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A SUGGESTION FOR FURTHER SIMPLIFYING THE THEORY OF MONEY

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Our suggestion consists of three postulates: assets are valued only in terms of their payoffs, perfect foresight, and complete and costless markets under laissez-faire. Together these postulates imply that the crucial anomaly, rate-of-return dominance of "money," is to be explained by legal restrictions.

Our defense of these postulates is two-fold. First we compare them with existing alternative theories. Second, we provide an illustrative model which: (a) is consistent with the postulates, (b) implies rate-of-return dominance under suitable legal restrictions, and (c) addresses monetary policy questions with standard welfare economics and, in particular, rationalizes in terms of price discrimination a debt management policy that "tailors debt issues to the needs of the market."

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A Suggestion for Further Simplifying the Theory of Money

In his well-known 1935 paper, "A Suggestion For Simplifying The Theory Of Money," Hicks recommended a new framework for monetary theory, a framework in which questions concerning money are addressed using the same techniques employed in other branches of economics. We are recommending a different, but related, framework with the same goal: to render monetary theory subject to standard modes of analysis. In our framework, monetary policy questions--questions concerning the role of alternative compositions of the government's portfolio and of alternative portfolio regulations on individuals and financial institutions--become public finance questions, many of which involve aspects of price discrimination.

Our framework consists of three postulates:

A. Assets are valued only in terms of their payoff distributions.

B. Anticipated payoff distributions are the same as actual payoff distributions.

C. Under laissez-faire, no transaction costs inhibit the operation of markets and, in particular, the law of one price.

The crucial and immediate implication of these postulates is that observations that fail to conform to the law of one price are to be explained by deviations from laissez-faire, for example, legal restrictions on who may issue what kind of liability and who may hold what kind of asset.

Postulate A is entirely within the spirit of Hicks' suggestion. When Hicks says (p. 15), "People do choose to have money rather than other things, and therefore, in the relevant sense, money must have a marginal utility. But merely to call that marginal utility X, and then to proceed to draw curves, would not be very helpful," he is rejecting theories that begin by making money and, perhaps, other assets arguments of utility functions. He is simultaneously rejecting what we will later describe as the macroeconomic approach: namely, theories that start out with "curves" or asset demand functions.

Postulate B, at least as a starting point, is also within the spirit of Hicks' suggestion. Hicks emphasized that current asset choices depend on views about asset prices in the future. He says (p. 27), "If I am right, the whole problem of applying monetary theory is largely one of deducing changes in anticipations from the changes in objective data which call them forth." Technical developments now allow us to relate anticipations to actual events via use of a perfect foresight or rational expectations equilibrium concept in fairly complicated models. Although we cannot say whether Hicks would have opted for this way of modeling anticipations if current tools were available to him, it is clear that Hicks was not willing to leave anticipations completely up in the air as free parameters. In that sense, postulate B is in accord with his suggestion.

It is with regard to postulate C that we part way with Hicks' suggestion and with the views accepted by most economists today. For us, as for Hicks, a crucial anomaly that any theory of "money" must confront is rate-of-return dominance (of money), a seeming violation of the law of one price. In Hicks' words,

> The critical question arises when we look for an explanation of the preference for holding money rather than capital goods. For capital goods will ordinarily yield a positive rate of return, which money does not. What has to be explained is the decision to hold assets in the form of barren money, rather than of interest- or profit-yielding securities . . . . So long as rates of interest are positive, the decision to hold money rather than lend it, or use it to pay off old debts, is apparently an unprofitable one. This, as I see it, is really the central issue in the pure theory of money. Either we have to give an explanation of the fact that people do hold money when rates of interest are positive, or we have to evade the difficulty somehow. It is the great traditional evasions which have led to Velocities of Circulation, Natural Rates of Interest, *et id genus omne*. (p. 18)

Hicks goes on to say that the explanation of the fact of rate-of-return dominance must lie in frictions.

> Of course, the great evaders would not have denied that there must be some explanation of the fact. But they would have put it down to "frictions," and since there was no adequate place for frictions in the rest of their economic theory, a theory of money based on frictions did not seem to them a promising field for economic analysis. This is where I disagree. I think we have to look the frictions in the face, and see if they are really so refractory after all. (p. 18)

But although the profession has accepted Hicks' view, adequate modeling of the frictions that inhibit the operation of the law of one price has indeed proved refractory. That is one of the reasons for proposing a different explanation of observations like rate-of-return dominance. Within our framework, such violations of the law of one price are to be explained by the presence of legal restrictions that inhibit arbitrage.

Thus, for example, consider U.S. currency and U.S. Treasury bills which are large-denomination, default-free titles to fixed amounts of U.S. currency in the future. According to our framework, the rate-of-return dominance implied by Treasury bills selling at substantial discounts must be explained by legal restrictions that prevent arbitrage between Treasury bills and currency-like assets. Absent legal restrictions, our framework predicts that arbitrage would wipe out the yield differential implied by the discount on Treasury bills. Absent legal restrictions, one form that such arbitrage could take is bank issues of small-denomination circulating notes--titles to, say, \$20 of U.S. currency payable to the bearer in, say, 30 days or thereafter--which are fully backed by holdings of Treasury bills. We are hypothesizing, first, that such circulating notes would sell at par because they would be regarded by the public as perfect substitutes for U.S. currency, and second, that the costs to "banks" of engaging in such arbitrage would be small enough so that they can be ignored.

Because such views represent a radical break with Hicks' suggestion and with existing views about money, we devote Section I of the paper to an extended discussion of why we are proposing postulates A-C. In particular, we will argue that our framework has very substantial advantages over its two existing rivals. One rival is what we call the macroeconomic approach, represented by, for example, the research program suggested by Tobin's, "General Equilibrium Approach to Monetary Theory" (1969). The other rival consists of models that invoke transactions costs at the level of markets, as represented, say, by Heller-Starr (1976), and Bryant-Wallace (1979, 1980). We will argue that neither of these represent defensible alternatives to our framework. The first constitutes an evasion in Hicks' sense, while the second is empirically implausible and, in a somewhat subtle way, also constitutes an evasion in Hicks' sense. While we agree with Hicks that there are important phenomena that require that we "look frictions in the face," we are suggesting, first, that existing models of financial systems have not done that adequately, and second, that we can go a long way toward a successful model of financial systems without addressing frictions.

Models consistent with postulates A-C have already been applied in several contexts, for example, to the study of international monetary systems (Kareken-Wallace, forthcoming) and to the study of government portfolio decisions under laissez-faire (Wallace, forthcoming). Most of this paper is devoted to setting out an illustrative model (Section II) and specific examples of it (Section III) that permit us to focus on rate-of-return dominance between yields on two government liabilities, "currency" and "bonds," and its implications for monetary policy. The main idea illustrated by the examples is that arbitrage-inhibiting legal restrictions and a multiplicity of government liabilities create possibilities for price discrimination, including beneficial (in the Pareto sense) "second best" price discrimination. The legal restrictions, which serve to separate markets, and the multiplicity of liabilities allow different prices (rates of return) to be offered in different markets. Given that an inflation tax is to be levied, such a financial system makes it possible to levy a discriminatory inflation tax; for example, one tax rate, in the form of a negative real yield on currency, for "poor" savers who, because they can save only in the form of currency, have a relatively inelastic demand for government debt, and a different, lower tax rate, in the form of a negative real yield on Treasury bills, for "rich" savers who, because they can save in other forms, have a relatively elastic demand for government debt. In a way, then, we provide a rationale for a debt management policy that attempts to "tailor debt issues to the needs of the market."

I. Alternatives to a No-Transaction Cost, Legal Restriction Theory of Financial Systems

Our purpose in this section is to argue that the reader is not giving up much by entertaining postulates A-C as a potential basis for a theory of financial systems. By not giving up much, we mean that existing alternative models of financial systems have taught us very little.[1/]

1. The Macroeconomic Approach

Despite Hicks' admonition, most modeling of financial systems still begins by "drawing curves" in the sense that the modeler begins by postulating one or more asset demand functions which specify that the quantities demanded depend on own and other interest rates, wealth, and, perhaps, income (see, for example, Friedman (1956) and Tobin (1969)). But there is, of course, a great deal of uneasiness about starting this way, uneasiness which is dealt with by asserting that the curves can be justified by appeal to an underlying model. Assertions, however, do not constitute valid arguments and, in this case, the assertions cannot be sustained.

To get at the meaning of an appeal to an underlying model, we must inquire into the logical relationship between the so-called underlying model U_j (U for underlying), and the "curves" being justified, M_j (M for macroeconomic). Here, U_j is a set of assumptions--often, a description of individual preferences and opportunities--and M_j is another set of assumptions--often, restrictions on quantities and prices labeled demand or supply functions. (The role of the subscript j will be made clear below.) For example, U_j might be a risk-aversion portfolio model (see, e.g., Tobin (1958)), while M_j is one or a set of asset demand functions. Alternatively, and we will comment further on this possibility below, U_j might be Baumol (1952), Tobin (1956), or Miller-Orr (1966) inventory theory of money demand and, again, M_j might be one or more asset demand functions. Of necessity, the modeler is satisfied to demonstrate the following logical relationship between U_j and M_j: U_j implies M_j. In particular, no claim that U_j and M_j are equivalent--U_j implies M_j <u>and</u> M_j implies U_j--could ever be made. However, as we now argue, because such equivalence does not hold, the appeal to U_j as justification for M_j is often misleading in macroeconomic contexts.

We attached a subscript to U_j and to M_j because M_j is not a complete model; it is one equation (restriction), or perhaps a set of equations (restrictions) that comprise at most one sector of a model. Suppose, then, that the complete (macroeconomic) model is $M = (M_1, M_2, \ldots, M_n)$, where many of the M_i have underlying models, U_i, that, at best, also satisfy the logical relationship, U_i implies M_i. If the notion of justifying underlying models means anything, it must mean that the user of the complete (macroeconomic) model, M, appeals to all the underlying U_i simultaneously.

However, such an appeal makes no sense if the U_i are mutually inconsistent, and inconsistency is possible and likely because of the nonequivalence between the U_i and the M_i and because of the sector-by-sector specification of the U_i's. Thus, for example, if U_j is a risk-aversion portfolio model, then, according to it, asset demands depend on the entire joint yield distribution, and, in particular, not only on mean rates of return. But in M_j, the dependence on aspects of the joint yield distribution other than means is usually subsumed in the functional form of the "curves." Thus, it is easy to impose other M_i--equilibrium conditions, specifications of the technology, and policy rules--which give rise to a solution of M which contradicts the second and possibly higher moment structure assumed in U_j, but not visible in M_j.[2/]

Such problems of inconsistency are not minor problems that are easily corrected. Most of the underlying models of macroeconomics, what we have been calling the U_i, are partial equilibrium models in which agents--*individuals or* entities arbitrarily labeled firms--are faced with complicated market environments. In order that such U_i be part of a general equilibrium model, it is necessary to specify a physical environment that ends up implying such complicated market environments as equilibria. In general, that is a difficult task.

We are led, therefore, to conclude that the appeal to an underlying model cannot be taken seriously. The approach we have been calling the macroeconomic approach really does start with "curves" that must stand on their own. Hicks dismissed starting this way and called it an evasion, but he only hinted at the reasons for his dismissal. As we see it, there are at least two related reasons for dismissing it. First, there are available no invariance arguments for the parameters of such "curves," one consequence being that no theoretical arguments are available to guide empirical research. Second, if one starts with "curves," one cannot appraise alternative policies using standard welfare criteria.

Empirical research on curves has mainly followed the program suggested by Tobin's "General Equilibrium Approach to Monetary Theory" (1969). (Monetarism, which we view as even less defensible, constitutes the special case of this approach in which the researcher tries to find some asset total, the quantity of which is dependent on very few interest rates.) This research, which its supporters assert is to be judged by its ability to find stable empirical relationships, has not even on that criterion been very successful. Our postulates suggest an explanation for this lack of success.

In the absence of any other guidance, the empirical researcher attempting to follow the research strategy of Tobin (1969) must take names seriously--names like demand deposits, time deposits, CDs, NOW accounts, francs, marks, and Eurodollars, to list a few. But postulate A, which is hardly controversial, says that assets are wanted because of the intertemporal trades in consumption that they make possible. (As applied to "money," this is the old dictum that one person gives up consumption for money only because the person expects to trade the money for consumption subsequently.) Therefore, it is entirely consistent with the possibility that changes in laws and features of the physical environment produce changes in the names of assets used to accomplish underlying intertemporal trades or produce large shifts in the demand for assets with particular names. Indeed, laissez-faire and postulates A-C imply name indeterminacy; instances of this indeterminacy are the Modigliani-Miller theorem for corporate liability structures and the indeterminacy of exchange rates between national fiat monies under laissez-faire (see Modigliani-Miller (1958), and Kareken-Wallace (forthcoming)).

To emphasize the futility of an approach which proceeds by taking the names of assets seriously, consider what the field of finance would look like if such an approach were used. A researcher trying, say, to explain the relative prices of shares on the New York Stock Exchange would, following this approach, list all the stocks and attempt to estimate a set of related asset demand functions. If time series were used, the researcher would, in addition to needing many observations, have to start over every time the list of traded shares changed; evidently, according to the approach being described, the disappearance of a firm or the appearance of a new firm amounts of an exogenous change in the list of available "substitutes." While this sounds ridiculous, it accurately depicts the difficulties faced by researchers who have tried to implement the research program advocated in Tobin's "General Equilibrium Approach to Monetary Theory" (1969) or that advocated in Friedman's "The Quantity Theory of Money: A Restatement" (1956). Of course, empirical work in finance does not proceed in the manner just described. Indeed, much of finance theory assumes laissez-faire and postulates A-C.[3/] We are, in effect, urging that the approach used in finance theory be applied to the study of financial systems in general.[4/]

Whether or not one is lucky enough to find some empirical regularities between the quantities of particular assets and other variables, the most serious defect of starting with curves is that such curves do not answer most questions. They do not, for example, tell us the effects of quantitative controls on bank loans, of international capital controls, or of technological developments that lower record-keeping costs and, therefore, make it easier to carry out private borrowing and lending using credit cards and charge accounts. The macroeconomic approach is bankrupt in two senses in dealing with such questions. First, the approach does not show how its curves shift in consequence of such changes. Second, it does not allow us to appraise such changes in terms of standard welfare economics. In contrast, a theory which makes explicit use of postulate A offers some chance of success. After all, we agree that interventions of the sort listed above impinge on behavior and on the welfare of individuals by affecting their ability to accomplish intertemporal trades. Strict adherence to postulate A will tell us how demands depend on interventions and how equilibrium consumption allocations depend on them. The macroeconomic approach offers no hope of doing this.

2. Market Transaction Cost Models

In the spirit of Hicks' suggestion that we "look frictions in the face," there is a large literature which proceeds by positing a technology that makes trading costly. Indeed, Hicks described what from hindsight is very much an outline of a Baumol (1952) or Tobin (1956) inventory model of money demand.

> Now, since the expected interest increases both with the quantity of money to be invested and with the length of time for which it is expected that the investment will remain untouched, while the costs of investment are independent of the length of time, and (as a whole) will almost certainly increase at a diminishing rate as the quantity of money to be invested increases, it becomes clear that with any given level of costs of investment, it will not pay to invest money for less than a certain period, and in less than

> certain quantities. It will be profitable to hold assets for short periods, and in relatively small quantities, in monetary form. (p. 19)

Since this is very familiar and sounds quite reasonable, it behooves us to say why we regard it as inadequate as a theory of financial systems.

Notice, of course, that Hicks is describing the situation of an agent in a given market environment. The key features of the environment are (i) the agent needs something called money in order to transact; and (ii) there are costs that display scale economies of switching between money and higher-yielding assets. Since Hicks wrote, there have appeared many detailed descriptions of such market environments and of an agent's optimizing response to it. (See, for example, Baumol (1952), Tobin (1956), and Miller-Orr (1966)). Such analyses of an individual agent in a given market environment have been used in two ways.

One way is what we above called the macroeconomic approach. That approach uses the above picture of the market environment of an individual as an underlying model, a U_j, in order to justify a money demand function. It takes the dependence between the individual's desired money holdings and <u>some</u> of the features of the market environment described by (i) and (ii) and uses it as a money demand function in a macroeconomic model.

The macroeconomic approach is deficient for the reasons cited above. Specifically, the money demand function, while an implication of the market environment consisting of (i) and (ii), is not equivalent to that model of the individual's environment. Thus, the money demand function and other relationships that together form a complete macroeconomic model may imply a market environment which bears no resemblance to (i) and (ii). One crucial feature missing from standard macroeconomic models is the transactions technology and the implied resources used up in switching between money and other assets. This is revealed by contrasting the analysis of open-market operations suggested by the underlying model with that implied by standard macroeconomic models.

The underlying transaction cost model suggests that an open-market sale--more bonds, less money--gives rise to a higher yield on bonds, which induces individuals to hold more bonds and less money and to make more transactions between bonds and money, more "trips to the bank." If interest on government bonds is financed by taxes, the model suggests that the presence of interest-bearing government bonds amounts to a tax-financed subsidy on trips to the bank, a resource-using activity. Although standard macroeconomic models claim to have the inventory models as underlying models of their money demand functions, they come to no such conclusion.

In order to avoid such anomalies, economists who take seriously the transaction cost market environment that Hicks and others proposed have tried to proceed in a way that ends up capturing all the critical assumptions of the transaction cost setup. This alternative way involves inventing a general equilibrium physical environment that implies as an equilibrium the market environment that Hicks assumed his individual to face. This is the way that primarily interests us and that we regard as the main existing rival to models built on postulates A-C.

Building on the approach of Hahn (1973), Kurz (1974), and others, Heller-Starr (1976) present a general equilibrium model in which transaction costs are imposed on market trading.

> In addition to a budget constraint, the agent's actions are restricted by a transactions technology. This technology specifies for each complex of purchases and sales at date to what resources will be consumed by the process of transaction: labor time, paper and pens, gasoline, telephone services, and so forth. It is because transactions costs may differ between spot and futures markets for the same good that we consider the reopening of markets allowed by the sequence economy model. (p. 197)

But, Heller-Starr recognize the provisional nature of such assumptions.

> Though we will take the individual's transactions technology as fixed for the purpose of this model, it should

> be recognized that unlike the production technology of the firm in standard competitive equilibrium models, an individual's transaction technology should be made to depend on the actions of others in the economy. Thus, the structure of the economy (including, for example, the legal system and contract enforcement) will affect an individual's transaction possibilities. A more general model would allow endogenous specification of the individual transaction technology. (p. 197)

In effect, they are saying that they would like to have an underlying theory of the transaction technology. And why is evident.

In order to use the Heller-Starr model to analyze monetary policy, the user must specify the transaction technology. Because the theory offers no guidance, the user must make as many decisions as someone who uses the starting-with-curves approach. The user must decide what money is and must describe the transaction technology for dealings in money and all sorts of other assets. In doing this, the user must end up, as Heller-Starr do, taking names seriously:

> Any durable good or futures contract can perform the function of shifting purchasing power forward or back, but transactions and storage costs associated with some commodities used for this purpose will be prohibitive. A distinguishing feature of money should be its low transactions and storage costs as compared to goods, bonds, and futures contracts. (p. 203)

It turns out, then, that a model with a transaction technology of the Heller-Starr sort has many of the defects of the macroeconomic approach. Both the curves of the macroeconomic approach and the Heller-Starr transaction technology turn out to be ex post rationales for what we have observed. Neither allows us to predict the effects of legal or technological changes.

The defects of a market transaction cost approach are revealed in some of our earlier work in which we studied monetary policy in a simple market transaction cost model. Bryant-Wallace (1979) contrast bond and currency issue financing of a given real deficit in the context of a simple overlapping generations model. The bonds are meant to resemble large-denomination titles to fiat currency in the future; they are meant to be like U.S. Treasury bills. Bryant-Wallace invoke postulates A and B, but not postulate C. Instead, they assume that there is a costly technology available for converting bonds into small-denomination assets that are indistinguishable from government issued currency, indistinguishable because they are fully backed by holdings of default-free bonds and, hence, are sure titles to government currency in the future. In terms of the criticisms we made above of models like the Heller-Starr model, one can object to the imposition of a costly intermediation technology in the Bryant-Wallace model on the following grounds.

In order that government bonds not sell at par--i.e., bear interest--in the Bryant-Wallace (1979) model, individuals must not be able to get together and share large-denomination bonds. What prevents them from doing that? What if there were many consumption goods and what if individuals had different patterns of intertemporal endowments or preferences so that there were many kinds of potential private trades in the model. How would Bryant-Wallace specify the transaction technology for all such trades? And what about transaction costs for the government? According to Bryant-Wallace, the government is indifferent in terms of real resources between supplying bonds and supplying currency, but bonds impose resource costs on the private sector. Why this asymmetry? If the government can costlessly produce many pieces of paper (small-denomination currency) instead of one piece (the large-denomination bond), then why can't the private sector do the same (costlessly intermediate bonds through issue of bank notes)? Such questions can be answered only by a theory of transactions costs, a theory which approaches like Heller-Starr and Bryant-Wallace do not provide.

There is another and for some readers, no doubt, more convincing ground for quarreling with a Heller-Starr or Bryant-Wallace (1979) theory of interest on default-free bonds. Such theories make the nominal yield on Treasury bills a spread between the yield on intermediary assets (the Treasury bills) and the yield on intermediary liabilities (bank notes), a spread determined by the intermediation technology. From observations on the spreads charged by mutual funds, we can infer something about the magnitude and variation of such spreads. Spreads charged by mutual funds tend to be small, less than 1 percent per year, and tend not to vary. This hardly seems descriptive of yields on Treasury bills. Yields on Treasury bills tend to be too high and to vary too much to be accounted for entirely by a plausible intermediation technology.

To summarize, then, in addition to viewing the imposition of transaction costs at the market level as being deficient because of the lack of any theoretical guidance about how such costs should be specified, we also view such theories as empirically implausible in the following sense. The specification of somewhat plausible intermediation costs--plausible in the light of spreads charged by mutual funds--does not seem able to account for the actual behavior of default-free interest rates.

There exists a class of models that is seemingly free of both these difficulties--namely, Clower-constraint models. These are models in which something called money is needed in order to consummate some or all trades. From such an assumption, it does indeed follow that nominal interest rates are quite free and that additional assumptions about the transaction technology are not needed. But how are we to interpret the Clower-constraint assumption? We think it must either be interpreted as a legal restriction or as a very extreme version of a Heller-Starr-type transaction technology. To support this view, we will comment on a particularly simple version of a Clower-constraint model.

Martins (1980) and Bryant (1980) study an overlapping generations model peopled by three-period-lived agents. The only assets are government-supplied currency and two-period bonds. In general, agents in the first period of their lives acquire both currency and bonds, but any dissaving by agents in the second period of their lives must be supported by sales of currency. Any bonds acquired in the first period of their lives are not marketable in the second period of their lives. It is the absence of such a market that allows bonds to sell at a discount, i.e., to bear interest. But how are we to interpret this nonmarketability of bonds? Absent legal restrictions or costs of the Heller-Starr or Bryant-Wallace sort, if bonds sell at a discount, then agents in the first period of their lives can make infinite profits by buying bonds and selling marketable default-free titles to currency in two periods, when their bonds mature. If such dealings are inhibited by the technology, then that technology must be of an extreme kind; as these models stand, there are no bounds on nominal interest rates and, hence, no bounds on the potential revenues from engaging in such intermediation. Moreover, it matters whether the nonmarketability is to be interpreted in terms of a costly technology or in terms of legal restrictions. It matters when we ask welfare-type questions about the desirability of different monetary policies. It also matters when we attempt to use the model to interpret experience. If we adopt the legal restriction interpretation, then we expect to observe different rate-of-return patterns under different kinds of legal restrictions.

Thus, we are led to conclude that Clower-constraint models are either extreme versions of Heller-Starr models or are legal restriction models. We view them as defective because we don't know what they are. Indeed, we view them as a step backward from Heller-Starr models and, indeed, as a reversion to those evasions that Hicks ridiculed.

3. The Defensible Alternatives

Although we reject imposing transaction costs at the level of markets as a sort of evasion, we agree that some sort of transaction cost model is necessary in order to explain important features of most market economies, features like the role of first-come first-serve as an allocative device, vertical integration and the role of firms, and, more generally, the allocation of interactions between market interactions and nonmarket interactions. We would insist, however, that a useful theory must be one that presents us with a theory of transaction costs. It must, for example, explain why some markets seem more "perfect" than other markets. Such a deep theory of transactions costs would be <u>the</u> general economic theory: postulates A-C and most of standard economic theory would be the special limiting case of no-transaction costs.

One can regard Hicks and most of the profession as asserting that the special limiting case cannot serve as a basis for a theory of financial systems. Our position is that the special limiting case has not been given a trial as a theory of financial systems and that it ought to be given a trial.

First of all, a deep transaction cost theory is not in hand. Second, there are grounds for suspecting that it alone will not suffice. The questions we raised above the inability of a plausible Bryant-Wallace (1979) technology to explain observed yield patterns on Treasury bills would also apply to any theory of transaction costs. Such a theory would have to reconcile the small and constant spreads charged by mutual funds with the relatively large and variable yields on Treasury bills, yields which in the absence of legal restrictions have to be interpreted as a spread between yields on intermediary assets and intermediary liabilities. Thus, we believe that even if we had an acceptable transaction cost theory, there would have to be appeal to legal restrictions to explain significant instances of rate-of-return dominance.

In principle, of course, nothing precludes studying the role of legal restrictions in the context of a model that includes a deep transaction cost theory. But we suspect that we can learn a lot by studying the role of legal restrictions in the special case of no transaction costs. One reason for proceeding that way is the same as the reason for studying international trade and tariffs in a model that abstracts from transport costs; we think we can more easily arrive at certain critical insights without cluttering up the model with transport costs. But the analogy between transport costs in the theory of international trade and the deep theory of transaction costs in the theory of financial systems is unfortunately not complete. We know how to handle transport costs, essentially by labeling goods by location and by positing a technology, the transport technology, for converting some of these goods into others. That puts us in the happy position of being able to check on whether propositions developed in the model that abstracts from transport costs do, in fact, hold in a model that does not abstract from them. In our case, a satisfactory transaction cost model does not exist. Hence, we see our choice to be between postulates A-C, on the one hand, and one of the kinds of models reviewed above. We have already described what we regard as serious defects of either starting with curves or starting with <u>market</u> transaction costs. We must now demonstrate that a model that builds on postulates A-C is less subject to these defects.

II. An Illustrative Model

The model is an overlapping generations model peopled by two-period-lived generations. We will describe equilibrium conditions for the model under laissez-faire (LF) and under portfolio restrictions that preclude all within-generation intertemporal trades, a regime which is labeled portfolio autarky (PA). Portfolio autarky is, of course, a very stringent form of legal restriction. We study it primarily because it is relatively easy to work with and because it allows us to illustrate some general principles.

1. Endowments, Preferences, and the Technology

The model is of a discrete-time economy.[5/] We let t, an integer, denote the date and let t=1 be the current or initial date. At each date t, a new generation, generation t, appears and is present in the economy at t and t+1. There is a single consumption good at each date t, and, in general, member h of generation t is endowed with some time t good, $w_t^h(t) \geq 0$, and some time t+1 good, $w_t^h(t+1) \geq 0$.

As for preferences, each member of generation 0 (those who at t=1 are in the second and last period of their lives) maximizes consumption of time 1 good, while each member h of generation t, t > 0, has preferences that are represented by a twice differentiable, increasing, and strictly concave utility function, $u_t^h[c_t^h(t), c_t^h(t+1)]$, where $c_t^h(t+i)$ is consumption of time t+i good by member h of generation t. Under uncertainty, expected utility is maximized.[6/]

We assume that different generations are identical both with regard to the pattern of endowments and preferences, but we allow and will make some use of intrageneration diversity.

There is also a technology for converting time t good into time t+1 good. The input is time t good, while the output is, in general, a probability distribution of time t+1 good. Thus, if k is the input of time t good, the output of time t+1 good is zero if $k < K$ and is $x(t+1)k$ if $k \geq K$, where $x(t+1) = x_j > 0$ with probability θ_j; j=1, 2, ..., J. It is assumed that x(t+1) is observed after the input decision at t is made, but before generation t+1 appears. Note that $K \geq 0$ is the minimum scale on which this technology can be operated. For inputs greater than this minimum, the technology is a constant returns to scale, stochastic (storage) technology. The minimum scale will play a role only under PA.

2. Government

We assume that the government attempts to consume $G(t) \geq 0$ units of time t good and that its only method of financing this expenditure is by way of a deficit. It can issue fiat currency, and it can issue one-period default-free discount bonds. Each bond issued at t is a title to a known amount of currency at t+1. Thus, the cash flow constraint of the government is

$$G(t) = p(t)[M(t)-M(t-1)] + p(t)P_b(t)B(t) - p(t)B(t-1) \tag{1}$$

where p(t) is the time t price of a unit of currency in terms of time t good (the inverse of the price level), M(t-1) is the stock of currency held by the public from t-1 to t, B(t-1) is the total face value in units of time t currency of the government bonds issued at t-1, and $P_b(t)$ is the price at t in terms of currency of an amount of bonds which pays one unit of currency at t+1 ($1/P_b(t)$ is unity plus the nominal interest rate on bonds issued at t).[7/]

We describe the government's financing scheme in terms of the ratio $B(t)/[B(t)+M(t)] \equiv \gamma(t) \in [0,1]$. The government also specifies a minimum size per bond, which, like the minimum scale for storage, plays a role only under PA. This minimum scale is in terms of a minimum expenditure on bonds in terms of time t good, F(t); that is, the minimum nominal face value at t, $\underline{b}(t)$, say, satisfies $p(t)P_b(t)\underline{b}(t) = F(t)$. The government also chooses whether to impose PA, the only alternative being LF.

3. Choice Problems and Equilibrium Conditions: Permanent Laissez-Faire

We describe the conditions for a perfect foresight competitive equilibrium in terms of time t markets for claims on time t+1 good in "state" $x(t+1) = x_j$. The members of generation t in their role as consumers deal only in such claims. "Firms," operated by members of generation t in their role as "producers," supply such claims by storing time t good, currency, and newly issued bonds.

As a consumer, member h of generation t is assumed to maximize

$$\sum_j \theta_j u_t^h[c_t^h(t), c_t^h(t+1,j)]$$

subject to

$$c_t^h(t) + \sum_j s_t(t+1,j)c_t^h(t+1,j) \leq w_t^h(t) + w_t^h(t+1)\sum_j s(t+1,j) \tag{2}$$

by choice of nonnegative $c_t^h(t)$ and $c_t^h(t+1,j)$; j=1, 2, ..., J, where $c_t^h(t+1,j)$ is consumption of time t+1 good in state $x(t+1) = x_j$ and $s_t(t+1,j)$ is the price of one unit of this good in units of time t good. Letting $s_t(t+1)$ be the J-element vector of these prices, the solution to this maximization problem is a set of demand functions, $c_t^h(t+1,j) = a_j^h(s_t(t+1))$; j=1, 2, ..., J. We let $A_j(s_t(t+1)) \equiv \sum_h a_j^h(s_t(t+1))$ be the set of aggregate demand functions, the summation being over the members of generation t.

In their role as producers, members of generation t may store the consumption good, currency, or bonds. Any producer maximizes profit as a price taker with regard to $s_t(t+1)$ and the time t and time t+1 prices of currency, which are taken to be state independent.

Profit in terms of time t good from storing $k \geq K$ units of the consumption good is $k\sum_j x_j s_t(t+1,j) - k$. Since this is linear in k, the condition that storage be finite in any equilibrium implies as an equilibrium condition

(3) $\sum_j x_j s_t(t+1,j) \leq 1$

a condition that must hold with equality if total storage is as large as $\bar{K}$.

Profit in terms of time t good from storing $m \geq 0$ units of currency is $mp(t+1)\sum_j s_t(t+1,j) - p(t)m$. Since this is linear in m, finiteness of the currency supply implies that prices in any competitive equilibrium satisfy

(4) $p(t+1)\sum_j s_t(t+1,j) \leq p(t)$

a condition which must hold with equality if firms store currency.

Profit in terms of time t good from storing bonds with nominal face value b such that $p(t)P_b(t)b \geq F(t)$ is $bp(t+1)\sum_j s_t(t+1,j) - p(t)P_b(t)b$. Since this is linear in b, for b satisfying the constraint we must have

(5) $p(t+1)\sum_j s_t(t+1,j) - p(t)P_b(t) \leq 0$

and with equality if $b > 0$.

Notice that if both bonds and currency are held, then, by (4) and (5), $P_b(t) = 1$.

We can now define a (perfect foresight competitive) equilibrium under LF.

Given $\{G(t)\}$, $\{F(t)\}$, $\{\gamma(t)\}$, and $M(0) + B(0)$, a LF equilibrium consists of positive $\{s_t(t+1)\}$ and nonnegative $\{p(t)\}$, $\{K(t)\}$, where $K(t)$ is total storage of time t good and $K(t) = 0$ or $K(t) \geq \bar{K}$, $\{M(t)\}$, and $\{B(t)\}$ such that for all $t \geq 1$

(6) $A_j[s_t(t+1)] = \sum_h w_t^h(t+1) + x_j K(t) + p(t+1)[M(t)+B(t)]$

for j=1, 2, ..., J and such that (1) and (3)-(5) (with their provisos) are satisfied. (The symbol $\{\cdot(t)\}$ is to be interpreted as a sequence defined for all $t \geq 1$.)

4. The Choice Problem and Equilibrium Conditions: Portfolio Autarky

Under PA, each member h of generation t again maximizes expected utility, but by choosing nonnegative consumption, nonnegative currency ($m^h(t)$), bonds ($b^h(t)$), and storage ($k^h(t)$) subject to

$$c_t^h(t) + p(t)m^h(t) + p(t)P_b(t)b^h(t) + k^h(t) \leq w_t^h(t),$$

$$c_t^h(t+1,j) \leq w_t^h(t+1) + p(t+1)m^h(t) + p(t+1)b^h(t) + x_j k^h(t),$$

$$p(t)P_b(t)b^h(t) \geq F(t) \text{ or } b^h(t) = 0, \text{ and } k^h(t) \geq K \text{ or } k^h(t) = 0.$$

It is convenient to redefine the currency and bond choice variables in real terms. Thus, let $q_1^h(t) = p(t)m^h(t)$ and $q_2^h(t) = p(t)P_b(t)b^h(t)$. Then, for $p(t) > 0$ and $P_b(t) > 0$, we may rewrite the above constraints as

$$c_t^h(t) + q_1^h(t) + q_2^h(t) + k^h(t) \leq w_t^h(t), \tag{7}$$

$$c_t^h(t+1,j) \leq w_t^h(t+1) + R_1(t)q_1^h(t) + R_2^h(t)q_2^h(t) + x_j k^h(t), \tag{8}$$

$$q_2^h(t) \geq F(t) \text{ or } q_2^h(t) = 0, \text{ and } k^h(t) \geq K \text{ or } k^h(t) = 0, \tag{9}$$

where $R_1(t) \equiv p(t+1)/p(t)$ and $R_2(t) = p(t+1)/p(t)P_b(t)$. The $R_i(t)$'s are real gross rates of return, which we will hereafter refer to simply as rates of return. Figure 1 depicts the upper boundary in consumption space implied by (7)-(9) for the case $x_j \equiv 0$ (no storage), $R_2(t) > R_1(t)$, and $0 < F(t) < w_t^h(t)$.

[INSERT FIGURE 1]

Note that it is PA which prevents any individual from earning $R_2(t)$ on saving of less than $w_t^h(t) - F(t)$. In other words, under PA, two or more agents cannot share a bond. Formally speaking, to do that one agent would have to buy the bond and issue IOU's to the others. Such intermediation is ruled out by assumption under PA.

The solution to this maximization problem consists in part of demand functions (possibly correspondences) $q_i^h(t) = d_i^h(R_1(t),R_2(t),F(t))$; i=1, 2. We define a PA equilibrium in terms of aggregate demand functions (correspondences) $D_i(R_1(t),R_2(t),F(t)) \equiv \sum_h d_i^h(R_1(t),R_2(t),F(t))$, i=1, 2.

Given $\{G(t)\}$, $\{F(t)\}$, $\{\gamma(t)\}$, and $M(0) + B(0) > 0$, a PA monetary equilibrium consists of positive $\{p(t)\}$ and $\{P_b(t)\}$ and nonnegative $\{M(t)\}$ and $\{B(t)\}$ such that for all $t \geq 1$,

(10) $$D_1(R_1(t),R_2(t),F(t)) = p(t)M(t)$$

(11) $$D_2(R_1(t),R_2(t),F(t)) = p(t)P_b(t)B(t)$$

and such that (1) holds, it being understood that the $R_i(t)$ are defined, as above, in terms of currency and bond prices.

5. Stationary Monetary Equilibria Under PA

In the next section we present examples that suggest the range of possibilities that can occur under PA. These examples present the stationary or constant inflation rate and bond yield equilibria for various specifications of the physical environment (tastes, endowments, and storage technologies) and for various constant values of $G(t)$, $\gamma(t)$, and $F(t)$, denoted, respectively, G, γ, and F. Our view is that G is given and that (monetary) policy under PA involves choosing γ and F.

We find it convenient to describe such equilibria in the following way. Letting R_i denote a constant value of $R_i(t)$, it follows from (1) for $t \geq 2$ and (10) and (11) that an equilibrium (R_1,R_2) must satisfy $G = (1-R_1)D_1(R_1,R_2,F) + (1-R_2)D_2(R_1,R_2,F)$, where $1-R_1$ should be interpreted as the tax rate on currency holdings and $(1-R_2)$ as the tax rate on bond holdings. Moreover, to be a monetary equilibrium, it must also satisfy $R_2 \geq R_1 > 0$ and $D_i(R_1,R_2,F) > 0$ for at least one value of i. In order to have a symbol to represent the set of (R_1,R_2)'s that satisfy these conditions and its dependence on G and F, we let

(12) $$S(G,F) = \{(R_1,R_2) \mid (1-R_1)D_1(R_1,R_2,F) + (1-R_2)D_2(R_1,R_2,F) = G,$$
$$R_2 \geq R_1 > 0 \text{ and } D_i(R_1,R_2,F) > 0 \text{ for at least one value of } i\}.$$

To go from a given G and F and a pair (R_1,R_2) in S(G,F) to equilibrium price sequences for currency and bonds, we need an associated initial price of currency, p(1). Using (10) and (11) for t = 1 and an initial condition for M(0) + B(0), we find an associated p(1) from equation (1) for t = 1; namely,

(13) $$G = D_1(R_1,R_2,F) + D_2(R_1,R_2,F) - p(1)[M(0)+B(0)].$$

Then, given G, F, and M(0) + B(0), a monetary equilibrium is any (R_1,R_2) in S(G,F), an associated solution for p(1) from (13), and the associated paths of nominal supplies of currency and bonds given by (10) and (11), respectively. Since $1/P_b(t)$, the nominal gross yield on bonds, is, by the definition of R_i's, a constant, R_2/R_1, it follows from (10) and (11) that those currency and bond sequences imply a constant ratio of currency to bonds, or equivalently, a constant $\gamma(t)$. Thus, we can first study the set S(G,F) and then find the nominal asset supplies that "support" various elements of S(G,F) as stationary monetary equilibria.

Our last task before turning to examples is to relate p(1) solutions to features of the S(G,F) sets. We are interested in p(1) because it determines the effects of alternative policies on the current old; p(1) determines the value of the given initial nominal wealth of the current old, M(0) + B(0).

Proposition 1: For given G and M(0) + B(0) > 0, if $(R^*,R^*) \in S(G,F^*)$, $(\tilde{R}_1,\tilde{R}_2) \in S(G,\tilde{F})$, $D_2(\tilde{R}_1,\tilde{R}_2,\tilde{F}) > 0$, and $\tilde{R}_2 > \tilde{R}_1 \geq R^*$, then $\tilde{p}(1) > p^*(1)$, where $\tilde{p}(1)$ is the p(1) solution to (13) for $(\tilde{R}_1,\tilde{R}_2,\tilde{F})$ and $p^*(1)$ is that for (R^*,R^*,F^*).

<u>Proof</u>: In view of (13), we need only show that $D_1^* + D_2^* < \tilde{D}_1 + \tilde{D}_2$ where $D_i^* \equiv D_i(R^*,R^*,F^*)$ and $\tilde{D}_i \equiv D_i(\tilde{R}_1,\tilde{R}_2,\tilde{F})$. Since $(R^*,R^*) \in S(G,F^*)$ and $(\tilde{R}_1,\tilde{R}_2) \in S(G,\tilde{F})$, we have $(1-R^*)[D_1^*+D_2^*] = (1-\tilde{R}_1)\tilde{D}_1 + (1-\tilde{R}_2)\tilde{D}_2 < (1-\tilde{R}_1)(\tilde{D}_1+\tilde{D}_2)$, where the inequality follows from $\tilde{R}_2 > \tilde{R}_1$. But then $\tilde{R}_1 \geq R^*$ implies $D_1^* + D_2^* < \tilde{D}_1 + \tilde{D}_2$.¤

Note that the "~" solution is a PA solution in which bonds bear interest, while the "*" solution is one in which bonds, if they exist, sell at par. Thus, proposition 2 says that if an interest-bearing bond solution has as low an inflation rate as a noninterest-bearing bond solution, then it has a lower initial price level.

III. Examples

The examples illustrate the freedom of the nominal interest rate on bonds under PA and some of the welfare aspects of PA and of the issuance of interest-bearing bonds. They provide and, to some extent, characterize instances in which PA and bond issue is Pareto superior to LF, instances in which it is Pareto noncomparable to LF, instances in which bond issue lowers the inflation rate and instances in which it raises it, and instances in which it depresses the initial price level and instances in which it does not. Perhaps the most striking features displayed are three: (i) a group which is discriminated against in terms of rate of return can be made better off thereby; (ii) bonds, despite being safe claims to currency, can substitute for storage rather than for currency; and (iii) bond issue is inflationary if the real interest rate on bonds is positive.

In our examples we consider only two cases with regard to within-generation diversity: either there is no diversity, or each generation is composed of two groups which differ only as regards their endowments. We refer to our two-group examples as poor-rich setups. Each member h of the poor group has an endowment $(w_t^h(t), w_t^h(t+1)) = (w_1^p, w_2^p)$, while each member h of the rich group has an endowment $(w_t^h(t), w_t^h(t+1)) = (w_1^r, w_2^r) = \lambda(w_1^p, w_2^p)$, for some $\lambda > 1$. Finally, we order examples in terms of the complexity of the storage technology; we go from no storage to nonstochastic storage to stochastic storage.

1. No Storage of Goods and No Diversity

With $x_j \equiv 0$ and no diversity within a generation, LF, on the one hand, and PA with $\gamma = 0$ (no bonds) or with $P_b(t) \equiv 1$ are equivalent in the following sense: Any equilibrium under PA with $\gamma = 0$ or with $P_b(t) \equiv 1$ is also an equilibrium under LF. We are interested in comparing such equilibria with those under PA in which bonds bear interest.

With no diversity, in equilibrium all individuals in a given generation must have consumption bundles on the same indifference curve. And if $\gamma \varepsilon (0,1)$ and if bonds bear interest, some of them ("money holders") must be situated at a point like A (see Figure 2), while the others ("bondholders") must be situated at a point like B.

[INSERT FIGURE 2]

Our discussion of the no-storage, no-diversity setup is built around proposition 2.

Before stating the proposition, some notation and explanation is needed. Let N be the size of each generation, let $q(R) = d_1^h(R,R,0) + d_2^h(R,R,0)$ (where $d_i^h(R_1,R_2,F)$ is the PA demand correspondence defined above), let $S(G,0) = \{R|N(1-R)q(R)=G\}$ and let $\underline{R}_1 = \min S(G,0)$ and $\overline{R}_1 = \max S(G,0)$. Moreover, let $(\underline{c}_1,\underline{c}_2)$ be the unique solution to the following three conditions: $\underline{c}_1 + \underline{c}_2 = w_1 + w_2 - G/N$ (where $(w_1,w_2) = (w_t^h(t),w_t^h(t+1))$; $u_t^h(\underline{c}_1,\underline{c}_2) = u_t^h[w_1-q(\underline{R}_1),w_2+\underline{R}_1q(\underline{R}_1)]$; and $\underline{c}_1 < w_1 - q(\underline{R}_1)$. (Hereafter, we drop the subscript and superscript on u.) And, finally, let $(\overline{c}_1,\overline{c}_2)$ be the unique solution to: $\overline{c}_1 + \overline{c}_2 = w_1 + w_2 - G/N$, $u(\overline{c}_1,\overline{c}_2) = u[w_1-q(\overline{R}_1),w_2+\overline{R}_1q(\overline{R}_1)]$ and $\overline{c}_1 < w_1 - q(\overline{R}_1)$.

[INSERT FIGURES 3 and 4]

Note that q(R) is per capita saving when all assets bear the rate-of-return R. Equivalently, it is per capita desired real money holding when money bears the rate-of-return R and there are no other assets. In Figure 3 we depict the function $(1-R)q(R)$, which is the real per capita revenue obtained by the government when R is the return on money and holding money is the only option. For any G, S(G,0) is the set of values of R that satisfy $N(1-R)q(R) = G$. Elements of the set S(G,0) can be interpreted as alternative money-only (γ=0) equilibria under PA and as alternative equilibria under LF. Of course, if S(G,0) is not empty, there are, in general, at least two elements in it.

Figure 4 depicts the allocations corresponding to the minimal and maximal elements of S(G,0) and the corresponding indifference curves, labelled $\underline{u}$ and $\bar{u}$, respectively. The 45-degree line shown represents consumption bundles, which if common to everyone in every generation t, $t \geq 1$, are consistent with the government consuming G in every period. Moreover, for consumption bundle profiles which are identical across all generations $t \geq 1$, if some of these bundles are outside the 45-degree line depicted, then, in order that the government consume exactly G, some other bundles must be inside the line. Thus, for example, in a stationary equilibrium in which bondholders end up outside the line, money holders must end up inside it, and vice versa. Finally, cases 1 and 2 in Figure 4 refer to first-period bondholder consumption implied by different ranges for F.

We can now state

<u>Proposition 2</u>: If S(G,0) is not empty, $F \in (q(\bar{R}_1), w_1-\underline{c}_1)$, and $w_1 > 0$ and $w_2 > 0$, then for any number of bonds $n \in \{1,2,\ldots,N\}$ there exists a constant inflation rate equilibrium with positive nominal interest on bonds and N-n money holders and n bondholders. Moreover, if $F \in (q(\bar{R}_1), w_1-\bar{c}_1]$ (case 1), then $R_2 < 1$ and every member of generation t, $t \geq 1$, is on an indifference curve at least as high as $\bar{u}$; while if $F \in (w_1-\bar{c}_1, w_1-\underline{c}_1)$ (case 2), then every member of generation t, $t \geq 1$, is on an indifference curve lower than $\bar{u}$.

The proof of proposition 2 is given in the Appendix. We now discuss some consequences of proposition 2 and of well-known optimality results for overlapping generations models.

<u>Correlary 2.1</u>: If $0 < n < N$, then any proposition 2 equilibrium is not Pareto optimal.

<u>Proof</u>: This is clear from Figure 2. With N-n members of generation t, $t \geq 1$, having allocation A and n members of the same generation having allocation

B, there exists a rearrangement of these that gives everyone in the generation a preferred allocation on the line segment that connects A and B. ¤

Correlary 2.2: Let (c_1^*, c_2^*) be the preferred point on the 45-degree line of Figure 4 (that is, $c_1^* + c_2^* = w_1 + w_2 - G/N$ and $u_1(c_1^*, c_2^*)/u_2(c_1^*, c_2^*) = 1$). If $n = N$ and $F \geq w_1 - c_1^*$, then any proposition 2 equilibrium is Pareto optimal.

Proof: With $n = N$, all members of generation t, $t \geq 1$, are bondholders, so the kind of within-generation misallocation that occurs when there are both money holders and bondholders is absent. Moreover, with $n = N$, the common consumption of every member of every generation $t \geq 1$ is on the 45-degree line of Figure 4. The lower bound on F insures that this bundle is either the most preferred point on that line, or is southeast of the most preferred point. Conditional on the government getting G per period, it is well known that all such allocations are Pareto optimal. (If the bundle is southeast of the most preferred bundle, then it is easily shown that no allocation improves the well-being of any member of generation t for any $t \geq 1$ without hurting the current old. (See, for example, the proof of proposition 5 in the appendix in Wallace (1980).)¤

Correlary 2.3: If $G > 0$, then case 1 is not empty and any case 1 proposition 2 equilibrium is Pareto superior to any LF (or $n = 0$ PA) stationary equilibrium.

Proof: Under the hypotheses of proposition 2, nonemptiness of case 1 is obvious if $G > 0$. Under the current setup, Pareto superiority of any proposition 2 case 1 equilibrium follows if we can establish that any such equilibrium satisfies the hypotheses of proposition 1.

The stationary LF equilibrium that puts all members of generation t, t ≥ 1, on the $\bar{u}$ indifference curve is Pareto superior to any other LF stationary equilibria (see Figure 4). But proposition 2 says that there exists a case 1 equilibrium which puts all the members of generation t, $t \geq 1$, on an indifference curve at least that high and that has rates of return on money and bonds, (R_1,R_2), that satisfy $R_2 > R_1 \geq \overline{R}_1$. These satisfy the conditions of proposition 1 and imply, therefore, that the initial price level in the case 1 equilibrium is lower than in the best stationary LF equilibrium. Thus, the initial old are better off in the case 1 equilibrium than in the best LF equilibrium. Note, by the way, that $R_2 < 1$ in any case 1 equilibrium; although bonds bear a positive nominal interest rate, they bear a negative real interest rate in any case 1 equilibrium.¤

Correlary 2.3 describes our first instance in which the imposition of PA and the use of bonds helps in an unambiguous way. The general idea is familiar from public finance or second-best theory. With $G > 0$, it is well known that LF gives rise to a nonoptimal equilibrium. (See, for example, proposition 7 of Wallace (1980).) It gives rise to an equilibrium with a uniform, distorting excise tax on second-period consumption. PA allows for the imposition of non-linear taxes. It is no surprise, then, that better allocations are possible under this broader set of possible tax schemes.

As this discussion suggests, it should not be possible to produce Pareto superior allocations with PA and bonds if $G = 0$. This is so. With $G = 0$, it is evident that case 1 is empty; $\overline{u}$ is tangent to the 45-degree line of Figure 4. Thus, if $G = 0$, only case 2 exists, and in any case 2 equilibrium, the members of generation t, $t \geq 1$, are worse off than under the best LF equilibrium. We have not been able to establish whether the current old are necessarily better off in a case 2 equilibrium than under LF. In other words, we have not been able to establish whether the initial price level is necessarily lower in a case 2 equilibrium than it is under LF.

2. No Storage of Goods and Diversity

In the case of no diversity, we studied price discrimination that involves facing individuals with nonlinear tax schedules, essentially, two-part pricing. Here, although the tax schedule is similar, we impose sufficient diversity so that, in effect, price discrimination involves facing <u>different</u> individuals with <u>different</u> linear tax schedules. We do this by choosing the endowments of the poor and of the rich and the value of F so that no poor person can hold a bond, while each rich person wants to save an amount greater than F.

Our first result says that if the rich and the poor are sufficiently alike, then there cannot be such an interior price discrimination solution that is Pareto superior to the best LF solution.

<u>Proposition 3</u>: If the common utility function is homothetic, if $(w_1^r, w_2^r) = \lambda(w_1^p, w_2^p)$, $\lambda > 0$, and if (G,F) is such that $(R_1, R_2) \in S(G,F)$, $R_2 > R_1$, and yields internal solutions for the poor at R_1 and the rich at R_2, then there exists R such that $(R,R) \in S(G,0)$ and $R > R_1$.

<u>Proof</u>: Let $D^p(R)$ and $D^r(R)$ be the aggregate saving functions of the poor and rich, respectively, when members of each group are faced with the single rate-of-return R. It follows from the preference and endowment assumptions that $D^r(R) = \lambda^* D^p(R)$ for some $\lambda^* > 0$. If the proposition is not true, then for all $R \in (R_1, R_2]$, $(1-R)D^p(R) + (1-R)\lambda^* D^p(R) < G$. But this implies $(1-R_2)D^p(R_2) < G/(1+\lambda^*)$ and $(1-R_1)D^p(R_1) \leq G/(1+\lambda^*)$. These inequalities, in turn, imply $(1-R_1)D^p(R_1) + (1-R_2)\lambda^* D^p(R_2) < G$, which contradicts $(R_1, R_2) \in S(G,F)$.¤

We now display a numerical example that shows that for nonhomothetic utility, there can exist interest-bearing bond solutions that are Pareto superior to the best LF equilibrium.

<u>Numerical Example 1</u>

Common (Nonhomothetic) Utility Function:

$$u(c_1, c_2) = z(c_1) + z(c_2) \text{ with } z(c) = c^{.875} + \ln c.$$

Endowments: 10×10^6 poor with $(w_1^p, w_2^p) = (.01, 0)$;

100 rich with $(w_1^r, w_2^r) = (1000, 0)$.

Government Policy: G = 25,000, F = 1.0, PA.

Note that we have imposed endowments such that in the relevant ranges, $z(c)$ for the poor is approximately $\ln c$, while $z(c)$ for the rich is approximately $c^{.875}$. Then, letting $d^p(R)$ and $d^r(R)$ be individual saving functions of poor and rich, respectively, Table 1 is generated by solving $(10\mathrm{x}10^6)(1-R_1)d^p(R_1) + 100(1-R_2)d^r(R_2) = 25{,}000$ for R_1 given various selected values of R_2. We know that there exists an equilibrium for each such (R_1,R_2) pair satisfying $R_2 \geq R_1$.

[INSERT TABLE 1]

The first row of Table 1 is the LF solution. Both poor and rich face a single rate-of-return (.523), an inflation rate of almost 100 percent. While each poor person saves almost half of w_1^p (each would save exactly half, namely, .005, if the utility function was exactly $\ln c_1 + \ln c_2$), each rich person saves only about 2.5 percent of w_1^r. Note that the movement of the initial price level across rows is implied by the movement of the sum $(10 \times 10^6)\ d^p + (100)\ d^r$ by way of equation (13); the higher is this sum, the lower is the initial price level.

Each of the rows for $R_2 = .55$ to $R_2 = .95$ depicts a discriminating solution that is Pareto superior to LF. In each case, both the poor and the rich face higher rates of return than under LF, and the value of the asset holdings of the current old is also higher than under LF. At $R_2 = 1.00$ (and at values of R_2 sufficiently close to 1.00), the PA solution is not Pareto superior to LF; although the rich and the current old are better off than under LF, the poor are worse off, which is to say that the inflation rate is higher.

Our next and last example for this setup shows that the initial price level need not be decreasing in R_2, as it is in example 1.

<u>Numerical Example 2</u>

Common Utility Function: $u(c_1,c_2) = c_1^{1/2} + c_2^{1/2}$.

Endowments: 1,000 poor with $(w_1^p,w_2^p) = (1.0,0)$;

100 rich with $(w_1^r,w_2^r) = (10,0)$.

Government Policy: $G = 0$, $F = 1.0$, PA.

[INSERT TABLE 2]

In this example, higher R_2 is accompanied by a higher inflation rate and by a higher initial price level ($1000\ d^p(R_1) + 100\ d^r(R_2)$ is decreasing in R_2).

We next turn to examples with a storage technology for goods.

3. A Nonstochastic Storage Technology

Here we assume that $x_j = x > 0$ for all j so that if $k \geq K$ units of time t goods are stored, output with certainty is xk units of time t+1 good. As might be expected, the existence of this linear nonstochastic storage technology severely restricts the kinds of monetary equilibria that can exist, certainly under LF and even under PA if K is not too large. Moreover, with a storage technology, LF and PA with $\gamma = 0$ (no bonds) need not be identical. Thus, here and in the next subsection, we distinguish at least three policies, LF, PA with $\gamma = 0$, and PA with $\gamma > 0$ and with F big enough so that bonds bear interest.

Under LF, x is a lower bound on the return on currency and bonds. One potential role of the imposition of PA is the removal of this lower bound. PA does this if K, the minimum storage scale, is large enough so that it is binding under PA. If K is so large that PA rules out storage of the good completely, then all our results and examples of the last two subsections apply with one proviso: the no-bond equilibrium described there must be interpreted as a PA equilibrium and not as an LF equilibrium.

With no diversity, if PA does not rule out storage of the good, then $\underline{u}$ in Figure 4 must be replaced by the maximum of $\underline{u}$ and the level of utility implied by maximization of utility given only the option of storing the good. Subject to this reinterpretation of $\underline{u}$, proposition 2 and the correlaries listed hold.

We now consider a poor-rich setup with F and K such that neither is binding for the rich and such that the poor can hold only currency. We will, in effect, compare PA with $\gamma = 0$ to PA with $\gamma > 0$. Not surprisingly, it matters greatly for inflation whether $x > 1$ or $x \leq 1$. We state some results in two propositions.

Proposition 4a: If $x > 1$ and if $(R_1, R_2) \varepsilon S(G,F)$ and $\gamma > 0$ (bonds are outstanding), then there exists a stationary monetary equilibrium with $\gamma = 0$ and a lower inflation rate.

Proof: Since only the rich hold bonds, we have $R_2 \geq x > 1$ and, hence, $(1-R_2)D_2(R_1,R_2,F) < 0$. Therefore, $(1-R_1)D_1(R_1,R_2,F) > G$, $R_1 < 1$, and $D_1(R_1,R_2,F) = D^p(R_1)$, where $D^p(R)$ is the aggregate saving function of the poor, as defined in the proof of proposition 3. Then, since $(1-R)D^p(R) > G \geq 0$ for $R = R_1 < 1$ and $(1-R)D^p(R) = 0$ for $R = 1$, continuity of $(1-R)D^p(R)$ implies the existence of an $R \varepsilon (R_1, 1]$, say R^*, with $(1-R^*)D^p(R^*) = G$. This is a $\gamma = 0$ equilibrium because, since $x > R^*$, the rich are content to have their saving entirely in the form of storage of the good.¤

Proposition 4b: If $x \leq 1$ and if $(R_1, x) \varepsilon S(G,F)$ with $R_1 < x$ and with $D_2(R_1,R_2,F) < D^r(R_2)$ (a PA equilibrium with saving of the rich, $D^r(R_2)$, composed partly or totally of storage of the good), then there exists a Pareto superior PA equilibrium (with more bonds and less storage).

Proof: We are given $(1-R_1)D^p(R_1) + (1-x)D_2(R_1,x,F) = G$ with $D_2(R_1,x,F) < D^r(x)$. It follows from the continuity of $(1-R)D^p(R)$ that there exist values of $B \varepsilon (D_2(R_1,x,F), D^r(x)]$ such that the R that satisfies $(1-R)D^p(R) + (1-x)B = G$, denoted $R(B)$, satisfies $x > R(B) \geq R_1$, where equality arises if and only if $x = 1$. It is evident that for any such B, $(R(B),x) \varepsilon S(G,F)$ and can be supported as an equilibrium. Pareto superiority follows by showing that the initial price level is lower for any such $(R(B),x)$ equilibrium than it is for the (R_1,x) equilibrium. From $(1-R_1)D^p(R_1) + (1-x)D_2(R_1,x,F) = [1-R(B)]\ D^p(R(B)) + (1-x)B$ and $x > R(B) \geq R_1$, we get $D^p(R_1) + D_2(R_1,x,F) < D^p(R(B)) + B$. Our conclusion for the price level follows from equation (13).¤

Although, as these propositions show, bond issue has different effects on the inflation rate depending on the value of x, in some other respects the value of x is not so critical. So long as $R_1 < x$, there is a range over which the demand for bonds is perfectly elastic at $R_2 = x$. Over this range, higher γ almost certainly implies a lower initial price level.[8/] Moreover, for small and positive γ, these economies are ones for which the nominal interest rate on bonds and the inflation rate satisfy the Fisherian relationship: the nominal interest minus the inflation rate is a constant, x.

As is to be expected, a linear, nonstochastic storage technology does not give rise to smooth substitution among assets, or, in other words, to individual portfolios that are diversified in a determinate way. In the next subsection, we produce smooth substitution.

4. A Stochastic Storage Technology

Here we present four numerical examples that again illustrate the possible beneficial role of PA and bond issue. All of them are poor-rich examples. The first two use the preference and endowment assumptions of numerical example one. They differ only as regards the minimum scale for storage. In the first, there is no such minimum scale, so that everyone is free to engage in real investment under PA, while in the second the minimum scale is large enough so that the poor cannot engage in real investment under PA.

Numerical Example 3

Common Utility Function:

$$u(c_1,c_2) = z(c_1) + z(c_2) \text{ with } z(c) = c^{.875} + \ln c.$$

Endowments: 10×10^6 poor with $(w_1^p, w_2^p) = (.01, 0)$;

100 rich with $(w_1^r, w_2^r) = (1000, 0)$.

Storage Technology: $K = 0$, $J = 2$, $x_1 = 1.975$, $x_2 = .05$, $\theta_1 = \theta_2 = .5$.

Government Policy: $G = 9{,}000$, $F = 1.0$, PA.

The storage technology of this example is one with two equally likely outcomes yielding an average return of 1.0125. As noted above, there is no minimum scale for storage. Note also that the deficit in this example is smaller than that of example 1.

Some of the possible PA equilibria are described in Table 3. In addition to R_2 and R_1, we here report time t saving in terms of time t good in the form of government debt--d_g^p for each poor person and d_g^r for each rich person--and saving, similarly measured, in the form of storage of the good--d_k^p for each poor person and d_k^r for each rich person. We also report the expected utility of each poor person EU^p.

[INSERT TABLE 3]

The first row of Table 3 happens to be both the lowest inflation rate LF equilibrium and the PA equilibrium with $\gamma = 0$ or with few enough bonds so that bonds do not bear interest. The remaining rows list other PA equilibria in which bonds bear interest. Note that every row except the last one gives rise to an equilibrium which is Pareto superior to the row 1 equilibrium. (A priori, within this table, expected utility of each poor person must be strictly increasing in R_1, while that of each rich person must be strictly increasing in R_2. We include the expected utility of each poor person in order to allow for comparisons between this example and the next one.) Note that within the range depicted, the main effects of higher R_2 are more total saving by the rich and less real investment by the rich. The initial price level is lower the higher is the sum $(10 \times 10^6)\ d_g^p + (100)\ d_g^r$.

The economy of our next example is identical, except that the minimum scale for storage is big enough so that poor people cannot store the good under PA. Thus, Numerical Example 4 Identical to example 3 except that $K = 1$.

We report selected PA equilibria for this economy in Table 4.

[INSERT TABLE 4]

Note that the LF equilibrium for this economy is the same as that for the economy of example 3 (see row 1 of Table 3). Thus, every PA equilibrium depicted in Table 4 is Pareto superior to the best LF equilibrium. (Since the poor cannot store the good in this example, the well-being of the poor between Tables 3 and 4 is not ordered simply in terms of the value of R_1. However, that of the rich is ordered by the value of R_2.) Moreover, within Table 4 all the rows except the last one depict PA equilibria in which bonds bear interest and which are Pareto superior to PA equilibrium with $\gamma = 0$.

We now describe two examples that emphasize feasibility or existence of equilibrium. In example 5, the deficit is so big that no LF equilibrium exists. In example 6, which differs from example 5 only in having a larger deficit, no equilibrium exists either under LF or under PA with $\gamma = 0$. As we show, however, there are such equilibria under PA with bonds.

Numerical Example 5

Common Utility Function: $u(c_1,c_2) = \ln c_1 + \ln c_2$.

Endowments: 10×10^6 poor with $(w_1^p,w_2^p) = (.01,0)$;

100 rich with $(w_1^r,w_2^r) = (1000,0)$.

Storage Technology: $K = 1$, $j = 2$, $x_1 = 2.0$, $x_2 = .35$, $\theta_1 = \theta_2 = .5$.

Government Policy: $G = 25{,}000$, $F = 1.0$, PA.

[INSERT TABLE 5]

Table 5 describes equilibria for selected values of R_2. Note that the rich hold no government liabilities if bonds do not bear interest. Note also that every solution displayed is Pareto superior to the PA solution with $\gamma = 0$.

Numerical Example 6 Identical to example 5 except that $G = 54,000$.

[INSERT TABLE 6]

Table 6 describes three possible equilibria. Note that in this example, the deficit exceeds the aggregate endowment of the poor so that any equilibrium must involve taxation of the rich.

Examples 5 and 6 are easy to produce in the sense that we get by with homothetic preferences. They work as they do because PA prevents the poor, but not the rich, from storing the consumption good. Under PA, this gives rise to more elastic demand for government indebtedness on the part of the rich than on the part of the poor.

Notice, finally, that Tables (3)-(6) depict solutions that display high substitutability between bonds and real investment and no substitutability between bonds and currency. This happens despite the fact that currency and bonds have certain rates of return while real investment in our examples has a very risky return distribution. On the basis of these rate-of-return distributions alone, one might expect that bonds and currency would be highly substitutable, and that bonds and real investment would not be highly substitutable (see Tobin (1963)). In these examples, the restriction that allows bonds to dominate currency in terms of rate of return also gives rise to high substitutability between bonds and storage of the good.

IV. Concluding Remarks

Despite the amount of space we have expended on the illustrative model and specific examples of it, it is important to keep in mind that it is only an illustrative model. The usefulness of postulates A-C is not to be judged by the applicability of our illustrative model to a particular economy. Our choice of an illustrative model was largely dictated by a desire to keep things simple.

We would certainly insist that any illustrative model has to be dynamic. Given that requirement, an overlapping generations model of two-period-lived agents seems to us to offer the simplest setting for examples. Note in this regard, that neither our postulates nor even our illustrative model commits us one way or another on the question of whether equilibria with valued fiat money can arise without legal restrictions. Our examples include both situations (the assumptions of proposition 5a rule out such equilibria, while those of 5b do not). Within the class of overlapping generations models, we chose setups so that portfolio autarky would be equivalent to a government monopoly on the making of small change; there seems to be a long history of governmental restrictions on the making of small change (see, for example, Timberlake (1978), Chapter 6). We have also, of course, kept things very simple in other regards. Our poor-rich examples could be made more complicated (and, perhaps, more realistic) by assuming that bonds can be purchased only in discrete amounts, instead of in any amount greater than some minimum. Also, we have simplified by abstracting from enforcement costs.

The view we are espousing is not to be judged by whether these simplifications depict any actual economy at any particular time. Our goal is not to get the reader to accept our illustrative model. It is to get the reader to entertain postulates A-C and the public-finance price-discrimination view of financial sector regulations and the composition of government indebtedness that goes along with those postulates. In saying this, we do not mean to suggest that all governments choose portfolio restrictions and debt compositions with the express intent of price discriminating. Sometimes, however, they seem to. For example, the attempts by the United States in recent years to sell mark-denominated bonds in Germany would seem to be an attempt to price discriminate. Clearly, the idea, possibly wrong, is that the U.S. and German markets are separate enough so that sales in Germany are less likely to substitute for holdings of other U.S. government liabilities than would sales of the same bonds in the United States.

But for many readers, no doubt, postulates A-C must seem to be a step backward from what Hicks suggested. Put another way, there is a sense in which an economist who adopts postulates A-C is giving up; he or she gives up seeking a "natural" explanation for a wide range of facts in favor of an "unnatural" explanation, one that relies on legal restrictions. To us, however, it is obvious that economists have tried to do too much with the kind of explanation suggested by Hicks. As we noted above, we would be prepared to interpret a 1 percent nominal yield on U.S. Treasury bills as arising from the frictions Hicks described. However, given the intermediation spreads charged by mutual funds, we are very reluctant to so interpret yields of 6 percent, 8 percent, and 10 percent. To take another example, we think it is a mistake to explain the existence of well-defined demands for different national fiat monies on the basis of such frictions (see Kareken-Wallace (forthcoming)). In both these instances, and in others, it seems clear that an adequate model must contain legal restrictions.

However, even if a model must contain legal restrictions, there is a potential alternative to what we do. We simply appeal to or impose legal restrictions. A preferable procedure is to explain legal restrictions with a deep public finance theory. For example, while we produce examples in which the costless imposition of portfolio autarky is beneficial vis-a-vis laissez-faire, we do not produce the underlying physical setting that rules out all forms of taxation other than those implied by a deficit. Nor do we have a theory that describes the feasible and best legal restrictions. We would like to have a theory that does all that and that is simultaneously the deep transaction cost theory alluded to in Section I.3. But in the absence of such a theory, we think postulates A-C deserve serious consideration as an alternative to the existing theories described in Sections I.1 and I.2. We have tried to demonstrate that postulates A-C provide a better foundation than do those theories for models that both explain rate-of-return dominance and address many of the policy questions we want to address.

Appendix: Proof of Proposition 3

Let $\overline{g}$ be the unique maximum of $(1-R)q(R)$ and let $\underline{R}$ be the unique value of R such that $q(R) = 0$ (see Figure 3). Also, let

$$S_{[a,b]}(R_2) = \{R \mid (N-n)(1-R)q(R)+n(1-R_2)F = G \text{ and } R \in [a,b]\}.$$

The crucial fact we use, which is implied by the continuity of $(1-R)q(R)$ in R, is as follows: if $[a,b] \subset [\underline{R},1]$, then $(R_2, S_{[a,b]}(R_2))$ is a continuous curve in $[1-G/nF, 1-(G-(N-n)\overline{g})/nF] \times [a,b]$.

Now, let $u^m(R) = u(w_1-q(R), w_2+Rq(R))$ and let $u^b(R) = u(w_1-F, w_2+RF)$, where u^m is to be interpreted as money-holder utility and u^b as bondholder utility. It follows that $(R_2, u^m[S_{[a,b]}(R_2)])$ is a continuous curve in $[1-G/nF, 1-(G-(N-n)\overline{g})/nF] \times [u^m(a), u^m(b)]$ and that $\Omega = \{(R_2,u) \mid R_2 > -w_2/F \text{ and } u \leq u^b(R_2)\}$ is a convex set in $(-\infty,\infty) \times (-\infty,\infty)$.

We now show that there exist points of $(R_2, u^m[S_{[a,b]}(R_2)])$ both outside of and in Ω, and thereby establish that there exist one or more points of the former which are on the boundary of Ω. We also show that there is an equilibrium corresponding to any such point.

Let $\hat{R}_2$ be such that $(1-\hat{R}_2)F = G/N$. (Note that $\hat{R}_2$ is such that $(w_1-F, w_2+\hat{R}_2F)$ is on the 45-degree line of Figure 4.) We consider two cases separately, cases which correspond to cases 1 and 2 in Figure 4.

<u>Case 1</u>: $u^b(\hat{R}_2) \geq \overline{u}$. Here we let $[a,b] = [\overline{R}_1, 1]$.

Since $(1-\overline{R}_1)q(\overline{R}_1) = G/N$ and $(1-\hat{R}_2)F = G/N$, $\overline{R}_1 \in S_{[\overline{R}_1,1]}(\hat{R}_2)$. And since $u^b(\hat{R}_2) \geq \overline{u} = u^m(\overline{R}_1)$, $(\hat{R}_2, u^m(\overline{R}_1))$ is a point of $(R_2, u^m[S_{[\overline{R}_1,1]}(R_2)])$ which is in Ω.

We now show that $(1-G/nF, u^m(1))$ is a point of $(R_2, u^m[S_{[\overline{R}_1,1]}(R_2)])$ which is not in Ω. First, $1 \in S_{[\overline{R}_1,1]}(1-G/nF)$, which implies that $(1-G/nF, u^m(1))$

is a point of $(R_2, u^m[S_{[\overline{R}_1,1]}(R_2)])$. Now, if $1 - G/nF \leq -w_2/F$, then, by definition, $(1-G/nF, u^m(1))$ is not in Ω. If $1 - G/nF > -w_2/F$, it follows from $1 - G/nF \leq 1$ and $F > q(\overline{R}_1)$ that $u^m(1) > u^b(1-G/nF)$. This also implies that $(1-G/nF, u^m(1))$ is not in Ω.

Having shown that there are points of $(R_2, u^m[S_{[\overline{R}_1,1]}(R_2)])$ both in and outside of Ω, it follows that there is a point on the former which is on the boundary of Ω. Let us denote by (R_2^*, R_1^*) the associated point on the curve $(R_2, S_{[\overline{R}_1,1]}(R_2))$.

We have shown that (i) $u^m(R_1^*) = u^b(R_2^*)$, (ii) $(N-n)(1-R_1^*)q(R_1^*) + n(1-R_2^*)F = G$, and (iii) $R_1^* \geq \overline{R}_1$. To establish that (R_2^*, R_1^*) is an equilibrium, it remains to show that $b = F$ maximizes $u(w_1-b, w_2+R_2^*b)$ subject to $b \geq F$. This follows from (i) if we can show that $F > q(R_1^*)$.

Suppose $F \leq q(R_1^*)$. If so, then since $R_2^* \geq R_1^*$ (this follows from (i)) (ii) implies $(1-R_1^*)q(R_1^*) \geq G/N$. But by the definition of $\overline{R}_1$, this implies $R_1^* \leq \overline{R}_1$. From (iii) we then conclude that $R_1^* = \overline{R}_1$ or that $F \leq q(\overline{R}_1)$, a violation of our hypothesis on F.

Our last task for case 1 is to show that $R_2^* \leq 1$. Since $R_1^* \geq \overline{R}_1$ $(1-R_1^*)q(R_1^*) \leq G/N$. This and (ii) imply $(1-R_2^*)F \geq G/N$ and, therefore, $R_2^* \leq \hat{R}_2$. Since $\hat{R}_2 \leq 1$, we have $R_2^* \leq 1$.

<u>Case 2</u>: $u^b(\hat{R}_2) < \overline{u}$. Here we let $[a,b] = [\underline{R}_1, \overline{R}_1]$.

Clearly, $\underline{R}_1 \in S_{[\underline{R}_1,\overline{R}_1]}(\hat{R}_2)$. Also, $u^m(\underline{R}_1) \leq u^b(\hat{R}_2)$. It follows that $(\hat{R}_2, u^m(\underline{R}_1))$ is a point of $(R_2, u^m[S_{[\underline{R}_1,\overline{R}_1]}(R_2)])$ which is in Ω. Since $\overline{R}_1 \in S_{[\underline{R}_1,\overline{R}_1]}(\hat{R}_2)$ and $u^b(\hat{R}_2) < u^m(\overline{R}_1)$ by assumption, it follows that $(\hat{R}_2, u^m(\overline{R}_1))$ is a point of $(R_2, u^m[S_{[\underline{R}_1,\overline{R}_1]}(R_2)])$ that is not in Ω. Therefore, as in case 1, there is a point of the former which is on the boundary of Ω. Let us again denote by (R_2^*, R_1^*) the corresponding point of the curve $(R_2, S_{[\underline{R}_1,\overline{R}_1]}(R_2))$.

We now show that $u^b(R_2^*) < \bar{u}$. Suppose to the contrary that $u^b(R_2^*) \geq \bar{u}$. Because $u^b(\hat{R}_2) < \bar{u}$, $R_2^* > \hat{R}_2$ and, therefore, $(1-R_2^*)F < G/N$. This implies $(1-R_1^*)q(R_1^*) > G/N$. And since $u^m(R_1^*) = u^b(R_2^*) \geq \bar{u}$, we also have $R_1^* \geq \bar{R}_1$. But this and $(1-R_1^*)q(R_1^*) > G/N$ contradict the assumption that $\bar{R}_1$ is the largest value of R satisfying $(1-R)q(R) = G/N$.

For case 2 we have now shown that (i) $u^m(R_1^*) = u^b(R_2^*) < \bar{u}$ and (ii) $(N-n)(1-R_1^*)q(R_1^*) + n(1-R_2^*)F = G$. To complete the argument we must, as for case 1, show that $F > q(R_1^*)$.

Suppose instead that $F \leq q(R_1^*)$. Then since $R_2^* \geq R_1^*$ (by the first equality of (i)), (ii) implies $(1-R_1^*)q(R_1^*) \geq G/N$ and $(1-R_2^*)F \leq G/N$. In words, money holders are on or inside the 45-degree line of Figure 4 and bondholders are on or outside it. This and (i) and $F \leq q(R_1^*)$ imply $F < q(\bar{R}_1)$, a violation of our hypothesis on F.

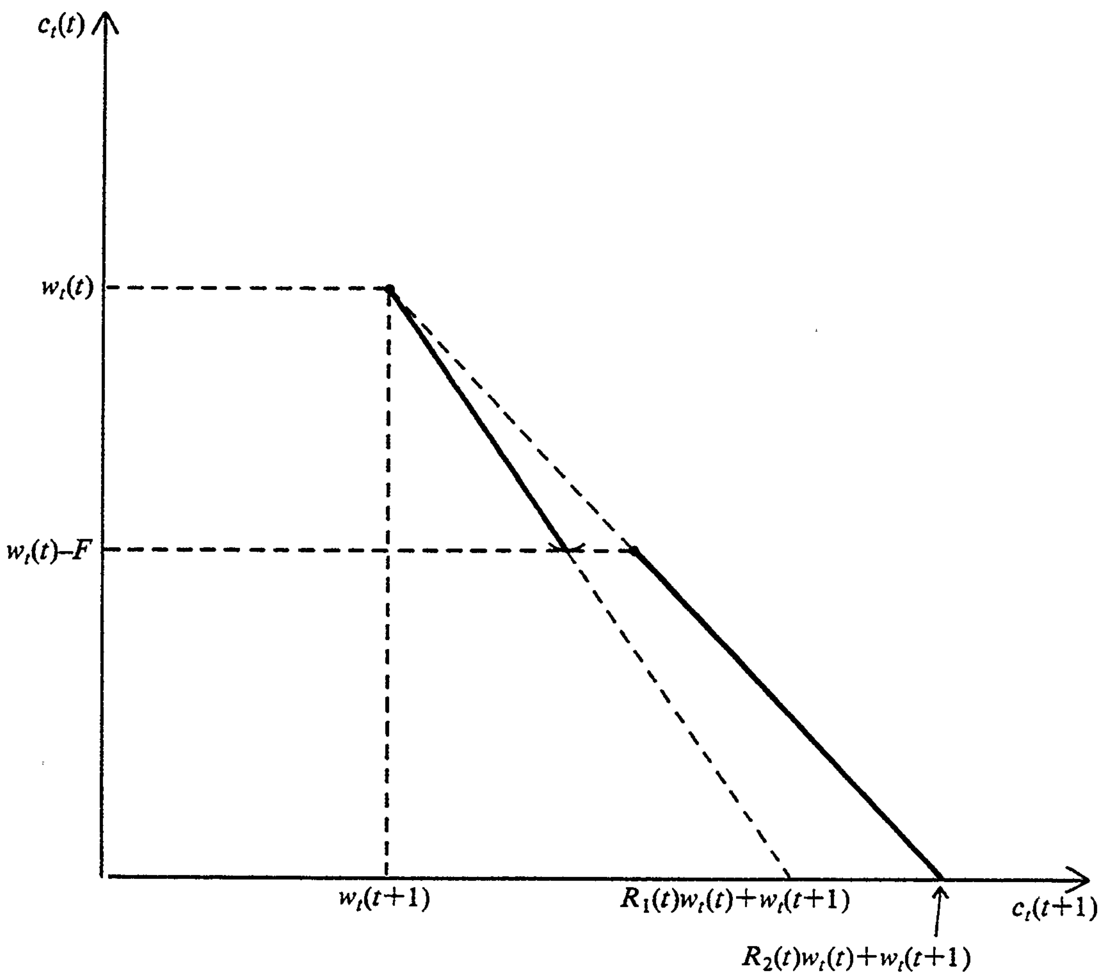


Figure 1

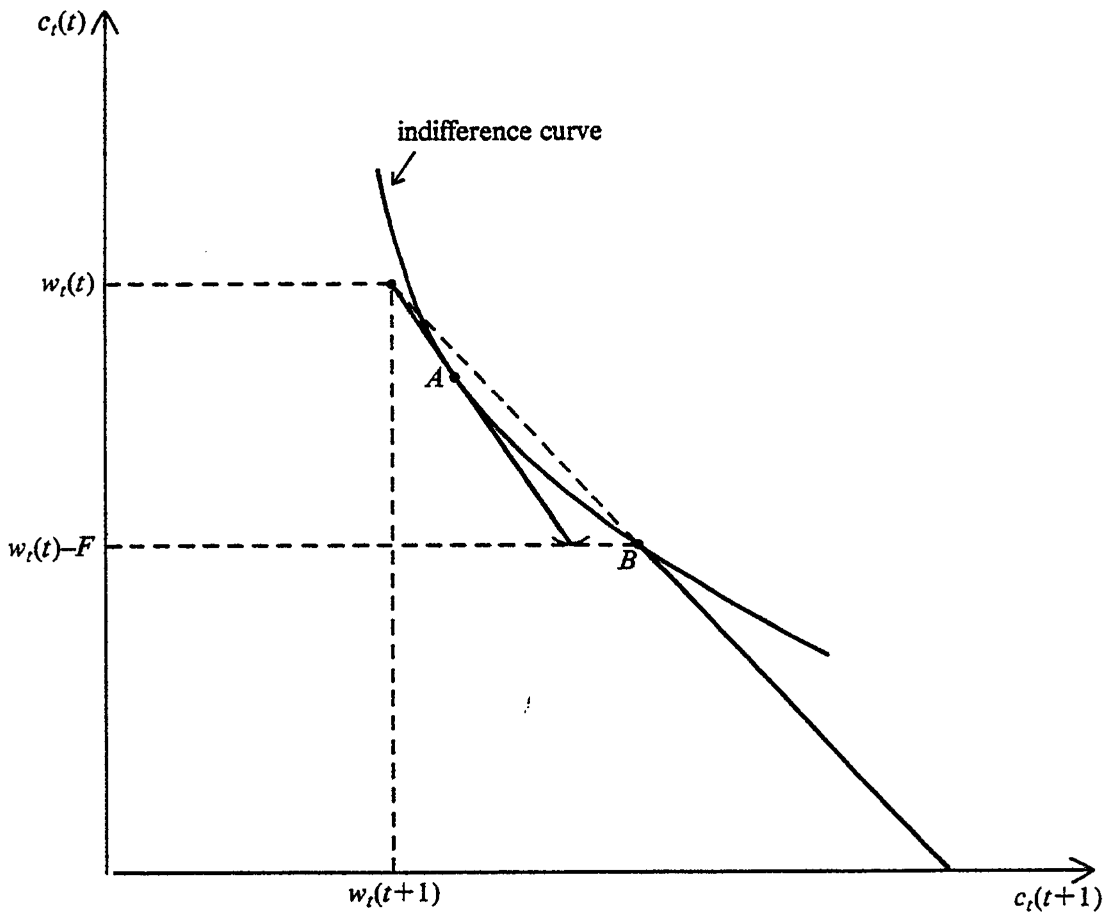


Figure 2

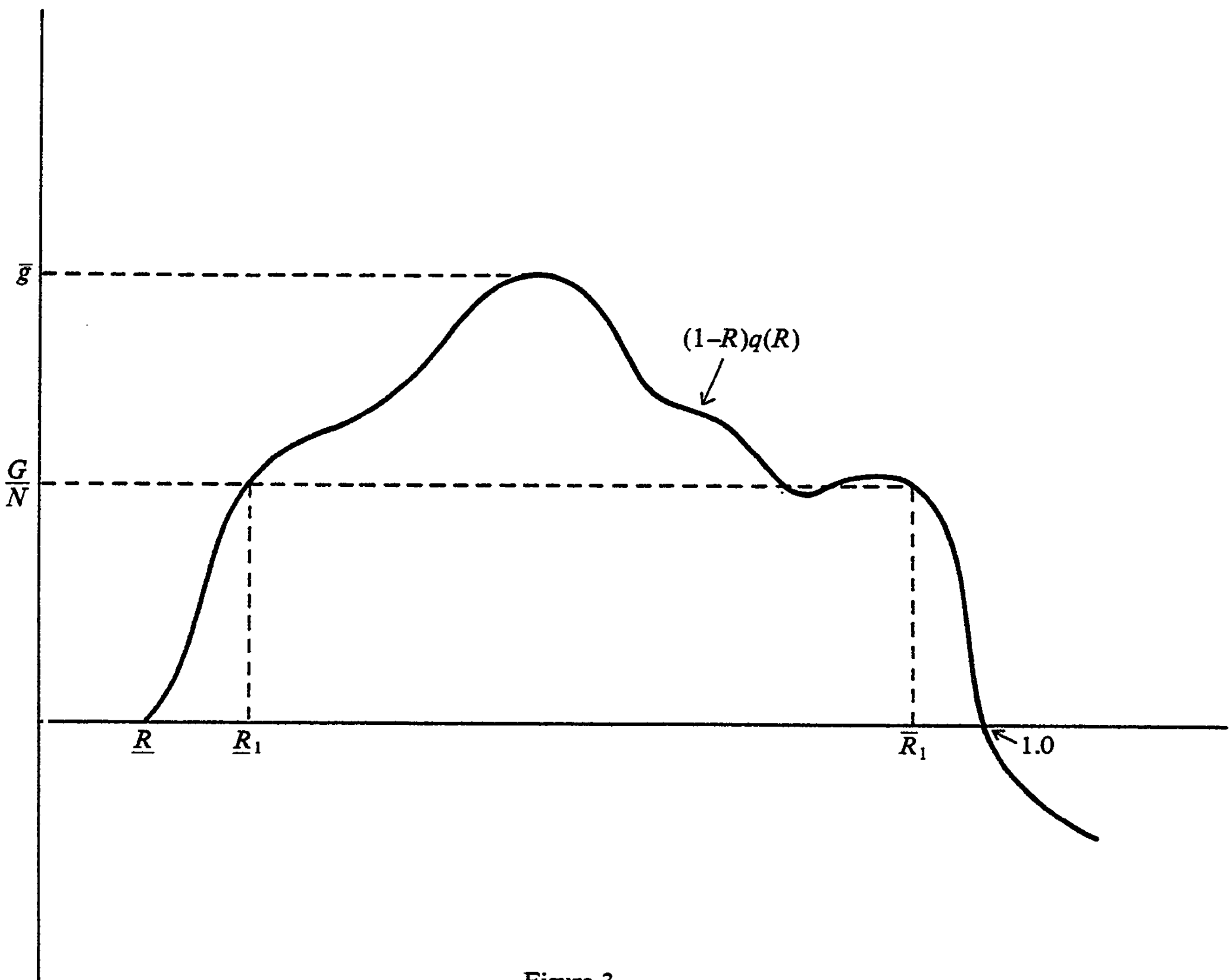
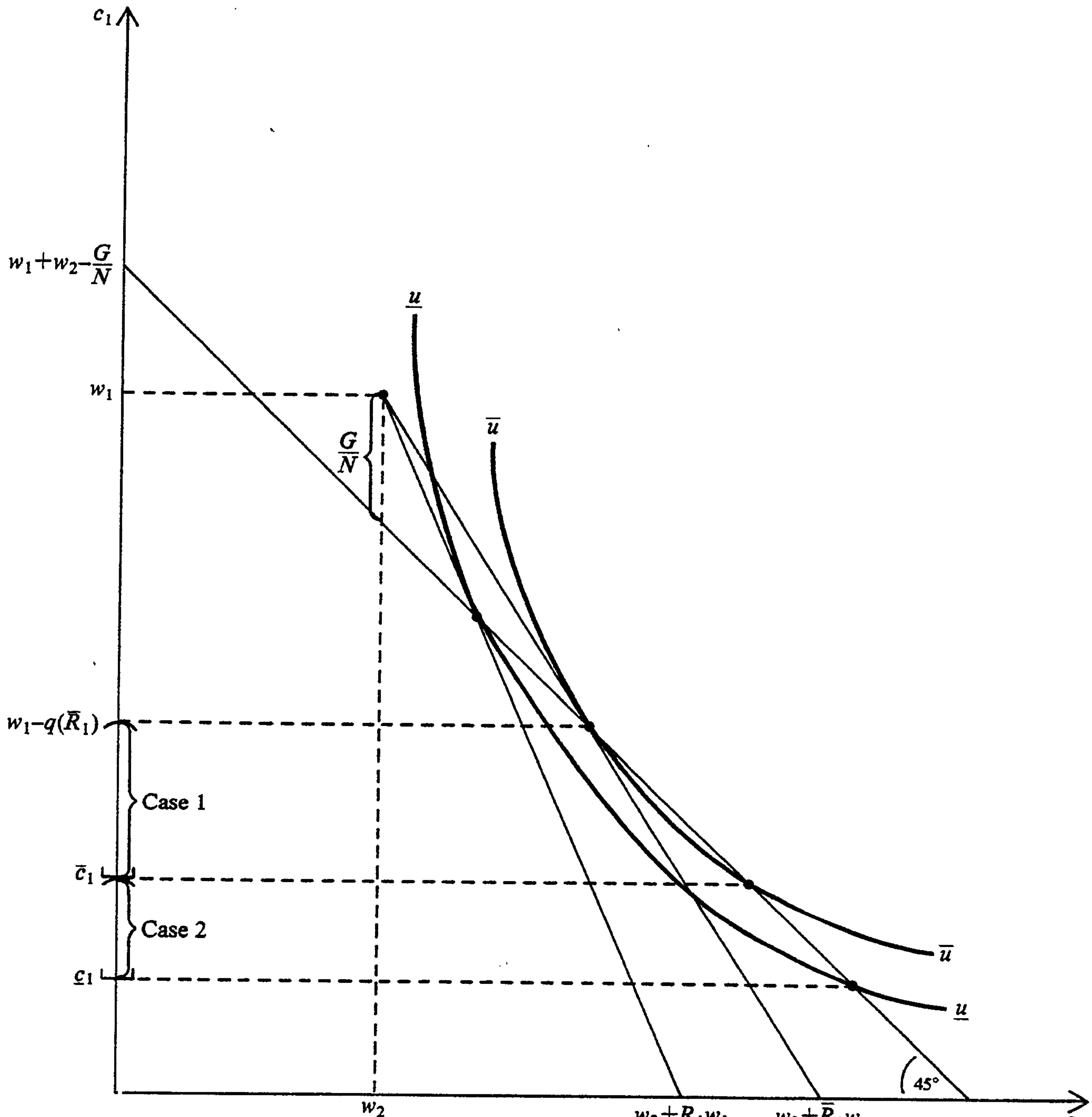


Figure 3



c_1
$w_1+w_2-\frac{G}{N}$
w_1
$\frac{G}{N}$
$\underline{u}$
$\overline{u}$
$w_1-q(\overline{R}_1)$
Case 1
$\overline{c}_1$
Case 2
$\underline{c}_1$
$\overline{u}$
$\underline{u}$
45°
w_2
$w_2+\underline{R}_1w_1$
$w_2+\overline{R}_1w_1$
c_2

Figure 4

TABLE 1
Some Alternative Equilibria for Example 1

R_2	R_1	$d^p(R_1)$	$d^r(R_2)$
.523	.523	.00499	25.24
.550	.526	.00499	30.35
.600	.534	.00499	43.57
.650	.544	.00499	63.40
.700	.554	.00499	92.33
.750	.566	.00499	132.76
.800	.573	.00499	186.32
.850	.575	.00499	252.98
.900	.565	.00499	330.51
.950	.541	.00499	414.65
1.000	.499	.00499	500.00

TABLE 2
Some Alternative Equilibria for Example 2

R_2	R_1	$d^p(R_1)$	$d^r(R_2)$
1.000	1.000	.5000	5.000
1.010	.990	.4975	5.025
1.050	.947	.4865	5.122
1.100	.889	.4705	5.238

TABLE 3
Some Alternative Equilibria for Example 3

R_2	R_1	d_g^p	d_g^r	d_k^p	d_k^r	EU^p
.773	.773	.00393	5	.00107	385	-10.804
.800	.780	.00395	16	.00104	373	-10.747
.850	.794	.00400	49	.00099	344	-10.783
.900	.806	.00405	113	.00095	295	-10.771
.950	.807	.00405	235	.00044	207	-10.770
.975	.796	.00401	328	.00098	140	-10.781
1.000	.770	.00391	449	.00108	52	-10.807

TABLE 4

Some Alternative Equilibria for Example 4

R_2	R_1	d_g^p	d_g^r	d_k^p	d_k^r	EU^p
.831	.831	.00500	34	0	357	-10.763
.850	.835	.00500	49	0	344	-10.759
.900	.843	.00500	113	0	295	-10.750
.950	.843	.00500	235	0	207	-10.749
.975	.836	.00500	328	0	140	-10.757
1.000	.820	.00500	449	0	52	-10.777

TABLE 5
Some Alternative Equilibria for Example 5

R_2	R_1	d_g^p	d_g^r	d_k^p	d_k^r
.5000	.5000	.005	0	0	500
.5957	.5000	.005	0	0	500
.6000	.5057	.005	7	0	493
.6500	.5551	.005	79	0	421
.7000	.5808	.005	135	0	365
.7500	.5906	.005	181	0	319
.8000	.5889	.005	222	0	278
.8500	.5779	.005	240	0	260
.9000	.5590	.005	295	0	205
.9500	.5330	.005	330	0	170
1.000	.5000	.005	365	0	135

TABLE 6
Some Alternative Equilibria for Example 6

R_2	R_1	d_g^p	d_g^r	d_k^p	d_k^r
.70	.0008	.005	135	0	365
.75	.0106	.005	181	0	319
.80	.0089	.005	222	0	278

Footnotes

1/ This section borrows heavily from Kareken-Wallace (1980).

2/ A contradiction is obvious if the solution of M implies nonrandom asset prices. More generally, since the asset price distributions implied by M depend on all the M_i, consistency with the distributions assumed in U_j seems farfetched in the extreme. (A necessary condition for the U_i being consistent is that solutions of M not contradict any of the U_i. Because U_i contains more than M_i, mere existence of a solution of M does not imply that the U_i are mutually consistent.) See Lucas (1976) for other examples of inconsistencies.

3/ Strangely enough, both finance theory and the starting-with-curves approach appeal to a risk-aversion portfolio diversification model. It seems unlikely that both appeals are valid.

4/ Fama (1980) makes a similar suggestion.

5/ The model is similar to those used in Bryant-Wallace (1980) and Wallace (forthcoming).

6/ If government consumption of time t good, G(t), affects individual welfare, it is assumed to do so in a separable way. That is, if $V_t[c_t^h(t), c_t^h(t+1), G(t), G(t+1)]$ is the utility function of member h of generation t, we assume that $V_t^h(\quad) = U_t^h[u_t^h(c_t^h(t), c_t^h(t+1)), v_t^h(G(t), G(t+1))]$, where U_t^h is increasing in its first argument.

7/ Note that (1) implies that explicit taxes are not levied and, in particular, are not levied to cover interest on debt. One interpretation of this is that the government has exhausted the possible use of explicit taxes and that G(t) represents government consumption in excess of that financed by explicit taxes.

8/ It is easy to produce examples in which higher γ implies a lower initial price level and a higher inflation rate. In such instances, casual observers could mistake the once-for-all price level effect for a favorable inflation rate effect.

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