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Online Appendix for:

International Evidence on Long-Run Money Demand

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A The Data

Here follows a detailed description of the dataset. Almost all of the data used in this paper are from original sources. Specifically, they are from either (*i*) original hard copy (books or, in the case of West Germany's M1, scanned PDFs of the Bundesbank's *Monthly Reports*, which are available from the Bundesbank's website), in which case we have entered the data manually into Excel; or (*ii*) central banks' or national statistical agencies' websites (these data are typically available in either Excel or simple text format). The few exceptions are discussed below. In those cases, we were not able to find the data we were looking for in original documents, and therefore we took them from either the International Monetary Fund's *International Financial Statistics* (henceforth, IMF and IFS, respectively) or the World Bank.

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A.1 Argentina

All of the series are from the *Banco Central de la República Argentina* (Argentina's central bank, henceforth, *Banco Central*). Specifically, a series for M1, available for the period 1900-2014, is from *Banco Central*'s Table 7.1.4 ("Agregados Monetarios"). A series for a short-term nominal interest rate, available for the period 1821-2018, is from Banco Central's Table 7.1.4 ("Tasas activas"). Interestingly, among all of the countries we consider in this paper, Argentina is the *only* one that directly provides an estimate of (the inverse of) the velocity of circulation of monetary aggregates. Specifically, Banco Central's Table 7.1.4 provides the ratios between either M1 and M3 and nominal GDP ("M1 % PBI" and "M3 % PBI," respectively; "PBI" is the Spanish acronym for GDP). Based on the ratio between M1 and GDP, and the series for M1, we then reconstructed a nominal GDP series.

A.2 Australia

An annual M1 series for the period 1900-2017 has been constructed in the following way. A series for the period 1900-1973 has been kindly provided by Cathie Close of the *Reserve Bank of Australia* (henceforth, *RBA*). A monthly seasonally unadjusted series, available since 1975, is from the RBA's website ("M1, $ billion, RBA, 42216"; the series' acronym is DMAM1N), and we converted it to the annual frequency by taking annual averages (since for the year 1975 the series is available from February, the average for that year has been computed for the period February-December). The missing observation for 1974 has been interpolated as in Bernanke, Gertler, and Watson (1997), using as an interpolator series the IMF's IFS series labeled as "Money," which, over the overlapping periods, closely comoves with both M1 series. A series for a "short rate," available for the period 1941-1989, is from Table 79 of Homer and Sylla (2005). A 90-day nominal interest rate for bank accepted bills and negotiable certificates of deposit is from the RBA's website ("90-day BABs/NCDs, Bank Accepted Bills/Negotiable Certificates of Deposit-90 days, Monthly, Original, Per cent, AFMA, 42156, FIRMMBAB90"). It is available since 1969. A series for nominal GDP, available since 1960, is from the Australian Bureau of Statistics ("Gross domestic product: Current prices; A2304617J; $ Millions"). An alternative series for nominal GDP, available for the period 1870-2012, is from the website of the *Global Price and Income History Group* at the University of California at Davis, at: http://gpih.ucdavis.edu/.

A.3 Austria

A monthly seasonally unadjusted M1 series, available since January 1970, is from the *European Central Bank*. The series has been converted to the annual frequency by taking simple annual averages. An annual series for the discount rate of the

Oesterreichische Nationalbank (Austria's central bank), available for the period 1957-1998, is from the *IMF*'s *IFS*. An annual series for nominal GDP is from *Statistics Austria* since 1995, and from the *IMF*'s *IFS* before then. Over the overlapping periods, the two nominal GDP series are nearly identical, which justifies their linking.

A.4 Bahrain

An annual series for M1, available since 1965, is from the website of the Financial Stability Directorate of the Central Bank of Bahrain. An annual series for "Interest Rates on BD Deposits & Loans," available since 1976, is from the central bank's Statistical Bulletin, available at: https://www.cbb.gov.bh/page-p-statistical_bulletin.htm. An annual series for nominal GDP is from the website GCC-Stat, a statistical database for Persian Gulf countries (at: http://dp.gccstat.org/en/DataAnalysis?215Jv283P0CFmaBBdivhQ) since 2008. Before that, it is from the World Bank.

A.5 Barbados

An annual series for nominal GDP in million Barbados dollars, available since 1975, is from Tables I7A and I7B of the *Barbados Statistical Service*. An annual series for M1 in million Barbados dollars, available since 1973, is from Table C1 from the *Central Bank of Barbados*. An annual series for the 3-month time deposits rate starting in 1961 has been computed as the average of the two series "3 month Time Deposits - Lower FIDR_TD3L" and "3 month Time Deposits - Upper FIDR_TD3U," from the *Central Bank of Barbados*.

A.6 Belgium

An annual M1 series ("Stock monétaire (milliards de francs)"), available for the period 1920-1990, is from the Séries rétrospectives, Statistiques 1920-1990 from *Banque Nationale de Belgique*'s (Belgium's central bank, henceforth *BNB*), Statistiques Economiques Belges, 1980-1990. For the period 1991-1998, M1 data are from the BNB's *Bulletin Statistique*. An annual series for nominal GDP ("Value Added at Market Prices in Current Prices, billion of francs"), available for the years 1920-1939 and 1946-1990, is from Smits, Woltjer, and Ma (2009). An annual series for the BNB's discount rate available for the period 1920-1990 is from the Séries rétrospectives, Statistiques 1920-1990 from the BNB's Statistiques Economiques Belges 1980-1990. For the period 1991-1998, the discount rate is from several issues of the BNB's *Annual Report*.

A.7 Belize

Annual series for M1 and for Belize's Treasury bill rate, both available since 1977, are from the *Central Bank of Belize*. An annual series for nominal GDP, available since 1970, is from the *Penn World Tables* Mark 7.0 until 2001, and from the *Central Bank of Belize* after that. Over the overlapping periods, the two nominal GDP series are near-identical, which justifies their linking.

A.8 Bolivia

Series for nominal GDP, M1, and a short-term nominal interest rate, all available for the period 1980-2013, are from the Unidad de Analisis de Politicas Sociales y Economicas (Bolivia's national statistical agency, known as UDAPE for short).

A.9 Brazil

Series for nominal GDP, M1, and GDP deflator inflation, all available for the period 1901-2000, are from IBGE's (the Brazilian Institute of Geography and Statistics) *Estatisticas do Seculo XX* (Statistics of the XX Century). The URL is http://seculoxx.ibge.gov.br/economicas. A series for nominal GDP for the period 2000-2017 is also from IBGE. A series for M1 for the period 2000-2017 is from the *Banco Central do Brasil* (Brazil's central bank, henceforth *Banco Central*). A series for a short-term nominal interest rate for the period 1974-2012 is from the *Banco Central*. Two series for a nominal government bond yield (for periods 1901-1913 and 1929-1959) and the *Banco Central*'s discount rate (for period 1948-1989) are both from Homer and Sylla (2005)'s Table 81, pages 629-631.

A.10 Canada

An annual series for nominal GDP, available since 1870, has been constructed by linking the Urquhart series (available from *Statistics Canada* (henceforth, *SC*), which is Canada's national statistical agency), for the period 1870-1924; series 0380-0515, v96392559 (1.1) from *SC*, for the period 1925-1980; and series 0384-0038, v62787311 (1.2.38) from *SC*, for the period 1981-2013. A short-term interest rate for the period 1871-1907 (specifically, the "Montreal call loan rate") is from Furlong (2001). A series for the official discount rate, available since 1926, has been constructed as follows. Since 1934, when the *Bank of Canada* (Canada's central bank) was created, it is simply the official bank rate ("Taux Officiel d'Escompte") from the *Bank of Canada*'s website. Before that, we use the Advance Rate, which had been set by the Treasury Department for the discounting of bills, from Table 6.1 of Shearer and Clark (1984).[1] As for the latter period, we use a series for the 3-month Treasury bill rate, which has been constructed by linking the series from the Historical Statistics

[1] To be precise, Shearer and Clark (1984) do not provide the actual time series for the Advance Rate, but rather the dates at which the rate had been changed (starting from August 22, 1914), together with the new value of the rate prevailing starting from that date. Based on this information, we constructed a daily series for the rate starting on January 1, 1915, via a straightforward MATLAB program, and we then converted the series to the annual frequency by taking annual averages.

of Canada, available for the period 1934-1935, to the series "Treasury Bill Auction - Average Yields - 3 Month, Per cent / en pourcentage" from the *Bank of Canada*. A monthly series for M1 starting in January 1872 is from Metcalf, Redish, and Shearer (1996). This series is available until December 1952. After that, we link it via splicing to the series labeled as "Currency and demand deposits, M1 (x 1,000,000), v37213" from *SC* until November 1981. Finally, from December 1981 until December 2006, we use the series from *SC* labeled as "M1 (net) (currency outside banks, chartered bank demand deposits, adjustments to M1 (continuity adjustments and inter-bank demand deposits) (x 1,000,000), v37200." An important point to stress is that over the overlapping periods, the three series are nearly-identical (up to a scale factor), which justifies their linking. For the period after December 2006, however, we were not able to find an M1 series that could be reliably linked to the one we use for the period December 1981-December 2006 (over the last several decades, Canada's monetary aggregates have undergone a number of redefinitions, which complicates the task of constructing consistent long-run series for either of them). As a result, for the most recent period we have decided to use another series that we consider in isolation (that is, without linking it to any other M1 aggregate). The series is "M1B (gross) (currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits) (x 1,000,000), v41552787," which is available since January 1967 from *SC*. Finally, we convert all monthly series to the annual frequency by taking simple annual averages.

A.11 Chile

Annual series for nominal GDP, the GDP deflator, and M1 are from Braun-Llona *et al.* (1998) for the period 1940-1995. As for the period 1996-2012, they are from the *Banco Central de Chile,* Chile's central bank (specifically, nominal GDP and the GDP deflator are from the *Banco Central*'s Anuarios de Cuentas Nacionales, whereas M1 is from *Banco Central*'s Base Monetaria y Agregados Monetarios Privados). A short-term nominal interest rate ("1-day interbank interest rate, financial system average (annual percentage)") from *Banco Central* is available for the period 1940-1995. In order to extend our analysis to the present as much as possible, we therefore also consider, as an alternative measure of the opportunity cost of money, GDP deflator inflation.

A.12 Colombia

Data for Colombia have been kindly provided by David Perez Reyna. Annual series for nominal GDP and a short-term nominal interest rate for the period 1905-2003 are from Junguito and RincÃşn (2007). As for the period 2004-2012, they are from Colombia's *Ministerio de Hacienda y Credito Publico.* An annual series for M1 for the period 1905-2012 is from the *Banco de la Republica*, Colombia's central bank.

A.13 Ecuador

All of the data for Ecuador are from the website of *Banco Central del Ecuador* (henceforth, *BCE*), Ecuador's central bank. Most of them are from "85 Años, 1927-2012: Series Estadísticas Históricas," a special publication celebrating *BCE*'s 85th anniversary. Specifically, a series for annual CPI inflation ("Variación Anual del Indice Ponderado de Precios al Cunsumidor por Ciudades y por Categorias de Divisiones de Concumo, Nacional"), available for the period 1940-2011, is from Chapter 4 of "85 Años." An annual series for a nominal interest rate has been constructed by linking the series "Tasas, Máxima Convencional, En porcentajes," available for the period 1948-1999; "Tasas de Interés Referenciales Nominales en Dólares, Máxima Convencional," available for the period 2000-2007; and "Tasas de Interés Referenciales Efectivas en Dólares, Máxima Convencional," available for the period 2007-2011. All of them are from from Chapter 1 of "85 Años." An annual series for nominal M1 in US dollars has been constructed by linking the M1 aggregate ("Oferta Monetaria M1, En millones de dólares al final del período"), available for the period 2000-2011, which is expressed in US dollars, and the M1 aggregate ("Medio Circulante (M1), Saldos en millones de sucres"), available for the period 1927-1999, which is expressed in Ecuador's national currency, the *sucre* (both series are from Chapter 1 of "85 Años"). The latter M1 aggregate has been converted in US dollars based on the series for the *sucre*/dollar nominal exchange rate found in Chapter 2 of "85 Años," which is available for the period 1947-1999. Specifically, the exchange rate series (*sucre* per dollar) has been computed as the average between the "Compra" (i.e., buy) and the "Venta" (i.e., sell) series. An annual series for nominal GDP in U.S. dollars ("Producto interno bruto (PIB), Miles de dólares"), available for the period 1965-2011, is from Chapter 4 of "85 Años". An important point to stress is that since we are working with M1 *velocity*—defined as the *ratio* between nominal GDP and nominal M1—the specific unit in which the two series are expressed (US dollars, or Ecuadorian *sucres*) is irrelevant.

A.14 Finland

Long-run monthly data for M1 for the period January 1866-December 1985 have been generously provided by Tarmo Haavisto. The data come from his Ph.D. dissertation (see Haavisto (1992)) and have been converted to the annual frequency by taking simple annual averages. A series for Finland's monetary policy rate (labeled as the "Base rate"), available since January 1867, is from *Suomen Pankki Finlands Bank* (Finland's central bank, henceforth, *Suomen Pankki*).[2] Finally, an annual series for

[2] To be precise, *Suomen Pankki* does not provide the actual time series for the base rate, but rather the dates at which the rate had been changed (starting from January 1, 1867), together with the new value of the base rate prevailing starting from that date. Based on this information, we constructed a daily series for the base rate starting on January 1, 1867, via a straightforward MATLAB program, and we then converted the series to the annual frequency by taking annual averages.

nominal GDP, available since 1860, is from Finland's Historical Statistics, which are available from the web page of *Statistics Finland* (Finland's national statistical agency). (To be precise, from the homepage of Statistics Finland, look at Home > Statistics > National Accounts > Annual national accounts > Tables.) Specifically, the nominal GDP series is B1GMHT ("Gross domestic product at current prices, 1860-1960, million. mk").

A.15 France

Annual series for nominal GDP, nominal M1, and the short rate are all from SaintMarc (1983). Specifically, the series for nominal GDP is the Toutain Index from Annexe I: Revenu national, Produit Interieur Brut, pages 99-100 of Saint Marc (1983), and it is available for the period 1815-1913. The series for M1 is from the table "Vitesse-Revenu, Vy, et taux de liquidite, TL,"pages 74-75 of Saint Marc (1983), and it is available for the period 1807-1913. The series for the short rate is from Section 7, "Evaluation des taux de l'interet," pages 93-96, of Saint Marc (1983), and it is available for the period 1807-1913. In our analysis, however, we focus on the period 1851-1913 because for the entire period 1820-1851, the short rate had been fixed at 4%.

A.16 Guatemala

All of the data are from the *Banco de Guatemala*'s website. A series for nominal GDP is available for the period 1950-2017. A series for M1 ("M1 Medio Circulante-Millones de quetzales") is available for the period 1980-2018. A series for a nominal short rate ("Interest rate, Domestic currency, borrowing (passive)") is available for the period 1980-2018.

A.17 Hong Kong

An annual series for nominal GDP for the period 1961-2017 is from the *Hong Kong Monetary Authority*'s (henceforth, *HKMA*) website (it is labeled as "Nominal GDP, HK$ million"). The series is from Table031 ("GDP and its main expenditure components at current market prices"). An annual series for M1 for the period 1985-2017 is from the *HKMA*'s website (the series is labeled as "M1, Total, HK$ million"). An annual series for a short-term interest rate for the period 1982-2018 is from the *HKMA*'s website. The series is labeled as "Overnight rate, Table 6.3: Hong Kong Interbank Offered Rates").

A.18 Israel

Series for nominal and real GDP, available since 1950, are from Israel's *Central Bureau of Statistics* (henceforth, *CBS*; special thanks to Svetlana Amuchvari of the *CBS* for help with the data). Specifically, starting from 1995, the data are from Table 17 of the National Accounts. For the period 1950-1994, they are from the *CBS*'s Statistical Abstract of Israel (see columns D and J of Table 6.1, "National Income and Expenditure: Resources and Uses of Resources"). The GDP deflator has been computed as the ratio between the two series. An annual CPI inflation series ("Change in Level of Price Indices, Percentages, Annual, average"), available since 1971, is from the *CBS* website (specifically, the series is from Table 13.1 of Statistical Abstract of Israel). For the period 1966-1975, the series for M1 is from Table 4.6, page 120, of Barkai and Liviatan (2007). For the period since April 1981, a monthly M1 series is from the *Bank of Israel*'s website (special thanks to Aviel Shpitalnik of the *Bank of Israel* for help with the data). The series is M1.M ("M1 = Money supply, Monthly (M), NIS, million, Current prices"), and it has been converted to the annual frequency by taking annual averages. A short-term interest rate for the period 1966-1974 is the "Nominal rate of return on MAKAM (3-month bills)" from Table 4.9, page 129, of Barkai and Liviatan (2007). Since 1989 it is the *Bank of Israel*'s "Actual effective rate of interest,"from the *Bank of Israel*'s website. For the period 1983-1988, we use the "Discount Rate" from the IMF's IFS. Over the overlapping periods (i.e., since 1989), the *Bank of Israel*'s actual effective rate of interest and the discount rate from the IMF are virtually identical, which justifies their linking.

A.19 Italy

Series for nominal GDP at current market prices, real GDP in chained 2005 euros, and the implied GDP deflator, all available for the period 1861-2010, are from the sheet "Tab_03' in the Excel spreadsheet 'Data_Na150-1.1.xls,"which is available at the *Banca d'Italia*'s website at http://www.bancaditalia.it/statistiche/tematiche/stat-storiche/index.html. The spreadsheet contains the estimates of the Italian National Accounts' aggregates, which are extensively discussed in Baffigi (2011). A series for M1, available for the period 1861-1991, is from the Data Appendix, pp. 49-52, of Fratianni and Spinelli (1997). Series for M1 and M2, available for the period 1948-1998, are from the table "Componenti della moneta dal 1948 al 1998" of BancadItalia (2013). In our analysis we use the M1 series from Fratianni and Spinelli (1997) for the gold standard period, and the one from *Banca d'Italia* for the post-WWII period (over the overlapping periods, however, the two series are very similar, so in practice this choice does not entail material implications). Short- and long-term interest rates for the period 1861-1996 are from Muscatelli and Spinelli (2000). A series for the "Tasso Ufficiale di Sconto"—that is, *Banca d'Italia*'s official discount rate—is from the tables "Tassi d'interesse delle principali operazioni della banca centrale" and "Variazione dei tassi ufficiali della Banca d'Italia, 1936-2003" of BancadItalia (2013).

A.20 Japan

Sources for Japanese data are as follows. A monthly series for the *Bank of Japan*'s (henceforth, *BoJ*) discount rate, available since January 1883, is from the *BoJ*'s long-run historical statistics, which are available at its website (the series is labeled as "BJ'MADR1M: The Basic Discount Rate and Basic Loan Rate"). Annual series for nominal GNP and M1 for the period 1885-1940 are from Table 48 of Tamaki (1995). As for the period since 1955, data for nominal GDP and M1 are as follows. Series for nominal GDP are from the *Economic and Social Research Institute* (henceforth, *ESRI*), Cabinet Office, Government of Japan. (The key URLs are http://www.stat.go.jp/english/data/chouki/03.htm and

http://www.stat.go.jp/english/data/nenkan/1431-03.htm.) An important point to stress here is the following. For the period before 1970, *ESRI* only provides tables for gross domestic *expenditure*, rather than gross domestic *product*. However, over the overlapping periods (that is, 1970-1998), the relevant series coming from Table 3-1 ("Gross Domestic Expenditure (At Current Prices, At Constant Prices, Deflators) - 68SNA, Benchmark year = 1990 (C.Y.1955–1998, F.Y.1955–1998), Value in billions of yen") and Table 3-3*b* ("3-3-b Gross Domestic Product Classified by Economic Activities (Medium Industry Group), (At Current Prices, At Constant Prices, Deflators) - 68SNA, Benchmark year = 1990 (1970–1998), Value in billions of yen") are either numerically identical (in the case of nominal series) or numerically identical up to a scale factor (in the case of real series and their deflators). This means that—as should be expected based on simple economic logic—the series that in Table 3-1 is labeled as "Gross Domestic Expenditure" (Column Y in the Excel spreadsheet 03-01.xls) is, in fact, nominal gross domestic product. As for M1, a monthly series for the period January 1955-December 2018 was constructed by linking, via splicing, the following three series from the *BoJ*'s website: MA'MAMS1EN01 ("(discontinued)_M1/Amounts Outstanding at End of Period/(Reference) Money Stock (Based on excluding Foreign Banks in Japan, etc., through March 1999)"); MA'MAMS3EN01 ("(discontinued)_M1/Amounts Outstanding at End of Period /(Reference) Money Stock (from April 1998 to March 2008)"); and MA'MAM1NEM3M1MO ("M1/Amounts Outstanding at End of Period/Money Stock"). An important point to stress is that, over the overlapping periods, the series are essentially identical (up to a scale factor), which justifies their linking. Finally, the resulting monthly M1 series was converted to the annual frequency by taking annual averages.

A.21 Mexico

A monthly interest rate series, available since January 1978, is from the *Banco de Mexico*'s "Indicadores de tasas de interes de Valores Publicos" (*Banco de Mexico*, henceforth *BdM*, is Mexico's central bank). It has been converted to the annual frequency by taking annual averages. Two annual interest rates series ("Interest Rate (%) Commercial loans" and "Interest Rate (%), Official discount rate,"respectively) are from Table 83, pages 639-640, of Homer and Sylla (2005). The first series is available for the periods 1942-1963 and 1978-1989. The second is available for the period 1936-1978. An annual series M1 for the period 1925-2000 is from the *Instituto Nacional de Estadistica y Geografia* (Mexico's national statistical agency, henceforth *INEGI*), "Estadisticas Historicas de Mexico, 2014,"and for the period 1985-2014 they are from the *BdM*'s website. The series from the *BdM* are available at the monthly frequency, and we converted them to the annual frequency by taking annual averages. Annual series for nominal GDP are from *INEGI*, "Estadisticas Historicas de Mexico 2014"for the period 1925-1970; from the IMF's IFS for the period 1970-1988; from *BdM* for the period 1988-2004; and from *INEGI* for the period since 2004. The four series have been linked via splicing. An annual CPI inflation series available since 1949 is from the IMF's IFS ("Mexico, Consumer Prices, All items, Percent Change over Corresponding Period of Previous Year").

A.22 Morocco

A monthly seasonally unadjusted series for M1, available since January 1985, is from the website of *Bank Al-Maghrib* (the central bank of Morocco, henceforth, *BAM*). The annual series has been computed by taking simple annual averages of the original monthly data. An annual series for nominal GDP, available since 1980, is from the "Comptes Nationaux" (National Accounts) from the website of the High Commission for Planning of Morocco. A series for the minimum rate applied to notebook accounts, available since January 1983, is from the website of *BAM*. *BAM* sets this interest rate two times a year, on January 1 and on July 1. The table at the central bank's website reports the values for the interest rate which have been set every January 1, and every July 1, starting from 1983. From this information we computed the annual average rates by taking a simple average within the year.

A.23 Netherlands

A series for the discount rate of *De Nederlandsche Bank* (the Dutch central bank, henceforth, *DNB*) for the period 1900-1992 is from Table 65 of Homer and Sylla (2005) until 1989 and from *DNB*'s website after that. Series for nominal and real net national income (NNI) and for the NNI deflator for the period 1900-1992 are from Table 1, pages 94-95, of Boeschoten (1992). A series for M1, available since 1864, has been constructed by linking the series from deJong (1967) and one from *DNB*.

A.24 New Zealand

A series for M1, available since 1934, is from the website of the *Reserve Bank of New Zealand* (henceforth, *RBNZ*). A series for nominal GDP in million of Australian dollars is from *Statistics New Zealand* (New Zealand's statistical agency). A series for a short-term nominal interest rate starting in 1934 has been constructed in the following way. Homer and Sylla's (2005) Table 79 contains a series for the *RBNZ*'s official discount rate for the period 1934-1989. Since 1999, the *RBNZ* has been using, as its monetary policy rate, the "Official Cash Rate,"which is available from the *RBNZ*'s website. Since these two short-term rates have been used by the *RBNZ* as its official monetary policy rate for the periods 1934-1989 and 1999 to the present, respectively, they are in fact conceptually the same and can therefore be linked. For the period in between (1990-1998), for which no official monetary policy rate is available, we have used the "Overnight Interbank Cash Rate" from the *RBNZ*. The rationale for doing so is that since 1999, this rate has been very close to the Official Cash Rate, which justifies the linking of the two series.

A.25 Norway

A series for M1, available since 1919, is from the Historical Statistics of *Norges Bank* (Norway's central bank), which are available at its website. Specifically, all historical statistics for Norway's monetary aggregates are from Klovland (2004). Series for nominal GDP and the GDP deflator, and for real GDP, real private consumption expenditures, and real gross investments (in millions of 2005 NOKs, i.e., kronas), all available since 1830, are from *Norges Bank*'s Historical Statistics (for all series, the period 1940-1945 is missing). As for the short-term nominal interest rate, ideally we would have liked to use *Norges Bank*'s discount rate. The problem is that, although the discount rate is available (from *Norges Bank*'s website) since 1819, it has missing observations for the period 1987-1990. As a result, we have resorted to using the Average Deposit Rate (again, from *Norges Bank*'s website), which is available since 1822, has no missing observations, and over the period that is analyzed herein has been quite close to the discount rate.

A.26 Paraguay

Annual series for CPI inflation ("Índice de Precios al Consumidor, Área Metropolitana de Asunción, Indice General"), available for the period 1951-2015, and for nominal M1 in thousands of *guaranies,* available since 1962, are both from the website of *Banco Central del Paraguay* (Paraguay's central bank, henceforth *BCP*). An annual series for nominal GDP in thousands of *guaranies*, available since 1960, is from the *International Monetary Fund*'s *International Financial Statistics.*

A.27 Peru

All of the data for Peru are from the website of the *Banco Central de Reserva del Perú*, Peru's central bank. An annual series for nominal GDP in million of *nuevos soles* is available since 1950. An annual series for inflation is available since 1901.

An annual series for nominal M1 in million of *nuevos soles*, available since 1959, has been constructed as the sum of currency in circulation ("Billetes y Monedas en Circulación") and deposits ("Depósitos a la Vista del Sistema Bancario en Moneda Nacional").

A.28 Portugal

An annual series for M1 for the period 1854-1998 is from Table 5 of Mata and Valerio (2011). Annual series for real and nominal GDP for the period 1868-2008 are from Table 4 of Mata and Valerio (2011). A series for the official discount rate of the *Banco de Portugal* (the Portuguese central bank), available for the period 1930-1989, is from Table 74 of Homer and Sylla (2005).

A.29 South Africa

All of the data for South Africa are from the website of its central bank, the *South African Reserve Bank* (*SARB*). Specifically, a series for the "Bank rate" ("Lowest rediscount rate at *SARB*"; code is KBP1401M) is available since 1923. A series for M1 ("Monetary aggregates / Money supply: M1, R millions"; code is KBP1371J) is available since 1967. A series for nominal GDP ("Gross domestic product at market prices, R millions"; code is KBP6006J) is available since 1946.

A.30 South Korea

A series for M1 ("M1, Narrow Money, Average, Billion Won") is available since 1970 from the website of the *Central Bank of Korea* (henceforth, *BOK*), at: http://ecos.bok.or.kr. The series is from Table 1.1. ("Money & Banking (Monetary Aggregates, Deposits, Loans & Discounts etc."). A series for nominal GDP ("Gross domestic product, current prices, Billion Won") is available since 1953, again from the BOK's website. A series for the central bank's discount rate ("Republic of Korea, Interest Rates, Discount Rate, Percent per Annum") is available since 1948 from the IMF's IFS.

A.31 Spain

An annual series for M1 for the period 1865-1998 is from Cuadro 9.16 "Agregados Monetarios, 1865-1998" of Barciela-LÃşpez, Carreras, and Tafunell (2005), pp. 697-699 (the series is labeled as "M1, datos a fin de ano, en millones de pesetas"; the years 1936-1940 are missing). An annual series for nominal GDP for the period 1850-2000 is from Cuadro 17.7 of Barciela-LÃşpez, Carreras, and Tafunell (2005), pp. 1338-1340 (the series is labeled as "El PIB a precios corrientes, 1850-2000, millones de pesetas"; PIB is the Spanish acronym of GDP). An annual series for the "Descuento comercial" of the *Banco de Espana* (Spain's central bank, henceforth, *BdE*) is from

Cuadro 9.17 of Barciela-LÃşpez, Carreras, and Tafunell (2005), pp. 699-701. The series is available for the periods 1874-1914, 1920-1935, and 1942-1985. An annual series for the official discount rate of the *BdE*, available for the period 1930-1989, is from Table 74, pp. 541-542, of Homer and Sylla (2005). A monthly series for the three-month Treasury bill rate available since March 1988 ("Tipo de interese hasta 3 meses. Conjunto del mercado. Op. simples al contado. Letras del Tesoro."), is from the *BdE*'s website, and it has been converted to the annual frequency by taking annual averages (the data for 1988 have been ignored, since the series starts in March of that year).

A.32 Switzerland

Annual series for M1 (based on the 1995 definition) and the official discount rate of the *Swiss National Bank* (Switzerland's central bank, henceforth *SNB*), all available at least since 1929, are from the *SNB*'s website. An annual series for nominal GDP available for the period 1948-2005 is from the website of the project *Economic History of Switzerland during the 20th century*—see at http://www.fsw.uzh.ch/histstat/main.php. (Q.16b Gross domestic product (expenditure approach) in real 1990 prices and nominal, 1948-2005 in Million Swiss Francs).

A.33 Taiwan

All of the data are from the *Central Bank of the Republic of China (Taiwan)*, that is, Taiwan's central bank (henceforth *CBRCT*). An annual series for nominal GDP ("GDP by expenditures at current prices") is available since 1951. An annual series for the *CBRCT*'s discount rate is available since 1962. Two annual series for M1 ("M1A (End of Period), M1A = Currency in circulation(currency held by the public)+Checking accounts and passbook deposits of enterprises, individuals and non-profit organizations held in banks and community financial institutions" and "M1B (End of Period), M1B = M1A + Passbook savings deposits of Individuals and non-profit organizations in banks and community financial institutions") are both available since 1962. In order to be sure that the series we use in this paper does not include components that go beyond a transaction purpose, we used the first one, M1A.

A.34 Thailand

An annual series for GDP at current prices in billions of *baht*, available for the period 1946-2005, is from Mitchell (2007). Since 1990 this series has been linked to the nominal GDP series from the Macro Economic Indicators of the *Bank of Thailand* (Thailand's central bank, henceforth *BoT*). Over the overlapping periods, the two series are very close, which justifies their linking. An annual M1 series in billions of *baht*, available since 1970, has been constructed by taking, for each year, the December observation from the series "Money supply (M1)" from Table 5 of the *BoT*'s monetary aggregates for the period up to 2005. Since then, we have taken the December observation from the monthly M1 series from the *BoT*'s Macro Economic Indicators. The reason for taking, for each year, the December observation, rather than computing the annual average, is that for the period 1970-1980 the December figure is the only one available for each year. An annual series for the 1-year maximum interest rate on fixed deposits, available since 1979, is from the *BoT*'s Macro Economic Indicators.

A.35 Turkey

A monthly series for M1, available since January 1964, is from the website of Turkey's central bank, *Turkiye Cumhuriyet Merkez Bankasi* (henceforth, *TCMB*). The series we use has been constructed by taking simple annual averages of the original monthly data. A series for the central bank's discount rate is from Homer and Sylla's (2005) Table 74, pages 541-542, until 1990. After that, it is from *TCMB*. Specifically, *TCMB*'s website reports the dates in which the discount rate was changed, together with the new values taken by the discount rate at each date. Based on this information, for each year since 1990 we have calculated the number of days in the year for which each value of the discount rate has been in effect, and based on this we have computed, for every year, a simple weighted average of the individual daily values of the discount rate. A series for the gross domestic product in current prices, available since 1967, is from the website of Turkey's statistical office, *TurkStat*.

A.36 United Kingdom

All U.K. data are from version 3.1 of the dataset "A millennium of macroeconomic data," which is available from the *Bank of England*'s website at:

http://www.bankofengland.co.uk/statistics/research-datasets. The first version of the dataset (which was called "Three centuries of macroeconomic data") was discussed in detail in Hills and Dimsdale (2010). Specifically, series for M1, available since 1922; the *Bank of England*'s monetary policy rate (known as the "Bank Rate"), available since 1694; and nominal GDP ("Nominal UK GDP at market prices"), available since 1700, are, respectively, from columns A.24, A.31, and A.9 of the sheet "A1. Headline series."

A.37 United States

The series for the 3-month Treasury bill rate; nominal GDP; both the "standard" M1 aggregate and the "New M1" one; and Money Market Deposits Accounts (MMDAs), are all from Lucas and Nicolini (2015) All series have been updated to 2017 based on either series' updated original data sources. The original source for the 3-month Treasury bill rate is the *Economic Report of the President* (henceforth, *ERP*), and the ones for nominal GDP are Kuznets and Kendrick's Table Ca184-191 before 1929, and Table 1.1.5 of the *National Income and Product Accounts* (henceforth, *NIPA*) after that. A series for Money Market Mutual Funds (MMMFs) starting in 1974 is from the Federal Reserve (the FRED II acronym is MMMFFAA027N, "Money market mutual funds, Total financial assets, Billions of dollars"). An annual series for nominal GDP at current prices is from Officer and Williamson (2015).

A.37.1 Adjusting for the share of currency held by foreigners

As documented, for example, by Judson (2017), over the last several decades, the fraction of US currency held by foreigners has significantly increased, and it stood, at the end of 2016, at around 50%-60% of total currency, depending on the methodology that was used to estimate it. Since the demand for M1, which is being investigated in the present work, is a demand on the part of US nationals, this raises the issue of how to adjust US currency in order to purge it of the fraction held by foreigners. This could be done in several ways, none of them ideal. One possibility would be, following Judson (2017), to estimate a model for the demand of *Canadian* currency as a function of Canadian nominal GDP and interest rates, and then to apply the estimated coefficients to U. nominal GDP and interest rates in order to back out a predicted level of US currency demanded by US nationals. As extensively discussed in Judson (2017), the rationale for doing this is that—most likely as a consequence of the similarity between the U.S. and Canadian economies—up until about 1990 the ratios between currency and nominal GDP in the two countries had tended to closely comove. Only since then has the demand for US currency on the part of non-US nationals skyrocketed, thus causing the traditional relationship between the demands for U.S. and Canadian currency, as fractions of their respective GDPs, to go off kilter. For our own purposes, this approach suffers from the limitation that, by definition, it produces a "fundamental," predicted value for the demand for US currency on the part of US nationals which does not reflect idiosyncratic, transitory factors that are not captured by either nominal GDP or the short rate. Because of this, we have adopted an alternative approach in which we estimate the fraction of US currency held by foreigners as the simple difference between the ratios between currency and nominal GDP in the United States and Canada. One problem with this approach is that since, during the early years of the Great Depression, Canada did not experience banking collapses of a magnitude comparable to the United States, the "flight to currency" there was much more muted. As a result, our approach mechanically interprets the increase in the demand for US currency as a fraction of GDP between the crash of 1929 and the inauguration of F.D. Roosevelt's presidency as an increase in demand on the part of foreigners. Our counterargument to this is that the spike in the demand for currency, although sizable, was very short-lived, as it only pertained to four years, from 1930 to 1933. As a result, since this only pertains to currency—which, in 1929, was just 14.6% of overall M1—it is reasonable to assume that the impact of this on our estimates should be negligible.

A.38 Venezuela

Annual data for nominal GDP ("Producto Interno Bruto, Millones de Bolívares a Precios Corrientes"), M1 ("Circulante, (M1), I.1, Circulante, Liquidez Monetaria y Liquidez Ampliada, Saldos al final de cada período en millones de bolívares"), and a short-term rate ("Tasas de Interes Activas Anuales Nominales Promedio, Ponderadas de los Bancos Comerciales y Universales, Porcentajes") are from the *Banco Central de Venezuela* (Venezuela's central bank). GDP is available since 1957, and M1 is available since 1940. The interest rate is available for the period 1962-1999. An alternative monthly interest series, available since July 1997 ("Tasa de Interés Aplicable al Cálculo de los Intereses Sobre Prestaciones Sociales (Porcentajes)"), cannot be linked to the other interest rate series because, over the overlapping periods, the two series are different. As a consequence, we limited our analysis to the period 1962-1999.

A.39 West Germany

Although data for post-WWII Germany are available, in principle, for the entire period 1950-1998, in the empirical work we have decided to only use data for West Germany for the period 1960-1989. The reason is that we are skeptical about the possibility of meaningfully linking the various series for nominal GDP in order to create a single series for the period 1950-1998 because (*i*) before 1960, GDP data did not include West Berlin and the Saarland, which, in 1960, jointly accounted for about 6% of overall GDP; and (*ii*) the reunification of 1990 created discontinuities in both GDP and M1 (we thought the problem could be side-stepped by focusing on M1 velocity, i.e. their ratio, but in fact this series also seems to exhibit a discontinuity around the time of reunification). Entering into details, an annual series for the *Bundesbank*'s monetary policy rate for the period 1949-1998 has been constructed by taking annual averages of the monthly series "BBK01.SU0112, Diskontsatz der Deutschen Bundesbank / Stand am Monatsende, % p.a.,"which is available from the *Bundesbank*'s website. As for nominal GDP, the original annual series are from Germany's Federal Statistical Office, and they are available for the period 1950-1960 ("Gross domestic product at current prices, Former Territory of the Federal Republic excluding Berlin-West and Saarland"); 1960-1970 ("Gross domestic product at current prices, Former Territory of the Federal Republic"); and 1970-1991 ("Gross domestic product at current prices, Former Territory of the Federal Republic, (results of the revision 2005)"). There is also a fourth series available for reunified Germany, but, as mentioned, it cannot be meaningfully linked to the series for the period 1970-1991 because of the discontinuity induced by the 1990 reunification. The second and third series can be linked because the difference between them is uniquely due to changes in the accounting system, rather than to territorial redefinitions. Linking the first and second series, on the other hand, is problematic because, as mentioned, before 1960 GDP data did not include West Berlin and the Saarland. Our decision has been to ignore the first GDP series, and therefore to start the sample in 1960, for the following two reasons. *First*, the dimension of West Berlin and the Saarland was not negligible. The value taken by nominal GDP in 1960 according to the first and second series was equal to 146.04 and 154.77, respectively, a difference equal to 6%. *Second*, this problem might be ignored if we had good reasons to assume that, during those years, West Berlin and the Saarland's nominal GDP was growing exactly at the same rate as in the rest of Germany. This, however, is pretty much a heroic assumption—especially for West Berlin. As a result, in the end we just decided to ignore the first series. Finally, turning to M1, this turned out to be the single most excruciating piece of data collection in the entire enterprise. German M1 data, which are available at the monthly frequency since 1948, can only be recovered from the Bundesbank's original *Monthly Reports*, which are available in scanned form at the Bundesbank's website. So we downloaded the scanned PDFs of the *Monthly Reports*, and we manually entered the data in Excel, one "piece" (that is, one *Monthly Report*) at a time. An important point to notice is that German monetary aggregates are *not revised*, so that it is indeed possible to link the figures coming from successive issues of the *Monthly Report*. With a few exceptions in 1940 and the early 1950s, each report contains about one year to one year and a half of data. There are a few discontinuities in the series, but other than that, the overlapping portions coming from successive issues are identical (over the entire sample we noticed about four to five exceptions, which means that those months were revised, and in those cases we took the values coming from the most recent *Monthly Report*). The discontinuities were just level shifts: we checked the log-differences of the two series pertaining to each discontinuity, and they were nearly identical. So in the end we linked the various pieces coming from the different issues of the *Monthly Report*, thus obtaining a single monthly series for the period up to December 1998. Finally, we converted the series to the annual frequency by taking annual averages.

B Mathematical Derivations

B.1 Interest rate rules and money rules

Note that (6) and (7) in the text imply

$$\beta E\left[\frac{V'(\omega')}{\pi(s')}\right] = \frac{\varepsilon}{R}$$

and

$$\delta = \frac{\varepsilon}{n}\left[1 - \frac{R^m}{R}\right].$$

Substituting this in equation (4), we obtain

$$U'(x) = \frac{\varepsilon}{R} + \frac{\varepsilon}{n}\left[1 - \frac{R^m}{R}\right]$$

or

$$\begin{aligned} \varepsilon &= \frac{U'(x)}{\left[\frac{1}{n} + \frac{1}{R}(1 - \frac{1}{n}R^m)\right]} \\ &= \frac{nU'(x)}{\left[1 + \frac{1}{R}(n - R^m)\right]}. \end{aligned}$$

Now, combining (7) and (9), we obtain

$$\beta E\left[\frac{\varepsilon'(s')}{\pi(s')}\right] R = \varepsilon$$

or, using the result above and noting that $x = z(1-\theta(n))$,

$$\beta E\left[\frac{n(s')U'\left[(z(s')(1-\theta(n(s')))\right]}{\left[1 + \frac{1}{R(s')}(n(s') - R^m(s'))\right]}\frac{1}{\pi(s')}\right] R = \frac{nU'(z(1-\theta(n)))}{\left[1 + \frac{1}{R}(n - R^m)\right]}.$$

But replacing the inflation rate $\pi(s') = \frac{M(s')x(s')}{Mx}\frac{n}{n(s')}$, we obtain

$$\beta E\left[\frac{U'\left[(z(s')(1-\theta(n(s')))\right]}{\left[1 + \frac{1}{R(s')}(n(s') - R^m(s'))\right]}\frac{M}{M(s')}\right] R = \frac{z(1-\theta(n))U'(z(1-\theta(n)))}{\left[1 + \frac{1}{R}(n - R^m)\right]}.$$

Now, if we let

$$\Omega = \frac{U'(z(1-\theta(n)))z(1-\theta(n))}{\left[1 + \frac{1}{R}(n - R^m)\right]},$$

we can write the expression above as

$$\beta E\left[\Omega(s')\frac{M}{M'}\right] R = \Omega.$$

But

$$M(s') = M + \mu(s')P,$$

so

$$\frac{M}{M(s')} = 1 - \frac{\mu(s')}{\pi(s')m(s')} = \left(1 - \frac{\mu(s')n(s')}{\pi(s')z(s')(1-\theta(n(s')))}\right).$$

Replacing the above,

$$\beta E\left[\Omega(s')\left(1 - \frac{\mu(s')n(s')}{\pi(s')z(s')(1-\theta(n(s')))}\right)\right] R = \Omega$$

or

$$\beta E\left(\frac{\Omega(s')}{\Omega}\right)-\beta E\left(\frac{\Omega(s')}{\Omega}\frac{\mu(s')n(s')}{\pi(s')z(s')(1-\theta(n(s')))}\right)=\frac{1}{R}.$$

In general, there are many solutions for the growth rate of money stochastic sequence $\mu(s')$ that are consistent with a given interest rate. This is so because the nominal interest rate pins down (weighted) expected inflation, but there are many distributions of future price levels that are consistent with the same expected value of inflation. Notice, however, that there exists a unique growth rate of money that is consistent with the interest rate sequence, and that is predetermined the period before, the solution, μ^*, satisfying

$$E\left(\frac{\beta\Omega(s')}{\Omega}\right)-\mu^* E\left(\frac{\beta\Omega(s')}{\Omega}\frac{n(s')}{\pi(s')z(s')(1-\theta(n(s')))}\right)=\frac{1}{R}.$$

B.2 The Bellman equation describing the decision problem

The Bellman equation describing the decision problem is

$$\begin{aligned} V(\omega) &= \max_{x,n,m,b,q(s')} U(x)-\varepsilon\left[m+b+E\left[q(s')\pi(s')\widetilde{P}^Q(s')\right]-\omega\right]-\delta\left[x-mn\right] \\ &\quad +\beta E\left[V(\frac{mR^m+bR+\left[1-\theta(n)\right]z-x}{\pi(s')}+\tau(s')+q(s'))\right], \end{aligned}$$

where, for simplicity, we omitted the dependence of current variables on the state, and where s' denotes the future state.

The first order conditions are

$$x:\quad U'(x)=\beta E\left[\frac{V'(\omega')}{\pi(s')}\right]+\delta \tag{1}$$

$$n:\quad \delta m=\beta E\left[\frac{V'(\omega')}{\pi(s')}\right]\theta_n(n)z \tag{2}$$

$$m:\quad \delta n+\beta E\left[\frac{V'(\omega')}{\pi(s')}\right]R^m=\varepsilon \tag{3}$$

$$b:\quad \beta E\left[\frac{V'(\omega')}{\pi(s')}\right]R=\varepsilon \tag{4}$$

$$q(s'):\quad \beta V'(\omega')=\varepsilon\pi(s')P^Q(s'), \tag{5}$$

and the envelope condition is

$$V'(\omega)=\varepsilon.$$

Note that (3) and (4) imply

$$\beta E\left[\frac{V'(\omega')}{\pi(s')}\right](R-R^m)=\delta n,$$

which in turn implies

$$\frac{m}{\theta_n(n)z}(R - R^m) = n.$$

In equilibrium,

$$m = \frac{x}{n} = \frac{z(1-\theta(n))}{n},$$

so if we replace the value of m in the previous equation and let $r^* \equiv (R - R^m)$, we obtain

$$r^* \equiv (R - R^m) = n^2 \frac{\theta_n(n)}{1-\theta(n)}.$$

B.3 The model with heterogeneous agents

Consider a model as the one above, with a unit mass of agents that are alike in all respects, except that they differ in their productivity and in their borrowing constraints. Let idiosyncratic productivity for agent j be equal to $\xi^j \in [\xi_l, \xi^h]$, where the mean of ξ^j is equal to one. In each period, the productivity of each agent is $\xi^j z(s_t)$. We also assume agent-specific upper bounds on debt, which we denote as b^{j*}, with $b^{j*} \in [b_l, b^h]$.

The common preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t^j).$$

Equilibrium in the labor market and the equality of production and consumption imply

$$\begin{aligned} 1 &= \int_0^1 l_t^j dj + \gamma \nu_t \int_0^1 n_t^{j\sigma} dj \\ \int_0^1 x_t dj &= z_t (1 - \gamma \nu_t \int_0^1 n_t^\sigma dj). \end{aligned}$$

These technologies imply that the real wage, per unit of efficiency, is equal to z_t.

The portfolio decision is constrained by an agent-specific equivalent to (6),

$$m_t^j + b_t^j + E_t \left[s_t^{jt+1} \pi_t^{t+1} Q_t^{t+1} \right] \leq w_t^j. \tag{6}$$

Finally, we impose a productivity-adjusted borrowing constraint for the agent of the form

$$b_t \geq z_t b^{*j}. \tag{7}$$

The agent's wealth next period, contingent on the actions taken in the current period and the realization of the exogenous shock, is given by

$$w_t^{jt+1} \leq \frac{m_t^j + b_t^j(1+r_t) + \left[1 - \gamma n_t^{j\sigma} \nu_t\right] z_t \xi^j - x_t}{\pi_t^{t+1}} + q_t^{t+1} + \tau_t^{t+1}, \tag{8}$$

where $\tau(s^t, s_{t+1})$ is the real value of the monetary transfer the government makes to the representative agent. Finally, the cash-in-advance constraint can be written in real terms as

$$x_t^j \leq m_t^j n_t^j. \tag{9}$$

We now consider the decision problem of a single, atomistic agent that maximizes utility subject to restrictions (6), (8),(7) and (9).

Consider now the solution given the distribution of ξ^j and given a distribution of initial wealth among the population. Using the same arguments as for the representative agent case, it is trivial to show that if the borrowing constraint does not bind for agent j, the solution is given by

$$r_t = n_t^{j2} \frac{\sigma \gamma n_t^{j\sigma - 1} \nu_t}{1 - \gamma n_t^{j\sigma} \nu_t}. \tag{10}$$

Thus, the individual money demand function can be well approximated by a log-log function with elasticity equal to $1/(1+\sigma)$. Note that this demand function only depends on aggregates, so the aggregate money demand for the group of agents for which the borrowing constraint does not bind is also a log-log function with the same elasticity. It trivially follows that if no agent is constrained in equilibrium, the aggregate money demand is as in the representative agent economy.

In an intermediate case in which some agents are constrained, the solution for them is given by

$$m_t^j = w_t^j + z_t b^* \xi^j,$$

so for them, n_t^j is locally invariant to movements in the interest rate. In this intermediate case, then, aggregate real money demand is a combination of a mass of agents for which the elasticity is zero and the complement mass for which the elasticity is $1/(1+\sigma)$.

The size of the mass of agents for which the constraint binds is weakly decreasing with the interest rate, a property thereby inherited by the aggregate elasticity. Eventually, if the constraint becomes binding for all agents at some interest rate, the aggregate elasticity becomes zero.

B.4 Proof of Lemma 1

Proof. For the first part, consider a pair $r_l < r_h$, and let (m_h/x_h) be the solution to the equation

$$r_t = \frac{\sigma \gamma n_t^{\sigma+1} \nu_t}{1 - \gamma n_t^{\sigma} \nu_t} \tag{11}$$

when the interest rate differential is r_h. Assume that constraint binds for r_h. It follows that

$$w - m_h < z b^*,$$

where we omitted the time subscripts for simplicity. Assume, toward a contradiction, that it does not bind for r_l. Then,

$$w - m_l > zb^*,$$

which then implies that

$$m_l < m_h. \tag{12}$$

However, as the ratio of money to output is decreasing on the net interest rate,

$$\frac{m_l}{x_l} > \frac{m_h}{x_h}.$$

But the number of trips to the bank is increasing with the net interest rate, so $n_l < n_h$. This implies that

$$z(1 - \gamma\nu n_l^{\sigma}) = x_l > z(1 - \gamma\nu n_h^{\sigma}) = x_h.$$

The last two conditions jointly imply that

$$\frac{m_l}{m_h} > \frac{x_l}{x_h} > 1$$

which contradicts (12). A symmetric argument proves the second part. QED. ■

C Integration Properties of the Data

Table C.1 reports bootstrapped p-values[3] for Elliot, Rothenberg, and Stock (1996) unit root tests for either the levels or the logarithms of M1 velocity and the short rate, and for the logarithms of nominal M1 and nominal GDP,[4] and Table C.2 reports the corresponding set of results for either the first differences or the log-differences of the series. For the logarithms of nominal GDP and nominal M1, which exhibit obvious trends, tests are based on models including an intercept and a time trend.[5] For (the logarithms of) the short rate and velocity, on the other hand, they are based

[3] For any of the series, p-values have been computed by bootstrapping 10,000 times estimated ARIMA(p,1,0) processes. In all cases, the bootstrapped processes are of a length equal to the series under investigation. As for the lag order, p, since, as it is well known, results from unit root tests may be sensitive to the specific lag order which is being used, for reasons of robustness we consider two alternative lag orders, either 1 or 2 years.

[4] The reason for not considering tests based on the levels of nominal M1 and nominal GDP is that either series' level is manifestly characterized by exponential-type growth. This would not be a problem if Elliot *et al.*'s tests allowed for the alternative of stationarity around an *exponential* trend rather than a linear one. Since this is not the case, for both GDP and M1 we are compelled to only consider tests based on the logarithms.

[5] The reason for including a time trend is that, as discussed, for example, by Hamilton (1994, pp. 501), the model used for unit root tests should be a meaningful one also under the alternative.

on models including an intercept but no time trend. For the short rate, the rationale for not including a trend is obvious: the notion that nominal interest rates may follow an upward path,[6] in which they grow over time, is manifestly absurd.[7] For velocity, on the other hand, things are at first sight less obvious. The reason for not including a trend has to do with the fact that we are focusing here on a demand for money *for transaction purposes* (so this argument holds for M1, but it would not hold for broader aggregates). The resulting natural assumption of unitary income elasticity logically implies that, if the demand for M1 is stable, M1 velocity should inherit the stochastic properties of the opportunity cost of money. In turn, this implies that the type of tests we run for velocity should be *the same* as those for the nominal rate.

The evidence in the two tables can be summarized as follows.

First, there is overwhelming evidence of unit roots in any of the series, with the bootstrapped p-values being near-uniformly greater than the 10% threshold which, throughout the entire paper, we take as our benchmark significance level and in most cases markedly so.[8] The handful of cases in which the null of a unit root is rejected based on either lag order has been highlighted, in Table C.1, in yellow.

Second, for both the first difference and the log-difference of either velocity or the short rate, the null of a unit root can be rejected almost uniformly, with the very few cases in which this is not the case—so that the relevant series should be regarded, according to Elliot *et al.*'s (1996) tests, as I(2)—having been highlighted in yellow in Table C.2. Accordingly, for these cases we will not run cointegration tests. As for nominal M1 and especially nominal GDP, on the other hand, the opposite is true, with the null of a unit root not being rejected most of the time. In all of these cases, we will therefore eschew unrestricted specifications for the logarithms of nominal M1, nominal GDP, and a short rate.

D Details of the Bootstrapping Procedures

As for the Johansen test, we bootstrap trace and maximum eigenvalue statistics via the procedure proposed by Cavaliere *et al.* (2012; henceforth, CRT). In a nutshell, CRT's procedure is based on the notion of computing critical and p-values by bootstrapping the model that is relevant under the null hypothesis. This means that for

[6] The possibility of a downward path is ruled out by the zero lower bound.

[7] This does *not* rule out the possibility that, over specific sample periods in which inflation exhibits permanent variation (such as post-WWII samples dominated by the Great Inflation episode), nominal interest rates are I(1), too. Rather, by the Fisher effect, we *should* expect this to be the case. Historically, however, a unit root in inflation has been the exception rather than the rule; see Benati (2008).

[8] In a few cases, results based on the two alternative lag orders we consider produce contrasting evidence. This is the case, for example, for the logarithm of nominal GDP for Austria, the Barbados islands, Hong Kong, Canada (1967-2017), Israel, and South Korea. In these cases, we regard the null of a unit root as not having been convincingly rejected, and in what follows we therefore proceed under the assumption that these series are I(1).

tests of the null of no cointegration against the alternative of one or more cointegrating vectors, the model that is being bootstrapped is a simple, noncointegrated VAR in differences. For the maximum eigenvalue tests of h versus h+1 cointegrating vectors, on the other hand, the model that ought to be bootstrapped is the VECM estimated under the null of h cointegrating vectors. All of the technical details can be found in CRT, to which the reader is referred. We select the VAR lag order as the maximum[9] between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria[10] for the VAR in levels.

As for the Wright (2000) test, since the test has been designed to be equally valid for data-generating processes (DGPs) featuring either exact or near unit roots, we consider two alternative bootstrapping procedures, corresponding to either of the two possible cases. (In practice, as a comparison between the results reported in Table 2 in the text and in Table E.1 in Appendix E makes clear, the two procedures produce nearly identical results.) The former procedure involves bootstrapping—as detailed in CRT and briefly recounted in the previous paragraph—the cointegrated VECM estimated (based on Johansen's procedure) under the null of one cointegration vector. This bootstrapping procedure is the correct one if the data feature exact unit roots. For the alternative possible case in which velocity and the short rate are near unit root processes, we proceed as follows. Based on the just-mentioned cointegrated VECM estimated under the null of one cointegration vector, we compute the implied VAR in levels. By construction, this VAR has one—and only one—eigenvalue equal to 1. Bootstrapping this VAR would obviously be exactly equivalent to bootstrapping the underlying cointegrated VECM, that is, it would be the correct thing to do if the data featured exact unit roots. Since, on the other hand, here we want to bootstrap under the null of a near unit root cointegrated DGP, we turn such exact unit root VAR in levels into its near unit root correspondent, by "shrinking down" the single unitary eigenvalue to λ=1-0.5$\cdot$(1/T), where T is the sample length. In particular, we do that via a small perturbation of the parameters of the VAR matrices B_j's in the cointegrated VECM representation $\Delta Y_t = A + B_1 \Delta Y_{t-1} + ... + B_p \Delta Y_{t-p} + GY_{t-1} + u_t$, where Y_t collects (the logarithms of) M1 velocity and the short rate, and the rest of the notation is obvious. By only perturbating the elements of the VAR matrices B_j's—leaving unchanged the elements of the matrix G (and therefore both the cointegration vector and the loading coefficients)—we make sure that both the long-run equilibrium relationship between velocity and the short rate, and the way in which disequilibria in such relationship map into subsequent adjustments in the two series, remain unchanged. The bootstrapping procedure we implement for the second

[9] We consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model misspecification) is more serious than the one resulting from choosing a lag order greater than the true one (overfitting).

[10] On the other hand, we do not consider the Akaike Information Criterion since, as discussed by Luetkepohl (1991), for example, for systems featuring I(1) series, the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

possible case in which the processes feature near unit roots is based on bootstrapping such near unit root VAR.

We now turn to discussing Monte Carlo evidence on the performance of the two bootstrapping procedures.

D.1 Monte Carlo evidence on the performance of the two bootstrapping procedures

D.1.1 Evidence for Johansen's test of the null of no cointegration

Table D.1 in this appendix reports Monte Carlo evidence on the performance of the bootstrapping procedure for Johansen's trace tests[11] proposed by CRT.[12] We perform the Monte Carlo simulations based on two types of DGP, featuring *no cointegration* and *cointegration*, respectively. As for the DGP featuring *no cointegration*, we simply consider two independent random walks. As for the one featuring *cointegration*, we consider the following bivariate process:

$$y_t = y_{t-1} + \epsilon_t, \text{ with } \epsilon_t \sim i.i.d.\ N(0,1) \tag{D.1}$$

$$x_t = y_t + u_t \tag{D.2}$$

$$u_t = \rho u_{t-1} + v_t, \text{ with } 0 \leq \rho < 1,\ v_t \sim i.i.d.\ N(0,1). \tag{D.3}$$

As for ρ, we consider six possible values, corresponding to alternative ranges of persistence of the cointegration residual between the three series, that is, $\rho = 0$, 0.25, 0.5, 0.75, 0.9, and 0.95. There are two reasons for using this specific DGP. *First*, it captures the essence of the problem at hand. Here we have two I(1) series—M1 velocity and a short rate—whose long-run dynamics might obey a cointegration relationship. *Second*, by parameterizing the extent of persistence of the deviation from the long-run equilibrium relationship, we can effectively explore how the performance of the test depends on such persistence, even in very large samples. This is key because, as we document in Online Appendix H, real-world ("candidate") cointegration residuals are indeed very highly persistent. Intuitively, for the reasons discussed by Engle and Granger (1987), we would expect that, *ceteris paribus*, the higher the persistence of the cointegration residual, the more difficult it is for any statistical test to detect cointegration. As we will see, this is indeed the case.

[11] Numerically near-identical evidence for Johansen's maximum eigenvalue tests is not reported for reasons of space, but it is available upon request.

[12] Extensive Monte Carlo evidence on the good performance of the CRT procedure was already provided by CRT themselves in their original paper. Benati (2015) also provided some (much more limited) evidence conditional on the specific DGPs he was interested in. The rationale for providing additional evidence here is the same as Benati (2015), that is, looking at how the procedure performs conditional on the DGPs we are interested in here.

Details of the Monte Carlo simulations are as follows. For either DGP, we consider five alternative sample lengths, $T = 50$, 100, 200, 500, and 1,000. For each combination of values of ρ and T, we generate 5,000 artificial samples of length T+100, and we then discard the first 100 observations in order to eliminate dependence on initial conditions (which we set to 0 for either series). For each individual simulation, we bootstrap the relevant test statistic based on 2,000 bootstrap replications.

Table D.1 reports the evidence for Johansen's trace test of the null of no cointegration against the alternative of one or more cointegration vectors. Specifically, the table reports, for either DGP, the sample length and (for the DGP featuring cointegration) the value of ρ; and the fraction of replications for which no cointegration is rejected at the 10% level. The following main findings clearly emerge from the table.

First, in line with the evidence reported by both CRT and Benati (2015), the procedure performs remarkably well conditional on DGPs featuring *no cointegration*. A key point that ought to be stressed is that the specific sample length used in the simulations does not appear to make any material difference for the final results, with the fractions of rejections ranging between 0.098 and 0.119 (with the ideal one being 0.1). This is testimony to the power of bootstrapping, which is capable of automatically controlling for the specific characteristics of the DGP under investigation.

Second, when the DGP does feature *cointegration*, ideally we would like the test to reject as much as possible. As the lower part of the table shows, the procedure indeed performs very well if ρ is small. If $\rho = 0$, for example, cointegration is already detected 100% of the time for $T = 100$, whereas if $\rho = 0.5$, it is detected 88.2% of the time for $T = 100$, and a sample length of $T = 200$ is already sufficient to detect cointegration 100% of the time. As ρ increases, however, the performance deteriorates. The intuition for this is straightforward: as the cointegration residual becomes more and more persistent, it gets closer and closer to a random walk (in which case there would be no cointegration), and the procedure therefore needs larger and larger samples to detect the truth (that the residual is highly persistent but ultimately stationary). In particular, as ρ increases, the fraction of rejections tends to converge, for each sample size, to the fraction of rejections under the DGP featuring no cointegration. This is especially apparent for $T = 50$ or 100, with the fractions being equal to 0.114 and 0.120, respectively. In the limit, for $\rho \rightarrow 1$, the procedure will tend to reject 10% of the time.

Comparison with the Monte Carlo evidence of Cavaliere *et al.* (2012) This evidence is qualitatively and also quantitatively in line with the Monte Carlo evidence reported in CRT's Tables I and II, pp. 1731-1732. Although the DGPs they used (either noncointegrated VARs or cointegrated VECMs featuring one cointegration vector) were different from the DGPs used herein, their results and ours turn out to be very close. Specifically, the results are as follows:

- The results in panel (b) of their Table I illustrate the excellent performance of their bootstrapping procedure for tests of the null of no cointegration when the true DGP features no cointegration. In line with the evidence reported in the first row of our Table E.1, their results illustrate how, at the 5% level, the empirical rejection frequencies (henceforth, ERF) are quite close to 5% irrespective of the sample size.

- Panel (a) in the same table reports qualitatively and quantitatively similar evidence for the maximum eigenvalue test of 1 versus 2 cointegrating vectors, conditional on DGPs featuring one cointegrating vector.

- Finally, CRT's Table II reports evidence on the ability of the sequential bootstrapped procedure to select the correct cointegration rank, which in their experiments is one (see the columns under the heading "Bootstrap (CRT)"). Those results are in line with the ones reported in our Table 1 in the main text conditional on DGPs featuring one cointegration vector. In either case, the larger the sample size, the more frequently CRT's procedure detects the truth, with ERFs converging toward 1 for sufficiently large samples. In comparatively small samples (e.g., for $T = 50$), ERFs are typically much below one—as we show, the more so, the more persistent is the cointegration residual.[13]

The bottom line is that our Monte Carlo evidence, although based on a set of DGPs that have been specifically tailored to the problem at hand, is in fact exactly in line, both qualitatively and quantitatively, with the evidence reported in CRT.

Summing up The proceeding can be summarized as follows:

- If the true DGP features *no cointegration*, CRT's procedure performs remarkably well irrespective of the sample size.

- If, however, the true DGP features *cointegration*, Johansen's tests—even bootstrapped as in CRT—perform well only if the persistence of the cointegration residual is sufficiently low and/or the sample size is sufficiently large. If, on the other hand, the cointegration residual is persistent and the sample size is small, the procedure will fail to detect cointegration a nonnegligible fraction of the time. For example, with $T = 100$, cointegration will be detected 43.3% of the time if $\rho = 0.75$ and just 12.0% of the time if $\rho = 0.95$.

[13] Different from the present work, CRT do not explore how the persistence of the cointegration residual affects the performance of their procedure. The results reported in their Table II, however, are quantitatively in line with ours. We found this in the following way. We simulated their VECM conditional on one cointegration vector 10,000 times for samples of length $T = 10{,}000$, and for each simulation we computed the implied cointegration residual, and based on it we estimated an AR(4) process (in fact, given the nature of their DGP, an AR(2) would have been enough). The sum of the AR coefficients is our measure of persistence. For their benchmark case of δ=0.1, both the mean and the median of the distribution were equal to 0.61. From their Table II, we can see that for δ=0.1 and $T = 50$, the ERF is 49.0%. In Table 1 of the main text we report, for $T = 50$ and ρ=0.5, an ERF of 35%.

All of this means that if Johansen's tests do detect cointegration, we should have a reasonable presumption that cointegration is indeed there. If, on the other hand, they do not detect it, a possible explanation is that the sample period is too short and/or the cointegration residual is highly persistent.

D.1.2 Evidence for Wright's (2000) test of the null of cointegration

Table D.2 reports evidence for the two bootstrapping procedures we use for Wright's test. Specifically, the top portion of the table reports evidence for the case of exact unit roots, with the true DGP that is being simulated being given by (D.1)-(D.3), and the bootstrapping procedure being the first one discussed in the second paragraph of this appendix, that is, being based on bootstrapping the VECM estimated conditional on one cointegration vector. The second portion of Table D.2 reports the corresponding Monte Carlo evidence for the near unit root case. Here the DGP that is being simulated is the near unit root version of (D.1)-(D.3), that is, the DGP that is obtained when the random walk (D.1) is replaced with

$$y_t = \lambda y_{t-1} + \epsilon_t, \text{ with } \epsilon_t \sim i.i.d.\ N(0,1) \tag{D.1}$$

with λ=1-0.5$\cdot(1/T)$, where T is the sample length. For each single bootstrapped replication, the test statistics are bootstrapped based on the second procedure discussed in the second paragraph of this appendix, that is, by bootstrapping the near unit root VAR in levels obtained by "shrinking down" to λ=1-0.5$\cdot(1/T)$ the unitary eigenvalue of the VAR in levels implied by the VECM estimated conditional on one cointegration vector.

All of the details of the Monte Carlo simulations are exactly the same as for Johansen test (grids of values for ρ and T, number of Monte Carlo simulations, and number of bootstrap replications). As a comparison between the top and bottom portions of Table D.2 shows, evidence for the two cases of exact and near unit roots is virtually identical and can be summarized as follows:[14]

First, if the true DGP features *cointegration*, the procedure works remarkably well if the sample size is sufficiently long, the persistence of the cointegration residual is sufficiently low, or both. For example, for T = 1,000, the empirical rejection frequencies (ERFs) at the 10% level range between 0.103 and 0.116, very close to the ideal of 0.1. As the sample size decreases, however, the ERFs systematically increase. For T = 200, for example, the ERF is still equal to 0.105 for $\rho = 0.75$, but for $\rho =$ 0.95 it becomes equal to 0.127. For T = 50 the ERF is still reasonably close to 0.1 for $\rho = 0.5$, but for $\rho = 0.9$ it is already equal to 0.181, and it further increases to 0.206 for $\rho = 0.95$.

Second, if the true DGP features *no cointegration* (i.e., two independent random walks), the ERFs range between 0.200 and 0.227 depending on sample size.

[14] In what follows, all of the numbers mentioned pertain to the case of exact unit roots.

Summing up The proceeding can be summarized as follows:

- If the true DGP features *cointegration*, in the case of either exact or near unit roots, the respective bootstrapping procedures perform remarkably well if the sample size is sufficiently long, the persistence of the cointegration residual is sufficiently low, or both. However, if the sample is sufficiently short *and* the cointegration residual is sufficiently persistent, the null of cointegration will be incorrectly rejected, in the worst possible scenario analyzed in the Monte Carlo experiment ($T = 50$ and $\rho = 0.95$) at about twice the nominal size. The explanation for this is straightforward, and it is, in fact, in line with the previously mentioned point made by Engle and Granger (1987): when the cointegration residual is highly persistent, only sufficiently long samples allow the test to detect the truth, that is, that the deviation between the two series is ultimately transitory, so that they are in fact cointegrated. On the other hand, under these circumstances the shorter the sample period, the more likely it will be to mistakenly infer that the deviation between the two series is, in fact, permanent, so that they are not, in fact, cointegrated.

- If, on the other hand, the true DGP features *no cointegration*, in the case of either exact or near unit roots, the test will reject the null at roughly twice the nominal size.

A key implication is that, in fact, lack of rejection of the null of no cointegration does not represent very strong evidence that cointegration truly is in the data. Since in the case of two independent random walks (or their near unit root correspondent) the null of cointegration is rejected about one time out of five irrespective of sample length, an alternative interpretation is simply that the data do not feature cointegration, but the test is not capable of detecting this.

E Additional Results from Wright's (2000) Test for M1 Velocity and the Short Rate

Table E.1 in this appendix reports results from Wright's (2000) test based on the second bootstrapping procedure previously discussed in Appendix D, that is, based on bootstrapping the near unit root VAR, which has been obtained by perturbating the coefficients of the AR matrices of the cointegrated VECM produced by Johansen's procedure (estimated conditional on one cointegration vector) in such a way that the unitary eigenvalue is "shrunk down" to its near unit root equivalent 1-0.5$\cdot(1/T)$, where T is the sample length. Evidence is nearly identical to that reported in Table 2 in the main text, and it points towards the presence of cointegration across the board between (the logarithms of) M1 velocity and the short rate.

F Unrestricted Tests of the Null of No Cointegration

Table F.1 in this appendix reports results from Johansen's cointegration test based on unrestricted specifications featuring the logarithms of M1, nominal GDP, and a short rate. On the other hand, we do not perform the corresponding set of tests based on the *levels* of the three series, since two of them—M1 and nominal GDP—exhibit obvious exponential (that is, nonlinear) trends, which makes linear cointegration tests, such as Johansen's, Shin's (1994), or Wright's (2000), pointless within the present context. Out of 18 samples, the tests detect cointegration only in 5 cases. Results for the United Kingdom, the United States, Argentina, Canada, Switzerland, Bolivia, Belize, Australia, Spain, and South Africa are in line with those based on the corresponding restricted specification for the logarithms of velocity and the short rate in Table 1 in the main text. On the other hand, for Colombia, New Zealand, and Belgium, where the tests based on velocity and the short rate did detect cointegration, the corresponding unrestricted tests in Table F.1 do not reject the null of no cointegration. A possible and likely explanation for this contrast is that failure to impose the (true) restriction of unitary income elasticity decreases the power of the test, leading it to incorrectly not reject the null. For Israel, Portugal, and Norway, relaxation of the constraint of unitary income elasticity leads the unrestricted test to detect cointegration, in contrast to the results from the unrestricted test in Table 1. In light of the evidence of a remarkably strong correlation between the logarithms of M1 velocity and the short rate for Israel in Figure 5 of the main text, and of the corresponding lack of any correlation between the levels of the two series, we regard the results for this country in either table as a fluke, likely due to the short sample length.

G Testing for Stability in Cointegration Relationships

In Section 6.2 in the main text of the paper we test for either breaks or, more generally, time variation in cointegration relationships based on the three tests discussed by Hansen and Johansen (1999): Two Nyblom-type tests for stability in the cointegration vector and the vector of loading coefficients, respectively; and a fluctuation test, which is essentially a joint test for time-variation in the cointegration vector and the loadings.

G.1 Monte Carlo evidence on the performance of Hansen and Johansen's (1999) tests

Table G.1 in this appendix reports Monte Carlo evidence on the performance of the three tests conditional on bivariate cointegrated DGPs featuring no time variation of any kind, for alternative sample lengths, and alternative degrees of persistence of the cointegration residual, which is modelled as an AR(1). Specifically, the DGP is given by

$$y_t = y_{t-1} + \epsilon_t, \text{ with } \epsilon_t \sim i.i.d.\ N(0,1) \tag{G.1}$$

$$x_t = y_t + u_t \tag{G.2}$$

with the cointegration residual being

$$u_t = \rho u_{t-1} + v_t, \text{ with } 0 \leq \rho < 1,\ v_t \sim i.i.d.\ N(0,1). \tag{G.3}$$

We consider $T = 50$, 100, 200, and $\rho = 0$, 0.25, 0.5, 0.75, 0.9, 0.95, and for each combination of T and ρ, we stochastically simulate the DGP based on each simulation, we perform either of the three tests on $[y_t\ x_t]'$, bootstrapping them as in Cavaliere *et al.* (2012) conditional on one cointegration vector (i.e., the true number of cointegration vectors in the DGP), based on 1,000 bootstrap replications. In performing of the three tests, we set the "trimming parameter" to the standard value in the literature of 0.15. For each combination of T and ρ, we perform 1,000 Monte Carlo simulations. The table reports, for each combination of T and ρ, the fraction of Monte Carlo simulations for which stability is rejected at the 10% level.

The main results in the table can be summarized as follows. The two Nyblom-type tests exhibit an overall reasonable performance, incorrectly rejecting the null of no time variation, most of the time, at roughly the nominal size. Crucially, this is the case irrespective of the sample length and of the persistence of the cointegration residual. The fluctuation test, on the other hand, exhibits a good performance only if the persistence of the cointegration residual is low. The higher the residual's persistence, however, the worse the performance, so that, for example, when the AR root of the residual is equal to 0.95, for a sample length $T = 50$, the test rejects at roughly twice the nominal size. This is clearly problematic since—as we mentioned in the main text, and we discuss more in detail in Appendix H —cointegration residuals are typically moderately to highly persistent. Because of this, in the main text we therefore focus on the results from the two Nyblom-type tests, whereas we eschew results from fluctuations tests (these results are, however, reported in table G.4).

G.2 Evidence on the stability of cointegration relationships

Table G.2 in this appendix reports the results from Hansen and Johansen's (1999) Nyblom-type tests for breaks in either the cointegration vector or the loading coefficients in the cointegrated VECMs estimated (based on Johansen's estimator) conditional on one cointegration vector. Specifically, the table reports p-values for testing stability in either feature of the cointegrated VECM, which have been bootstrapped as in Cavaliere *et al.* (2012), that is, based on the model estimated conditional on one cointegration vector. The main result emerging from the table is that evidence of breaks in either feature is weak to nonexistent. In particular, focusing on the cointegration vector—which, for the purpose of addressing the question of whether there is a stable long-run relationship between velocity and the short rate is clearly the key feature—at the 10% level we detect a break in four cases (Canada, Thailand, Turkey, and South Africa) based on the Selden-Latané specification, and in *just two* cases (Belgium and Finland) based on the log-log. Evidence of breaks in the vector of loading coefficients is slightly stronger—seven instances based on Selden-Latané and three based on log-log—but breaks in this feature bear *no implication* for the presence of a stable long-run relationship between the two series, as they uniquely hinge upon the way the system converges toward the long-run equilibrium.

Table G.3 reports, for the handful of cases in which a break in either feature has been detected, the estimated break dates, together with the values which the model feature which has been subject to breaks—either the cointegration vector or the loading coefficients—has taken in the two subsamples.

Finally, Table G.4 reports p-values (again, bootstrapped as in Cavaliere *et al.* (2012) based on the VECM estimated conditional on one cointegration vector) for Hansen and Johansen's (1999) fluctuations tests based on the cointegrated VECM. Results are qualitatively in line with those for the two just-discussed Nyblom-type tests for breaks in either the cointegration vector or the loading coefficients, with evidence of time variation identified only in a handful of cases.

H Evidence on the Persistence of 'Candidate' Cointegration Residuals

Tables H.1 and H.2 in this appendix report Hansen's (1999) "grid bootstrap" median-unbiased (henceforth, MUB) estimates of the sum of the AR coefficients in AR(2) representations for the "candidate cointegration residuals" in our dataset.[15] By "candidate cointegration residual" (henceforth, CCR), we mean the linear combination of the I(1) variables in the system which will indeed be regarded as a cointegration residual *if* cointegration is detected.[16] For reasons of robustness, for either the Selden-Latané specification (Table H.1) or the log-log (Table H.2), we consider two

[15] Results are based on 2,000 bootstrap replications for each possible value of the sum of the AR coefficients in the grid. Bootstrapping has been performed as in Diebold and Chen (1996). For reasons of robustness, we report results based on two alternative estimators of the cointegration vector, Johansen's, and Stock and Watson's (1993).

[16] We label it as the candidate cointegration residual because, as the Monte Carlo evidence in the previous section has shown, if a residual is highly persistent, cointegration might well *not* be detected *even if it is present*, which would prevent the candidate from being identified as a true cointegration residual.

alternative estimators of the cointegration residual, either Johansen's or Stock and Watson's (1993).

Evidence points toward both a nonnegligible degree of persistence of the CCRs and a wide degree of heterogeneity across countries. Focusing on results based on the log-log specification, the MUB estimate based on Johansen's estimator of the cointegration vector—let's label it as $\hat{\rho}^{J}_{MUB}$—ranges from a minimum of 0.27 for Belize to a maximum of 1.17 for the Barbados islands. By classifying the $\hat{\rho}^{J}_{MUB}$'s, in an admittedly arbitrary fashion, as "highly persistent" ($\hat{\rho}^{J}_{MUB} \geq 0.8$), "moderately persistent" ($0.4 < \hat{\rho}^{J}_{MUB} < 0.8$), and "not very persistent" ($\hat{\rho}^{J}_{MUB} \leq 0.4$), we end up with 22 $\hat{\rho}^{J}_{MUB}$'s in the first group, 14 in the second, and 4 in the third. Results based on Stock and Watson's estimator are qualitatively the same, with the three groups comprising respectively 25, 13, and 2 countries.

I Evidence on the Functional Form

Figures I.1 and I.2 provide simple, informal evidence on which specifications—Selden-Latané or log-log —provides the most plausible description of the data at low and, respectively, high interest rates. In both figures, the top row shows the levels of M1 velocity and the short rate, and the bottom row shows the logarithms of the two series. The evidence in the two rows therefore corresponds to a Selden-Latané and, respectively, a log-log specification for the demand for real M1 balances with unitary income elasticity, linearly relating either the level or the logarithm of M1 velocity to the level or, respectively, the logarithm of the short rate. Figure I.2 reports evidence for all of the countries that we classified as high-inflation countries, arranged in descending order according to the level of the interest rate. For reasons of space, Figure I.2 only reports evidence for 10 of the remaining countries. This is however without loss of generality, as this evidence is representative of the entire set of low-inflation countries (this evidence is available upon request).

Two broad patterns emerge from Figures I.1 and I.2. First, at low interest rates the Selden-Latané specification appears to provide a more plausible description of the data than the log-log, in the sense that, in several cases, the correlation between the levels of velocity and the short rate is manifestly stronger than that between the logarithms of the two series. This is especially clear for the United States, the United Kingdom, Switzerland, the Barbados islands, and, to a certain extent, Belize, whereas evidence for Japan is slightly weaker, and it crucially hinges on the period since the beginning of the new millennium. On the other hand, for countries such as Canada, South Korea, the Netherlands, and New Zealand, neither specification is manifestly superior, as the correlation between the levels of the series appears as equally strong as that between their logarithms. Another way of stating this is that, among low-inflation rate countries, in no single case does the log-log specification provide a manifestly better description of the data than the Selden-Latané specification, whereas in several cases the data clearly prefer the latter to the former.

Second, at high or very high interest rates, the opposite is true: in no single case is the Selden-Latané specification clearly preferred by the data, whereas in a few cases, the log-log specification provides a manifestly better description of the data. This is especially clear for Israel, Argentina, and Brazil, and to a lesser extent for Chile, Bolivia, and Ecuador, whereas for Turkey and Venezuela, the correlations between the levels and, respectively, the logarithms of the two series appear as equally strong or weak, depending on the period. Interestingly, the data's preference for the log-log specification appears stronger the higher the level of the interest rate (which is reported, in the top row, in the right-hand side scale in either panel in black), whereas it becomes progressively weaker for countries characterized by comparatively lower interest rates.

References

Baffigi, A. (2011): "Italian National Accounts: A Project of Banca d'Italia, ISTAT and University of Rome Tor Vergata," *Economic History Working Papers, Banca d'Italia, n. 18*.

BancadItalia (2013): "Tavole Storiche, Indicatori Monetari e Finanziari," Dicembre 2013.

Barciela-LÃşpez, C., A. Carreras, and X. Tafunell (2005): *EstadÃηsticas HistÃşricas de EspaÃśa: Siglos XIX-XX, Vol. 3*. Fundacion BBVA.

Barkai, H., and N. Liviatan (2007): *The Bank of Israel Volume 1: A Monetary History*. New York: Oxford University Press.

Benati, L. (2008): "Investigating Inflation Persistence Across Monetary Regimes," *Quarterly Journal of Economics*, 123(3), 1005–1060.

——— (2015): "The Long-Run Phillips Curve: A Structural VAR Investigation," *Journal of Monetary Economics*, 76, 15–28.

Bernanke, B. S., M. Gertler, and M. Watson (1997): "Systematic Monetary Policy and the Effects of Oil Price Shocks," *Brookings Papers on Economic Activity*, 1997(1), 91–157.

Boeschoten, W. (1992): *Hoofdlijnen van de economische geschiedenis van Nederland 1900-1990*. Amsterdam: NIBE.

deJong, A. (1967):.

Diebold, F. X., and C. Chen (1996): "Testing Structural Stability with Endogenous Breakpoint: A Size Comparison of Analytic and Bootstrap Procedures," *Journal of Econometrics*, 70(1), 221–241.

Elliot, G., T. J. Rothenberg, and J. H. Stock (1996): "Efficient Tests for an Autoregressive Unit Root," *Econometrica*, 64(4), 813–836.

Fratianni, M., and F. Spinelli (1997): *A Monetary History of Italy*. Cambridge: Cambridge University Press.

Furlong, K. (2001): "The Montreal Gazette Call Loan Rate, 1871-1907," *Canadian Journal of Economics*, 34(1), 165–173.

Haavisto, T. (1992): *Money and Economic Activity in Finland, 1866-1985*. Lund Economic Studies.

Hansen, H., and S. Johansen (1999): “Some Tests for Parameter Constancy in Cointegrated VAR Models,” *Econometrics Journal*, 2(2), 306–333.

Hills, S., and R. T. N. Dimsdale (2010): “The UK Recession in Context: What Do Three Centuries of Data Tell Us?,” *Bank of England Quarterly Bulletin*, (2010 Q4), 277–291.

Homer, S., and R. Sylla (2005): *A History of Interest Rates*. New York: John Wiley.

Judson, R. (2017): “The Death of Cash? Not So Fast: Demand for U.S. Currency at Home and Abroad, 1990-2016,” Federal Reserve Board, Division of International Finance Working Paper, forthcoming.

Junguito, R., and H. RincÃşn (2007): “La polÃŋtica fiscal en el siglo XX en Colombia,” *in J. Robinson and M. Urrutia (eds.), EconomÃŋa Colombiana del Siglo XX: Un AnÃąlisis Cuantitativo, Banco de la RepÃžblica y Fondo de Cultura EconÃşmica, Colombia*, pp. –.

Klovland, J. T. (2004): “Monetary aggregates in Norway 1819-2003,” *in ÃŸyvind Eitrheim, Jan T. Klovland and Jan F. Qvigstad, eds., Historical Monetary Statistics for Norway 1819ÂŰ2003, Norges Bank, Occasional Paper N. 35/2004.*

Luetkepohl, H. (1991): *Introduction to Multiple Time Series Analysis, 2nd ed.* Berlin: Springer-Verlag.

Mata, E., and N. Valerio (2011): *The Concise Economic History of Portugal: A Comprehensive Guide*. Coimbra: Almedina.

Metcalf, C., A. Redish, and R. Shearer (1996): “New Estimates of the Canadian Money Stock: 1871-1967,” *University of British Columbia Discussion Paper No. 96-17*, July 1996.

Mitchell, B. R. (2007): *International Historical Statistics: Africa, Asia, and Oceania, 1750-2000*. New York: Palgrave Macmillan.

Muscatelli, A., and F. Spinelli (2000): “The Long-Run Stability of the Demand for Money: Italy 1861-1996,” *Journal of Monetary Economics*, 45(3), 717–739.

Officer, L. H., and S. H. Williamson (2015): “The Annual Consumer Price Index for the United States, 1774-2014,” *available at: http://www.measuringworth.com/uscpi/.*

SaintMarc, M. (1983): *Histoire Monetarie de la France, 1880-1980*. Paris, Presses Universitaires de la France.

Shearer, R. A., and C. Clark (1984): "Canada and the Interwar Gold Standard, 1920-35: Monetary Policy without a Central Bank," *in M. D. Bordo and A. J. Schwartz, eds., A Retrospective on the Classical Gold Standard, 1821-1931, University of Chicago Press*, pp. 277–310.

Smits, J.-P., P. Woltjer, and D. Ma (2009): "A Dataset on Comparative Historical National Accounts, ca. 1870-1950: A Time-Series Perspective," .

Tamaki, N. (1995): *Japanese Banking: A History 1859-1959*. Cambridge: Cambridge University Press.

Table C.1 Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Logarithm of:								*Level of:*			
	nominal GDP		nominal M1		M1 velocity		short rate		M1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Argentina, 1914-2009	0.227	0.903	0.201	0.847	0.686	0.746	0.316	0.332	0.562	0.724	0.012	0.009
Australia												
1941-1989	0.771	0.607	0.622	0.978	0.931	0.854	0.784	0.872	0.947	0.932	0.915	0.984
1969-2017	0.100	0.465	0.955	0.982	0.856	0.884	0.905	0.966	0.840	0.872	0.596	0.815
Austria, 1970-1998	0.024	0.128	0.125	0.685	0.970	0.977	0.509	0.351	0.961	0.968	0.361	0.198
Bahrain, 1980-2017	0.525	0.466	0.368	0.059	0.652	0.769	0.448	0.365	0.544	0.689	0.335	0.216
Barbados, 1975-2016	0.060	0.114	0.572	0.317	0.802	0.748	0.992	0.968	0.552	0.528	0.418	0.428
Belgium, 1946-1990	0.861	0.885	0.211	0.236	0.036	0.166	0.130	0.670	0.497	0.733	0.414	0.664
Belize, 1977-2017	0.234	0.580	0.302	0.138	0.942	0.943	0.359	0.956	0.876	0.904	0.811	0.796
Bolivia, 1980-2013	0.090	0.067	0.114	0.062	0.915	0.849	0.866	0.837	0.627	0.674	0.139	0.188
Brazil, 1934-2014	0.304	0.581	0.258	0.559	0.747	0.744	0.367[b]	0.340[b]	0.554	0.534	0.007[b]	0.054[b]
Canada												
1926-2006	0.132	0.148	0.340	0.144	0.792	0.793	0.607	0.730	0.790	0.806	0.384	0.479
1967-2017	0.087	0.330	0.022	0.032	0.971	0.969	0.792	0.824	0.854	0.837	0.617	0.681
Chile												
1940-1995	0.399	0.544	0.374	0.261	0.134	0.050	0.341	0.263	0.212	0.124	0.133	0.090
1941-2017	0.922	0.913	0.906	0.692	0.450	0.303	0.205[b]	0.280[b]	0.302	0.127	0.047[b]	0.017[b]
Colombia, 1960-2017	0.107	0.974	0.090	0.942	0.244	0.263	0.827	0.905	0.162	0.199	0.718	0.822
Ecuador, 1980-2011	0.493	0.375	0.097	0.188	0.877	0.815	0.793	0.870	0.795	0.761	0.437	0.727
Finland, 1946-1985	0.288	0.100	0.057	0.071	0.776	0.594	0.541	0.521	0.914	0.897	0.512	0.511
France, 1852-1913	0.001	0.001	0.896	0.891	0.642	0.803	0.051	0.037	0.522	0.743	0.027	0.040
Guatemala, 1980-2017	0.950	0.965	0.992	0.993	0.672	0.585	0.596	0.508	0.622	0.520	0.573	0.516
Hong Kong, 1985-2017	0.023	0.113	0.649	0.805	0.925	0.942	0.622	0.596	0.716	0.849	0.495	0.481

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M1, and with an intercept and no time trend for the other series.
[b] For this period we consider inflation, rather than the short rate.

Table C.1 (continued) Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]												
	Logarithm of:								*Level of:*			
	nominal GDP		nominal M1		M1 velocity		short rate		M1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Japan, 1955-2017	0.144	0.623	0.090	0.314	0.945	0.936	0.697	0.748	0.749	0.757	0.578	0.566
Israel, 1983-2016	0.000	0.144	0.001	0.001	0.792	0.040	0.477	0.065	0.318	0.257	0.106	0.057
Italy, 1949-1996	0.794	0.889	0.993	0.945	0.333	0.648	0.857	0.899	0.234	0.643	0.805	0.848
Mexico, 1985-2014	0.013	0.021	0.066	0.016	0.767	0.100	0.629	0.289	0.679	0.027	0.346	0.023
Morocco, 1985-2017	0.119	0.193	0.400	0.654	0.319	0.267	0.806	0.717	0.059	0.048	0.761	0.595
Netherlands, 1950-1992	0.985	0.996	0.703	0.783	0.100	0.194	0.194	0.450	0.232	0.297	0.243	0.347
New Zealand, 1934-2017	0.960	0.981	0.479	0.527	0.825	0.814	0.665	0.651	0.815	0.798	0.351	0.337
Norway, 1946-2014	0.955	0.995	0.123	0.153	0.898	0.880	0.802	0.755	0.846	0.837	0.774	0.723
Paraguay, 1962-2015	0.719	0.920	0.733	0.706	0.426	0.447	0.032	0.067	0.342	0.394	0.125	0.249
Peru, 1959-2017	0.767	0.857	0.738	0.794	0.599	0.427	0.488	0.564	0.600	0.419	0.112	0.116
Portugal, 1914-1998	0.634	0.614	0.209	0.145	0.594	0.407	0.716	0.714	0.607	0.430	0.596	0.469
South Africa, 1965-2015	0.995	0.995	0.751	0.839	0.918	0.927	0.289	0.484	0.875	0.882	0.283	0.332
South Korea, 1970-2017	0.080	0.245	0.101	0.417	0.664	0.610	0.643	0.745	0.384	0.290	0.061	0.258
Spain, 1941-1989	0.632	0.504	0.154	0.505	0.187	0.440	0.828	0.878	0.363	0.512	0.589	0.720
Switzerland, 1948-2005	0.949	0.930	0.498	0.712	0.425	0.359	0.156	0.177	0.453	0.417	0.186	0.120
Taiwan, 1962-2017	0.309	0.788	0.141	0.574	0.314	0.264	0.662	0.713	0.057	0.034	0.408	0.513
Thailand, 1979-2016	0.291	0.867	0.944	0.936	0.907	0.916	0.619	0.523	0.890	0.898	0.589	0.418
Turkey, 1968-2017	0.856	0.827	0.879	0.903	0.735	0.767	0.644	0.668	0.653	0.673	0.727	0.770
United Kingdom, 1922-2016	0.140	0.805	0.080	0.391	0.831	0.746	0.926	0.942	0.779	0.728	0.345	0.575
United States, 1915-2017												
M1	0.702	0.385	0.482	0.158	0.626	0.783	0.443	0.248	0.578	0.697	0.302	0.319
M1 + MMDAs	0.699	0.380	0.488	0.159	0.833	0.811	0.443	0.251	0.713	0.702	0.293	0.326
Venezuela, 1962-1999	0.521	0.752	0.738	0.817	0.574	0.729	0.744	0.730	0.543	0.786	0.691	0.706
West Germany, 1960-1989	0.844	0.963	0.662	0.840	0.752	0.739	0.067	0.137	0.721	0.719	0.069	0.138

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M1, and with an intercept and no time trend for the other series.

Table C.2 Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Log-difference of:								*First difference of:*			
	nominal GDP		nominal M1		M1 velocity		short rate		M1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Argentina, 1914-2009	0.038	0.050	0.023	0.035	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
Australia												
1941-1989	0.046	0.061	0.004	0.009	0.011	0.024	0.000	0.004	0.003	0.023	0.000	0.032
1969-2017	0.245	0.415	0.002	0.012	0.001	0.024	0.000	0.004	0.002	0.022	0.000	0.001
Austria, 1970-1998	0.350	0.227	0.111	0.258	0.071	0.098	0.023	0.067	0.041	0.084	0.025	0.057
Bahrain, 1980-2017	0.007	0.085	0.028	0.260	0.002	0.004	0.005	0.004	0.001	0.003	0.001	0.001
Barbados, 1975-2016	0.117	0.144	0.033	0.075	0.017	0.048	0.069	0.073	0.006	0.011	0.001	0.003
Belgium, 1946-1990	0.006	0.002	0.009	0.050	0.002	0.000	0.000	0.004	0.001	0.001	0.001	0.002
Belize, 1977-2017	0.006	0.033	0.011	0.031	0.008	0.023	0.074	0.055	0.004	0.007	0.003	0.007
Bolivia, 1980-2013	0.125	0.157	0.135	0.150	0.044	0.085	0.007	0.032	0.019	0.051	0.017	0.054
Brazil, 1934-2014	0.133	0.197	0.070	0.205	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
Canada												
1926-2006	0.005	0.003	0.012	0.019	0.002	0.011	0.000	0.000	0.003	0.018	0.000	0.000
1967-2017	0.132	0.379	0.007	0.006	0.020	0.031	0.000	0.001	0.023	0.020	0.000	0.001
Chile												
1940-1995	0.153	0.079	0.361	0.317	0.000	0.002	0.001	0.001	0.001	0.004	0.005	0.002
1941-2017	0.126	0.053	0.328	0.221	0.000	0.000	0.000[b]	0.000[b]	0.000	0.000	0.000[b]	0.000[b]
Colombia, 1960-2017	0.001	0.003	0.000	0.004	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
Ecuador, 1980-2011	0.016	0.111	0.043	0.061	0.022	0.020	0.007	0.076	0.016	0.019	0.003	0.036
Finland, 1946-1985	0.014	0.051	0.010	0.007	0.001	0.001	0.000	0.003	0.000	0.002	0.000	0.003
Guatemala, 1980-2017	0.053	0.120	0.011	0.053	0.005	0.020	0.001	0.042	0.005	0.014	0.001	0.081
Hong Kong, 1985-2017	0.251	0.282	0.023	0.095	0.041	0.156	0.010	0.015	0.045	0.133	0.002	0.005

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and no time trend.
[b] For this period we consider inflation, rather than the short rate.

Table C.2 (continued) Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Log-difference of:								*First difference of:*			
	nominal GDP		nominal M1		M1 velocity		short rate		M1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Japan, 1955-2017	0.475	0.716	0.136	0.342	0.009	0.046	0.000	0.002	0.004	0.011	0.000	0.000
Israel, 1983-2016	0.009	0.001	0.051	0.020	0.002	0.042	0.003	0.021	0.006	0.017	0.012	0.053
Italy, 1949-1996	0.205	0.565	0.152	0.394	0.002	0.031	0.002	0.120	0.001	0.057	0.000	0.031
Mexico, 1985-2014	0.239	0.002	0.100	0.129	0.009	0.029	0.009	0.036	0.013	0.019	0.006	0.009
Morocco, 1985-2017	0.019	0.279	0.094	0.207	0.027	0.329	0.020	0.083	0.013	0.216	0.022	0.097
Netherlands, 1950-1992	0.068	0.437	0.007	0.099	0.001	0.057	0.000	0.000	0.001	0.042	0.000	0.000
New Zealand, 1934-2017	0.001	0.014	0.000	0.032	0.004	0.022	0.000	0.000	0.002	0.004	0.000	0.000
Norway, 1946-2014	0.001	0.026	0.005	0.047	0.003	0.033	0.001	0.002	0.003	0.035	0.001	0.003
Paraguay, 1962-2015	0.101	0.225	0.029	0.117	0.002	0.005	0.000	0.000	0.001	0.003	0.000	0.000
Peru, 1959-2017	0.120	0.066	0.062	0.058	0.009	0.018	0.000	0.002	0.007	0.016	0.000	0.001
Portugal, 1914-1998	0.026	0.039	0.039	0.010	0.000	0.000	0.003	0.089	0.000	0.000	0.001	0.003
South Africa, 1965-2015	0.037	0.090	0.001	0.015	0.001	0.008	0.000	0.001	0.001	0.009	0.000	0.001
South Korea, 1970-2017	0.664	0.717	0.076	0.255	0.001	0.006	0.000	0.002	0.000	0.001	0.000	0.001
Spain, 1941-1989	0.001	0.006	0.011	0.011	0.011	0.051	0.000	0.000	0.027	0.095	0.000	0.002
Switzerland, 1948-2005	0.028	0.087	0.000	0.005	0.000	0.009	0.000	0.000	0.000	0.007	0.000	0.001
Taiwan, 1962-2017	0.221	0.553	0.018	0.026	0.001	0.002	0.000	0.000	0.001	0.000	0.000	0.000
Thailand, 1979-2016	0.131	0.164	0.007	0.059	0.006	0.028	0.003	0.005	0.007	0.032	0.002	0.003
Turkey, 1968-2017	0.462	0.562	0.122	0.405	0.001	0.006	0.007	0.116	0.001	0.005	0.002	0.055
United Kingdom, 1922-2016	0.008	0.060	0.007	0.020	0.001	0.006	0.000	0.001	0.000	0.004	0.000	0.000
United States, 1915-2017												
M1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M1 + MMDAs	0.000	0.000	0.001	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.000
Venezuela, 1962-1999	0.171	0.305	0.035	0.344	0.001	0.051	0.031	0.037	0.000	0.064	0.061	0.039
West Germany, 1960-1989	0.106	0.243	0.011	0.175	0.007	0.090	0.007	0.077	0.005	0.114	0.005	0.054

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and no time trend.

Table D.1 Monte Carlo evidence on the performance of Johansen's tests of the null of no cointegration, bootstrapped as in Cavaliere *et al.*'s (2012):[a] fractions of replications for which no cointegration is rejected[b] at the 10 per cent level

	Sample length:				
	$T = 50$	$T = 100$	$T = 200$	$T = 500$	$T = 1000$
	True data-generation process: no cointegration[c]				
	0.116	0.098	0.105	0.107	0.119
Persistence of the cointegration residual:	*True data-generation process: cointegration*				
$\rho = 0$	0.774	1.000	1.000	1.000	1.000
$\rho = 0.25$	0.584	0.993	1.000	1.000	1.000
$\rho = 0.5$	0.350	0.882	1.000	1.000	1.000
$\rho = 0.75$	0.184	0.433	0.937	1.000	1.000
$\rho = 0.9$	0.117	0.167	0.328	0.958	1.000
$\rho = 0.95$	0.114	0.120	0.164	0.533	0.966

[a] Based on the trace test of the null of no cointegration against the alternative of 1 or more cointegrating vectors. [b] Based on 5,000 Monte Carlo replications, and, for each of them, on 2,000 bootstrap replications. [c] Two independent random walks.

Table D.2 Monte Carlo evidence on the performance of Wright's tests of the null of cointegration:[a] fractions of replications for which cointegration is rejected at the 10 per cent level

Persistence of the cointegration residual:	Sample length: $T = 50$	$T = 100$	$T = 200$	$T = 500$	$T = 1000$
	True DGP: exact unit root processes Bootstrapped process: cointegrated VECM				
	True data-generation process: cointegration				
$\rho = 0$	0.113	0.115	0.103	0.098	0.107
$\rho = 0.5$	0.119	0.115	0.109	0.097	0.109
$\rho = 0.75$	0.143	0.115	0.105	0.102	0.103
$\rho = 0.9$	0.181	0.133	0.122	0.112	0.116
$\rho = 0.95$	0.206	0.167	0.127	0.120	0.103
	True data-generation process: no cointegration[b]				
	0.227	0.215	0.223	0.202	0.200
	True DGP: near unit root processes Bootstrapped process: near unit root VAR				
	True data-generation process: cointegration				
$\rho = 0$	0.125	0.109	0.087	0.084	0.093
$\rho = 0.5$	0.117	0.116	0.111	0.094	0.094
$\rho = 0.75$	0.146	0.121	0.111	0.099	0.091
$\rho = 0.9$	0.174	0.140	0.124	0.110	0.108
$\rho = 0.95$	0.209	0.170	0.142	0.125	0.105
	True data-generation process: no cointegration[b]				
	0.228	0.217	0.229	0.216	0.220

[a] Based on 5,000 Monte Carlo replications, and, for each of them, on 2,000 bootstrap replications. [b] Two independent random walks.

Table E.1 Additional results from Wright's (2000) tests: 90% coverage confidence intervals for the second element of the normalized cointegration vector (based on bootstrapping a near-unit root VAR)

Country	*Period*	*Selden-Latané*	*Log-log*
United Kingdom	1922-2016	[-0.537; -0.409]	NCD
U.S. - M1 + MMDAs	1915-2017	[-0.609; -0.397]	[-0.706; 0.171]
U.S. - M1	1915-2017	[-1.401; -0.837]	[-0.352; -0.108]
Argentina	1914-2009	[-0.111; -0.087]	[-0.537; -0.205]
Brazil	1934-2014	[-0.065; -0.009]	[-1.302; 0.216]
Canada	1926-2006	[-1.526; -1.021]	[-0.739; -0.579]
	1967-2017	[-0.586; -0.486]	[-0.417; -0.313]
Colombia	1960-2017	[-0.247; -0.183]	NCD
Guatemala	1980-2017	[-0.760; -0.440]	[-0.686; -0.398]
New Zealand	1934-2017	NCD	[-0.669; -0.196]
Switzerland	1948-2005	NCD	NCD
Bolivia	1980-2013	[-0.357; -0.201]	[-0.524; -0.384]
Israel	1983-2016	NCD	[-0.392; -0.316]
Mexico	1985-2014	[-0.264; -0.180]	[-0.426; -0.310]
Belgium	1946-1990	[-0.473; -0.277]	[-1.411; -0.702]
Belize	1977-2017	[-1.144; -0.388]	[-2.567; 1.433]
Austria	1970-1998	[-0.729; 0.208]	[-1.040; 0.618]
Bahrain	1980-2017	NCD	[-0.262; -0.186]
Barbados	1975-2016	[-2.115; -0.636]	[-2.899; 0.101]
Ecuador	1980-2011	NCD	NCD
Netherlands	1950-1992	NCD	[-0.495; -0.303]
South Korea	1970-2017	[-0.613; -0.525]	[-0.643; -0.334]
Thailand	1979-2016	[-0.449; -0.405]	[-0.514; -0.366]
Venezuela	1962-1999	[-0.031; -0.003]	[-0.301; 0.404]
Australia	1941-1989	[-0.695; -0.518]	[-0.848; -0.191]
	1969-2017	[-0.500; -0.388]	[-0.514; -0.302]
Chile	1940-1995	[-0.112; -0.060]	[-0.382; -0.278]
	1941-2017	[-0.110; 0.047]	[-0.235; 0.105]
Finland	1946-1985	[-0.558; -0.390]	[-2.557; -1.917]
Japan	1955-2017	[-0.544; -0.292]	[-0.537; -0.077]
Spain	1941-1989	[-0.175; -0.151]	[-0.384; -0.320]
Taiwan	1962-2017	[-0.453; -0.337]	[-0.465; -0.229]
Turkey	1968-2017	NCD	NCD
West Germany	1960-1989	[-0.991; 0.959]	[-0.581; 0.929]
Italy	1949-1996	[0.032; 0.208]	[0.123; 0.567]
Norway	1946-2014	[-1.001; 1.025]	[-0.255; 1.155]
Paraguay	1962-2015	[-0.360; 0.157]	[-0.236; 0.017]
Peru	1959-2017	[-0.038; 0.022]	[-0.533; 0.748]
Portugal	1914-1998	NCD	[-0.046; 0.246]
South Africa	1965-2015	NCD	[-0.096; 1.281]

NCD = No cointegration detected.

Table F.1 Bootstrapped *p*-values for Johansen's maximum eigenvalue tests[a] between the logarithms of M1, nominal GDP, and a short rate[b]

Country	*Period*	*Bootstrapped p value*
United Kingdom	1922-2016	0.841
U.S. - M1 + MMDAs	1915-2017	0.922
U.S. - M1	1915-2017	0.924
Argentina	1914-2009	0.004
Canada	1926-2006	0.775
Colombia	1960-2017	0.309
New Zealand	1934-2017	0.238
Switzerland	1948-2005	0.001
Bolivia	1980-2013	0.183
Israel	1983-2016	0.000
Belgium	1946-1990	0.124
Belize	1977-2017	0.020
Australia	1941-1989	0.356
Spain	1941-1989	0.556
Norway	1946-2014	0.637
Portugal	1914-1998	0.941
South Africa	1965-2015	0.362

[a] Tests of 0 versus 1 cointegration vectors.
[b] Based on 10,000 bootstrap replications.

Table G.1 Monte Carlo evidence on the performance of Hansen and Johansen's tests for time-variation in cointegrated VARs, bootstrapped as in Cavaliere *et al.*'s (2012):[a] fractions of replications for which stability is rejected[b] at the 10 per cent level

	Sample length:		
Persistence of the cointegration residual:	$T = 50$	$T = 100$	$T = 200$
	I: Nyblom test for stability in the cointegration vector		
$\rho = 0$	0.124	0.099	0.109
$\rho = 0.25$	0.140	0.106	0.115
$\rho = 0.5$	0.125	0.114	0.121
$\rho = 0.75$	0.107	0.144	0.127
$\rho = 0.9$	0.091	0.137	0.130
$\rho = 0.95$	0.102	0.119	0.146
	II: Nyblom test for stability in the loading coefficients		
$\rho = 0$	0.083	0.072	0.072
$\rho = 0.25$	0.088	0.087	0.070
$\rho = 0.5$	0.086	0.108	0.070
$\rho = 0.75$	0.093	0.097	0.099
$\rho = 0.9$	0.077	0.100	0.111
$\rho = 0.95$	0.080	0.098	0.109
	III: Fluctuation tests		
$\rho = 0$	0.105	0.119	0.117
$\rho = 0.25$	0.118	0.124	0.120
$\rho = 0.5$	0.135	0.139	0.131
$\rho = 0.75$	0.169	0.142	0.133
$\rho = 0.9$	0.206	0.166	0.156
$\rho = 0.95$	0.200	0.196	0.165

[a] Based on 1,000 Monte Carlo replications, and, for each of them, on 1,000 bootstrap replications.

Table G.2 Bootstrapped p-values[a] for testing stability in the cointegration relationship between (log) M1 velocity and (the log of) a short-term term rate

		Tests for stability in:			
		cointegration vector		*loading coefficients*	
Country	*Period*	*Selden-Latané*	*Log-log*	*Selden-Latané*	*Log-log*
United Kingdom	1922-2016	0.444	0.771	0.543	0.612
U.S. - M1 + MMDAs	1915-2017	0.811	0.348	0.250	0.592
Argentina	1914-2009	–	0.638	–	0.065
Brazil	1934-2014	–	0.506	–	0.822
Canada	1926-2006	0.048	0.489	0.187	0.463
	1967-2017	0.657	0.600	0.075	0.176
Colombia	1960-2017	0.599	0.359	0.968	0.913
Guatemala	1980-2017	0.239	0.194	0.912	–[b]
New Zealand	1934-2017	0.440	0.601	0.183	0.717
Switzerland	1948-2005	0.903	0.470	0.272	0.594
Bolivia	1980-2013	0.362	0.180	0.645	0.346
Israel	1983-2016	0.504	0.441	0.242	0.358
Mexico	1985-2014	0.107	0.350	0.883	–[b]
Belgium	1946-1990	0.632	0.049	0.611	0.225
Belize	1977-2017	0.668	0.999	0.185	0.380
Austria	1970-1998	–[b]	0.619	–[b]	–[b]
Bahrain	1980-2017	0.472	0.509	0.509	0.086
Barbados	1975-2016	0.138	–[b]	0.786	–[b]
Ecuador	1980-2011	–[b]	0.288	–[b]	–[b]
Netherlands	1950-1992	0.347	0.355	0.093	0.941
South Korea	1970-2017	0.491	0.714	0.934	0.853
Thailand	1979-2016	0.066	0.249	0.894	0.974
Venezuela	1962-1999	0.318	0.897	0.974	0.975
Australia	1941-1989	0.220	0.544	0.479	0.716
	1969-2017	0.747	0.781	0.815	0.877
Chile	1940-1995	0.430	0.301	0.090	0.166
	1941-2017	0.593	0.102	0.947	0.163
Finland	1946-1985	0.279	0.062	0.485	0.028
Japan	1955-2017	0.637	0.134	0.936	0.106
Spain	1941-1989	0.714	0.597	0.134	0.981
Taiwan	1962-2017	–	0.889	–	0.501
Turkey	1968-2017	0.073	–	0.004	–
West Germany	1960-1989	–	0.361	–	0.527
Italy	1949-1996	0.973	–	0.685	–
Norway	1946-2014	0.126	0.327	0.059	0.398
Paraguay	1962-2015	0.411	0.332	0.915	0.723
Peru	1959-2017	0.292	0.651	0.013	0.081
Portugal	1914-1998	0.745	0.800	0.017	0.909
South Africa	1965-2015	0.086	0.113	0.337	0.648

[a] Based on 10,000 bootstrap replications. [b] In these cases the test could not be run.

Table G.3 Estimated break dates for the cointegration relationship between (log) M1 velocity and (the log of) a short-term term rate[a]

Country	*Period*	Break date	$\hat{\beta}_1'$, $\hat{\beta}_2'$
		I: Tests for stability in the cointegration vector	
			Selden-Latané
Canada	1926-2006	1979	[1 -1.013]′, [1 -1.351]′
Thailand	1979-2016	1992	[1 -0.541]′, [1 -0.370]′
Turkey	1968-2017	1994	[1 -0.073]′, [1 -0.169]′
South Africa	1965-2015	1981	[1 -0.951]′, [1 -0.476]′
			Log-log
Belgium	1946-1990	1982	[1 -0.599]′, [1 -0.680]′
Finland	1946-1985		[1 -3.048]′, [1 -2.901]′
		II: Tests for stability in the loading coefficients	
		Break date	$\hat{\alpha}_1'$, $\hat{\alpha}_2'$
			Selden-Latané
Canada	1967-2017	1980	[-0.008; 1.306]′, [-0.160; 0.900]′
Netherlands	1950-1992	1975	[0.234; 0.647]′, [-0.056; 0.862]′
Chile	1940-1995	1973	[-0.009; -3.041]′, [0.014; 1.309]′
Turkey	1968-2017	2001	[0.007; -0.309]′, [-0.137; -0.612]′
Norway	1946-2014	1980	[-0.011; 0.032]′, [-0.017; -0.004]′
Peru	1959-2017	1988	[-0.002; -4.308]′, [-0.006; 3.353]′
Portugal	1914-1998	1983	[-0.001; 0.022]′, [0.000; -0.056]′
			Log-log
Argentina	1914-2009	1987	[-0.035; -0.076]′, [-0.082; 0.513]′
Bahrain	1980-2017	2005	[-0.699; -1.704]′, [-0.845; -2.059]′
Finland	1946-1985	1930	[-0.101; 0.096]′, [-0.125; 0.119]′
Peru	1959-2017	1988	[-0.010; -0.331]′, [-0.060; 0.248]′

[a] Based on 10,000 bootstrap replications.

Table G.4 Bootstrapped *p*-values[a] for fluctuations tests for the cointegrated VECM for for (log) M1 velocity and (the log of) a short-term rate[a]

Country	*Period*	*Selden-Latané*	*Log-log*
United Kingdom	1922-2016	0.518	0.255
U.S. - M1 + MMDAs	1915-2017	0.544	0.183
Argentina	1914-2009	–	0.702
Brazil	1934-2014	0.454	0.521
Canada	1926-2006	0.326	0.133
	1967-2017	0.175	0.171
Colombia	1960-2017	0.056	0.032
Guatemala	1980-2017	0.652	0.512
New Zealand	1934-2017	0.013	0.530
Switzerland	1948-2005	0.693	0.732
Bolivia	1980-2013	0.502	0.692
Israel	1983-2016	0.556	0.532
Mexico	1985-2014	0.437	0.733
Belgium	1946-1990	0.068	0.421
Belize	1977-2017	0.311	0.571
Austria	1970-1998	0.082	0.071
Bahrain	1980-2017	0.398	0.391
Barbados	1975-2016	0.121	0.192
Ecuador	1980-2011	0.367	0.515
Netherlands	1950-1992	0.011	0.039
South Korea	1970-2017	0.013	0.352
Thailand	1979-2016	0.885	0.491
Venezuela	1962-1999	0.249	0.201
Australia	1941-1989	0.037	0.209
	1969-2017	0.120	0.008
Chile	1940-1995	0.548	0.307
	1941-2017	0.756	0.155
Finland	1946-1985	0.514	0.106
Japan	1955-2017	0.294	0.373
Spain	1941-1989	0.649	0.659
Taiwan	1962-2017	0.552	0.812
Turkey	1968-2017	0.192	–
West Germany	1960-1989	–	0.547
Italy	1949-1996	0.530	–
Norway	1946-2014	0.573	0.287
Paraguay	1962-2015	0.523	0.850
Peru	1959-2017	0.129	0.469
Portugal	1914-1998	0.606	0.493
South Africa	1965-2015	0.745	0.158

[a]Based on 10,000 bootstrap replications.

Table H.1 Estimates of the sum of the AR coefficients for the candidate cointegration residual based on Selden-Latanè[a]

		Estimates based on:	
Country	*Period*	*Johansen*	*Stock and Watson*
United Kingdom	1922-2016	0.55 [0.39; 0.71]	0.57 [0.41; 0.74]
U.S. - M1 + MMDAs	1915-2017	0.70 [0.58; 0.83]	0.75 [0.63; 0.87]
U.S. - M1	1915-2017	0.92 [0.85; 1.01]	1.00 [0.96; 1.02]
Argentina	1914-2009	0.33 [0.19; 0.47]	0.47 [0.32; 0.62]
Brazil	1934-2014	0.58 [0.40; 0.77]	0.86 [0.75; 1.01]
Canada	1926-2006	0.76 [0.63; 0.91]	0.81 [0.68; 0.95]
	1967-2017	0.32 [0.10; 0.53]	0.32 [0.11; 0.53]
Colombia	1960-2017	0.90 [0.75; 1.02]	0.91 [0.75; 1.02]
Guatemala	1980-2017	0.62 [0.35; 0.95]	0.63 [0.37; 1.01]
New Zealand	1934-2017	0.78 [0.67; 0.90]	0.84 [0.74; 0.95]
Switzerland	1948-2005	0.70 [0.49; 0.93]	0.78 [0.60; 0.99]
Bolivia	1980-2013	0.42 [0.12; 0.77]	0.56 [0.28; 0.98]
Israel	1983-2016	0.36 [0.33; 0.40]	0.35 [0.32; 0.39]
Mexico	1985-2014	0.46 [0.26; 0.69]	0.51 [0.29; 0.71]
Belgium	1946-1990	0.57 [0.38; 0.79]	0.63 [0.45; 0.84]
Belize	1977-2017	0.70 [0.50; 0.96]	0.74 [0.53; 1.01]
Austria	1970-1998	0.67 [0.37; 1.02]	1.01 [0.89; 1.05]
Bahrain	1980-2017	0.67 [0.50; 0.86]	0.59 [0.36; 0.84]
Barbados	1975-2016	0.62 [0.39; 0.88]	0.71 [0.52; 0.95]
Ecuador	1980-2011	0.79 [0.53; 1.03]	0.99 [0.71; 1.04]
Netherlands	1950-1992	0.60 [0.35; 0.89]	0.71 [0.49; 1.01]
South Korea	1970-2017	0.49 [0.30; 0.69]	0.51 [0.32; 0.70]
Thailand	1979-2016	0.66 [0.47; 0.87]	0.66 [0.46; 0.88]
Venezuela	1962-1999	0.91 [0.74; 1.03]	0.88 [0.69; 1.03]
Australia	1941-1989	0.80 [0.61; 1.02]	0.78 [0.58; 1.01]
	1969-2017	0.41 [0.17; 0.67]	0.42 [0.18; 0.68]
Chile	1940-1995	0.74 [0.64; 0.85]	0.75 [0.58; 0.98]
	1941-2017	0.66 [0.55; 0.78]	0.83 [0.74; 0.93]
Finland	1946-1985	0.38 [0.09; 0.67]	0.46 [0.17; 0.76]
Japan	1955-2017	0.82 [0.70; 0.97]	0.87 [0.76; 1.01]
Spain	1941-1989	0.59 [0.39; 0.82]	0.61 [0.41; 0.82]
Taiwan	1962-2017	0.90 [0.83; 0.98]	0.81 [0.72; 0.91]
Turkey	1968-2017	1.01 [0.84; 1.04]	1.01 [0.84; 1.04]
West Germany	1960-1989	0.39 [0.11; 0.71]	1.01 [0.83; 1.04]
Italy	1949-1996	0.97 [0.80; 1.03]	0.98 [0.85; 1.03]
Norway	1946-2014	1.00 [0.95; 1.02]	1.01 [0.97; 1.02]
Paraguay	1962-2015	0.70 [0.52; 0.90]	0.81 [0.66; 1.00]
Peru	1959-2017	0.36 [0.17; 0.59]	0.89 [0.76; 1.02]
Portugal	1914-1998	0.99 [0.93; 1.02]	0.99 [0.93; 1.02]
South Africa	1965-2015	0.87 [0.75; 1.01]	1.01 [0.96; 1.03]

[a] Median and 90% confidence interval (based on 10,000 bootstrap replications).

Table H.2 Estimates of the sum of the AR coefficients for the candidate cointegration residual based on log-log[a]

		Estimates based on:	
Country	*Period*	*Johansen*	*Stock and Watson*
United Kingdom	1922-2016	0.94 [0.84; 1.02]	0.93 [0.83; 1.02]
U.S. - M1 + MMDAs	1915-2017	0.86 [0.78; 0.95]	0.90 [0.81; 1.00]
U.S. - M1	1915-2017	0.94 [0.88; 1.01]	1.00 [0.97; 1.01]
Argentina	1914-2009	0.82 [0.72; 0.93]	0.86 [0.77; 0.99]
Brazil	1934-2014	0.94 [0.84; 1.02]	1.01 [0.97; 1.03]
Canada	1926-2006	0.77 [0.62; 0.95]	0.80 [0.66; 1.00]
	1967-2017	0.38 [0.18; 0.57]	0.38 [0.19; 0.57]
Colombia	1960-2017	0.89 [0.76; 1.02]	0.91 [0.79; 1.02]
Guatemala	1980-2017	0.56 [0.29; 0.85]	0.58 [0.31; 0.89]
New Zealand	1934-2017	0.88 [0.78; 1.01]	0.91 [0.81; 1.01]
Switzerland	1948-2005	0.73 [0.56; 0.91]	0.81 [0.67; 1.00]
Bolivia	1980-2013	0.73 [0.53; 0.99]	0.72 [0.53; 0.97]
Israel	1983-2016	0.60 [0.34; 0.93]	0.62 [0.35; 0.95]
Mexico	1985-2014	0.75 [0.59; 0.96]	0.74 [0.55; 0.98]
Belgium	1946-1990	0.53 [0.32; 0.76]	0.56 [0.35; 0.79]
Belize	1977-2017	0.27 [0.23; 0.44]	0.26 [0.07; 0.44]
Austria	1970-1998	0.75 [0.41; 1.02]	1.01 [0.81; 1.04]
Bahrain	1980-2017	0.60 [0.36; 0.87]	0.57 [0.31; 0.91]
Barbados	1975-2016	1.17 [1.10; 1.48]	1.04 [1.00; 1.16]
Ecuador	1980-2011	0.94 [0.75; 1.03]	0.98 [0.78; 1.04]
Netherlands	1950-1992	0.64 [0.41; 1.00]	0.68 [0.45; 1.00]
South Korea	1970-2017	1.00 [0.84; 1.03]	0.90 [0.73; 1.02]
Thailand	1979-2016	0.67 [0.48; 0.88]	0.66 [0.46; 0.90]
Venezuela	1962-1999	1.00 [0.93; 1.04]	0.94 [0.75; 1.03]
Australia	1941-1989	0.71 [0.48; 1.01]	0.71 [0.48; 1.01]
	1969-2017	0.85 [0.65; 1.02]	0.84 [0.63; 1.02]
Chile	1940-1995	0.76 [0.58; 0.94]	0.76 [0.59; 0.98]
	1941-2017	0.84 [0.72; 0.99]	0.88 [0.78; 1.00]
Finland	1946-1985	0.36 [0.08; 0.64]	0.50 [0.22; 0.82]
Japan	1955-2017	0.92 [0.84; 1.01]	0.94 [0.85; 1.01]
Spain	1941-1989	0.84 [0.69; 1.01]	0.84 [0.68; 1.01]
Taiwan	1962-2017	0.84 [0.75; 0.96]	0.86 [0.76; 0.96]
Turkey	1968-2017	1.01 [0.90; 1.04]	1.01 [0.88; 1.04]
West Germany	1960-1989	0.38 [0.09; 0.71]	1.01 [0.85; 1.09]
Italy	1949-1996	0.96 [0.80; 1.03]	0.97 [0.84; 1.02]
Norway	1946-2014	1.00 [0.94; 1.02]	1.01 [0.98; 1.03]
Paraguay	1962-2015	0.58 [0.34; 0.89]	0.69 [0.48; 0.94]
Peru	1959-2017	0.93 [0.82; 1.02]	0.98 [0.89; 1.02]
Portugal	1914-1998	0.97 [0.91; 1.01]	0.97 [0.91; 1.02]
South Africa	1965-2015	0.92 [0.81; 1.01]	1.01 [0.98; 1.04]

[a] Median and 90% confidence interval (based on 10,000 bootstrap replications).

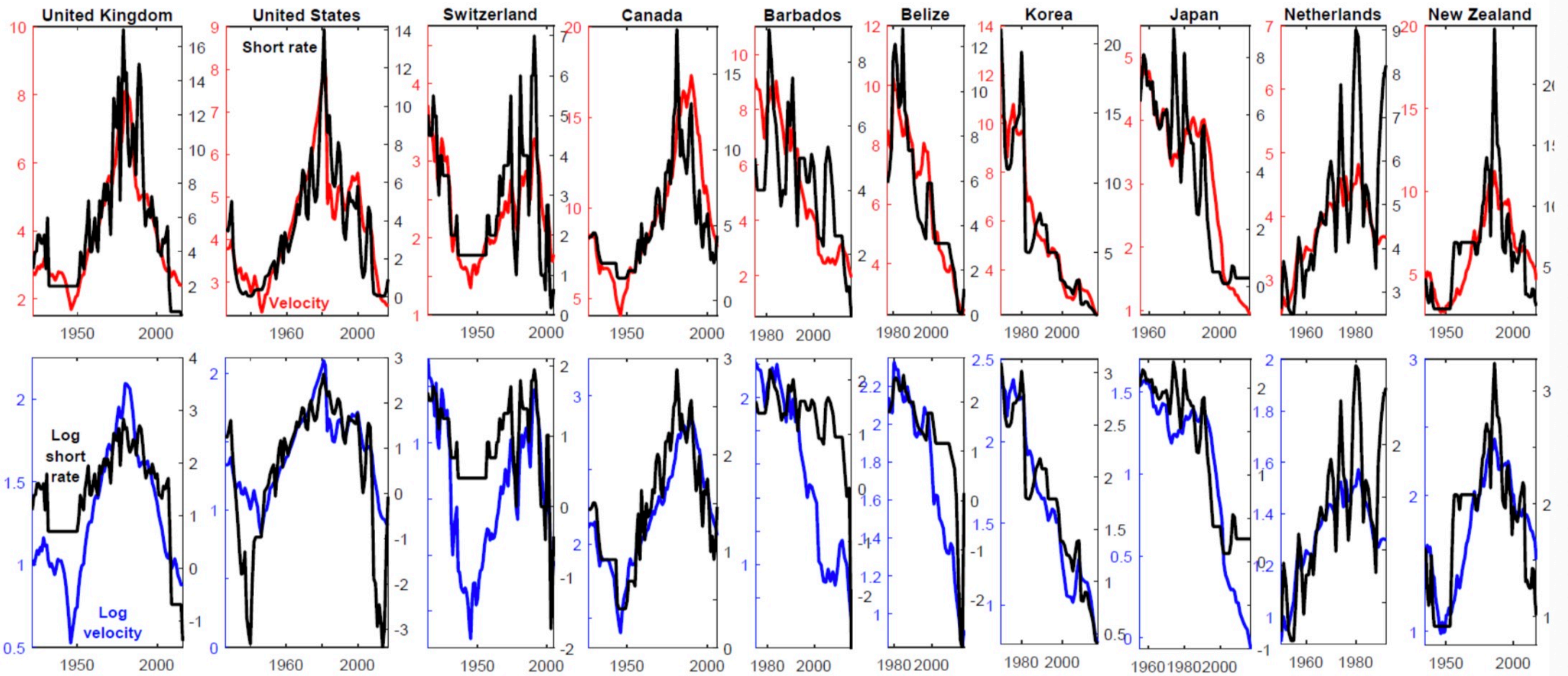

Figure I.1 Comparing the Selden-Latané and log-log specifications: selected evidence for low-inflation countries

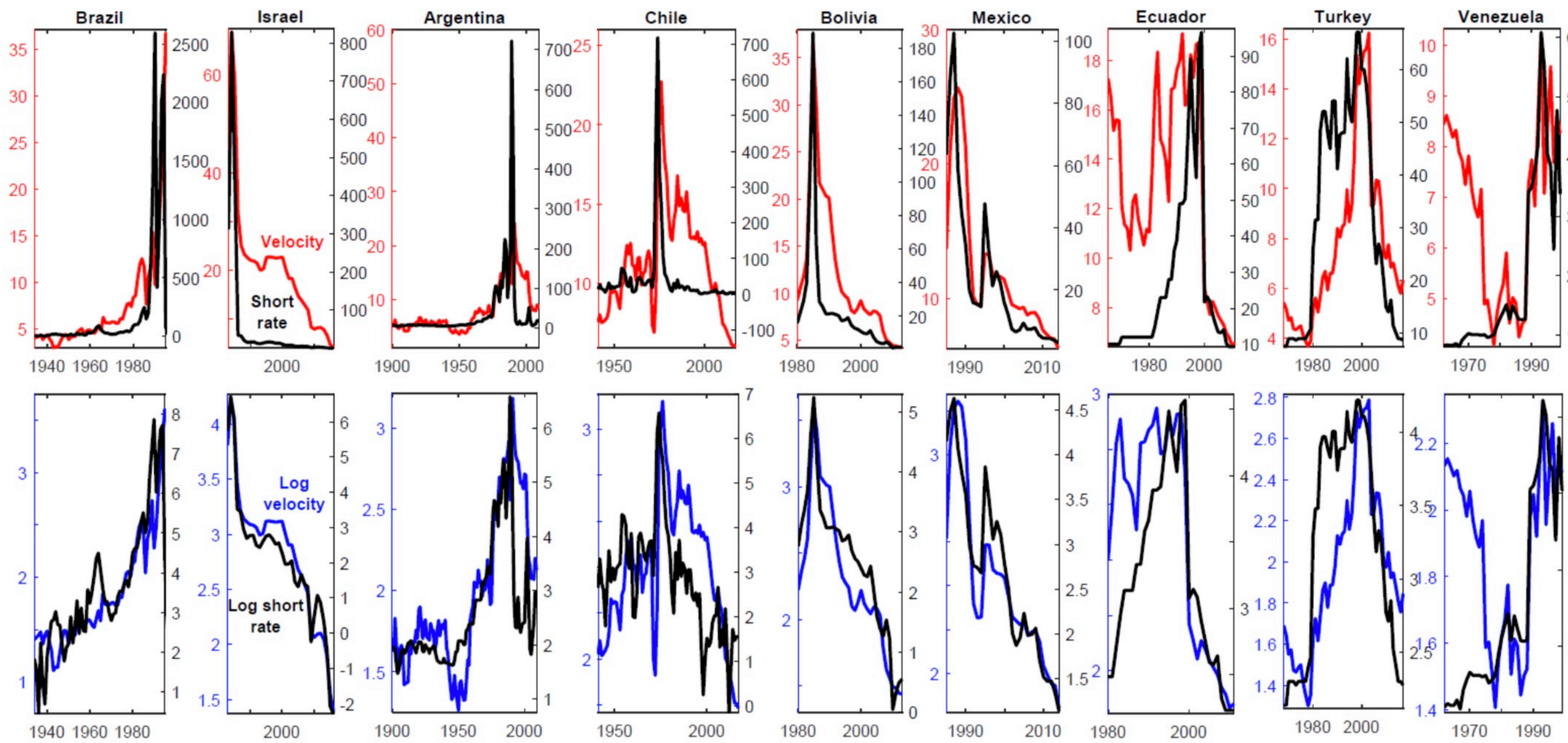

Figure I.2 Comparing the Selden-Latané and log-log specifications: selected evidence for high-inflation countries