, g/y)$ varies very little over the sample. If this term is replaced by a constant, the results reported by GLZ are barely changed. Note also that $\ln(i_t/(1+i_t))$ is approximately equal to $\ln i_t$ for interest rates on the order of those observed. Therefore, as in the earlier money demand literature, GLZ's theory implies a log-linear relationship of the form

$$\ln v_t = b_0 + b_1 \ln i_t. \tag{3}$$

That is, theory implies that secular movements in velocity are directly tied to secular movements in interest rates.

One important distinction should be noted, however. For GLZ, the interest elasticity (b_1) is equal to one rather than something on the order of one-third. A high elasticity implies that small changes in interest rates lead to large changes in velocity. For example, if the interest rate changed from 2 percent to 15 percent as it did between

1960 and 1980, equation (3) predicts that the change in the logarithm of velocity is on the order of 2 (or $\ln(15) - \ln(2)$). Figure 1 shows that the change in the logarithm of velocity between 1960 and 1980 is on the order of $\frac{1}{2}$.[2]

But GLZ never worry about this high interest elasticity because, unlike Lucas (1988) and Meltzer (1963), they do not focus on the static relationship in (2). The reason they say is that their model does not do a good job explaining liquidity effects. Lucas (1988), faced with this problem as well, noted that one could work with either annual data or longer-term interest rates. The focus is, after all, *secular* movements in velocity. In fact, if one compared longer rates with GLZ's velocity measure, one would find a similar pattern as in Figure 1. In Figure 2, I plot the logarithm of GLZ's velocity measure over the period 1960–1997 along with 10-year Treasury note rates. Both series have been logged. Notice again that the trend in interest rates tracks the trend in base velocity. Again, this suggests to me that there is no puzzle with regard to secular movements in base velocity.

3 What GLZ actually do

GLZ choose to ignore (2) and instead focus their attention on another necessary condition of their model. In the model of GLZ, velocity can also be expressed in terms of expectations of future policies as follows:

$$\ln v_t = \ln \left(E_t \phi \left(\left\{ \frac{g_s}{y_s} \right\}_{s=t}^{\infty}, \{\tau_s\}_{s=t+1}^{\infty} \right) \right) - b_1 \ln \left(E_t \left(1 + \beta \frac{M_t}{M_{t+1}} + \beta^2 \frac{M_t}{M_{t+2}} + \dots \right) \right) \tag{4}$$

where τ_t is the tax rate on net output in period t, M_t is nominal money balances in period t, and $b_1 = 1$. Of course, for the model to be internally consistent, it has to be true that the first terms on the right side of (2) and (4) are equal to each other and that the second terms on the right side of (2) and (4) are equal to each other. In other words, it has to be true that the capital-output and government spending ratios summarize expectations of future fiscal policies and that interest rates summarize expectations of future monetary growth rates.

What GLZ do is construct expectations of future policy variables by estimating the parameters of a second model, a BVAR model

$$Y_t = B(L)Y_{t-1} + u_t, \tag{5}$$

where Y_t is a vector consisting of four variables: base money, the ratio of federal government spending to output, the ratio of federal government transfers to output, and the ratio of federal tax receipts to nominal output. From this BVAR, they calculate expectations of future policies and substitute them into the right side of equation (4). (This is the series marked 'Bayesian Updating' in Figure 4 of GLZ's paper). This series represents the main result of their paper. In Figure 3, I plot this series along with the the prediction of velocity constructed using the sample mean for $\ln(E_t\phi(\{g_s/y_s\}, \{\tau_s\})$ in (4) in place of the estimated series. I do this to see what role the fiscal variables play. Notice that the two curves lie almost on top of each other. The fiscal variables play no role as is typically assumed in the money demand literature.

To get to this point, I have done little with GLZ's original monetary model.

Thus far, I have only used the assumption that velocity and policy expectations are related functionally as in (4). But GLZ do not use anything more from the original model in estimation. GLZ's prediction for velocity over the period 1960–1997 is based on the expectations from the BVAR model.

4 Flawed logic

At this point, I will recap. From the money demand literature, it is well-known that trends in base velocity are well explained by trends in interest rates when one assumes interest elasticities on the order of one-third, i.e.,

$$\ln v_t = \text{constant} + \frac{1}{3} \ln i_t. \tag{6}$$

GLZ's theory implies the following:

$$\ln v_t = \text{constant} + \ln i_t. \tag{7}$$

Equation (7) is not consistent with the data because the interest elasticity is too high.[3] But GLZ actually work with

$$\ln v_t = \text{constant} - \ln E_t \left(1 + \beta \frac{M_t}{M_{t+1}} + \beta^2 \frac{M_t}{M_{t+2}} + \dots \right) \tag{8}$$

where the expectations are found by estimating a BVAR model. They conclude that the right side of (8) does "surprisingly well" when compared to velocity data. But I find that (7) does poorly when I fill in v and i from the data. This must mean that the policy expectations in (8) do poorly when compared to interest rates—despite the fact that theory would predict otherwise.

5 What about interest rates?

In an equilibrium of the monetary model, (2) must hold. What if we substitute the predicted velocity from the BVAR into this equation to derive an implied interest rate? Does the implied interest rate look like interest rates in the data? The answer is no. In Figure 4, I plot the interest rate i_t^{BVAR} that satisfies

$$\ln v_t^{BVAR} = \ln \psi_t^{BVAR} + \ln \left(\frac{i_t^{BVAR}}{1 + i_t^{BVAR}} \right) \tag{9}$$

(which is the series plotted in Figure 5 of GLZ's paper) along with the 3-month Treasury bill rate. The interest rate implied by the BVAR is much smoother and has a significantly higher mean.

The figure looks this way for two reasons. First, as I noted earlier, the money demand equation of GLZ has an interest elasticity of one (i.e., $b_1 = 1$), which is much higher than the empirical estimates reported in Lucas (1988) and elsewhere. In this case, small changes in interest rates are consistent with large changes in velocity. Second, the fiscal variables play almost no role. Although GLZ use federal government spending which has fallen significantly over the period 1960–1997, changes in g/y have a very small impact on $\psi(k/y, g/y)$. If fiscal variables make a difference for the trend in velocity, then GLZ's theory would say that it is due to secular movements in the capital-output ratio. Over the period 1960–1997, there has been little change in the capital-output ratio.

6 Conclusions

As I noted earlier, GLZ argue that it makes sense to work with policy expectations from a BVAR model rather than interest rates directly because "observed rates will to some degree reflect the liquidity aspects abstracted from in the model." Since the topic of their paper is trend velocity, this argument seems to me to be a nonsequitur.

Because interest rates summarize the relevant policy expectations in the model, I see no reason for using a BVAR model. There may be other applications in which one might have to forecast expectations of future policies, because no current price summarizes these expectations or because we have no analogue price in the data. However, even if this were true, I am not swayed by the argument of GLZ that estimating such expectations with a BVAR model allows one to be agnostic about policy. The BVAR model is not assumption-free.

Notes

[1]Lucas (1988) estimates a semi-elasticity equal to 0.07. Using a mean of 4.67 for the period 1900–1985 implies an interest elasticity around one-third.

[2]Despite the fact that their interest elasticity is outside the range of estimates found in empirical studies of money demand, GLZ issue warnings about the fact that standard money demand regressions could have biased estimates due to the absence of terms like $\psi(k_t/y_t, g_t/y_t)$ in (2).

[3]This relationship and the one that follows are only approximately true because I am assuming that $\ln(i_t) \approx \ln(i_t/(1+i_t))$ and that the term with arguments k_t/y_t and g_t/y_t is essentially constant. See Figure 3.

References

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Meltzer, A.H.

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Fig. 1. Velocity and Short-Term Interest Rate, 1960-97

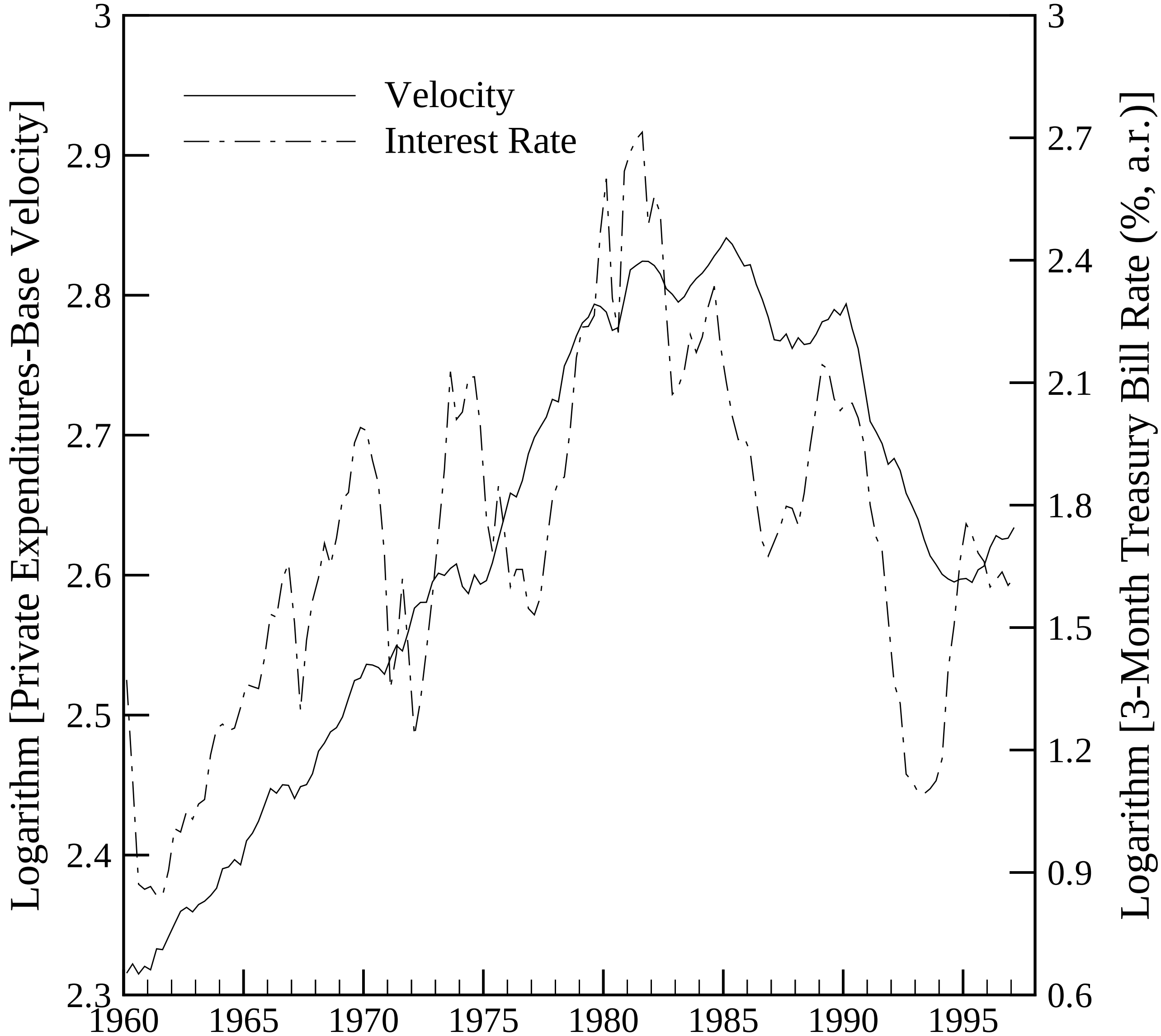

Fig. 2. Velocity and Long-Term Interest Rate, 1960-97

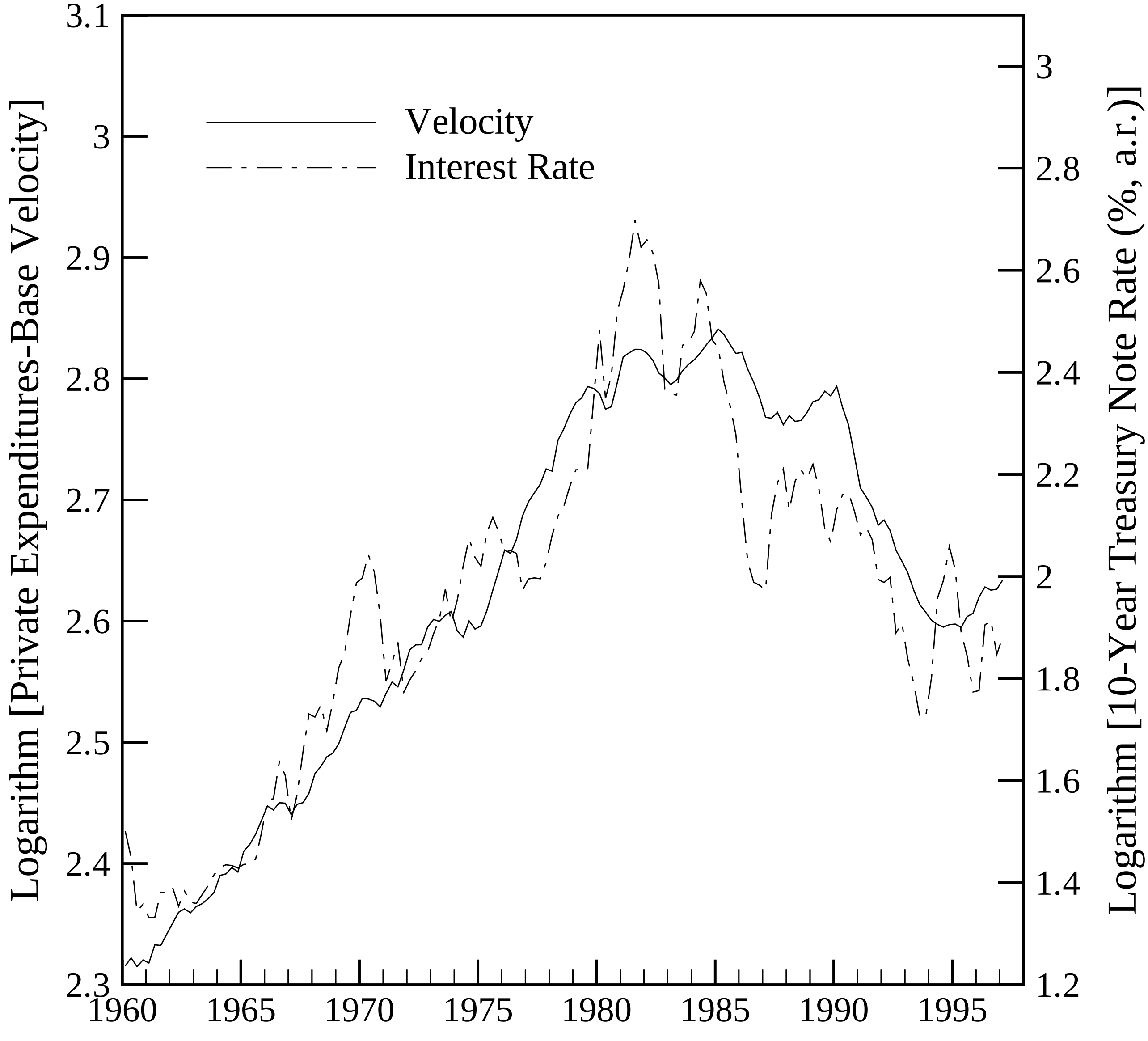

Fig. 3. Actual and Predicted Base Velocity, 1960-97

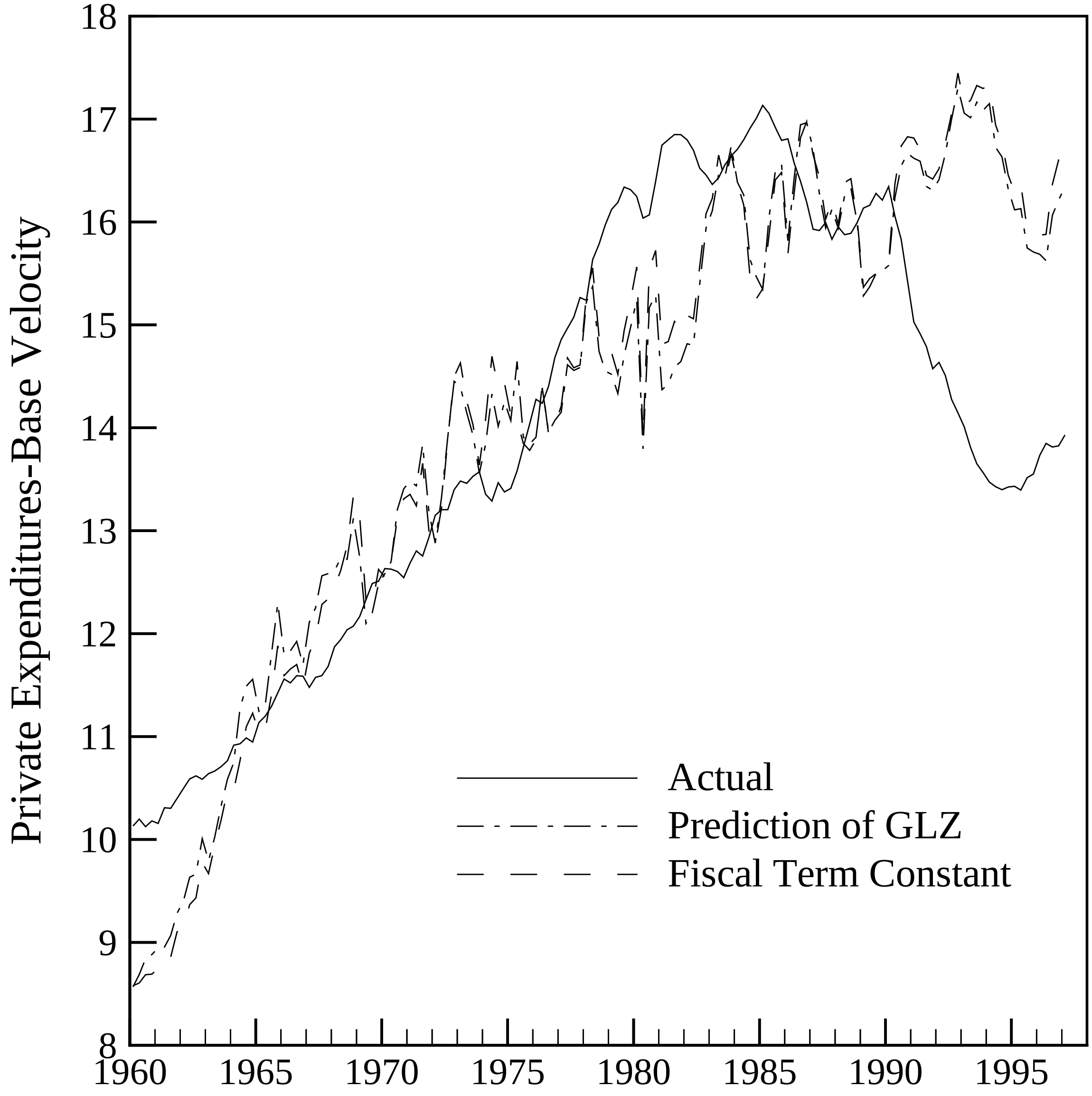

Fig. 4. Actual and Predicted Short-Term Rate, 1960-97