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Appendix for: Optimal Cooperative Taxation in the Global Economy

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Appendix for Optimal Cooperative Taxation in The Global Economy by Chari, Nicolini and Teles

A Competitive Equilibrium with Consumption, Labor and Trade Taxes

The first-order conditions of the household's problem include

(A.1) $-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{(1+\tau^c_{it})\,q_{it}}{(1-\tau^n_{it})\,w_{it}},$

(A.2) $u^i_{c,t} = \frac{Q_t q_{it}\,(1+\tau^c_{it})}{Q_{t+1} q_{it+1}\left(1+\tau^c_{it+1}\right)}\beta u^i_{c,t+1},$

for all $t \geq 0$, where $u^i_{c,t}$ and $u^i_{n,t}$ denote the marginal utilities of consumption and labor in period t. Note that (A.2) can be used to recover the familiar interest rate parity condition,

$$\frac{u^i_{c,t}\left(1+\tau^c_{it+1}\right)e^i_{t+1}}{\beta u^i_{c,t+1}\,(1+\tau^c_{it})\,e^i_t} \text{ is the same for all } i$$

where $e^i_t \equiv q_{it}/q_{1t}$ denotes the price of the final goods in country i in units of final goods in, say, country 1–namely, the bilateral real exchange rate relative to country 1.

The first-order conditions of the firms' problems are, for all i and all $t \geq 0$,

(A.3) $p_{iit}F^i_{n,t} = w_{it},$

(A.4) $\frac{Q_t}{Q_{t+1}} = \frac{p_{iit+1}}{q_{it}}F^i_{k,t+1} + \frac{q_{it+1}}{q_{it}}\,(1-\delta)\,,$

where $F^i_{n,t}$ and $F^i_{k,t}$ denote the marginal products of capital and labor in period t,

(A.5) $p_{iit} = \left(1-\tau^x_{ijt}\right)p_{ijt},\ i \neq j,$

(A.6) $q_{it}G^i_{i,t} = p_{iit},$

(A.7) $q_{it}G^i_{j,t} = \left(1+\tau^m_{jit}\right)p_{jit}$, and $i \neq j$.

If we combine the household's and firm's equilibrium conditions, it can be shown that the value of the firm in (11) is

$$V_{i0} + d_{i0} = q_{i0}\left[1 - \delta + G^i_{i,0} F^i_{k,0}\right] k_{i0}.$$

We can obtain the familiar condition that the returns on capital adjusted for the real exchange rates are equated across countries. To obtain this condition, note that (A.4) and (A.6) can be combined to obtain that

$$\left[G^i_{i,t+1} F^i_{k,t+1} + 1 - \delta\right] \frac{e^i_t}{e^i_{t+1}} \text{ is the same for all } i.$$

We now derive the equilibrium conditions (15)-(18). Using conditions (A.1), (A.3), and (A.6), we obtain (15). Using (A.2), (A.4), and (A.6), we obtain (16).

Using (A.7),

$$\frac{G^i_{j,t}}{G^i_{l,t}} = \frac{\left(1 + \tau^m_{jit}\right) p_{jit}}{\left(1 + \tau^m_{lit}\right) p_{lit}},$$

and (A.5),

$$\frac{p_{jjt}}{p_{llt}} = \frac{\left(1 - \tau^x_{jit}\right) p_{jit}}{\left(1 - \tau^x_{lit}\right) p_{lit}},$$

we have that

$$\frac{\left(1 - \tau^x_{jit}\right)\left(1 + \tau^m_{lit}\right) G^i_{j,t}}{\left(1 + \tau^m_{jit}\right)\left(1 - \tau^x_{lit}\right) G^i_{l,t}} = \frac{p_{jjt}}{p_{llt}}$$

is the same for all i, which is condition (17).

Using (A.4), (A.6), and (A.7), we have

$$\frac{Q_t}{Q_{t+1}} = \frac{\left(1 + \tau^m_{jit+1}\right) p_{jit+1} G^i_{j,t}}{\left(1 + \tau^m_{jit}\right) p_{jit} G^i_{j,t+1}} \left[G^i_{i,t+1} F^i_{k,t+1} + 1 - \delta\right],$$

and using (A.5), we obtain that

$$\frac{Q_t p_{jjt}}{Q_{t+1} p_{jjt+1}} = \frac{\left(1+\tau^m_{jit+1}\right)\left(1-\tau^x_{jit}\right) G^i_{j,t}}{\left(1-\tau^x_{jit+1}\right)\left(1+\tau^m_{jit}\right) G^i_{j,t+1}} \left[G^i_{i,t+1} F^i_{k,t+1} + 1 - \delta\right]$$

is the same for all i, which is (18).

A.1 Balance of Payments Conditions

Here, we show that in an economy with two countries $i = 1, 2$, the following balance of payments conditions must hold:

$$\sum_{t=0}^{\infty} Q_t \left[p_{ijt} y_{ijt} - p_{jt} y_{jit}\right] = -\left(1 + r_0^f\right) f_{i,0}, \text{ for } j \neq i,$$

with $\left(1 + r_0^f\right) f_{1,0} + \left(1 + r_0^f\right) f_{2,0} = 0$.

The budget constraints of the household and the government, with equality, in each country,

$$\sum_{t=0}^{\infty} Q_t \left[q_{it}\left(1+\tau^c_{it}\right) c_{it} - \left(1-\tau^n_{it}\right) w_{it} n_{it}\right] = \left(1-\tau^W_i\right) a_{i0},$$

$$a_{i0} = V_{i0} + d_{i0} + Q_{-1} b_{i0} + \left(1 + r_0^f\right) f_{i0},$$

and

$$\sum_{t=0}^{\infty} Q_t \left[\tau^c_{it} q_{it} c_{it} + \tau^n_{it} w_{it} n_{it} + \tau^x_{ijt} p_{ijt} y_{ijt} + \tau^m_{ijt} p_{jit} y_{jit} - q_{it} g_{it}\right] + \tau^W_i a_{i0} = Q_{-1} b_{i0},$$

imply

$$\sum_{t=0}^{\infty} Q_t \left[q_{it} c_{it} + q_{it} g_{it} - \tau^x_{ijt} p_{ijt} y_{ijt} - \tau^m_{ijt} p_{jit} y_{jit} - w_{it} n_{it}\right] = V_{i0} + d_{i0} + \left(1 + r_0^f\right) f_{i0}.$$

Using the expression for the value of the intermediate good firm,

$$V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t \left[p_{iit} y_{iit} + \left(1-\tau^x_{ijt}\right) p_{ijt} y_{ijt} - w_{it} n_{it} - q_{it} x_{it}\right],$$

we get

$$\begin{aligned}
&\sum_{t=0}^{\infty} Q_t \left[q_{it}c_{it} + q_{it}g_{it} - \tau^x_{ijt}p_{ijt}y_{ijt} - \tau^m_{ijt}p_{jit}y_{jit}\right] \\
= &\sum_{t=0}^{\infty} Q_t \left[p_{iit}y_{iit} + \left(1-\tau^x_{ijt}\right)p_{ijt}y_{ijt} - q_{it}x_{it}\right] + \left(1+r^f_0\right) f_{i0},
\end{aligned}$$

or

$$\sum_{t=0}^{\infty} Q_t \left[q_{it}\left(c_{it}+g_{it}+x_{it}\right) - p_{iit}y_{iit} - p_{ijt}y_{ijt} - \tau^m_{ijt}p_{jit}y_{jit}\right] = \left(1+r^f_0\right) f_{i0}.$$

Using the zero profits condition of the final good firms,

$$\sum_{t=0}^{\infty} Q_t \left[q_{it}\left(c_{it}+g_{it}+x_{it}\right) - p_{iit}y_{iit} - \left(1+\tau^m_{jit}\right)p_{jit}y_{jit}\right] = 0,$$

we have

$$\sum_{t=0}^{\infty} Q_t \left[p_{jit}y_{jit} - p_{ijt}y_{ijt}\right] = \left(1+r^f_0\right) f_{i0}, \text{ for } i=1,2, \text{ and } i \neq j,$$

which is the balance of payments condition, with $\left(1+r^f_0\right) f_{10} + \left(1+r^f_0\right) f_{20} = 0$.

Using the final goods firms' conditions, (A.5)-(A.7), repeated here,

$$p_{iit} = \left(1-\tau^x_{ijt}\right)p_{ijt},\ i \neq j,$$

$$q_{it}G^i_{i,t} = p_{iit},$$

$$q_{it}G^i_{j,t} = \left(1+\tau^m_{jit}\right)p_{jit}, \text{ and } i \neq j.$$

together with the household's intertemporal condition, (A.2),

$$u^i_{c,t} = \frac{Q_t q_{it}\left(1+\tau^c_{it}\right)}{Q_{t+1}q_{it+1}\left(1+\tau^c_{it+1}\right)}\beta u^i_{c,t+1},$$

we obtain the balance of payments condition,

$$\sum_{t=0}^{\infty} \frac{(1+\tau_{i0}^{c})}{(1+\tau_{it}^{c})} \frac{\beta^{t} u_{c,t}^{i}}{u_{c,0}^{i}} \left[\frac{G_{j,t}^{i} y_{jit}}{(1+\tau_{jit}^{m})} - \frac{G_{i,t}^{i} y_{ijt}}{(1-\tau_{ijt}^{x})} \right] = \left(1+r_{0}^{f}\right) \frac{f_{i0}}{q_{i0}}, \text{ for } i=1,2, \text{ and } i \neq j,$$

where

$$\frac{u_{c,0}^{i}(1+\tau_{it}^{c})}{\beta u_{c,t}^{i}(1+\tau_{i0}^{c})} = \Pi_{s=0}^{t}\left[G_{i,s}^{i} F_{k,s}^{i} + 1 - \delta\right],$$

$$\sum_{t=0}^{\infty} \frac{1}{\Pi_{s=0}^{t}\left[G_{i,s}^{i} F_{k,s}^{i} + 1 - \delta\right]} \left[\frac{G_{j,t}^{i} y_{jit}}{(1+\tau_{jit}^{m})} - \frac{G_{i,t}^{i} y_{ijt}}{(1-\tau_{ijt}^{x})} \right] = \left(1+r_{0}^{f}\right) \frac{f_{i0}}{q_{i0}}, \text{ for } i=1,2, \text{ and } i \neq j.$$

B Production Efficiency

B.1 Proof of Proposition 2

Here, we show that trade taxes can be chosen to satisfy both production efficiency and the balance of payments conditions (25). We begin by setting the trade taxes so that $\tau_{ijt}^{x} = -\tau_{ijt}^{m}$, to satisfy production efficiency. We define variables $\varkappa_{ijt}$, g_{ijt}, and h_{jit} as

$$\varkappa_{ijt} \equiv \frac{1}{(1-\tau_{ijt}^{x})},\ g_{ijt} \equiv \frac{1}{\Pi_{s=0}^{t}\left[G_{i,s}^{i} F_{k,s}^{i} + 1 - \delta\right]} G_{i,t}^{i} y_{ijt}, \text{ and } h_{jit} \equiv \frac{1}{\Pi_{s=0}^{t}\left[G_{i,s}^{i} F_{k,s}^{i} + 1 - \delta\right]} G_{j,t}^{i} y_{jit}$$

and rewrite (25) as

$$\text{(B.1)} \quad \sum_{t=0}^{\infty} \sum_{j \neq i} \left[\varkappa_{ijt} g_{ijt} - \varkappa_{jit} h_{jit}\right] = R_{i} \text{ for all } i,$$

where R_i is the right-hand side of (25). Proving proposition 2 amounts to finding $\varkappa_{ijt}$ that satisfy $(B.1)$. We find it useful to restate definitions from graph theory.

Definition 1: (Direct link) We say that there is direct link between a pair of countries (i,j) if there exists some t such that either $g_{ijt} \neq 0$ or $h_{jit} \neq 0$.

Definition 2: (Indirect link) We say that a pair of countries (i,j) is indirectly linked if there is a sequence of countries $\{i,..,j\}$ such that every pair of consecutive elements in the sequence is directly linked.

Definition 3: (Connectedness) Countries are connected if for every pair of countries, there is a direct or indirect link between them.

Definition 4: (Complete cover) A sequence is a complete cover if

1. every country is an element of the sequence;
2. every pair of consecutive countries in the sequence is directly linked.

Remark 1: Notice that sequences that are a complete cover may contain the same country several times.

Lemma 1: If countries are connected, there exists a finite complete covering.

Proof: Consider a sequence that begins with country 1 and ends with country 2. Such a link exists because the countries are connected. Append to the sequence a sequence that begins with country 2 and ends with country 3. Proceed in this fashion until we end with country N.

We measure the length of a sequence by the number of elements in it. Since a finite complete covering exists, it immediately follows that there is a shortest finite complete cover.[1]

Lemma 2: The first country in a shortest complete cover appears only once in the sequence.

Proof: Suppose the first country appears more than once. Then, consider a new sequence that omits the first element. This new sequence is a complete cover, since all countries appear on it and are connected. ■

We now describe an algorithm to construct a set of policies $\{\varkappa_{ijt}\}$ for all i, j, t that satisfy $(B.1)$. The first main step is to fix a shortest complete cover for countries 1 to N and to relabel the countries so that the first element in this complete cover is relabeled as country 1 and the second element as country 2. Since country 1 has a direct link with country 2, either $g_{1,2,t} \neq 0$ or $h_{2,1,t} \neq 0$ for some t. Set $\varkappa_{1jt} = 1$ and $\varkappa_{j1t} = 1$ for all $j > 2$, and set $\varkappa_{12t}$ and $\varkappa_{21t}$ so as to satisfy the balance of payment condition for country 1. Note that $\varkappa_{12t}$ and $\varkappa_{21t}$ appear only in the balance of payment condition for countries 1 and 2. Thus the balance of payments condition for country 2 can be written as

[1]Clearly, there may be many shortest complete covers.

$$\text{(B.2)} \quad \sum_{t=0}^{\infty} \sum_{j>2} [\varkappa_{2jt} g_{2jt} - \varkappa_{j2t} h_{j2t}] = R_2 - [\varkappa_{12t} g_{12t} - \varkappa_{21t} h_{21t}] \,,$$

and the balance of payment conditions for all other countries are suitably adjusted.

Consider a new sequence that is obtained from the given shortest complete cover, but omitting country 1. This sequence is a complete cover for countries 2 to N, but it may not be the shortest complete cover for these $N-1$ countries.

The second main step in the algorithm is to fix a new shortest complete cover for these $N-1$ countries. Notice that this implies relabeling the remaining $N-1$ countries. Suppose that in the first stage of the procedure, country 2 becomes, say, country l with this relabeling. Repeat the procedure within the first main step to construct policies for the first element in this new sequence, recognizing that the balance of payment condition for country l (which was labeled 2 in the first stage of the procedure) is now given by the analogue of $(B.2)$, and all other conditions are suitably adjusted.

Proceeding in this fashion, we construct policies for all the countries that satisfy both production efficiency and $(B.1)$.

B.2 Restrictions on Trade Taxes and Efficiency

Here, we consider restrictions on trade taxes similar to the ones imposed in Keen and Wildasin (2004). We first consider a static version of our economy with two goods and four countries. Countries 1, 2, and 3 produce good 1, while country 4 produces good 2. The static model has no capital and no assets, and labor is inelastically supplied. We then consider a dynamic version in which for simplicity, we also ignore capital.

The static economy

Assume that at the relaxed Ramsey allocation, countries 1, 2, and 3 export good 1 to country 4, which also exports good 2 to countries 1, 2, and 3. Countries 1, 2, and 3 do not directly trade with each other. Thus, the countries are connected.

Using the notation in the appendix above, the balance of payment conditions $(B.1)$

for countries 1 to 3 can be written as

$$\text{(B.3)} \quad [\varkappa_{14} g_{14} - \varkappa_{41} h_{41}] = R_1$$

$$\text{(B.4)} \quad [\varkappa_{24} g_{24} - \varkappa_{42} h_{42}] = R_2$$

$$\text{(B.5)} \quad [\varkappa_{34} g_{34} - \varkappa_{43} h_{43}] = R_3,$$

where $\varkappa_{ij} = 1/\left(1 - \tau^x_{ijt}\right)$. Walras law implies that the balance of payment condition for country 4 will be satisfied.

We first show that if no restrictions are imposed on the policy terms $\varkappa_{ij}$, they can be chosen to satisfy all balance of payment conditions. This is just an application to this particular case of the proof of proposition 2. To do so, first note that as countries 1 and 4 have a direct link, then either $g_{14} \neq 0$, or $h_{41} \neq 0$. Then, set the corresponding $\varkappa_{14}$ and $\varkappa_{41}$ so as to satisfy $(B.3)$. Set $\varkappa_{12} = \varkappa_{21} = \varkappa_{13} = \varkappa_{31} = 1$. Countries 2 and 3 also have a direct link with 4, so proceed accordingly.

Remark: Each country has three policy instruments. These are three export subsidies/taxes (the import tariffs are then pinned down by the production efficiency conditions $\tau^x_{ij} = -\tau^m_{ij}$). The twelve available instruments, together with the connectedness assumption, ensure that they are enough to satisfy the balance of payments conditions.

Restrictions on trade taxes

Now, we impose the restriction that trade taxes imposed by any given country can only depend on the physical characteristics of the goods and not on the origin-destination pair. This restriction is imposed on export taxes, τ^x_{ij} so

$$\text{(B.6)} \quad \tau^x_{ij} = \tau^x_i \text{ for } j = 1, 2, 3, \text{and } 4,$$

which implies that there are only four instruments, $\varkappa_1, \varkappa_2, \varkappa_3$, and $\varkappa_4$.

The restriction is also imposed on tariffs. This implies that

$$\text{(B.7)} \quad \tau^m_{14} = \tau^m_{24} = \tau^m_{34} = \tau^m_4.$$

But production efficiency requires that

(B.8) $\tau_{14}^{x} = -\tau_{14}^{m},\ \tau_{24}^{x} = -\tau_{24}^{m}$ and $\tau_{34}^{x} = -\tau_{34}^{m}$.

If we combine $(B.7)$ and $(B.8)$,

$$\tau_{14}^{x} = \tau_{24}^{x} = \tau_{34}^{x} = \tau_{4}^{m},$$

which implies two additional restrictions

$$\varkappa_1 = \varkappa_2 = \varkappa_3 \equiv \varkappa'.$$

These restrictions reduce the number of independent policy instruments to two, $\varkappa_4$ and $\varkappa'$, which in general will not be sufficient to satisfy the balance of payment conditions $(B.3)$ - $(B.5)$.

The dynamic economy

Consider now an economy that consists of repeating the economy above an infinite number of periods. The balance of payments conditions are given by $(B.1)$ and repeated here:

$$\begin{aligned}
\sum_{t=0}^{\infty} [\varkappa_{14t} g_{14t} - \varkappa_{41t} h_{41t}] &= R_1 \\
\sum_{t=0}^{\infty} [\varkappa_{24t} g_{24t} - \varkappa_{42t} h_{42t}] &= R_2 \\
\sum_{t=0}^{\infty} [\varkappa_{34t} g_{34t} - \varkappa_{43t} h_{43t}] &= R_3.
\end{aligned}$$

We maintain the restriction that trade taxes cannot depend on the origin-destination pair. Thus, following the analysis of the static case, we have that

$$\begin{aligned}
\varkappa_{41t} &= \varkappa_{42t} = \varkappa_{43t} = \varkappa_{4t} \\
\varkappa_{14t} &= \varkappa_{24t} = \varkappa_{34t} = \varkappa_t',
\end{aligned}$$

which implies that there are two independent instruments each period, $\varkappa_{4t}$ and $\varkappa'_t$. If we impose these restrictions, the balance of payment conditions can be written

$$\begin{aligned}
\sum_{t=0}^{\infty} [\varkappa_{4t} g_{14t} - \varkappa'_t g_{41t}] &= R_1 \\
\sum_{t=0}^{\infty} [\varkappa_{4t} g_{24t} - \varkappa'_t g_{42t}] &= R_2 \\
\sum_{t=0}^{\infty} [\varkappa_{4t} g_{34t} - \varkappa'_t g_{43t}] &= R_3,
\end{aligned}$$

so there are now an infinite number of instruments to satisfy the three conditions.

To characterize a sufficient condition for the relaxed Ramsey allocation to be implementable, set $\varkappa'_t = \varkappa_{4t} = 1$ for all $t > 1$. Then, the conditions can be written as

$$\begin{aligned}
\sum_{t=0}^{1} [\varkappa_{4t} g_{14t} - \varkappa'_t h_{41t}] + \sum_{t=2}^{\infty} [g_{14t} - h_{41t}] &= R_1 \\
\sum_{t=0}^{1} [\varkappa_{4t} g_{24t} - \varkappa'_t h_{42t}] + \sum_{t=2}^{\infty} [g_{24t} - h_{42t}] &= R_2 \\
\sum_{t=0}^{1} [\varkappa_{4t} g_{34t} - \varkappa'_t h_{43t}] + \sum_{t=2}^{\infty} [g_{34t} - h_{43t}] &= R_3,
\end{aligned}$$

or by properly defining R'_1,

$$\begin{aligned}
\sum_{t=0}^{1} [\varkappa_{4t} g_{14t} - \varkappa'_t h_{41t}] &= R'_1 \\
\sum_{t=0}^{1} [\varkappa_{4t} g_{24t} - \varkappa'_t h_{42t}] &= R'_2 \\
\sum_{t=0}^{1} [\varkappa_{4t} g_{34t} - \varkappa'_t h_{43t}] &= R'_3.
\end{aligned}$$

This can be written as

$$\begin{bmatrix} g_{140} & h_{410} & g_{141} & h_{411} \\ g_{240} & h_{420} & g_{241} & h_{421} \\ g_{340} & h_{430} & g_{341} & h_{431} \end{bmatrix} \begin{bmatrix} \varkappa_{40} \\ -\varkappa_0' \\ \varkappa_{41} \\ \varkappa_1' \end{bmatrix} = \begin{bmatrix} R_1' \\ R_2' \\ R_3' \end{bmatrix},$$

or, in matrix notation,

$$G\varkappa = R.$$

A sufficient condition for the relaxed Ramsey allocation to be implementable as a Ramsey equilibrium is that the matrix G be of rank 3. It is obvious that the choice of the first two periods was arbitrary, so it is required only that there exist two different periods for which the condition above holds. This argument can clearly be extended to have an arbitrary number of countries N, so that we have the following proposition.

Proposition B1: Consider a dynamic economy like the one above, extended to have $N-1$ type-1 countries. Consider the infinite-dimensional matrix formed by the coefficients g_{ijt} and h_{jit}. Suppose that there exist $N-1$ periods so that the submatrix G, induced by considering the coefficients for only these $N-1$ periods, has rank $N-1$. Then, the solution to the relaxed Ramsey problem can be implemented as a Ramsey equilibrium.

C Optimality of Explicit Free Trade with Zero Transfers

In this appendix, we show that a cooperative Ramsey solution is implemented with zero transfers across countries. We use consumption and labor income taxes, set trade taxes to zero, and solve for the optimal level of government consumption. Note that (13) can be written as

$$\left[\sum_{t=0}^{\infty} Q_t q_{it} g_{it} + Q_{-1} b_{i0}\right] - \left[\sum_{t=0}^{\infty} Q_t \left(\tau_{it}^c q_{it} c_{it} + \tau_{it}^n w_{it} n_{it}\right) + \tau_i^W a_{i0}\right] = T_{i0}.$$

The Ramsey problem is to maximize

$$\sum_{i=1}^{2} \omega^i U^i,$$

subject to the conditions

$$\sum_{t=0}^{\infty} \left[\beta^t u^i_{c,t} c_{it} + \beta^t u^i_{n,t} n_{it}\right] \geq \bar{\mathcal{W}}_i$$

$$c_{it} + g_{it} + k_{it+1} - (1-\delta)\, k_{it} \leq G^i\left(\overline{y}_{it}\right)$$

$$\sum_j y_{ijt} \leq F^i\left(k_{it}, n_{it}\right).$$

Let λ^i, ε_{it}, and δ_{it} be the multipliers on these three conditions. We prove the proposition for the case in which $\bar{\mathcal{W}}_i = 0$ for $i = 1, 2$. The result follows by continuity.

Proposition C1: Let $\mathcal{W}_{10} = 0$. Then there exists a weight ω^1 small enough such that $T_{10} < 0$.

Proof: The first-order conditions of the Ramsey problem include

$$\omega^1 h'(g_{1t}) = \varepsilon_{1t}.$$

Thus, as $\omega^1 \to 0$, $g_{1t} \to 0$ for all t.

Preliminary result 1.

The first-order conditions for an interior solution are

$$\omega^1 \beta^t u^1_{ct} + \lambda^1 \beta^t u^1_{ct} + \lambda^1 \beta^t \left[u^1_{cct} c_{1t} + u^1_{cnt} n_{1t}\right] = \varepsilon_{1t}$$

$$\omega^1 \beta^t u^1_{nt} + \lambda^1 \beta^t u^1_{nt} + \lambda^1 \beta^t \left[u^1_{nct} c_{1t} + u^1_{nnt} n_{1t}\right] = -\delta_{1t} F^1_{nt}$$

$$\varepsilon_{1t} G^1_{1t} = \delta_{1t}$$

$$\varepsilon_{1t} G^1_{2t} = \delta_{2t}$$

$$\varepsilon_{2t} G^2_{1t} = \delta_{1t}$$

$$\varepsilon_{2t} G_{2t}^2 = \delta_{2t}$$

$$\varepsilon_{1t} = \varepsilon_{1t+1} (1 - \delta) + \delta_{1t+1} F_{kt}^1.$$

Now, replace δ_{1t} and multiply the first-order conditions by quantities

$$\omega^1 \beta^t u_{ct}^1 c_{1t} + \lambda^1 \beta^t u_{ct}^1 c_{1t} + \lambda^1 \beta^t \left[u_{cct}^1 c_{1t}^2 + u_{cnt}^1 n_{1t} c_{1t} \right] = \varepsilon_{1t} c_{1t}$$

$$\omega^1 \beta^t u_{nt}^1 n_{1t} + \lambda^1 \beta^t u_{nt}^1 n_{1t} + \lambda^1 \beta^t \left[u_{nct}^1 c_{1t} n_{1t} + u_{nnt}^1 n_{1t}^2 \right] = -\varepsilon_{1t} G_{1t}^1 F_{nt}^1 n_{1t}.$$

Add them up:

$$\beta^t \left[u_{ct}^1 c_{1t} + u_{nt}^1 n_{1t} \right] \left[\omega^1 + \lambda^1 \right] + \lambda^1 \beta^t \left[u_{cct}^1 c_{1t}^2 + 2 u_{cnt}^1 n_{1t} c_{1t} + u_{nnt}^1 n_{1t}^2 \right] = \varepsilon_{1t} \left[c_{1t} - G_{1t}^1 F_{nt}^1 n_{1t} \right].$$

Add over time:

$$\left[\omega^1 + \lambda^1 \right] \sum_{t=0}^{\infty} \beta^t \left[u_{ct}^1 c_{1t} + u_{nt}^1 n_{1t} \right] + \lambda^1 \sum_{t=0}^{\infty} \beta^t \left[u_{cct}^1 c_{1t}^2 + 2 u_{cnt}^1 n_{1t} c_{1t} + u_{nnt}^1 n_{1t}^2 \right] = \sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G_{1t}^1 F_{nt}^1 n_{1t} \right].$$

Note that, since the multiplier λ^1 is non-negative and the function u is concave, the term

$$\lambda^1 \sum_{t=0}^{\infty} \beta^t \left[u_{cct}^1 c_{1t}^2 + 2 u_{cnt}^1 n_{1t} c_{1t} + u_{nnt}^1 n_{1t}^2 \right]$$

is negative.[2] It follows that

$$\text{(C.1)} \quad \left[\omega^1 + \lambda^1 \right] \mathcal{W}_{i0} > \sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G_{1t}^1 F_{nt}^1 n_{1t} \right].$$

Preliminary result 2.

[2]The non-negativity of the multiplier is directly implied by the Kuhn-Tucker conditions once we allow each government to make non-negative lump-sum transfers to the private agents. We omitted those transfers from the problem for simplicity.

We relate the term on the right-hand side,

$$\sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G_{1t}^{1} F_{nt}^{1} n_{1t} \right],$$

to a term involving the present value of trade balances.

Owing to constant returns to scale, the Euler theorem implies

$$\text{(C.2)} \quad c_{1t} + g_{1t} + k_{1t+1} - (1-\delta) k_{1t} = G^{1}(y_{11t}, y_{21t}) = G_{1t}^{1} y_{11t} + G_{2t}^{1} y_{21t}$$

$$\text{(C.3)} \quad y_{11t} + y_{12t} = F^{1}(k_{1t}, n_{1t}) = F_{kt}^{1} k_{1t} + F_{nt}^{1} n_{1t}.$$

The trade balance (in units of the intermediate good produced in country 1) satisfies

$$y_{21t} q_{2t} = y_{12t} q_{1t} - TB_{1t} q_{1t},$$

or dividing by q_{1t},

$$y_{21t} \frac{q_{2t}}{q_{1t}} = y_{12t} - TB_{1t}.$$

But in a Ramsey allocation $\frac{q_{2t}}{q_{1t}} = \frac{G_2^1}{G_1^1}$, so

$$y_{21t} \frac{G_{2t}^{1}}{G_{1t}^{1}} = y_{12t} - TB_{1t}.$$

If we replace in $(C.2)$ above,

$$\begin{aligned} c_{1t} + g_{1t} + k_{1t+1} - (1-\delta) k_{1t} &= G_{1t}^{1} y_{11t} + G_{1t}^{1} y_{12t} - G_{1t}^{1} TB_{1t} \\ &= G_{1t}^{1} (y_{11t} + y_{12t}) - G_{1t}^{1} TB_{1t}, \end{aligned}$$

and using $(C.3)$,

$$c_{1t} + g_{1t} + k_{1t+1} - (1-\delta) k_{1t} = G_{1t}^{1} F_{kt}^{1} k_{1t} + G_{1t}^{1} F_{nt}^{1} n_{1t} - G_{1t}^{1} TB_{1t},$$

so

$$c_{1t} - G^1_{1t} F^1_{nt} n_{1t} = G^1_{1t} F^1_{kt} k_{1t} - G^1_{1t} TB_{1t} - g_{1t} - \left[k_{1t+1} - (1-\delta)\, k_{1t}\right].$$

Multiplying each term by ε_{1t} and adding up for all t,

$$\sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G^1_{1t} F^1_{nt} n_{1t}\right] = \sum_{t=0}^{\infty} \varepsilon_{1t} \left[G^1_{1t} F^1_{kt} k_{1t} - G^1_{1t} TB_{1t} - g_{1t} - \left[k_{1t+1} - (1-\delta)\, k_{1t}\right]\right].$$

Recall that the first-order condition with respect to k_{1t+1} implies

$$-\varepsilon_{1t} + \left[G^1_{1t+1} F^1_{kt+1} + (1-\delta)\right] \varepsilon_{1t+1} = 0,$$

so we obtain the preliminary result 2:

$$\text{(C.4)} \quad \sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G^1_{1t} F^1_{nt} n_{1t}\right] = -\sum_{t=0}^{\infty} \varepsilon_{1t} \left[G^1_{1t} TB_{1t} + g_{1t}\right] + \left[G^1_{10} F^1_{k0} + (1-\delta)\right] \varepsilon_{10} k_{10}.$$

Proof: Using $(C.4)$ with $(C.1)$, and noting that when $\omega^1 \to 0$, $g_{1t} \to 0$ for all t, we obtain

$$\left[\omega^1 + \lambda^1\right] \mathcal{W}_{10} > -\sum_{t=0}^{\infty} \varepsilon_{1t} G^1_{1t} TB_{1t} - \left[G^1_{10} F^1_{k0} + (1-\delta)\right] \varepsilon_{10} k_{10},$$

or

$$\text{(C.5)} \quad \sum_{t=0}^{\infty} \varepsilon_{1t} G^1_{1t} TB_{1t} = \sum_{t=0}^{\infty} \delta_{1t} TB_{1t} > -\left[\omega^1 + \lambda^1\right] \mathcal{W}_{10} + \left[G^1_{10} F^1_{k0} + (1-\delta)\right] \varepsilon_{10} k_{10}.$$

As we assumed that $\mathcal{W}_{10} = 0$, it follows that

$$\sum_{t=0}^{\infty} \delta_{1t} TB_{1t} > \left[G^1_{10} F^1_{k0} + (1-\delta)\right] \varepsilon_{10} k_{10}.$$

As the right-hand side is positive, this equation implies that

$$\sum_{t=0}^{\infty} \delta_{1t} TB_{1t} > 0.$$

The δ_{1t} are the multipliers of constraints

$$(\delta_{1t}) y_{11t} + y_{12t} \leq F^1 (k_{1t}, n_{1t}),$$

which is the value for the planner of the intermediate goods. Because of production efficiency, the private and social values of the intermediate goods are the same, so the present value of the trade balance is positive, which means that the transfer is negative since f_{i0} are zero.

Remark: Equation $(C.4)$ makes clear that, given that $\omega^1 \to 0$, a weaker sufficient condition is

$$-\left[\omega^1 + \lambda^1\right] \mathcal{W}_{10} + \left[G_{10}^1 F_{k0}^1 + (1-\delta)\right] \varepsilon_{10} k_{10} \geq 0,$$

or

$$\lambda^1 \mathcal{W}_{10} \leq \left[G_{10}^1 F_{k0}^1 + (1-\delta)\right] \varepsilon_{10} k_{10},$$

which is weaker than the one assumed in the proposition. This condition, however, involves multipliers, which are endogenous.

To understand the role of restricting the value for $\mathcal{W}_{10}$, imagine that it takes a value that is higher than the present value of current plus all future national incomes in country 1, when all taxes are set to zero and all government expenditures are set to zero. Any feasible allocation therefore requires transfers of resources from country 2 to country 1, independently of the values of the weights ω^i. This logic also makes clear that there are high enough values for $\mathcal{W}_{10}$ and $\mathcal{W}_{20}$ such that the set of implementable allocations is empty.

Thus far we have focused on interior allocations. It is possible to extend the proof to situations in which the solution is at corner; details are available upon request.

D Taxes on Assets

In this appendix, we show that it is possible to implement the solution of the Ramsey problem in Section III.A as a competitive equilibrium.

We consider a system with income taxation of labor and assets, including a corporate income tax. We consider a common tax on the household's returns from foreign assets and on equity returns including capital gains.

We now describe the problems of the firms and the household in each country and define a competitive equilibrium. We maintain the assumption that ownership of firms is domestic, but we will see that this is without loss of generality.

Firm

The representative intermediate good firm in each country produces and invests in order to maximize the present value of dividends, $V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$. Dividends, in units of the numeraire, d_{it}, are given by

$$\text{(D.1)} \quad d_{it} = p_{it} F(k_{it}, n_{it}) - w_{it} n_{it} - \tau_{it}^k \left[p_{it} F(k_{it}, n_{it}) - w_{it} n_{it} - q_{it} \delta k_{it} \right] - q_{it} \left[k_{it+1} - (1-\delta) k_{it} \right],$$

where τ_{it}^k is the tax rate on capital income net of depreciation.

The first-order conditions of the firm's problem are now $p_{iit} F_{n,t}^i = w_{it}$, together with

$$\text{(D.2)} \quad \frac{Q_t q_{it}}{Q_{t+1} q_{it+1}} = 1 + \left(1 - \tau_{it+1}^k\right) \left(\frac{p_{it+1}}{q_{it+1}} F_{k,t+1}^i - \delta \right).$$

Substituting for d_{it} from $(D.1)$ and using the firm's first-order conditions, it is easy to show that the present value of the dividends at time zero in units of the numeraire is given by

$$\text{(D.3)} \quad V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it} = \left[1 + \left(1 - \tau_{i0}^k\right) \left(\frac{p_{i0}}{q_{i0}} F_{ik,0} - \delta \right) \right] p_{i0} k_{i0}.$$

The problem of the final good firm is as it was before.

Households

The flow of funds constraint in period t for the household in country i in units of the numeraire is given by

$$\text{(D.4)} \qquad \begin{aligned} & q_{it}c_{it} + b_{it+1} + f_{it+1} + V_{it}s_{it+1} \\ = \; & (1-\tau_{it}^{n}) w_{it}n_{it} + \left[1 + r_t^f - \tau_{it}\left(r_t^f - \frac{q_{it}-q_{it-1}}{q_{it-1}}\right)\right](b_{it}+f_{it}) \\ & + (V_{it}+d_{it}) s_{it} - \tau_{it}\left(d_{it} + V_{it} - V_{it-1} - \frac{(q_{it}-q_{it-1}) V_{it-1}}{q_{it-1}}\right) s_{it}. \end{aligned}$$

In period 0, the constraint is

$$\text{(D.5)} \qquad \begin{aligned} & q_{i0}c_{i0} + b_{i1} + f_{i1} + V_{i0}s_{i1} \\ = \; & (1-\tau_{i0}^{n}) w_{i0}n_{i0} + \left(1-\tau_i^W\right)\left[1 + r_0^f - \tau_{i0}\left(r_0^f - \frac{q_{i0}-q_{i-1}}{q_{i-1}}\right)\right](b_{i0}+f_{i0}) \\ & \left(1-\tau_i^W\right)\left[(V_{i0}+d_{i0}) s_{i0} - \tau_{i0}\left(d_{i0} + V_{i0} - V_{i-1} - \frac{(q_{i0}-q_{i-1}) V_{i-1}}{q_{i-1}}\right) s_{i0}\right]. \end{aligned}$$

Dividends and capital gains are taxed at rate τ_{it} with an allowance for numeraire inflation. Returns on domestic and foreign bonds are also taxed at the same rate, τ_{it}, also with an allowance for numeraire inflation.

The household's problem is to maximize utility (1), subject to $(D.4)$; $(D.5)$; and no-Ponzi-scheme conditions, $\lim_{T\to\infty} Q_{iT+1}b_{iT+1} \geq 0$ and $\lim_{T\to\infty} Q_{iT+1}f_{iT+1} \geq 0$ with

$$\text{(D.6)} \quad \frac{Q_{it}}{Q_{it+1}} = (1-\tau_{it+1})\left(1 + r_{t+1}^f\right) + \tau_{it+1}\frac{q_{it+1}}{q_{it}} \text{ with } Q_{i0} = 1.$$

The first-order conditions of the household's problem in each country are, for $t \geq 0$,

$$\text{(D.7)} \quad -\frac{u_{c,t}^i}{u_{n,t}^i} = \frac{q_{it}}{(1-\tau_{it}^n) w_{it}},$$

$$\text{(D.8)} \quad u_{c,t}^i = \frac{Q_{it}q_{it}}{Q_{it+1}q_{it+1}}\beta u_{c,t+1}^i,$$

and

$$\text{(D.9)} \quad \frac{Q_{it}}{Q_{it+1}} = \frac{(V_{it+1} + d_{it+1}) - \tau_{it+1}\left(V_{it+1} - V_{it} + d_{it+1} - \frac{q_{it+1}-q_{it}}{q_{it}}V_{it}\right)}{V_{it}}.$$

Condition $(D.9)$ implies that

$$1 + r^f_{t+1} = \frac{V_{it+1} + d_{it+1}}{V_{it}}.$$

This condition on the two returns can be written, using $1 + r^f_{t+1} = \frac{Q_t}{Q_{t+1}}$, as

$$Q_t V_{it} = Q_{t+1} V_{it+1} + Q_{t+1} d_{it+1}.$$

Imposing that $\lim_{T\to\infty} Q_{T+1} V_{iT+1} = 0$, then

$$V_{it} = \sum_{s=0}^{\infty} \frac{Q_{t+1+s}}{Q_t} d_{it+1+s}.$$

The present value of dividends for the households of country i is a different expression from the one above because they pay taxes on the asset income. Using (D.9), we have that

$$V_{i0} = \sum_{t=0}^{\infty} \left(1 - \hat{\tau}^a_{it+1}\right) Q_{it+1} d_{it+1},$$

where $1 - \hat{\tau}^a_{it+1} = \Pi^t_{s=0}\left(1 - \hat{\tau}_{is+1}\right)$, and $1 - \hat{\tau}_{it+1} = \frac{(1-\tau_{it+1})}{\left(1-\tau_{it+1}\frac{q_{it+1}Q_{it+1}}{q_{it}Q_{it}}\right)}$. The values are the same, since $\left(1 - \hat{\tau}^a_{it+1}\right) Q_{it+1} = Q_{t+1}$. This condition is obtained from (D.6).

The value of the firm for the households in country i, including the dividends in period 0, is

$$\begin{aligned} \text{(D.11)} \qquad & V_{i0} + d_{i0} - \tau_{i0}\left(V_{i0} + d_{i0} - \frac{q_{i0}V_{i-1}}{q_{i-1}}\right) \\ = \quad & (1 - \tau_{i0})(V_{i0} + d_{i0}) + \tau_{i0}\frac{q_{i0}V_{i-1}}{q_{i-1}}. \end{aligned}$$

Notice that the market price of the firm before dividends, $V_{i0} + d_{i0}$, is a linear function of the value for the firm for the households of each country, so that the solution of the maximization problem of the firm also maximizes shareholder value. That would also be the case if the stocks were held by the households of the foreign country. This means that the restriction that firms are owned by the domestic households is without loss of generality.

Using the no-Ponzi-games condition, the budget constraints of the household, (D.4) and (D.5), can be consolidated into the single budget constraint,

$$\sum_{t=0}^{\infty} Q_{it} \left[q_{it} c_{it} - \left(1 - \tau_{it}^{n}\right) w_{it} n_{it} \right] = \left(1 - \tau_{i}^{W}\right) a_{i0},$$

where

$$a_{i0} = \left(1 - \tau_{i0}\right)\left(V_{i0} + d_{i0}\right) + \tau_{i0} \frac{q_{i0} V_{i-1}}{q_{i-1}} + \left[1 + r_0^f - \tau_{i0}\left(1 + r_0^f - \frac{q_{i0}}{q_{i-1}}\right)\right]\left(b_{i0} + f_{i0}\right). \tag{D.12}$$

Using (D.3) as well as $s_0 = 1$, the initial asset holdings in (D.12) can be written as

$$\begin{aligned} a_{i0} &= \left(1 - \tau_{i0}\right) q_{i0} \left[1 + \left(1 - \tau_{i0}^{k}\right)\left(G_{i,0}^{i} F_{ik,0} - \delta\right)\right] k_{i0} + \tau_{i0} \frac{q_{i0} V_{i-1}}{q_{i-1}} \\ &+ \left[1 + r_0^f - \tau_{i0}\left(1 + r_0^f - \frac{q_{i0}}{q_{i-1}}\right)\right]\left(b_{i0} + f_{i0}\right). \end{aligned}$$

The interest rate parity condition is obtained from

$$\frac{Q_t}{Q_{t+1}} = \frac{q_{it+1}}{q_{it}} \left[1 + \left(1 - \tau_{it+1}^{k}\right)\left(\frac{p_{it+1}}{q_{it+1}} F_{k,t+1}^{i} - \delta\right)\right]$$

for $i = 1, 2$, or

$$\frac{q_{1t+1}}{q_{1t}} \left[1 + \left(1 - \tau_{1t+1}^{k}\right)\left(\frac{p_{1t+1}}{q_{1t+1}} F_{k,t+1}^{1} - \delta\right)\right] = \frac{q_{2t+1}}{q_{2t}} \left[1 + \left(1 - \tau_{2t+1}^{k}\right)\left(\frac{p_{2t+1}}{q_{2t+1}} F_{k,t+1}^{2} - \delta\right)\right].$$

Using the first-order conditions of the firms to replace the relative prices of the intermediate and final goods, it follows that

$$\text{(D.13)} \qquad \frac{G^1_{j,t}}{G^1_{j,t+1}} \left[1 + \left(1 - \tau^k_{1t+1}\right) \left(G^1_{1,t+1} F^1_{k,t+1} - \delta\right)\right]$$
$$= \frac{G^2_{j,t}}{G^2_{j,t+1}} \left[1 + \left(1 - \tau^k_{2t+1}\right) \left(G^2_{2,t+1} F^2_{k,t+1} - \delta\right)\right], \text{ for } j = 1, 2.$$

To get production efficiency–that is, to satisfy (9)–we need to either set the two tax rates to zero or pick τ^k_{1t+1} and τ^k_{2t+1} according to

$$\tau^k_{1t+1} \left(G^1_{1,t+1} F^1_{k,t+1} - \delta\right)$$
$$= \tau^k_{2t+1} \left(G^1_{1,t+1} F^1_{k,t+1} - \delta - \left(\frac{G^1_{j,t+1}/G^2_{j,t+1}}{G^1_{j,t}/G^2_{j,t}} - 1\right)\right), \text{ for } j = 1, 2.$$

Using the intertemporal condition of the household (D.8) and

$$\frac{Q_{it}}{Q_{it+1}} = (1 - \tau_{it+1}) \frac{Q_t}{Q_{t+1}} + \tau_{it+1} \frac{q_{it+1}}{q_{it}}$$

obtained from (D.6), together with $\frac{Q_t}{Q_{t+1}} = 1 + r^f_{t+1}$, and combining them with the firm's condition (D.2), together with the first-order conditions of firms' production decisions, we obtain

$$\text{(D.14)} \quad \frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = 1 + (1 - \tau_{it+1}) \left(1 - \tau^k_{it+1}\right) \left(G^i_{i,t+1} F^i_{k,t+1} - \delta\right).$$

The marginal conditions in this economy can be summarized by

$$\text{(D.15)} \quad -\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{1}{(1 - \tau^n_{it})\, G^i_{i,t} F^i_{n,t}},$$

as well as the intertemporal condition (D.14), the interest rate parity condition (D.13), and condition (8), for all $t \geq 0$.

The Ramsey allocation can be implemented with a (possibly time-varying) common tax on home and foreign assets. Corporate income taxes in both countries either must be set to zero or must be set according to the difference in real returns in the goods of the two countries to ensure production efficiency. In this economy with a common tax on domestic equity and foreign returns, firms use a common price to value dividends. If relaxed, the restriction that firms are owned by the domestic residents would not change the implementable allocations.

D.1 Corporate Income Taxes with Deductibility

Here, we consider an implementation with taxes on assets in which the corporate income taxes allow for the deduction of investment expenses. We will show that, as long as the tax rate is the same across countries or constant over time, the Ramsey allocation can be implemented with such taxes.

The representative intermediate good firm in each country produces and invests in order to maximize the present value of dividends, $V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$, where Q_t is the pretax discount factor. Dividends, d_{it}, in units of the numeraire, are now given by

$$d_{it} = \left(1 - \tau_{it}^k\right) \left[p_{it} F(k_{it}, n_{it}) - w_{it} n_{it}\right] - \left(1 - \tau_{it}^k\right) q_{it} \left[k_{it+1} - (1-\delta) k_{it}\right],$$

where τ_{it}^k is the tax rate on corporate income net of investment expenses.

The first-order conditions of the firm's problem are now $p_{iit} F_{n,t}^i = w_{it}$, together with

$$\frac{Q_t}{Q_{t+1}} \frac{q_{it}}{q_{it+1}} = \frac{\left(1 - \tau_{it+1}^k\right)}{\left(1 - \tau_{it}^k\right)} \left[\frac{p_{it+1}}{q_{it+1}} F_k(k_{it+1}, n_{it+1}) - (1-\delta)\right].$$

This implies the following interest-rate parity condition:

$$\frac{q_{it+1}\left(1 - \tau_{it+1}^k\right)}{q_{it}\left(1 - \tau_{it}^k\right)} \left[\frac{p_{it+1}}{q_{it+1}} F_k(k_{it+1}, n_{it+1}) - (1-\delta)\right] \text{ has to be the same across } i.$$

The profit maximization conditions for the final goods producers are, for all i,

$$p_{ii,t} = p_{ij,t} \equiv p_{i,t},\ i \neq j,$$

$$q_{i,t} G_{i,t}^i = p_{ii,t},$$

$$q_{i,t} G_{j,t}^i = p_{ji,t},\ i \neq j.$$

This implies

$$q_{i,t}G^i_{j,t} = p_{j,t}, \text{ for all } i \text{ and } j.$$

It follows that the interest rate parity condition can be written as

$$\frac{G^i_{j,t}\left(1-\tau^k_{it+1}\right)}{G^i_{j,t+1}\left(1-\tau^k_{it}\right)}\left[G^i_{i,t+1}F_k(k_{it+1},n_{it+1})-(1-\delta)\right] \text{ has to be the same across } i.$$

The dynamic production efficiency condition is satisfied if τ^k_{it} is the same across countries or if it is constant over time.

The households conditions are

$$u^i_{c,t} = \frac{Q_{it}q_{it}}{Q_{it+1}q_{it+1}}\beta u^i_{c,t+1},$$

with

$$\frac{Q_{it}}{Q_{it+1}} = \left(1-\tau_{it+1}\right)\frac{Q_t}{Q_{t+1}} + \tau_{it+1}\frac{q_{it+1}}{q_{it}}.$$

These conditions, together with

$$\frac{Q_t}{Q_{t+1}}\frac{q_{it}}{q_{it+1}} = \frac{\left(1-\tau^k_{it+1}\right)}{\left(1-\tau^k_{it}\right)}\left[\frac{p_{it+1}}{q_{it+1}}F_k(k_{it+1},n_{it+1})-(1-\delta)\right],$$

imply

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \left(1-\tau_{it+1}\right)\frac{\left(1-\tau^k_{it+1}\right)}{\left(1-\tau^k_{it}\right)}\left[G^i_{i,t+1}F_k(k_{it+1},n_{it+1})-(1-\delta)\right]+\tau_{it+1}.$$

E Value-Added Taxes

E.1 Algebra for Border-Adjusted VAT

Here, we display the algebra needed to prove proposition 7. The first-order conditions of the household's problem now include

$$\text{(E.1)} \quad -\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{q_{it}}{\left(1-\tau^n_{it}\right)w_{it}}, \; t \geq 0$$

and

(E.2) $u^i_{c,t} = \dfrac{Q_t q_{it}}{Q_{t+1} q_{it+1}} \beta u^i_{c,t+1}$, $t \geq 0$.

The first-order conditions of the firms' problems for an interior solution are

(E.3) $p_{iit} \left(1 - \tau^v_{it}\right) F^i_{n,t} = w_{it}$

(E.4) $Q_t q_{it} \left(1 - \tau^v_{it}\right) = Q_{t+1} p_{iit+1} \left(1 - \tau^v_{it+1}\right) F^i_{k,t+1} + Q_{t+1} q_{it+1} \left(1 - \tau^v_{it+1}\right) (1 - \delta))$

(E.5) $p_{iit} \left(1 - \tau^v_{it}\right) = p_{ijt}$, for $j \neq i$

(E.6) $q_{it} G^i_{i,t} = p_{iit}$

(E.7) $q_{it} \left(1 - \tau^v_{it}\right) G^i_{j,t} = p_{jit}$, for $j \neq i$.

The households' and firms' conditions can be manipulated to obtain (44) and (45), together with (8) and (9).

Conditions $(E.1)$, $(E.3)$, and $(E.6)$ can be used to obtain (44). Conditions $(E.2)$, $(E.4)$, and $(E.6)$ can be used to obtain (45). To see that the conditions $(E.3)$-$(E.7)$ imply (8) and (9), note that $(E.5)$-$(E.6)$ imply

$$q_{it} \left(1 - \tau^v_{it}\right) G^i_{j,t} = p_{jit} = p_{jjt} \left(1 - \tau^v_{jt}\right)$$

and

$$q_{it} G^i_{i,t} = p_{iit} = \frac{p_{ijt}}{\left(1 - \tau^v_{it}\right)},$$

implying

$$\frac{G^i_{j,t}}{G^i_{i,t}} = \frac{p_{jit}}{p_{ijt}} = \frac{p_{jjt} \left(1 - \tau^v_{jt}\right)}{q_{jt} \left(1 - \tau^v_{jt}\right) G^j_{i,t}} = \frac{G^j_{j,t}}{G^j_{i,t}}.$$

Note also that $(E.3)$ and $(E.6)$ imply

$$\frac{Q_t}{Q_{t+1}} = \frac{q_{it+1}\left(1-\tau^v_{it+1}\right)}{q_{it}\left(1-\tau^v_{it}\right)}\left[G^i_{i,t+1}F^i_{k,t+1}+1-\delta\right],$$

so that

$$\frac{q_{it+1}\left(1-\tau^v_{it+1}\right)}{q_{it}\left(1-\tau^v_{it}\right)}\left[G^i_{i,t+1}F^i_{k,t+1}+1-\delta\right] = \frac{q_{jt+1}\left(1-\tau^v_{jt+1}\right)}{q_{jt}\left(1-\tau^v_{jt}\right)}\left[G^j_{j,t+1}F^j_{k,t+1}+1-\delta\right].$$

Since, from $(E.5)$ - $(E.6)$,

$$q_{jt}\left(1-\tau^v_{jt}\right)G^j_{i,t} = p_{ijt} = p_{iit}\left(1-\tau^v_{it}\right) = q_{it}G^i_{i,t}\left(1-\tau^v_{it}\right), \text{ for } j \neq i,$$

we obtain

$$\frac{G^i_{i,t}}{G^i_{i,t+1}}\left[G^i_{i,t+1}F^i_{k,t+1}+1-\delta\right] = \frac{G^j_{i,t}}{G^j_{i,t+1}}\left[G^j_{j,t+1}F^j_{k,t+1}+1-\delta\right].$$

Comparing the four equilibrium conditions, (44) - (9), with the corresponding ones in the economy with consumption, labor income, and trade taxes, (15)-(18), we obtain proposition 7.

E.2 Algebra for VAT Without BA

The first-order conditions in the economy with VAT without border adjustments (with trade taxes) include the households' conditions $(E.1)$ and $(E.2)$, which are the same as with border adjustments, repeated here,

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{q_{it}}{\left(1-\tau^n_{it}\right)w_{it}},\ t \geq 0,$$

and

$$u^i_{c,t} = \frac{Q_t q_{it}}{Q_{t+1}q_{it+1}}\beta u^i_{c,t+1},\ t \geq 0;$$

and the first-order conditions for the final good firms, $(E.3)$ and $(E.4)$, which are also the same as with border adjustments,

$$p_{iit}\left(1-\tau_{it}^{v}\right)F_{n,t}^{i}=w_{it},$$

$$\frac{Q_t}{Q_{t+1}}=\frac{p_{iit+1}\left(1-\tau_{it+1}^{v}\right)}{q_{it}\left(1-\tau_{it}^{v}\right)}F_{k,t+1}^{i}+\frac{q_{it+1}\left(1-\tau_{it+1}^{v}\right)}{q_{it}\left(1-\tau_{it}^{v}\right)}(1-\delta));$$

as well as the conditions for the intermediate good firms, $(A.5)$-$(A.7)$, repeated here,

$$p_{iit}=\left(1-\tau_{ijt}^{x}\right)p_{ijt},\ i\neq j,$$

$$q_{it}G_{i,t}^{i}=p_{iit},$$

$$q_{it}G_{j,t}^{i}=\left(1+\tau_{jit}^{m}\right)p_{jit},\ i\neq j.$$

In order to show that these conditions can be written as (44)-(48), note first that (44) and (45) can be obtained as in the case with border adjustments, using $(E.1)$, $(E.2)$, $(E.3)$, $(E.4)$, and $(A.6)$. In order to obtain (47), note that $(A.5)$-$(A.7)$ imply

$$\frac{q_{it}G_{j,t}^{i}}{q_{it}G_{i,t}^{i}}=\frac{\left(1+\tau_{jit}^{m}\right)p_{jit}}{p_{iit}}=\frac{p_{jjt}}{\left(1-\tau_{ijt}^{x}\right)p_{ijt}}=\frac{q_{jt}G_{j,t}^{j}}{q_{jt}G_{i,t}^{j}},\ i\neq j.$$

Condition (48) is obtained using $(E.4)$ and $(A.6)$, so that

$$\frac{q_{it+1}\left(1-\tau_{it+1}^{v}\right)}{q_{it}\left(1-\tau_{it}^{v}\right)}\left[G_{i,t+1}^{i}F_{k,t+1}^{i}+1-\delta\right]=\frac{q_{jt+1}\left(1-\tau_{jt+1}^{v}\right)}{q_{jt}\left(1-\tau_{jt}^{v}\right)}\left[G_{j,t+1}^{j}F_{k,t+1}^{j}+1-\delta\right],$$

and from $(A.5)$-$(A.7)$,

$$q_{jt}G_{i,t}^{j}=\left(1+\tau_{ijt}^{m}\right)p_{ijt}=\frac{\left(1+\tau_{ijt}^{m}\right)q_{it}G_{i,t}^{i}}{\left(1-\tau_{ijt}^{x}\right)},\ i\neq j,$$

so that

$$\frac{\left(1-\tau_{it+1}^{v}\right)}{\left(1-\tau_{it}^{v}\right)}\left[G_{i,t+1}^{i}F_{k,t+1}^{i}+1-\delta\right]=\frac{\frac{\left(1+\tau_{ijt+1}^{m}\right)G_{i,t+1}^{i}}{\left(1-\tau_{ijt}^{x}\right)G_{i,t+1}^{j}}\left(1-\tau_{jt+1}^{v}\right)}{\frac{\left(1+\tau_{ijt}^{m}\right)G_{i,t}^{i}}{\left(1-\tau_{ijt}^{x}\right)G_{i,t}^{j}}\left(1-\tau_{jt}^{v}\right)}\left[G_{j,t+1}^{j}F_{k,t+1}^{j}+1-\delta\right],\ i\neq j.$$

E.3 Border Adjustments and Lerner Symmetry

Lemma 1 We start by proving Lemma 1. Consider that country 1 introduces an import tariff, τ_{21t}^{m}, and an export tax on all goods, τ_{12t}^{x}. The conditions for the household and firms in country 1 are

(E.8) $-\dfrac{u_{c,t}^{1}}{u_{n,t}^{1}}=\dfrac{\left(1+\tau_{1t}^{c}\right)q_{1t}}{\left(1-\tau_{1t}^{n}\right)w_{1t}},$

(E.9) $\dfrac{u_{c,t}^{1}}{\left(1+\tau_{1t}^{c}\right)}=\dfrac{Q_{t}q_{1t}}{Q_{t+1}q_{1t+1}}\dfrac{\beta u_{c,t+1}^{1}}{\left(1+\tau_{1t+1}^{c}\right)},$

(E.10) $F_{n,t}^{1}=\dfrac{w_{1t}}{p_{11t}},$

(E.11) $\dfrac{Q_{t}}{Q_{t+1}}=\dfrac{p_{11t+1}}{q_{1t}}F_{k,t+1}^{1}+\dfrac{q_{1t+1}}{q_{1t}}\left(1-\delta\right),$

(E.12) $G_{1,t}^{1}=\dfrac{p_{11t}}{q_{1t}},$

(E.13) $p_{11t}=\left(1-\tau_{12t}^{x}\right)p_{12t},$

(E.14) $q_{1t}G_{2,t}^{1}=\left(1+\tau_{21t}^{m}\right)p_{21t}.$

The proof of Lemma 1 follows by inspecting the first-order conditions above, $(E.8)$ through $(E.14)$, as well as the household budget constraints written as (21) and (22) and satisfied with an appropriate choice of $\hat{\tau}_{1}$,

$$\mathcal{W}_{10}=\left(1-\hat{\tau}_{1}\right)\frac{u_{c,0}^{1}}{\left(1+\tau_{10}^{c}\right)}\left[\left(1-\delta+G_{1,0}^{1}F_{k,0}^{1}\right)k_{10}+Q_{-1}\frac{b_{10}}{\hat{q}_{10}}+\left(1+r_{0}^{f}\right)\frac{f_{1,0}}{\hat{q}_{10}}\right].$$

Conditions $(E.8)$ through $(E.14)$ are satisfied in the economy with $\left(1-\hat{\tau}_{12t}^{x}\right)=\kappa_{s}\left(1-\tau_{12t}^{x}\right)$ and $\left(1+\hat{\tau}_{21t}^{m}\right)=\kappa_{s}\left(1+\tau_{21t}^{m}\right)$ with $\hat{p}_{11t}=\kappa_{s}p_{11t}$, $\hat{q}_{1t}=\kappa_{s}q_{1t}$, $\hat{w}_{1t}=\kappa_{s}w_{1t}$ for $\kappa_{t}=\kappa_{s}$ Here, we have assumed that b_{10} and $f_{1,0}$ are fixed in units of the world numeraire. Notice that the proof goes through even if these initial conditions are fixed in real terms. The higher price of the final good in country 1 (and the price of the imported good after the tariff, together with the price of the exported good after the subsidy) reduces the value of domestic and foreign assets, so that the government must compensate that with a lower tax on initial wealth $\hat{\tau}_1$. There is no need to adjust transfers to satisfy the balance of payments condition for country $i = 1$ in (14).

Lemma 2 Let tildes denote prices in terms of domestic currency. Let E_t denote domestic currency per numeraire. Then, for example, $\tilde{p}_{11t} = E_t p_{11t}$. Now, when we multiply all the trade policy terms by κ, it is equivalent to letting $\hat{E}_t = \frac{E_t}{\kappa}$ (if $\kappa > 1$, the domestic currency appreciates) and leaving all domestic prices denoted in domestic currency unaffected.

Then, conditions $(E.8)$ through $(E.14)$ can be written as

$$-\frac{u^1_{c,t}}{u^1_{n,t}} = \frac{(1+\tau^c_{1t})\,\tilde{q}_{1t}}{(1-\tau^n_{1t})\,\tilde{w}_{1t}},$$

$$\frac{u^1_{c,t}}{(1+\tau^c_{1t})} = \frac{Q_t}{Q_{t+1}}\frac{\tilde{q}_{1t}}{\tilde{q}_{1t+1}}\frac{e_{t+1}}{e_t}\frac{\beta u^1_{c,t+1}}{\left(1+\tau^c_{1t+1}\right)},$$

$$F^1_{n,t} = \frac{\tilde{w}_{1t}}{\tilde{p}_{11t}},$$

$$\frac{Q_t}{Q_{t+1}} = \frac{\tilde{p}_{11t+1}}{\tilde{q}_{1t}}\frac{e_t}{e_{t+1}}F^1_{k,t+1} + \frac{\tilde{q}_{1t+1}}{\tilde{q}_{1t}}\frac{e_t}{e_{t+1}}(1-\delta),$$

$$G^1_{1,t} = \frac{\tilde{p}_{11t}}{\tilde{q}_{1t}},$$

$$\tilde{p}_{11t} = E_t\,(1-\tau^x_{12t})\,p_{12t},$$

$$\tilde{q}_{1t}G^1_{2,t} = E_t\,(1+\tau^m_{21t})\,p_{21t}.$$

The proof of Lemma 2 follows by inspecting the first-order conditions above, as well as the household budget constraints written as (21) and (22) and satisfied with an appropriate choice of $\hat{\tau}_1$, as long as foreign assets are denominated in the world numeraire, so as to satisfy

$$\mathcal{W}_{10} = (1-\hat{\tau}_1)\frac{u^1_{c,0}}{(1+\tau^c_{10})}\left[\left(1-\delta+G^1_{1,0}F^1_{k,0}\right)k_{10} + Q_{-1}\frac{b_{10}}{\tilde{q}_{10}} + \left(1+r^f_0\right)\frac{f_{1,0}}{\tilde{q}_{10}}\frac{e_0}{\kappa}\right].$$

There is no need to adjust transfers to satisfy the balance of payments condition for country $i = 1$ in (14).

Suppose now that net foreign assets were denominated in the domestic numeraire. The value of initial wealth is given by

$$\mathcal{W}_{10} = (1-\tau_1)\frac{u^1_{c,0}}{(1+\tau^c_{10})}\left[\left(1-\delta+G^1_{1,0}F^1_{k,0}\right)k_{10} + Q_{-1}\frac{b_{10}}{\tilde{q}_{1,0}} + \left(1+r^f_0\right)\frac{f_{1,0}}{\tilde{q}_{1,0}}\right].$$

Note that in this case, there is no change in the real value of domestic public debt and foreign assets, so that there is no need to change τ_1. On the other hand, there is a need to change the level of international transfers, since the balance of payments condition is now

$$\sum_{t=0}^{\infty} Q_t\left[p_{12t}y_{12t} - p_{21t}y_{21t}\right] = -\left(1+r^f_0\right)\frac{f_{1,0}\kappa}{E_0} - \hat{T}_{10}.$$

Since the foreign assets are denominated in domestic currency, they are now worth more in units of foreign currency, and country 1 would have to receive lower transfers.

Nonuniform changes in trade taxes

We start by taking international prices p_{21t}, p_{12t}, and Q_t and allocations as given. We multiply the trade taxes in country 1, $(1+\tau^m_{21t})$ and $(1-\tau^x_{12t})$, by $\kappa_t > 0$. The equilibrium conditions become

$$-\frac{u^1_{c,t}\left(1-\tau^n_{1t}\right)}{u^1_{n,t}\left(1+\tau^c_{1t}\right)} = \frac{q_{1t}}{w_{1t}},$$

$$\frac{u^1_{c,t}}{(1+\tau^c_{1t})} = \frac{q_{1t}Q_t}{q_{1t+1}Q_{t+1}}\frac{\beta u^1_{c,t+1}}{\left(1+\tau^c_{1t+1}\right)},$$

$$F^1_{n,t} = \frac{w_{1t}}{p_{11t}},$$

$$\frac{Q_t}{Q_{t+1}} = \frac{p_{11t+1}}{q_{1t}}F^1_{k,t+1} + \frac{q_{1t+1}}{q_{1t}}\left(1-\delta\right),$$

$$\frac{1}{\left(1-\tau^x_{12t}\right)p_{12t}} = \frac{\kappa_t}{p_{11t}},$$

$$G^1_{1,t} = \frac{p_{11t}}{q_{1t}},$$

$$\frac{G^1_{2,t}}{p_{21t}\left(1+\tau^m_{21t}\right)} = \frac{\kappa_t}{q_{1t}}.$$

In order for κ_t to be neutral, it must be that $\frac{\kappa_t}{q_{1t}}$, $\frac{\kappa_t}{p_{11t}}$, $\frac{q_{1t}}{w_{1t}}$, and $\frac{q_{1t}}{q_{1t+1}}$ are kept constant. This can happen only if $\kappa_t = \kappa$.

Changes in trade taxes may also be neutral if both countries change them in particular ways. To see this, let both countries multiply $\left(1+\tau^m_{jit}\right)$ and $\left(1-\tau^x_{ijt}\right)$ by κ_{it}, for $i = 1, 2$ and $j \neq i$. The equilibrium conditions can be written as

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{\left(1+\tau^c_{it}\right)}{\left(1-\tau^n_{it}\right) G^i_{i,t} F^i_{n,t}},$$

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \frac{\left(1+\tau^c_{it}\right)}{\left(1+\tau^c_{it+1}\right)} \left[G^i_{i,t+1} F^i_{k,t+1} + 1 - \delta\right],$$

$$\frac{G^1_{2,t}}{G^1_{1t}} = \frac{\kappa_{1t}\left(1+\tau^m_{21t}\right)}{\kappa_{1t}\left(1-\tau^x_{12t}\right)} \frac{\kappa_{2t}\left(1+\tau^m_{12t}\right)}{\kappa_{2t}\left(1-\tau^x_{21t}\right)} \frac{G^2_{2t}}{G^2_{1,t}},$$

$$\frac{\kappa_{2t}\left(1+\tau^m_{12t}\right)}{\kappa_{2t+1}\left(1+\tau^m_{12t+1}\right)} \frac{\kappa_{1t+1}\left(1-\tau^x_{12t+1}\right)}{\kappa_{1t}\left(1-\tau^x_{12t+1}\right)} \frac{G^1_{1t}}{G^1_{1t+1}} \left[G^1_{1,t+1} F^1_{k,t+1} + 1 - \delta\right] = \frac{G^2_{1,t}}{G^2_{1,t+1}} \left[G^2_{2,t+1} F^2_{k,t+1} + 1 - \delta\right].$$

If the adjustments are such that $\frac{\kappa_{1t+1}}{\kappa_{1t}} = \frac{\kappa_{2t+1}}{\kappa_{2t}}$, the policy is neutral. The nominal intertemporal price, $\frac{Q_t}{Q_{t+1}}$, is adjusting by the same amount, $\frac{\kappa_{1t+1}}{\kappa_{1t}}$.

F Non-Cooperative Foundations of Cooperative Equilibria

Here, we provide explicit non-cooperative foundations for the cooperative Ramsey equilibria in our dynamic environment.

We begin by describing a static model that is a two-country version of our dynamic model. The static model has no capital, no assets, and no government consumption; labor is inelastically supplied.

The households in each country i have preferences over consumption of the country specific final good c_i, labor n_i, and public consumption g_i, $u^i\left(c_i, n_i\right) + h^i\left(g_i\right)$. Firms in country

i produce a country-specific intermediate good y_i, according to

$$\text{(F.1)} \quad \sum_{j=1}^{N} y_{ij} = y_i = F^i n_i,$$

where y_{ij} denotes the quantity of intermediate goods produced in country i and used in country j and F^i is a parameter. The technology for producing the final good is

$$\text{(F.2)} \quad c_i + g_i \leq G^i \left(y_{1i}, y_{2i}\right),$$

where G^i is constant returns to scale.

If lump-sum taxes in each country, as well as transfers across countries, are available, the allocations on the Pareto frontier satisfy the following efficiency conditions:

$$\text{(F.3)} \quad -\frac{u_c^i}{u_n^i} = \frac{1}{G_i^i F_n^i},$$

$$\text{(F.4)} \quad \frac{G_j^i}{G_i^i} \text{ is the same across countries } i,\ j \neq i,$$

which, together with the resource constraints, characterize the Pareto frontier.

Consider now the economy with distorting labor income taxes, τ_i^n; taxes levied on exports shipped from country i to country j, τ_{ij}^x; and a tariff, τ_{ij}^m, levied on imports shipped from country i to country j.

Firms

Each country has two representative firms. The *intermediate good firm* in each country uses the technology in $(F.1)$ to produce the intermediate good using labor. The intermediate good firm maximizes profits given by

$$\text{(F.5)} \quad p_{ii} y_{ii} + \left(1 - \tau_{ij}^x\right) p_{ij} y_{ij} - w_i n_i, \text{ for } j \neq i$$

subject to $(F.1)$. Here, p_{ij} is the price of the intermediate good produced in country i and sold in country j and w_i is the wage rate, all in units of a common world numeraire.

The *final goods firm* of country i chooses the quantities of intermediate goods to maximize profits,

$$q_i G^i \left(y_{ii}, y_{ji}\right) - p_{ii} y_{ii} - \left(1 + \tau_{ji}^m\right) p_{ji} y_{ji}, \text{for } j \neq i.$$

Households

The household problem in country i is to maximize utility subject to the budget constraint

(F.6) $q_i c_i - \left(1 - \tau_i^n\right) w_i n_i \leq 0.$

Governments

The budget constraint of the government of country i is given by

(F.7) $\tau_i^n w_i n_i + \tau_{ji}^m p_{ji} y_{ji} + \tau_{ij}^x p_{ij} y_{ij} = q_i g_i,\ j \neq i.$

Combining the budget constraints of the government and the household (with equality) in each country, we obtain the balance of payments condition of country i:

(F.8) $p_{ij} y_{ij} - p_{ji} y_{ji} = 0,\ j \neq i.$

A *competitive equilibrium* is defined in the usual fashion.

Next, we characterize the competitive equilibrium. To do so, note that the first-order conditions of the household's problem include

(F.9) $-\dfrac{u_c^i}{u_n^i} = \dfrac{q_i}{\left(1 - \tau_i^n\right) w_i}.$

The first-order conditions of the firms' problems are, for all i, $p_{ii} F^i = w_i$,

(F.10) $p_{ii} = \left(1 - \tau_{ij}^x\right) p_{ij},\ i \neq j,$

(F.11) $q_i G_i^i = p_{ii},$

(F.12) $q_i G^i_j = \left(1+\tau^m_{ji}\right) p_{ji},\ i \neq j.$

The first-order conditions can be rearranged as

$$\text{(F.13)}\quad -\frac{u^i_c}{u^i_n} = \frac{1}{\left(1-\tau^n_i\right) G^i_i F^i},$$

$$\text{(F.14)}\quad \frac{G^i_j}{G^i_i} = \frac{\left(1+\tau^m_{ji}\right)\left(1+\tau^m_{ij}\right)}{\left(1-\tau^x_{ji}\right)\left(1-\tau^x_{ij}\right)} \frac{G^j_j}{G^j_i},\ i \neq j.$$

The balance of payments condition can be written as

$$\text{(F.15a)}\quad \frac{G^i_i y_{ij}}{1-\tau^x_{ijt}} - \frac{G^i_j y_{ji}}{1+\tau^m_{ji}} = 0.$$

Next we define and characterize the non-cooperative equilibrium of a game. The timing is that the two governments simultaneously choose their policies. Given these policies, we then have a competitive equilibrium in which households and firms optimize and prices clear markets. Let $\pi_i = \left\{\tau^n_i, \tau^m_{ij}, \tau^x_{ij}\right\}$ denote the policies chosen by the government of country i, and let $\pi = (\pi_1, \pi_2)$. Let $x(\pi) = (x_1(\pi), x_2(\pi))$ denote the resulting competitive equilibrium allocations for the two countries, $x_1(\pi)$ and $x_2(\pi)$, and let $p(\pi)$ denote the associated prices. The government of country i chooses π_i to maximize

$$\text{(F.16)}\quad u^i\left(c_i(\pi_1,\pi_2), n_i(\pi_1,\pi_2)\right) + h^i\left(g_i(\pi_1,\pi_2)\right)$$

subject to its budget constraint,

$$\begin{aligned} & \tau^n_i w_i(\pi_1,\pi_2) n_i(\pi_1,\pi_2) + \tau^m_{ji} p_{ji}(\pi_1,\pi_2) y_{ji}(\pi_1,\pi_2) + \tau^x_{ij} p_{ij}(\pi_1,\pi_2) y_{ij}(\pi_1,\pi_2) \\ \text{(F.17)}\quad = \ & q_i(\pi_1,\pi_2) g_i(\pi_1,\pi_2),\ j \neq i, \end{aligned}$$

taking as given the policies of the other country.

A non-cooperative equilibrium consists of policies, π^*, and allocations and pricing rules, $x(\pi), p(\pi)$, such that for each i, taking π^*_j as given for $j \neq i$, π^*_i maximizes $(F.16)$ over the set of policies, and for all π, $(\pi, x(\pi), p(\pi))$ is a competitive equilibrium.

Proposition F1: Non-cooperative equilibria of the static game do not satisfy production efficiency. The proof is by contradiction. Suppose country 2 sets all trade taxes to zero; then, country 1 can improve its welfare by deviating from zero trade taxes.

The proof of this proposition follows the standard logic in the optimal tariff literature.

For future use, let $z^s = (\pi^*, x^*, p^*)$ and $u^{s,i}$ denote the equilibrium outcomes and utilities in the non-cooperative equilibrium of the static economy.

Dynamic formulation

Consider now an infinite repetition of the static economy. In this infinite repetition, neither consumers nor governments can borrow or lend across periods. The only link between periods is strategic. To develop these strategic links, let h_t denote the history of policies and allocations, up to the beginning of period t. These histories are recursively defined by starting at the null history and constructing h_{t+1} as follows. Let the history for private agents be denoted by $h_{p,t} = (h_t, \pi_t)$, where $\pi_t = (\pi_{1,t}, \pi_{2,t})$. Let $h_{t+1} = (h_{p,t}, x_t, p_t)$, where x_t and p_t denote allocations and prices in period t.

A strategy for government i is given by a sequence of functions $\sigma_{i,t}(h_t)$, which maps histories into period t policies, with $\sigma_t = (\sigma_{1,t}, \sigma_{2,t})$. Allocation and pricing rules are denoted by sequences of functions $x_t(h_{p,t})$ and $p_t(h_{p,t})$, which map histories for private agents into allocations and prices. Strategies, allocations, and pricing rules induce future histories from past histories in the natural way. For example, induced history $h_{p,t}$ from some arbitrary history h_t is given by $h_{p,t} = (h_t, \sigma_t(h_t))$, and the induced history h_{t+1} from some arbitrary history $h_{p,t}$ is given by $h_{t+1} = (h_{p,t}, x_t(h_{p,t}), p_t(h_{p,t}))$. Let $V_t^i(h_t)$ denote the discounted utility for the residents of country i associated with the strategies, allocations, and pricing rules.

A *sustainable equilibrium* of this game consists of strategies, allocation rules, and pricing rules such that (1) for all periods t and for all histories $h_{p,t}$, the induced allocations and prices are a competitive equilibrium; and (2) for all periods t and for all histories h_t, the strategy for, say, government 1 in period t, maximizes

$$\text{(F.18)}\quad u^1(c_{1,t}(h_t, \pi_{1,t}, \pi_{2,t}), n_{1,t}(h_t, \pi_{1,t}, \pi_{2,t})) + h^1(g_{1,t}(h_t, \pi_{1,t}, \pi_{2,t})) + \beta V_{t+1}^1(h_{t+1}),$$

subject to the analog of the budget constraint for the static case, (F.17), where $\pi_{2,t} = \sigma_{2,t}(h_t)$,

$h_{p,t} = (h_t, \pi_{1,t}, \sigma_{2,t}(h_t))$, and $h_{t+1} = (h_{p,t}, x_t(h_{p,t}), p_t(h_{p,t}))$.

A sustainable outcome is defined as an infinite sequence of policies, allocations and prices, $\{\pi_t, x_t, p_t\}_{t=0}^{\infty}$, induced from the null history by a sustainable equilibrium.

Next, we provide a characterization of the set of sustainable outcomes. We restrict ourselves to equilibria that can be sustained by reversion to static outcomes. Formally, we restrict ourselves to equilibria such that for all histories h_{t+1},

$$V_{t+1}^{i}(h_{t+1}) \geq \frac{u^{s,i}}{1-\beta}.$$

These equilibria are the analogs of equilibria in repeated games that are sustained by reversion to the Nash equilibria of the static game.[3]

We then have the following lemma.

Lemma F1: Characterization of sustainable equilibria

An arbitrary sequence $\{\pi_t, x_t, p_t\}_{t=0}^{\infty}$ is a sustainable outcome if and only if (1) it is a competitive equilibrium; and (2) for all periods r, and for, say, country 1

$$\begin{aligned} & \sum_{t=r}^{\infty} u^1(c_{1,t}, n_{1,t}) + h^1(g_{1,t}) \\ \text{(F.19)} \quad \geq \; & u^1\left(c_1^s(\hat{\pi}_{1,t}, \pi_{2,t}), n_1^s(\hat{\pi}_{1,t}, \pi_{2,t})\right) + h^1\left(g_{1,t}^s(\hat{\pi}_{1,t}, \pi_{2,t})\right) + \frac{\beta u^{s,1}}{1-\beta}, \end{aligned}$$

for all $\hat{\pi}_{1,t}$, where $c_1^s(\cdot,\cdot)$, $n_1^s(\cdot,\cdot)$, and $g_{1,t}^s(\cdot,\cdot)$ are the functions associated with the static equilibrium.

The proof of this lemma is a straightforward adaptation of the arguments in Chari and Kehoe (1990). We then have the following proposition:

Proposition F2: Sustainability of the cooperative Ramsey equilibrium

There is some $\tilde{\beta} < 1$ such that for all $\beta \geq \tilde{\beta}$, the cooperative Ramsey outcome is a sustainable outcome.

[3] While we could prove the theorem for other equilibria as well, using the techniques of Abreu, Pearce, and Stachetti (1990), the proof described here is simpler to follow.