Default and the Maturity Structure in Sovereign Bonds[*]

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ABSTRACT

This paper studies the maturity composition and the term structure of interest rate spreads of government debt in emerging markets. In the data, when interest rate spreads rise, debt maturity shortens and the spread on short-term bonds rises more than the spread on long-term bonds. To account for this pattern, we build a dynamic model of international borrowing with endogenous default and multiple maturities of debt. Long-term debt provides a hedge against future fluctuations in interest rate spreads, while short-term debt is more effective at providing incentives to repay. The trade-off between these hedging and incentive benefits is quantitatively important for understanding the maturity structure in emerging markets. When calibrated to data from Brazil, the model accounts for the dynamics in the maturity of debt issuances and its comovement with the level of spreads across maturities.

1   Introduction

Debt crises in emerging economies are often blamed on governments borrowing extensive quantities of short-term debt in international capital markets. Short-term borrowing leaves an economy with large amounts of debt to roll over, which becomes problematic when interest rates rise and access to external credit is restricted. This idea has motivated several recent empirical studies on debt and financial crises. For example, Rodrik and Velasco (2003) show, using evidence from a broad set of countries, that a high level of short-term foreign debt increases the likelihood of a crisis.[1] From this *ex post* point of view, it seems desirable to implement policies that would lengthen the maturity structure of emerging market economies' external liabilities. However, as documented by Broner, Lorenzoni, and Schmukler (2008), emerging market governments actively shift to shorter-maturity debt in a crisis and issue long-term debt in normal times. Although this pattern may leave countries more exposed to roll-over crises in bad times, it suggests that there must be some benefits, *ex ante*, of shortening the maturity structure of debt precisely in such times. By the same token, the benefits of long-term debt must outweigh those of short-term debt in normal times.

This paper develops a model of sovereign debt in which the borrowing country endogenously chooses a time-varying maturity structure of debt. Bond prices compensate lenders for the expected loss from default, and interest rate spreads - i.e., yields on defaultable bonds minus yields on riskless bonds of equal maturity - are jointly determined along with the maturity structure of debt. The model accounts for the following observation in the data: when interest rate spreads rise, the maturity of newly issued debt shortens and the short-term spread rises more than the long-term spread. In the absence of commitment to repayment and future debt choices, short-term debt is always more effective at providing incentives to repay. During times of high spreads, the maturity shortens because incentives to repay become more valuable, and long spreads rise by less because of mean reversion in the probability of repayment.

We present data on prices and issuances of foreign-currency denominated bonds for four emerging market countries: Argentina, Brazil, Mexico, and Russia. We estimate *spread curves* – interest rate spreads over U.S. Treasury bonds across maturity – as well as the *duration* of bonds issued – a measure of the average time to maturity of payments on coupon-paying bonds. Across these four countries, within periods in which 1-year spreads are below

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[1] Other detailed studies on individual cases include Calvo and Mendoza (1996) for Mexico; Radelet and Sachs (1998) for the East Asian economies; and Bevilaqua and Garcia (2002) for Brazil. Cole and Kehoe (1996) use a model of self-fulfilling crises to argue that the 1994 Mexican debt crisis could have been avoided if the maturity of government debt had been longer.

their 50th percentile, the average duration of new debt is 7.3 years, and the average difference between the 10-year spread and the 1-year spread – the *slope* of the spread curve – is 2 percentage points. When the 1-year spreads are above their 50th percentile, the average duration shortens to 5.8 years, while the slope of the spread curve is 0.99 percentage points. From this evidence, we conclude that in times of high spreads, the maturity of debt shortens and short spreads rise more than long spreads.

In our model, the borrowing country faces persistent income shocks and can issue long and short duration bonds. The country can default on debt at any point in time but faces costs of doing so, in the form of lower income and exclusion from international financial markets. In equilibrium, default tends to occur in low-income, high-debt times, when the cost of debt payments outweighs the costs of default. Bond prices are functions of the levels of each maturity of debt and income, which determine the borrower's probability of repaying in the future.

Our model generates the dynamics of spread curves observed in the data because the endogenous probability of repayment is persistent, yet mean reverting, as a result of the dynamics of debt and income. In times of low debt and high income, default is unlikely in the near future, so spreads are low. Long-term spreads are higher than short-term spreads because default may become likely in the far future if the borrower receives a sequence of bad shocks and accumulates debt. Conversely, in times of low income and high debt, default is likely in the near future, so spreads are high. Long-term spreads rise less than short-term spreads because the borrower's likelihood of repaying may rise over a longer time horizon if a sequence of good shocks occurs and debt is reduced.

The characteristics of the bond price functions which generate these dynamics in spreads also determine the optimal maturity of debt. The maturity structure reflects a trade-off between the relative *incentive benefit* of short-term debt and the *hedging benefit* of long-term debt reflected in these price functions. When the country issues more debt, incentives to repay fall. The increase in default probability in the near future lowers the price of short- and long-term debt. The price of long-term debt, however, falls more than the price of short-term debt because in addition it depends heavily on future decisions to issue more debt. When issuing long-term debt, the borrower cannot commit to a future sequence of debt issuances, and the probability to default on long-term debt depends on these future choices. Since future borrowing reduces the lender's expected payoff, the price of long-term debt effectively penalizes this lack of commitment. Short-term debt is therefore a better instrument to provide the country with incentives to repay.

However, by the same reasoning, long-term debt provides a hedge. Since the price of

long-term debt is more sensitive to changes in default risk than the price of short-term debt, the value of outstanding long-term debt falls in bad states, while the value of short-term debt remains unchanged. Long-term debt is a good hedge because it lowers the total debt burden in bad states relative to good states, essentially preventing the country from having to roll over large amounts of short-term debt when interest rates rise.

The time-varying maturity structure in the model is a result of the time-varying valuation of the incentive benefit of short-term debt relative to the hedging benefit of long-term debt. Periods of low default probabilities and upward-sloping spread curves correspond to states where incentives to repay debt in the future are high. These are good times to issue more long-term debt to take advantage of its hedging properties. Periods of high default probabilities and flatter or inverted spread curves correspond to states where incentives to repay debt in the future are expected to be low. These are times when the incentive to repay provided by short-term debt is most valuable, and thus the portfolio shifts toward shorter-term debt.

We calibrate the model to Brazil and find that it fits the pattern of the observed dynamics of spreads and maturity. For example, when the spread on 1-year debt is below its 10th percentile, the 10-year spread is on average 1.8 percentage points higher than the 1-year spread, compared to a slope of 1.5 percentage points in Brazil. In periods when the 1-year spread is above its 90th percentile, the slope of the spread curve in the model inverts to -2.5 percentage points, while in the Brazilian data the slope inverts to -0.7 percentage points. The model also mirrors the data in that the average duration of new debt issuances is lower when spreads are high. Specifically, we find that the average duration of debt is about 1.2 years shorter in periods when spreads are below the median than when spreads are above the median. This difference in duration in the Brazilian data across these periods equals 2.4 years. We can therefore rationalize higher short-term debt positions in times of crises as an optimal use of the incentives for repayment provided by short-term debt relative to long-term debt.

We extend the benchmark model to allow for risk premia in the prices of sovereign bonds. Risk premia change the equilibrium bond price functions and provide a tighter fit to the data on spreads. Risk premia also change borrowing and default behavior, lowering the average probability of default and shortening the average duration. Nevertheless, the difference in duration and spread curves across periods with high and low spreads is similar to the benchmark. With risk premia, the bond price functions are influenced by the correlation between the lenders' pricing kernel and the country's default probability, but the incentive and hedging benefits determined by the shapes of these price functions continue to determine the dynamics of the maturity composition.

The model in this paper builds on the work of Aguiar and Gopinath (2006) and Arellano (2008), who model equilibrium default with incomplete markets, as in the seminal paper on sovereign debt by Eaton and Gersovitz (1981). In recent work, Chatterjee and Eyigungor (2009) and Hatchondo and Martinez (2009) show that exogenously lengthening the maturity of debt in these models allows a better fit of emerging market data in terms of the volatility and mean of the country spread and debt levels. Our paper generalizes this framework by including two types of debt, short- and long-term, to analyze the optimal maturity structure.

Several papers study the maturity structure of debt in the presence of default, and we discuss them in comparison to our model in detail in Section 4.4 below. Most closely related to our work is Broner, Lorenzoni, and Schmukler (2008), who argue that emerging markets shift to short-term debt in a crisis because shocks to lenders' risk aversion raise the risk premium on long-term bonds more than on short-term bonds. We consider the effects of time-varying risk in the extended model, but we highlight a tradeoff that exists even with risk-neutral lenders.

The outline of the rest of the paper is as follows. Section 2 presents data on the dynamics of the spread curve and maturity composition for four emerging markets. Section 3 presents the theoretical model. Section 4 illustrates the mechanisms that determine the optimal debt portfolio by considering simplified versions of the model. Section 5 presents quantitative results for our benchmark model. Section 6 contains the results for our extension with risk premia, and Section 7 concludes the paper.

2   Emerging Markets Bond Data

We examine data on sovereign bonds issued in international financial markets by four emerging-market countries: Argentina, Brazil, Mexico, and Russia. We look at the behavior of the interest rate spreads over default-free bonds, across different maturities, and at the way the maturity of new debt issued covaries with spreads. We find that when spreads are low, governments issue longer term bonds and long-term spreads are higher than short-term spreads. When spreads rise, the maturity of bond issuances shortens and short-term spreads rise more than long-term spreads.

2.1   Spread Curves

We define the n-year *spread* for an emerging market country as the difference between the yield on a defaultable, zero-coupon bond maturing in n years issued by the country and the yield on a zero-coupon bond of the same maturity with negligible default risk (for example,

a U.S. Treasury note). The spread is the implicit interest rate premium required by investors to be willing to purchase a defaultable bond of a given maturity. The *spread curve* refers to the spread as a function of maturity.

We denote the continuously compounded yield at date t on a zero-coupon bond issued by country i, maturing in n years, as $r_{t,i}^n$. The yield is related to the price $p_{t,i}^n$ of an n-year zero-coupon bond, with face value 1, through

$$p_{t,i}^n = \exp(-n \times r_{t,i}^n). \tag{1}$$

The n-year spread for country i at date t is given by: $s_{t,i}^n = r_{t,i}^n - r_{t,rf}^n$, where $r_{t,rf}^n$ is the yield of an n-year default-free bond.[2]

Since governments do not issue zero-coupon bonds in a wide range of maturities, we estimate a country's spread curve by using secondary market data on the prices at which coupon-bearing bonds trade. We describe this procedure further in the Appendix and illustrate that the resulting pricing errors are small.

We compute spreads starting in March 1996 at the earliest and ending in April 2011 at the latest, depending on the availability of data for each country. Figure 1 displays the estimated spreads for 1-year and 10-year bonds for Argentina, Brazil, Mexico, and Russia.

Spreads are very volatile, and the difference between long-term and short-term spreads varies substantially over time. When spreads are low, long-term spreads are generally higher than short-term spreads. However, when the level of spreads rises, the gap between long and short-term spreads tends to narrow and sometimes reverses: the spread curve flattens or inverts. The time series in Figure 1 show sharp increases in interest rate spreads associated with Russia's default in 1998, Argentina's default in 2001, and Brazil's financial crisis in 2002.[3] The expectation that the countries would default in these episodes is reflected in the high spreads charged on defaultable bonds.

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[2] Our data include bonds denominated in U.S. dollars and European currencies, so we take U.S. and Euro-area government bond yields as default-free.

[3] For Argentina and Russia, we do not report spreads after default on external debt, unless a restructuring agreement was largely completed at a later date. We use dates taken from Sturzenegger and Zettelmeyer (2005). For Russia, we report spreads until the second week of August 1998 and beginning again after August 2000 when 75% of external debt had been restructured. For Argentina, we report spreads until the last week of December 2001, when the country defaulted. Although 76% of the debt was restructured by 2005, we do not have data for enough of the restructured bonds to calculate spreads since then.
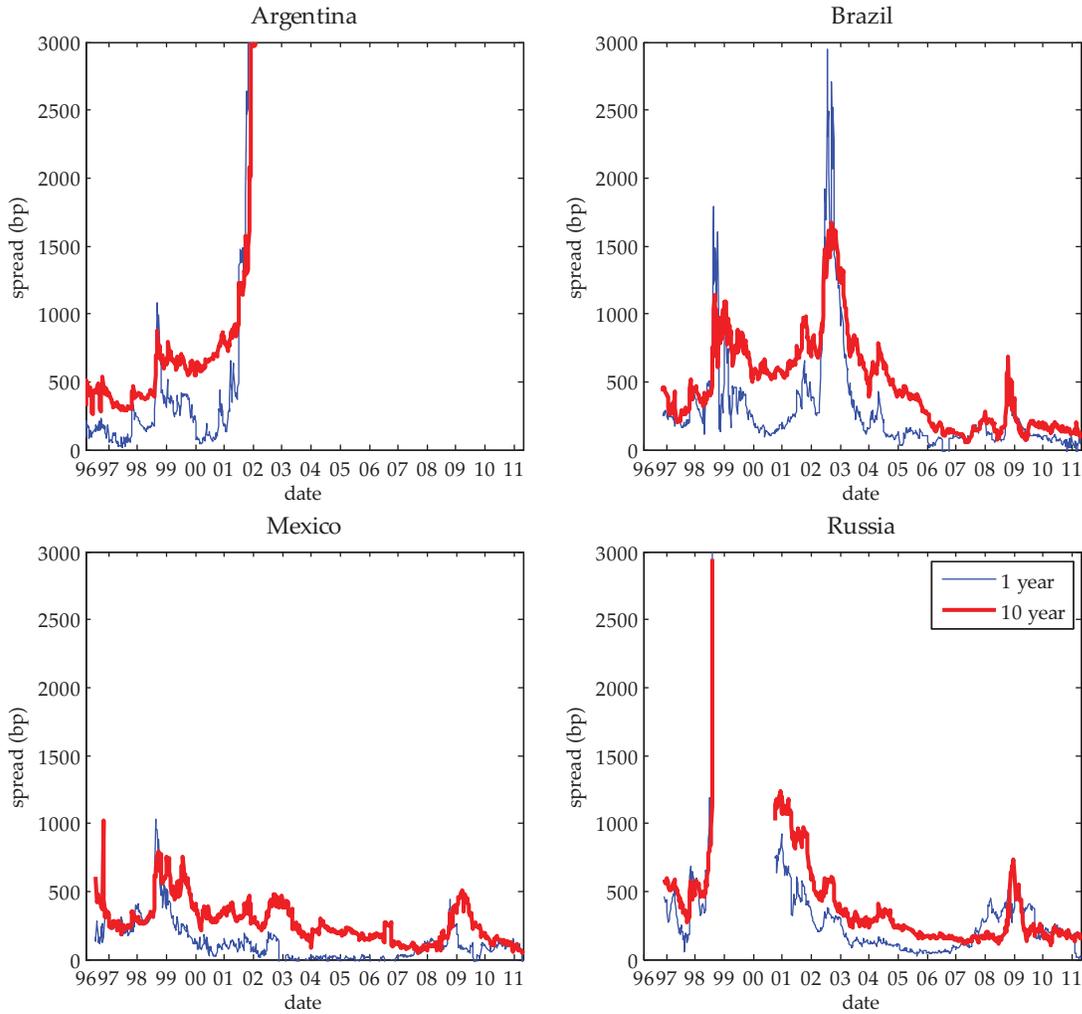
Figure 1: Time Series of Short and Long Spreads

2.2 Spreads and the Maturity Composition of Debt

We now examine the maturity of new debt issued by the four emerging market economies during the sample period, and relate the changes in the maturity of debt to changes in spreads.[4]

In each week in the sample, we measure the maturity of debt as a quantity-weighted average maturity of bonds issued that week. We measure the maturity of a bond using two alternative statistics. The first is simply the number of years from the issue date until the

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[4]In addition to external bond debt, emerging countries also have debt obligations with multilateral institutions and foreign banks. However, marketable debt constitutes a large fraction of the external debt. The average marketable debt from 1996 to 2004 is 56% of total external debt in Argentina, 59% in Brazil, and 58% in Mexico (Cowan et al. 2006).

maturity date. The second is the bond's *duration*, defined in Macaulay (1938) as a weighted average of the number of years until each of the bond's future payments. A bond issued at date t by country i, paying annual coupon c at dates $n_1, n_2, \ldots, n_J$ years into the future, and face value of 1 has duration $d_{t,i}(c)$ defined by

$$d_{t,i}(c) = \frac{1}{p_{t,i}(c)} \left(\sum_{j=1}^{J} \exp(-n_j r_{t,i}^{n_j}) n_j c + \exp(-n_J r_{t,i}^{n_J}) n_J \right), \qquad (2)$$

where $p_{t,i}(c)$ is the coupon bond's price, and $r_{t,i}^n$ is the zero-coupon yield curve. In (2), the time until each future payment is weighted by the discounted value of that payment relative to the price of the bond. Therefore, a zero-coupon bond has duration equal to the number of years until its maturity date, but a coupon-paying bond has duration shorter than its time to maturity. We consider duration as a measure of maturity because it is more comparable across bonds with different coupon rates.

We calculate the average maturity and average duration of new bonds issued in each week by each country. Table 1 displays each country's averages of these weekly maturity and duration series, along with 1-year and 10-year spreads. The table contains overall averages, as well as averages within periods of high (above median) and low (below median) 1-year spreads.

Table 1: Average Spreads, Maturity and Duration, Conditional on Levels of 1-Year Spread

		Argentina	Brazil	Mexico	Russia
Maturity	overall	9.25	11.32	11.91	10.84
	< median	10.36	16.12	12.03	8.18
	≥ median	7.42	9.97	12.94	11.14
Duration	overall	6.11	6.47	6.69	6.45
	< median	6.65	8.23	7.45	6.70
	≥ median	4.86	5.82	6.79	5.64
1-year spread	overall	4.22	2.94	1.46	2.70
	< median	1.12	0.96	0.34	1.11
	≥ median	7.32	4.91	2.58	4.28
10-year spread	overall	6.29	4.75	2.63	3.71
	< median	4.65	2.78	1.99	2.24
	≥ median	7.93	6.72	3.26	5.17

First, the table shows that duration tends to be much shorter than maturity. Because the yield on an emerging market bond is typically high, the principal payment at the maturity

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date is severely discounted, and much of the bond's value comes from coupon payments made before maturity.

Second, the average duration of debt is shorter when spreads are high than when they are low. Brazil, for example, issues debt that averages about 2 and a half years longer in duration when the 1-year spread is below its median than when it is above its median. In Mexico, the difference is about 8 months. For Argentina and Brazil, this pattern also holds for the average time-to-maturity of bonds issued during periods of high spreads compared to low spreads: Brazil issues bonds that mature about 5 years sooner when spreads are high, regardless of the coupon rate.

In Table 2, we report the results of several univariate panel regressions of duration on spread measures. We pool the data for the four countries and include country fixed effects. Since debt is issued infrequently compared to the frequency of the spread data, we smooth the data by running regressions with averages within 20 quantiles of the 1-year spread, so that one observation is, for example, the average duration of debt Brazil issued during periods when the 1-year spread is between the 5th and 10th percentiles of the series. (Using 10 or 50 quantiles yields similar results.) All coefficients are significant at the 1% level, and robust standard errors are reported in parentheses. Column I reports the effect of the 1-year spread on the duration of new issuances. The coefficient means that a 1 percentage point increase in the 1-year spread is associated with a decrease in the duration of new debt issued by about four months. Column II shows a similar effect for the 10-year spread. These figures indicate that the covariation between the duration of new debt issuance and interest rate spreads is both economically large and statistically significant. Moreover, Column III of Table 2 shows that the duration of new debt is positively associated with how large the 10-year spread is relative to the 1-year spread. When the ratio of the long spread to the short spread is reduced by half, the duration of new debt falls by about $1.094 \times \log(2) =$ three quarters of a year. For example, in Mexico, the average difference between the 10-year and the 1-year spread below the 5th percentile of the 1-year spread is about 1.84 percentage points, and the average duration of debt issued in these periods is 7.1 years. Moving above the 95th percentile of the short spread, the slope of the spread curve falls to -1.24 percentage points, and the duration of new debt falls to 6.2 years.

One concern with using duration as defined in equation (2) is that the current period's interest rates are used to discount future payments. In periods of high spreads, the spread curve flattens (or inverts), which by itself can affect the duration of debt issued. In the appendix we show the statistics in Tables 1 and 2 computed with an analogue of duration using the default-free yield curve; the results are similar under this alternative measure of

Table 2: Regressions of Duration of New Issuances on Spreads

Variable	I	II	III
1-year spread	−0.348 (0.026)		
10-year spread		−0.439 (0.029)	
$\log \frac{10 \text{ year spread}}{1 \text{ year spread}}$			1.094 (0.048)
Constant	7.323 (0.060)	8.293 (0.111)	5.684 (0.052)
R^2	0.256	0.296	0.281
No. of obs.	60	60	60

Notes: The dependent variable is the average duration of debt issued within each of 20 quantiles of the 1-year spread. Independent variables are analogues of the relevant spread measure. All coefficients are siginificant at the 1% level.

duration.

2.3 Summary

The message of this section is that the spread curve and the maturity of bond issuances in emerging markets are systematically time-varying. In particular, the level of spreads covaries negatively with the duration of new debt, and with the slope of the spread curve: when spreads are low, the slope of the spread curve is higher, and the duration of new debt is longer, than when short-term spreads are high.

Our findings confirm the earlier results of Broner, Lorenzoni, and Schmukler (2008), who show in a sample of eight emerging economies that debt maturity shortens when spreads are very high. They show that a high spread level is a statistically significant determinant for a shorter maturity of debt issuances even after controlling for selection effects since the timing of debt issuances is very irregular. Their empirical work treats the choice of issuing short-term or long-term debt as a discrete variable, whereas we use the continuous variable of duration as a measurement of maturity. Moreover, they ultimately focus on the relationship between the term structure of *risk premia* (compensation for risk aversion) and the average *maturity* of debt, whereas we focus on the term structure of *yield spreads* and the average *duration* of debt, because we use these statistics to quantitatively assess our model.

3 The Model

We consider a dynamic model of defaultable debt that includes bonds of short and long duration. A small open economy receives a stochastic stream of income, y, of a tradable good. Income follows a Markov process with support Y and transition function $f(y_t, y_{t+1})$. The economy trades two bonds of different duration with international lenders. Financial contracts are unenforceable, so the economy can default on its debt at any time. If the economy defaults, it temporarily loses access to international financial markets and also incurs direct costs.

The representative agent in the small open economy (henceforth, the "borrower") receives utility from consumption c_t and has preferences given by

$$E \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{3}$$

where $0 < \beta < 1$ is the time discount factor and $u(\cdot)$ is increasing and concave.

The borrower issues short- and long-term debt. Short-term debt is a one period discount bond. The short-term contract specifies a price q_{St} and a loan face value b_{St+1} such that the borrower receives $q_{St} b_{St+1}$ units of goods in period t and promises to pay, conditional on not defaulting, b_{St+1} units of goods in period $t+1$. We model long-term debt as a perpetuity contract with coupon payments that decay geometrically at rate δ, as in Hatchondo and Martinez (2009). The perpetuity contract specifies a price q_{Lt} and a value ℓ_t such that the borrower receives $q_{Lt} \ell_t$ units of goods in period t and promises to pay, conditional on not defaulting, $\delta^{n-1} \ell_t$ units of goods in every future period $t+n$.

At every date t, the economy has outstanding the history of all past bond issuances. Define b_{Lt}, the stock of long-term bonds at time t, as the total payments due in period t on all past issuances, conditional on not defaulting:

$$b_{Lt} = \sum_{j=1}^{t} \delta^{j-1} \ell_{t-j} = \ell_{t-1} + \delta \ell_{t-2} + \delta^2 \ell_{t-3} + \dots + \delta^t \ell_0 + b_{L0},$$

where b_{L0} is given. The law of motion for the stock of long-term debt can be written recursively as

$$b_{Lt+1} = \delta b_{Lt} + \ell_t. \tag{4}$$

With this definition, we can compactly write the borrower's budget constraint conditional on not defaulting. Purchases of consumption are constrained by income less payments on

outstanding debt, $b_{St} + b_{Lt}$, plus the issues of new short bonds b_{St+1} at price q_{St} and long bonds ℓ_t at a price q_{Lt}:

$$c_t = y_t - b_{St} - b_{Lt} + q_{St}b_{St+1} + q_{Lt}\ell_t. \tag{5}$$

The borrower chooses new issues short- and long-term debt from a menu of contracts where prices q_{St} and q_{Lt} for are quoted for each pair (b_{St+1}, b_{Lt+1}).

If the borrower defaults, we assume that all outstanding debt (b_{St}, b_{Lt}) is erased from the budget constraint, and the economy cannot borrow or save, so that consumption equals output. In addition, the country incurs output costs:

$$y_t^{def} = \begin{cases} y_t & \text{if } y_t \leq (1-\lambda)\bar{y} \\ (1-\lambda)\bar{y} & \text{if } y_t > (1-\lambda)\bar{y} \end{cases},$$

where $\bar{y}$ is the mean level of income. This specification, following Arellano (2008), assumes that borrowers lose a fraction λ of income if their income is above a certain threshold.

3.1 Recursive Problem

We focus on recursive Markov equilibria and represent the borrower's infinite horizon decision problem as a recursive dynamic programming problem. The model has two endogenous states – the stocks of each type of debt, b_{St} and b_{Lt} – and one exogenous state, the income of the economy, y_t. The state of the economy at date t is then given by $(b_S, b_L, y) \equiv (b_{St}, b_{Lt}, y_t)$.

At any given state, the value of the option to default is given by

$$v^o(b_S, b_L, y) = \max_{c,d} \left\{ v^c(b_S, b_L, y), v^d(y) \right\}, \tag{6}$$

where $v^c(b_S, b_L, y)$ is the value associated with not defaulting and staying in the contract and $v^d(y)$ is the value associated with default.

Since we assume that default costs are incurred whenever the borrower fails to repay its obligations in full, the model will only generate complete default on all outstanding debt, both short and long term. When the borrower defaults, income falls to y^{def}, and the economy is temporarily in financial autarky; θ is the probability that it will regain access to international credit markets each period. The value of default is then given by the following:

$$v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^o(0, 0, y') + (1-\theta)v^d(y') \right] f(y, y')dy'. \tag{7}$$

We are taking a simple route to model both costs of default that seem empirically relevant:

exclusion from financial markets and direct costs in income. Moreover, we assume that the default value does not depend on the maturity composition of debt prior to default. This captures the idea that the maturity composition of defaulted debt is not relevant for the restructuring procedures that allow the economy to reenter the credit market.[5]

When the borrower chooses to remain in the contract, the value is the following:

$$v^c(b_S, b_L, y) = \max_{\{b'_S, b'_L, \ell, c\}} \left(u(c) + \beta \int_{y'} v^o(b'_S, b'_L, y') f(y, y') dy' \right) \qquad (8)$$

subject to the budget constraint:

$$c = y - b_S - b_L + q_S\left(b'_S, b'_L, y\right) b'_S + q_L\left(b'_S, b'_L, y\right) \ell \qquad (9)$$

and to the law of motion for the stock of long bonds:

$$b'_L = \delta b_L + \ell.$$

The borrower decides on optimal debt levels b'_S and b'_L to maximize utility. The borrower takes as given that each contract $\{b'_S, b'_L\}$ comes with specific prices $\{q_S, q_L\}$ that are contingent on today's state y. The decision of whether to remain in the credit contract or default is a period-by-period decision, so that the expected value from next period forward in (8) incorporates the option to default in the future.

The default policy can be characterized by default sets and repayment sets. Let the repayment set, $R(b_S, b_L)$, be the set of income levels for which repayment is optimal when short- and long-term debt are (b_S, b_L):

$$R(b_S, b_L) = \left\{ y \in Y : v^c(b_S, b_L, y) \geq v^d(y) \right\}, \qquad (10)$$

and let the complement, the default set $D(b_S, b_L)$, be the set of income levels for which default is optimal for debt positions (b_S, b_L):

$$D(b_S, b_L) = \left\{ y \in Y : v^c(b_S, b_L, y) < v^d(y) \right\}. \qquad (11)$$

When the borrower does not default, optimal new debt takes the form of two decision

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[5]This is consistent with empirical evidence regarding actual restructuring processes, where the maturity composition of the new debt obligations is part of the restructuring agreement (Sturzenegger and Zettelmeyer 2005).

rules mapping today's state into tomorrow's debt levels:

$$b'_S = \tilde{b}_S(b_S, b_L, y) \tag{12}$$
$$b'_L = \tilde{b}_L(b_S, b_L, y)$$

Given this characterization of debt and default decisions, we can now define equilibrium bond prices.

3.2 Bond Prices and Equilibrium

Lenders are perfectly competitive and, for the bulk of the analysis, we assume they are risk neutral and discount the future at the rate r^*. We relax this assumption in the quantitative results and consider the case of risk-averse lenders.

Each new issue of short-term debt b_{St+1} is a promise by the borrower to pay the face value the next period conditional on not defaulting. A new issue of long-term debt $\ell_t > 0$ is a promise to pay a coupon payment every period in the future, conditional on not defaulting up to that period. If $\ell_t < 0$ the borrower is repurchasing some of its long-term debt. If the borrower defaults in period t, we assume that lenders receive zero.

The price of a new debt issue is the discounted sum of the value of the promised future payments, adjusted by the cumulative probability of repayment. We can decompose the price of a long-term debt issue in period t into the expected payoff in period $t + 1$ plus the discounted sum of the value of coupon payments from period $t + 1$ on. Given the recursive structure of debt, this discounted sum from period $t + 1$ on is equal to the price of new debt at period $t + 1$. Hence, we can write the long-term bond price recursively by assuming that the lender forecasts future debt levels using the borrower's decision rules, defined in (12). Prices for short- and long-term debt then satisfy the functional equations:

$$q_S(b'_S, b'_L, y) = \frac{1}{1 + r^*} \int_{R(b'_S, b'_L)} f(y, y') \, dy' \tag{13}$$

$$q_L(b'_S, b'_L, y) = \frac{1}{1 + r^*} \int_{R(b'_S, b'_L)} \left[1 + \delta q_L \left(\tilde{b}_S(b'_S, b'_L, y'), \tilde{b}_L(b'_S, b'_L, y'), y' \right) \right] f(y, y') \, dy'. \tag{14}$$

where $R(b_S, b_L), \tilde{b}_S, \tilde{b}_L$ are the repayment set and policy functiosn defined in (10) and (12).

In our recursive Markov equilibria all decision rules are functions only of the state variables b_S, b_L, and y. A recursive equilibrium for this economy is (i) a set of policy functions for consumption $\tilde{c}(b_S, b_L, y)$, new issuances for short-term debt $\tilde{b}_S(b_S, b_L, y)$ and long-term debt

$\tilde{\ell}(b_S, b_L, y)$, perpetuity stocks for long-term debt $\tilde{b}_L(b_S, b_L, y)$, repayment sets $R(b_S, b_L)$, and default sets $D(b_S, b_L)$, and (ii) price functions for short debt $q_S(b'_S, b'_L, y)$ and long debt $q_L(b'_S, b'_L, y)$, such that

1. Taking as given the bond price functions $q_S(b'_S, b'_L, y)$ and $q_L(b'_S, b'_L, y)$, the policy functions $\tilde{b}_S(b_S, b_L, y)$, $\tilde{b}_L(b_S, b_L, y)$, $\tilde{\ell}(b_S, b_L, y)$ and $\tilde{c}(b_S, b_L, y)$, repayment sets $R(b_S, b_L)$, and default sets $D(b_S, b_L)$ satisfy the borrower's optimization problem.

2. The bond price functions $q_S(b'_S, b'_L, y)$ and $q_L(b'_S, b'_L, y)$ satisfy equations (13) and (14).

3.3 Optimal Maturity Structure[6]

We analyze the optimal maturity structure that results from solving the borrower's problem in (8). For illustration, we assume that the distribution function f is continuous and that the bond price functions q_S, q_L and the value of repaying v^c are differentiable. We can then write the first-order necessary conditions for an interior solution to the borrower's problem as

$$u'(c)\left[q_S + \frac{\partial q_S}{\partial b'_S}b'_S + \frac{\partial q_L}{\partial b'_S}(b'_L - \delta b_L)\right] = \beta \int_{R'} u'(c')f(y, y')\, dy' \tag{15}$$

$$u'(c)\left[q_L + \frac{\partial q_S}{\partial b'_L}b'_S + \frac{\partial q_L}{\partial b'_L}(b'_L - \delta b_L)\right] = \beta \int_{R'} (1 + \delta q'_L)u'(c')f(y, y')\, dy'. \tag{16}$$

where R' is shorthand for the repayment set in the next period. The right-hand sides of these first-order conditions reflect how debt affects consumption in the next period in repayment states. In equilibrium, these expressions do not depend on how b'_S and b'_L affect the default decision because changes in the repayment and default sets have zero net effect on the borrower's future value, as v^c and v^d are equal at the boundary between these sets.[7]

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[6] This section draws heavily on the insightful comments of an anonymous referee.

[7] To see this, suppose that the default and repayment sets took the form of a single threshold: $R(b_S, b_L) = [\hat{y}(b_S, b_L), y_H]$ and $D(b_S, b_L) = [y_L, \hat{y}(b_S, b_L))$, where $y_L < y_H$ are the lower and upper bounds on the shock. Then, the borrower's expected future value of choosing (b'_S, b'_L) is

$$\int_{\hat{y}(b'_S, b'_L)}^{y_H} v^c(b'_S, b'_L, y')\, f(y, y')\, dy' + \int_{y_L}^{\hat{y}(b'_S, b'_L)} v^d(y')\, f(y, y')\, dy'.$$

Since $v^c(b'_S, b'_L, \hat{y}(b'_S, b'_L)) = v^d(\hat{y}(b'_S, b'_L))$, the derivative of the above expression with respect to b'_S is

$$\int_{\hat{y}(b'_S, b'_L)}^{y_H} \frac{\partial v^c(b'_S, b'_L, y')}{\partial b_S}\, f(y, y')\, dy'$$

and $\frac{\partial v^c}{\partial b_S} = u'(c')$ by the Envelope theorem. The argument extends to the case in which default and repayment sets are composed of an arbitrary set of nonoverlapping intervals.

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The first-order condition for each maturity of debt equates the marginal gain in utility today from borrowing to the marginal reduction in utility from repaying tomorrow. The terms in square brackets capture how the price of debt - and hence the marginal benefit of borrowing - depends on the quantity of debt. Given that incentives to repay decline with the quantity of debt, bond prices are decreasing in debt levels, $\frac{\partial q_m}{\partial b'_n} \leq 0$ for $m, n = \{S, L\}$. Long and short-term debt differ in the sensitivities of the bond price functions and also in the marginal value of repaying: one unit of short-term debt is worth one unit tomorrow, but one unit of long-term debt is worth $1 + \delta q'_L$ tomorrow. If we divide each first-order condition by the corresponding price q_S or q_L, and use the definitions of the price functions from (13)-(14), then

$$u'(c) \left[1 + \frac{\partial q_S}{\partial b'_S} \frac{b'_S}{q_S} + \frac{\partial q_L}{\partial b'_S} \frac{(b'_L - \delta b_L)}{q_S} \right] = \beta (1 + r^*) E\left[u'(c') | R' \right] \tag{17}$$

$$u'(c) \left[1 + \frac{\partial q_S}{\partial b'_L} \frac{b'_S}{q_L} + \frac{\partial q_L}{\partial b'_L} \frac{(b'_L - \delta b_L)}{q_L} \right] = \beta (1 + r^*) E\left[u'(c') | R' \right] \frac{E\left[(1 + \delta q'_L) u'(c') | R' \right]}{E\left[(1 + \delta q'_L) | R' \right] E\left[u'(c') | R' \right]} \tag{18}$$

where the notation $E[\cdot | R']$ denotes the conditional expectation across the repayment states in the next period.

These first-order conditions illustrate how the hedging benefit of long-term debt and the incentive benefit of short-term debt determine the maturity structure. The right-hand side of (18) captures the hedging benefit of long-term debt. If the price of long-term debt tomorrow q'_L tends to go down in bad states when c' is low, then long-term bonds are a good hedge, since their value falls in bad times. This effect is absent in (17), since the value of short-term debt in the future does not vary with the state. In other words, the returns to long-term debt, $\frac{1 + \delta q'_L}{q_L}$ and short-term debt, $\frac{1}{q_S}$, do not span each other.

The terms in brackets on the left-hand sides of equations (17) and (18) capture the effects of issuing short- and long-term debt on the incentive to repay, as reflected in the sensitivities of the bond price functions (13) and (14). The price of short-term debt depends on the borrower's incentive to repay in the next period, whereas the price of long-term debt depends on the incentives to repay as well as the decision to borrow again in the future. If there were only short-term debt, issuing an additional unit of debt decreases its price, because the incentive to repay falls (the repayment set R' shrinks). With long-term debt, the same effect is present, but there is an additional effect on the price of long term debt q_L through the policy functions for debt next period, $\tilde{b}_S(b'_S, b'_L, y')$ and $\tilde{b}_L(b'_S, b'_L, y')$ as seen in (14). The borrower cannot commit to a particular level of debt for the next period but instead takes the future debt policy functions as given. If these policy functions are increasing in debt levels,

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then q_L is more sensitive to changes in debt levels than q_S . We also find in our numerical results that both price schedules are more sensitive to changes in b'_L than b'_S.

Taking the ratio of these two first-order conditions, the optimal maturity structure in our model equates the hedging benefit of long-term debt, defined as

$$\text{Hedging benefit} = \frac{E\left[(1 + \delta q'_L)u'(c')|R'\right]}{E\left[(1 + \delta q'_L)|R'\right]E\left[u'(c')|R'\right]} \tag{19}$$

to the relative incentive benefit of short-term debt:

$$\text{Relative incentive benefit} = \frac{1 + \frac{\partial q_S}{\partial b'_L}\frac{b'_S}{q_L} + \frac{\partial q_L}{\partial b'_L}\frac{(b'_L - \delta b_L)}{q_L}}{1 + \frac{\partial q_S}{\partial b'_S}\frac{b'_S}{q_S} + \frac{\partial q_L}{\partial b'_S}\frac{(b'_L - \delta b_L)}{q_S}}. \tag{20}$$

Both of these forces are driven by the shapes of the bond prices as functions of long- and short-term debt as well as income.

In illustrating these mechanisms, we have argued that short-term debt has an incentive benefit if bond prices are less sensitive to short-term than long-term debt and that long-term debt is a good hedge if q_L falls in bad times. The following examples illustrate in detail how these features arise and how they vary across good and bad times in an environment with endogenous default.

4   Highlighting the Link between Default and Maturity

The optimal maturity structure in our model is the outcome of a trade-off between the incentive benefit of short-term debt and the hedging provided by long-term debt. In this section, we use simple three-period versions of our model to illustrate these forces and how they can generate cyclical variation in the optimal maturity structure. We label the periods $t = 0, 1, 2$, and assume throughout that the borrower can issue one-period bonds in periods 0 and 1 (b_0^1 and b_1^1, respectively) and a two-period zero-coupon bond in period 0 (b_0^2).

4.1   The Incentive Benefit of Short-Term Debt

This example shows that short debt allows larger borrowing because of an incentive benefit relative to long-term debt. Consider a borrower who has linear utility over consumption in the three periods: $U(c_0, c_1, c_2) = c_0 + \beta c_1 + \beta^2 c_2$. Suppose that the risk-free interest rate is $r^* = 0$ and that the borrower's discount factor $\beta < 1$. Income is deterministic and equal to $y_0 = 0$, y_1, and y_2. If the borrower defaults in any period, consumption is equal to $y^{def} = 0$

from then on. In this deterministic setting, the price of a bond is equal to 1 if the borrower will repay and 0 otherwise. Since $\beta(1 + r^*) < 1$, the borrower would like to consume its lifetime income in period 0. Hence, the optimal allocation in this environment is $c_0 = y_1 + y_2$ and $c_1 = c_2 = 0$.

This allocation can be implemented with short- and long-term debt in period 0 and short-term debt in period 1. In particular, short-term debt transfers income from period 1 and long-term debt transfers income from period 2: $b_0^1 = y_1$, $b_0^2 = y_2$, and $b_1^1 = 0$. There is no default and the prices of both bond issues are equal to 1. Alternatively, the same allocation can be implemented without long-term debt in period 0, and with short-term debt in periods 0 and 1: $b_0^1 = y_1 + y_2$, $b_0^2 = 0$, and $b_1^1 = y_2$.

Can the same allocation be implemented without short-term debt in period 0? No. Short-term debt provides an incentive for the borrower to repay in period 1 that long-term debt does not. If the borrower chooses any $b_0^2 > y_2$, this debt could be repaid only by saving in period 1 – choosing $b_1^1 < 0$. Once period 1 arrives, however, the borrower has no incentive to save: the optimal decision is to set $b_1^1 = 0$ and then default in period 2. Taking into account this policy for b_1^1, lenders effectively constrain b_0^2 by setting a price of zero for any long-term bond bigger than y_2. In this sense, prices are more sensitive to long debt than to short debt, as argued in the general model with equations (17) and (18).[8]

The two key assumptions behind the incentive benefit of short-term debt are the lack of commitment in debt policies inherent in the Markov equilibria we consider, and that the punishment arises only in the event of an explicit default. If lenders could instead impose a punishment for deviating from a given debt policy or could enrich debt contracts with future debt limits then the incentive benefit of short-term debt would be diminished. The extent to which the benefits of short-term debt change would depend on the details of these punishments or the enforceability of future debt limits.[9]

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[8]It is easy to extend this example to an infinite horizon environment with deterministic and time varying output. A one-period bond economy can deliver higher initial consumption than a longer-term bond –two-period or perpetuity – economy. The main idea is again that the threat of punishment can be used more effectively with one-period bonds because longer-term contracts might require savings in the future which are impossible to induce with default punishments.

[9]Empirically, bond contracts rarely specify contingencies based on future borrowing. Bizer and DeMarzo (1992) argue that in corporate debt markets it is very unusual that lending contracts limit future borrowing behavior. Hatchondo et al. (2011) argue that the same is true for sovereign debt markets, because countries borrow from several lenders, and debt seniority clauses are uncommon.

4.2 The Hedging Benefit of Long-Term Debt

To highlight the hedging benefit of long-term debt, we now assume the borrower has concave utility, and we introduce uncertainty, such that default risk varies across states. Here the borrower attempts to spread lifetime wealth evenly across states, and doing so requires a combination of short-term and long-term debt. Essentially, when default risk varies across states, short- and long-term debt span different states, and hence a risk-averse borrower uses both. However, we show that when wealth is sufficiently low, long-term borrowing becomes constrained as in the deterministic example above, and short-term debt is used more intensively.

Consider a borrower whose utility is

$$U\left(c_0, c_1, c_2\right) = E_0 \left[\log c_0 + \beta \log c_1 + \beta^2 \log c_2\right]. \tag{21}$$

The risk-free rate is $r^* = 0$, and the borrower's discount factor $\beta < 1$. Income in period 0 is y_0. In period 1, there are two states, indexed by p and g. Income in period 1 is equal to y_1 in either state, but the borrower receives a signal about income in period 2 that differs across states. In state p, the borrower learns that $y_2 = y_2^H$ with probability p, while $y_2 = y_2^L < y_2^H$ otherwise. In state g, the probability of getting y_2^H is g. The probability of state p occurring is α. Consumption in the event of a default is now equal to $y^{def} > 0$.

First, suppose that the endowments and the default punishment satisfy

$$\frac{\beta^2}{R} W\left(y_0, y_1, y_2^H\right) - \left(y_2^H - y_2^L\right) < y^{def} \leq \frac{\beta^2}{R} W\left(y_0, y_1, y_2^H\right), \tag{22}$$

where $W\left(y_0, y_1, y_2^H\right) = y_0 + y_1 + \left(\alpha p + (1 - \alpha) g\right) y_2^H$ and $R = 1 + \beta + \beta^2 \left(\alpha p + (1 - \alpha) g\right)$. W can be interpreted as the market value of the borrower's income in all states except when $y_2 = y_2^L$.

Under these conditions, the equilibrium is as follows. The borrower defaults if income in period 2 is y_2^L and repays in all other states. The default patterns imply that bond prices are $q_0^1 = 1$, $q_0^2 = \alpha p + (1 - \alpha) g$, and importantly that bond prices are different across states in period 1 with $q_1^1\left(\pi\right) = \pi$ for $\pi = p, g$.

The optimal consumption allocations decline over time at rate β but are constant across

states in periods 1 and 2:

$$c_0 = \frac{1}{R}W \tag{23}$$

$$c_1(p) = c_1(g) = \frac{\beta}{R}W$$

$$c_2^H(p) = c_2^H(g) = \frac{\beta^2}{R}W.$$

This is an equilibrium because the allocation maximizes expected utility subject to the borrower's budget constraints: $c_0 = y_0 + b_0^1 + (\alpha p + (1-\alpha)g) b_0^2$, $c_1(\pi) = y_1 - b_0^1 + \pi b_1^1(\pi)$, $\pi = p, g$, and $c_2^H(\pi) = y_2^H - b_0^2 - b_1^1(\pi)$, $\pi = p, g$.

To see this, write the first-order conditions for b_0^1, b_0^2, and b_1^1 of maximizing (21) subject to the budget constraints as

$$\frac{1}{c_0} = \beta \left(\alpha \frac{1}{c_1(p)} + (1-\alpha) \frac{1}{c_1(g)} \right) \tag{24}$$

$$\frac{1}{c_0}(\alpha p + (1-\alpha)g) = \beta^2 \left(\alpha p \frac{1}{c_2^H(p)} + (1-\alpha) q \frac{1}{c_2^H(g)} \right) \tag{25}$$

$$\pi \frac{1}{c_1(\pi)} = \pi \beta \frac{1}{c_2^H(\pi)}, \pi = p, g. \tag{26}$$

From (26) it is immediate that $c_2^H(\pi) = \beta c_1(\pi)$ for $\pi = p, g$. Then combining (24) and (25) yields $c_1(p) = c_1(q) = c_1 = \beta c_0$. The only way to equate consumption across states in period 1 and have $c_1 = \beta c_0$ is to borrow nothing in period 1, i.e., to set $b_1^1(p) = b_1^1(g) = 0$.[10] The budget constraints then yield the consumption allocation (23) and the debt allocation in period 0:

$$b_0^1 = y_1 - \frac{\beta}{R}W \tag{27}$$

$$b_0^2 = y_2^H - \frac{\beta^2}{R}W \tag{28}$$

The default and repayment decisions are optimal at each date given the allocations and assumption (22).

The borrower uses debt to spread its lifetime income so as to equalize marginal utility within each period across non defaulting states and achieve a smoothly declining path of consumption over time. Note that using both long- and short-term debt in period 0 is

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[10]The fact that $b_1^1(p)$ and $b_1^1(g)$ are zero is a result of the assumption that y_1 does not differ across states. More generally, $b_1^1(p) = b_1^1(g)$, but it may differ from zero if $y_1(p) \neq y_1(g)$. Consumption in period 1 would still be equalized across states.

essential to keep consumption constant across states in period 1.

To relate this result to the hedging benefit we derived in the general model above in (18), note that the first-order conditions (24) and (25) and the fact that $q_0^2 = E\left[q_1^1(\pi)\right]$ imply that

$$E\left[\frac{1}{c_1(\pi)}\right] E\left[q_1^1(\pi)\right] = E\left[q_1^1(\pi)\frac{1}{c_1(\pi)}\right].$$

This equation says that short- and long-term debt are used such that the marginal utility is uncorrelated with the price of debt in period 1. The two assets attain perfect hedging in this example, which contains two possible values for default risk in period 1. Moreover, for the range of income levels satisfying (22), default probabilities do not change with the amount of debt issued, so the incentive benefit of short-term debt is not apparent from the first-order conditions, while the hedging benefit of long-term debt determines the maturity structure.

4.3 Cyclical Maturity Structure

We now consider how changing the borrower's income changes the maturity structure.

First, the ratio of short- to long-term debt, b_0^1/b_0^2, is increasing in y_1 and decreasing in y_2^H: when y_1 is high relative to y_2^H, the borrower uses short-term debt more intensively to achieve the optimal consumption allocation, and vice versa. This comparative static illustrates how the hedging benefit of the maturity structure works: short-term and long-term debt are used to borrow from period-1 and period-2 income, respectively, so as to keep consumption in period 1 constant.

We now consider how the maturity structure of debt in period 0 changes with income in period 0, y_0. As long as y_0 is sufficiently high, debt holdings are still given by (27)-(28), and $b_1^1(p) = b_1^1(g) = 0$.[11] Here, both short-term debt and long-term debt rise as y_0 falls.

Once y_0 falls sufficiently low, however, the allocation derived above is no longer an equilibrium because the borrower would prefer to default in period 2 even when income is y_2^H. The key point is that default is more attractive in period 2 than in period 1 because consumption optimally declines with time, which implies that $c_2 - y^{def} < c_1 - y^{def}$. Nevertheless, default in period 2 when income is y_2^H is costly because defaulting would prevent the borrower from transferring this income to period 0.

Hence, when y_0 falls enough, the equilibrium continues to be to borrow nothing in period 1, but now long-term debt in period 0 is constrained to what the borrower is willing to repay in period 2 state y_2^H; that is,

$$b_0^2 = y_2^H - y^{def}$$

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[11]The threshold is $y_0 \geq \frac{1}{\beta^2}y^{def}\left(1+\beta+\beta^2\left(\alpha p+(1-\alpha)g\right)\right)-y_1-\left(\alpha p+(1-\alpha)g\right)y_2^H$.

Figure 2: Short-term debt (b_0^1) and long-term debt (b_0^2) in a numerical example
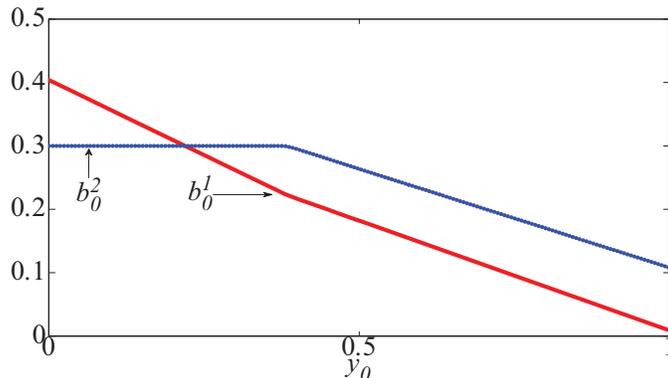
and b_0^1 is determined from a first-order condition similar to (24) and the budget constraints, giving

$$b_0^1 = y_1 - \frac{\beta}{1+\beta} \left(W - (\alpha p + (1-\alpha) g) y^{def} \right).$$

This is the equilibrium because it is still optimal to borrow or save nothing in period 1.

The forces that determine why b_0^2 is constrained are similar to those in the deterministic example in subsection 4.1 above, and rely crucially on the inability of the borrower to commit to save. Once the borrower enters period 1, it is not optimal to save in order to avoid default in period 2. If the borrower would save just enough to be indifferent between defaulting and repaying in period 2, it would reduce consumption in period 1 without changing consumption in period 2 as $c_2 = y^{def}$. Hence, this policy is not optimal in period 1.

Under this constrained allocation, the usefulness of the incentive benefit of short-term debt increases as y_0 falls further, because b_0^2 cannot be increased any more. As y_0 falls, short-term debt rises and long-term debt is flat, so the maturity structure shifts to short-term debt. The debt portfolio is plotted for a numerical example in Figure 2.[12] As income in period 0 falls, moving from the right-hand side of the figure, borrowing needs in period 0 rise, and more of both types of debt are issued. When income reaches a certain level, long-term debt becomes constrained, and the borrower shifts to issuing short-term debt more heavily. This comparative static illustrates the mechanism at work in generating cyclical variation in the maturity structure in our full model.

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[12]The parameter values for the figure are $\beta = 0.9$, $\alpha = 0.2$, $g = 0.9$, $p = 0.7$, $y_1 = 1$, $y_2^H = 1$, $y_2^L = 0.5$, and $y^{def} = 0.7$.

4.4 Discussion

In our model, short-term debt has an incentive benefit relative to long-term debt that arises solely from the borrower's lack of commitment (to repayment and to future debt decisions), and how it affects price functions for short-term and long-term debt. We discuss here how this role of short-term debt relates to others that have been considered in the literature.

The trade-off we consider is related to Hart and Moore (1989, 1994), who consider a contracting problem between an investor and an entrepreneur in which the only friction is lack of enforcement. In their model, short-term debt provides creditors with the ability to liquidate a project early in bad states, so they are willing to lend larger amounts short-term than long-term. The effect resembles the incentive benefit in our model. The use of short-term debt to provide incentives is also a feature of the model in Jeanne (2009): short-term debt gives incentives for sovereign governments to implement creditor-friendly policies, because creditors can discipline the government by rolling over the debt only after desired policies are implemented. This approach is related to that of Diamond and Rajan (2001), who show that short-term liabilities are the optimal way for banks to finance investments, because they provide incentives for extracting returns from the investments in order to pay creditors.

The incentive benefit of short-term debt in our model is related to the idea of long-term debt "dilution", i.e. that issuing debt reduces the value of the outstanding stock of debt, discussed in Bizer and DeMarzo (1992). Chatterjee and Eyigungor (2009) emphasize this effect in explaining why a borrower would prefer short-term debt. Bi (2007) focuses on the effect of dilution in the renegotiation process on the maturity structure. In our model, the decision to issue new debt affects the value of outstanding long-term debt in the future, even in the absence of renegotiation.

Alternative mechanisms give short-term debt a role in the models in Broner, Lorenzoni, and Schmukler (2008) and Niepelt (2008). In Broner, Lorenzoni, and Schmukler (2008), creditors prefer to lend short-term because they are risk averse and face more uncertainty when lending long-term. Niepelt (2008) shows that in a model in which the default decision is independent across maturities, issuing multiple maturities smooths the costs of issuing debt. In our model with endogenous default, short-term debt has an incentive benefit even if lenders are risk-neutral, and even if borrowers do not selectively default on certain maturities.

The hedging benefit of long-term debt that we consider is an implication of the results in Kreps (1982) and Duffie and Huang (1985), regarding the role of long-term debt in completing markets. Angeletos (2002), Buera and Nicolini (2004), and Lustig, Sleet, and Yeltekin (2008) emphasize this mechanism in models of the optimal maturity structure of government debt

with incomplete markets. In these models, short- and long-term interest rate dynamics reflect the variation in the representative agent's marginal rate of substitution, which changes with the state of the economy. The difference in our model is that bond prices vary across states – and hence there is a role for hedging – due to default risk, even in the absence of variation in the lender's marginal rate of substitution.

5   Quantitative Analysis

5.1   Parameterization

We solve the model numerically to evaluate its quantitative predictions regarding the dynamic behavior of the optimal maturity composition of debt and the spread curve in emerging markets. We calibrate the model to the Brazilian economy. We first calibrate the model with risk-neutral lenders and then generalize it in Section 6 below.

Table 3 summarizes the parameter values. The utility function of the borrower is $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The borrower's risk aversion coefficient σ is set to 2, which is a common value used in real business cycle studies. The length of a period is one year. We set the risk free interest rate r to 3.2%, which is the average 1-year yield of U.S. bonds from 1996 to 2011. The stochastic process for income is a log-normal AR(1) process, $\log(y_{t+1}) = \rho \log(y_t) + \varepsilon_{t+1}^y$ with $E[(\varepsilon^y)^2] = \eta^2$. We discretize the shocks into a six-state Markov chain using a quadrature-based procedure (Tauchen and Hussey, 1991). We use annual series of GDP growth for 1960–2010 taken from the World Development Indicators (WDI) to calibrate the volatility of income. Due to the short sample, rather than estimating the autocorrelation coefficient we choose an autocorrelation coefficient for the income process of 0.9, which is in line with standard estimates for developed countries. The decay parameter δ is chosen so that the default-free duration of the long-term bond equals 10 years. We choose the probability of reentering financial markets, θ, so that the average length of time in exclusion is 6 years, consistent with data presented in Benjamin and Wright (2009) on the median length of sovereign debt renegotiations.

We calibrate the model by choosing the parameters that control the income cost after default λ and time preference β, to match the average 1-year spread and the volatility of the trade balance relative to income in the Brazilian data.

We define the yield on each bond in the model as we do in the data. Yields are the implicit constant interest rates at which the discounted value of the bond's coupons equal its price and the spread as the difference between the yield on the defaultable bond relative to

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Table 3: Parameter Values

	Value	Target
Lenders' discount rate	$r^* = 3.2\%$	U.S. interest rate
Borrower's risk aversion	$\sigma = 2$	Standard value
Perpetuity decay factor	$\delta = 0.936$	Default-free duration of 10 years
Stochastic structure	$\rho = 0.9, \eta = 0.017$	Brazil GDP
Probability of reentry	$\theta = 0.17$	Benjamin and Wright (2009)
Calibrated parameters		
Output after default	$\lambda = 0.045$	Brazil average 1-year spread
Borrower's discount factor	$\beta = 0.935$	Volatility of trade balance

the risk-free rate. Thus, the short-term yield and spread are $r_S = \log\left(\frac{1}{q_S}\right)$ and $s_S = r_S - r^*$. The long-term yield and spread are $r_L = \log\left(\frac{1}{q_L} + \delta\right)$ and $s_L = r_L - r^*$.

Table 4 presents the calibrated moments as well as other statistics from the model and data. The data series for consumption, trade balance, and debt are annual for 1960–2010 taken from the WDI. For consumption we use log consumption growth rates, and for the trade balance we detrend the ratio of the trade balance to GDP. The series for debt are for total stocks of external debt and short-term external debt, defined in the WDI data as debt with maturity of one year or less. The data for spreads and duration of new debt issuances are quarterly averages constructed from the data in Section 2.

The model matches the calibrated moments. It also predicts an average long spread of 2.5%, which is below the empirical counterpart of 4.8%. Since pricing is actuarially fair, the model predicts a similar average spread on long and short bonds, which closely mirrors the default probability in the economy of 2.4%. The average debt to GDP ratio in the model is 16%, which is about half of that observed in the data. Nevertheless, the model matches the data in the majority of the stock of debt is long-term: about 85% of the total debt is long-term in the model, whereas in the data it is 87%.

As in the data, consumption in the model is more variable than output, which is a well documented fact in emerging economies. The model also predicts that consumption is negatively correlated with short and long spreads, with correlations equal to -0.65 and -0.81, respectively.

Table 4: Model Statistics

	Model	Data
mean s_S (percent)	2.5	2.9
mean s_L (percent)	2.5	4.8
std(trade balance)/std(y)	0.36	0.36
std(c)/std(y)	1.03	1.10
all debt: $(q_{St}b_{St} + q_{Lt}b_{Lt})/y)$	0.16	0.30
short: $(q_{St}b_{St})/(q_{St}b_{St} + q_{Lt}b_{Lt})$	0.15	0.13
long: $(q_{Lt}b_{Lt})/(q_{St}b_{St} + q_{Lt}b_{Lt})$	0.85	0.87

5.2 Bond Prices and Policy Functions for Debt

In this section we illustrate the trade-off between the incentive benefit of short-term debt and the hedging benefit of long-term debt with the bond price functions and decision rules from the calibrated model.

Figure 3 plots the bond price functions for short- and long-term debt as a function of the choice of debt. The values in the x-axis are reported relative to the mean output for the economy. The steps in the functions reflect the discrete number of income states.

The left panel plots the bond price function for short-term debt as a function of the choice of short-term debt, when we fix $b'_L = 0$ for a low and high income (which are 1.8 standard deviations above and below the mean): $q_S(b'_S, 0, \{y^H, y^L\})$. When loans are small, bond prices are risk-free, $q_S = 1/(1 + r^*)$, because default probabilities are zero. Prices decline for larger loans because default probabilities rise with debt. The price declines faster as a function of debt when income is low than when income is high, because income is persistent, and default is more likely when future income is low. The right panel plots the bond price function for long-term debt as a function of the choice of the long-term debt, when we fix $b'_S = 0$: $q_L(0, b'_L, \{y^H, y^L\})$. Unlike in the case of short-term debt, the price of long-term debt is never at the risk-free level (which in our economy equals $10.1 = 1/(1 + r^* - \delta)$), even for small loans. As equation (14) shows, the long-term price incorporates an additional premium contained in the price tomorrow q'_L, which depends on the choice of debt tomorrow. The price of long-term debt also declines with the loan size and declines faster in low income states.

Given that bond price functions fall with debt, loan contracts contain borrowing limits, defined as the level of debt that provides the maximum amount of resources today.[13] One way

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[13]Arellano (2008) shows that a one-period-bond version of our model generates an endogenous Laffer
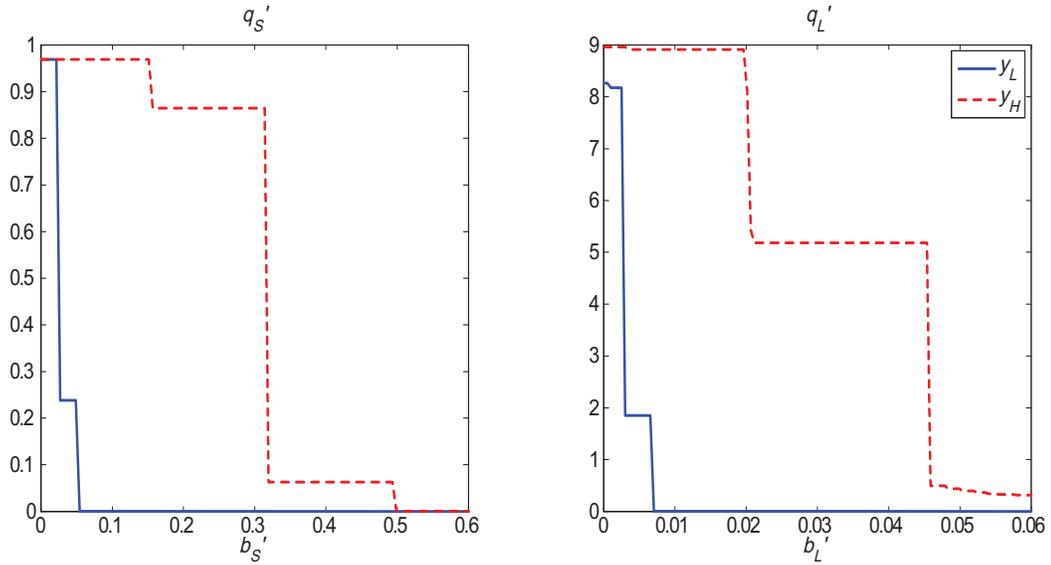
Figure 3: Bond Price Functions

to compare how lenient these bond price functions are across types of debt is by computing borrowing limits, $\max(q_i b_i')$ when $b_{-i}' = 0$ for $i = \{S, L\}$ and $b_L = 0$. By this metric, bond price functions for short debt are more lenient because they provide more resources for consumption. For short-term debt the $\max(q_S b_S') = \{0.05, 0.40\}$ for $y = \{y^L, y^H\}$ corresponding in the figure to $b_S' = \{0.05, 0.49\}$. Long-term debt is more restricted: the $\max(q_L b_L') = \{0.05, 0.33\}$ for $y = \{y^L, y^H\}$ corresponding in the figure to $b_L' = \{0.007, 0.045\}$. As discussed above, short-term debt has more lenient borrowing limits because the incentive to repay is less sensitive to the level of short-term debt than long-term debt.

The top panels in Figure 4 plot the debt decision rules for b_S' and b_L' as a function of short-term debt for the same high and low income states when $b_L = 0$: $\tilde{b}_S(b_S, 0, \{y^L, y^H\})$ and $\tilde{b}_L(b_S, 0, \{y^L, y^H\})$. The bottom panels in Figure 4 plot the incentive effects of both types of debt as a function of the potential choices for debt b_S' and b_L' when $b_L = 0$ and $y = \{y^L, y^H\}$. These incentive effects are defined as the denominator and numerator of equation (20).[14]

First consider the case $y = y^L$. Borrowing is restricted for both short- and long-term debt

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Curve for borrowing, which generates a debt limit.

[14]Due to our discrete grid, we approximate the incentive effect of short-term debt $1 + \frac{\partial q_S}{\partial b_S'} \frac{b_S'}{q_S} + \frac{\partial q_L}{\partial b_S'} \frac{(b_L' - \delta_L b_L)}{q_L}$ by the difference

$$\frac{q^S(b_S', b_L', y)b_S' + q^L(b_S', b_L', y)b_L' - \left[q^S(0, b_L', y) \times 0 + q^L(0, b_L', y)b_L'\right]}{q^S(0, b_L', y)b_S'}$$

evaluated at $b_L' = 0$ for $y = \{y_L, y_H\}$. The incentive effect of long-term debt is defined analogously.
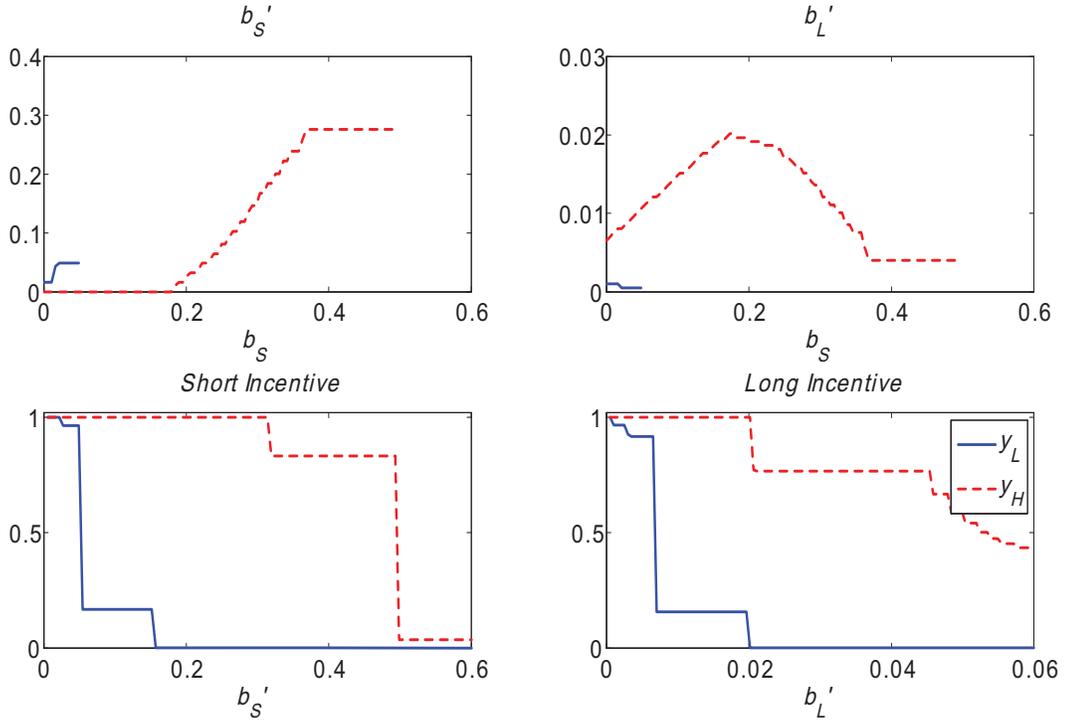
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Figure 4: Debt Policy Rules and Repayment Incentives

because the incentive to repay falls to zero quickly with larger loans. Nevertheless, short-term debt is used more heavily because the incentive to repay falls faster as a function of long-term loans than as a function of short-term loans. In the top left panel of Figure 4, the choice for short-term debt b'_S plateaus at 0.05, which is the borrowing limit described above. Now consider the case $y = y_H$. Borrowing possibilities are ample for both types of debt because the incentive effects fall slowly to zero. In this case, borrowing is primarily long-term and is increasing in the level of the state b_S. Borrowing is larger when wealth is lower (i.e., b_S is larger), as in standard precautionary savings models. However, when long-term borrowing b'_L reaches 0.02 at $b_S = 0.19$, the long-term incentive benefits fall. At this point, the borrower substitutes some b'_L for some b'_S, leading to a fall in long-term debt and a rise in short-term debt. This shift towards short-term issuance when the level of debt is high is reminiscent of the example depicted in Figure 2, where short-term debt was used more heavily for low levels of wealth. The shapes of the bond price functions are at the heart of the maturity composition of debt as reflected in the incentive effects in Figure 4.

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5.3 Simulation Results

We now compare our model results to the data for Brazil. We simulate the model and report statistics on the dynamic behavior of spreads and the maturity composition of debt from the limiting distribution of debt holdings. We first show that the model matches the data in generating time-varying differences in the pricing of short- and long-term debt, due to movements in the probability of default. We then show that the model rationalizes the dynamics in the maturity composition observed in the data.

Spread Curves

We now compare spread dynamics in the model to the data. For this comparison, we organize the data into quantiles based on the level of the short spread. Table 5 presents average spreads for short and long debt across periods when short spreads are below their 10th and 50th percentiles and above their 50th and 90th percentiles.

Table 5: Spread Curves

	Model		Data	
s_S percentile	s_S	s_L	s_S	s_L
< 10	.002	1.78	0.30	1.78
< 50	.004	1.53	0.96	2.78
≥ 50	5.08	3.54	4.91	6.72
≥ 90	6.72	4.22	12.42	11.72
Mean	2.54	2.54	2.94	4.75

The first two columns of Table 5 present the short and long spreads in the model, and the last two columns present the data. In the model, when default is unlikely, both spreads are low, and the spread curve is upward-sloping: when the short spread is below its 10th percentile, for example, the average short spread is nearly zero, and the average long spread is 1.78%. In contrast, when the probability of default is higher, both spreads rise, and the spread curve becomes downward-sloping: when the short spread is above the 90th percentile, the average short spread is 6.72%, and the average long spread is 4.22%. Compared to the data for Brazil, the model captures the dynamics of spread curves and in particular the difference in the slope of the spread curve in periods of high and low short spreads. The model misses, however, on the average slope of the spread curve as well as the periods with high short spreads.

The important feature of our model for generating the observed dynamics of the spread curve is that the probability of default is mean-reverting and persistent: a period with a high probability of default is followed by a period with a lower probability of default, and vice versa. The effects of mean-reverting and persistent default probabilities on the spread curve are the same as those highlighted by Merton (1974) in the case of credit spreads for corporate debt. In contrast to Merton's model, which directly assumes a stochastic process for the default probability, the probability of default in our model is *endogenously* mean-reverting and persistent as a result of the dynamics of the income process and debt accumulation.

Our model, nevertheless, generates too much persistence spreads, especially short spreads. The autocorrelation of long and short spreads in the model equal 0.72 and 0.67, whereas in the data they equal 0.67 and 0.33 respectively.

Maturity Composition

We now present the quantitative predictions for the maturity composition of debt. To compare new issuances of long- and short-term debt between the model and data, we compute conditional averages of the duration of new debt issuances, based on the level of the short spread. We define the duration of each bond issued at each date as the weighted average of the time until each coupon payment, with the weights determined by the fraction of the bond's value on each payment date. Hence, in the model the duration for short and long bonds equal

$$d_S = 1, \quad \text{and} \quad d_L = \frac{1}{q_L} \sum_{n=1}^{\infty} n e^{-n r_L} \delta^{n-1} = \frac{1}{1 - \delta e^{-r_L}}. \tag{29}$$

Average duration in the model is the sum of the duration of each bond issuance in (29) weighted by their share in total new debt issued, $q_S b'_S / (q_S b'_S + q_L \ell)$ and $q_L \ell / (q_S b'_S + q_L \ell)$, respectively. Moreover, given that in the data we compute only the duration of debt issuances and not repurchases, we do the same in the model.

Table 6: Average Duration of New Debt Issuances

	Model	Data
s_S pctile		
< 50	4.9	8.2
≥ 50	3.7	5.8
Mean	4.3	6.5

Table 6 reports the average duration of new debt issuances when spreads are above their median relative to when spreads are below their median in the model and in the Brazilian data. In our examples in Section 4, we discussed the relationship between wealth and the maturity structure. In our simulations, the probability of default tends to be high, and hence spreads are high, when wealth is low. We report results conditioned on spreads because we observe them in the data. The dynamics in the duration of new debt issuances in the model mirrors that in Brazil. In the model, average duration when spreads are low equals 4.9 years and shortens to 3.7 years when spreads are high. In Brazil, the average duration when spreads are low equals 8.2 years and shortens to 5.8 years when spreads are high. Although the model matches the comovement of duration and spreads, it underpredicts the average and volatility of duration of debt.

Table 7 quantifies the incentive and hedging benefits that determine the maturity composition. The first row of Table 7 shows the model's portfolio – the fraction $\frac{q_S b'_S}{q_S b'_S + q_L \ell}$ of the value of new debt issuances that is short-term – conditional on different levels of the short spread. When spreads are low, the borrower issues on average 54% in short-term bonds (and 46% of debt in long-term bonds). When spreads are high, the maturity composition shifts to 60% in short-term bonds (and 40% in long-term bonds).

The optimal portfolio depends on the valuations of the hedging benefits of long-term debt relative to the incentive benefit of short-term debt. Table 7 reports several metrics to evaluate these benefits. The second and third rows of the table report the relative incentive benefit and hedging benefit defined in equations (20) and (19).[15] The relative incentive term computes the effect of increasing long-term debt on the incentive to repay in the future relative to the effect of increasing short-term debt. In our model, short-term debt always has an incentive benefit because this ratio is always less than 1. The smaller is this number, the more long-term debt reduces the incentive to repay relative to short-term debt. The hedging benefit as defined in equation (19) is related to the conditional covariance between future marginal utility $u'(c')$ and the future price of long-term debt, q'_L. The subsequent rows of the table also show the correlation between these two variables and between q'_L and income, and the volatility of q'_L. Long-term debt potentially provides a hedge because, as the table shows, the value of outstanding long-term debt is volatile and negatively correlated with marginal utility. The greater the hedging achieved, the closer this correlation is to zero.

The second row of the table shows that when spreads are high, the incentive benefit of

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[15]We approximate the incentive effects from our policy rules as described in the construction of Figure 4. The hedging term $\frac{E\left[(1+\delta q'_L)u'(c')|R'\right]}{E\left[(1+\delta q'_L)|R'\right]E[u'(c')|R']}$ at each period is computed using conditional expectations over next period's repayment states. These two terms do not have to be equal to one another because of the approximations using a discrete grid.

short-term debt is larger than when spreads are low: the ratio of incentive terms equals 0.60 in the first case and 0.85 in the second, on average. On the other hand, as shown in the fourth row, when spreads are high, the hedging benefit provided by long-term debt is less than when spreads are low, as the correlation between marginal utility and the price of long-term debt is -0.90 compared to -0.71. Therefore, time-variation in the maturity structure reflects variation in the importance of the incentive benefit of short-term debt and the hedging benefit of long-term debt.

Table 7: Model Maturity Composition

	Level of short spread		
	$<$ median	$\geq$ median	overall
mean($q_{St}b_{St}$ / ($q_{St}b_{St} + q_{Lt}\ell_t$))	0.54	0.60	0.57
Relative incentives	0.85	0.60	0.80
Hedging benefits	0.997	0.998	0.998
corr($(1 + \delta q_{Lt})$, $u'(c_t)$)	-0.71	-0.90	-0.83
corr($(1 + \delta q_{Lt})$, y_t)	0.91	0.94	0.88
std(q_{Lt})	0.69	1.01	1.00
spread curve $s_L - s_S$	1.52	-1.54	-0.01

The last row of the table reports the average spread curve, defined as the difference between the long and short spreads. These statistics show that in our model the maturity shortens precisely in periods the short spread rises more than the long spread. This positive correlation between debt duration and the difference between the long to short spread matches the data as reported in Table 2. In our model, the maturity of debt is determined by the shapes of the bond price functions, both in terms of how fast they decline with short or long term and its correlation with marginal utility. The equilibrium levels of these prices, summarized in the level of spreads, contain some of the information but are not a comprehensive measure for the cost of debt. In fact, times of higher short spreads relative to long spreads correspond to periods when the incentive benefit of short debt is higher.

In summary, through the lens of our model, the maturity structure of defaultable debt in emerging markets and its covariation with spreads can be rationalized by two factors: a hedging advantage of long-term debt for insuring against fluctuations in future default risk, and the benefit of short-term debt in providing incentives to repay during times of high default risk.

6   Risk Premia in Sovereign Bonds

In our benchmark calibration, we considered actuarially fair pricing for bonds to illustrate the main mechanisms driving the maturity structure. However, Longstaff et al. (2011) and Borri and Verdelhan (2009), among others, argue that risk premia are important for understanding sovereign bond prices. If emerging market governments tend to default in states when foreign investors have high marginal utility, then bond prices reflect compensation for this risk. In this section, we evaluate how risk premia change the determination of bond price functions, equilibrium spreads and the maturity structure.

We model risk premia through a pricing kernel $M(y_t, y_{t+1})$ that is a function of the borrower's income. In this modified model, the pricing formulas (13) and (14) are replaced by

$$q_S(b'_S, b'_L, y) = \int_{R(b'_S, b'_L)} M(y, y') f(y, y') dy' \tag{30}$$

$$q_L(b'_S, b'_L, y) = \int_{R(b'_S, b'_L)} \left[1 + \delta q_L\left(\tilde{b}_S(b'_S, b'_L, y'), \tilde{b}_L(b'_S, b'_L, y'), y' \right) \right] M(y, y') f(y, y') dy'. \tag{31}$$

The rest of the model is the same as in the case with actuarially fair pricing.

Following the specification for pricing kernels in affine term-structure models, described for example in Cochrane and Piazzesi (2008), we assume that the pricing kernel takes the following form:

$$M(y_t, y_{t+1}) = \exp\left(-r^* - \gamma_t \varepsilon^y_{t+1} - \frac{1}{2} \gamma_t^2 \eta^2 \right), \tag{32}$$

where $\gamma_t = \alpha_0 + \alpha_1 \log y_t$, r^* is the risk-free rate, $\varepsilon^y_{t+1} = \log y_{t+1} - \rho \log y_t$ is the shock to the borrower's income, and η^2 is its variance. The term γ_t is the market price of risk in state y_t. We allow γ_t to depend on $\log y_t$ to allow for time variation in the market price of risk. The risk premium in our model comes from the interaction of the lenders' pricing kernel with default outcomes and future bond prices. To see this, we can rewrite equation (31) (and analogously, (30)) as a typical asset-pricing equation $q_L = E[M' x'_L]$, where $x'_L = 1 + \delta q'_L$ in states in which the borrower repays and $x'_L = 0$ in states in which the borrower defaults. Written this way, the price of a bond is composed of the lender's discounted expected payoff plus a risk premium term:

$$q_L = E[M'] E[x'_L] + cov(M', x'_L). \tag{33}$$

Equation (33) shows that to the extent that payoffs are negatively correlated with the pricing kernel, investors are compensated for this risk by paying a lower price q_L for debt. If $\alpha_0 > 0$,

then $\gamma_t > 0$ on average, making M and x_L negatively correlated. A negative shock ε^y_{t+1} to future income lowers both the repayment probability and future prices while it increases M. This correlation implies that prices have to be lower, and spreads higher, to compensate for this risk. In addition, if $\alpha_1 < 1$, then the risk premium is higher in states when the borrower has low income.

We define the pricing kernel as a function of only the borrower's income because it is a parsimonious way to model risk premia that vary with the probability of default.[16] An alternative way to model the pricing kernel would be to explicitly use measures of foreign (e.g., U.S.) investors' consumption as the state variable of the pricing kernel. In this setting, the risk premium U.S. investors demand on Brazilian bonds would depend on the correlation between the Brazilian default probability and U.S. consumption. For our purposes, the method we use has the benefit that we do not need to introduce an additional exogenous state variable into the model. In addition, we document in the appendix that spreads on Brazilian debt are correlated with and predict excess returns on U.S. Treasury debt, an indicator of the risk premium demanded by US investors. Therefore, times when spreads on Brazilian debt are high are times of high risk aversion for U.S. investors. A similar point is demonstrated for a cross section of emerging market economies by Borri and Verdelhan (2009).

To choose values for the parameters α_0 and α_1, we draw on evidence in Longstaff et al. (2011), who estimate the market prices of risk for several emerging market countries (including Brazil) as functions of the risk-neutral probabilities of default. The appendix describes in detail how we map their estimated parameters to our pricing kernel parameters in (32). The resulting values for the parameters are $\alpha_0 = 11$, $\alpha_1 = -141$. To make results comparable to the benchmark results, we recalibrate the other parameters to match the same targets as in the risk-neutral case. Namely, we calibrate the output cost after default λ and time preference β to match the average short spread and the volatility of the trade balance. The values are $\beta = 0.905$ and $\lambda = 0.015$. In the appendix we also conduct a sensitivity analysis on the risk parameters.

6.1   Results

Table 8 presents the main results for the economy with risk premia and compares them to the benchmark model with actuarially fair prices and to the data. The model with risk premia

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[16]Note that in the absence of default risk, this pricing kernel generates a flat term structure with zero risk premium. Hence, risk premia reflect compensation for the risk associated with default losses and the changes in default probabilities reflected in changes in y, not simply the risk associated with fluctuations in the borrower's income.

improves the fit of spreads over the benchmark model. First, the model with risk premia delivers a positive average spread curve slope of 0.6%, getting closer to the data counterpart of 1.8%. Second, the volatility of the short spread is reduced, making the dynamics of the short spread closer to the data. Third, risk premia break the tight connection between average spreads and default probabilities thereby addressing the so-called credit spread puzzle (see, e.g. Chen, Collin-Dufresne, and Goldstein (2009) for a discussion of the credit spread puzzle for corporate bonds).

Introducing risk premia in our model tightens price schedules for all levels of debt, which reduces the average stock of debt of the economy from 16% of income to 5% of income. Moreover, risk premia disproportionately affect the price schedule for long-term debt by magnifying the effect of issuing long-term debt on the incentive to repay. This effect lowers the average duration from 4.3 to 1.8 years. Nevertheless, as in the benchmark, the model with risk premia predicts that short-term debt is used more aggressively when spreads are high.

Table 8: Risk Premia, Spreads, and the Maturity Composition

	Risk Premia $\alpha_0 = 11, \alpha_1 = -141$			Benchmark Actuarially fair			Data		
s_S pctile:	< 50	≥ 50	mean	< 50	≥ 50	mean	< 50	≥ 50	mean
s_S, percent	0.3	4.5	2.4	0.004	5.1	2.5	1.0	4.9	2.9
s_L, percent	1.9	4.1	3.0	1.5	3.6	2.5	2.8	6.7	4.8
rp_S, percent	0.2	1.6	0.9	0.0	0.0	0.0	–	–	–
rp_L, percent	0.4	1.2	0.8	0.0	0.0	0.0	–	–	–
duration	2.9	1.1	1.8	4.9	3.7	4.3	8.2	5.8	6.5
$\text{std}(tb)/\text{std}(y)$			0.34			0.36			0.36
$\text{mean}(q_{St}b_{St} + q_{Lt}b_{Lt})/y))$			0.05			0.16			0.30
default probability			1.4			2.5			–

We can decompose the interest rate spreads into two parts: the actuarially fair compensation for expected losses from default, and the risk premium. The actuarially fair prices q_S^{AF}, q_L^{AF} are defined by computing risk-neutral versions (no M) of the bond pricing equations (30)-(31) using the debt and default decisions from the model with risk premia. Then, the actuarially fair yields are given by $r_S^{AF} = \log\left(1/q_S^{AF}\right)$, $r_L^{AF} = \log\left(1/q_L^{AF} + \delta\right)$, and the *spread risk premium* for each maturity m is defined as $rp_m = r_m - r_m^{AF}$. Rows 3-4 of Table 8 show that risk premia are positive: the borrower tends to default in states with low income, and the lender's pricing kernel M is negatively correlated with the borrower's income. Risk premia are sizable and constitute about 40% (0.9/2.4) of the short spread and 30% (0.8/3.0)

of the long spread on average.

Overall, we find that having risk premia provides a better fit to data on spreads. However, we find that risk premia change the equilibrium quantities of debt in a way that worsen the fit to data on quantities. In particular, risk premia lower debt levels and shorten average maturity. Nevertheless, the dynamics in the maturity structure continue to be determined by the incentive and hedging benefits illustrated in the benchmark model. These forces depend on the shape of the bond price functions, which in the model with risk premia also depend on the correlation between default probabilities and the lender's pricing kernel.

7   Conclusion

In this paper, we have developed a dynamic model to study the maturity composition of defaultable sovereign bonds. In data for emerging markets, changes in the maturity composition of debt comove with changes in the term structure of spreads: when spreads on short-term debt are low, long-term spreads are higher than short-term spreads, and the maturity of debt issued is long. When short-term spreads rise, long-term spreads rise less, and the maturity of debt shortens. Our model simultaneously reproduces the patterns observed in the term structure of spreads and the maturity composition of debt. Changes in the spread curve, which reflects the average default probability at different time horizons, result from the income dynamics and the endogenous dynamics of debt. Issuing long-term debt hedges future fluctuations in consumption that come from changes in default risk. Short-term debt is beneficial because the incentives to repay it are less sensitive to the level of debt than for long-term debt. With these two forces, the model generates the pattern of issuances observed in the data.

Our main innovation has been to highlight this trade-off between the benefits of short-term and long-term debt in a quantitative, dynamic model with endogenous default and multiple, long-term assets. We view the resulting framework as useful for addressing a variety of other questions for which it is important to analyze a trade-off in maturity choice with defaultable debt. Natural applications are the maturity structure of consumer and corporate debt. The literature on consumer bankruptcy thus far has focused on modeling very short-term unsecured credit (Chatterjee et al. (2007) and Livshits, MacGee, and Tertilt (2007)). However, it would be interesting to analyze both long-term and short-term defaultable loans, such as mortgages and credit card debts. In addition, the mechanisms in our model are likely to be relevant in corporate debt given the similarity between our facts on emerging market spread curves and the cross section of corporate debt spread curves (e.g. Sarig and

Warga, 1989). Default risk has been shown to have important implications for firm dynamics (Cooley and Quadrini (2001) and Arellano, Bai, and Zhang (2011)). The model of this paper can be used to further understand how the maturity choice can influence the entry, exit, and growth of firms. Overall, our paper provides a tractable quantitative framework to study defaultable debt of multiple maturities appropriate for these questions, and has highlighted the relevant economic trade-offs important for understanding debt maturity choice in the presence of default.

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Appendix

Data Description

All the sovereign bond data are from Bloomberg. For the four countries we examine, we use all US dollar and european currency bonds with prices quoted at some point between March 1996 and April 2011, with the following exceptions. We exclude all bonds with floating-rate coupon payments, and at every date, we exclude bonds that are less than three months to maturity, following Gurkaynak, Sack, and Wright (2007). For each country, we estimate spreads starting from the first week for which at least four bond prices are available every week through the end of the sample. We use data from 110 bonds for Argentina, 71 for Brazil, 63 for Mexico, and 25 for Russia. To estimate default-free yield curves, we use data on U.S. and European government bond yields. The U.S. data are from the Federal Reserve Board, and the European data are from the European Central Bank.[17] For constructing the quarterly maturity and duration statistics, we also include bonds issued during the sample period that did not have prices quoted, and use the estimated spread curve to construct their prices according to equation (34).

Spread Curve Estimation

We use a method proposed by Svensson (1994) and used by Gurkaynak, Sack, and Wright (2007) for the United States, and Broner, Lorenzoni, and Schmukler (2008) for a sample of emerging markets, to fit a spread curve to this data using a simple functional form suggested by Nelson and Siegel (1987).

A coupon bond is priced as a collection of zero-coupon bonds, each with maturity given by a coupon payment date, and face value given by the cash flow on that payment date. The price at date t of a bond issued by country i, paying an annual coupon rate c at dates $n_1, n_2, \ldots n_J$ years into the future, is

$$p_t^i(c, \{n_j\}) = \sum_{j=1}^{J} \exp(-n_j r_t^i(n_j))c + \exp(-n_J r_t^i(n_J)) \tag{34}$$

with the face value of the bond paid on the last coupon date.

Spreads are defined as $s_t^i(n) = r_t^i(n) - r_t^*(n)$, where $r_t^*(n)$ is a default-free yield curve.

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[17]The U.S. data are the Treasury constant maturities yields, available at http://www.federalreserve.gov/releases/h15/data.htm.

The European data are Euro area benchmark government bond yields, which is an average of European national government bond yields available at http://sdw.ecb.europa.eu.

We define spreads as a parametric function of maturity following Nelson and Siegel (1987)

$$s_t^i(n; \beta_t^i) = \beta_{1t}^i + \beta_{2t}^i \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) + \beta_{3t}^i \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right) \tag{35}$$

for each country i, where $\beta_t^i = (\beta_{1t}^i, \beta_{2t}^i, \beta_{3t}^i)$ and λ are parameters. For default-free bonds, we define

$$r_t^{\$}(n; \beta_t) = \beta_{1t}^{\$} + \beta_{2t}^{\$} \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) + \beta_{3t}^{\$} \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right) \tag{36}$$

and

$$r_t^{\euro}(n; \beta_t) = \beta_{1t}^{\euro} + \beta_{2t}^{\euro} \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) + \beta_{3t}^{\euro} \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right) \tag{37}$$

for US (\$) and Euro area (€) bonds.

As described by Nelson and Siegel (1987) and Diebold and Li (2006), the three components of this curve correspond to a "long-term," or "level" factor (the constant), a "short-term," or "slope" factor (the term multiplying β_2) and a "medium-term," or "curvature" factor (the term multiplying β_3). Linear combinations of these factors can capture a broad range of shapes for the spread curve.

We first estimate the parameters $\beta_t^{\$}$ and $\beta_t^{\euro}$ by OLS, using U.S. and Euro area bond yields. Throughout, we set the parameter $\lambda = 0.089$, so that the term multiplying β_3 in all countries' spread curves is maximized when $n = 20$ years.

Then, given a set of parameters β_t^i, we use equation (34) to price each of country i's bonds at date t using the risk-free yield given by (36) or (37) and the spread given by (35):

$$p_t^i(c, \{n_j\}; \beta_t^i) = \sum_{j=1}^{J} \exp(-n_j \left(s_t^i(n_j; \beta_t^i) + r_t^*(n_j) \right))c + \exp(-n_J \left(s_t^i(n_J; \beta_t^i) + r_t^*(n_J) \right)),$$

where r_t^* refers to $r_t^{\$}$ if the bond is denominated in U.S. dollars, or $r_t^* = r_t^{\euro}$ if the bond is denominated in a European currency.

We estimate the parameters β_t^i by nonlinear least squares to minimize the sum of squared deviations of the predicted yields to maturity from their actual values. That is, our estimated parameters solve

$$\min_{\beta_t^i} \sum (y_t^i(c, \{n_j\}; \beta_t^i) - y_t^i(c, \{n_j\}))^2,$$

where $y_t^i(\cdot)$ is the yield to maturity of a bond with price $p_t^i(\cdot)$, and the summation is taken over all bonds issued by country i with prices available at date t.

The following features present in the data require modification of the basic bond pricing

equation (34):

1. Between coupon periods, the quoted price of a bond does not include accrued interest, so we subtract from the bond price the portion of the next coupon's value that is attributed to accrued interest.

2. For bonds with principal payments guaranteed by U.S. Treasury securities, we discount the payment of principal by the risk-free yield only, without the country spread.

3. For bonds with coupon payments that increase or decrease over time with certainty ("step-up" and "step-down" bonds, respectively), we modify the sequence of payments in equation (34) accordingly.

Table 9 below displays mean absolute errors in matching bond yields. Yield errors are moderate, averaging under 1 percentage point across the four countries. During periods when spreads are high, errors tend to be larger; for example, the mean absolute error in matching yields of Argentinean bonds is 2.65 percentage points in periods when the 1-year spread is above its 90th percentile.

Table 9: Bond Pricing Errors

	Mean absolute yield error, %	percentile of 1-year spread					
		$<$ 10th	$<$ 25th	$<$ 50th	$\geq$ 50th	$\geq$ 75th	$\geq$ 90th
Argentina	0.93	0.61	0.70	0.63	1.22	1.68	2.65
Brazil	0.66	0.56	0.47	0.49	0.83	0.98	1.20
Mexico	0.39	0.45	0.33	0.32	0.46	0.53	0.56
Russia	0.60	0.41	0.39	0.39	0.82	1.08	1.47

Further Statistics on Duration and Spread Curves

Table 10 reports the analogues of the duration statistics and regression coefficients from Tables 1 and 2 for a measure of duration that uses the default-free yield curve to discount future coupon payments. The results are largely similar to those in the main tables in the text, and all the regression coefficients are significant at the 1% level.

Table 10: Statistics for Default-Free Measure of Duration

Average duration	Argentina	Brazil	Mexico	Russia
overall	6.43	7.11	7.34	7.11
< median	7.07	9.09	8.26	6.78
≥ median	5.38	6.52	7.53	6.77

Regression Variable	I	II	III
1-year spread	−0.333 (0.036)		
10-year spread		−0.361 (0.044)	
$\log \frac{10 \text{ year spread}}{1 \text{ year spread}}$			1.528 (0.085)
Constant	8.037 (0.083)	8.733 (0.168)	6.098 (0.072)
R^2	0.189	0.181	0.292
No. of obs.	60	60	60

Table 11 reports further spread curves and spread volatilities for all countries.

7.1 Risk Premia Parameters

Longstaff et al. (2011) estimate a pricing kernel of the form $M_{t,t+1} = \exp\left(-r - \tilde{\gamma}_t \varepsilon_{t+1}^\lambda - \frac{1}{2}\tilde{\gamma}_t^2\right)$, where $\tilde{\gamma}_t = \delta_0 + \delta_1 \log \lambda_t$, and λ_t is the risk-neutral probability of default, which follows an AR(1) process in logs with innovations $\sigma_\lambda \varepsilon_{t+1}^\lambda$ and $\varepsilon_{t+1}^\lambda \sim N(0,1)$. We use their estimates for δ_0, δ_1, and σ_λ for Brazil to construct parameters for our pricing kernel as follows: we first project the quarterly one-year spread in our data (as a proxy of the one-year risk-neutral default probability) on a constant and log deviations of Brazilian GDP from a linear trend, $\log \lambda_t = A + B \log y_t$. Then, plugging the fitted levels of and innovations to $\log \lambda_t$ into the Longstaff et al. (2011) pricing kernel, we arrive at (32) with $\alpha_0 = \frac{B}{\sigma_\lambda}(\delta_0 + A\delta_1)$ and $\alpha_1 = \frac{B^2}{\sigma_\lambda}\delta_1$. Our estimates of A and B are $A = -3.98$ and $B = -7.22$, and both are signifi-

Table 11: Average Spreads and Volatility

	Maturity (years)	Overall (%)	Std. Dev	When 2-year spread is above/below nth percentile					
				< 10th	< 25th	< 50th	≥ 50th	≥ 75th	≥ 90th
Argentina	1	4.22	8.28	0.46	0.70	1.12	7.32	11.85	22.85
	2	4.56	6.53	1.12	1.52	1.89	7.22	10.99	19.90
	5	5.39	3.69	2.60	3.29	3.54	7.24	9.55	14.49
	10	6.29	3.45	3.87	4.62	4.65	7.93	9.79	13.56
Brazil	1	2.93	3.86	0.30	0.63	0.97	4.89	7.44	12.42
	2	3.47	3.88	0.65	0.86	1.38	5.55	8.07	13.02
	5	4.47	3.95	1.37	1.37	2.25	6.70	9.07	13.59
	10	4.75	3.37	1.78	1.80	2.78	6.72	8.65	11.72
Mexico	1	1.16	1.43	-0.43	-0.13	0.19	2.12	3.05	4.27
	2	1.33	1.21	0.16	0.31	0.54	2.13	2.91	3.95
	5	1.87	1.08	1.43	1.30	1.35	2.39	2.94	3.63
	10	2.73	1.48	2.32	2.12	2.13	3.32	3.98	4.56
Russia	1	2.85	5.02	0.44	0.68	1.11	4.59	6.26	9.15
	2	3.14	3.84	0.87	1.09	1.49	4.77	6.38	9.13
	5	3.65	2.53	1.65	1.83	2.18	5.12	6.66	9.17
	10	3.73	2.84	1.75	1.92	2.25	5.20	6.85	9.43

cant at the 10% level. The resulting estimates for α_0 and α_1, using their esimates of δ_0, δ_1, and σ_λ, are $\alpha_0 = 11$, $\alpha_1 = -141$.

They estimate pricing kernel parameters for a cross section of countries and find that the market price of risk for Brazil is one of the largest from their sample. Table 12 presents the main results from our model using the average risk parameters across the countries in their sample. These parameters correspond to $\alpha_0 = 6$, $\alpha_1 = -70$. To highlight the role of these parameters, we present two sets of results. The middle of the table shows the main results when we change the risk parmeters to $\alpha_0 = 6$, $\alpha_1 = -70$ while keeping the other parameters as in the calibration with risk premia in the main text. The right section shows the results when we recalibrate the discount factor β and the default costs λ to target an equal value of the short spread and trade balance volatility as in the model with risk premia.

Consider first the comparative static results. Unlike models with exogenous default risk, where a higher market price of risk raises spreads by increasing risk premia, in our model changing the market price of risk changes default behaviour. When the parameters controlling the market price of risk, $[\alpha_0, \alpha_1]$, are reduced from [11,-141] to [6,-70] the bond price functions become more lenient. With less expensive borrowing, default probabilities increase and hence both short and long spreads increase by about 3 and a half percentage points. The spread

	Baseline Risk Premia $\alpha_0 = 11, \alpha_1 = -141$			Comparative Static $\alpha_0 = 6, \alpha_1 = -70$			Recalibration		
s_S pctile:	< 50	≥ 50	mean	< 50	≥ 50	mean	< 50	≥ 50	mean
s_S, percent	0.3	4.5	2.4	0.7	11.2	5.9	0.3	5.2	2.7
s_L, percent	1.9	4.1	3.0	4.4	9.0	6.7	2.4	4.8	3.6
rp_S, percent	0.2	1.6	0.9	0.5	3.2	1.8	0.3	1.9	1.1
rp_L, percent	0.4	1.2	0.8	1.4	3.0	2.2	1.1	2.1	1.6
duration	2.9	1.1	1.8	3.5	1.2	2.3	4.9	2.5	3.7
std(trade balance)/std(y)			0.34			0.37			0.34
mean($q_{St}b_{St} + q_{Lt}b_{Lt})/y$))			0.05			0.08			0.12
default probability, percent			1.4			4.0			1.6

curves are more volatile; in periods of low spreads, $s_L - s_S = 4.3$, and in periods of high spreads $s_L - s_S = -2.2$, compared to 1.6 and -0.4 in the baseline risk premia calibration. The average contribution of risk premia to spreads is similar although the contribution increases slightly for the long spread and decreases slightly for the short spread.

More lenient bond prices also increase the average level of debt and the average duration of new issuances. The dynamics of debt duration are amplified, with a difference in duration of 2.3 years. This amplification happens because more volatile and lenient price functions accentuate the hedging benefits long spreads in low spreads times.

Consider now the result of the recalibrated model. The average spreads and the dynamics of spread curves are very similar to the results in the baseline risk premia calibration although the average spread curve, $s_L - s_S$, increases from 0.6 to 0.9 . One statistic that differs somewhat is the contribution of risk premia to the long spread, which is 0.44 in the recalibration as opposed to 0.27 in the baseline risk premia calibration. In terms of quantity measures, the average debt and duration are higher in the recalibration. Relative to the risk premia baseline, bond prices in the recalibration are more lenient leading more usage of all debt and stronger dynamics of duration of debt.

From these results, we conclude our results with risk premia are robust to using the parameters from Longstaff et al (2011) estimated for Brazil versus those estimated for the average country.

7.2 Foreign Investor Risk and Brazilian Spreads

Our use of Brazilian GDP in the pricing kernel (32) proxies for time-varying risk aversion of foreign investors. The aim is to capture the extent to which bad times in Brazil correspond

to times of high risk aversion for foreign investors. To provide some evidence of this link, we regress a quarterly average of the Brazilian spreads on excess returns on U.S. treasury bonds. The excess return on n-year U.S. bonds at date t, $rx_t^\$(n)$ is defined from prices $p_t^\$(n)$ and the 1-year yield $r_t^\$(1)$ by

$$rx_t^\$(n) = \frac{p_{t+1}^\$(n-1)}{p_t^\$(n)} - r_t^\$(1).$$

Table 13 reports the results of several univariate regressions of US excess returns for various maturities on different maturities of Brazilian spreads. The coefficients in the table show that the Brazilian spreads we estimate are significant predictors of excess returns on US treasuries, with R^2's of up to 13%.

Table 13: US Excess Returns and Brazilian Spreads

Dependent variable: n-year US excess return				
	$n = 1$	$n = 2$	$n = 5$	$n = 10$
Regression on 1-year Brazilian spread				
constant	0.005**	0.010**	0.021**	0.032**
spread	0.136**	0.249**	0.471**	0.589**
R^2	0.118	0.124	0.123	0.075
Regression on 2-year Brazilian spread				
constant	0.005**	0.010**	0.019**	0.029**
spread	0.133**	0.243**	0.458**	0.575**
R^2	0.124	0.130	0.128	0.079
Regression on 5-year Brazilian spread				
constant	0.004	0.008	0.017	0.026
spread	0.115**	0.209**	0.395**	0.498
R^2	0.109	0.114	0.112	0.069

Note: ** denotes significance at the 5% level