VARIABLE RATE SUBSIDIES:
THE INEFFICIENCY OF IN-KIND TRANSFERS REVISITED

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ABSTRACT

The inefficiency of fixed rate consumer price subsidies, relative to cash transfers, is one of the best-known propositions in welfare economics. It has also been used to show that matching grants are a more inefficient intergovernmental aid than are lump sum grants. Furthermore, the cost of fixed rate subsidies cannot be controlled without providing a "cap" beyond which amount no subsidy is received. This paper reports, both qualitatively and quantitatively, that a broad class of variable rate price subsidies also dominates fixed rate subsidies on both counts. The relative inefficiency of matching grants compared to the variable rate Federal General Revenue Sharing program is estimated.

INTRODUCTION

The inefficiency of fixed rate price subsidies for consumer goods, that is, subsidies which pay a fixed percentage of the unit price of a good, is one of the best-known propositions in welfare economics. Among others, Aaron and Von Furstenberg (1971) and Smolensky (1968) have used this proposition to examine the inefficiency of housing subsidies when compared to cash transfers. Haskell (1964), Thurow (1966), and Wilde (1971) have also used it to show that recipient governments will prefer intergovernmental aid with no strings attached over equally costly fixed rate matching grants. Finally, Friedman and Friedman (1980) have used the proposition to promote the negative income tax as a replacement for food stamps and other in-kind transfers.

In addition, these studies often cite another drawback of fixed rate price subsidies. In the absence of a good estimate for the price elasticity of the subsidized good, the sponsoring government cannot determine the total amount of aid to be distributed. While the sponsoring government can place a "cap" on the total amount of aid to be distributed, doing so introduces a kink into the budget constraint of the recipient. This kink complicates the problem of predicting the recipient response to changes in the subsidy rate.[1]

In this paper, it is shown that a broad class of price subsidies, dubbed "variable rate subsidies," are more efficient than conventional fixed rate subsidies, although modestly so. In contrast to fixed rate subsidies, these variable rate subsidies vary the percentage rate of price reduction with the amount of the

subsidized good purchased. Because of this characteristic, recipients always find them somewhat more desirable than equally costly fixed rate subsidies, and the total amount of aid can be fixed in advance without introducing troublesome kinks into the recipients' budget constraints.

The variable rate subsidies examined herein have an additional advantage: the rate of subsidy can be designed to vary among recipients, depending on their socioeconomic characteristics. For example, a special case of the variable rate subsidy is found in the federal government's three-factor General Revenue Sharing (FGRS) formula. Through a tax effort factor incorporated into it, FGRS lowers the price of government services purchased (that is, provided) by recipient state and local governments. The rates of price reduction depend not only on the amounts of government services purchased, but also on the recipient governments' populations and personal incomes.

After a brief review of the theory of fixed rate subsidies, the variable rate subsidies to be studied are defined and their dominance over fixed rate subsidies in the areas cited above is demonstrated using the assumption that all recipients are identical. This assumption is not necessary for the results and is dropped in the application section. There, as an application of the general theory, an estimation is made of the differences between FGRS in 1972 and a hypothetical fixed rate subsidy calculated to provide each state government with its actual 1972 FGRS allotment.

THEORY OF FIXED RATE SUBSIDIES

A brief review of the standard inefficiency argument is in order. There are N recipients of a price subsidy, the ith of which is assumed to allocate its disposable income $(1-t)M_i$ by maximizing a utility U^i of the subsidized good G_i and a composite unsubsidized good C_i. Units are chosen so that prices are initially equal to one. In addition, the analyst splits spending on G_i into two components: the recipient's own-source expenditure T_i and the dollar amount of subsidy R_i, so that

$$G_i = T_i + R_i. \tag{1}$$

A fixed rate price subsidy is calculated as a fixed rate r_i of T_i, that is,

$$R_i = r_i T_i. \tag{2}$$

Consumption of the unsubsidized composite good is then given by

$$C_i = (1-t)M_i - T_i. \tag{3}$$

The problem faced by the ith recipient is to maximize $U^i(C_i, G_i)$ subject to (1), (2), and (3). Solving (3) for T_i and substituting into (2), find

$$R_i = r_i[(1-t)M_i - C_i]. \tag{4}$$

Substituting for T_i and R_i in (1) and simplifying yields the budget constraint:

$$C_i + \frac{1}{1 + r_i} G_i = (1-t)M_i \tag{5}$$

which clearly shows that the fixed rate subsidy lowers the price of G by a fixed amount which is independent of G. The ith recipient is thus assumed to

$$\max U^i(C_i, G_i) \tag{6}$$

$$\text{s.t. } C_i + \frac{1}{1 + r_i} G_i = (1-t)M_i.$$

Assuming an interior solution (C*,G*) exists, it is characterized by

$$\frac{\partial U^i}{\partial C_i} \bigg/ \frac{\partial U^i}{\partial G_i} (C_i^*, G_i^*) = 1 + r_i. \tag{7}$$

The solution of (7) for a typical recipient occurs at the tangency P* depicted in Figure 1. This, of course, is nothing more than the standard graphical solution to the choice problem faced by a consumer with income (1-t)M who pays 1 dollar for each unit of C and 1/(1+r) dollars for each unit of G. Kay (1980), Rosen (1978), and Stutzer (1982) measure recipient i's deadweight loss with an equivalent variation-based measure, which for a subsidy is

$$W_i = R_i - EV_i \tag{8}$$

where EV_i is the equivalent variation in income needed to produce the same utility U_0 as the price subsidy produced. W_i is the largest amount of money recipient i would be willing to forgo in order to obtain a lump sum subsidy rather than a price subsidy. Thus, W_i measures the savings the sponsoring government could attain by replacing a price subsidy with an equal utility producing lump sum subsidy EV_i. The deadweight loss resulting from a

fixed rate subsidy, denoted W*, is depicted in Figure 1 for a typical recipient.

Finally, it is clear that the total amount of subsidies, denoted Q, cannot be fixed in advance without detailed knowledge of each recipient's problem (6) and its solution (7) because

$$Q = \sum_{i=1}^{N} R_i = \sum r_i T_i^* \qquad (9)$$

where $T_i^* = G_i^* - R_i$ varies with r_i according to (7).

VARIABLE RATE SUBSIDIES

A broad class of variable rate subsidies for both consumer and intergovernmental aid can be created by generalizing the so-called three-factor, or Senate, formula used, in part, to distribute Federal General Revenue Sharing funds to the states.[2] The formula is

$$R_i = I_i(T_i)Q = \frac{w_i T_i}{\sum\limits_{j=1}^{51} w_j T_j} Q; \quad i = 1, \ldots, 51 \qquad (10)$$

where R_i is the aid to recipient (that is, state) i.[3] Note that $\sum_{i=1}^{51} R_i \equiv Q$, so that Q can be fixed in advance. $I_i(T_i)$ is thus a function giving the fraction of Q distributed to recipient i. It depends on the level of T_i (for governments, T_i is taxes levied) chosen by recipient i and a weight w_i reflecting the socioeconomic characteristics of recipient i. In Federal General Revenue Sharing, for example, the weight w_i equals the square of the reciprocal of state i's per capita income.[4] The good being subsidized is $G_i = T_i + R_i$. The subsidy is called "variable rate" because, in contrast to (2), the rate of subsidy $\partial R_i / \partial T_i$ for (10), that is,

$$\frac{\partial R_i}{\partial T_i} = \frac{\partial I_i}{\partial T_i} Q = \frac{w_i \sum_{j \neq i} w_j T_j}{(w_i T_i + \sum_{j \neq i} w_j T_j)^2} Q \qquad (11)$$

varies with the level of T_i, ceteris paribus. Note that the rate of subsidy to recipient i declines as T_i increases because

$$\frac{\partial^2 R_i}{\partial T_i^2} = \frac{-2w_i^2 \sum_{j \neq i} w_j}{(\sum_j w_j T_j)^3} Q < 0. \qquad (12)$$

Thus, unlike the fixed rate subsidy (2), (10) is both increasing and strictly concave in T_i.

Similar formulae have been used in some states to distribute state revenue to local governments. More generally, for both consumer and intergovernmental programs, one could define a system

$$R_i = I_i(T_i)Q = \frac{w_i f_i(T_i)}{\sum_j w_j f_j(T_j)} Q; \quad i = 1, \ldots, N \qquad (13)$$

where f_i is strictly concave and increasing in T_i, the recipient's own-source expenditure on the subsidized good G_i.

THEORY OF VARIABLE RATE SUBSIDIES

By substituting (10) for (2), a model of recipient response to variable rate subsidies can be created. Following steps (1)-(5), the ith of N recipients is assumed to solve[5/]

$$\max U^i(C_i, G_i) \tag{14}$$

$$\text{s.t. } C_i + G_i = (1-t)M_i - I_i[(1-t)M_i - C_i]Q$$

$$= (1-t)M_i - \frac{w_i[(1-t)M_i - C_i]}{\sum_{j=1}^{N} w_j[(1-t)M_j - C_j]} Q$$

$$i = 1, \ldots, N$$

treating C_j, $j \neq i$, parametrically.

The assumption that each recipient treats C_j, $j \neq i$, parametrically implies that recipients do not collude to maximize joint utility or to attain some other common objective. Each recipient assumes that its spending does not affect the spending decisions of other recipients. As in models of noncooperative oligopoly, this assumption seems realistic when N is not "too small." In the empirical application discussed later, N = 51, which seems large enough to rule out collusion of this kind.

The solution to (14) is characterized by

$$\frac{\partial U^i}{\partial C_i} \bigg/ \frac{\partial U^i}{\partial G_i} (C_i, G_i) = 1 + \frac{\partial I_i}{\partial T_i} Q \tag{15}$$

$$= 1 + \frac{w_i \sum_{j \neq 1} w_j[(1-t)M_j - C_j]}{\{\sum_j w_j[(1-t)M_j - C_j]\}^2} Q$$

$$i = 1, \ldots, N.$$

Once the budget constraint from (14) is solved for G_i and substituted into (15), there result N simultaneous equations in N unknowns $C_1, \ldots, C_N$. Because the ith recipient is assumed to treat C_j, $j \neq i$, parametrically, a simultaneous solution

$(C_i^!, G_i^!)$, i = 1, ..., N, to (15) is a Nash equilibrium for the noncooperative game described by (14).[6/] Given values $C_j^!$, j ≠ i, in a Nash equilibrium, a typical recipient's budget constraint in (14) is represented in Figure 1 as the concave curve tangent to U_0 at P'. The concavity of the curve follows from the concavity of R_i in T_i, demonstrated in (12). In the figure, the parameters Q and w_1, ..., w_N have been set so that the utility U_0 attained in (14) is the same as that attained under a fixed rate subsidy in (6). At least in this figure, we see that

(a) The deadweight loss of the variable rate subsidy, denoted W', is smaller than that of an equal utility fixed rate subsidy.

(b) Variable rate subsidies stimulate less spending on the subsidized good G than do equal utility fixed rate subsidies.

In Chapter 3 of Stutzer (1981a), both of these properties are shown to hold for general formulae $I_i(T_i)$ which are concave and increasing in T_i, like (13) is. Furthermore, under the additional mild assumption that $\partial^2 U_i / \partial C_i \partial G_i > 0$ in some representation of the preference ordering of recipient i, it is also rigorously proven there that properties (a) and (b) hold for equal cost, rather than equal utility, subsidies.[7/] How much less the deadweight loss and the spending on the subsidized good are depends on all the utility functions U^i and all the parameters r_i, w_i, $(1-t)M_i$, and Q.

QUANTITATIVE COMPARISON OF FIXED AND VARIABLE RATE SUBSIDIES

In order to isolate the inherent differences between fixed and variable rate subsidies, one must first control for variations in the utility functions and parameters. To do so, I follow Aaron and Von Furstenberg's (1971) study of fixed rate subsidies in assuming that recipients possess identical utility functions, have identical disposable incomes, and face the same fixed rate subsidy r. Also, as in Fisher (1981), the assumption is made that the weights w_i in (10) have a common value w.[8/] These assumptions will be relaxed in the empirical application which follows this section, though. Because recipients are, for the moment, assumed to be identical, one can drop the subscript i and sum the right-hand side of (15) to obtain the Nash equilibrium condition for the variable rate subsidy (10):

$$\frac{\partial U}{\partial C} \Big/ \frac{\partial U}{\partial G} (C',G') = 1 + \frac{(N-1)Q}{N^2 T}. \tag{16}$$

Making use of the budget constraint in (14) and the fact that N recipients with identical weights w and tax levels T will each obtain R = Q/N, the following nonlinear equation in C' results:

$$\frac{\partial U}{\partial C} \Big/ \frac{\partial U}{\partial G} [C', (1-t)M + Q/N - C'] \tag{17}$$

$$= 1 + \frac{(N-1)Q}{N^2[(1-t)M - C']}.$$

Similarly, a fixed rate subsidy r would result in a common level of C, denoted C*, solving

$$\frac{\partial U}{\partial C} \Big/ \frac{\partial U}{\partial G} \{C*, (1+r)[(1-t)M - C*]\} = 1 + r. \tag{18}$$

Also following Aaron and Von Furstenberg (1971), one assumes a CES
utility function

$$U(C,G) = \left[aC^v + (1-a)G^v\right]^{1/v} \qquad (19)$$

where $v = 1 - 1/\sigma$ and σ is the constant elasticity of substitution
of C for G. Denote the share of income the recipient allocates to
G, in the absence of any subsidy ($r = 0$), by b. Then it is easy
to show from (18) that

$$a = \left[1 + (\frac{1 - b}{b})^{-1/\sigma}\right]^{-1}. \qquad (20)$$

Then, for any levels of $(1-t)M$ and b, the solution of (18) depends
solely on r and σ. Thus, given levels of these four parameters,
one can compute C* from (18), the cost of the subsidy per recipi-
ent $R = R^*$ from (4), the consumption of the subsidized good $G^* =$
$(1+r)[(1-t)M - C^*]$, and the fixed rate deadweight loss $W = W^*$ per
recipient from (8), using $R = R^*$ and $EV = EV^*$.

To compare a fixed rate subsidy r with an equal cost
variable rate subsidy for various σ and b, Q is set in (17) equal
to (4) times N, and (17) is then solved to obtain C'. From this,
G' is computed from the budget constraint in (14) and the variable
rate subsidy's deadweight loss per recipient $W = W'$ from (8),
using $R = R' = R^*$ and $EV = EV'$. To compute the degree of ineffi-
ciency of fixed rate subsidies relative to equally costly variable
rate subsidies, the relative inefficiency index W^*/W' is com-
puted. A useful index of inefficiency should be invariant to lump
sum transfers of income. Results in Stutzer (1982) establish that
this index is invariant to income changes, at least for homothetic
utilities such as (19).

Finally, in keeping with convention from other studies, the fixed rate subsidy r is represented as a percentage price reduction in G. Noting from (5) that the price of G is 1/(1+r), the percentage price reduction S is

$$S = \frac{r}{1 + r}.$$ (21)

The comparisons for N = 50 recipients with $10,000 disposable income who spend b = .25 of their income on G in the absence of a subsidy are shown in Table 1.[9/] There, note that spending on G increases with the price reduction S and the elasticity of substitution σ. As was claimed earlier, G' < G*, and, because the relative inefficiency exceeds one, W' < W*. While the quantitative differences per recipient are small, they will be magnified in the aggregate. Also, although both W' and W* generally increase with σ, the relative inefficiency decreases with σ. The relative inefficiency approaches one from above as σ → ∞, because then C and G are perfect substitutes, in which case there is no difference between cash and in-kind transfers.[10/]

In Table 2, the same comparisons are made for N = 10,000. The differences between fixed and variable rate subsidies have narrowed substantially. Fixed rate subsidies, which were at most 3.9 percent more inefficient when N = 50, are always less than .1 percent more inefficient when N = 10,000. These computations suggest that there would be no differences between equal cost fixed and variable rate subsidies (10) in the limit as N → ∞, at least when the recipients are identical. In the appendix, a simple proof of this claim is presented which is valid for any

utility function.[11/] This result suggests that the welfare and spending differences between fixed and variable rate subsidies may be more important for high cost programs with a small number of recipients, such as federal aid to states, than for consumer welfare programs like food stamps or housing assistance.

AN APPLICATION:
GENERAL REVENUE SHARING VS. FIXED RATE MATCHING GRANTS

In this section, state-by-state and aggregate impacts of Federal General Revenue Sharing in 1972 are simulated and contrasted with a system of hypothetical equally costly fixed rate matching grants. To do so, the assumption of identical recipients must be dropped because states have varying equilibrium values for C_i, G_i, R_i, and so forth and the weights w_i do vary among states.

In the presence of FGRS in any year, assume that state i would behave as if it solved problem (14) with w_1, ..., w_{51}, with Q given by data obtained for that year, and with U^i given by the CES form:

$$U^i(C_i,G_i) = [a_i C_i^v + (1 - a_i)G_i^v]^{1/v} \qquad (22)$$

$$i = 1, ..., 51.$$

Thus, the distribution parameter a_i is permitted to vary across recipients, while the elasticity of substitution $\sigma = 1/(1-v)$ is not. For each state i, the distribution parameter a_i must be estimated.

To estimate a_i for 1972, assume that the advent of FGRS was not anticipated prior to recipient government budgeting for 1972. Then, in 1972, state i acted as if it maximized (22) sub-

ject to $C_i + G_i = (1-t)M_i$. For any $\sigma = 1/(1-v)$, the first-order conditions for this problem can be solved for a_i in terms of the observed C_i and G_i in 1972:

$$a_i = \frac{G_i^{v-1}}{G_i^{v-1} + C_i^{v-1}}. \qquad (23)$$

Thus, for any assumed common elasticity of substitution σ in 1972, one can obtain the a_i from 1972 NIPA data on C_i and G_i.

After the model is calibrated by this method, (15) is solved simultaneously to obtain the Nash equilibrium (C_i', G_i'), i = 1, ..., 51. The equilibrium FGRS allocation to state i is then $R_i' = I_i[(1-t)M_i - C_i']Q$ given in (13). The deadweight loss W_i' for each state i is calculated by subtracting a computed equivalent variation from R_i' and is summed to obtain a total deadweight loss estimate for FGRS in 1972.

To obtain the comparison between the revenue sharing equilibrium and the fixed rate matching grants, compute an equally costly matching grant rate r_i for each state i by solving the following two equations in the unknowns r_i and T_i^*:

$$\frac{\partial U^i}{\partial C_i} \Big/ \frac{\partial U^i}{\partial G_i} \left[(1-t)M_i - T_i^*, (1+r_i)T_i^* \right] = 1 + r_i \qquad (24)$$

$$r_i T_i^* = R_i'$$

i = 1, ..., 51.

Then numerically compute an equivalent variation in income for (24) and subtract from R_i' to obtain the deadweight loss resulting from state i receiving an equally costly fixed rate matching grant

at the rate r_i. This is denoted W_i^* in Table 3 and is summed to obtain a total fixed rate deadweight loss.

The data in Table 3 indicate that the 1972 deadweight loss from the \$5.3 billion FGRS program would have been \$242.8 million, which is 4.6 percent of the program cost, had the common elasticity of substitution σ been 2. An equally costly system of fixed rate subsidies would have generated a larger deadweight loss of \$258.5 million, which is 6.6 percent larger than the loss due to FGRS. Both of these figures would have been lower had σ been .67, thus confirming the evidence from the identical recipients' case. However, the relative inefficiency of 1.066 is somewhat larger than one would have inferred from the identical recipients' evidence of Table 1.

A QUALIFICATION

It has been assumed to this point that the total cost Q of either subsidy program is not financed by the recipients of the program, that is, that t is independent of Q. This may be a valid assumption for consumer welfare programs, but is surely not as valid for intergovernmental aid programs. Letting M be a recipient's disposable income gross of its contribution to finance Q, its contribution to a fully funded program must be

$$tM = \frac{Q}{N}. \tag{25}$$

One could argue that this contribution is treated as a lump sum tax by the recipient in (6) or (14), in which case the recipients will still prefer equal cost variable rate subsidies to fixed rate ones. However, as has been noticed by both Teeples (1966) and

Fisher (1981), funding fixed rate subsidies by recipient contributions introduces another distortion, for in even the simple case of identical recipients

$$tM = \frac{Q}{N} = \frac{r \sum_{j=1}^{N} [(1-t)M_j - C_j]}{N} \tag{26}$$

which results in the maximization condition (18) being modified to

$$\frac{\partial U}{\partial C} / \frac{\partial U}{\partial G} \{C^*, (1+r)[(1-t)M - C^*]\} = \frac{1 + r}{1 + r/N}. \tag{27}$$

Clearly, as $N \to \infty$, there is no difference between (27) and (18). Because of this, the limiting equivalence of fixed and variable rate subsidies proven in the appendix is still valid when the program is fully funded by its recipients.[12/]

CONCLUSION

Variable rate subsidies are price subsidies which have two properties. First, the rate of subsidy varies smoothly with the amount of the subsidized good purchased; that is, the price changes with the amount purchased. Second, the total cost of the subsidy program can be set in advance without introducing kinks into recipients' budget constraints. Both of these properties are not present in conventional fixed rate price subsidies. A broad class of variable rate subsidies defined herein is preferred by recipients to equally costly fixed rate subsidies, a fact illustrated graphically by Johnson (1975) and proved in Stutzer (1981a). While the associated decrease in deadweight loss per recipient is modest, it is not inconsequential. However, it is proven that when recipients are identical, the decrease in dead-

weight loss per recipient shrinks to zero as the number of recipients increases to infinity. A special case of this more general result was previously reported by Fisher (1981). In an empirical application of these methods, where recipients are not identical, a variable rate subsidy feature present in the Federal General Revenue Sharing formula was shown to be 6.6 percent more efficient than an equally costly system of fixed rate subsidies (that is, matching grants).

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NOTES

[1]/See Waldauer (1973) for further discussion of this problem.

[2]/Federal General Revenue Sharing is a far more complicated system than just the Senate formula. See Nathan et al. (1975) for a more detailed description.

[3]/Puerto Rico is counted as the 51st state.

[4]/See Johnson (1975) for details.

[5]/Also see Johnson (1977) and Fisher (1977).

[6]/For a formal existence proof, see Stutzer (1981a).

[7]/See Johnson (1975) for a graphical illustration of these propositions.

[8]/These assumptions are not necessary for actual applications. In Stutzer (1981b), large-scale simulation with recipients differing in their weights and incomes is shown to be a practical technique.

[9]/A computer program calculating the comparisons for arbitrary parameter values is available from the author.

[10]/The author is indebted to Henry Aaron for pointing this out.

[11]/For a more lengthy proof of this, valid only for Cobb-Douglas utilities, see Fisher (1981).

[12]/Fisher (1981) has argued that this distortion brings variable rate subsidies closer to fixed rate subsidies for finite N as well.

APPENDIX

Denoting the "demand" function solving (17) by $C'(N)$, substitute the equal cost condition $Q = Nr[(1-t)M - C^*]$ into (17) and take the limit as $N \to \infty$ in (17) to obtain

$$\frac{\partial U}{\partial C} \Big/ \frac{\partial U}{\partial G} \{C'(\infty), (1-t)M + r[(1-t)M - C^*] - C'(\infty)\} \qquad \text{(i)}$$

$$= 1 + r \frac{(1-t)M - C^*}{(1-t)M - C'(\infty)}$$

where C^* solves the fixed rate subsidy maximization condition (18). A simple substitution verifies that $C'(\infty) = C^*$ is the unique solution to (i).

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