Open-Market Operations in a Model of
Regulated, Insured Intermediaries

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In "The Inefficiency of Interest-Bearing National Debt," (JPE, April 1979) we argued that private sector transaction costs are needed in order to explain interest on government debt. It follows that if the government's transaction costs do not depend on its portfolio, then, barring special circumstances, an open-market purchase is deflationary and welfare improving. In this paper we show that this result can survive a potentially relevant special circumstance: reserve requirements which limit the size of insured intermediaries.

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In our earlier paper, "The Inefficiency of Interest-Bearing National Debt," we argued that transaction costs are necessary in order to account for positive interest on safe government debt. The particular model studied was a version of Samuelson's overlapping-generations model in which (i) all government debt has to be intermediated by way of a resource-using, constant-average cost technology, (ii) individuals hold as assets only currency and intermediary liabilities, and (iii) nothing distinguishes currency from intermediary liabilities when their yields are the same.

The intermediaries of the earlier paper are like bond mutual funds. A once-for-all open-market purchase, a central bank purchase of outstanding government bonds, decreases the amount of bonds and the liabilities of these bond funds and increases the amount of currency held by the public. Under the assumption that central bank resource costs are independent of its portfolio, the only effect is to free some resources; less resources are used in intermediation. Hence, our conclusion: the open-market purchase is deflationary and welfare-improving.[1] A simple price-theoretic way to state the inefficiency result is to note that if central bank resource costs do not vary with the relative amounts of currency and bonds it supplies, then barring special circumstances relative market prices of currency and bonds ought to reflect this resource cost equivalence, which they do not if bonds bear interest.

We are occupied in this paper with studying one potentially relevant special circumstance: the role of currency reserve requirements in limiting the size of insured intermediation. The presence of improperly priced deposit insurance--FDIC insurance in which a bank's premium does not depend on its portfolio or insurance in the form of Federal Reserve lender-of-last resort activity--could give rise to a second-best situation in which limiting the supply

of currency (the monetary base) is beneficial because it limits the size of a distorting activity, namely, risk taking by insured intermediaries.

As in our first paper, we pursue the investigation within a version of Samuelson's overlapping-generations model. But the version used here is richer in several respects. First, individual currency holdings are here assumed subject to a random proportional uninsurable loss. At some relative prices, this gives rise to portfolios that are diversified in a determinate way between currency holdings and safe intermediary liabilities. Put differently, this assumption gives rise to a more standard demand function for individual currency holdings than was implied by the setup of our earlier paper. Second, we here assume that the single consumption good of the model is storable with a stochastic storage technology so that storage of the consumption good constitutes a risky real investment. Without a risky investment opportunity, it would not be possible to investigate the distortions caused by improperly priced deposit insurance. Third, we make entry into insured intermediation contingent on the purchase of a marketable license. Since the number of licenses is fixed by the government, this allows us to study both limited and free entry into insured intermediation. And, of course, each insured intermediary is regulated; it is subject to both a reserve requirement and a capital requirement. These and other aspects of the model are described in detail in Section I.

In Section II we study how the equilibrium depends on some of the parameters by way of a numerical example. For a particular economy, we study how the stationary monetary equilibrium depends (a) on the relative amounts of currency and government bonds outstanding--our way of studying open-market operations, (b) on whether entry into insured intermediation is limited or not, and (c) on whether or not interest is paid on the currency reserves of insured intermediaries. The examples show, among other things, that although the

interest rate is higher and investment lower the greater the amount of bonds outstanding, there is no presumption that our earlier results on the inefficiency and inflationary effects of bonds are overturned.

In the concluding section we offer some general remarks in defense of our approach to modeling the effects of open-market operations. We also argue that there is no obvious evidence that contradicts our view.

I. The Model

As noted above, our model is a complicated version of Samuelson's pure consumption loans model. Our particular assumptions are chosen to be consistent with the existence of an equilibrium in which all prices remain the same from period to period. And, in order to conserve on notation, we will omit time subscripts except when to do so would be confusing.

1. Tastes, Technology, Resources, and Government

Time is discrete. At any date t, the model is peopled by N old people and N young people, the latter being the members of generation t. At t, each member of generation t acts to maximize expected utility, utility being common to all members of all generations and given by $U(e_1, e_2) = \sum u(e_j)$ where e_j is age $j(=1,2)$ consumption of the single consumption good in the model. The function u satisfies $u' > 0$, $u'' < 0$, and $u'(0) = \infty$.

Each member of generation t has an endowment at t of w units of the consumption good. This is perfectly divisible and may be consumed at t or traded.

Only intermediaries may store the consumption good, hold government bonds, and store currency safely. At each date, intermediaries acquire assets and sell claims to the payoffs that accrue at the next date. We express the intermediation technology in the form of a total cost function

$$G(y) = g_0 y + g_1 [\max(0, y-\bar{y})]^2; \quad g_0, \ g_1, \ \bar{y} > 0 \tag{1}$$

where y is the total value of the assets acquired at t. Both y and $G(y)$ are in units of the time t consumption good. The nonlinearity of $G(y)$ is introduced in order to make a restriction on the number of insured intermediaries matter.

The return on storage of the consumption good is constant-returns-to-scale and stochastic. We let $s \in [0,1]$ be a state variable and $x(s)$, a non-negative, nondecreasing bounded function, be the gross return function; if k units are stored at t, $kx(s)$ units are available at $t+1$. The value of s is drawn independently from period to period according to the density function $f(s)$. We assume that the drawing of s occurs so as to preclude intergeneration risk sharing. Thus, the realization of s that determines the return to storage from t to $t+1$ occurs after generation $t-1$ disappears and after storage at t is determined, but before generation $t+1$ appears.

The government issues currency and one-period, zero-coupon bonds that are titles to specific amounts of currency. It also operates an insurance program for licensed intermediaries. The government behaves so that the stocks of currency, H, and of government bonds, B, are constant over time. At each date t, the government sells bonds with an aggregate face value of B; that is, bonds that in the aggregate promise to pay B units of currency next period. If the bonds sell at less than face value, then the government levies taxes payable by the young equal to total interest on the debt. We assume that resources are not required in order to carry out these government activities.

While there is free entry into intermediation, there is not into insured intermediation. Instead, there are J licenses outstanding, each license being a permit to operate a limited liability corporation that issues insured liabilities subject to certain regulations and the cost function G described above.

We denote the price paid for each license by V. The government taxes the transfer of ownership of these licenses collecting $J\gamma V$ each period where $g_0/(1+g_0) < \gamma \leq 1$. These taxes are returned to the current old via a lump-sum scheme. The insuring of the liabilities of these J intermediaries may for some realizations, s, imply payments by the government, as may interest on reserves. These payments are financed by preannounced lump-sum (state-dependent) taxes levied on the old. This is equivalent to making the payments be those of an insurance policy purchased by the government in the preceding period with the cost of the policy financed by a lump-sum tax levied on the young.

As is spelled out in detail below, the model is described in terms of time t markets in claims on time t+1 consumption in each state s. Consumers use these markets to arrange in the first period of their lives for consumption in the second period of their lives in each state s, while taking into account the preannounced state s taxes to which they are subject. Intermediaries use these markets to sell claims at t on time t+1 consumption in each state s.

2. The Choice Problem of the Young

Our first task is to describe the riskiness of individual currency holdings. We divide the N members of each generation into two groups: $N_1 = N_2 = N/2$. Each period a fair coin is tossed. If "heads," then each member of N_1 loses a fraction δ of his or her currency holdings. The loss shows up as a lump-sum transfer to one member of N_2. If "tails," then vice versa. The crucial restriction is that bets cannot be placed on the outcome of the coin tossing. Such bets or contingent contracts would prevent the coin tossing from producing uncertainty.

The symmetry between members of N_1 and N_2 is imposed for convenience. It implies that everyone is identical prior to the coin tossing. Therefore, each young person at t maximizes

$$EU = u(e_1) + .5\int u[e_2(s,\delta)]f(s)ds + .5\int u[e_2(s,0)]f(s)ds \qquad (2)$$

where

$$e_2(s,\delta) = q(s) + (1-\delta)P(t+1)c \qquad (3)$$

$$e_2(s,0) = q(s) + P(t+1)c + \delta P(t+1)c'. \qquad (4)$$

Here $e_2(s,\delta)$ is second-period consumption in the state (s, "heads"); $e_2(s,0)$ is second-period consumption in the state (s, "tails"); one unit of q(s) is a claim

on one unit of second-period consumption in state (s, "heads") <u>and</u> in state (s, "tails"), c (a choice variable) is currency holding, c' is per capita currency holding of the other group, and P(t+1) is the price of currency at t+1 in units of the consumption good at t+1.

EU is maximized by choice of $e_1 \geq 0$, a function q(s), and $c \geq 0$ subject to

$$e_1 + P(t)c + \int p(s)q(s)ds - (w-T) \leq 0 \tag{5}$$

with P(t), P(t+1), p(s), c', and (w-T) treated as parameters. Here p(s) is the price in units of first-period consumption of one unit of q(s), while T is the present value of direct taxes, an expression for which is given below.

For P(t) = P(t+1) = P > 0 and p(s) > 0, the unique maximizing values of e_1, and c, and the almost unique maximizing function q(s) satisfy (5) with equality and the first-order conditions

$$u'(e_1) - \lambda = 0 \tag{6}$$

$$(1-\delta)\int u'[e_2(s,\delta)]f(s)ds + \int u'[e_2(s,0)]f(s)ds - 2\lambda \leq 0 \tag{7}$$

$$u'[e_2(s,\delta)]f(s) + u'[e_2(s,0)]f(s) - 2\lambda p(s) = 0; \text{ almost all s} \tag{8}$$

where λ is the nonnegative multiplier associated with (5) and where (7) holds with equality if c > 0.

We will use (3) through (8) to find implicit demand functions for c and q(s).

First, we examine what is implied by c = 0. By symmetry, if c = 0, then c' = 0. Then, (3) and (4) imply $e_2(s,\delta) = e_2(s,0) = e_2(s)$, so (7) becomes

$$(2-\delta)\int u'[e_2(s)]f(s)ds - 2\lambda \leq 0 \tag{9}$$

while (8) becomes

$$u'[e_2(s)]f(s) - \lambda p(s) = 0. \tag{10}$$

If we integrate (10) over s and substitute the result into (9), we get

$$(2-\delta)\int p(s)ds - 2 \leq 0. \tag{11}$$

Letting $1/r \equiv \int p(s)ds$, we may write (11) as

$$r \geq (2-\delta)/2 \tag{12}$$

where r has the interpretation of the safe gross rate of interest.

This proves that $r < (2-\delta)/2$ implies $c > 0$. It is also easily shown that (12) implies $c = 0$.

Noting that $c = c'$ at equilibrium, (5), (6), and (8) imply

$$u'[q(s)+(1-\delta)Pc]f(s) + u'[q(s)+(1+\delta)Pc]f(s) = \tag{13}$$

$$2p(s)u'[w-T-Pc-\int p(s)q(s)ds]; \text{ for all } s$$

while (5), (6), and (7) at equality imply

$$(1-\delta)\int u'[q(s)+(1-\delta)Pc]f(s)ds + \int u'[q(s)+(1+\delta)Pc]f(s)ds = \tag{14}$$

$$2u'[w-T-Pc-\int p(s)q(s)ds].$$

If (12) holds, then $c = 0$, and we equate $q(s)$ in (13) to the supply of $q(s)$ which we describe below.

If (12) does not hold, then we equate $q(s)$ and c in (13) and (14) to their respective supplies.

3. Uninsured Intermediaries

Free entry into uninsured intermediation provides bounds on S, the nominal market value of government debt per dollar of face value, and on certain

functions of $p(s)$. Free entry implies that profits from holding any asset and operating on the linear portion of the cost function G must be nonpositive.

For storage of the consumption good, this implies

$$k \int p(s)x(s)ds - k - g_0 k \leq 0 \text{ for } k \leq \bar{y}$$

or

$$\int p(s)x(s)ds \equiv px \leq 1 + g_0 \qquad (15)$$

which holds with equality if uninsured intermediaries store some of the consumption good.

For holdings of currency, R', the nonpositive profit condition gives

$$P(t+1)R'/r - P(t)R' - g_0 P(t)R' \leq 0 \text{ for } P(t)R' \leq \bar{y} \text{ or at } P(t) = P(t+1) = P > 0$$

$$1/r \leq 1 + g_0 \qquad (16)$$

which holds with equality if uninsured intermediaries store currency.

And, finally, for government bonds with face value b, we have

$$P(t+1)b/r - P(t)Sb - g_0 P(t)Sb \leq 0 \text{ for } P(t)Sb \leq \bar{y} \text{ or at } P(t) = P(t+1) = P > 0$$

$$1/r \leq (1+g_0)S \qquad (17)$$

which holds with equality if uninsured intermediaries hold government bonds.

Notice that if uninsured intermediaries store currency so that $1/r = 1+g_0$, then $S = 1$. More generally, we will have $S \leq 1$.

4. Insured Intermediaries

We begin by expressing $V(t)$, the value in terms of time t consumption of a license to operate an insured intermediary, in terms of its value at t+1, $V(t+1)$, the portfolio it holds and the prices it faces. The required expression is

$$V(t) = \int p(s)\max[zx(s)+P(t+1)(\lambda_1 R/S+(1-\lambda_1)R+b)+(1-\gamma)V(t+1)-P(t+1)D,0]ds \quad (18)$$

$$- G[z+P(t)(R+Sb)] - [z+P(t)(R+Sb)] + P(t+1)D/r$$

where $z \geq 0$ is the amount of the consumption good the intermediary stores from t to t+1; $R \geq 0$ is the amount of currency it stores from t to t+1, λ_1 being the fraction of such currency that earns interest at the rate on government bonds; $b \geq 0$ is the face value of government bonds stored from t to t+1 and $D \geq 0$ is total insured liabilities in terms of the amount of currency to be paid out at t+1.

The RHS of (18) is simply revenue minus costs. One component of revenue is the last term $P(t+1)D/r$; since deposits are insured, they pay off in every state and are valued as a safe asset. The second component of revenue is the integral; it is the value of state-specific net receipts. Note that $(1-\gamma)V(t+1)$ is after-tax receipts from the sale of the license. Owners get all of this in state s only if the state s value of assets, $zx(s) + P(t+1)[\lambda_1 R/S+ (1-\lambda_1)R+b]$, exceeds the state s value of liabilities, $P(t+1)D$. Costs consist of operating costs, $G[z+P(t)(R+Sb)]$, and the costs of assets, $z + P(t)(R+Sb)$.

Before we can state the optimizing problem of the insured intermediary, we need a preliminary proposition.

For given nonnegative D, b, R, and z, denote the RHS of (18) by $F[V(t+1)]$. The proposition is about bounded V sequences that satisfy (18) for all t at constant prices and at an unchanging portfolio. (We are concerned only with bounded sequences, because, as will be seen below, any equilibrium V sequence must be bounded.)

<u>Proposition 1</u>. For a portfolio and prices that are constant over time:

    (a) If $F(0) \geq 0$, there exists one and only one nonnegative bounded $V(t)$ sequence that satisfies (18). This is the constant sequence, the constant being the unique solution to $V = F(V)$.

    (b) If $F(0) < 0$, no nonnegative bounded V sequence satisfies (18).

This proposition, proved in the appendix, follows from the properties of F, inequalities (15) and (16), and our assumptions about the tax, Υ, on the sale of V. In the absence of such a tax, the licenses, which can be costlessly stored, would dominate currency. Indeed, as we shall see, even with the tax, their costless storage plays a role.

In light of Proposition 1 and our goal of describing stationary equilibria, we state the problem of an insured intermediary as follows. Choose nonnegative values of D, b, R, and z treating P, S, and $p(s)$ parametrically to maximize the solution to $V = F(V)$ subject to three regulatory constraints,

$$\lambda_1 R/S + (1-\lambda_1)R + b \geq \alpha D; \quad 0 < \alpha < 1, \; 0 \leq \lambda_1 \leq 1 \qquad \text{(i)}$$

$$(\lambda_1/S+1-\lambda_1)R \geq \lambda_2 \alpha D; \quad 0 \leq \lambda_2 \leq 1 \qquad \text{(ii)}$$

$$z \geq \beta P(t+1)[D-(\lambda_1/S+1-\lambda_1)R-b]/r; \quad \beta \geq 1. \qquad \text{(iii)}$$

The first, a reserve requirement, says that safe assets, counting interest on reserves, must be at least the fraction α of insured liabilities. The second, also a reserve requirement, says that currency holdings must be at least the fraction $\lambda_2 \alpha$ of insured liabilities. The third is our version of a capital requirement. It says that storage of the consumption good, our only risky asset, must be a multiple, β, of the value of the difference between insured liabilities and safe assets.

In the appendix we prove:

<u>Proposition 2.</u> Portfolios satisfying (i)-(iii) with equality maximize the solution to V = F(V).

Roughly speaking, insured intermediaries hold the riskiest allowable portfolio of assets.[2/] Upon imposing (i)-(iii) at equality, the stationary version of (18) can be written in terms of prices and deposits, D, as

$$V = \int p(s)\max[(1-\alpha)PD(\beta x(s)/r-1)+(1-\gamma)V,0]ds - G(PD\theta) \tag{19}$$

$$+ PD(1/r-\theta) \equiv F(V)$$

where

$$\theta = (1-\alpha)\beta/r + \alpha[S(1-\lambda_2)+\lambda_2/(\lambda_1/S+1-\lambda_1)] > 0.$$

The critical function of prices and cost parameters for determining the profitability of an insured intermediary is

$$\Psi \equiv \int p(s)\max[(1-\alpha)(\beta x(s)/r-1),0]ds - (1+g_0)\theta + 1/r \tag{20}$$

because F(0) = PDΨ if PD$\theta \leq \bar{y}$ and F(0) < PDΨ if PD$\theta > \bar{y}$. The role of Ψ is summarized in the following proposition proved in the appendix.

<u>Proposition 3.</u>

(a) If $\Psi > 0$, there exists one or more values of D that maximize the solution to V = F(V). Any maximizing D implies V > 0 and is such that $0 < D \leq \bar{D}$, where $\bar{D}$ maximizes F(0).

(b) If $\Psi = 0$, V $\leq$ 0 for all D. Any D consistent with $0 \leq PD\theta \leq \bar{y}$ implies V = 0, and, hence, is a maximizing D.

(c) If $\Psi < 0$, D = 0 is the unique D that maximizes V. V = 0 and insured intermediaries do not operate.

Our last task in this section is to derive an expression for the value of net taxes implied by the operation of insured intermediaries.

Letting $L(s)$ denote the insurer's payout in state s including interest on reserves held as currency, we have

$$-L(s) = \min[zx(s)+P(\lambda_1 R/S+(1-\lambda_1)R+b)+(1-\gamma)V-PD,0] - \lambda_1 PR(1/S-1).$$

Upon subtracting $\int p(s)L(s)ds$, the current value of these payments, from both sides of the stationary version of (18) we have

$$V - \int p(s)L(s)ds = z(px-1) + PR(1/r-1) + Pb(1/r-S) + (1-\lambda)V/r - G[z+P(R+Sb)].$$

This may be written as

$$\int p(s)L(s)ds - \gamma V/r = z(1+g_0-px) + PR(1+g_0-1/r) \tag{21}$$

$$+ Pb[S(1+g_0)-1/r] + V(1-1/r) + g_1[\max(0,y-\bar{y})]^2$$

which is the net tax in current consumption attributable to the operation of each insured intermediary.

Each term on the RHS of (21) has a straightforward interpretation. The first term contributes to taxes by an amount proportional to the discrepancy between $1+g_0$ and px. Thus, by inequality (15), this contribution is nonnegative and is zero if uninsured intermediaries are also storing the consumption good.

The second term has an analogous interpretation for currency. By inequality (16), this term is nonnegative and is zero only if uninsured intermediaries hold currency. By inequality (17), the third term is also nonnegative. It is zero if uninsured intermediaries hold government bonds.

The fourth term is negative if $r < 1$. Note that by (16), $1 - 1/r \leq -g_0$ with equality if uninsured intermediaries hold currency. If the licenses have value, they provide for the economy as a whole (but not for individuals) a costless means of carrying wealth from period to period in the form of a safe asset. In general, the alternative cost per unit of safe second-period

consumption is given by (17) at equality, which implies $1/r - 1 = (1+g_0)S - 1$. This is a maximum, g_0, at $S = 1$. The presence of the term in V gives rise to the possibility that the LHS of (21) is negative. When this happens the existence of insured intermediaries implies a net transfer and happens to be socially beneficial. This possibility arises only because the claims to intermediaries are a superior form of money for the economy.[3/]

The fifth term is simply the total of resource costs in excess of those that would be borne if the same assets were held by uninsured intermediaries.

Although it is not true that the LHS of (21) is always positive, it is true that if $L(s) = 0$ for all s, then $V \leq 0$. In other words, in order that licenses have value, it is necessary that the insurer have a positive gross liability. (This follows from (21) and the lower bound imposed on Υ.)

5. Equilibrium

To begin we write out an expression for the value in units of time t consumption of the total of net lump-sum taxes levied on members of generation t:

$$NT = PB(1-S) + J\{PD[\lambda_1\lambda_2\alpha(1-S)/r[\lambda_1+(1-\lambda_1)S]+\theta-\beta(1-\alpha)px/r-\alpha/r]$$

$$+V(1-1/r)+G(PD\theta)\} \tag{22}$$

where the first component represents interest on government bonds and the second arises from the operations of J insured intermediaries (see (21) and Proposition 2).

For equilibrium, we equate c and q(s) in (13) and (14) to supplies given as follows:

$$Nc = H - R' - J\lambda_2\alpha D/(\lambda_1/S+1-\lambda_1) \tag{23}$$

$$Nq(s) = x(s)[K+J\beta(1-\alpha)PD/r] + P[R'+B+J\lambda_2\alpha D/(\lambda_1/S+1-\lambda_1)] + JV \tag{24}$$

where R' and K denote total currency and total goods holdings, respectively, of uninsured intermediaries. Note that two restrictions on D and V are implied by the solution to the optimizing problem of each insured intermediary. We get V as a function of D. And we get a correspondence between D and the prices, p(s) and P.

A stationary equilibrium in which currency has value consists of a positive value of P, a value of S satisfying $0 < S < 1$, nonnegative functions p(s) and q(s), and nonnegative values of R', K, c, D, and V that satisfy the two restrictions just mentioned, (13), (23), (24), and (15) through (17) with the following provisos: if $c > 0$, then (14) holds; if $R' > 0$, then $1/r = 1+g_0$; if $B > (1-\lambda_2)\alpha D$, then $1/r = (1+g_0)S$; and if $K > 0$, then $px = 1+g_0$.

The parameters are the per capita endowment, w; the size of a generation, N; the possible loss on individual currency holdings, δ; the regulatory parameters, J, α, β, λ_1, λ_2, and γ; those that determine the functions u, G, x(s), and f(s); and H and B.

6. "Neutrality," Open-Market Operations, and Welfare Comparisons

It is easily verified that only the ratio of H to B matters in our model. Put formally, if (X*,P*,D*) is a stationary equilibrium for the vector of parameters (H*,B*,μ*)--where X, here, represents the vector of all endogenous variables other than P and D and μ represents the vector of all parameters other than H and B--then for any $\sigma > 0$, (X*,P*/σ,σD*) is a stationary equilibrium for the vector of parameters (σH*,σB*,μ*). In the light of this property--which we will henceforth call "neutrality"--we will be describing how the stationary equilibrium for a given set of parameters μ depends on the ratio h = H/(H+B). Moreover, as we now argue, subject to strict qualifications we can interpret these alternative stationary equilibria as achievable in a single economy through open-market operations.

For us, an open-market operation proceeds as follows. If B(t-1) is the face value of bonds sold at t-1 and B(t) = B(t-1) - Δ is the amount sold at t, expenditures and receipts for the government at t are as follows:

Expenditures	Receipts
B(t-1)	S(t)[B(t-1)-Δ]
	[1-S(t)][B(t-1)-Δ]

The expenditure item is what must be handed out to pay off maturing bonds. The first receipt item is the proceeds from the sale at t of bonds with face value B(t). The second receipt item is the component of taxes levied on the young at t for interest on government bonds. As we assume, this is interest implied by the new stock of bonds and the new price. Note that expenditures minus receipts equal Δ, thus giving us H(t) = H(t-1) + Δ and, therefore, H(t) + B(t) = H(t-1) + B(t-1).

So our open-market operation holds the sum H + B constant.[4] In the examples described below, the different values of h are achieved in this way.

Given our assumptions about how the young are taxed, the past matters hardly at all in our model. Aside from consumption of the old at time t, all the endogenous variables at time t are determined solely by the values of parameters at t, so long as it is believed that these values will prevail forever. Thus, we can interpret different stationary equilibria as achievable by way of the open-market operation described above if we assume that the change from B(t-1) to B(t) is viewed as a once-for-all surprise. To be specific, at t-1 it must have been believed that B(t-1) was going to be maintained forever, while at t it must be believed that B(t) is going to be maintained forever.

This suggests a way to evaluate the welfare effects of once-for-all (surprise?) changes. The new stationary equilibrium values imply a value of expected utility for the current and future young. They also imply asset values different from those that would have prevailed, values that affect the

consumption of the current old. Since open-market operations (and changes in regulatory parameters) do not affect the return on or amount of the consumption good stored from the last period, the implied change in the consumption of the current old is equal to the change in $P(H+B) + JV$. And since we hold $H + B$ constant, the value of money, P, and the value of licenses, V, are the only determining factors.

II. Some Examples

We now describe how the stationary monetary equilibrium depends on the relative quantities of H and B under five policy regimes. The regimes differ with regard to the number of insured intermediaries and with regard to whether interest is paid on their currency reserves. One regime has no insured intermediaries. Two have limited entry—in particular, $J/N = .3$—and two have free entry into insured intermediation. Aside from λ_1, the parameter that determines interest on reserves, only one set of regulatory parameters is studied: $(\alpha,\beta,\gamma,\lambda_2) = (.8,1.15,.5,1.0)$; that is, the reserve requirement is .8, the capital requirement is 1.15, the tax on a license to operate an insured intermediary is 50 percent, and bonds cannot be held as reserves.

Individuals in the economy of our example are described by $u(e_j) = 85 - e_j^{-6}$, $w = 1$, and $\delta = 0.1$. The intermediation cost function is given by $g_0 = 1/12$, $g_1 = 1.0$, and $\bar{y} = .15$, while the return-to-storage function is $x(s) = 0.5$ for $0 \leq s < .5$ and $x(s) = 2.0$ for $s \geq .5$ with $f(s) \equiv 1.0$. Since $x(s)$ takes on only two values—with probability one-half goods stored halve and with probability one-half they double—$p(s)$ and $q(s)$ each take on two values. It is convenient, therefore, to define $p_1 = .5p(s)$ and $q_1 = q(s)$ for $0 \leq s < .5$, and $p_2 = .5p(s)$ and $q_2 = q(s)$ for $s \geq .5$.

This economy and the regimes were designed to have equilibria with the following properties: (a) there is a monetary equilibrium ($P>0$) with positive

storage of the consumption good, (b) individuals hold some currency and some safe intermediary liabilities for some values of H/(H+B) = h, and (c) insured intermediaries operate at most values of h.

Properties (b) and (c) guided our choices of the regulatory parameters, primarily (α, β). To keep individual holdings of currency positive, the gross return offered on insured deposits must remain below .95, the average of 1-δ and 1. That requires what might be called stringent regulation that among other things holds down the size and profitability of insured intermediaries. But if the regulation is too stringent, then insured intermediaries do not operate.

We chose a utility function that would imply positive holdings of safe intermediary liabilities for all values of h. In the absence of insured inter- mediation and with h = 1, individuals face a safe gross rate-of-return of 12/13. (See (16).) Substantial risk aversion is required to make individuals hold positive amounts of an asset with this rate-of-return given that holdings of currency offer gross returns of .9 and unity each with probability 1/2.

1. No Insured Intermediaries

This economy is described in Table 1. In the tables, EU is expected utility of a young person; I is per capita total storage of the consumption good, the sum of storage by insured and uninsured intermediaries divided by N; and i is $100[1/S-1]$, the interest rate on bonds. All other variables are as defined in the text.

Note that columns 3-8 describe aspects of the equilibrium portfolio in units of the consumption good. For example, at h = 1 (no bonds), each individual (with an endowment of unity) holds currency with real value .4280, holds risky stored consumption good by way of claims on uninsured intermediaries in an amount .0318, and holds safe claims on uninsured intermediaries with real value

.4676-.4280, column 3 minus column 6. Thus, at h = 1 in Table 1, individual currency holdings constitute a very large fraction of total assets.

Table 1 is consistent with the results of our earlier paper. If bonds bear interest, then they constitute a distortion. When bonds bear interest (S<1), their presence amounts to a subsidy, financed by lump-sum taxes, on the provision of safe assets by intermediaries. Too many resources are used in that activity, and expected utility is lower than it is in the absence of bonds.

Note that bonds do not bear interest at h = .95. This is because bonds serve as well as currency in the "vaults" of the uninsured intermediaries. Bonds begin to bear interest--that is, to sell at a discount--only when uninsured intermediaries must charge a service fee lower than g_0 on their safe deposits in order to attract sufficient deposits to enable them to hold all the bonds supplied. And once bonds bear interest, uninsured intermediaries do not hold currency.

The greater the bond supply (and the smaller the currency supply), the greater the equilibrium return on safe deposits and the greater the interest rate on bonds. Accompanying this is a smaller amount of storage of the consumption good since safe assets seem more and more attractive. In other words, open-market sales drive the interest rate up and drive investment (storage) down. The increased safety of the aggregate portfolio is reflected in an altered relative price p_1/p_2. The greater the amount of bonds outstanding, the cheaper is a claim on consumption in the bad state relative to a claim on consumption in the good state.

The only surprising aspect of this scenario is the behavior of the value of money and, perhaps, of expected utility. The value of money, P--the movement of which is given by that of P(H+B)/N since (H+B)/N is a constant--moves hardly at all and in what many would regard as a perverse direction.

It is obvious from Table 1 and our earlier discussion that a once-for-all (surprise?) change from h < 1 to h = 1 in this economy is a welfare-improving move. Since the change increases the value of money, the consumption of the current old increases. The effects on the current and future young are given by the expected utility outcomes in Table 1.

2. Insured Intermediaries with Noninterest-Bearing Reserves

We begin our discussion with the limited-entry regime (J/N = .3). With this number of insured intermediaries and with the particular regulatory parameters in effect, insured intermediaries in the aggregate are "small" in terms of their holdings of the real risky asset (storage of the consumption good), but "not small" in terms of their holdings of reserves (currency, which bears no interest).

Insured intermediaries are small with regard to investment in that for all values of h in Table 2 uninsured intermediaries store the consumption good. Equivalently, (15) holds with equality. Thus, the first term on the RHS of (21) is zero for all h, and total storage in the economy at each value of h is not much different from that in Table 1.

Insured intermediaries are not small as regards their holdings of reserves in that for all h in Table 2, uninsured intermediaries hold.no currency and (16) holds with strict inequality. It follows that the second term on the RHS of (21) is positive for all h and that currency holdings of insured intermediaries constitute a distortion.

The pattern within Table 2 as h decreases from 1.0 to 0.1 is qualitatively like that in Table 1. Lower h must be accompanied by higher interest rates to induce the public to hold additional safe intermediary liabilities and to give up individual currency holdings. Since insured intermediaries must pay the same rate on deposits as uninsured intermediaries pay on their safe deposits,

insured intermediaries are smaller the smaller is h. From h = 1.0 to h = 0.1, the patterns of the value of money and of expected utility are similar to those in Table 1.

Notice that at each value of h, the interest rate is higher in Table 2 than in Table 1. Insured intermediaries constitute an additional source of demand for currency. It therefore takes a higher interest rate at each value of h to induce the private sector to hold the bonds. This higher rate of subsidy gives rise to a greater distortion. Thus, at each h expected utility is lower in Table 2 than in Table 1.

In Table 2 decreases in h are in general accompanied by two kinds of shifts: there is a shift of assets from insured intermediaries to uninsured intermediaries and a shift from individual currency holdings to holdings of safe uninsured assets. The first shift is beneficial, while the second (which is the only one at work in Table 1) is not. In Table 2, the second effect dominates until h is so small that individuals hold no currency. At that point, h = 0.1, further declines in h produce only the shift of assets from insured to uninsured intermediaries. This effect shows up in the last four rows of Table 2. And, in this range, the value of money rises as h declines, although only moderately. Note that at h = .05 insured intermediaries have been forced back on to the linear portion of their cost curves.

It is important to note that with different parameters, in particular, more stringent regulatory parameters, insured intermediaries would be forced out completely before individuals are induced to give up all their currency.

We now turn to free entry into insured intermediation with the same set of regulatory parameters. These results appear in Table 3. Free entry implies that $\Psi \leq 0$ (see (20)).

It turns out that for most values of h (h $\geq$ 0.2), free entry implies that insured intermediation is "large" both in terms of storage of the consumption good and in terms of currency holdings. For h near unity, total storage—all of it held by insured intermediaries—is approximately double what it is in Tables 1 and 2. The contingent claims prices reflect this additional riskiness of the economy's portfolio. The large size of insured intermediation is also reflected in the interest rate. Even at h = 1 there is a substantial subsidy on safe storage by intermediaries. Both these distortions are reflected in taxes—which at h = 1 come entirely from the operation of insured intermediaries. (With free entry, such taxes are entirely attributable to the first two terms on the RHS of (21). The other three terms on the RHS of (21) are zero.) And, of course, the distortions are also reflected in expected utility. At each h, with one exception (h = 0.1), expected utility is lower in Table 3 than in Table 2.

Free entry makes the demand for currency even greater than in Table 2. This is reflected in higher interest rates and less currency holding by individuals at each value of h. By h = .35 individuals do not hold currency. From that point on further declines in h are beneficial. In the vicinity of h = .35, declines in h reduce overinvestment in the risky real asset. This effect disappears by h = .15 when uninsured intermediaries hold some of the risky real asset. (Note that once uninsured intermediaries store some of the consumption good, the contingent claims prices and, hence, the interest rate, are pinned down by (15) at equality and Ψ = 0.) Throughout the range h $\leq$ .35, declines in h produce shifts of safe assets from insured to uninsured intermediaries. These are beneficial because uninsured storage costs depend on the market value of bonds, while insured costs depend, in effect, on the real value of their face value which is greater. This difference is reflected in the second term on the RHS of (21).

At h = 0.1, expected utility is higher with free entry than with the limited entry of Table 2. The higher interest rate with free entry makes uninsured intermediary costs per unit of real bond holdings lower. Although this effect is present at all h < 1, it becomes more important the greater the bond holdings.

With free entry the value of money is decreasing in h for all values of h. However, in the range where individuals hold currency, h ≥ 0.40, this effect is tiny. Even in the range where individuals do not hold currency it is only of moderate size.

3. Interest on Reserves

Here the regulatory parameters are the same as those above except that λ_1 = 1.0: the government pays interest on currency reserves at the market rate on its bonds.

We begin with limited entry, J/N = .3, and compare Table 4 with Table 2. When interest is paid on reserves, decreases in h are not in general accompanied by decreases in the amount of insured intermediation. In the absence of interest on reserves, decreases in h are in general accompanied by higher rates-of-return on the safe liabilities of uninsured intermediaries, the holders of bonds. Since insured intermediaries must match these rates-of-return, their profitability and, hence, their size decreases as h decreases. The payment of interest on reserves tends to offset this effect. The result is that at each value of h, the interest rate is (slightly) higher in Table 4 than in Table 2 and real (and nominal) individual currency holdings (slightly) lower in Table 4 than in Table 2. In other words, more of any reduction in H comes from individual currency holdings in Table 4 than in Table 2. Thus, at any given h < 1, there is more of a subsidy on the holding of safe intermediary liabilities in Table 4 than in Table 2. So long as individual currency holdings are positive, the magnitude

of the implied distortion varies with the magnitude of the subsidy. Thus, expected utility is lower at each value of $h \geq 0.1$ in Table 4 than in Table 2.

When individuals hold no currency, at $h < 0.1$, this is no longer true. Further decreases in h are accompanied by decreases in the amount of insured intermediation. In our example, these are accompanied by very sizable increases in interest rates. As in Table 2, these have two effects: (1) the distorting effect is to push the economy out of the risky real asset; (2) the beneficial effect is to lower the real resource costs of storing safe assets. The latter dominates in the range $h \leq .1$ and produces increases in expected utility and moderate increases in the value of money.

These effects are magnified with free entry. (See Table 5.) As in Table 4, decreases in h do not force a contraction in insured intermediation until individuals no longer hold currency. Once individuals are out of currency, further declines in h are welfare improving and are accompanied by moderate increases in the value of money.

4. Summary of the Examples

Several features show up under all five regimes. The more bonds that are outstanding, the higher is the interest rate on bonds, the lower is the nominal money supply as measured by the sum of individual currency holdings plus insured intermediary deposits, and the lower is real investment. Moreover, within the range in which individual currency holdings are positive, the distortion created by bonds that show up when there are no insured intermediaries prevails under all regimes. The more bonds that are outstanding, the greater is the subsidy on safe intermediary liabilities and the lower is expected utility. In this range paying interest on reserves reduces expected utility as it increases the subsidy on safe intermediary liabilities. And, with one exception, within the range in which individual currency holdings are positive, the value of money

falls as the amount of bonds increases. The one exception is in Table 3, but there while the amount of H--the monetary base--goes from 100 percent of the sum of H + B to only 40 percent of this sum, the value of money rises by less than 1 percent. In the range where individuals hold no currency, which does not seem to be the range of practical interest, all decreases in currency come from the reserves of insured intermediaries. Within this range, there is a negative relation between h and the value of money. However, the elasticity of the value of money with respect to currency outstanding is always close to zero.

III. Concluding Remarks

If there is a startling implication of our earlier paper and of the model of this paper, it is the absence of anything like neutrality for open-market operations; once-for-all changes in the monetary base brought about by way of asset exchanges (open-market operations) have real effects--for example, on expected utility and investment--and do not give rise to nearly proportional changes in the price level. But this should not be a surprise. There are neither theoretical nor empirical grounds for expecting neutrality, by which we mean absence of real effects and proportional changes in the price level.

Ours is far from the first model in which neutrality holds for once-for-all proportional increases in bonds and money.[5/] Indeed, it is hard to imagine a model without this property. But this being so, there is, if anything, a strong theoretical presumption against neutrality for equal and opposite changes in money and government bonds, namely, for open-market operations.

Our model says that the price level need bear no relationship to the size of the monetary base, H. Some may think that this is contradicted by a wide range of evidence that shows a close association between the monetary base and the price level. We think not. Our model does not deny an association between the monetary base and the price level if the changes in H are accompanied by

changes in the sum of H and B in the same direction. (The neutrality implication for proportional changes in H and B is an extreme example.) And our claim is that historical episodes that reveal a close association between changes in the monetary base and changes in the price level fall into this class. Gold discoveries and deficits and surpluses partly financed by changes in the base clearly do. And in a somewhat less obvious way, so do the sharp declines in measures of the money supply that accompany banking panics. None of these qualify as open-market operations.

The purported evidence of macroeconometric models suffers from the same confusion. The so-called money demand functions of those models consist of little more than simple correlations between various measures of money, on the one hand, and measures of nominal income, on the other hand. These are the same simple correlations that we assert result primarily from changes in H that come about in association with changes in the sum, H + B. Our claim is that these correlations would change drastically if sizable changes in H were, in fact, brought about through open-market operations.

Nor for a whole host of reasons do we take seriously the attempts to compare monetary and fiscal policy by way of time series regressions of output and/or the price level on distributed lags of measures of the money supply and of fiscal policy. In such studies, regression results are treated as invariant to the rules generating the monetary and fiscal policy variables. No argument supporting such an interpretation has been given. On the contrary, no response has even been made to devastating criticisms of such an interpretation.[6/]

But none of this is to say that we have the right detailed model for the analysis of open-market operations. While we think ours is the only existing model that qualifies as providing a rigorous analysis of open-market operations and of other aspects of monetary policy, we have no great attachment to most of

the details of the model--to the way individual currency demand is modeled, to the way intermediation costs are modeled, and so on. That is why we have presented only a limited examination of its implications.

This is not to say, though, that our investigation is without serious implications. The general message of this and our earlier paper is that the effects of open-market operations depend entirely on transaction-cost details. Moreover, we have constructed several examples of coherent economies in which widely held notions about the effects of open-market operations do not hold. The standard monetarist view is that changes in the quantity of money have similar effects no matter how they are brought about. It seems to us that it is now up to those who hold this view to present their case. Needless to say, we doubt that such a case can be made.

Appendix

Proof of Proposition 1. Under the hypotheses of the proposition, we may write the RHS of (18) as

$$F(V) = \int \phi(s,V) ds + \phi_0$$

where ϕ_0 is a constant and ϕ is the function of s which is integrated in (18). For a fixed s, $\phi(s,V)$ consists of two connected line segments:

$$\phi(s,V) = \begin{cases} 0 \text{ for } V \leq \bar{V} \\ \\ p(s)(1-\gamma)V \text{ for } V > \bar{V} \end{cases}$$

where $\bar{V}$, which may be positive or negative, is such that the first argument of the maximum function in (18) is zero.

It follows that $F(V)$ is continuous and monotone increasing for all V. Moreover, for fixed s and any $V_2 > V_1$,

$$\phi(s,V_2) - \phi(s,V_1) \leq p(s)(1-\gamma)(V_2-V_1)$$

so that

$$F(V_2) - F(V_1) = \int [\phi(s,V_2)-\phi(s,V_1)] ds \leq (1-\gamma)(1+g_0)(V_2-V_1)$$

where the last inequality follows from inequality (16). Since $(1-\gamma)(1+g_0) < 1$ by assumption, proposition 1 is an immediate consequence.

Proof of Proposition 2. This is proved by establishing three facts.

Fact 1. If $S \leq 1$, any feasible portfolio--one satisfying (i)-(iii) and nonnegativity conditions--with (ii) at strict inequality can be weakly dominated by a feasible portfolio with (ii) at equality.

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It follows that $F(V)$ is continuous and monotone increasing for all V. Moreover, for fixed s and any $V_2 > V_1$,

$$\phi(s,V_2) - \phi(s,V_1) \leq p(s)(1-\gamma)(V_2 - V_1)$$

so that

$$F(V_2) - F(V_1) = \int [\phi(s,V_2) - \phi(s,V_1)] ds \leq (1-\gamma)(1+g_0)(V_2 - V_1)$$

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<u>Proof of Proposition 2.</u> This is proved by establishing three facts.

<u>Fact 1.</u> If $S \leq 1$, any feasible portfolio—one satisfying (i)-(iii) and nonnegativity conditions—with (ii) at strict inequality can be weakly dominated by a feasible portfolio with (ii) at equality.

Proof: We denote by "0" the given portfolio and by "*" the weakly dominating portfolio.

Let $D* = D^0$, $z* = z^0$, $R*$ be such that $[\lambda_1/S+(1-\lambda_1)]R* = \lambda_2\alpha D^0$ and $b*$ be such that

$$[\lambda_1/S+(1-\lambda_1)]R* + b* = [\lambda_1/S+(1-\lambda_1)]R^0 + b^0. \tag{a}$$

Obviously, feasibility of the "0" portfolio implies feasibility of the "*" portfolio.

To prove dominance we show that $F*(V) \geq F^0(V)$ for all V. From the relationship between the "*" and "0" portfolio, we have from (18)

$$\text{Sign } [F*(V)-F^0(V)] = - \text{Sign } [(R*+Sb*)-(R^0+Sb^0)] \geq 0,$$

where the inequality follows from (a) and $S \leq 1$. $F*(V) = F^0(V)$ if either $S = 1$ or $\lambda_1 = 1$. Otherwise, there is strict inequality.

Fact 2. If $S \leq 1$, then any feasible portfolio with (i) at strict inequality and (ii) at equality can be weakly dominated by one with both (i) and (ii) at equality.

Proof: We consider two cases, again denoting the given feasible portfolio by "0" and the weakly dominating portfolio by "*."

Case 1. $b^0 < (1-\lambda_2\alpha)D^0$. Let $z* = z^0$, let $b*$ and $D*$ be the solution to (a) $b - (1-\lambda_2\alpha)D = b^0 - (1-\lambda_2\alpha)D^0$ and (b) $b = \alpha(1-\lambda_2)D$, and let $R* = \lambda_2\alpha D*/[\lambda_1/S+(1-\lambda_1)]$.

It follows from this specification that $0 < b* < b^0$, $0 < D* < D^0$, and $R* < R^0$ and that the "*" portfolio satisfies (i) and (ii) with equality and is feasible. Since it also follows that the integral term in (18) is the same for the "0" and "*" portfolios, we have

$$F*(V) - F^0(V) = P(t+1)(D*-D^0)/r - P(t)(R*+Sb*-R^0-Sb^0)$$

$$- G[z^0+P(t)(R*+Sb*)] + G[z^0+P(t)(R^0+Sb^0)]$$

$$\geq P(t+1)(D*-D^0)/r - P(t)(R*+Sb*-R^0-Sb^0)(1+g_0)$$

$$\geq P(D*-D^0)\{1/r-(1+g_0)S[\lambda_2\alpha/(\lambda_1+(1-\lambda_1)S)+1-\lambda_2\alpha]\}$$

$$\geq P(D*-D^0)[1/r-(1+g_0)S] \geq 0.$$

The first inequality follows from $R* < R^0$, $b* < b^0$, and our specification of G. The next one (actually an equality) follows from (a). The next to the last one follows from $D* < D^0$ and $S \leq 1$ while the last one is (17).

Case 2. $b^0 \geq (1-\lambda_2\alpha)D^0$. It follows that the maximum term in the integral in (18) is the first argument for all s. Then inequalities (15) and (16) imply that $F^0(0) \leq 0$ so that the trivial portfolio of no assets and no liabilities is a weakly dominating portfolio.

Fact 3. Any portfolio with (i) and (ii) at equality and (iii) at strict inequality can be weakly dominated by one with (i)-(iii) at equality.

Proof: Letting "0" again denote the given portfolio and "*" the weakly dominating portfolio, we let the two be identical except that $z* < z^0$ so that (ii) holds at equality for the "*" portfolio. At any $V(t+1)$, the integral term for the "*" portfolio is less than that for the "0" portfolio by no more than $(z^0-z*)(px)$. But, then, inequality (15) implies that for any V, $F*(V) \geq F^0(V)$.

Proof of Proposition 3. Parts (b) and (c) are immediate from proposition 1 and the inequality $F(0) \leq (PD)\Psi$, which holds with strict inequality if $\theta PD > \bar{y}$ and with strict equality if $\theta PD \leq \bar{y}$.

To begin the proof of part (a), it is helpful to make the dependence of $F(V)$ on D explicit by defining $\bar{\underline{\phi}}(V,D)$ to be the RHS of (19).

It follows that

$$\underline{\Phi}(0,D) = \begin{cases} (PD)\Psi & \text{for } 0 \leq \theta PD \leq \overline{y} \\[2em] (PD)\Psi - g_1(PD\theta-\overline{y})^2 & \text{for } PD\theta \geq \overline{y}. \end{cases}$$

It is immediate, then, that for $\Psi > 0$, $\overline{D}$ is unique and satisfies $P\overline{D}\theta > \overline{y}$ and $\underline{\Phi}(0,\overline{D}) > 0$.

Our first task is to prove that for any $D' > \overline{D}$, $\overline{\Phi}(V,D') - \overline{\Phi}(V,\overline{D}) < 0$ for each $V \geq 0$. (This implies that any D that maximizes the solution to $V = F(V)$ is no greater than $\overline{D}$.)

For any $V > 0$, let Ω' be the set of states for which $PD'n(s) + (1-\gamma)V \geq 0$ and let $\overline{\Omega}$ be the set for which $P\overline{D}n(s) + (1-\gamma)V \geq 0$, where $n(s) \equiv (1-\alpha)(\beta x(s)/r-1)$. From $D' > \overline{D}$, we have $\Omega'C\overline{\Omega}$. Therefore,

$$\overline{\Phi}(V,D') - \overline{\Phi}(V,\overline{D}) \leq (PD'-P\overline{D})\int_{\Omega'}p(s)n(s)ds$$

$$- [G(PD'\theta)-G(P\overline{D}\theta)] - (PD'-P\overline{D})(\theta-1/r)$$

$$\leq (PD'-P\overline{D})\int p(s)\max[n(s),0]ds - [G(PD'\theta)-G(P\overline{D}\theta)]$$

$$- (PD'-P\overline{D})(\theta-1/r) = \overline{\Phi}(0,D') - \underline{\Phi}(0,\overline{D}) < 0.$$

This shows that any $D \geq 0$ that implies a maximal solution, V, to $V = \underline{\Phi}(V,D)$ is in the interval $(0,\overline{D}]$. Therefore, the sought-after solution is the solution to the following problem: choose $D \epsilon [0,\overline{D}]$ to maximize the solution to $V = \underline{\Phi}(V,D)$. This problem has a solution, because since $\underline{\Phi}(V,D)$ is a continuous function of D, the solution, V, to $V = \overline{\Phi}(V,D)$ is a continuous function of D.

Footnotes

John Bryant is an economist at the Federal Reserve Bank of Minneapolis, and Neil Wallace is a professor at the University of Minnesota and an advisor to the Federal Reserve Bank of Minneapolis. The views expressed herein do not necessarily represent those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The authors would like to thank Thomas Doan and Robert Litterman for computational assistance and anonymous referees for helpful comments on the exposition. An earlier version of this paper was presented at the University of Florida Conference on "Models of Expectations," Gainesville, May 1978.

[1/] Our answer is different from that of others not so much because we give a transactions cost explanation of positive interest on safe government bonds. This, in a sense, is standard. See, for example, the inventory models of money demand, Tobin (1956) and Miller and Orr (1956). Our answer differs mainly because we carry the transactions costs into our analysis of open-market operations. Most macroeconomic models leave them out.

[2/] A similar result is found in Kareken and Wallace (1978).

[3/] This seeming anomaly could be avoided by a requirement that licenses be intermediated by uninsured intermediaries.

[4/] One ordinarily thinks of an open-market operation as one that holds the sum H + SB constant. But, as is well known, this cannot describe the actions of a consolidated Treasury-Federal Reserve. Since such an operation necessarily implies a change in the net interest obligations of the Treasury, gross interest payments minus payments by the Federal Reserve to the Treasury, it must be accompanied by a change in fiscal policy. We assume a change in taxes such that the interest payment continues to be financed by taxes.

[5/] See Patinkin (1961).

6/Studies that treat such regressions as invariant include Anderson and Jordan (1968), Stein (1976), and Perry (1978). One of the more serious criticisms that can be raised against this interpretation is the following. In any sensible model, individuals respond to the rule generating the deficit. The regression studies use time-series data for periods during which the deficit was in part random or unpredictable. The regression coefficients reflect this and almost certainly understate the price effects of alternative average deficits. To take an extreme example, the invariance interpretation of these regression equations implies that nothing happens at time t if it is announced at that time that a huge average deficit will be run starting at time t+1. Does anyone believe that? Moreover, there is good reason to believe that the estimated coefficients depend on the monetary policy that was being followed. For example, did the Federal Reserve have a money supply or interest rate target? See also Lucas (1972, 1976) and Tobin (1970).

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Table 1

No Insured Intermediaries

h	EU	$P[H+B]/N$ $\times 10^2$	T $\times 10^2$	I $\times 10^2$	Pc $\times 10^2$	PJD/N $\times 10^2$	JV/N $\times 10^2$	i	P_1	P_2
1.00	30.43	46.76	0	3.18	42.8			.00	.722	.361
.95	30.43	46.76	0	3.18	42.8			.00	.722	.361
.90	30.28	46.72	.002	3.18	42.0			.04	.722	.361
.85	29.73	46.57	.012	3.16	39.6			.17	.720	.362
.80	29.14	46.42	.028	3.15	37.1			.30	.718	.362
.75	28.49	46.29	.051	3.14	34.7			.44	.716	.363
.70	27.80	46.16	.081	3.12	32.3			.54	.714	.363
.65	27.05	46.03	.117	3.10	29.9			.73	.712	.364
.60	26.25	45.91	.161	3.08	27.5			.88	.710	.364
.55	25.39	45.80	.212	3.07	25.2			1.04	.707	.365
.50	24.48	45.69	.270	3.05	22.8			1.20	.705	.365
.45	23.50	45.59	.336	3.03	20.5			1.36	.703	.366
.40	22.47	45.49	.410	3.00	18.2			1.52	.701	.367
.35	22.36	45.40	.491	2.98	15.9			1.69	.698	.367
.30	20.19	45.31	.580	2.96	13.6			1.86	.696	.368
.25	18.95	45.23	.676	2.93	11.3			2.03	.693	.368
.20	17.64	45.16	.780	2.91	9.0			2.21	.691	.369
.15	16.25	45.09	.892	2.88	6.8			2.38	.689	.370
.10	14.78	45.03	1.012	2.85	4.5			2.56	.686	.370
.05	13.23	44.97	1.139	2.85	2.2			2.74	.684	.371

Note:

h	=	the ratio of currency to the sum of currency and bonds.
EU	=	expected utility.
P[H+B]/N	=	per capita value of currency and bonds.
T	=	real per capita tax.
I	=	per capita storage of the consumption good.
PC	=	per capita value of individually held currency.
PJD/N	=	per capita value of insured deposits.
JV/N	=	per capita value of licenses.
i	=	interest rate on bonds.
P_j	=	real price of a unit of state j second-period consumption.

Table 2

30 Insured Intermediaries

No Interest on Reserves

h	EU	P[H+B]/N $\times 10^2$	T $\times 10^2$	I $\times 10^2$	Pc $\times 10^2$	PJD/N $\times 10^2$	JV/N $\times 10^2$	i	P_1	P_2
1.00	30.40	46.43	-.005	3.18	42.4	5.03	.309	.02	.722	.361
.95	29.86	46.29	.004	3.17	40.0	5.02	.303	.15	.720	.362
.90	29.28	46.15	.020	3.15	37.5	5.00	.294	.28	.718	.362
.85	28.65	46.03	.042	3.14	35.1	4.99	.288	.42	.716	.363
.80	27.97	45.90	.070	3.12	32.7	4.97	.279	.56	.714	.363
.75	27.23	45.78	.105	3.11	30.4	4.96	.273	.71	.712	.364
.70	26.45	45.67	.148	3.09	28.0	4.94	.264	.85	.710	.364
.65	25.61	45.56	.197	3.07	25.7	4.92	.258	1.01	.708	.365
.60	24.71	45.46	.254	3.05	23.4	4.91	.249	1.16	.706	.365
.55	23.76	45.37	.318	3.03	21.0	4.89	.240	1.32	.703	.366
.50	22.74	45.28	.389	3.01	18.7	4.87	.234	1.48	.701	.366
.45	21.66	45.19	.468	2.99	16.5	4.86	.225	1.66	.699	.367
.40	20.51	45.12	.555	2.96	14.2	4.84	.216	1.82	.696	.368
.35	19.30	45.05	.649	2.94	11.9	4.82	.207	1.99	.694	.368
.30	18.01	44.98	.751	2.91	9.7	4.80	.198	2.16	.692	.369
.25	16.65	44.92	.860	2.89	7.4	4.78	.192	2.33	.689	.369
.20	15.21	44.86	.977	2.86	5.2	4.76	.183	2.51	.687	.370
.15	13.70	44.81	1.101	2.83	2.9	4.75	.174	2.68	.684	.371
.10	12.10	44.77	1.232	2.81	.7	4.73	.165	2.86	.682	.371
.081	11.94	45.09	1.914	2.60	0	4.57	.093	4.40	.661	.376
.080	12.03	45.19	2.103	2.54	0	4.52	.072	4.85	.655	.378
.05	12.40	45.56	2.779	2.33	0	2.85	0	6.47	.634	.383

Table 3

Free Entry into Insured Intermediation

No Interest on Reserves

h	EU	P[H+B]/N $\times 10^2$	T $\times 10^2$	I $\times 10^2$	Pc $\times 10^2$	PJD/N $\times 10^2$	JV/N $\times 10^2$	i	P_1	P_2
1.00	20.40	42.40	1.726	6.47	21.4	26.30	0	1.26	.853	.217
.95	19.83	42.42	1.719	6.35	19.6	25.85	0	1.39	.848	.221
.90	19.22	42.43	1.717	6.24	17.9	25.41	0	1.52	.842	.225
.85	18.56	42.45	1.720	6.12	16.1	24.96	0	1.65	.837	.229
.80	17.86	42.47	1.729	6.00	14.4	24.50	0	1.78	.831	.233
.75	17.11	42.50	1.743	5.88	12.7	24.04	0	1.91	.826	.237
.70	16.32	42.53	1.763	5.76	10.9	23.58	0	2.05	.820	.241
.65	15.48	42.56	1.789	5.64	9.2	23.12	0	2.18	.815	.246
.60	14.58	42.60	1.820	5.52	7.4	22.65	0	2.32	.808	.250
.55	13.63	42.64	1.857	5.39	5.7	22.18	0	2.46	.803	.254
.50	12.63	42.68	1.900	5.27	4.0	21.71	0	2.60	.797	.258
.45	11.56	42.73	1.950	5.15	2.2	21.24	0	2.74	.792	.263
.40	10.44	42.78	2.005	5.03	.5	20.77	0	2.88	.786	.267
.35	10.74	43.23	2.087	4.55	0	18.91	0	3.47	.761	.286
.30	11.44	43.85	2.222	3.93	0	16.45	0	4.28	.727	.312
.25	11.99	44.51	2.411	3.30	0	13.90	0	5.13	.691	.339
.20	12.12	45.19	2.659	2.66	0	11.30	0	6.01	.654	.368
.15	12.18	45.55	2.802	2.33	0	8.54	0	6.47	.634	.383
.10	12.29	45.56	2.791	2.33	0	5.69	0	6.47	.634	.383
.05	12.40	45.56	2.779	2.33	0	2.85	0	6.47	.634	.383

Table 4

30 Insured Intermediaries

Interest on Reserves

h	EU	P[H+B]/N $\times 10^2$	T $\times 10^2$	I $\times 10^2$	Pc $\times 10^2$	PJD/N $\times 10^2$	JV/N $\times 10^2$	i	P_1	P_2
1.00	30.40	46.43	-.005	3.18	42.4	5.03	.310	.02	.722	.361
.95	29.85	46.28	.005	3.17	39.9	5.04	.310	.15	.720	.362
.90	29.26	46.14	.021	3.15	37.5	5.05	.311	.28	.718	.362
.85	28.62	46.00	.043	3.14	35.1	5.06	.311	.42	.716	.363
.80	27.93	45.86	.072	3.12	32.7	5.06	.311	.56	.714	.363
.75	27.19	45.74	.108	3.11	30.3	5.07	.312	.71	.712	.364
.70	26.40	45.62	.152	3.09	27.9	5.08	.312	.86	.710	.364
.65	25.55	45.50	.202	3.07	25.6	5.09	.313	1.02	.708	.365
.60	24.64	45.39	.260	3.05	23.2	5.10	.313	1.17	.705	.365
.55	23.26	45.25	.353	3.02	20.9	5.11	.313	1.40	.702	.366
.50	22.64	45.19	.397	3.01	18.6	5.12	.314	1.50	.701	.366
.45	21.54	45.10	.478	2.98	16.3	5.12	.314	1.66	.699	.367
.40	20.38	45.01	.565	2.96	14.0	5.13	.315	1.83	.696	.368
.35	19.15	44.93	.661	2.94	11.7	5.14	.315	2.00	.694	.368
.30	17.85	44.86	.764	2.91	9.4	5.15	.315	2.18	.691	.369
.25	16.48	44.79	.875	2.88	7.2	5.16	.316	2.35	.689	.369
.20	15.02	44.72	.993	2.86	4.9	5.17	.316	2.53	.687	.370
.15	13.49	44.66	1.119	2.83	2.7	5.18	.317	2.71	.684	.371
.10	11.87	44.61	1.252	2.80	.4	5.19	.317	2.88	.682	.371
.09	11.93	44.87	1.930	2.59	0	5.27	.320	4.44	.661	.376
.089	12.56	45.44	3.264	2.18	0	5.44	.326	7.63	.620	.387
.085	13.97	46.90	7.320	1.24	0	5.89	.295	18.12	.520	.398
.08	15.87	47.52	11.110	1.18	0	6.17	.188	29.71	.470	.365
.05	18.37	49.22	19.21	.77	0	5.00	0	62.38	.360	.307

Table 5

Free Entry into Insured Intermediation

Interest on Reserves

h	EU	$P[H+B]/N$ $\times 10^2$	T $\times 10^2$	I $\times 10^2$	Pc $\times 10^2$	PJD/N $\times 10^2$	JV/N $\times 10^2$	i	P_1	P_2
1.00	15.75	41.20	2.53	7.58	16.9	30.91	0	1.57	.886	.180
.95	14.68	41.11	2.60	7.57	14.8	30.89	0	1.73	.884	.181
.90	13.55	41.03	2.68	7.55	12.7	30.86	0	1.89	.882	.181
.85	12.37	40.96	2.77	7.53	10.6	30.83	0	2.05	.881	.181
.80	11.14	40.88	2.86	7.51	8.6	30.80	0	2.22	.879	.181
.75	9.85	40.82	2.95	7.49	6.6	30.76	0	2.38	.877	.181
.70	8.50	40.76	3.06	7.46	4.6	30.71	0	2.54	.875	.182
.65	7.10	40.70	3.16	7.44	2.6	30.66	0	2.70	.873	.182
.60	5.63	40.65	3.28	7.41	.6	30.60	0	2.87	.871	.182
.55	6.62	41.10	4.81	7.04	0	30.17	0	6.78	.827	.187
.50	8.60	41.78	6.82	6.51	0	29.36	0	12.43	.769	.195
.45	10.48	42.48	8.68	5.95	0	28.24	0	18.16	.714	.203
.40	12.22	43.21	10.39	5.38	0	26.78	0	23.95	.661	.213
.35	13.81	43.97	11.97	4.79	0	24.96	0	29.76	.612	.223
.30	15.21	44.75	13.42	4.18	0	22.75	0	35.54	.565	.234
.25	16.41	45.57	14.76	3.55	0	20.12	0	41.25	.520	.247
.20	17.36	46.42	15.99	2.89	0	17.04	0	46.83	.478	.260
.15	18.03	47.32	17.14	2.21	0	13.51	0	52.24	.437	.275
.10	18.38	48.26	18.21	1.50	0	9.50	0	57.44	.398	.290
.05	18.37	49.22	19.21	.77	0	5.00	0	62.38	.360	.307