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A Simple General Equilibrium Model of Depression

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A Simple General Equilibrium
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This paper presents a simple model of a financial collapse which reduces output and employment. The real disruption resulting from a financial collapse does not depend upon price rigidities but rather is the direct consequence of reduced efficiency of transacting.

Models of fiat money are characterized by at least two equilibria, a fiat money and a nonfiat money equilibrium. We model financial collapse as a move from a fiat money to a nonfiat money solution, where the nonfiat money solution is a low output, unemployment equilibrium. While we do not have a coherent model of financial institutions, is it not possible that their role is to provide an efficient means of transacting, and their collapse is a move to a less efficient means of transacting as in this model? In our simple model of fiat money there is global instability in output and unemployment. A fortiori such pathological behavior can characterize a more sophisticated model of the financial system.

The model provides no explanation for which of the multiple equilibria will characterize the economy at a point in time, and therefore is devoid of dynamics. Moreover, the presented model is bare bones and thus does not have the richness to explain many phenomena, and it should be viewed as a polar case. First the model is presented, then some alternative ways to enrich the model will be sketched. However,

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the simplicity of the model is a virtue as it isolates the important elements and eliminates the extraneous.

The Model

The model is a simple variation on the Samuelson pure consumption loans model.[1/] There are N identical individuals born each period and they live two periods. They have perfect foresight. In his first period an individual is endowed with L units of nontransferable leisure, while in the second he is endowed with nothing. There is a linear zero intercept technology available to the individual to transform leisure hours into a transferable but nonstorable consumption good. One hour of work yields w units of good where $w < 1$. The consumption good and leisure are perfect substitutes in the utility function of the individual where one hour of work equals one unit of goods. The individual maximizes his two-period utility using utility function $U(C_1,C_2)$. U is two-smooth, increasing in its arguments, concave, $U_1(0,C_2) = \infty = U_2(C_1,0)$, C_1 and C_2 are strictly noninferior and strictly gross substitutes, and there exists an $S > 0$ such that $\lim_{\varepsilon\to 0} \varepsilon U_2(L-S,\varepsilon S) > U_1(L-S,0)$. There exists a quantity of NM dollars of fiat money which the young get from the old in exchange for goods.

We consider the representative consumer of generation t where the subscript t is dropped for simplicity. Let P be the current rate of exchange of goods for dollars and P' be the next period value of that variable. 1/P is the price level as usually interpreted. The individual must choose hours of work, W, and dollars of money holding, m, to maximize his utility given P, P'.

His problem is

$$\max_{W,m} U(C_1,C_2)$$

subject to

$$C_1 = L - W + wW - Pm$$

$$C_2 = P'm$$

$$Pm \leq wW$$

$$W \leq L.$$

If $P' = 0$, $W = 0$ and $C_2 = 0$. As $w < 1$, the individual will never produce for consumption, but only for sales, $wW = Pm$. As $U_1(0,C_2) = \infty$, $W < L$ always. As $U_2[L,0] = \infty$, $P' > 0$ implies $W > 0$.

For P, $P' > 0$ the problem can be written

$$\max_{W} U[L-W, \frac{P'}{P} w W].$$

The first-order condition is

$$-U_1[L-W, \frac{P'}{P} w W] + \frac{P'}{P} w U_2[L-W, \frac{P'}{P} w W] = 0.$$

This can be written as $W = f(\frac{P'}{P})$. f is continuous and single valued by strict noninferiority of C_1, C_2, is strictly increasing by the strict gross substitutes assumption, and is bounded below by the assumption that $\lim_{\varepsilon \to 0} \varepsilon U_2(L-S,\varepsilon S) > U_1(L-S,0)$. The domain of f is $(0,\infty)$ and its range is within $[S,L]$.

The current old get no benefit from dollar holding so they trade all their NM dollars to the young for goods, NwW goods. Our equilibrium condition is that

$$\text{(I)} \qquad \frac{NwW}{P} = NM.$$

Substituting in the optimal decision rule for W and rearranging yields

$$\text{(II)} \qquad Nwf(\frac{P'}{P}) = PNM.$$

We are now ready for our central proposition.

Theorem I:

A. There is a unique monetary equilibrium characterized by a constant price level and $W > 0$.

B. There is a nonmonetary equilibrium (an equilibrium with $P = 0$ in all periods) characterized by $W = 0$.

C. The monetary equilibrium is Pareto superior to the nonmonetary equilibrium.2/

Proof:

A. From (II) there is a unique constant positive price equilibrium at price $\overline{P} = \frac{w}{M}f(1) = \frac{w}{M}W$. Consider any other positive equilibrium price sequence $\{P_t\}$ with $P_t \neq \overline{P}$ in some period t. Suppose $P_t > \overline{P}$. Then from (II) and the monotonicity of f, $P_{t+1}/P_t > 1$. Indeed, by using (II) iteratively we see that $\{P_{t+k}\}$ must be growing at an increasing percentage rate. But this is not feasible as $\{P_t\}$ is bounded above uniformly by $\frac{w}{M}L$ from (II) and the upper bound of f. Suppose $0 < P_t < \overline{P}$. Then $\{P_{t+k}\}$ must be falling at an increasing percentage rate. This implies $\lim_{k\to\infty} P_{t+k} = 0$, but this is impossible from (II) and the lower bound on f. Suppose that for some equilibrium price sequence $\{P_t\}$ not zero in every period, for some k_0, $P_{k_0+1} = 0$. Then $W_{k_0} = 0$, which from the equilibrium condition (I) implies $P_{k_0} = 0$. Let the first nonzero element occur at time $k_1 + 1$ where $k_1 > k_0$. Then $W_{k_1} > 0$, which implies $P_{k_1} > 0$ from the equilibrium condition (I), contradiction.

B. We have seen that for $\{P_t\} = \underline{0}$, $W = 0$ satisfies the equilibrium condition and the individual maximization problem.

C. As $W = 0$ is feasible for the individual in the fiat money equilibrium, the fiat money equilibrium is revealed preferred to the nonfiat money equilibrium for the current young and future generations. As the current old consume only the real value of money holdings, the fiat money equilibrium is superior for them as well. Note the role of the unusual assumption on utility which yields the lower bound on f. Without this assumption there could be multiple monetary equilibrium price sequences characterized by different inflation rates--all Pareto superior to the nonmonetary equilibrium, however.

We take "depression" to be a completely surprise shift from the monetary to the nonmonetary equilibrium.

Refinements

In this model people work only for future consumption, all they can consume in the future is the fruits of current labor, and fiat money is the only means of transacting for the fruits of one's labor. In the nonmonetary equilibrium transactions are impossible, yielding zero output, employment, and second-period consumption. This should be viewed as a polar case of what we do see in reality. We do transact for the fruits of our labor, and fiat money is an efficient means of transacting. The model can easily be modified to include endowments in the second period of existence and other means of transacting.[3/]

As with the "new-new" labor economics, people not working at all does depend upon the utility of "leisure."[4/] For example, in our simple model if $C_1 = wW - Pm$, then $W = L$ if $w > 0$. However, as unemployment does not imply starvation, this assumption does not seem unreasonable. That unemployment means that there are better opportunities available outside the economy than in it is, nonetheless, cold comfort!

In the simple model "firms" are factored into the individuals' problem. This can be changed to have separate firm entities.

In the nonfiat money equilibrium nobody works at all, but they are not searching for work and therefore are not involuntarily unemployed. This is easily fixed in a model with multiple means of transaction and two-period endowments by having individuals drawn from a pool and randomly assigned to one of two production technologies, more profitable and less profitable. Individuals have to pay a small "search cost" to belong to the pool. With the fiat money equilibrium both technologies are used, but in the nonfiat money equilibrium only the more profitable is used (if the model is rigged right). In this way not everyone is idle in a "depression," but those who are are involuntarily unemployed. And, of course, productivity is higher in a "depression." If there are separate firms, a similar result can be achieved by having individuals come in two kinds, skilled and unskilled, but having the only means of discriminating being to employ them a short time.

The model has financial collapse, the change from fiat to nonfiat money, a complete surprise. Instead, one could assume that there are subjective probabilities of moving from one to the other. One could further suppose that the probabilities are not independent and assume learning.

In short, there are an innumerable number of ways to enrich the model to make it more "realistic."

Conclusions

We have produced a simple general equilibrium model with multiple equilibria, one of them being a low output, unemployment equilibrium. Moreover, if so simple a model can generate multiple equilibria, *a fortiori* more complex models of the financial system can exhibit instability.

In the model inadequate demand *per se* is not the problem but the collapse of the financial system. We treat financial collapse as a shift to a less efficient means of transacting. This implies that demand management may not be the way to avoid global instability, rather careful regulation (or deregulation!) of the financial markets may be the answer. It also raises the possibility that depression and cyclical downturn are very different phenomena rather than essentially similar events of different magnitude.

Footnotes

1/ See [4].

2/ A similar theorem is proved in [3] and [5].

3/ See [5].

4/ See [1] and [2].

References

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