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INCOME STABILITY AND ECONOMIC EFFICIENCY UNDER ALTERNATIVE TAX SCHEMES

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ABSTRACT

The relative efficiency of alternative income tax systems is analyzed in a dynamic, general equilibrium model having an endogenous labor supply and imperfect risk sharing. This theoretical model allows different tax systems to be compared with respect to their labor distortion effects, their automatic income stability properties, and the welfare they provide on average to a representative consumer-laborer. The comparisons are done for the optimal tax parameters under each given tax system. Despite a role for income stabilization, the optimal income tax schedule turns out to be regressive.

In the last year, proposals for a flat-rate income tax system have been advanced or proclaimed by prominent economists (including Robert Hall and Milton Friedman), President Reagan, and representatives and senators of both political parties.[1/] Arguments in favor of these proposals point out the distortions caused by our current, progressive tax system. But because they fail to consider the automatic income stabilization our current system provides, the arguments are incomplete. The purpose of this paper is to analyze the desirability of alternative income tax systems in a setting where income taxes distort individual labor-leisure decisions and where there is a role for income stabilization. For the specific examples examined in this paper, it is favorable to give up some income stability for reduced labor distortion so that the arguments for moving away from a progressive tax system still seem valid.

The proposals that have been advanced generally consist of two parts: first, to raise a given amount of revenue by broad-

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[1/]In the Wall Street Journal article "Simpler Tax Laws Are a Top Priority: Flat Rate Is among Ideas," Kenneth Bacon reported that ". . . Mr. Reagan called the simplification of the tax laws a 'top priority' of his administration. One of the possible changes the White House is looking at, he said, is a move toward a 'flat rate' tax . . ." (January 21, 1983, p. 4). Meanwhile, in the Ninety-seventh Congress, 12 comprehensive income tax bills were introduced that proposed moves toward a proportional tax. Included among these are the Flat Rate Tax Act of 1982 H.R. 5513 (Rep. Crane, R.) and S.2200 (Sen. Helms, R.); the Income Tax Simplification Act of 1982 H.R. 6070 (Rep. Panetta, D.); and the Fair Tax Act of 1982 H.R. 6944 (Rep. Gephardt, D.) and S.2817 (Sen. Bradley, D.).

ening the tax base and lowering the average tax rate and, second, to move from a progressive tax rate structure to a single flat rate. The first part of the proposals conforms to a general tenet of optimal taxation and seems desirable. Since it can be implemented with any tax rate structure, it is not addressed in this paper. The second part, which is addressed, has consequences for both income distribution and efficiency that make its desirability difficult to evaluate.

A change from the current progressive income tax structure to a flat rate requires for a given level of revenue that the rich pay relatively less and the poor pay relatively more. The desirability of this change in income distribution or of offsetting it through other taxes and transfers is a political consideration.

But even if income distribution is held constant, the proposed change in tax structures has opposing effects, which make an efficiency ranking of the two taxes difficult to determine. On the one hand, a flat-rate structure provides a smaller disincentive to work and thus interferes less with the decisions of consumer-laborers. On the other hand, a flat-rate structure provides less automatic income stabilization, which can be welfare reducing when people are risk averse and there is imperfect risk sharing. An efficient tax structure must give weight to both types of effects.

In this paper, the relative efficiency of alternative income tax systems is analyzed in a dynamic, general equilibrium model having an endogenous labor supply and imperfect risk sharing. This theoretical model allows different tax systems to be compared with respect to their labor distortion effects, their automatic income stability properties, and the welfare they provide on average to a representative consumer-laborer. The comparisons are done for the optimal tax parameters under each given tax system.

The model incorporates a primitive business cycle. Labor productivity in each period is assumed to be an independent, identically distributed random variable. Individuals choose how many hours to work and, hence, how much income to earn, based on knowledge of the state of labor productivity and taxes. Thus, income can vary from period to period according to the state of labor productivity and the response of workers to different returns to labor.

Taxes can stabilize income in this model by moderating the response of labor supply to productivity changes. It is assumed that individual preferences imply that labor supply is positively related to the after-tax return to work. One tax stabilizes income relative to another when the stationary distribution of income it implies has a smaller variance.

The different tax systems that are compared include lump sum, flat tax, contingent flat tax (where the contingency is with respect to the state of either productivity or income), and quadratic tax schedule (allowing a progressive or regressive tax structure).[2/] The lump-sum tax system is included to provide a

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[2/] A tax structure is defined in this study to be progressive, proportional, or regressive as the second derivative of (footnote continued) standard for comparison: it delivers the optimal allocation of goods. It is ruled out as a practical alternative, however, so that some form of income tax must be used.

Comparison of the contingent and single flat taxes demonstrates a role for income stabilization. The optimal contingent flat tax is found to have a higher rate when the economy is strong than when it is weak, thus stabilizing income relative to a flat-rate system. Moreover, because a contingent flat tax includes a single flat tax as a special case, it is clear that the optimal contingent flat tax increases welfare relative to the optimal single flat tax.

Despite this role for income stabilization, the optimal income tax schedule turns out to be regressive. Although a regressive tax schedule reduces income stability relative to a flat-rate system, it more than offsets that loss with a decrease in labor distortion. The optimal regressive tax structure then improves on welfare relative to either the single flat-rate or contingent flat-rate systems.

Several implications follow from the model and results. First, it is possible to construct a dynamic model of business fluctuations that permits a welfare analysis of alternative tax structures. Second, such an analysis is necessary in determining the desirability of alternative income tax structures. Judging

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tax revenue with respect to income is positive, zero, or negative, respectively. A tax is regressive, for example, if the marginal tax rate declines as income increases. A tax structure describes the taxes a given individual faces at different levels of income. Thus, in this study, progressivity and regressivity do not refer to the distribution of tax burdens across individuals in different income classes.

desirability based solely on macroeconomic criteria, such as income stabilization properties, can be misleading. Third, the relative efficiency of a nonprogressive tax system in a dynamic setting is consistent with findings of previous studies done in a static setting.[3/] If nothing else, this strengthens the argument for moving away from a progressive tax system.

In the next section, the model and methodology are described under somewhat general assumptions about utility and production functions. In the following section, those assumptions are specialized to allow derivation of explicit expressions for demand and supply functions and for optimal tax rates. The specialized utility functions allow average welfare to be divided into parts relating to the means and variances of consumption and leisure in each period of an individual's life. It can also be divided into the loss due to labor distortion and the loss due to instability. A comparison of alternative tax systems with respect to average welfare and its components is then carried out for numerical parameter values. The paper concludes with comments on the limitations and possible extensions of the analysis.

THE GENERAL MODEL AND METHODOLOGY

The model is populated by overlapping generations of two-period lived agents. Each generation consists of N identical agents with discounted utility functions

$$W(t) = U[c_1(t),\tilde{L}_1(t)] + \beta\, U[c_2(t),\tilde{L}_2(t)],$$

[3/] See Seade 1977.

where for each individual born in period t,

$c_i(t)$ = consumption in i^{th} period of life,

$\tilde{L}_i(t)$ = leisure in i^{th} period of life,

$\tilde{L}_i(t) \in [0,1]$, and the contemporaneous utility function U is assumed to be concave.[4/]

Each individual is endowed with one unit of time each period. In the first period, that time can be divided between leisure $\tilde{L}_1$ and labor L: $L + \tilde{L}_1 = 1$. In the second period, all the time must go to leisure: $\tilde{L}_2 \equiv 1$.

A perishable consumption good y(t) is produced with constant returns to labor:

$$y(t) = \mu(t)L(t),$$

where μ is a serially uncorrelated random variable. A higher value of μ corresponds to higher labor productivity and a better state of the economy. The young at time t observe $\mu(t)$ before they decide how much to work.

In this economy, no private exchanges will occur. Because the N young are identical and face identical production possibilities, they cannot gain by trading among themselves. And because the N old have only leisure, they have nothing to exchange.

[4/] The t notation will be dropped when there is no confusion.

With no private exchanges, there is a role for government tax-transfer schemes. It is assumed that the government has available balanced schemes under which it taxes the working young and transfers the proceeds to the retired old.

The taxes that are considered are of the general form:

$$T(I,y) = k_0(I) + k_1(I)y + k_2(I)y^2,$$

where T is the tax collected from a representative worker and I is the information available to the government. It is assumed that I = μ (full information) or I = y (incomplete information).

Given production and tax transfers, we then have for consumption,

$$c_1(t) = y(t) - T(I,y) = \mu(t)L(t) - T(I,y) \text{ and}$$

$$c_2(t-1) = T(I,y).$$

In general, the labor supply function L(t) will depend on labor productivity $\mu(t)$ and the tax system T(I,y). It is assumed that the second-period transfer is independent of the individual's first-period work effort.

The following tax systems are considered:

I. Lump-sum tax: $I = \mu$, $k_1 = k_2 = 0$

II. Income taxes: $k_0 = 0$

   A. Proportional: $k_2 = 0$

      1. Single flat rate: $k_1(I) = k_1(I')$

      2. State-contingent flat rate: $I = \mu$

      3. Income-contingent flat rate: $I = y$

B. Nonproportional: $I = y$, $k_1(y) = k_1$, $k_2(y) = k_2$.

The properties of alternative tax systems are found in two steps. In the first step, labor supply functions are derived for given labor productivity and given tax systems. Thus, L is found by

$$\max_{L} \{ \underset{\mu(t+1)}{E}[W|\mu(t)] = U(c_1, 1-L) + \beta \underset{\mu(t+1)}{E} U(c_2, 1)\}$$

subject to $c_1 = \mu(t)L - T(I,y)$ and c_2 independent of L. The consumption of an individual in the second period of life $c_2(t)$ depends on the productivity of labor of the younger generation $\mu(t+1)$, the young's labor supply $L(t+1)$, and the tax-transfer system $T(I,y)$; it does not depend on the individual's work effort in the first period of life. Thus, the L found in the first step depends only on observed productivity $\mu(t)$ and the tax system T: $L = L(\mu,T)$.

In the second step of the solution, optimal parameters for a given tax system are found by assuming that the government chooses fixed parameters for all time in order to maximize the average welfare of all generations from time t on:

$$\max_{T} \{ \underset{\mu(t),\mu(t+1)}{EW} = \underset{\mu(t)}{E} U(c_1, 1-L) + \beta \underset{\mu(t+1)}{E} U(c_2, 1)\}$$

subject to

$$L(t) = L[\mu(t),T], \; 0 \leqq L(t) \leqq 1$$

$$y(t) = \mu(t)L(t)$$

$$c_1(t) = y(t) - T[I,y(t)]$$

$$c_2(t) = T[I,y(t+1)].$$

Some defense of this second step is required. First, the maximization is actually with respect to the expected welfare of a representative individual of generation t, where the expectation is taken before $\mu(t)$ is observed. Because all generations are composed of identical agents, the solution to the former problem must also be the maximizer for all future generations. If the government observed $\mu(t)$, then the young at time t and all future generations could not be treated symmetrically. Second, it is assumed that the labor supply functions are time invariant, but that is a clear implication of the stationary setup of the model. Similarly, the stationary setup of the model justifies considering tax systems with fixed parameters over all time. Finally, the optimality criterion ignores the welfare of the old at time t. This was done to avoid multiple Pareto optimal allocations and to focus on unique steady-state optima. The old want only the largest transfer possible and do not care about labor incentive effects. Thus, there is a broad range of tax parameters that makes the young and all future generations worse off but makes the old better off as it raises their transfers. Such a range would correspond to Pareto noncomparable allocations if the welfare of the old were included. (Because the optimality criterion gives equal weight to the welfare of the young and all future generations, extending the criterion to the old would give them a very small [zero] weight, anyway.)

Once optimal parameters have been determined for given tax systems, it is possible to compare economies operating under different systems. The model generates distributions of consumption, labor, and income and allows a ranking of the tax systems with respect to the average welfare they imply.

SOLUTIONS UNDER SPECIAL ASSUMPTIONS

In this section, relationships are derived under special assumptions about utility and production functions, and then solutions are reported given numerical parameter values for these special functions. The specific, numerical examples illustrate the feasibility of the approach, the types of micro and macro information it can provide, and the optimality of nonprogressive income taxes in a well-specified model of the economy.

Simplifications

Contemporaneous utility is assumed to be a quadratic function of consumption and leisure:

$$U(c,\tilde{L}) = -A(c-c^*)^2 - B(\tilde{L}-\tilde{L}^*)^2; \; A > 0 \text{ and } B > 0.$$

Although this form for the contemporaneous utility function was chosen partly for mathematical convenience, it was also chosen to conform to objective functions used in standard models of income stabilization policy.[5/] The parameters c^* and $\tilde{L}^*$ can be considered to be either the target or satiation levels of consumption and leisure, respectively. In order for the utility function to be nicely behaved, it must be assumed that c^* and $\tilde{L}^*$ are at least as large as any attainable levels. Given this interpretation of

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[5/] A cross-product term $-C(c-c^*)(\tilde{L}-\tilde{L}^*)$ was initially included, but it seemed to complicate the analysis without providing additional insights. It, therefore, was dropped.

c* and $\tilde{L}^*$, it both makes sense and further simplifies the analysis to set $\tilde{L}^* \equiv 1$.

An advantage of a quadratic utility function is that expected utility can be decomposed into the squared deviations of expected consumption and leisure from their targets and the variances of consumption and leisure. Given the special assumptions made above, it follows that

$$\underset{\mu(t),\mu(t+1)}{EW} = \underset{\mu(t)}{E} U(c_1,1-L) + \beta \underset{\mu(t+1)}{E} U(c_2,1)^2 =$$

$$-A(\bar{c}_1-c^*)^2 - A\sigma^2_{c_1} - B\bar{L}^2 - B\sigma^2_L - \beta A(\bar{c}_2-c^*)^2 - \beta A\sigma_{c_2}^2,$$

where $(\bar{\cdot})$ and $\sigma^2_{(\cdot)}$ can be considered unconditional expectations and variances, respectively, because μ is i i d. Given this utility function, the utility loss from labor distortion can be associated with the terms:

$$\text{Distortion} = -A(\bar{c}_1-c^*)^2 - B\bar{L}^2 - \beta A(\bar{c}_2-c^*)^2$$

and the utility loss from income instability can be associated with the remaining terms:

$$\text{Instability} = -A\sigma^2_{c_1} - B\sigma^2_L - \beta A\sigma_{c_2}^2.$$

As the last expression makes clear, it is the variance of consumption and leisure, and not the variance of income per se, that leads to a loss in utility.

The productivity shock is assumed to be a Bernoulli random variable with probability of either state occurring being 1/2:

$$p(\mu_t=\mu_B) = 1/2$$

$$p(\mu_t=\mu_G) = 1/2.$$

Without loss of generality, the "bad" state productivity μ_B is set equal to 1, and the "good" state productivity μ_G is set equal to $\theta > 1$.

Basic Relationships Under Alternative Tax Systems

Given the special assumptions, it is straightforward to derive distributions for consumption, labor, and income under postulated tax systems. These distributions indicate the level of each variable in each state of the economy. For the lump-sum and proportional tax systems, it is also possible to derive explicit expressions for the optimal taxes. The distributions and tax expressions are derived below for the different tax systems.

Lump-sum

Labor supply functions conditional on the observed state of the economy are derived from the problem:

$$\max_L \left[\underset{\mu_{t+1}}{EW}|\mu \right] = \max_L \left[-A(c_1-c^*)^2 - BL^2\right]$$

subject to

$$c_1 = \mu L - k_0(\mu) \text{ and}$$

$$0 \leqq L \leqq 1.$$

Letting 'B' and 'G' superscripts denote functions or taxes conditional on the state μ_B or μ_G, respectively, and assuming an interior solution, we obtain the labor supply functions:

$$L^B = \frac{A(k_0^B+c^*)}{A+B} \text{ and } L^G = \frac{A\theta(k_0^G+c^*)}{A\theta^2+B}.$$

These labor supply functions indicate that lump-sum taxes are not neutral; labor supply, in fact, increases as the lump-sum tax increases. The reason is that although a change in the lump-sum tax has no substitution effect with respect to leisure, it does have an income effect. With both consumption and leisure being normal goods, a rise in the lump-sum tax lowers income and decreases the demands for both goods.

Given the labor supply functions, income and consumption conditional on each state are

$$\begin{aligned} y^B &= L^B & y^G &= \theta L^G \\ c_1^B &= y^B - k_0^B & c_1^G &= y^G - k_0^G \\ c_2^B &= k_0^B & c_2^G &= k_0^G. \end{aligned}$$

The means and variances of consumption, labor, and income can easily be calculated given their distributions.

The optimal lump-sum taxes contingent on the observed state of the economy are found as solutions to the problem:

$$\max_{k_0^B, k_0^G} \{EW = -A(\bar{c}_1-c^*)^2 - A\sigma_{c_1}^2 - B\bar{L}^2 - B\sigma_L^2 - \beta A(\bar{c}_2-c^*)^2 - \beta A\sigma_{c_2}^2\}.$$

The maximizing values of k_0^B and k_0^G are found from the first-order conditions to be

$$\hat{k}_0^B = \frac{[\beta(A+B)-B]c^*}{\beta(A+B)+B} \text{ and } \hat{k}_0^G = \frac{[\beta(A\theta^2+B)-B]c^*}{\beta(A\theta^2+B)+B}.$$ 6/

The distributions of consumption, labor, and income can then be found by substituting the tax rates $\hat{k}_0^B$ and $\hat{k}_0^G$ in the functions derived in the earlier step. Maximum expected utility is the value of the objective function calculated for the means and variances of consumption and labor from these distributions.

The lump-sum tax system serves as a standard of comparison for the income tax systems that follow. In deriving optimal income taxes, it is assumed that no lump-sum taxes are feasible.

Proportional

The optimal tax rates $k_1(I)$ for the three types of proportional tax systems considered (single flat rate, state-contingent flat rate, and income-contingent flat rate) are all found from small variations of the same problem. In order to avoid repetition, therefore, only the solution to the state-contingent flat tax is described in detail. The differences in the other solutions are simply noted.

Labor supply functions contingent on the observed state of the economy are derived from the problem:

$$\max_L [\underset{\mu_{t+1}}{EW} | \mu_t] = \max_L [-A(c_1-c^*)^2 - BL^2]$$

6/ In order to have an interior solution, the maximizing values of k_0 must imply $c_1 \geq 0$, $c_2 \geq 0$, and $0 \leq L \leq 1$ for each state of the economy. These conditions depend on all the parameters of the utility and production function. The examples that follow prove that parameter values exist under which all conditions are satisfied as strict inequalities.

subject to $c_1 = [1-k_1(\mu)]\mu L$ and $0 \leqq L \leqq 1$. Assuming an interior solution, the state-contingent labor supply functions, L^B and L^G, are given by

$$L^B = \frac{Ac^*(1-k_1^B)}{A(1-k_1^B)^2+B} \quad \text{and} \quad L^G = \frac{Ac^*\theta(1-k_1^G)}{A\theta^2(1-k_1^G)^2+B} .$$

Labor supply can either increase or decrease with respect to an increase in a proportional income tax. That is because a change in a proportional income tax has both a substitution effect and an income effect. It is assumed here, and in all numerical examples that follow, that an interior solution obtains and that the substitution effect dominates to imply

$$L^i(k_1) > L^i(k_1+\Delta k),\ \Delta k > 0, \text{ and } i = B \text{ or } i = G$$

and

$$L^B(k_1^B) < L^G(k_1^G) \text{ if } k_1^B = k_1^G.$$

An interior solution along with the two conditions immediately above is assured when

$$\frac{Ac^*}{A\theta^2+B} \leqq 1 \text{ and } B > A\theta^2.$$

Given the labor supply functions, income and consumption conditional on each state are

$$y^B = L^B \qquad y^G = \theta L^G$$

$$c_1^B = (1-k_1^B)y^B \qquad c_1^G = (1-k_1^G)y^G$$

$$c_2^B = k_1^B y^B \qquad c_2^G = k_2^G\, y^G.$$

Again, the means and variances of consumption, labor, and income can easily be calculated given their distributions.

The optimal state-contingent flat rates are found as solutions to the problem:

$$\max_{k_1^B, k_1^G} \{EW = -A(\bar{c}_1 - c*)^2 - A\sigma_{c_1}^2 - B\bar{L}^2 - B\sigma_L^2 - \beta A(\bar{c}_2 - c*)^2 - \beta A\sigma_{c_2}^2\}.$$

The maximizing values of k_1^B and k_1^G are found from the first-order conditions to be the single roots in the interval (0,1) that satisfy the equations:

$$(1) \qquad 2\beta A^2(\hat{k}_1^B)^4 + [-7\beta A - 4\beta B + B]A(\hat{k}_1^B)^3 + [9\beta A + 9\beta B - 3B]A(\hat{k}_1^B)^2$$
$$+ [-5\beta A^2 - 7\beta AB - 2\beta B^2 + 3AB + B^2](\hat{k}_1^B) + [\beta A^2 + 2\beta AB + \beta B^2 - AB - B^2] = 0$$

$$(2) \qquad 2\beta A^2\theta^4(\hat{k}_1^G)^4 + [-7\beta A\theta^2 - 4\beta B + B]A\theta^2(\hat{k}_1^G)^3$$
$$+ [9\beta A\theta^2 + 9\beta B - 3B]A\theta^2(\hat{k}_1^G)^2$$
$$+ [-5\beta A^2\theta^4 - 7\beta AB\theta^2 - 2\beta B^2 + 3AB + B^2](\hat{k}_1^G)$$
$$+ [\beta A^2\theta^4 + 2\beta AB\theta^2 + \beta B^2 - AB\theta^2 - B^2] = 0.$$

Single roots in (0,1) satisfy these equations when

$$\beta > \frac{B}{A+B}.$$ 7/

Probably the most interesting feature of these equations in $\hat{k}_1^B$ and $\hat{k}_1^G$ is their complexity, given the extremely simple setup of the model. There is virtually no hope of determining analytically the

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7/ This condition is also required to guarantee that the optimal lump-sum taxes (and, thus, second-period consumption), k_0^B and k_0^G, are positive.

distributions of consumption, labor, and income and expected welfare under the optimal tax rates $\hat{k}_1^B$ and $\hat{k}_1^G$.

The optimal single flat tax can be found using the state-contingent labor supply functions above and then requiring $k_1^B = k_1^G = k_1$. Maximizing EW with respect to k_1 gives the optimizing value of k_1 as the single root of the equation found from the first-order condition:

$$
\begin{aligned}
(3)\qquad & 2\beta A^2(1+\theta^4)\hat{k}_1^4 + [-7\beta A^2(1+\theta^4)-4\beta AB(1+\theta^2)+AB(1+\theta^2)]\hat{k}_1^3 \\
& + [9\beta A^2(1+\theta^4)+9\beta AB(1+\theta^2)-3AB(1+\theta^2)]\hat{k}_1^2 \\
& + [-5\beta A^2(1+\theta^4)-7\beta AB(1+\theta^2)-4\beta B^2+3AB(1+\theta^2)+2B^2]\hat{k}_1 \\
& + [\beta A^2(1+\theta^4)+2\beta AB(1+\theta^2)+2\beta B^2-AB(1+\theta^2)-2B^2] = 0.
\end{aligned}
$$

The optimal income-contingent flat tax can duplicate the consumption and leisure distributions of the optimal state-contingent flat tax, when individuals have the incentive to work longer in the good state. Let $\hat{y}^B$ be the level of income in the bad state under the optimal state-contingent tax $\hat{k}_1^B$. The income-contingent tax system then can be defined:

$$
k_1(y) = \begin{cases} \hat{k}_1^B & \text{for } y \leqq \hat{y}^B \\ \hat{k}_1^G & \text{for } y > \hat{y}^B \end{cases}.
$$

Given this tax schedule, the individual will decide to work $\hat{L}^B$ hours in the bad state and generate income $\hat{y}^B$. In the good state, however, the individual has two choices. He can face the tax rate $\hat{k}_1^G$ and decide to work $\hat{L}^G$ hours as under the state-contingent system. Or, he can work $L^{G'} = \hat{L}^B/\theta$ hours to produce income $\hat{y}^B$ and face the tax rate $\hat{k}_1^B$. The income-contingent tax system is incentive compatible if the individual chooses the first option and that requires

$$-A[(1-\hat{k}_1^G)\theta\hat{L}^G-c^*]^2 - B\hat{L}^{G2} \geqq -A[(1-\hat{k}_1^B)\hat{L}^B-c^*]^2 - B(\hat{L}^B/\theta)^2.$$

If this condition is satisfied, the optimal income-contingent and state-contingent systems will be equivalent. If it is not satisfied, the optimal income-contingent flat tax system will be either the best incentive-compatible income-contingent rates or the best low rate in an incentive-incompatible system. (The low rate will be the only rate paid in an incentive-incompatible system.) The choice between the two is based on which yields higher expected utility.

Nonproportional

Under the nonproportional tax system, an individual faces the tax rate $k_1 + k_2y$. Thus, first-period consumption is given by $c_1 = (1-k_1-k_2y)y$. In terms of state-contingent labor input, it is given by

$$c_1(\mu) = \alpha(\mu)L(\mu) - \delta(\mu)L(\mu)^2,$$

where

$$\alpha(\mu) = (1-k_1)\mu \text{ and}$$

$$\delta(\mu) = k_2\mu^2.$$

Labor supply functions conditional on the state of the economy are derived from the problem:

$$\max_{L} \underset{\mu(t+1)}{E}[W|\mu(t)] = \max_{L}[-A(c_1-c^*)^2 - BL^2]$$

subject to

$$c_1 = \alpha(\mu)L - \delta(\mu)L^2 \text{ and}$$

$$0 \leqq L \leqq 1.$$

Assuming an interior solution, the state-contingent labor supply functions are given by the single real roots to the equations:

$$L^3 - \left(\frac{3\alpha}{2\delta}\right)L^2 + \left[\frac{A(\alpha^2+2\delta c^*)+B}{2A\delta^2}\right]L - \frac{\alpha c^*}{2\delta^2} = 0,$$

where

$$\text{for } L^B \begin{cases} \alpha = 1-k_1 \text{ and} \\ \delta = k_2 \end{cases}$$

$$\text{for } L^G \begin{cases} \alpha = (1-k_1)\theta \\ \delta = k_2\theta^2 \end{cases}.$$

A single real root is implied if $B > A\theta^2$, a condition required with proportional taxes to imply a stronger substitution effect than income effect with respect to leisure.

Explicit solutions could be found for L^B and L^G from the equations above, and then distributions for consumption and income could be derived. The optimal tax parameters $\hat{k}_1$ and $\hat{k}_2$ could then be found from maximizing EW.

That strategy was not followed, however, for two reasons. It would be extremely tedious for one. And the expressions would be so complex that it would be difficult to interpret or manipulate them for another. Instead, labor supply is calculated for the specific numerical examples that follow, and then optimal values of k_1 and k_2 are found using a simple grid search routine with an EW criterion.

The Effectiveness of Automatic Stabilizers

As McCallum and Whitaker (1979) showed, policies that affect the slopes of aggregate demand or supply curves can dampen the response of output to shocks and, therefore, can be effective automatic stabilizers, even with rational expectations. In this analysis, income taxes affect the slopes of labor supply curves and can dampen the response of output to productivity shocks. Thus, income taxes can be designed to effectively stabilize income. Because people correctly perceive the taxes, the effectiveness of the automatic stabilizers does not depend on any surprise element, and the Lucas critique does not apply.

When the government has as many policy instruments as states of the economy, it potentially can perfectly stabilize income. That indeed is the case for the contingent proportional income taxes analyzed above. The state-contingent flat taxes that perfectly stabilize income are given parametrically by

$$k_1^B = 1 - \left[\frac{B}{A\theta^2}\right]^{1/2} \cdot \left[\frac{\theta^2\lambda-1}{\lambda-1}\right]^{1/2} \cdot \left[\frac{1}{\lambda}\right]^{1/2},$$

$$k_1^G = 1 - \left[\frac{B}{A\theta^2}\right]^{1/2} \cdot \left[\frac{\theta^2\lambda-1}{\lambda-1}\right]^{1/2} \cdot \lambda^{1/2}, \text{ and}$$

$$\frac{A+B-[(A+B)^2-4AB/\theta^2]^{1/2}}{2A} \leqq \lambda \leqq \frac{1}{\theta^2}.$$

They are the tax rates that imply $L^G = L^B/\theta$.

The state-contingent income taxes that perfectly stabilize income are necessarily incentive incompatible if they are used as income-contingent taxes. The individual could have both more consumption and more leisure by paying the lower tax in the good state.

The income-contingent tax system can perfectly stabilize income by exploiting the incentive incompatibility condition. If k_1^G is set equal to one, then the individual will always decide to work L^B/θ in the good state to produce income y^B.

In the numerical examples that follow, the optimal state-contingent and income-contingent taxes from those that perfectly stabilize income are reported in addition to the rates that maximize EW with no side constraint. Although it is intuitively clear that perfect stabilizers cannot be optimal over the whole feasible set of tax rates, it is instructive to examine why they lead to a loss in welfare.

Numerical Examples

Parameter values of the utility and production functions were chosen to imply internal solutions and a negative relationship between labor supply and tax rates under a proportional tax system. The initial parameter set is $\langle A,B,c^*,\beta,\theta\rangle = \langle 2,10,4,.9,1.5\rangle$, and the results under given tax systems are displayed in Table 1 (p. 27).

Outcomes under six different taxes are listed. The first two columns are the outcomes under the optimal lump-sum and optimal single flat tax, respectively. The third column is the outcome under either the optimal state-contingent or the optimal income-contingent flat tax because the taxes are incentive compatible in the latter case. The fourth column is the outcome under the optimal nonproportional tax. The last two columns are the outcomes under the best state-contingent and income-contingent flat taxes, respectively, that perfectly stabilize income.

Rows 1-2 list average tax rates; rows 3-6 list means of consumption, labor, and income; and rows 7-10 list variances of these variables. Rows 11-14 list expected welfare, broken down by the parts attributable to first-period consumption $EW[c_1] \equiv -A(\bar{c}_1-c^*)^2 - A\sigma^2_{c_1}$; labor $EW[L] \equiv -B\bar{L}^2 - B\sigma^2_L$; and second-period consumption $EW[c_2] \equiv -\beta A(\bar{c}_2-c^*)^2 - \beta A\sigma^2_{c_2}$. The last two rows break expected utility into the loss due to distortion and the loss due to instability, where these terms are defined on pages 10 and 11.

In comparing the outcomes across columns, we see that the single flat-rate tax is inferior to the lump-sum tax solely because of its adverse incentive effects. It actually reduces instability relative to the lump-sum tax. The relative loss in expected utility occurs entirely with respect to second-period consumption. In order to effect the same second-period transfer as a lump-sum tax, the flat-rate tax would have to be too high, resulting in too great a loss in labor supply and, consequently, in first-period consumption.

The contingent flat-rate tax improves on expected welfare over the single flat-rate tax, and the gain is entirely due to reduced instability. It causes a larger loss with respect to labor distortion. The contingent flat-rate tax raises the mean and lowers the variance of labor for a net loss in terms of expected welfare. But it lowers the mean and variance of first-period consumption while raising both for second-period consumption, and each period's change in consumption results in a gain in expected welfare.

The optimal nonproportional tax improves on expected welfare over either proportional tax system and comes in second to the lump-sum tax. The nonproportional tax is regressive ($\hat{k}_2$ = -.13), and the regressivity increases instability relative to the proportional taxes. It greatly reduces the labor distortion effect, however, and, consequently, allows a larger second-period transfer than do the proportional taxes.

Although the perfect stabilizing taxes reduce income instability, they actually increase the instability of consumption and leisure with respect to the optimal contingent flat-rate tax. In this case, the perfect stabilizers are destabilizing with respect to the things people care about: consumption and leisure. Moreover, because the perfect stabilizers require a very high tax rate when the state of the economy is good, they result in large losses due to labor distortion. These losses show up as lower mean consumption in each period.

Tables 2 and 3 (pp. 28-29) report the same type of output as in Table 1 except for differences in parameters. Table 2 has the same parameter values as Table 1 except that β is increased from 0.9 to 1.0. Table 3 has the same parameter values as Table 1, but θ is increased from 1.5 to 2.0.

The relationships observed in Table 1 also hold up in Tables 2 and 3. In particular, the rankings of the different tax systems according to the EW criterion and their relative contributions to labor distortion and instability are unchanged.

An increase in β raises the average tax transfer and, hence, second-period consumption. When β = 1, the optimal risk-sharing arrangement implicit in the lump-sum tax makes first- and second-period consumption equal. That equality is not remotely approached under any income tax system.

An increase in θ raises the tax transfer in the good state because there is more income that can be shared. Although the regressive income tax manages to reduce disincentives--even relative to the lump-sum tax--it does so by creating considerably more instability.

CONCLUSIONS

This paper demonstrates that automatic income stabilizers can be effective, but income stabilization alone is not sufficient in judging the desirability of policies. First, it is not the stability of income that is important to individuals' welfare; it is the stability of their consumption and leisure that counts. The numerical examples in Tables 1 and 2 indicate that one policy can generate more stability of income than a second, while generating less stability of consumption and leisure. Second, the effects of policies on stabilization must be weighed with their effects on economic distortion in judging their desirability. For all the numerical examples studied in this paper, when taxes were optimally levied, the income tax system that implied the most instability was the most desirable. The reason is that it was able to more than offset the cost of increased instability with a reduction in labor distortion.

Although the conclusion that policies must be judged in terms of expected welfare seems general, the conclusion about the desirability of a regressive income tax could be a result of the many simplifying assumptions made in constructing the model. The conclusion undoubtedly could be overturned based on concerns about income distribution. But even if income distribution is held constant, the question remains whether a regressive income tax can effect a favorable trade between less labor distortion and more instability under more general assumptions about tastes, production, and trade.

The answer at first appears to be "no." If an optimal income tax system must balance the losses from distortion and instability, a progressive tax system would seem desirable when individuals are highly risk averse. Yet, this answer may be too simple. A quadratic utility function can locally approximate more general utility functions, and the regressive income tax conclusion holds locally: a small move to regressivity in these examples increases expected welfare relative to the optimal flat tax. Moreover, it is not clear that risk aversion can be increased while maintaining the other conditions assumed about labor supply functions. With the quadratic utility function, risk aversion could be raised by increasing $\tilde{L}^*$. But, in order for L to be between 0 and 1 and for the leisure substitution effect to dominate, A and c* also would have to be raised. Thus, the relative weights given to distortion and instability might not be much changed.

Without adding much complexity to the model, it should be possible to include money as a store of value. The model then would closely resemble that of Enders and Lapan (1982). As they show, even with money in this type of model, there is a role for a tax-transfer scheme. It may be interesting to see if the existence of money changes any of this paper's conclusions.

On the whole, though, the analysis of this paper suggests it will be difficult to obtain results under general assumptions. Even under the extremely simple assumptions used in this paper, the mathematics become very messy very fast. The optimal proportional taxes are roots of a fourth-degree polynomial, and to compare tax systems it is necessary to determine which of the roots are relevant. Labor supply functions under nonproportional taxes, meanwhile, are the single real roots of third-degree polynomials, suggesting that the optimal tax parameters for the nonproportional tax are roots to very high order polynomials.

Although it may be difficult to do theoretical analysis under more general assumptions, it should be possible to do empirical analysis under assumptions that are more closely in accord with the data. It should be possible to analyze as in this paper the desirability of alternative tax systems in an estimated or calibrated model, such as that of Kydland and Prescott (1982). It is an open question whether the optimal income tax would be regressive in that model.

Table 1

Tax scheme/ outcome	Lump-sum	Single flat-rate	Contingent flat-rate	Nonpro-portional	State-contingent flat-rate $\sigma_y^2 = 0$	Income-contingent flat-rate $\sigma_y^2 = 0$
T^B/y^B	0.2222	0.1277	0.0615	0.2755	0.0002	0.0000
T^G/y^G	0.3765	0.1277	0.1704	0.1804	0.6035	1.0000
$\bar{c}_1$	0.7075	0.7742	0.7725	0.7421	0.4654	0.6667
$\bar{L}$	0.8147	0.6927	0.6992	0.7213	0.5555	0.5555
$\bar{c}_2$	0.3416	0.1134	0.1168	0.1966	0.2012	0.0000
$\bar{y}$	1.0490	0.8876	0.8893	0.9387	0.6666	0.6667
$\sigma^2_{c_1}$	0.0285	0.0605	0.0301	0.1070	0.0404	0.0000
σ^2_L	0.0150	0.0076	0.0037	0.0220	0.0123	0.0123
$\sigma^2_{c_2}$	0.0353	0.0013	0.0060	0.0015	0.0404	0.0000
σ^2_y	0.1272	0.0795	0.0630	0.1339	0.0000	0.0000
$EW[c_1]$	-21.7388	-20.9321	-20.8938	-21.4416	-25.0677	-22.2222
$EW[L]$	-6.7870	-4.8740	-4.9263	-5.4228	-3.2091	-3.2090
$EW[c_2]$	-24.1553	-27.1927	-27.1539	-26.0413	-26.0485	-28.8000
EW	-52.6811	-52.9987	-52.9740	-52.9057	-54.3253	-54.2312
Distortion	-52.4106	-52.7994	-52.8660	-52.4690	-54.0488	-54.1082
Instability	-.2705	-.1993	-.1080	-.4367	-.2765	-.1230

Parameters

$A = 2$

$B = 10$

$c^* = 4$

$\beta = 0.9$

$\theta = 1.5$

Table 2

Tax scheme/ outcome	Lump-sum	Single flat-rate	Contingent flat-rate	Nonpro-portional	State-contingent flat-rate $\sigma_y^2 = 0$	Income-contingent flat-rate $\sigma_y^2 = 0$
T^B/y^B	0.5000	0.1857	0.1327	0.3816	0.0002	0.0095
T^G/y^G	0.5000	0.1857	0.2219	0.2450	0.6035	1.0000
$\bar{c}_1$	0.5492	0.6938	0.6898	0.6603	0.4654	0.6561
$\bar{L}_1$	0.8535	0.6639	0.6685	0.7011	0.5555	0.5520
$\bar{c}_2$	0.5492	0.1582	0.1621	0.2613	0.2012	0.0063
$\bar{y}$	1.0984	0.8520	0.8519	0.9216	0.6666	0.6624
$\sigma_{c_1}^2$	0.0344	0.0508	0.0278	0.1148	0.0404	0.0000
σ_L^2	0.0159	0.0079	0.0043	0.0328	0.0123	0.0122
$\sigma_{c_2}^2$	0.0344	0.0026	0.0067	0.0040	0.0404	0.0000
σ_y^2	0.1377	0.0767	0.0619	0.1613	0.0000	0.0000
$EW[c_1]$	-23.8848	-21.9639	-21.9709	-22.5366	-25.0677	-22.3629
$EW[L]$	-7.4430	-4.4859	-4.5110	-5.2434	-3.2091	-3.1691
$EW[c_2]$	-23.8848	-29.5236	-29.4718	-27.9630	-28.9427	-31.8995
EW	-55.2126	-55.9734	-55.9537	-55.7430	-57.2196	-57.4315
Distortion	-54.9160	-55.7876	-55.8417	-55.1774	-56.9350	-57.3095
Instability	-.2966	-.1858	-.1120	-1.5656	-.2846	-.1220

Parameters

$A = 2$

$B = 10$

$c^* = 4$

$\beta = 1.0$

$\theta = 1.5$

Table 3

Tax scheme/ outcome	Lump-sum 1/	Single flat-rate	Contingent flat-rate	Nonpro-portional	State-contingent flat-rate $\sigma_y^2 = 0$	Income-contingent flat-rate $\sigma_y^2 = 0$
T^B/y^B	0.2222	0.1981	0.0615	0.3862	0.0000	0.0000
T^G/y^G	0.4211	0.1981	0.2588	0.2596	0.7838	1.0000
$\bar{c}_1$	0.8482	0.9074	0.9102	0.8502	0.4056	0.6667
$\bar{L}_1$	0.8462	0.7078	0.7311	0.7167	0.5001	0.5000
$\bar{c}_2$	0.4980	0.2241	0.2328	0.3397	0.2612	0.0000
$\bar{y}$	1.3462	1.1314	1.1430	1.1900	0.6668	0.6667
$\sigma^2_{c_1}$	0.0959	0.2039	0.0968	0.3041	0.0683	0.0000
σ^2_L	0.0237	0.0194	0.0086	0.0529	0.0278	0.0278
$\sigma^2_{c_2}$	0.1184	0.0214	0.0375	0.0230	0.0683	0.0000
σ^2_y	0.4275	0.3170	0.2547	0.4945	0.0000	0.0000
$EW[c_1]$	-20.0594	-19.5366	-19.2876	-20.4501	-25.9766	-22.2222
$EW[L]$	-7.3970	-5.2046	-5.4310	-5.6650	-2.7789	-2.7780
$EW[c_2]$	-22.2890	-25.6859	-25.6122	-24.1569	-25.2841	-28.8000
EW	-49.7454	-50.4271	-50.3307	-50.2720	-54.0396	-53.8002
Distortion	-49.1035	-49.8030	-49.9836	-49.0934	-53.5021	-53.5222
Instability	-.6419	-.6241	-.3471	-1.1786	-.5375	-.2780

Parameters

$A = 2$

$B = 10$

$c^* = 4$

$\beta = 0.9$

$\theta = 2.0$

1/ The constraint $L^G \leqq 1$ was active.

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