The Inefficiency of Interest-Bearing

National Debt

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The coexistence of money and default-free interest-bearing goverment bonds is explained by transaction costs; the private sector absorbs money with less real difficulty than it absorbs bonds. Under the assumption that the costs of issuing money and issuing bonds are identical, it follows that the presence of government bonds is inefficient. Further, the steady-state inflation rate is higher with bond financing of a given real deficit because there is less net output, less real saving, and hence the need for the government to inflate faster. This is demonstrated in a version of Samuelson's pure consumption loans model.

The Inefficiency of Interest-Bearing National Debt

In what sense, if any, does interest on the national debt
represent a cost? And could the cost be avoided by financing any
deficit by issuing money instead of issuing bonds? The conventional
responses seem to be (i) interest on the national debt constitutes a
transfer payment rather than a cost in terms of resources; and (ii)
financing a given deficit by money issue is more inflationary than
financing it by bond sales and, perhaps, so inflationary that the
financing cannot be accomplished by money issue. Our answers are
considerably different.

In Section I we argue that almost any way of explaining positive
interest on government bonds implies that part or all of the interest
bill on the national debt represents a real resource cost. Since govern-
ment bonds are default-free, real transactions costs must be invoked to
explain the coexistence of fiat money paying zero nominal interest and
bonds selling at prices implying positive nominal interest. In the
simplest case we describe, the nominal interest rate on bonds reflects
only the proportional real transactions costs of transforming bonds into
intermediary liabilities that have all the properties of money.

Given that the presence of interest bearing bonds implies real
transaction costs and that money does not, bond financing of deficits is
inefficient (because resources are wasted chopping bonds up into money).[1]
Further, the steady-state inflation rate is higher with bond financing
because there is less net output, less real saving at each rate of
return, and hence, a need for the government to inflate faster to finance
a given real deficit. All of this is demonstrated in Section II in the
context of a version of Samuelson's pure consumption-loans model.

In a sense, our resource cost explanation of interest on goverment bonds is not new. The famous Baumol-Tobin and Miller and Orr inventory models of the demand for money are consistent with this explanation. Those models account for positive money holdings in the presence of safe "higher yielding" assets by positing transaction costs--trips to the bank, phone calls to the broker, etc. What is new is our insistence that the transaction costs be carried along into the analysis of debt vs. money financing of deficits. While most macroeconomic analyses implicitly or explicitly accept the asset demand functions implied by the inventory models, all fail to take into account the transaction costs and the variations in them that accompany various positive interest rates on safe assets.

Can the omission of transaction costs be unimportant on the ground, say, that in terms of total available resources, the resources involved in "trips to the bank" are insignificant? We think not. In the inventory models, for example, if the costs are left out, then money and bonds become perfect substitutes when their yields are the same. It follows immediately, then, that it does not matter how deficits are financed.

In what follows we work with transaction-cost models that are simpler than the inventory models. Moreover, our models lack a great deal of the the richness that economists attempt to incorporate into their models. But our models do represent coherent economies. Hence, at a minimum, they constitute valid counter-examples to widely held notions about the effects of different ways of financing deficits.

I. The Real Resource Cost of a National Debt

Why do government bonds--which everyone regards as certain or default-free titles to fiat money in the future--pay interest? Put more precisely, given that individuals are willing to hold fiat money from time t-1 to time t, how can the price at t-1 of a government bond that matures and pays X units of fiat money at t be other than X? And coupons aside, if the price of the bond at t-1 must be X, then its price at t-2 must be X and so on.

We regard certain kinds of answers as unacceptable.

For us, it is not an answer to say that the future price of government bonds is random and, hence, that risk aversion implies that such bonds must bear a positive expected rate of return. Why is the price random? One way to make the nominal price of the bond described above random is to make the payment at time t random. One could assume, for example, that the government at t decides whether or not holders of bonds at t will get a bonus in addition to X.[2] Or one could assume that the government at t decides whether or not holders of money at t will get a bonus. Since such random bonuses seem not to be important, we will stick to the notion that government bonds (and fiat money) are titles to known and nonrandom streams of fiat money.

Nor do we regard as acceptable the answer that government bonds pay interest because money is a medium of exchange and government bonds are not. We need an explanation of why government bonds do not serve as a medium of exchange.

Our claim is that almost any way of accounting for positive nominal interest on government bonds implies that part or all of the

total interest bill on the outstanding debt represents a real resource cost. Put formally, our claim is that most models that have an equilibrium in which government bonds pay interest display the property that part or all of the interest bill represents a real resource cost.

We will support this claim by describing the implications of several ways of accounting for positive interest on government bonds. All the models we describe are discrete time in the sense that all economic activity—in particular, the operations of all markets—occur at discrete points in time: t, t+1, ...,. In addition, there is only a single nonstorable consumption good in all the models we study. And since we assume that government bonds are finite maturity titles to fiat money, they have value in terms of the consumption good if and only if fiat money has value. That being so, we will be making observations about properties of equilibria in which both fiat money and government bonds have value. We call such equilibria monetary equilibria.[3]

1. Indivisible Government Bonds and Intermediation

Our first model of positive interest on government bonds assumes:

(i) Government bonds are zero coupon one-period bonds. They are issued in denominations so large that no individual can afford to hold them if they sell at no discount.

(ii) There is a constant cost intermediation technology available to everyone that converts government bonds into perfectly divisible intermediary liabilities.[4]

(iii) All potential savers are identical.

(iv) Everyone regards zero-interest intermediary liabilities as identical to fiat money.

In order to draw out the implications of these assumptions, it is necessary to elaborate assumption (ii) on intermediation costs. It is helpful to begin by setting out the consolidated income statement of the intermediation industry at time t under the assumption that it holds all the government bonds issued and sold at t, the number of which is denoted $J(t)$.

<div align="center">

Intermediation Income Statement

Expenditures	Receipts
$J(t)S(t)$	$J(t)F(t)$
$cJ(t)/P(t)$	

</div>

The entries are all in units of fiat money. Thus the first item on the expenditure side represents purchases of government bonds; $S(t)$ is the market price at t of a government bond in units of fiat money. The second item on the expenditure side represents the dollar value of resource costs of intermediation; c is the cost in terms of the date t consumption good of making one government bond divisible and $P(t)$ is the value of money in units of the consumption good (the inverse of the price level). The single item on the receipt side represents the proceeds of the sale by the intermediaries of divisible titles to time t+1 fiat money; $F(t)$ is the face value in units of fiat money of each of the government bonds sold at t.

To see why $J(t)F(t)$ constitutes period t receipts, note that the intermediaries will receive $J(t)F(t)$ units of fiat money from the government at t+1 when they turn in their bonds. Thus, they will have that amount to turn over to their customers. But assumption (iv) says that if there is an equilibrium in which both fiat money and intermediary liabilities have value, then intermediary liabilities pay a zero nominal

interest rate. It follows that $J(t)F(t)$ represents time t intermediary receipts. (We assume that the time t+1 paying-off activities of the intermediaries are carried out without cost.)

But free entry into intermediation implies that any equilibrium must display zero profits for the intermediation industry. Therefore, we have

(1) $S(t) = F(t) - c/P(t)$

or

$P(t)[J(t)F(t)-J(t)S(t)] = cJ(t).$

The first expression says that the current nominal price of each bond is less than its face value by the dollar value of resource costs per bond. The equivalent second expression says that the real value of total interest payments equals the total resource cost of carrying out intermediation.

The above results depend upon our assumption (i) that no individual can afford to hold the bonds directly. To see that we are not ignoring possible equilibria in which the government bonds are held directly, we now show that the assumption of such an equilibrium leads to a contradiction for appropriately chosen parameter values.

First of all, if bonds are held directly, then $S(t) = F(t)$, for at any other price of government bonds all individuals want to specialize in either money ($S(t) > F(t)$) or bonds ($S(t) < F(t)$). But if $S(t) = F(t)$, then the total real value of bonds and money is $P(t)[M(t)+J(t)F(t)]$, where $M(t)$ is the number of units of fiat money outstanding. Now suppose per capita saving in real terms is constant

and equal to s. (In Section II this will be seen to be a special case
of the general model we examine.) Then in such an equilibrium

$$P(t)[M(t)+J(t)F(t)] = Ns$$

where N is the number of savers. But it is easy to produce values of
M(t), J(t), F(t), N, and s for which the solution for P(t) is such that
the value of a single bond, P(t)F(t), exceeds the saving of any one
person. Thus, if s=1, J(t)=9, F(t)=1,000, M(t)=10,000, N=1,000; then
P(t)=.1 and P(t)F(t)=100.

So for this model we can easily choose parameter values that
imply that any equilibrium in which both bonds and money have value is
such that the total interest bill on the national debt constitutes a
real resource cost.

2. Customer Transaction Costs

Here we want to indicate briefly the consequences of abandoning
assumption (iv). While maintaining assumptions (i) through (iii), we
now suppose that each purchaser of intermediary liabilities bears a
fixed cost in units of the consumption good, the cost, say, of the
notorious trip to the bank.

Given such a customer cost, intermediary liabilities must bear
interest in any monetary equilibrium. In terms of the above income
statement of the intermediation industry, the single receipt item is
not J(t)F(t) but something smaller. Then, the zero-profit condition
implies an equilibrium value of S(t) smaller than that given by equation (1).
In other words, the interest rate on government bonds must be high
enough to cover both intermediation costs and direct customer costs.
And, although the details are somewhat complicated and of no particular

interest, for parameter values that prevent bonds from being held directly, any equilibrium for this model has the following features: (i) some savers hold only intermediary liabilities and the rest hold only fiat money; (ii) the real value of the total interest bill on the government debt equals the sum of total intermediary costs and the total of the direct costs of those who end up holding intermediary liabilities.

3. Transfer Payment Components of the Interest Bill

As we noted in the introduction, conventional wisdom has it that interest on the national debt constitutes a transfer payment from tax payers in general to holders of the debt. We are now going to present a model that is at least partly consistent with this view. Our point is that complete consistency with this view is hard to come by.

As is to be expected, the way to get transfer elements into the interest bill is to drop the assumption that all potential savers are identical. So let us do that while maintaining assumptions (ii) and (iv) and a version of assumption (i).

Suppose there are N_1 rich potential savers and N_2 poor potential savers. For the moment, let s_1 be the constant amount of per capita real saving of the rich and s_2 be that of the poor.

For this model, an equilibrium in which government bonds pay interest and in which the entire interest bill represents a transfer has the following features.

(a) The rich must be able and want to hold the bonds directly. The poor must be unable to hold the bonds directly. (If the poor can hold them directly, then by assumption (iv) the bonds sell at face value.)

(b) All bonds must be held directly by the rich. Intermediation must not be profitable.

(c) All money must be held by the poor. (If the rich hold money, then, again, by assumption (iv) bonds must sell at face value.)

These conditions imply that there must be positive values of P(t) and S(t) that satisfy the following five conditions.

(2) $\quad S(t) < F(t)$

(3) $\quad s_2 < P(t)S(t) \leq s_1$

(4) $\quad J(t)S(t)P(t) = N_1 s_1$

(6) $\quad S(t) \geq F(t) - c/P(t)$

(5) $\quad P(t)M(t) = N_2 s_2$

where the first two conditions are a consequence of (a), the next two are a consequence of (b), and the last is a consequence of (c). Dropping t's, and letting $j = J/N_1$ and $m = M/N_2$, by (4) we can write (3) as

(7) $\quad s_2/s_1 \leq 1/j \leq 1.$

By (2) and (5)

$$P(t)S(t) \leq P(t)F(t) \leq P(t)S(t) + c$$

which by (4) and (6), we may write as

(8) $\quad (s_1/j)/(s_2/m) \leq F \leq (c+s_1/j)/(s_2/m).$

In terms of the size of the debt, parameter values for which (7) and (8) hold constitute a "shelf-edge." The debt must be big enough so that the

rich cannot afford to hold it at face value but not so big that they can afford to hold it only at a discount so large that intermediation becomes profitable.

If the debt is smaller than this "shelf-edge," then a monetary equilibrium displays zero interest on government debt, all of which is held by the rich. If the debt is larger, then a monetary equilibrium displays positive interest on government debt with the rich holding only debt and the poor holding all the money and the intermediary liabilities. In the latter kind of equilibrium, the interest bill is partly a real resource cost and partly a transfer.

If the supply of debt varies over time or fluctuates sufficiently, the nominal yield on it may also fluctuate. This fluctuation in the nominal yield is consistent with the existence of a demand for government debt that varies directly with the nominal yield on that debt. Of course, with a constant size debt, varying numbers of rich and poor over time could also produce a varying nominal yield on government debt. In any case, nominal yields are positive only because of the presence of transaction costs. If intermediation costs are eliminated from the model we have been describing, then no matter how the supply of debt or the number of rich and poor fluctuates over time, there could not be interest on the debt.

4. Uncertainty and Diversified Portfolio

At the beginning of this section we stated that we do not consider a random return per se an adequate explanation of a positive expected rate of return on government bonds. Indeed, the models presented so far do not exhibit both a random rate of return on government bonds and diversified portfolios. Since our argument depends upon our models

being, in some sense, realistic, we now want to indicate how to alter
the last model so that individuals face uncertain returns on government
bonds.

Suppose the outstanding supply of government bonds at time t
consists of two-period bonds--promises to pay X units of fiat money at
time t+2. At time t+1, these two-period bonds become perfect substitutes
for newly issued one-period bonds. Let the supply of such newly issued
bonds be unknown as of time t. For simplicity, assume also that no new
two-period bonds will be issued and sold at time t+1. Then, from our
analysis in the last section, we know what will happen at time t+1
conditional on the total amount of one-period bonds outstanding at that
time. If that amount is small--and if we suppose there is a Federal
Reserve that can engage in open market purchases, then the amount could
be very small--then one-period bonds will sell at par at t+1. If the
amount is large--so large that intermediaries necessarily hold some of
them--then they sell at less than par, say, at $\overline{S}(t+1) < X$. Suppose
these are the only two possibilities.

Now what can we say about the situation at t? A rich person
chooses a portfolio consisting of fiat money and two-period bonds.
Letting $S(t)$ be the price of such bonds at t, we have several facts
about the demands of the rich: if $S(t) \leq \overline{S}(t+1)$, then two-period bonds
dominate fiat money. If $S(t) \geq X$, then fiat money dominates bonds so
this is not possible. Suppose that at $S(t) = \overline{S}(t+1)$, the demand for
bonds on the part of the rich--who at such a price want to specialize in
bonds--exceeds the supply of bonds. It follows, then, that in a mone-
tary equilibrium, $\overline{S}(t+1) < S(t) < X$ and that each risk-averse rich
person holds a diversified portfolio consisting of some fiat money and
some two-period bonds.

5. Summary

We have now sketched several models that imply interest on "safe" government debt. In some of them the entire interest bill represents a real resource cost. In others it is only partly a resource cost and can at times be entirely a transfer payment. But situations in which there is interest on government debt and in which the interest bill has no real resource cost component can arise, it would seem, only by accident and only because the model is such that there is a resource cost component to the interest bill at some supplies of debt.

II. Money- and Bond-Financing of Deficits in Samuelson's Pure Consumption Loans Model

This is a model of a discrete time economy in which N new two-period lived people appear each period.[5] Generation t is young at t and old at t+1, and each member of generation t maximizes $u[C_1(t), C_2(t)]$, $C_j(t)$ being age j consumption of the one nonstorable consumption good. The $C_j(t)$ are superior goods, and u is twice differentiable and has strictly convex upper contours. Each young person is endowed with y units of the nonstorable consumption good.

At any date t, there are 2N people in this economy, the N members of generation of t-1 (the old) and the N members of generation t (the young). We take as our task the description of the evolution of this economy from some arbitrary initial date which we label t=1. At t=1 the current old--the members of generation 0--own among them M(0) units of fiat money. This stuff is perfectly divisible and neither depreciates nor appreciates physically with the passage of time. The preferences of the old are simple; they attempt to maximize $C_2(0)$. Toward that end, they offer all their money holdings. In general, the government also offers money or bonds in an amount sufficient to allow it to get command of a given amount of the consumption good, an amount we denote D. We assume that assumptions (i)-(iv) of Section I.1 hold, although, as will be obvious, the specific form that intermediation costs take does not matter.

For us, the evolution of this economy is described by a competitive, perfect foresight equilibrium or by an equilibrium, for short. An allocation is a description of life-time consumption for all members of all generations $t \geq 1$ and second-period consumption for the

current old. An allocation and a sequence of values of money are an equilibrium if they are consistent with optimizing behavior of the young in every period given perfect foresight and with period-by-period market clearing: in each period the assets supplied must equal the assets demanded. As already noted, such an equilibrium is called monetary if the money of the current old has value in terms of the consumption good.

1. The Choice Problem of the Young

If follows from assumptions (i), (iii), and (iv) that no matter how the deficit is financed, each young person faces a single gross real rate-of-return on savings $P(t+1)/P(t) \equiv R(t)$. Then assuming that there are lump-sum taxes levied on the young in an amount $0 \leq T(t) < y$ and that $R(t) > 0$, lifetime consumption for this young person is given by

$$C_1(t) = y - T(t) - s(t)$$

$$C_2(t) = R(t)s(t)$$

where $s(t)$ is real savings, the single choice variable of the young. Let $v(C_1, C_2) \equiv u_1(C_1, C_2)/u_2(C_1, C_2)$ be the marginal rate of substitution function. Then assuming perfect foresight, for any $R(t) > 0$ the unique value of $s(t)$ that maximizes u is the solution to

$$v[y-T(t)-s(t), R(t)s(t)] = R(t)$$

which is

(9) $$s(t) = H[R(t),T(t)].$$

Here H, a real valued function, is defined for each pair $R(t) \epsilon (0, \infty)$ and $T(t) \epsilon [0, y)$, is differentiable with $H_2 \epsilon (-1, 0)$ and is such that $0 < H < y$.[6]

2. Equilibrium

Let $Z(t) \geq 0$ be total resources used up in intermediation at date t. Then in any equilibrum in which the government succeeds in financing its deficit, we have as an identity (at t+1)

$$(10) \qquad Ny = NC_2(t) + NC_1(t+1) + D + Z(t+1); \quad t \geq 0.$$

But in equilibrium, we also have for $t \geq 1$

$$NC_2(t) = NR(t)H[R(t),T(t)]$$

and

$$Ny - NC_1(t+1) = Ns(t+1) + NT(t+1).$$

So letting $d = D/N$ and $z(t) = Z(t)/N$, we have in equilibrium

$$(11) \qquad R(t)H[R(t),T(t)] = s(t+1) + T(t+1) - d - z(t+1).$$

A monetary equilibrium is a pair of positive sequences $R(t)$ and $s(t)$ that satisfy (9) and (11) for all $t \geq 1$.

3. A Money-Financed Deficit

Here, by assumption, there are no government bonds and no taxes and so $T(t) = Z(t) = 0$ for all t.

Proposition 1: There exists a monetary equilibrium with money-financing of the deficit D if and only if H is such that

$$(12) \qquad (1-R)H(R,0) \geq d \text{ for some } R > 0.$$

Proof: Sufficiency. If (12) holds, then there exists R* > 0 such that $(1-R^*)H(R^*,0) = d$. But then $s(t) = H(R^*,0)$ for all $t \geq 1$ satisfies (9) and (11).

Necessity. We suppose that there is a monetary equilibrium with money financing of D and that (12) does not hold. As we show, this gives rise to a contradiction.

Subtracting $H[R(t),0] = s(t)$ from both sides of the applicable version of (11), we have

$$[1-R(t)]H[R(t),0] = d - \Delta s(t)$$

where $\Delta s(t) \equiv s(t+1) - s(t)$. Thus, by the negation of (12), $\Delta s(t) > 0$ for all t. Then since any equilibrium $s(t)$ sequence is bounded (by y), we have $s^* \equiv \lim s(t) > d$. The inequality follows from $s(t) > d$ which is implied by (11). But the existence of such a limit s^* that satisfies (9) and (11) implies satisfaction of (12), a contradiction.

Before going on to bond-financed deficits, we should, perhaps, comment on what it means for there not to exist a monetary equilibrium. In this model--and in any good model of fiat money--there exist nonmonetary equilibria, equilibria in which the value of fiat money is zero in every period. If the government attempts to finance too large a deficit, then the nonmonetary equilibrium is the only equilibrium.[7]

4. A Bond-Financed Deficit

Here, by assumption, $z(t) > 0$ for all t. Also $0 \leq T(t) \leq z(t)$. Thus, we assume that bond financing uses up some resources and that taxes are bounded by this resource cost.

Proposition 2: If there exists a monetary equilibrium with bond-financing of the deficit D, then (12) holds.

Proof: We derive a contradiction from assuming that there
exists such an equilibrium and that (12) does not hold. Upon subtract-
ing $H[R(t), T(t)] = s(t)$ from both sides of (11) we have

(13) $[1-R(t)]H[R(t),T(t)] = d + [z(t+1)-T(t+1)] - \Delta s(t).$

We will first prove that $\Delta s(t) > z^*(t+1) \equiv z(t+1) - T(t+1) \geq$
0. If $R(t) \geq 1$ this is immediate from (13). If $R(t) < 1$, then by $H_2 <$
0 and the negation of (12), we have

(14) $[1-R(t)]H[R(t),T(t)] \leq [1-R(t)]H[R(t),0] < d$

But (13) and (14) imply $\Delta s(t) > z^*t+1$). Now if $z^*(t+1)$ does not approach
0, this is a contradiction since any equilibrium $s(t)$ sequence is bounded.
If $z^*(t+1)$ does approach 0, then there must exist a limit $\bar{s} > d$ and
limits $\bar{T}$ and $\bar{R}\varepsilon(0, 1)$ such that $s(t) = s(t+1) = \bar{s} = H(\bar{R},\bar{T})$ satisfies (9)
and (11) with $z^*(t+1) = 0$. But (14) rules this out.

Having proved Proposition 2, the inefficiency of a monetary
equilibrium with bond-financing is an immediate consequence. Inefficiency
is established by producing a feasible allocation with a total consump-
tion path that dominates that of the bond-financing equilibrium. Proposi-
tion 2 says that if a bond-financing allocation is feasible, then a
money-financing allocation is feasible. The fact that the total consump-
tion path of the latter dominates that of the former is immediate by
(10); bond financing involves a real resources cost.

5. Inflation Rates

As something of an aside, we now prove

Proposition 3: If there is a monetary equilibrium with bond-financing of D and with limiting values $\bar{z}$ = limit z(t+1) > 0, $\bar{s}_B$ = limit s_B(t) and $\bar{R}_B$ = limit R_B(t), then there exists a solution, R*, to (1-R*)H(R*,0) = d with R* > $\bar{R}_B$.

Proof: By (13), $\bar{R}_B$ and $\bar{s}_B$ satisfy

$$(1-\bar{R}_B)H(\bar{R}_B,\bar{T}) \geq d$$

with equality only if limit z*(t+1) = 0, and, hence, only if $\bar{T}$ > 0. Then, by H_2 < 0, (1-$\bar{R}_B$)H($\bar{R}_B$,0) > d, which implies the existence of the required R*.

Proposition 3 says that the limiting value of the inflation rate under bond-financing of D (if it exists) exceeds the inflation rate of an equilibrium with money-financing of D.

6. An Example

Our purpose here is to show that propositions 2 and 3 are not vacuous in the following sense. What we have done so far is consistent with the possibility that bond-financing is never feasible. We now show by example that there exist inefficient (and convergent) monetary equilibria with bond-financing of the deficit D > 0.

We assume u(C_1, C_2) = lnC_1 + b*lnC_2, b* > 0. This implies that s(t) = H[R(t),T(t)] = b[y-T(t)] where b = b*/(1+b*) ϵ (0, 1). And we will now use assumptions (i)-(iv) and the implied zero-profit condition, equation (1). Finally, we add assumption (v): P(t)F(t) = F where F > max(c, yb) where, as above, c is the real intermediation cost per bond.

Assumption (v) says that the government varies the nominal face value of its bonds to keep the real value of the face value at a sufficiently high constant value.

We examine, in turn, three financing schemes: (a) money-financing; (b) full bond-financing, that is, $T(t) = 0$ for all $t \geq 1$ and $M(0) = M(t)$ for all $t \geq 1$; and (c) bond-financing with $T(t)$ equal to the per capita time t cost of intermediation, $z(t)$, and with $M(0) = M(t)$ for all $t \geq 1$. In all cases, we assume $J(0) = 0$.

(a) Money-Financing

In this case, since $T(t) = z(t) = 0$ by assumption, we simply substitute $s(t) = yb$ for all $t \geq 1$ into the implied form of (11) to get the following result:

> If $1 - d/yb \equiv R* > 0$, then there is a monetary equilibrium with money-financing of the deficit and $R(t) = R*$ for all $t \geq 1$.

(b) Full Bond-financing

Under any bond-financing scheme $z(t) = cj(t)$ where, as above, $j(t)$ is the per capita number of bonds sold by the government at t. Since in this case $T(t) = 0$, (11) takes the form

$$(15) \qquad R(t)yb = yb - d - cj(t+1).$$

Now we must describe the evolution of $j(t)$. Since $T(t) = 0$ and no new money is issued, bond sales at t+1 must be sufficient to finance the deficit and to pay off previously issued bonds at face value; or

$$(16) \qquad P(t+1)S(t+1)J(t+1) = D + P(t+1)F(t)J(t), \quad t \geq 0$$

where, as above, $S(t)$ is the price in units of fiat money of a bond issued at t.

Now using equation (1), the zero-profit condition for $t+1$, we may write (16) as

(17) $P(t+1)F(t+1)J(t+1) - cJ(t+1) = D + R(t)P(t)F(t)J(t)$.

Then using assumption (v)—$P(t)F(t) = F$—and dividing by N, we have

(18) $(F-c)j(t+1) = d + R(t)Fj(t)$.

Upon substituting $R(t)$ for $t \geq 1$ from (15) into (18) we have

(19) $(F-c)j(t+1) = d + Fj(t)[yb-cj(t+1)-d]/yb; \quad t \geq 1$.

Study of this first-order difference equation—for which, by (18), we have as an initial condition $j(1) = d/(F-c)$—implies the following result:

> If $1 - d/yb - c/F \equiv \bar{R} > 0$, then there is a unique j sequence
> that satisfies (19) and the initial condition; this sequence
> is strictly increasing with limit $j(t) = yb/F$ and, by (15),
> implies $R(t) > 0$ for all t and limit $R(t) = \bar{R}$.

(c) Bond-financing With Taxes Equal to Intermediation Costs

Here, by assumption, $T(t) = z(t) = cj(t)$ for all $t \geq 1$. Therefore, (11) takes the form

(20) $R(t)b[y-cj(t)] = b[y-cj(t+1)] - d$.

And, again, we must describe the evolution of $j(t)$. Now in place of (16) we have that taxes plus new bond sales must be sufficient to finance the deficit and pay off previously issued bonds, or at t+1

(21) $cJ(t+1) + P(t+1)S(t+1)J(t+1) = D + P(t+1)F(t)J(t)$.

But, then, proceeding as we did for (16)--that is, using (1) and assumption (v), we may write (21) as

(22) $Fj(t+1) = d + R(t)Fj(t)$.

Then for $j(t) > 0$, we may solve (22) for $R(t)$ for $t \geq 1$ and substitute the result into (20) to get

(23) $[j(t+1)-d/F]b[y-cj(t)] = b[y-cj(t+1)]j(t) - dj(t)$.

This is a linear first-order difference equation in $j(t)$ for which we have an initial condition $j(1) = d/F$. Study of this difference equation implies the following result:

> If $1 - (d/yb)(1+bc/F) \equiv R > 0$, then there is a unique j
> sequence that satisfies (23) and the initial condition; this
> sequence is strictly increasing with limit $j(t) = yb/(F+bc)$,
> implies $R(t) > 0$ for all $t \geq 1$ and limit $R(t) = \tilde{R}$.

Since it is easy enough to choose values of b, c, d, F, and y consistent with both $\bar{R} > 0$ and $\tilde{R} > 0$ and with $F > \max(c, yb)$, we have now demonstrated that there are convergent monetary equilibria with bond-financing.

Notice that under money financing the inflation rate is $1/R^*$, which is lower than the inflation rate under either bond financing scheme. And yet, under the U.S. institutional setup everyone would say

that the central bank is following an extremely easy monetary policy

under the money financing scheme and an extremely tight monetary policy

under the two bond schemes.

Conclusion

How robust are the conclusions of propositions 2 and 3 that bond financing is less efficient and more inflationary than money financing? Will they emerge from other models?

An essential ingredient of our argument is the costliness in terms of real resources of bond issue relative to fiat money issue. And we do not prove this relative costliness. We assume it. Thus, our results depend on the reader not being able to produce models of positive nominal interest on government bonds without invoking something like the sort of real costs assumed in our Section I models. But the form that this relative costliness takes in our model is not necessary for propositions 2 and 3. Thus, for example, one could assume that intermediation resource costs depend on the number of the customers served, not on the number of bonds.

And what about the institutional setting? Would the conclusions of propositions 2 and 3 emerge for temporary or stochastic deficits, and, in a model with other assets? In particular, would they emerge in a model in which intermediaries hold as assets private debt and government debt?

Our guess is that the nature of the deficit does not matter so long as expectations are treated consistently in analyzing both money- and bond-financing. Thus, for example, while the financing by bonds at time t of a deficit known to be temporary is less inflationary at t than the financing by bonds of a permanent deficit of equal magnitude, this also holds for money-financing of temporary and permanent deficits.

Nor do we think that the conclusions of propositions 2 and 3 depend in an important way on whether other kinds of assets exist.

Whether they do or not, the model must, as it were, confront the margin between fiat money and "safe" government bonds.[8]

But this is not to say that the conclusions of propositions 2 and 3 will emerge from any model. In particular, we believe that by abandoning the assumption that everyone is the same, one can produce models for which these propositions do not hold.

First, note that if all the young face the same rate of return on saving and taxes are not varied in different directions for different individuals, then our proofs of propositions 2 and 3 hold when individuals are not identical. But if individuals differ, then for some ways of modeling intermediation costs different people can be faced with different rates of return on saving. This opens up possibilities that work like price discrimination.

Consider the rich-poor setup in Section I.3 in which bond financing implies a higher rate of return on savings for the rich than for the poor. If these groups also exhibit the "right" systematic differences in saving behavior, then bond-financing can succeed when money-financing cannot. Thus, suppose saving of the rich increases with increases in the rate of return while that of the poor decreases with increases in the rate of return. Then for each rate of return faced by the rich, total savings is larger if the poor face a lower rate of return (as under bond-financing) than if they face the same rate of return as do the rich (as under money-financing). And that should be enough to overturn propositions 2 and 3.

While one can construct counter examples to propositions 2 and 3, there is a standard price-theoretic interpretation of these propositions that, we think, argues for their robustness.

In the market, a Treasury bill with $10,000 face value sells for less than one thousand ten dollar bills. Barring special circumstances, this is inefficient--there are too many bonds outstanding--if, as we have been assuming, the government is indifferent in terms of real administrative costs between printing and selling the Treasury bill and printing the currency. Put simply, there is, then, a discrepancy between relative prices and relative production costs.

Of course, we are not sure that the governmental costs are the same. More generally, why should the real private costs of converting large denomination bonds into small denomination assets that people want be so different from the cost to the government?[9] If intermediation consists in doing no more than printing many pieces of paper instead of one, then such asymmetry would not seem to be justified. We have in mind that private intermediation involves rounding up customers, having a place of business, keeping records, and so on. But we must concede that we have not presented an explicit analysis of these aspects of the technology of intermediation. If we did, it is conceivable that the strong asymmetry we assume would disappear. We suspect there would remain an asymmetry since bond financing of the deficit seems to require both a bond sale and the disbursement of the proceeds, while money financing involves only the disbursement step. But again, we must concede that we have not modeled this. If asymmetry does not hold up, then our analysis implies that it does not matter, even for the price level, how a given deficit is financed. Such a conclusion is no kinder to the standard views about bond and money financing than is our inefficiency result.

Even with the government having a cost advantage in this particular kind of intermediation, there are special circumstances under

which it is not inefficient for bonds to sell at a discount. The price

discrimination possibility mentioned above is one. The proposition that

efficiency requires equality between relative prices and relative production

costs does not hold if price discrimination is allowed. It is well

known that perfect price discrimination is consistent with efficient

resource allocation. In addition, the presence of other distortions may

also vitiate the proposition.

One potentially relevant distortion is deposit insurance--that

provided by the FDIC and that provided by Federal Reserve lender-of-

last-resort activity. If the insurance is improperly priced--as it seems

to be--and if required reserves in the form of money are important in

limiting the size of insured intermediaries in the aggregate, then bonds

may have a role. While we plan to study this possibility in a subsequent

paper, it should be obvious that even in such a model there is no a

priori presumption in favor of the conventional view of the effects of

different ways of financing government deficits.

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[1]A path is inefficient if there exists another feasible path
that gives greater consumption in at least one period and no less consumption in any period.

[2]Indeed, we are aware that the Treasury has at times announced
"refunding" terms that face holders of maturing issues with a price of
new issues different from that faced by everyone else. And since these
terms are not announced until close to the new issue date, it seems as
if random bonuses have at times been in effect.

[3]In this section we describe only those features of the
models that we need in order to determine the nature of interest on
government bonds.

[4]For a detailed description of this role of intermediation,
see Klein 1973.

[5]For a general defense of this model and for a description of
some generalizations, see Wallace 1977.

[6]All this follows from the $C_j(t)$ being superior goods.

[7]Not only may there be no equilibrium that succeeds in raising
the revenue D, but there may be several monetary equilibria that succeed
in raising the revenue D. Neither is surprising. For example, consider

the analogous taxation problem: find the tax rate on cigarettes that raises the revenue D. Obviously there may be no such tax rate and there may be many such tax rates.

[8] Of course, the model must also confront the margin between fiat money and any real assets in it. And since we insist on maintaining the intrinsic uselessness of fiat money, just as bonds cannot dominate valued fiat money so real assets cannot dominate it. (Asset A dominates Asset B if A has a higher rate of return than B in all circumstances.)

[9] We are indebted to Robert E. Lucas of the University of Chicago and Christopher Sims of the University of Minnesota for bringing this question to our attention.