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FORECASTING AND CONDITIONAL PROJECTION USING REALISTIC PRIOR DISTRIBUTIONS*

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ABSTRACT

This paper develops a forecasting procedure based on a Bayesian method for estimating vector autoregressions. We apply the procedure to 10 macroeconomic variables and show that it produces more accurate out-of-sample forecasts than univariate equations do. Although cross-variable responses are damped by the prior, our estimates capture considerable interaction among the variables.

We provide unconditional forecasts as of 1982:12 and 1983:3. We also describe how a model such as this can be used to make conditional projections and analyze policy alternatives. As an example, we analyze a Congressional Budget Office forecast made in 1982:12.

While no automatic causal interpretations arise from models like ours, such models provide a detailed characterization of the dynamic statistical interdependence of a set of economic variables. That information may help evaluate causal hypotheses without containing any such hypotheses.

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We approach the analysis of a group of economic time series as the problem of using a prior joint distribution for the observed values of the series with future values to obtain a posterior distribution for future data conditional on observed data. The methods we suggest are Bayesian in spirit. We do not, however, attempt to make our prior distributions fully reflect our personal a priori knowledge and uncertainty. Instead we aim at a prior distribution that is easily standardized and reproduced by other researchers, one that reflects aspects of prior distributions that are likely to be similar in the work of many researchers. The posterior distribution produced by our analysis is, of course, just the likelihood function weighted by the prior probability density function (p.d.f.). Our methods can be thought of as a more useful way of reporting the likelihood function to other researchers, who themselves put little prior probability on regions of the parameter space that are given low probability by our prior.

We regard conventional methods of developing probability models for econometric time series as unreliable because they do not give probabilistic treatment to the uncertainty arising from inexact knowledge of the true model specification. Conventional approaches produce models that can be helpful adjuncts to judgment in producing forecasts, but the implied probability distributions about the forecast that such models generate are almost invariably too optimistic. (The ideas in these first two paragraphs are discussed at length in Sims 1982 and Litterman 1982.)

Specifying a joint distribution over the hundreds or thousands of interrelated data points available in most applications is a complex task. Any explicit joint probability model is likely to contain hidden implications that we would reject if we confronted them. Yet there is no joint distribution representing ignorance on which we can rely as being, in some sense, conservative. For example, if we take a large-variance joint normal prior on the coefficients of an unrestricted vector autoregressive model for the data as representing ignorance, we are in fact putting high probability on models with very large coefficients. These models produce erratic, poor forecasts and imply explosive behavior of future data. Most researchers would think it unlikely that such models actually characterize the data, yet the use of non-Bayesian estimation methods is roughly equivalent to the use of flat priors that put high probability on these models. Researchers who make practical use of non-Bayesian methods are forced to impose arbitrary or conventional restrictions to simplify their models, eliminating many parameters that admittedly are not known to be zero.

Because we (and the profession) have little experience in specifying joint distributions for these contexts, in this paper we experiment with a range of prior distributions that is indexed by a set of eight parameters. We believe that a good standard public prior may well be some weighted average of the priors indexed by these parameters. Since the priors with all parameters fixed are much more tractable than would be a weighted integral over the parameters, we hope to show that for many purposes good results can be obtained with a single setting of the parameters, without the extensive explorations that underlie this paper's results. We will be successful if, over a wide range of reasonable settings for the parameters, the model generates similar conditional distributions of the future when given past values of the variables in the system.

Another possibility is that conditional distributions are sensitive to the parameter setting, but the data are fit well only by parameter values in a certain narrow range and within this range conditional distributions of the future are all similar. In this case, although we need to search to find a good parameter vector, once one is found we can use it to generate conditional distributions. The inconvenience of computing many conditional distributions and then taking weighted averages of the results would be avoided.

While our explorations are in some ways like fitting the parameters of a conventional model--we examine various points in a parameter space and check how well the resulting models reproduce the data--the motivation and implications of the results are different in important respects. Our ideal conclusion would be that the parameters are ill-determined--that the fit is similar across a wide range of parameter settings having similar implications.

Of course, one may ask what we mean by a particular prior fitting the data well or badly. The Bayesian interpretation of fitting a prior to data is that we have specified our prior incompletely. The usual Bayesian formulation has a model for the data, y, specified as a density function, $p(y|\theta)$, for y conditional on the parameters, θ, yielding a joint density for y and θ as the product $p(y|\theta)q(\theta)$, where q is a prior density on θ. We are introducing an extra layer of parameterization. We specify a model for the data conditional on the parameters, θ, that we call <u>coefficients</u>. We specify a prior over θ conditional on a second set of parameters, π, so that our joint density for the data and the coefficients conditional on π is $p(y|\theta)q(\theta|\pi)$. We leave inexplicit our prior over π, which we need to fully specify the probability distribution of the data. We can in principle integrate $p(y|\theta)q(\theta|\pi)$ with respect to θ to obtain the marginal distribution for y given π, which we could call $m(y|\pi)$. If we are not directly interested in θ, we can treat $m(y|\pi)$ as our model for the data. For a fixed set of observed data, y, the behavior of $m(y|\pi)$ as a function of π plays the formal role of a likelihood function. As usual in such a context, if our prior density is flat in the region where $m(y|\pi)$ is large, our posterior p.d.f. for π will be proportional to $m(y|\pi)$, and we can think of ourselves as making inferences about the likely values of π. But since here π is interesting mainly for its implications about θ, we do not focus on estimating π.

Our posterior p.d.f. on θ, for a fully specified prior, is obtained by first forming the marginal joint p.d.f. for θ and y by integrating over π and then applying Bayes' rule. In the case where our prior p.d.f. on π is flat in the relevant region, this leads to a posterior p.d.f. for θ that is a weighted average of those obtained conditional on π, with the relative weight on π given by $m(y|\pi)$. Thus, when we measure the fit of the model, we ought naturally to use the relative size of $m(y|\pi)$, which is formally much like using the likelihood function. We will occasionally henceforth refer to $m(y|\pi)$ as the <u>likelihood</u>, but it is nonetheless a Bayesian notion because it is derived by taking the coefficients, θ, as a priori random. We will be searching over values of π to find high values of $m(y|\pi)$. This looks like a process of estimating our prior from the data, a most un-Bayesian notion. But we mean this search as an informal numerical integration over π. Conclusions are meant to be averaged across priors determined by different π's, with weights given by $m(y|\pi)$, the measure of fit.

In fact, we shall see that this Bayesian notion of how well a prior fits the data corresponds to measuring the fit by forecasting performance. That is, with a particular setting of π and data through t, we can generate recursively through the sample one-step-ahead forecasts of data at $t + 1$. The measure of fit based on our Bayesian likelihood turns out under our assumptions to be a weighted sum of squares of the one-step-ahead forecast errors. Readers uncomfortable with the Bayesian terminology can think of what we are doing as using π to index forecasting procedures, choosing among procedures by how well they forecast in the sample period. From this perspective, we are taking the large parameter space indexed by θ and reducing it to a smaller one indexed by π. What we are doing is quite different, however, from the conventional parsimonious parameterization approach, which would use some subspace of the θ-space, judiciously chosen, as if it were the whole parameter space. Our approach will, for any given choice of π, allow the θ used in forecasting to be more strongly data-determined as data accumulate through time, with no subspaces of the θ-space ruled out.

THE FORECASTING PROCEDURE

The procedure we are about to describe in detail was developed by Litterman (1980a,b; 1982) and Sims (1980, 1982). Although here we describe the procedure in general terms, the reader might find it helpful to remember that later we will apply it to a specific set of monthly data--ten variables measuring output, prices, money, federal government revenues and outlays, stock prices, interest rates, the value of the dollar, the flow of total nonfinancial debt, and the change in business inventories. Observations begin in January 1948 (1948:1) and end in March 1983 (1983:3). All but two variables are logged; the two exceptions are changes in business inventories and the interest rate. All but three variables are seasonally adjusted; these exceptions are the interest rate, the stock price index, and the trade-weighted dollar, none of which shows evidence of a seasonal pattern. (The data are described fully in the Appendix.)

We start from an unrestricted, time-varying, m'th-order vector autoregressive representation for the n-vector, X:

$$X_t = A_t(L)X_{t-1} + C_t + \varepsilon_t \tag{1}$$

where $A_t(L)$ is for each t a polynomial of order m in strictly positive powers of the lag operator, L, and ε_t is a zero-mean vector of jointly normal disturbances independent of X_s, for all $s < t$. We express our prior separately for each equation as a distribution over the coefficients in A and C. In principle we should also treat the variance of ε_t as uncertain, but instead we treat it as one of the parameters of our prior. Our approach can be thought of as imposing fuzzy restrictions on the equation, striking a balance between decreasing variance and increasing bias as the restrictions are tightened. What we do thus has antecedents in the literature on shrinkage estimation and its Bayesian interpretation (for example, Hoerl and Kennard 1970; Stein 1974; Shiller 1973; and Leamer 1972, 1978).

The prior is specified as a multivariate normal distribution for the coefficients of the vector autoregression. We refer to changes in the parameters of the prior that lead to smaller (or larger) variances of coefficients as <u>tightening</u> (or <u>loosening</u>) the prior. The prior means for all coefficients are zero, except for a mean of one at the first lag of the dependent variable in each equation. Thus, in the limit as the prior is tightened around its mean, each equation takes the form of a random walk:

$$x_t = x_{t-1} + \varepsilon_t. \tag{2}$$

Because most of the variables we use have persistent trends, we always keep the prior for the initial constant, C_0, in each equation flat in the relevant region of the parameter space. Therefore, the limiting form for each equation is essentially a random walk with drift fit to the data:

$$x_t = x_{t-1} + c + \varepsilon_t. \tag{3}$$

While we recognize that a more accurate representation of generally held prior beliefs would give systems with explosive roots less weight than is implied by our symmetric distributions around this mean, we doubt that the gain that could be achieved by abandoning the Gaussian form for our prior would be worth the price. In particular, the likelihood function for data that are not exploding will be quite clear in its rejection of roots significantly outside the unit circle.

We denote by θ_t the vector obtained by stacking up all the coefficients in one equation (or row) of the vector autoregression. The initial vector, θ_0, is given a multivariate normal prior density function with mean $\bar{\theta}$. The covariance matrix of the prior, denoted Σ_0, is generated as a function, F, of a vector of prior parameters, π. Thus, at t = 0, we have

$$\Sigma_0 = F(\pi) \tag{4}$$

$$\theta_0 \sim N(\bar{\theta}, \Sigma_0). \tag{5}$$

We postulate change in the coefficients of the autoregression over time according to

$$\theta_t = \pi_8 \theta_{t-1} + (1 - \pi_8)\bar{\theta} + \mu_t. \tag{6}$$

The parameter π_8 controls the rate of decay toward the prior mean. When it is set to one, as in a number of our experiments, we are modeling the coefficient variation as a random walk. The random change in the parameter vector, μ_t, is assumed to be drawn from a distribution with zero mean and covariance matrix proportional to Σ_0, independent of ε_t. 1/

1/ An exception is that the variance of changes in the constant term is kept equal to the variance of changes in the coefficient on the first own lag rather than set proportional to the effectively infinite prior variance on the constant term.

The factor of proportionality, π_7, which scales Σ_0 to determine the covariance matrix of μ_t, determines the amount of time variation allowed in the parameter vector.

Having specified the probability model, we apply the Kalman filter to each equation to obtain recursively posterior modes $\hat{\theta}_t$ for θ_t based on data through t - 1. When we have passed through the full sample this way, we end up with a value for the likelihood of the sample and with a full-sample estimate of the parameter vector applying at the first postsample date.

The Kalman filter is easiest to understand when the prior is normal with a fixed covariance matrix and the equation disturbance terms ε_t have known variance. In practice, of course, we do not know the equation disturbance variances a priori. Our procedure is to begin by using σ^2, .9 times the vector of estimated variances of residuals from least squares estimates of linear univariate autoregressions of order six, as if it were exactly the vector of variances of equation disturbances for the multivariate system. Suppose at t we have a normal probability distribution for θ_t, the coefficient vector for the i'th equation, so that (suppressing i subscripts)

$$\theta_t | t \sim N(\hat{\theta}_t, \Sigma_t). \tag{7}$$

Here the notation "X|Z" stands for "X conditional on Z," "~" means "is distributed as," and "t" in the conditioning set refers to all data observed up to and including date t. Then from (6)

$$\theta_{t+1} | t, \theta_t \sim N\left(\pi_8 \theta_t + (1-\pi_8)\bar{\theta},\ \pi_7 \Sigma_0\right) \tag{8}$$

and from (1)

$$X_{t+1} | t, \theta_{t+1} \sim N(Z_t \theta_{t+1},\ \sigma^2) \tag{9}$$

where Z_{t-1} is the list of right-hand-side variables in (1).

Equations (7)-(9) determine a joint normal distribution for X_{t+1}, θ_{t+1}, θ_t conditional on data through time t. The Kalman filter is a set of formulas for using this joint distribution to construct the conditional distribution of θ_{t+1} given data through t + 1. The same joint distribution can also be marginalized to produce the conditional distribution of X_{t+1} given data through t and the joint p.d.f.'s. To be specific,

$$X_{t+1}|X_t,X_{t-1},\ldots \sim N\{Z_t(\pi_8\theta_t + (1-\pi_8)\bar{\theta}, \qquad\qquad (10)$$
$$Z_t(\pi_8^2\Sigma_t + \pi_7\Sigma_0))Z_t' + \sigma^2\}.$$

Let

$$s_{it}^2 = Z_t(\pi_8^2\Sigma_{it} + \pi_7\Sigma_{i0})Z_t' + \sigma_i^2 \qquad\qquad (11)$$

be the variance of the one-step-ahead forecast of the i'th component of X_{t+1} using data through t. (Note that i subscripts are now back in use.) Let

$$\hat{\varepsilon}_{i,t+1} = X_{i,t+1} - Z_t(\pi_8\hat{\theta}_{it} + (1-\pi_8)\bar{\theta}_i). \qquad\qquad (12)$$

Then the log of the conditional p.d.f. determined by (10) is

$$-.5(\log s_{it}^2 - \hat{\varepsilon}_{i,t+1}^2/s_{it}^2) \qquad\qquad (13)$$

and the sample log likelihood is the sum over t of the terms given by (13).

It is not hard to check that if we multiply Σ_{i0} and σ_i^2 by the same scalar constant h, then Σ_{it} will also be multiplied by h for all t. Furthermore, if Σ_{it} and σ_i^2 are multiplied by the same constant, the equations of the Kalman filter are unaffected, producing the same conditional distribution for $\theta_{i,t+1}|t$ and therefore the same forecasts and forecast errors at each t. Nonetheless, the sample log p.d.f. value is affected by the choice of h, having the form

$$L = -.5[\sum_{t=0}^{T-1}\log s_{it}^2 - T\log h - \sum_{t=0}^{T-1}\hat{\varepsilon}_{i,t+1}^2/(hs_{it}^2)]. \qquad\qquad (14)$$

Maximizing (14) with respect to h implies

$$h = T^{-1}\sum_{t=0}^{T-1}\hat{\varepsilon}_{it}^2/s_{it}^2$$

$$L = -.5\left[\sum_{t=0}^{T-1}\log s_{it}^2 - T\log\left(T^{-1}\sum_{t=0}^{T-1}\hat{\varepsilon}_{it}^2/s_{it}^2\right) - T\right]. \tag{15}$$

Denoting by $\bar{s}_i^2$ the geometric mean over t of s_{it}^2, we have from (15)

$$L = .5\ T\log\left[T^{-1}\sum_{t=0}^{T-1}\hat{\varepsilon}_{it}^2/(s_{it}^2/\bar{s}_i^2)\right]. \tag{16}$$

Thus, the sample likelihood (actually the height of the p.d.f. for the sample conditional on the π parameters) is proportional to a kind of weighted average of one-step-ahead squared prediction errors. Equation (16) shows that prediction errors are given smaller weight when the model gives them a large standard error.

In reporting our results, we give π values without the likelihood-maximizing rescalings. We do so only because we were not computing the necessary statistics at early stages of the search. At the π values giving good fits, the values of h were about one, so the implied rescalings were not important. This was not true for every π we tried, however; occasionally h maximized the likelihood far from one, so that direct interpretation of π as determining implied standard errors of disturbances and of coefficients at time zero was not appropriate.

We have not seriously explored the potential gains from treating the equations of the system jointly. Least squares equation by equation is fully asymptotically efficient for an unconstrained vector autoregression because the same variables appear on the right-hand side of each equation. The Bayesian posterior mode is not correctly captured by single-equation methods, however, even if priors are normal and independent across equations, unless the prior covariance matrices are the same multiple of equation disturbance variance in each equation. Furthermore, in a system as large as the one we examine (with 10 variables), there are many (55) free parameters in the disturbance covariance matrix, all of which affect the posterior distribution. By imposing an informative prior on the 600 coefficients on lagged variables while using a flat prior on the 55 parameters of the covariance matrix, we are probably missing an avenue for improving reliability of these methods. [2]

The single-equation measures of fit that emerge naturally from the Kalman filter have a multivariate analog, but it cannot be computed without using a multivariate version of the Kalman filter. We have therefore put primary emphasis on a different class of multivariate measures of fit, the log-determinants of matrices of cross-products of k-step-ahead, out-of-sample forecast errors. The likelihood measure of fit would differ from one based on the determinant of the cross-product of one-step-ahead forecasts mainly in weighting the errors by the inverses of their conditional variances.

The log-determinants of the matrices of summed cross-products of k-step-ahead, out-of-sample forecast errors that we rely on as our primary measures of fit are defined by

$$ {}_t\hat{\varepsilon}_{t+k} = {}_t\hat{X}_{t+k} - X_{t+k} \tag{17}$$

$$ E_k = \sum_{s=1}^{T} \left({}_s\hat{\varepsilon}_{s+k} \; {}_s\hat{\varepsilon}'_{s+k} \right) \tag{18}$$

$$ \text{k-step-ahead log-determinant} = \log\left(|E_k|\right). \tag{19}$$

Eight parameters determine the general form of our joint density function for data and coefficients. The parameters and their roles are as follows:

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[2] We could parameterize the model initially in recursive form, with the j'th equation expressing X_{jt} as a linear function of lagged X_t's and current X_{it}'s for $i < j$ and with the covariance matrix of equation disturbances specified as diagonal. In such a model, single-equation procedures would coincide with multiple-equation procedures because of the diagonality of the disturbance matrix, and most of the free parameters of the covariance matrix of residuals would become coefficients on right-hand-side variables. The difficulty with this approach is that normal prior distributions on coefficients in such a recursive system cannot be chosen to treat variables symmetrically. The potential advantages of including contemporaneous relations among disturbances in the prior distribution are great enough, however, to encourage the exploration of this approach.

Parameter	What It Controls
π_1	Relative tightness on own lags
π_2	Relative tightness on lags of other variables
π_3	Relative tightness on constant term
π_4	Differential tightness among other variables
π_5	Overall tightness
π_6	Looseness on sums of coefficients
π_7	Tightness on time variation
π_8	Rate of coefficient decay toward prior mean

For π_1 through π_5 and π_7, smaller π_i means smaller variance and therefore increased tightness. The opposite holds for π_6. More precise definitions of the π's are given below.

Let the i'th component of X, x^i, have this scalar representation:

$$
\begin{aligned}
x_t^i = {} & a_{1,1}^i x_{t-1}^1 + a_{1,2}^i x_{t-2}^1 + \cdots + a_{1,m}^i x_{t-m}^1 \\
& + a_{2,1}^i x_{t-1}^2 + a_{2,2}^i x_{t-2}^2 + \cdots + a_{2,m}^i x_{t-m}^2 \\
& + a_{n,1}^i x_{t-1}^n + a_{n,2}^i x_{t-2}^n + \cdots + a_{n,m}^i x_{t-m}^n + c^i + \varepsilon_t .
\end{aligned}
\tag{20}
$$

The first five components of π, together with the elements of σ^2 and a set of relative weights, ω_j^i, for i = 1,...,n and j = 1,...,n, define a diagonal matrix of variances for the coefficients. For coefficients of own lags, that is, $a_{i,k}^i$ for k = 1,2,...,m, we assume the variance is given by

$$
\text{Var}\,(a_{i,k}^i) = \frac{\pi_5 \pi_1}{k \exp(\pi_4 \omega_i^i)} . \tag{21}
$$

For lags of other variables in a given equation, that is, $a^i_{j,k}$ for $k = 1,2,\ldots,m$ and $i \neq j$, we assume the variance is given by

$$\text{Var}\ (a^i_{j,k}) = \frac{\pi_5 \pi_2 \sigma_i^2}{k\ \exp(\pi_4 \omega^i_j)\sigma_j^2}. \tag{22}$$

For the constant term in each equation, we assume the variance is given by

$$\text{Var}\ (c^i) = \pi_5 \pi_3 \sigma_i^2. \tag{23}$$

The σ_i^2 scale factors are present to take account of the units of the data in determining the prior tightness for coefficients on different variables.

The relative weights, ω^i_j, are a set of numbers that we specify to reflect our a priori knowledge about the likelihood that lags of variable j will have nonzero coefficients in equation i. The larger ω^i_j is, the closer to zero we believe that coefficient is likely to be. For most of the variables, we specify $\omega^i_i = 0$ and $\omega^i_j = 1$ for $i \neq j$. For the interest rate and the trade-weighted dollar, we specify $\omega^i_i = 1$ and $\omega^i_j = 2$ for $i \neq j$. These weights, relative to the others, reflect our belief that these variables are a priori more likely to behave like random walks. Finally, for the stock price index, we specify $\omega^i_i = 1$ and $\omega^i_j = 5$ to reflect our strong belief that this variable behaves like a random walk.

Given the above tightnesses on individual coefficients, based on π_1 through π_5, we also want to impose a prior belief that the sums of coefficients on own lags are close to 1 and that those on lags of other variables are close to 0. This does not affect the mean of our prior. Consider a diagonal block of variances, M, for a vector of coefficients, θ, on lags of variable j in equation i, defined by parameters π_1 through π_5. Let the vector S be defined by

$$S = \{\frac{\pi_6 \sigma_j}{\sigma_i}\}[1\ 1\ \ldots\ 1]. \tag{24}$$

Then following the heuristic logic of Theil's mixed estimation procedure, we can introduce a dummy observation of the form

$$S\theta = v \tag{25}$$

with the variance of v set to one, by modifying M to take the new form

$$N = M - \left\{\frac{MS'SM}{1 + SMS'}\right\}. \tag{26}$$

IMPROVING FORECAST ACCURACY

Before we search over the prior parameters, we generate a set of benchmark univariate, linear, fixed-coefficient autoregressions. Based on the results in Litterman (1982), which viewed out-of-sample forecast performance as a function of lag length for many of these variables, we chose to include six lags and a constant term in each equation, and we estimated them by single-equation least squares. For this set of equations, and all subsequent specifications, we calculate sets of forecast errors one, three, six, and twelve steps ahead for each month from 1951:1 through 1980:12. We compute log-determinant measures of fit and standard errors for each variable, and we look at three 10-year subperiods, as well as the overall fit, in order to gauge the consistency of the results. The overall measure of forecast accuracy to which we give primary attention is the full-period log-determinant of the covariance matrix of one-step-ahead forecast errors. The univariate results are presented in table I.

The extent of our investigation of different settings of the π vector was constrained by the expense of evaluating the forecast performance for each value. Although our calculations were performed on a Cray-1 computer at the University of Minnesota that is both extremely fast and inexpensive, each evaluation of forecast performance for a given value of π required approximately 60 seconds and cost about $30. About half of the time for a given run was involved in the recursive estimation of the $\hat{\theta}_t$'s; the rest was used to generate the 12-step-ahead forecasts for each period and to do the accounting necessary to generate forecast accuracy statistics.

We chose to focus primarily on two dimensions of the prior: the overall tightness and the degree of time variation of the parameters. Our previous experience with priors of this form has suggested that the degree of parameterization of an equation is an important determinant of forecast accuracy. When we view the specification of a forecasting equation as the construction of a signal extraction filter, it is clear that equations with too many free parameters tend to pick up excess noise and to generate poor out-of-sample forecasts. Equations with too few parameters fail to pick up the signal. The specification of a prior provides a flexible format through which we can confront the trade-off between increasing signal extraction capabilities and over-fitting the data. By adjusting the tightness of the prior, we can tune the filter along this dimension.

We focus on the forecast performance as a function of the amount of time variation in order to investigate the degree to which results might be improved by relaxing the usual assumption of constant coefficients. We hope not only to increase forecast accuracy, but also to generate a more realistic description of the uncertainty of forecasts, particularly of those at multistep horizons.

As a first step in this investigation, we focused on the degree to which forecasting would be improved by searching along these two dimensions. Taking as given the parameter values $\pi_1 = .05$, $\pi_2 = .001$, $\pi_3 = 10^5$, $\pi_4 = 2$, $\pi_6 = 0$, and $\pi_8 = 1$, we began by minimizing the one-step-ahead log-determinant as a function of π_5 and π_7. An informal search requiring about 50 function evaluations led us to the values $\pi_5 = 1.4$ and $\pi_7 = .23 \times 10^{-7}$.

Over the range we examined, forecast performance varied little as we changed π_5 and π_7. It was clear, though, that at the conclusion of this search we had found values of π_5 and π_7 no more than a few percentage points from the point at which our one-step log-determinant measure was minimized.

The amount of parameter variation allowed at this specification is small. The implied standard error of the change in the first own lag, for example, over the entire sample is about .001. Since the prior mean of this parameter is 1, parameter drift might be taken as negligible. This result may seem surprising at first, but it should not be. In a model with 61 coefficients on the right-hand side, any very substantial amount of parameter drift implies large standard errors of one-step-ahead forecasts. The fact that simple random walk models forecast economic time series as well as they do over relatively long time spans is inconsistent with large amounts of parameter variability. In other words, allowing for parameter drift improves forecasts very little; since doing so is expensive, in many applications it will be reasonable to use fixed-coefficient models.

In a model with 61 coefficients on the right-hand side of each equation, however, even small amounts of variance in parameter changes can contribute a substantial amount to forecast error. Furthermore, in multistep forecasts, Markov parameter drift of the type our model allows builds up very rapidly in the estimated standard errors of forecast. Thus, any attempt to obtain more than point forecasts must allow for parameter drift.

One puzzle we found is that the 12-step-ahead log-determinant reached a minimum with priors that both were tighter and allowed less time variation than the prior that was best at the shortest forecast horizon. Although the differences in fit are small, the pattern of tighter priors leading to relatively better performance at distant horizons motivated our making further experiments with the form of the prior.

Since our conclusion about the amount of time variation seems important, we examined the possibility that it is dependent on the particular form in which we allow parameter variation. We experimented by comparing the forecasting performance of two constant-coefficient specifications, the first of which uses all available observations at each point in time and the second of which uses only the 120 most recent observations (if that many are available). By the one-step-ahead log-determinant measure, the first specification performs better. Thus, dropping observations, even those more than 10 years old, causes the log-determinant to rise. Interestingly, the forecasting performance at longer horizons did improve when the old observations were dropped. The conclusion that time variation is small relative to sampling error in coefficient estimates seems to be upheld. Since dropping observations gives more weight to the prior, long-horizon forecasts might be improved by assuming decay of parameters toward their prior means.

An additional restriction that we considered in the hope that it would allow more time variation in parameters was a limitation on the variance of sums of coefficients on lags of each variable in each equation. We found that if this restriction was imposed very tightly, then considerably more time variation in individual coefficients was possible before the forecasting performance worsened. However, none of these specifications performed as well as those without the tight restriction. [3/] The best performance along this added dimension was achieved when π_6 was between 5 and 1, that is, with standard deviations around sums of coefficients (aside from scaling) between .2 and 1. In choosing π_6 we also considered various values of π_7, but the returns to this search were not large. Of the combinations of values that we tried, the best was $\pi_6 = 1$ and $\pi_7 = 10^{-7}$. At this specification, the standard error of parameter change over the full sample was approximately double what it was at the previous best-fitting specification.

We imposed a second type of structure on the time variation of parameters by specifying that the coefficients slowly decay toward the

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[3/] After the empirical work for this paper was completed, we discovered that we had inadvertently switched the σ_i and σ_j in the computer code implementing equation (24). Thus, in equations where, because of the scales of variation, we intended to impose the sum of coefficients restriction relatively less tightly, instead we imposed it relatively more tightly. Most of the results reported here impose this aspect of the prior only very loosely; thus those results should be only marginally affected by this mistake. Obviously, no negative conclusion about the usefulness of this aspect of the prior should be drawn from this work. In more recent experiments, we have found that in systems with longer lags (we tried specifications with 12 and 15 lags) the sum of coefficients constraint, correctly imposed, can play a crucial role. With $\pi_6 = 250$, we have found significant forecast performance improvements in systems with longer lags than the system in this paper. Larger systems have some obvious disadvantages, however, since both memory and computing time increase approximately with the square of the number of lags. The generation of forecast performance statistics for the 15-lag specifications required 600,000 words of core storage and took 21.1 minutes of CPU on an IBM 3033, 5.3 times as long as our 6-lag specification.

prior mean. This structure was implemented by choosing values of the decay parameter, π_8, slightly less than 1. We reestimated the coefficients with each new observation, but cost considerations prevented us from revising the coefficient estimates at each step in the forecasting recursion. In one sample forecast where we did take account of parameter decay at the .9975 per period rate, we found that the forecasts changed only by about .1 percent at the 12-step horizon and about 1.5 percent at the 48-step horizon.

Letting $\pi_8 = .999$ in this type of specification was somewhat successful in terms of improving forecast performance, but it did not provide much room for allowing a larger degree of time variation. At this value for π_8, doubling the time-variation parameter, π_7, to $.2 \times 10^{-6}$ marginally improved the one-step-ahead forecasts but led to a much larger decrease in accuracy at longer horizons. Increasing the rate of decay to .9975 caused the forecast performance at a one-step horizon to worsen by about the same amount as that at a twelve-step horizon improved, a very small amount. Larger amounts of decay caused decreases in accuracy at all horizons.

Based on these findings, we adopted as our preferred specification the following parameter values: $\pi_1 = .05$, $\pi_2 = .005$, $\pi_3 = 10^5$, $\pi_4 = 2$, $\pi_5 = 1.4$, $\pi_6 = 1$, $\pi_7 = 10^{-7}$, and $\pi_8 = .999$. The forecast accuracy statistics at this specification are given in table II.

When we compare the performance of different systems, we see that, aside from covariance terms, changes in the log-determinant represent a sum of the percentage changes in the variance of forecast errors from each equation. Multiplying the change by 5 (dividing by 20 to get standard errors for 10 variables and multiplying by 100 to get a percentage) gives a rough estimate of the average percentage of change in forecast standard errors of the equations. Thus, in going from the univariate to the final specification, we observe an average improvement of about 2 percent in the one-step-ahead forecast errors and about 12 percent at the twelve-step horizon.

In searching informally over parameters of our prior, we were encouraged to find that forecast performance was generally insensitive to variation in the parameters. All of our sets of parameter values had log-determinants closer to our final choice than to the univariate, indicating the lack of sensitivity of forecast performance over our range of priors.

To investigate this sensitivity more carefully, however, we looked at forecast performance over a larger grid of values for the overall tightness and time-variation parameters of our prior. The grid was chosen to cover a region several orders of magnitude wide along both dimensions, far outside the range we would consider reasonable.

Our preferred prior overall tightness of 1.4 represents a 40 percent scaling up of the variances of all coefficient prior distributions from our original specification. For our grid search we chose to look at the values .014, .14, .7, 1.4, 2.8, and 14. The final value for our time-variation parameter was 10^{-7}. We chose a grid along this dimension of 10^{-15}, 10^{-7}, 10^{-6}, and 10^{-5}. The first value represents essentially no parameter variation, whereas the last specifies an order of magnitude larger than our preferred value.

The overall accuracy of forecasts generated by our vector autoregressions turns out to be a well-behaved function of the prior parameters over which we searched. We present the results of the grid search as a series of charts. The overall forecast accuracy is shown from two different views in charts 1 and 2. Here forecast accuracy is represented by the height of a surface for each point on our grid. The height is given by

$$5[\log|E_1| - \log|E_1(\pi_5,\pi_7)|] \tag{27}$$

where E_1 is the cross-product matrix of the one-step-ahead forecast errors for our preferred specification and $E_1(\pi_5,\pi_7)$ is the cross-product matrix of one-step-ahead forecast errors for the point on the grid (π_5,π_7).

These charts clearly show that the accuracy surface is not sensitive to even order-of-magnitude changes in these parameters of our prior. Because we would give low weight to regions of our grid away from the center, we interpret this result as indicating that if we think of ourselves as having a prior that is a mixture of normal priors indexed by the values of π, we will end up with a posterior much like that for our final chosen specification.

A slightly more detailed picture of the forecast performance over our grid is given in charts 3-8. Here we display the accuracy surfaces for each of our three nonoverlapping subperiods for the one-step-ahead and twelve-step-ahead horizons. The consistency of the shape of this surface over the different periods is reassuring. We can reasonably assume that any choice of values for π_5 and π_7 in a wide range around the center of this grid would remain close to the optimal choice, at least for one-step-ahead forecasts.

The results for the twelve-step-ahead horizon, displayed in charts 6-8, are less consistent over time. In general, though, they reflect the finding that tighter priors with less time variation of parameters appear to forecast better over longer horizons.

What have we accomplished through this specification search? By some standards, the answer would appear to be not much. After a complex and somewhat expensive search (a total computing cost of about $3,000), we find a specification that generates out-of-sample forecast errors averaging a few percentage points smaller than simple univariate autoregressions. Yet, as we pointed out earlier, our search here has been aimed at testing the usefulness of certain ways of specifying a prior. Nearly all the advantages of the multivariate procedures over the univariate procedures in forecasting performance could have been obtained without allowing for parameter drift (a major source of computational expense) and without searching over most of the dimensions we explored. A more difficult question is whether our search has given us a reliable probability distribution for future data.

Despite the small absolute gain in forecast accuracy, it is significant that we have documented a consistent gain from the use of a formally explicit multivariate method in a system of this size. This has not been done before, to our knowledge. The difference in accuracy that we find between multivariate and univariate methods is substantial relative to differences in forecast accuracy ordinarily turned up in comparisons across methods, even though it is not large relative to total forecast error. Moreover, if we think of a decomposition of movements in the data into signal and noise, with noise being the dominant component, then a 2 percent increase in forecast accuracy must represent a much larger percentage increase in the amount of signal that is being captured. With a multivariate probability model that has some claim to accuracy, we can generate conditional distributions of future time paths of a vector of economic variables that capture the most important cross-variable relations.

FORECASTS AND CONDITIONAL PROJECTION

The main purpose of generating a model like ours is to use it and the data available at a given date t to assess what is likely to happen after t. We describe here ideas for making assessments that are in some ways new but that could be applied to any time series model.

Obviously, one can construct a forecast of the most likely path of the economy. For our model, this is just a matter of recursively forecasting one-step-ahead with the autoregressive equations, using forecast values as if they were actual data as the date is advanced into the future. The appropriate procedure is to use the most recent estimate of the randomly varying parameters and vary them during the forecasting recursion according to their equation of evolution (6), ignoring the random term in that equation. Of course, when π_8 is 1, this amounts to holding the parameters constant. Because the forecasts after the first period with data are nonlinear functions of the parameters, they are not unbiased; that is, they do not represent the conditional expectation of future data. One can, at considerable expense, evaluate the conditional expectation by stochastically simulating the model and integrating the posterior distribution of forecasts by Monte Carlo methods. In one experiment using data through 1982:12 we found that the differences in forecasts based on time-invariant coefficients (coefficients decaying at the rate .999) and coefficients generated by Monte Carlo integration were quite small relative to the uncertainty in the forecasts.

We present in charts 9-24 two forecasts from the model for 1983 through 1986. The first is based on data through December 1982; the second, on data through March 1983. The charts in both cases show a forecast of an extremely vigorous recovery. This is quite different from the published forecasts circulating in February 1983. The Congressional Budget Office (CBO), for example, forecasted real GNP growth during calendar 1983 of only 4 percent, with inflation at 4.7 percent and the Treasury bill rate at 6.8 percent. With data through December 1982, our model forecasted real GNP growth at 8.8 percent during 1983 combined with inflation of 5.9 percent and an interest rate of 8.7 percent (charts 10, 12, and 17). Data for the first quarter suggested that the recovery began with less strength than the model had anticipated. These observations did not significantly change the growth rates forecasted for future quarters, however.

Cross-Variable Interactions

After preparing a forecast, perhaps the most obvious next step in using a model to evaluate future prospects is to ask how likely other paths are. We can ask, for example, whether the CBO's projected output and price level growth rates and Treasury bill rates are likely to be realized. In answering these questions, however, we will be taking seriously the cross-variable relationships estimated by the model. Before considering the questions, therefore, investigating those aspects of the model might be useful.

The favorable comparison between the forecast performance of our final specification and that of the univariate equations suggests that the cross-variable interactions that are captured by our equations represent predictable responses. Moreover, our tests indicate that these responses explain a significant proportion of the variation in most of the variables in the model and, with a few exceptions, that they remain fairly stable across different subperiods of the sample.

One measure of the size of the cross-variable interactions is the proportion of the forecast error variance of a variable explained by orthogonalized innovations in the other variables in the system. This measure is based on a decomposition of the variance of the k-step forecast into a sum of components associated with each of a set of orthogonal innovations (see Sims 1981). Although the decomposition depends on the ordering chosen for the orthogonalization, our point here is merely to demonstrate the extent to which interactions among variables are captured. We have looked at several orderings, and this aspect of the decomposition is not affected.

For some variables, such as the stock price index, our prior against cross-variable response is so strong that virtually none is allowed. Own innovations explain over 95 percent of stock price forecast errors, even at the fairly long 48-month horizon. For other variables in the system, however, the cross-variable responses are significant. The percentages of forecast variance explained by own innovations at a 48-month horizon (given in the order of orthogonalization for this decomposition) are as follows: M1, 29.2; stock price index, 95.1; Treasury bill rate, 59.9; flow of total nonfinancial debt, 76.9; GNP deflator, 28.4; change in inventories, 76.0; real GNP, 11.7; federal outlays, 79.7; federal receipts, 65.1; and trade-weighted dollar, 54.0.

We next display the responses of real GNP to the orthogonalized innovations. These responses also demonstrate the extent to which the model is capable of incorporating multivariate interactions, as well as the extent to which such responses are stable over time. The responses, shown in charts 25-34, were estimated independently over three nonoverlapping subperiods, the same prior being imposed at the beginning of each. Many of the responses are substantial relative to the response to own innovations, and for the more significant responses there appear to be strong similarities across the time periods.

The responses are scaled to show percentage movements in real GNP following orthogonalized innovations in each of the other variables. The size of the shock, which is the same for each period, is normalized to be one standard error of the distribution of innovations over the entire period. The largest responses of real GNP are to innovations in real GNP, the change in business inventories, and the stock price index (charts 34, 32, and 25). These responses are all similar across the different subperiods. The responses of output to interest rates and money innovations (charts 26 and 27) are also substantial, and they are relatively similar in their dynamic pattern in different subperiods. The other responses are not particularly consistent over time periods, but for the most part they are not large.

With regard to the question of consistency across subperiods, some readers will undoubtedly be more impressed at first glance by the variations in some of the responses than by the similarities in others. Perhaps the most natural metric for the degree of stability of the responses through time, though, is the measure of out-of-sample forecasting accuracy that we have already stressed. We know that our specification does well by that measure. What we find encouraging in looking at these response patterns, and the earlier decompositions of forecast variance, is that the prior that led to relatively accurate forecasts is also capable of capturing significant cross-variable interaction even in these three subperiods, each of which includes only a very limited amount of data.

We next present the response functions for all ten variables, plus those implied for federal deficits, in charts 35-45. These response functions are generated from the posterior mode coefficient estimates at the end of the sample. In each case, the responses of the variable are given to a one-standard-error orthogonalized innovation in each variable in the system. The responses are presented in the order of orthogonalization.

In contrast to the earlier responses shown for real GNP, here the same scale is used for all responses of a given variable in order to facilitate comparison of the relative magnitudes of different responses. It is clear in chart 35, for example, that real GNP's responses to the GNP deflator and to federal receipts and outlays are insignificant compared to its responses to money, stock prices, and interest rates.

In chart 36, a sustained positive response of the price level to money innovations can be seen. The price level exhibits a large negative response to interest rate innovations, but only after a year lag. A large negative response of money to interest rate innovations is shown in chart 37. Finally, it is clear in chart 44 that little of the variance of the budget deficit is accounted for by innovations in economic variables other than receipts and outlays.

Measuring the Likelihood of Alternative Paths

There is no unambiguously correct way to measure the likelihood that a particular condition on the projected future path of the economy will be realized. Of course, the probability that any set of equality restrictions will be exactly realized is zero. When we ask how likely a projected path is we ordinarily mean to ask how likely it is that the actual path will differ from the model's most likely projection as much as the projected path and in the same direction. There is no mechanical way to determine, from the path alone, how to interpret "in the same direction." In our example, we might be interested in the probability that real GNP growth will be at least as low as the CBO's 4 percent and that inflation and the interest rate will also be at least as low as its projections. But one might instead consider that only the GNP growth rate differences are interesting, so that forecasts differing in the same direction as the CBO's are all those with growth rates at least as low. Or one might suppose that the critical thing about the CBO forecast is its lower real interest rate, leading one to check the plausibility of its projected gap between inflation rates and interest rates.

If a class of future paths is specified, one can measure the probability of the class directly--by stochastically simulating the model--if no computationally cheaper analytic method is available. This method is expensive, however, both in computer time and in its requirement for careful thought about the class of paths to be assessed. Instead, one can mechanically construct a class of paths from specified restrictions. A natural way of doing this is available when the joint density function of future paths [4/] is unimodal and has convex level surfaces (like a normal density). We can first construct the most likely path satisfying the restrictions, then consider the class of all paths lying on the downhill side of the tangent plane to the level surface at that point in the space of future paths. Chart 46 shows the

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[4/] Here we are thinking of future paths of the economy as long lists of numbers, made up of the values of the variables in the system at each future date considered.

nature of the set of paths whose probability would be measured in a two-dimensional special case.

For a normal p.d.f., this leads to using the square root of the usual chi-squared statistic as if it were a normal random variable and measuring plausibility by the probability in the upper tail of the normal p.d.f. at the level of this statistic. One might wonder why it is not best, for the case of a normal distribution over future paths, to measure the plausibility of a set of linear restrictions directly by the significance level of its associated chi-squared statistic, using as degrees of freedom the number of restrictions applied. This is, after all, the form of a classical test of the restrictions. Such a procedure treats as the class of paths whose probability is to be measured all paths with lower likelihood than the most likely path satisfying the restrictions. Thus, if the model asserts that real growth will be 8 percent and inflation 6 percent and someone claims that instead growth will be 4 percent and inflation 9 percent, the claim is in some sense different from the model assertion in one direction: it is more pessimistic. The standard use of the chi-squared statistic would assess the likelihood of the pessimistic forecast by looking at the probability of all paths at least as unlikely, including those that are unlikely because they are much more optimistic than the model. The index we use here instead looks only at paths lying on one side of the claimed path. This class of paths includes some with less inflation and much less real growth as well as some with more inflation and less real growth, so it is not as narrow a class as that of paths with both less real growth and more inflation. However, the trade-off between inflation and real growth implicit in defining the class of paths more pessimistic than that claimed is constructed mechanically from the covariance matrix of paths. This will at best approximate the way we would construct a class of more pessimistic paths if we thought about it carefully. Nonetheless, we apply this measure of plausibility here.

To do so, we must first find the model's projection of the most likely future path for the economy subject to the condition that the CBO forecasts for annual average growth rates are satisfied. Such conditional projections may be interesting in their own right as part of a description of the likelihood function and for other applications we will mention below.

The principle is that the model provides a joint conditional density function for future paths of the process. We simply use that function to find likelihood-maximizing paths subject to certain restrictions on the future paths. These computations cannot in general be carried out recursively forward in time as can the point forecasts, because a constraint on future values of a variable in the system can carry information about the likely current value of all variables. If, for example, we know that the money stock will grow slowly between 12 and 18 months from now, and if we know that the money stock is negatively correlated with disturbances in the interest rate from 12 to 18 months earlier, then we should think it likely that interest rates will rise soon.

The computations are simplest when the model is stationary and concerned only with second-order properties, so we first describe our procedure within the confines of the prediction problem for covariance stationary processes. The vector stochastic process $\{x_t: \ t = \ldots,-2,-1,0,1,2,\ldots\}$ is assumed to be covariance stationary and linearly regular. The moving average representation (MAR) is

$$x_t = \sum_{s=0}^{\infty} B_s u_{t-s} \tag{28}$$

where the innovations u_t are uncorrelated both across time and contemporaneously. The MAR is normalized so that $E(u_t u_t') = I$.

A linear constraint upon future values of x is a linear constraint upon future values of the innovations process u. The constraint on x is transformed into the equivalent constraint on u. This has some computational advantages when, as is likely for models of this type, we have already computed the coefficients of the MAR in any case. The least squares estimate of the constrained u's is computed, and the least squares projection of x subject to this constraint is obtained by constructing the path for x implied by the computed innovations.

Let $[y|Q]$ denote the orthogonal projection of the random variable y onto the closed subspace Q in the Hilbert space of finite variance random variables on the underlying probability space. If y is a vector, the projection is done component by component. $H_x(t)$ is the closure of the subspace of finite linear combinations of x_s for $s \le t$. Consider the projection of x_{t+k} on the space spanned by $H_x(t)$ and

$$S^*x = \sum_{j=-\infty}^{\infty} S_j x_{t-j} \tag{29}$$

where the sequence S contains the coefficients on past and future x values in a set of constraints.

We assume

$$\sum_{j=-\infty}^{\infty} S_j S_j' < \infty \tag{30}$$

and S_j is dimension $q \times n$, where q is the number of constraints and n is the dimension of x_t. The projection we are considering can be thought of as the best linear predictor of x_{t+k}, given knowledge of x values up to time t and also knowledge of the linear combinations of past and future x's whose coefficients are in S. In practice, the S sequence will be zero except for a finite number of terms. Applying the law of recursive projections results in the following:

$$\left[x_{t+k} | H_x(t) + \mathrm{span}(S^*x)\right] = \tag{31}$$

$$\left[x_{t+k} | H_x(t)\right] + \left[\left(x_{t+k} - [x_{t+k} | H_x(t)]\right) | \mathrm{span}\left(S^*x - [S^*x | H_x(t)]\right)\right]$$

and

$$x_{t+k} - \left[x_{t+k} | H_x(t)\right] = \sum_{s=0}^{k-1} B_s u_{t+k-s}. \tag{32}$$

Now

$$S^*x - \left[S^*x | H_x(t)\right] = S^*\left(x - \left[x | H_x(t)\right]\right) \tag{33}$$

$$x_s - \left[x_s | H_x(t)\right] = 0 \tag{34}$$

for $s \le t$, and

$$x_s - \left[x_s | H_x(t)\right] = \sum_{j=0}^{s-t-1} B_j u_{s-j} \tag{35}$$

for $s > t$, so

$$S^*(x - [x|H_x(t)]) = \sum_{m=1}^{\infty} S_{-m}\left(\sum_{j=0}^{m-1} B_j u_{t+m-j}\right) \tag{36}$$

$$= \sum_{k=1}^{\infty} \left(\sum_{j=0}^{k-1} S_{j-k} B_j\right) u_{t+k} \tag{37}$$

$$= R^*u = \sum_j R_j u_{t-j}. \tag{38}$$

Now the second projection on the right-hand side of (31) can be written

$$\sum_{s=0}^{k-1} B_s [u_{t+k-s} | R^*u]. \tag{39}$$

It can be verified, using the orthogonality principle for projections, that the projection $[u_{t+k-j}|R^*u]$ is

$$\hat{u}_{t+k-j} = R_{j-k}\left\{\sum_{j=-\infty}^{-1} R_j R_j'\right\}^{-1}(R^*u). \tag{40}$$

These are the least squares projections of the future innovations. The right-hand side of (31) is thus the sum of the unconstrained forecast plus

$$\sum_{s=0}^{k-1} B_s \hat{u}_{t+k-s} \tag{41}$$

which implies that the conditional projections can be obtained by simulating the model beginning at $t + 1$, using the $\hat{u}$'s as the innovations. In a particular application, a value for S^*x is usually supplied; the equivalent value for R^*u is the difference between S^*x and the forecast value for S^*x.

To see how this works in a simple case, suppose that the level of the money supply (M_s) is available only with a two-week delay, while the interest rate for Treasury bills (TB) is available daily. We have a weekly model that we want to use to forecast, but at t we have data on M_s only for $s < t - 1$. Here, purely for forecasting purposes, we need to make a projection conditional on TB_{t-1} and TB_t.

With the vector autoregression normalized so that B_0 is lower-triangular and TB comes above M_s in the ordering of variables, the moving average coefficients needed are the responses of R to orthogonalized shocks in itself at zero and one step and in M_s at one step; call these b_{b0}, b_{b1}, and b_{m1}. With v_t and w_t as the innovations in TB and M_s at t,

$$\begin{bmatrix} TB_{t-1} - \hat{TB}_{t-1} \\ TB_t - \hat{TB}_t \end{bmatrix} = \begin{bmatrix} b_{b0} & 0 & v_{t-1} \\ b_{b1} & b_{m1} & w_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & v_t \\ b_{b0} & 0 & v_t \end{bmatrix} . \tag{42}$$

The 2×2 matrices in this are, respectively, R_{-1} and R_{-2} in the notation above. The most convenient way to do this computation is to stack the set of innovations. With

$$\hat{U} = [v_{t-1} w_{t-1} v_t w_t]' \tag{43}$$

$$\underline{R} = \begin{bmatrix} b_{b0} & 0 & 0 & 0 \\ b_{b1} & b_{m1} & b_{r0} & 0 \end{bmatrix} \tag{44}$$

$$\underline{r} = \begin{bmatrix} TB_{t-1} - \hat{TB}_{t-1} \\ TB_t - \hat{TB}_t \end{bmatrix} \tag{45}$$

the formula for the constrained U vector becomes $\hat{U} = \underline{R}'(\underline{RR}')^{-1}\underline{r}$, which is the solution of the problem: min U'U subject to $\underline{R}U = \underline{r}$.

In general, $\underline{R}$ is the matrix with as many rows as there are constraints formed by arranging horizontally the R_j's through the end of the constraint horizon, k,

$$\underline{R} = [R_{-1}\ R_{-2}\ \cdots\ R_{-k}] \tag{46}$$

and $\underline{r}$ is the vector of differences between the constrained values of S*x and the unconditional forecasts of S*x.

One important variant on this procedure is to add the additional constraint that only certain innovations are allowed to be nonzero. We might want to do this if we had in mind interpretations for certain innovations. For example, if we regarded money as a monetary policy variable, we might suppose that innovations in that variable represented changes in policy. Most monetarist rational expectations models make exactly this assumption. Then a forecast conditional on low inflation and on innovations being zero in all variables other than money would display the most likely way for monetary policy to generate low inflation. [5]

Holding certain innovations to zero in the conditional projection can be accomplished simply by eliminating the columns in the $\underline{R}$ matrix that correspond to the variables whose innovations must be zero. For computing constrained paths, the normalization of the MAR used to obtain orthonormal u's has no effect except on the computational burden: if $E(u_t u_t') = \Sigma$, the formula--using the stacking from above--is $\hat{U} = (\Sigma \otimes I)\underline{R}'[\underline{R}(\Sigma \otimes I)\underline{R}']^{-1}\underline{r}$, where a different $\underline{R}$ matrix is obtained using the nonorthogonalized MAR. Orthogonalization eliminates the need for the $\Sigma \otimes I$ by incorporating a factorization of Σ into the MAR and thus into the $\underline{R}$ matrix. However, when innovations for certain variables are constrained to be zero, orthogonalization is no longer innocuous, since the definition of a variable's innovations depends on the orthogonalization. For example, the least squares constrained path may prove to be obtained primarily through innovations in the policy variables in one ordering, but through innovations in the nonpolicy variables in another.

Although the proofs above are limited to covariance-stationary processes, the method will still work if, for example, x has an invariant autoregressive representation with unstable roots.

<u>Estimating Likely Forecast Errors</u>

Our experience suggests that, though models with time-invariant coefficients generate reasonable forecasts, they have a tendency to

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[5] A number of models in the literature identify innovations in certain variables as generated by policy or go still further and treat certain policy variables as exogenous, hence Granger-causally prior, and as entirely determined by policy. In fact, this kind of assumption is probably the norm in models used to generate implications for policy. We regard such assumptions as frequently being interesting speculative hypotheses, but seldom solidly justifiable as a priori knowledge.

generate unreasonably optimistic estimates of the likely size of future forecast errors--even when allowing for sampling error in the estimated coefficients (which we ignored above). One of the objectives of our research has been to discover whether our random parameter specification avoids this optimistic tendency.

We will compare four different ways of estimating the covariance matrix of forecast errors. The first matrix, F, is generated from the usual innovation covariance matrix, Σ, which is estimated by taking cross-products of in-sample residuals based on a fixed-coefficient model. The k-step forecast error covariance matrix is given as

$$\Sigma_k = \sum_{s=0}^{k-1} B_s \Sigma B_s' \tag{47}$$

where the B_s's are the coefficients in the MAR associated with the fixed-coefficient model.

Our second forecast error covariance matrix we call O, the estimate obtained by using a time-varying coefficient model, but taking the end-of-period coefficient estimates as fixed and using the recursively computed, one-step-ahead forecast errors to estimate the covariance matrix of innovations.

A third forecast error covariance matrix is obtained by a Monte Carlo simulation of the full random parameter model from the end-of-sample initial conditions. This estimate we call M. To reduce the expense of this simulation experiment, we used a prior with no sum of coefficients restriction imposed. This simplifies the simulations since it implies a diagonal covariance matrix for the coefficient changes.

Finally, a fourth, somewhat conservative way to assess likely forecast accuracy is to generate forecasts recursively over a range of horizons at each sample point, using data only up to the forecast date in making each set of forecasts. Forming the sample covariance matrix, V, of realized forecast errors at various horizons gives us a direct measure of likely forecast error variances at those horizons. This procedure assumes that the stochastic process for the vector of forecast errors by horizon is jointly stationary, but it requires no assumption that the model justifying the forecast procedure is also generating the data.

Covariance matrices F and M take no account of uncertainty in the estimated coefficients (though, of course, M accounts for uncertainty about future coefficients due to parameter change). Matrix O takes some account of parameter uncertainty by using the recursively generated one-step-ahead forecast errors to estimate equation residual error. And V takes full account of all sources of forecast error.

Our experiments with these four different ways of estimating forecast error covariance matrices gave no clear ranking of the methods. The estimated standard errors of forecasts at 12- and 48-month horizons are shown in table III. Each of our different estimated matrices--F, O, V, and M--at times gives both the largest and the smallest estimated standard errors. This result is certainly due in part to the small samples we used. In our Monte Carlo estimates we used only 200 draws, and for the generation of historical second moments in V we used 240 observations, which represent only five nonoverlapping 48-month periods.

Our original suspicion, that estimates of uncertainty such as F, which are based on fixed-coefficient models, would badly underestimate the average out-of-sample, multistep forecast errors as measured in V, was only occasionally verified. At the 48-step horizon, F badly underestimates the size of observed errors only for money and prices. In those two cases the Monte Carlo matrix M, based on the time-varying specifications, was much closer to the observed results in V. More often, however, the estimates in F were larger than the observed forecast variance, and M in some of those cases gave even larger estimates. It is possible, of course, that the use of V as a standard of comparison is inappropriate. When parameters are varying through time, the uncertainty also varies, and at a given time it may be very different from an estimate based on average errors in the past. For the trade-weighted dollar the Monte Carlo estimates suggest much less uncertainty than the others, and it is certainly conceivable that this is correct.

Conditional Projections

The time-varying parameters specification used in this paper implies a conditionally heteroscedastic non-Gaussian distribution for the forecast errors. If we form the sample covariance matrix, V, of forecast errors and form conditional projections as minimum mean square error predictions using V, we are therefore contradicting the probability model that justifies our forecasting procedure. However, it is not clear whether that model is more realistic than one that uses V to form conditional projections.

Using V to form conditional projections is only in a sense conservative: it is unlikely to underestimate greatly the magnitude of errors, even at long horizons. But when we estimate the whole of V without applying Bayesian methods, we are losing the stability provided by Bayesian shrinkage toward a prior mean. In particular, when we start comparing conditional projections to form conclusions about how much variables respond to each other, use of V may give an exaggerated view of the strength of interaction among variables in the data.

A Gaussian covariance-stationary process generates a normal joint distribution for future paths given the past, with some covariance matrix. However, that covariance matrix has a special structure. To take the simplest case, consider the covariance matrix of one- and two-step-ahead forecasts for a univariate process. If innovation variance is 1, the variance of two-step-ahead forecast errors, s_{22}, is $1 + b^2$; that for one-step-ahead forecast errors, s_{11}, is 1; and the covariance of one- and two-step-ahead forecasts, s_{12}, is b, where b is the coefficient on the first lagged innovation in the MAR. Thus, the square root of $s_{22} - s_{11}$ is s_{12}. But for a process such that minimum variance forecasts are nonlinear functions of the data, such a restriction on the covariance matrix of forecast errors is not in general satisfied. For example, suppose $y(t) = e(t) + \mathrm{sgn}[e(t - 1)]$, where e(t) is i.i.d. uniformly on (-.5,.5) and the function sgn has value 1 if its argument is positive and -1 if its argument is negative. Clearly, we can determine $\mathrm{sgn}[e(t - 1)]$ from knowledge of $y(t - 1)$, so the one-step-ahead forecast error variance is the variance of e(t), that is, 1/12. The variance of the two-step-ahead forecast error is $1 + (1/12)$, and the covariance of one- and two-step-ahead forecasts is not $(s_{22} - s_{11})^{.5}$ = 1, but instead .25.

Finding the best linear forecast for a given fixed V not generated by a covariance-stationary process therefore requires some modification of our procedure. For this case, the difference between the constrained forecast and the unconstrained forecast is $V\underline{S}'(\underline{S}V\underline{S}')^{-1}\underline{r}$, where $\underline{S}$ is a matrix with as many rows as there are constraints and consists of blocks given by the S_j's [defined in (29)] over the interval of the constraint, arranged horizontally, and $\underline{r}$ is the same vector as above. A restriction corresponding to the one that only certain variables have nonzero innovations can be obtained by examining the meaning of a Choleski factorization of V into LL', L lower-triangular. If E(UU') = I, then, if W = LU, E(WW') = V. The Choleski factorization transforms the forecast error W into LU, where each component of U is created as that part of the corresponding element of W that is uncorrelated with the previously defined U's. This is precisely how the orthogonalized innovations decompose the forecast error in the covariance-stationary case: the innovation for variable j at step k is the (normalized) part of the forecast error that is orthogonal to the innovations in all variables for steps < k and for variables < j at step k. L describes an analog of the MAR: each column gives the response of the system to a unit shock in the corresponding component of U. If $W = V\underline{S}'(\underline{S}V\underline{S}')^{-1}\underline{r}$, then $U = L^{-1}W$, and if $\underline{R}$ [analogous to the $\underline{R}$ defined in (46)] is defined as $\underline{S}L$, then $U = \underline{R}'(\underline{R}\underline{R}')^{-1}\underline{r}$. Again, by cutting the appropriate columns out of the matrix $\underline{R}$, restrictions that certain innovations remain at zero can be implemented.

In this paper we have made our analysis conditional on constraints that involve projections 48 periods into the future. Because of the size of the system, a full V or M matrix would be 480×480. Rather than attempt to operate with such a huge matrix, we have restricted ourselves to the conditional projections for a nonconsecutive sequence of horizons between 1 and 48 steps into the future, with all constraints being put only on the included horizons. That is, instead of forming the covariance matrix for forecasts 1 through 48 steps ahead, we form the covariance matrix for forecasts 1, 2, 3, 6, 9, 12, 18, 24, 30, 36, and 48 steps ahead. The restrictions we consider on future paths must then be defined in terms of these horizons.

We presented the unconditional forecasts earlier in charts 9-24. Here we report the unconditional forecasts as of 1982:12 again, in table IV in a format to match the way we will report the conditional forecasts. In table IV we also include the actual values for 1983, which have been observed since the forecasts were made.

Charts 47-57 each show those actual values, four conditional forecasts produced by the model, and a confidence interval for the unconditional forecasts. The first three conditional forecasts are constrained to match the CBO's 1983 and 1984 averages for real growth, inflation, and interest rates, using three covariance matrices: M, V, and O. The confidence interval includes the modal 50 percent of the posterior density.[6/] The interval is based on the 200 simulations used to form the covariance matrix M. Although they were not constrained to match the CBO projections for the deficit, these forecasts agree with them fairly closely. The CBO projects the deficit to be $194 billion, $197 billion, $214 billion, and $231 billion in fiscal years 1983-86, and all the model's conditional projections are in this range. The charts also include a conditional projection from an experiment in which the deficit was constrained to reach zero at 1984:6 and stay there.

The charts for variables expressed in terms of growth rates use step functions to emphasize that the growth is an annualized rate over the interval of the step. Note that the length of the interval increases with the horizon of the forecast. This variation in the interval length causes the confidence bands to narrow at the longer horizons despite increased uncertainty about month-to-month growth rates. The three variables expressed in terms of their levels--the Treasury bill rate, the change in business inventories, and the federal government deficit--are shown with lines connecting the points reflect-

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[6/]The modal 50 percent of the posterior density is found by ordering the values of the simulations for a given variable at a given time period. In this case, based on 200 simulations, the lower 50 percent bound is defined to be the 51st value of the 200; the upper bound is the 150th value.

ing the values for the last month of the intervals considered (charts 51, 52, and 56). The actual numerical values of the constrained forecasts are given in tables V through VIII.

All three forecasts conditioned on the CBO projections are implausible, based on their implausibility index (calculated as the root sum of squares of the standardized shocks required to generate the forecast). The conditional forecast using the fixed-coefficient 0 matrix has an index of 4.4, improbable if treated as a one-tailed normal or t-test statistic. For the forecast generated from the V matrix, the index is 3.0, and for the M matrix it is 3.3--both smaller than for the fixed-coefficient model, but still in the range of implausibility.

All the forecasts conditional on the CBO projections show an initial sharp contraction in both receipts and outlays (charts 54 and 55), and all show slower money growth than the unconditional forecast (chart 49). However, the degree to which money growth is reduced is much larger in the V forecast than in either of the other two, and the reduction in stock prices (chart 50) is much greater in the fixed-coefficient model 0 than in the other two. We should note that the results from the simulation-based M matrix differ noticeably between an M matrix based on 200 random draws and one based on 100 random draws. Because the empirical V matrix is also based on a sample of only a few hundred highly dependent, observed forecast errors, it too is probably infected by substantial sampling error. Thus, though noticeable differences exist, they may be inherent statistical error rather than fundamental differences in the results based on these different approaches.

To understand why the forecasts emerge as implausible, it may help to examine the time sequence of standardized shocks implied by them, as displayed in table IX for the empirical V version. (Note that there are no standardized shocks after 1984:12 because the constraints involved no dates after that.) Because the model shows a strong connection from both M1 and stock price innovations to subsequent output and (to a lesser extent) price movements, both these variables show a sequence of fairly large negative standardized shocks. One possible interpretation of the projection is what might be called an _irrational monetarist_ one. A less expansionary monetary policy than the unconstrained forecast of the model leads to correct anticipations of lower future inflation and to lower nominal interest rates. Because of some kind of price rigidity or money illusion (perhaps an inability of wage contracts to lower their rates of increase fast enough), the lower inflation rate leads to persistently lower output.

As Sims (1983) has recently argued, though, the practice of identifying policy actions with innovations in policy variables, which underlies much standard manipulation of econometric models for policy analysis as well as some rational expectations macroeconomics, requires a justification that may not be easy to find. One could interpret the forecasts conditioned on the CBO projections as showing the response of the economy to public recognition that capacity utilization is likely to remain low and unemployment high, due to continued slow adaptation of the industrial economies to high energy prices and to the nominal inertia of the wage and price setting mechanism. On this interpretation, new information appears first in the financial variables of money, the bill rate, and the stock price index because all three (with a partially accommodative monetary policy) react quickly to the public's anticipations of the future. They therefore do not reflect policy decisions, and the difference between the CBO and the central model projections cannot be read as displaying the effect of contractionary monetary policy.

One interpretation that is not consistent with the model is the idea that deficits might be critical to the difference between the model's expansionary central forecast and the less vigorous CBO forecast. Differences between the deficit predictions of these conditional projections and those of the model's central forecast are slight. Furthermore, in an experiment we do not report here in detail, we tried imposing a constraint that the deficit be down to 2 percent of GNP by 1984:12. That projection shows expenditures lower and revenues higher, with hardly any other change in the forecast relative to the model's unconstrained forecast. The implausibility index for this forecast ranges from .62 to 1.2 for the three methods, indicating that the forecast is not at all unlikely.

In a more extreme experiment, which we also show in charts 47-57 and table VIII, the deficit was constrained to reach zero in 1984:6 and stay there. The shocks associated with this projection, which used the M matrix, are shown in table X. This projection has an implausibility index ranging from 3.6 to 12, with the lowest value coming from this variable-parameters projection. The range is large, but of course all the index values show the constraint to be in the region of great implausibility.

The constraint of a zero deficit by 1984:6 produces noticeable effects on the projections for other variables, with even more rapid expansion than in the central forecast in the period before 1984:6, followed by a sharp reduction in output growth rate (chart 47) and a rise in interest rates (chart 51) when the deficit takes its sharpest drop. This course is consistent with a Keynesian interpretation that expansion tends to reduce deficits by raising the tax base faster than it raises government spending plans, at least in the short run, and that after taxes are raised and expenditures reduced, there are contractionary effects on the economy. The model, then, can be interpreted as saying that the most likely way to arrive at a zero deficit is to have a lucky expansion in output soon, followed by an unusually large rise in taxes and decline in expenditures.

Note also that there are several ways to model the effects of a correctly anticipated future reduction in the deficit that imply that it would have both current expansionary effects on demand and contractionary effects when it actually occurs. Since there is more than one way to get such a result, none of them simple, we do not lay out such a theory. We only point out that the initial expansion in the projection with small future deficit can be interpreted as directly produced by anticipations of that deficit.

The model shows less impact of drastic changes in future deficits than many economists would think likely. The modest implausibility index for the drastic deficit reduction of table VIII indicates that the model's deficit forecasts have shown substantial error in the historical sample, but announced and believed changes of such magnitude probably have not occurred before. If so, the conditional forecast in this table would not be a good guide to the likely effects of an announced and believed change. Yet, if changes of this magnitude have not been announced and believed before, that is reason to question whether a believable announcement of this type is possible.

As a kind of consistency check of these results, we also directly investigated the posterior distribution using the Monte Carlo method to integrate various regions and to evaluate conditional expectations. For example, to judge the plausibility of the CBO forecast in another way, we counted the number of our 200 simulations that had real GNP growth lower than projected by the CBO in 1983 and 1984. We found only 4 such simulations, confirming the implausibility of the CBO forecast according to our model. In a similar experiment, we found 37 simulations that had the price level growing less rapidly than the CBO forecast. There was only 1 simulation that had both real GNP and price level growth lower than that of the CBO forecast.

A forecast conditioned on low deficits was formed by averaging the 60 simulations with the lowest deficit forecasts for the period from 1984:6 to 1986:12. The average deficit path for this group is negative for the period, smoothly declining from current levels to zero in March 1985 and ending the period with a $100 billion surplus. Consistent with the conditional projections above, this subset of the simulations has lower interest rates; higher stock prices; and more rapid growth of money, prices, and output. The deficit forecast here is not forced to zero as in the earlier experiment, and growth in output stays above the overall average until late in 1986.

CONCLUSION

Our examples show that Bayesian vector autoregressive models yield no automatic causal interpretations when used for policy analysis. They do, though, provide a detailed characterization of dynamic statistical interdependence of a set of economic variables, information that may help in evaluating causal hypotheses without itself containing any such hypotheses.

APPENDIX: THE DATA

The data for this study consist of 10 series of monthly observations for the period 1948:1 through 1983:3. We took some of the series directly from the sources described below and constructed others by interpolating quarterly data. Data published in seasonally adjusted form we used in that form. Data published only in not seasonally adjusted form for which there was evidence of a seasonal pattern we adjusted ourselves. Details of the data construction and adjustment procedures are given below. (The data set itself, which is based on data published as of May 1983, is available from Litterman for a nominal charge.)

The four series that we had to interpolate are real GNP, the change in business inventories, the GNP deflator, and the flow of total nonfinancial debt. Real GNP is published as the sum of nine components, three of which are components of consumption and are available on a monthly basis. The other six components--one of which, the change in business inventories, we included separately--and the GNP deflator we interpolated from the quarterly national income and product accounts. The flow of total nonfinancial debt we interpolated from a quarterly series included in the Federal Reserve flow of funds accounts. The interpolations used related monthly series following the procedures of Chow and Lin (1971) and Litterman (1983).

Seasonal adjustment was required for federal government receipts and expenditures and several of the monthly series used in the interpolations.

INTERPOLATION

We used two interpolation procedures: the Chow-Lin (CL) procedures, in which errors are assumed to follow a first-order Markov process, and a variation of the CL procedure, in which the error process follows a random walk with a first-order Markov driving process (the RW procedure). When interpolation was required, we first tried the RW procedure. It is based on the assumption that the unobserved monthly series of interest, y_t, is related to a vector of observed monthly series, x_t, in this way:

$$y_t = x_t\beta + u_t.$$

The error process, u_t, is assumed to follow a random walk:

$$u_t = u_{t-1} + e_t$$

where e_t is a first-order Markov process:

$$e_t = \alpha e_{t-1} + v_t.$$

Litterman (1983) shows how to estimate α, β, and the monthly values of y, given quarterly averages of y and monthly values of x. He finds that, relative to other standard approaches, this procedure reduces the interpolation error in several cases where quarterly averages of observed monthly data are considered. In cases where the estimated Markov parameter, α, for this procedure was negative, however, the RW procedure did not perform well. Therefore, in such cases, we used the CL procedure. In the CL model, the error term, u_t, itself follows a first-order Markov process.

SEASONAL ADJUSTMENT

Where seasonal adjustment was necessary, we followed a frequency domain method based on the work of Nerlove (1964) and Geweke (1978). These are the steps:

1. Remove the deterministic constant, trend, and monthly seasonals.
2. Use a short-order autoregressive representation with seasonal lags to forecast and backcast two years of data.*

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*Early versions of the data set, including those used for the out-of-sample forecasting experiments, left this step out and padded with zeros rather than forecasts. The seasonally adjusted series generated without this step suffered at the ends of the data periods from a detectable modulation of the seasonal pattern. This led us to adopt the second step above.

3. Fourier-transform the series with the deterministic part removed and extensions appended, and estimate the spectrum.
4. Divide the Fourier transform of the data at seasonal frequencies by the ratio of the estimated spectrum to an estimate of the nonseasonal spectrum at that frequency. Obtain the estimate of the nonseasonal spectrum as a quadratic curve fit across seasonal frequencies to periodogram ordinates at each end of the seasonal band.
5. Transform the adjusted Fourier transform back to the time domain, and add the constant and trend.

INDIVIDUAL SERIES

Money Supply: Seasonally adjusted monthly values for the M1 measure of the money supply, as published by the Board of Governors of the Federal Reserve System, were used for the period from 1959:1 to 1983:3. Values for M1 during the period 1948:1 through 1958:12 were generated by scaling the old M1 series by the ratio of the new to the old value for 1959:1.

Treasury Bill Rate: This series is monthly averages of yields on three-month U.S. Treasury securities.

Stock Price Index: This series is monthly averages of the Standard and Poor's index of 500 securities prices.

Flow of Total Nonfinancial Debt: This is an interpolated version of the quarterly flow of total nonfinancial debt published in the flow of funds data set by the Federal Reserve Board. We constructed the quarterly series by summing seasonally adjusted nonfinancial sector credit market debt and foreign corporate equities and subtracting credit market funds raised by foreigners. These series are labeled F394104005, F263164003, and F264102005, respectively, in the flow of funds accounts.

The related monthly series used in the CL interpolation are commercial and industrial loans, the change in consumer credit outstanding, the seasonally adjusted consumer price index, the Treasury bill rate, M1, stock prices, and a constant and trend.

Because flow of funds data are released with essentially a one-quarter lag, the equation relating monthly variables to the quarterly variable together with the projected residuals was used to extend the data set through the first quarter of 1983, for which no quarterly observation was yet available. Also, the flow of debt series begins in 1952, requiring the use of the equation in a similar manner to extend observations back over the first four years of our sample.

Trade-Weighted Value of the U.S. Dollar: The U.S. Commerce Department's index of the weighted average exchange value of the U.S. dollar was used for the period for which it is available, 1967:1 through 1983:3. For the earlier period, a trade-weighted dollar was constructed following the usual formula and weights, except that it was based on exchange rates only between the United States and Germany, France, and the United Kingdom, rather than between the 10 countries in the current index. The constructed series was scaled so that the value for 1967:1 coincides with the current index. Over the period 1967:1 through 1969:12, the actual and the constructed indices moved quite closely, differing at any point by less than .3 percent.

Federal Government Outlays: Federal government budget outlays on a unified basis, not seasonally adjusted, are available from the U.S. Treasury Department monthly from 1968:2. Annual values are published for the prior years in our sample. We linearly interpolated the earlier annual data using the monthly outlays series on a cash basis, which is available for this period. The entire monthly series was then seasonally adjusted as described above.

Federal Government Receipts: The federal government budget receipts series was constructed using data analogous to that available for outlays, and it was also seasonally adjusted as described above.

GNP Deflator: The monthly GNP deflator is based on an RW interpolation using monthly data on the consumer price index, the producer price index, and a constant and trend. The two monthly price indices are published as levels and not seasonally adjusted and thus were seasonally adjusted as described above prior to use in the interpolation.

<u>Change in Business Inventories</u>: The monthly change in business inventories was generated by summing monthly series on nondurable and durable changes in business inventories, which were each separately interpolated. The nondurable inventories series is based on a CL interpolation. The related monthly series are the net change in inventories on hand and on order, wholesale inventories on nondurable goods, total inventories of nondurable goods, finished inventories of nondurable goods, and a constant, trend, and dummies for constant and trend over the period 1948:1 through 1957:12, during which the finished goods inventories are not available.

The change in business inventories of durable goods series was generated using a CL interpolation. The related monthly series are the net change in inventories on hand and on order and the series for durable goods corresponding to those used in the nondurables interpolation.

<u>Real GNP</u>: In addition to the change in business inventories, five other components of real GNP were interpolated: real business fixed investment, residential investment, government purchases, exports, and imports. Real business fixed investment was interpolated using the CL method. The related monthly series include the index of industrial production, the level of contracts and orders for plant and equipment in 1972 dollars, the composite index of capital investment commitments, new orders for capital goods, the Treasury bill rate, commercial and industrial loans, and a constant and trend.

The interpolation of residential investment used the RW method. The related monthly series are new private construction in constant dollars; total private construction put in place, which was seasonally adjusted and deflated using the GNP deflator; expenditures on private construction of residential buildings, which was deflated using the GNP deflator; and a constant, trend, and dummies for periods over which the monthly series were not available.

The interpolation of government purchases presented a problem because we could not find series that would explain its movements. We decided to use the RW interpolation method with a constant and trend.

Exports and imports were interpolated using the CL method. The related series are merchandise trade exports and imports, respectively, with constant and trend. Both trade series were deflated using the GNP deflator.

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TABLES AND CHARTS

Table I

Univariate Forecasting Performance

Variable	Period 1	Period 2	Period 3	Overall
1-step horizon				
Real GNP	.965	.796	.818	.863
GNP deflator	.204	.136	.208	.186
M1	.292	.321	.498	.381
Stock price index	2.895	3.280	3.695	3.306
Treasury bill rate	.231	.233	.871	.537
Trade-weighted dollar	.295	.355	1.981	1.174
Flow of nonfinancial debt	18.863	8.945	7.155	12.741
Change in inventories	4.967	4.981	5.531	5.542
Federal outlays	5.209	3.839	4.383	4.512
Federal receipts	4.300	4.330	3.905	4.183
Log-determinant	-69.035	-70.741	-62.819	-64.395
3-step horizon				
Real GNP	1.492	1.059	1.337	1.309
GNP deflator	.506	.368	.495	.460
M1	.651	.616	1.015	.782
Stock price index	5.901	6.608	7.532	6.714
Treasury bill rate	.502	.588	1.849	1.157
Trade-weighted dollar	.562	.753	4.429	2.614
Flow of nonfinancial debt	40.328	16.797	14.865	26.642
Change in inventories	5.868	5.507	8.185	6.627
Federal outlays	5.647	4.819	4.532	5.022
Federal receipts	5.279	5.263	4.178	4.934
Log-determinant	-58.277	-60.593	-52.840	-53.523
6-step horizon				
Real GNP	2.549	1.490	2.035	2.071
GNP deflator	.968	.673	.885	.851
M1	1.151	1.137	1.303	1.208
Stock price index	9.598	10.751	11.487	10.641
Treasury bill rate	.910	.902	2.410	1.576
Trade-weighted dollar	.882	1.060	6.811	4.012
Flow of nonfinancial debt	46.127	21.069	18.809	31.227
Change in inventories	6.700	5.866	9.282	7.427
Federal outlays	9.336	6.986	6.224	7.631
Federal receipts	8.073	6.090	4.932	6.496
Log-determinant	-50.336	-54.638	-47.249	-46.909
12-step horizon				
Real GNP	4.564	2.749	3.176	3.581
GNP deflator	2.880	1.349	1.722	2.088
M1	2.335	2.041	1.875	2.093
Stock price index	17.924	14.725	15.505	16.109
Treasury bill rate	1.184	1.340	2.717	1.878
Trade-weighted dollar	2.315	1.275	9.811	5.866
Flow of nonfinancial debt	64.439	32.025	24.778	43.939
Change in inventories	7.680	6.562	10.340	8.346
Federal outlays	16.970	11.555	10.316	13.265
Federal receipts	19.904	10.006	7.516	13.574
Log-determinant	-39.560	-48.346	-42.100	-38.284

Table II

Final Specification Forecasting Performance

Variable	Period 1	Period 2	Period 3	Overall
1-step horizon				
Real GNP	.925	.800	.759	.831
GNP deflator	.199	.131	.210	.183
M1	.269	.320	.474	.365
Stock price index	2.832	3.294	3.734	3.308
Treasury bill rate	.229	.241	.847	.525
Trade-weighted dollar	.319	.323	1.957	1.160
Flow of nonfinancial debt	18.087	8.248	6.694	12.110
Change in inventories	4.839	4.963	6.446	5.465
Federal outlays	4.879	3.500	4.485	4.327
Federal receipts	4.267	4.325	4.008	4.202
Log-determinant	-69.497	-71.310	-63.114	-64.829
3-step horizon				
Real GNP	1.355	1.083	1.178	1.211
GNP deflator	.506	.329	.518	.459
M1	.580	.625	.944	.735
Stock price index	5.841	6.567	7.556	6.692
Treasury bill rate	.493	.560	1.746	1.096
Trade-weighted dollar	.670	.677	4.339	2.565
Flow of nonfinancial debt	36.154	13.336	13.101	23.499
Change in inventories	5.610	5.402	8.247	6.549
Federal outlays	5.568	4.204	4.416	4.767
Federal receipts	4.931	5.151	4.440	4.850
Log-determinant	-59.240	-61.680	-53.379	-54.349
6-step horizon				
Real GNP	2.172	1.677	1.729	1.872
GNP deflator	.826	.567	.976	.808
M1	1.073	1.205	1.234	1.173
Stock price index	9.709	10.563	11.721	10.696
Treasury bill rate	.810	.793	2.273	1.466
Trade-weighted dollar	1.135	.990	6.666	3.946
Flow of nonfinancial debt	39.045	15.522	14.429	25.649
Change in inventories	6.140	5.899	9.290	7.276
Federal outlays	8.794	5.145	5.539	6.696
Federal receipts	6.900	5.969	5.670	6.202
Log-determinant	-52.102	-55.858	-47.723	-48.051
12-step horizon				
Real GNP	3.500	3.235	3.113	3.287
GNP deflator	1.627	1.122	1.992	1.620
M1	2.150	2.339	1.925	2.145
Stock price index	17.891	14.460	16.367	16.300
Treasury bill rate	1.033	1.108	2.469	1.672
Trade-weighted dollar	1.984	1.187	9.510	5.651
Flow of nonfinancial debt	53.879	20.877	8.594	35.045
Change in inventories	6.644	6.800	10.318	8.100
Federal outlays	17.870	7.174	8.061	12.052
Federal receipts	14.370	9.269	9.113	11.187
Log-determinant	-44.207	-50.265	-41.723	-40.680

Table III

Four Estimates of Standard Errors of Forecasts

	1-month forecasts		12-month forecasts				48-month forecasts			
Variable*	F**	O, M, V	F	O	M	V	F	O	M	V
M1	.00314	.00361	.0136	.0111	.0129	.0233	.0352	.0277	.0882	.0834
STOCKS	.03265	.03373	.1360	.1130	.0999	.1136	.2422	.2399	.2191	.1255
TBILL	.48101	.51455	1.4113	1.3225	1.3039	1.2549	1.5435	1.5542	1.8737	1.3493
DEBT	.15080	.13172	.2666	.2822	.2917	.2394	.3010	.3359	.4500	.1906
PGNP	.00184	.00221	.0120	.0088	.0136	.0202	.0245	.0240	.0948	.0871
CBI	5.52078	5.81678	7.9992	8.3999	7.8663	7.2900	8.2767	8.9611	11.0070	7.1102
RGNP	.00808	.00896	.0233	.0188	.0206	.0293	.0651	.0612	.1083	.0589
OUTL	.06902	.05457	.0857	.0890	.0949	.0826	.1048	.1128	.2054	.1164
RCPT	.07515	.04428	.0958	.1008	.0952	.0927	.1218	.1354	.2136	.1530
TRDOL	.01202	.01285	.0457	.0400	.0412	.0293	.0889	.0891	.0344	.0826

*M1 = money supply, STOCKS = stock price index, TBILL = Treasury bill rate, DEBT = flow of total nonfinancial debt, PGNP = GNP deflator, CBI = change in business inventories, RGNP = real GNP, OUTL = federal outlays, RCPT = federal receipts, and TRDOL = trade-weighted dollar.

**The four methods used to generate these estimates are designated by letter as follows: F = fixed-coefficient model, using in-sample residuals to estimate innovation covariance matrix; O = use of end-of-sample, time-varying coefficient estimates as if fixed, treating historical out-of-sample, one-step forecast error, second-moment matrix as if it were the innovation covariance matrix; M = Monte Carlo estimates of forecast errors based on time-varying coefficient model started up from end-of-sample initial conditions; and V = historical second moments of out-of-sample forecast errors.

Table IV

Unconditional Forecast as of 1982:12 and Outcome for 1983

(Except as noted, continuously compounded percentage changes from previous period at annual rates)

	1983						1984		1985		1986
Variable	1	2	3	6	9	12	6	12	6	12	12
M1											
(Forecast)	9.38	9.46	9.60	9.81	9.36	9.11	8.86	8.66	8.57	8.56	8.61
(Actual)	12.11	15.89	13.75	12.21	6.41	5.03					
STOCKS	14.38	15.70	11.86	7.47	6.38	6.03	6.01	6.08	6.17	6.28	6.45
	51.38	23.20	50.42	44.05	1.86	-6.53					
TBILL*	8.01	8.13	8.27	8.59	8.85	9.08	9.44	9.74	9.98	10.20	10.60
	7.86	8.11	8.35	8.79	9.00	9.00					
DEBT	144.54	111.67	59.06	37.94	30.21	16.31	9.60	6.80	7.84	9.96	12.93
	-57.83	9.96	223.16	4.01	-7.08	.43					
PGNP	4.08	4.26	4.97	5.48	5.91	6.33	6.78	7.20	7.47	7.66	7.85
	9.02	4.21	2.30	2.87	4.62	3.42					
CBI**	-15.61	-13.49	-7.70	-3.11	.49	3.24	6.04	7.19	7.57	7.63	7.48
	-21.55	-9.81	-14.84	-6.08	3.53	12.17					
RGNP	9.77	8.38	10.62	9.00	8.47	7.83	7.06	6.33	5.85	5.53	5.28
	-4.75	2.74	4.22	13.33	5.77	6.17					
OUTL	-35.04	.12	7.08	2.56	1.40	4.04	5.64	7.80	9.38	10.58	11.75
	40.34	-48.93	36.09	9.62	-33.98	42.96					
RCPT	-45.36	-3.96	10.32	2.72	5.64	8.40	10.30	11.44	11.88	12.06	12.19
	80.58	-65.47	17.28	21.70	20.89	-1.85					
TRDOL	-.13	4.02	5.60	5.81	4.24	3.23	2.23	1.41	.97	.74	.59
	-14.01	22.03	10.61	15.58	15.46	9.82					
DEFICIT**	201.24	203.22	202.86	203.93	198.40	193.86	184.75	179.95	179.71	183.87	203.03
	211.25	218.50	231.33	221.18	110.78	183.07					

*The Treasury bill rates are percent per annum.

**The changes in business inventories and federal deficits are billions of dollars at annual rates.

Table V

Using the Simulated Random Coefficient M Matrix:
Model Forecast Conditional on the CBO's Average Forecast of Real Growth, Inflation, and Interest Rates for 1983 and 1984

(Except as noted, percentage changes from previous period at annual rates)

	1983						1984		1985		1986
Variable	1	2	3	6	9	12	6	12	6	12	12
M1	7.56	7.20	7.68	8.72	8.48	10.08	8.68	7.18	6.72	6.34	4.61
STOCKS	-11.52	.24	6.00	-15.08	12.60	-.64	2.60	8.94	13.96	9.18	4.02
TBILL*	7.89	7.63	7.43	6.96	6.56	6.24	7.06	7.74	8.78	9.76	11.03
DEBT	134.16	91.68	82.68	29.68	18.00	10.12	40.16	19.42	-12.52	-5.68	-11.47
PGNP	4.32	3.84	4.44	5.04	5.20	4.36	4.06	5.14	4.74	5.46	5.64
CBI**	-15.83	-14.26	-11.10	-7.24	-7.50	-7.05	2.44	.12	3.55	6.87	5.34
RGNP	4.68	6.60	4.20	6.16	1.92	2.76	6.56	2.84	5.20	3.90	2.37
OUTL	-65.40	2.40	15.60	6.00	4.80	12.00	-1.60	4.00	3.80	-1.20	12.70
RCPT	-63.24	42.00	8.40	-7.60	-7.20	5.20	4.80	-2.00	6.60	12.80	9.30
TRDOL	3.24	.12	6.12	3.04	2.28	-2.60	.14	.44	2.08	2.80	4.51
DEFICIT**	190.34	171.62	177.43	200.22	220.02	236.64	216.09	238.32	234.46	189.37	239.55

*The Treasury bill rates are percent per annum.

**The changes in business inventories and federal deficits are billions of dollars at annual rates.

Table VI

Using the Empirically Estimated V Matrix:
Model Forecast Conditional on the CBO's Average Forecast of Real Growth, Inflation, and Interest Rates for 1983 and 1984

(Except as noted, percentage changes from previous period at annual rates)

	1983						1984		1985		1986
Variable	1	2	3	6	9	12	6	12	6	12	12
M1	4.56	5.28	7.08	8.20	7.56	7.24	6.66	6.68	5.56	6.32	5.40
STOCKS	-3.00	5.40	9.84	6.52	9.72	6.64	8.42	12.68	8.50	13.54	8.84
TBILL*	7.73	7.38	7.04	6.56	6.79	6.81	7.42	7.38	8.04	8.57	9.85
DEBT	155.76	113.64	49.80	30.84	24.76	23.96	25.04	15.68	11.76	-3.32	-3.95
PGNP	4.08	4.08	4.68	4.68	4.92	4.92	4.46	4.74	5.54	6.08	6.40
CBI**	-15.22	-13.82	-9.12	-6.93	-5.24	-2.09	3.34	1.78	8.81	9.17	7.69
RGNP	2.28	1.44	4.80	3.20	4.44	5.52	5.00	4.40	6.50	5.86	4.44
OUTL	-34.20	-3.60	-1.20	1.60	-5.60	2.00	6.00	4.80	.20	2.80	8.50
RCPT	-46.44	-12.00	7.20	-.40	1.60	4.40	3.80	6.20	4.20	6.60	11.30
TRDOL	.84	2.64	3.12	4.32	1.04	.40	2.00	-3.36	2.96	.26	.58
DEFICIT**	202.31	205.71	201.50	205.18	192.06	189.54	201.87	202.51	190.35	180.88	176.97

*The Treasury bill rates are percent per annum.

**The changes in business inventories and federal deficits are billions of dollars at annual rates.

Table VII

Using the Fixed-Coefficient O Matrix:
Model Forecast Conditional on the CBO's Average Forecast of Real Growth, Inflation, and Interest Rates for 1983 and 1984

(Except as noted, percentage changes from previous period at annual rates)

	1983						1984		1985		1986
Variable	1	2	3	6	9	12	6	12	6	12	12
M1	6.48	6.48	7.32	8.36	8.32	8.44	7.72	7.78	9.06	8.58	8.43
STOCKS	-14.16	-13.20	-15.24	-18.64	-14.56	-8.88	-2.76	5.06	8.30	8.48	8.52
TBILL*	7.76	7.53	7.28	6.81	6.54	6.57	7.24	7.56	8.39	9.04	9.91
DEBT	109.20	76.08	24.48	18.64	28.32	27.36	15.46	13.62	16.96	5.16	6.44
PGNP	3.72	3.60	4.32	4.56	4.92	5.44	4.36	4.84	7.90	8.12	8.26
CBI**	-15.71	-13.97	-8.84	-6.78	-5.16	-5.04	.48	-.22	5.48	7.50	8.09
RGNP	4.92	4.08	6.36	4.16	3.88	2.84	5.58	3.82	8.32	6.82	5.58
OUTL	-39.00	-1.20	6.00	1.20	-.40	.80	6.00	6.20	5.20	8.00	10.70
RCPT	-41.64	-3.60	8.40	.00	1.20	2.00	3.40	4.40	11.00	12.80	13.50
TRDOL	.24	3.48	4.32	4.48	2.32	1.20	.72	-1.18	-.48	-.66	-.28
DEFICIT**	196.89	197.85	197.68	200.02	197.49	196.13	209.90	222.01	209.48	201.77	202.89

*The Treasury bill rates are percent per annum.

**The changes in business inventories and federal deficits are billions of dollars at annual rates.

Table VIII

Using the M Matrix Again: Model Forecast Conditional on a Zero Deficit in 1984:6 and Thereafter

(Except as noted, percentage changes from previous period at annual rates)

	1983						1984		1985		1986
Variable	1	2	3	6	9	12	6	12	6	12	12
M1	9.48	9.48	10.08	10.24	9.92	9.96	9.22	9.22	9.62	9.42	9.40
STOCKS	13.80	18.84	26.40	16.92	9.00	12.64	7.66	4.34	7.18	5.64	4.87
TBILL*	8.05	8.09	8.28	8.56	8.72	8.93	9.70	9.85	9.93	10.34	10.58
DEBT	130.08	113.52	93.36	42.24	-6.12	18.92	18.20	-9.40	10.60	15.08	10.08
PGNP	4.20	4.44	5.04	5.76	6.08	7.52	8.34	9.06	9.00	9.04	9.31
CBI**	-15.27	-12.49	-7.14	-3.20	-.12	4.68	8.87	9.76	8.68	7.50	7.82
RGNP	9.96	6.96	11.88	9.48	9.04	9.44	8.04	6.72	5.08	5.40	5.29
OUTL	-33.00	-7.20	1.20	3.60	-4.80	-18.00	-4.40	11.60	10.20	12.20	15.60
RCPT	-39.24	-7.20	7.20	5.20	12.00	11.60	29.80	11.60	10.20	12.20	15.60
TRDOL	-.96	3.96	1.92	5.40	4.80	2.40	2.44	2.12	2.34	1.08	1.58
DEFICIT**	199.62	198.42	195.73	195.16	167.95	116.14	.00	.00	.00	.00	.00

*The Treasury bill rates are percent per annum.

**The changes in business inventories and federal deficits are billions of dollars at annual rates.

Table IX

Standardized Shocks Generating the Model Forecast
Conditional on the CBO's Projections, Using the V Matrix

	1983						1984	
Variable	1	2	3	6	9	12	6	12
M1	-.9	-.4	-.3	-.3	-.3	-.4	-.7	-.3
STOCKS	-.4	-.3	-.3	-.3	-.3	-.3	-.2	-.1
TBILL	-.6	-.6	-.5	-.9	-.6	-.4	.1	-.2
DEBT	.1	.1	.1	.3	.2	-.0	-.1	-.1
PGNP	.3	.3	.3	.4	.3	.2	-.1	.1
CBI	-.0	-.0	-.0	.1	-.1	-.0	-.2	-.4
RGNP	-.8	-.7	-.5	-.8	-.6	-.4	-.6	-.5
OUTL	.0	.0	.0	-.1	-.2	-.0	.0	.0
RCPT	-.0	-.0	.0	.0	.1	.1	.0	.0
TRDOL	.2	.1	.2	.2	.2	.0	.0	.0

Table X

Standardized Shocks Generating the Model Forecast Conditional on a Zero Deficit in 1984:6 and Thereafter, Using the M Matrix

	1983						1984		1985		1986
Variable	1	2	3	6	9	12	6	12	6	12	12
M1	.0	.0	.1	.2	.1	.3	.0	-.0	-.0	.0	-.0
STOCKS	-.0	.1	.3	.3	.1	.1	.2	.1	.0	.0	.0
TBILL	.1	-.2	.2	-.1	-.1	-.1	.3	-.0	-.1	.0	.0
DEBT	-.1	-.0	.1	.1	-.4	-.2	.1	-.1	-.1	-.1	-.0
PGNP	.0	.1	-.0	.1	-.0	.3	.3	.2	.1	.1	.0
CBI	.0	.1	.1	-.0	-.1	.3	.0	.1	-.0	.0	-.0
RGNP	-.0	-.4	.0	.0	.1	-.0	-.1	.0	.0	.0	.0
OUTL	.1	-.0	-.1	-.0	-.1	-.6	-.7	-.3	-.1	-.2	-.0
RCPT	.1	.0	.0	.1	.1	.3	.9	.4	.2	.3	.0
TRDOL	-.1	-.0	-.3	-.0	.1	-.1	-.0	-.0	.0	-.0	.0

Charts 1 and 2

How Forecast Accuracy Varies With Two Dimensions of the Prior

Chart 1. Front View

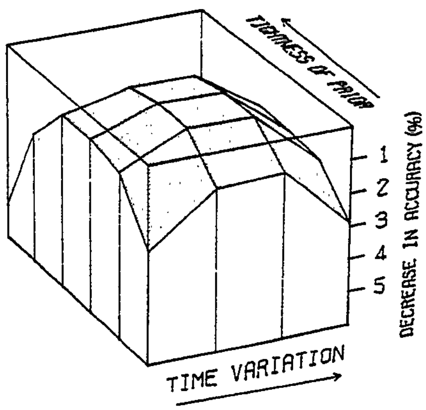
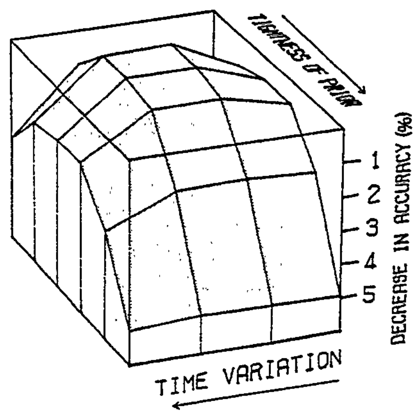


Chart 2. Back View



Charts 3–8

Forecast Accuracy Surfaces in Three Nonoverlapping Subperiods

All axes of these charts have the same scales as those of Chart 1.

Charts 3–5. One-Step-Ahead Forecast Horizon

Chart 3. From January 1952 to December 1961

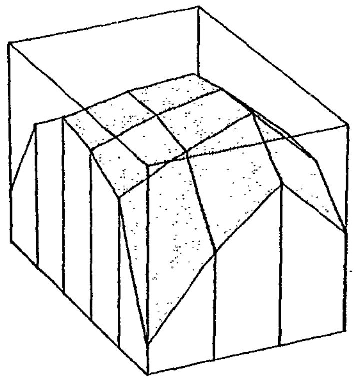


Chart 4. From January 1962 to December 1971

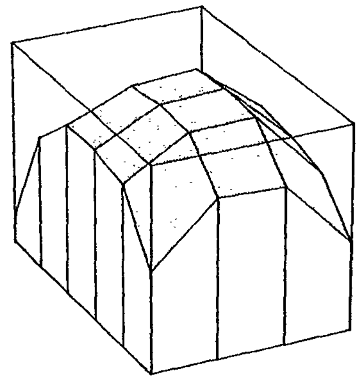
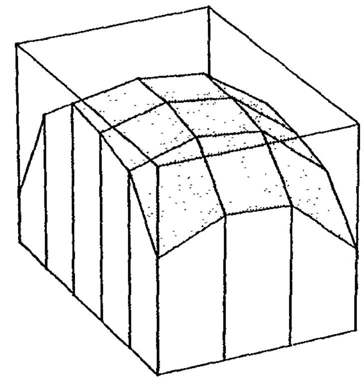


Chart 5. From January 1972 to December 1981



Charts 6–8. Twelve-Step-Ahead Forecast Horizon

Chart 6. From January 1952 to December 1961

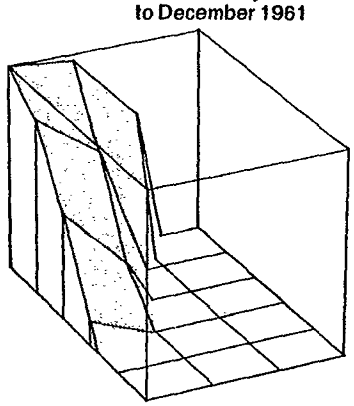


Chart 7. From January 1962 to December 1971

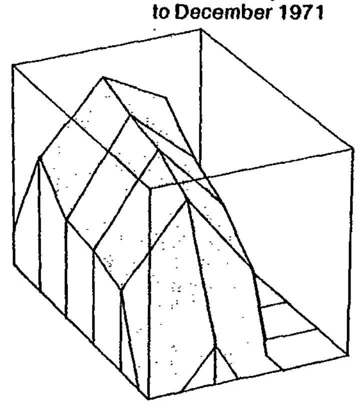
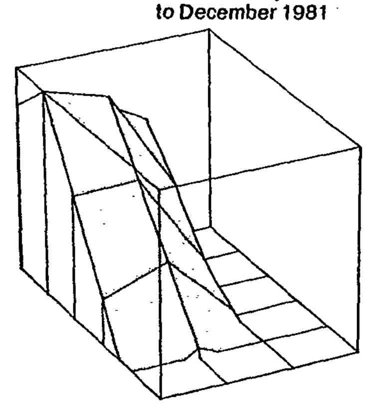


Chart 8. From January 1972 to December 1981



Charts 9–24
Forecasts

——— Actuals From January 1981 to March 1983

Forecasts:
- - - - - As of December 1982 (From January 1983 to December 1986)
— — — As of March 1983 (From April 1983 to December 1986)

Chart 9. Real GNP Level (1972 Dollars)

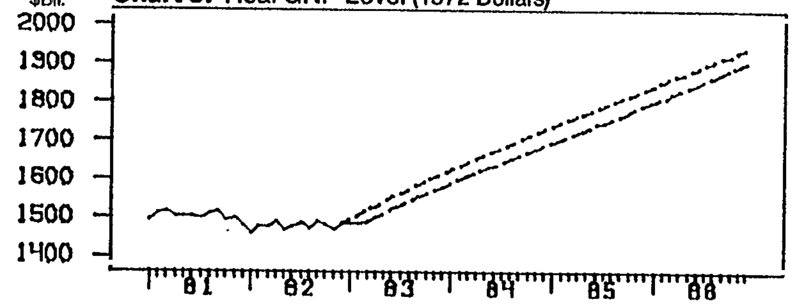


Chart 10. Real GNP Growth

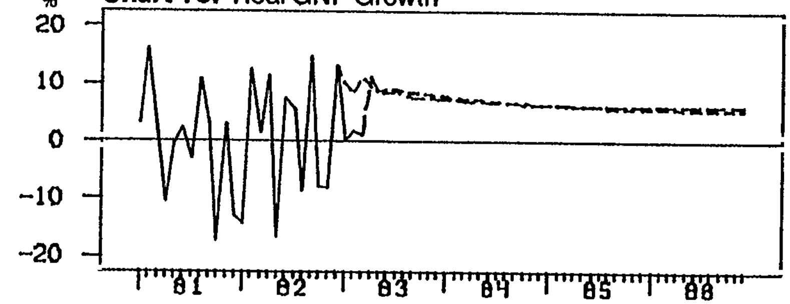


Chart 11. Price Level (GNP Deflator)

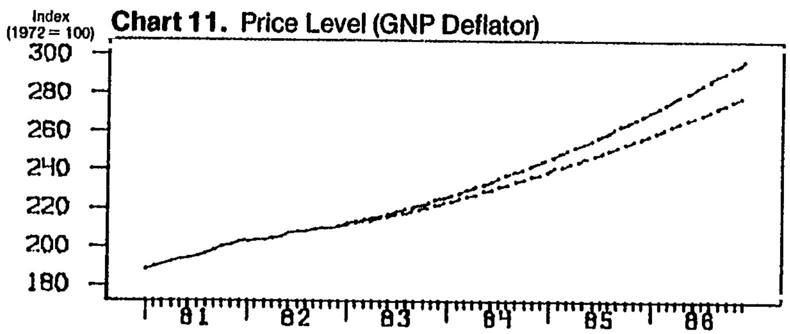


Chart 12. Inflation

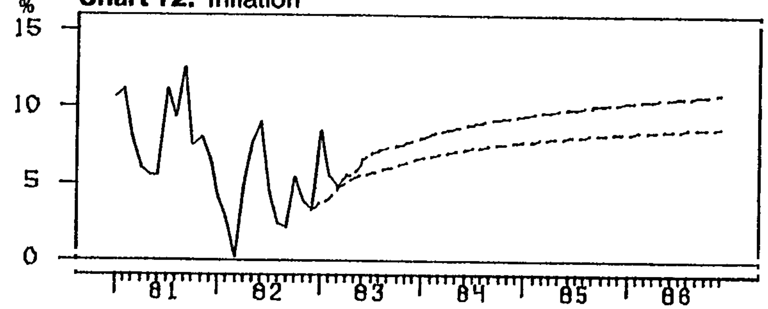


Chart 13. Money Supply (M1)

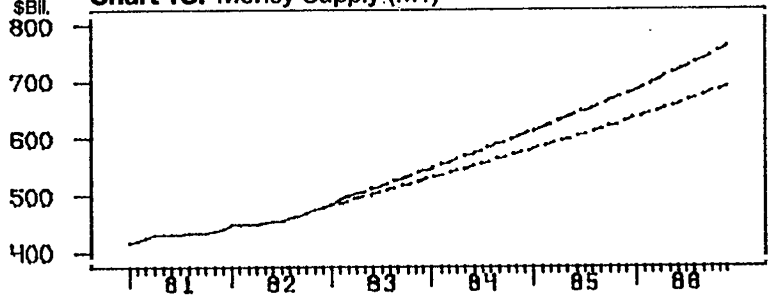


Chart 14. Money Supply Growth

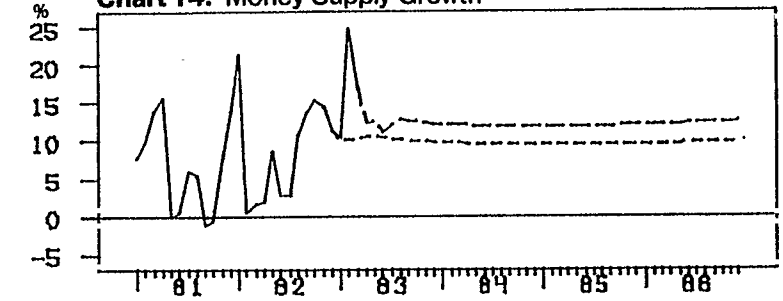


Chart 15. Stock Price Level (Standard & Poor's 500 Index)

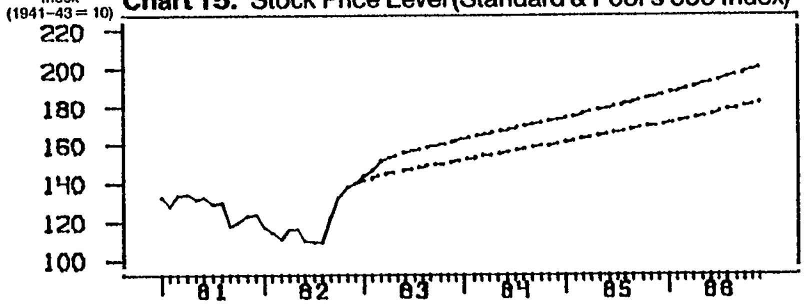


Chart 16. Stock Price Growth

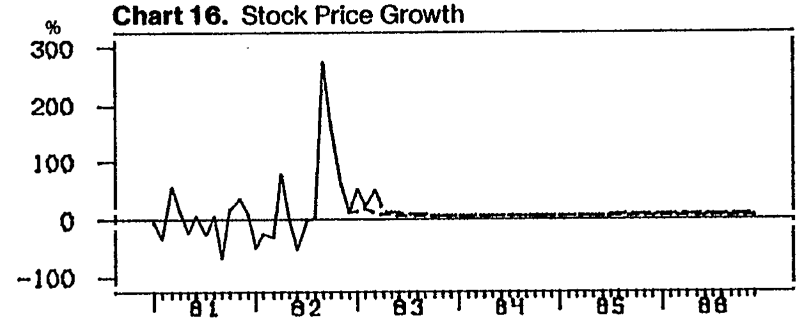


Chart 17. Treasury Bill Rate (3-Month)

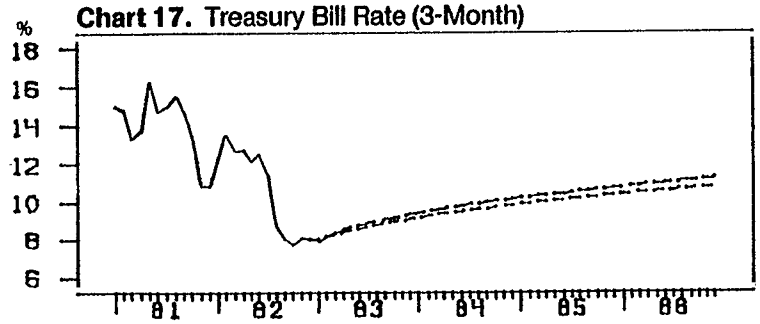


Chart 18. Change in Business Inventories (1972 Dollars)

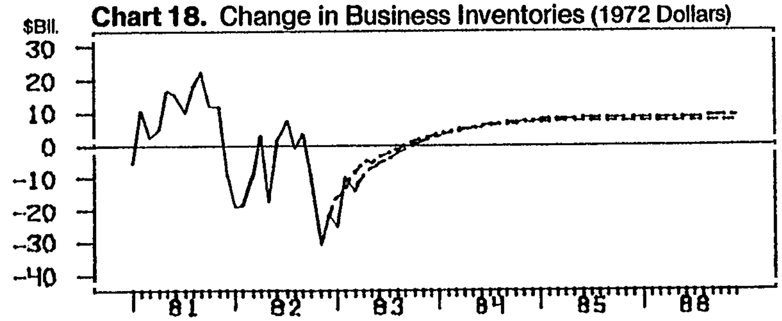


Chart 19. Value of the Trade-Weighted Dollar

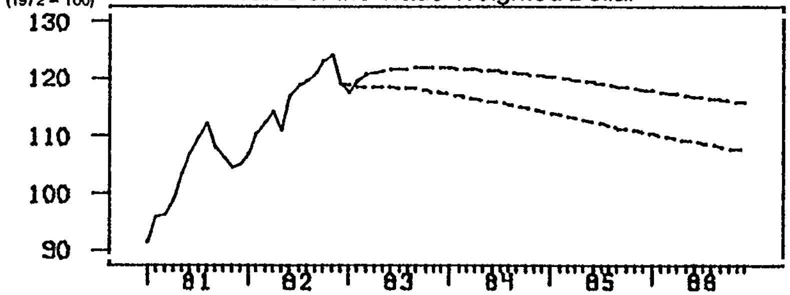


Chart 20. Growth in the Value of the Trade-Weighted Dollar

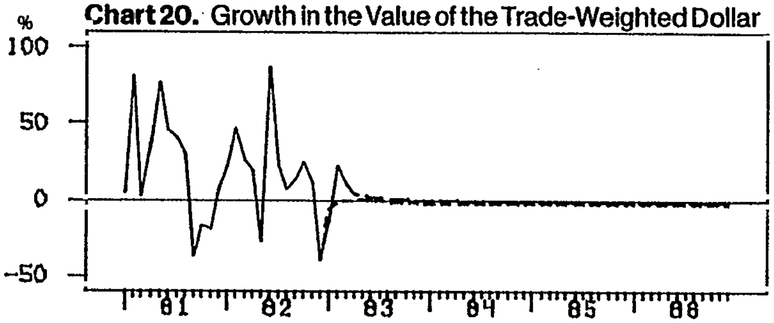


Chart 21. Federal Government Receipts

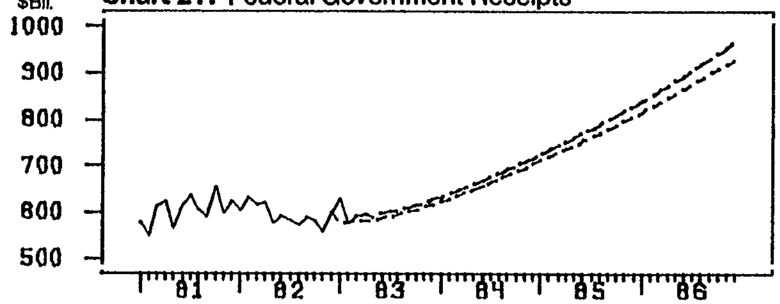


Chart 22. Federal Government Outlays

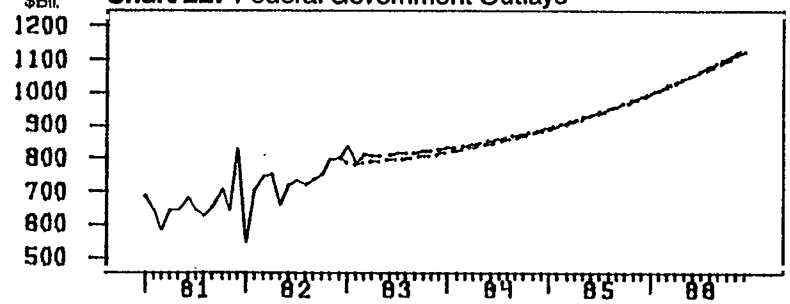


Chart 23. Federal Government Deficit

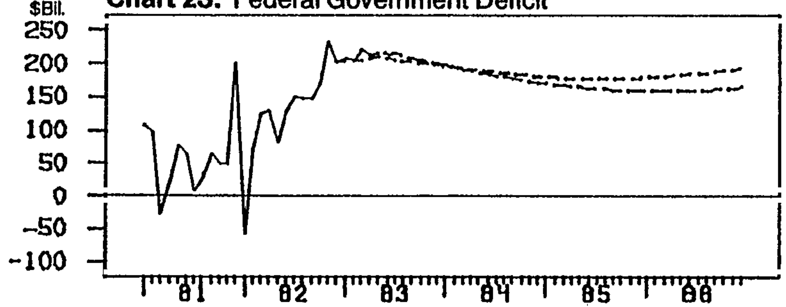
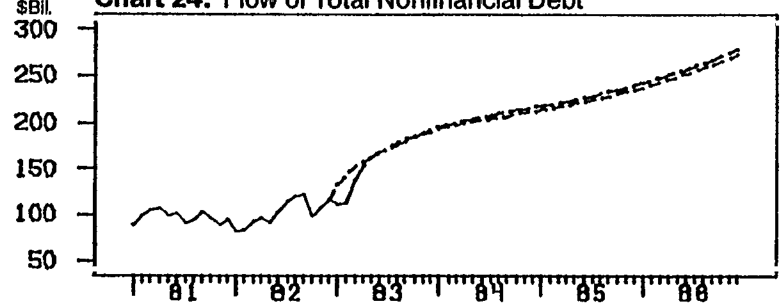


Chart 24. Flow of Total Nonfinancial Debt



Charts 25–34

Responses of Real GNP in Three Nonoverlapping Periods

——— From July 1948 to January 1960   - - - - - From February 1960 to August 1971   — — — From September 1971 to March 1983

Responses are to orthogonalized innovations of the same magnitude for each period. The graphs are displayed in the order that the innovations are orthogonalized. The size of the shock for each variable is normalized to be one standard error of the distribution of the residuals for that variable over the full period. Responses are displayed over a horizon of 48 months in units of percent.

Chart 25. To an Innovation in Stock Prices

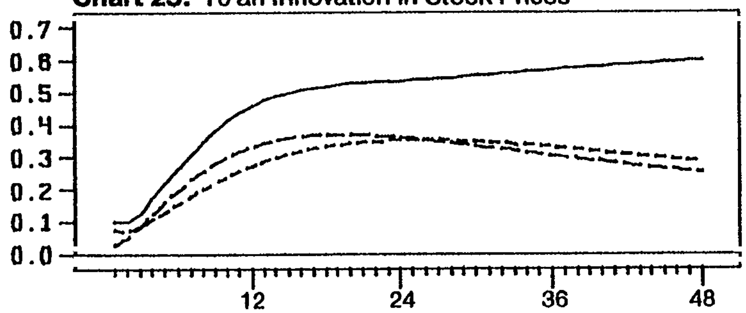


Chart 26. To an Innovation in Treasury Bill Rates

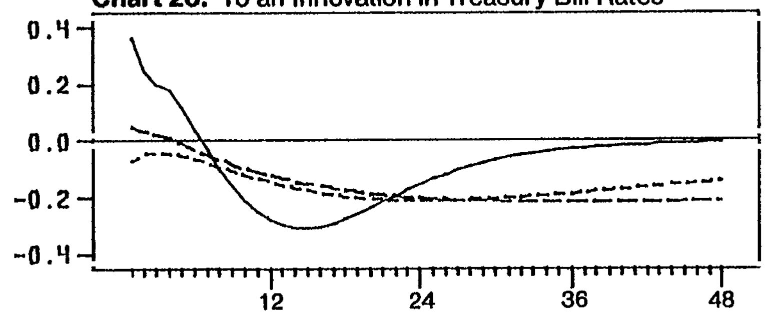


Chart 27. To an Innovation in the Money Supply

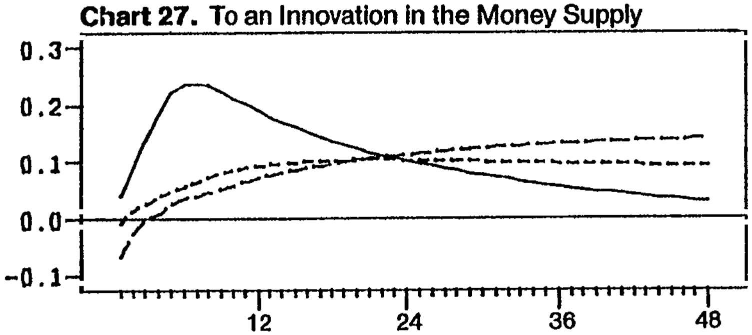


Chart 28. To an Innovation in the Trade-Weighted Dollar

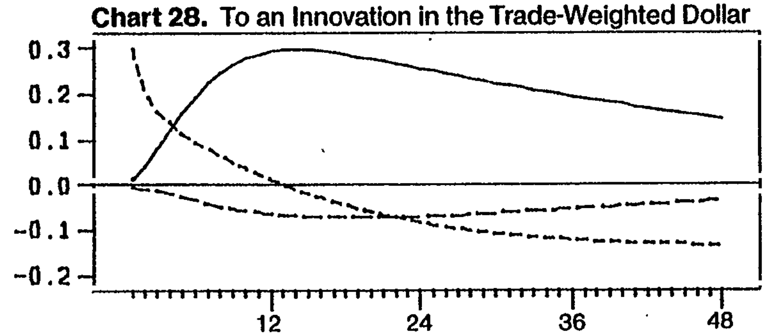


Chart 29. To an Innovation in the Flow of Total Nonfinancial Debt

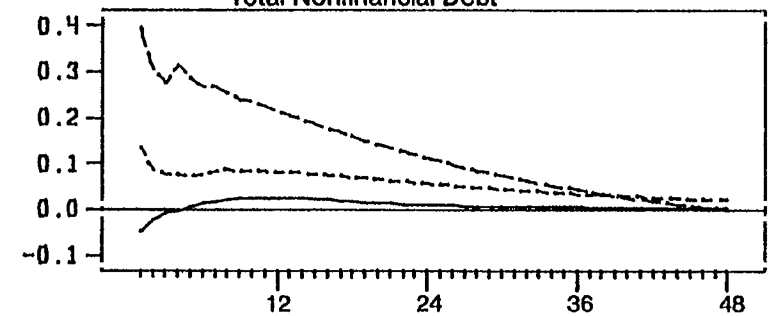


Chart 30. To an Innovation in Federal Government Outlays

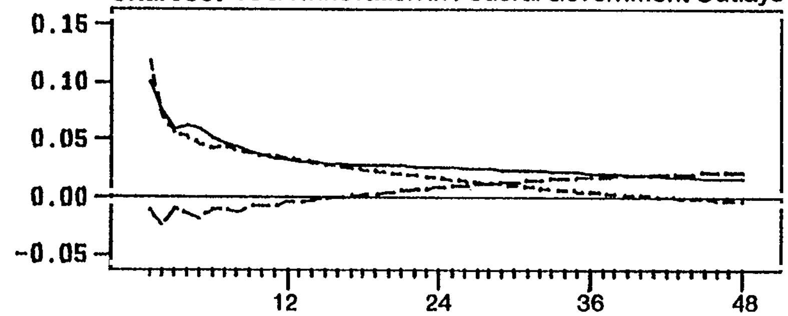


Chart 31. To an Innovation in Federal Government Receipts

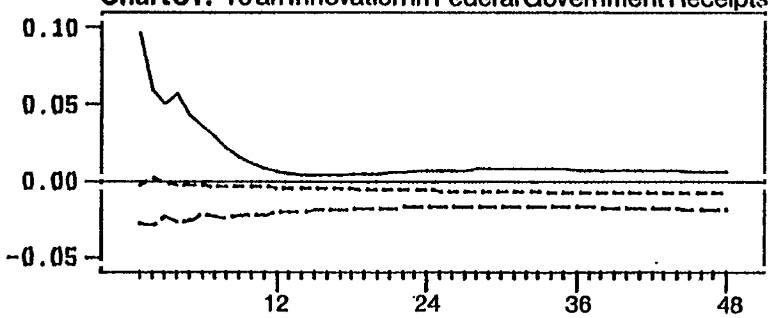


Chart 32. To an Innovation in the Change in Business Inventories

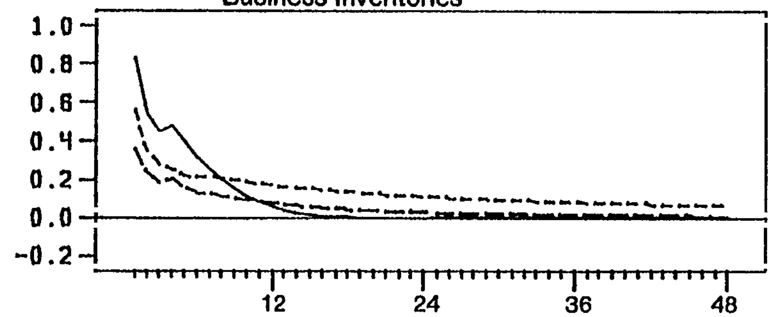


Chart 33. To an Innovation in the Price Level

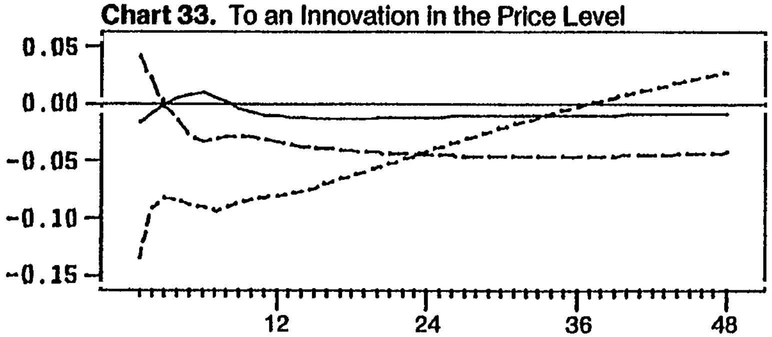
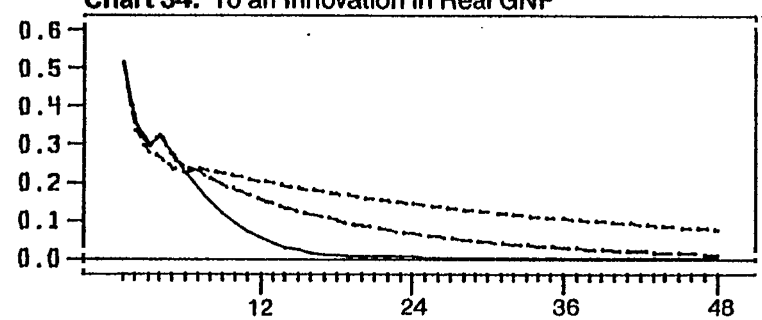


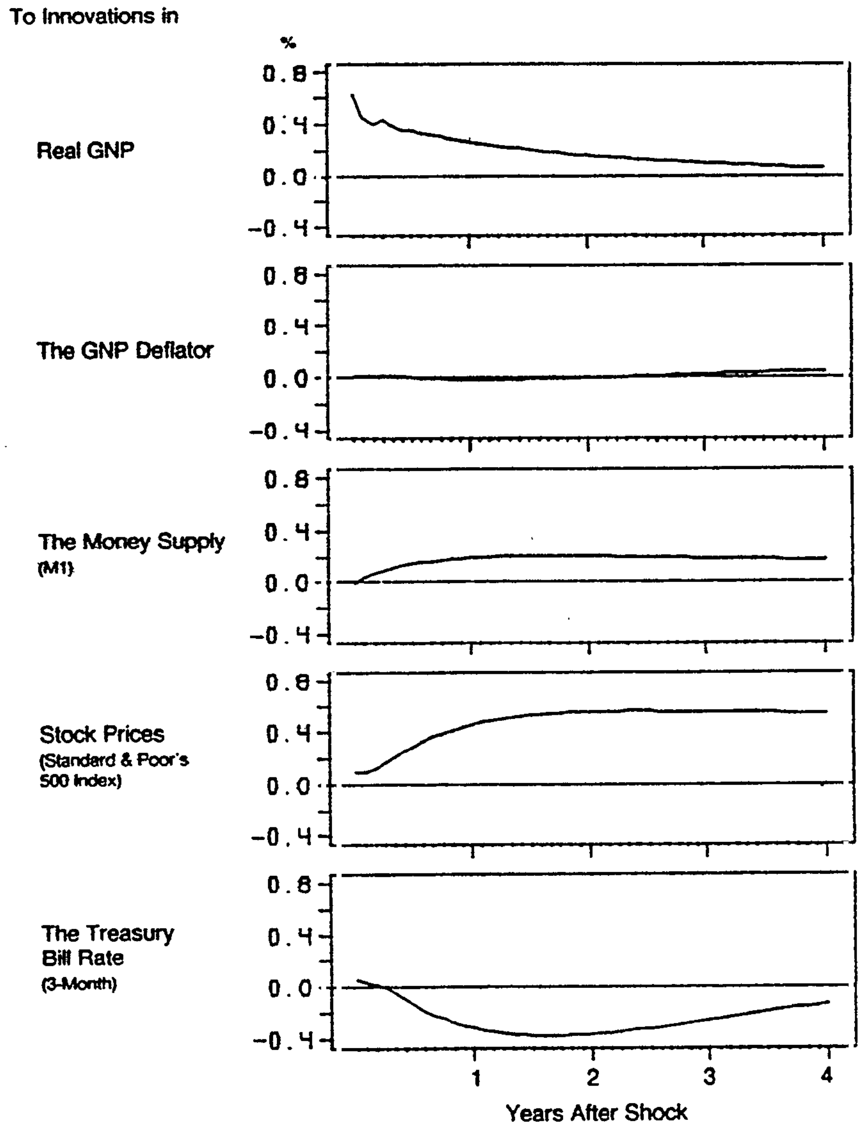
Chart 34. To an Innovation in Real GNP



Charts 35–45

Impulse Response Functions

Chart 35. Responses of Real GNP



(Continued)

Chart 35. Responses of Real GNP

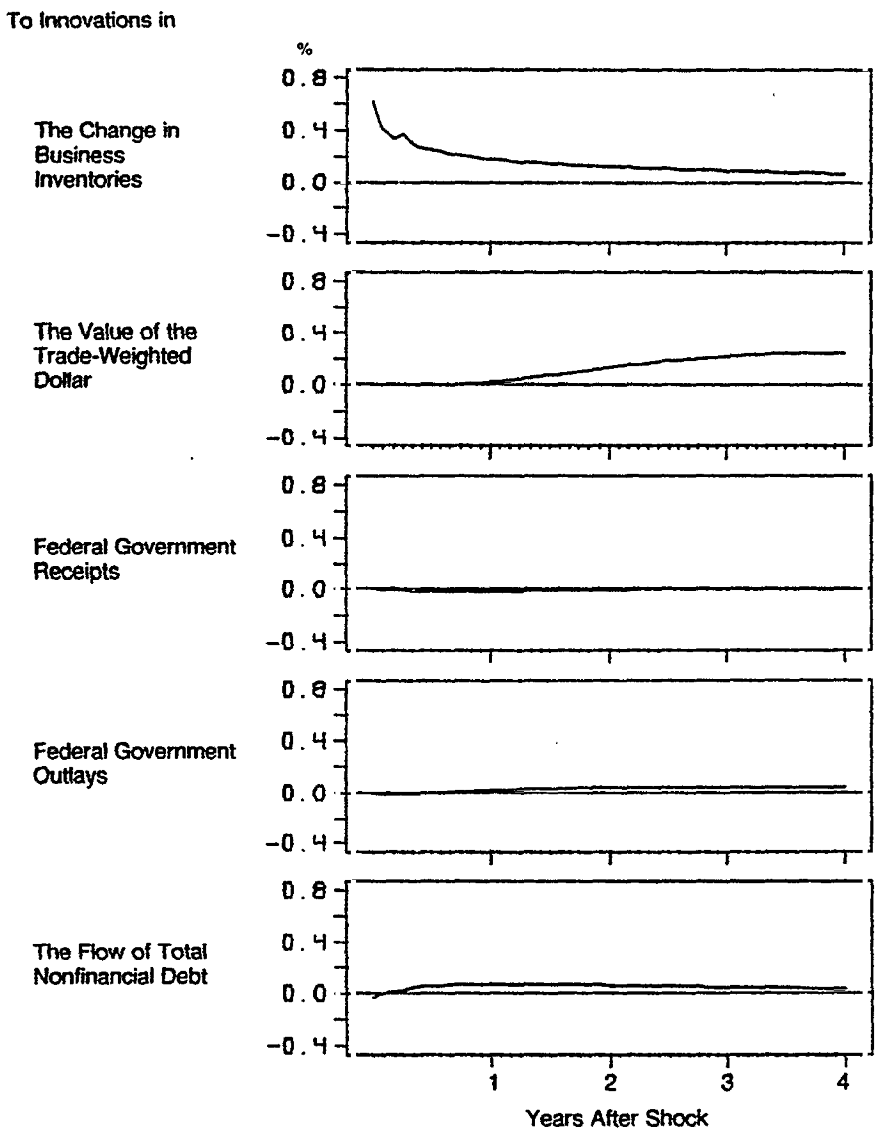
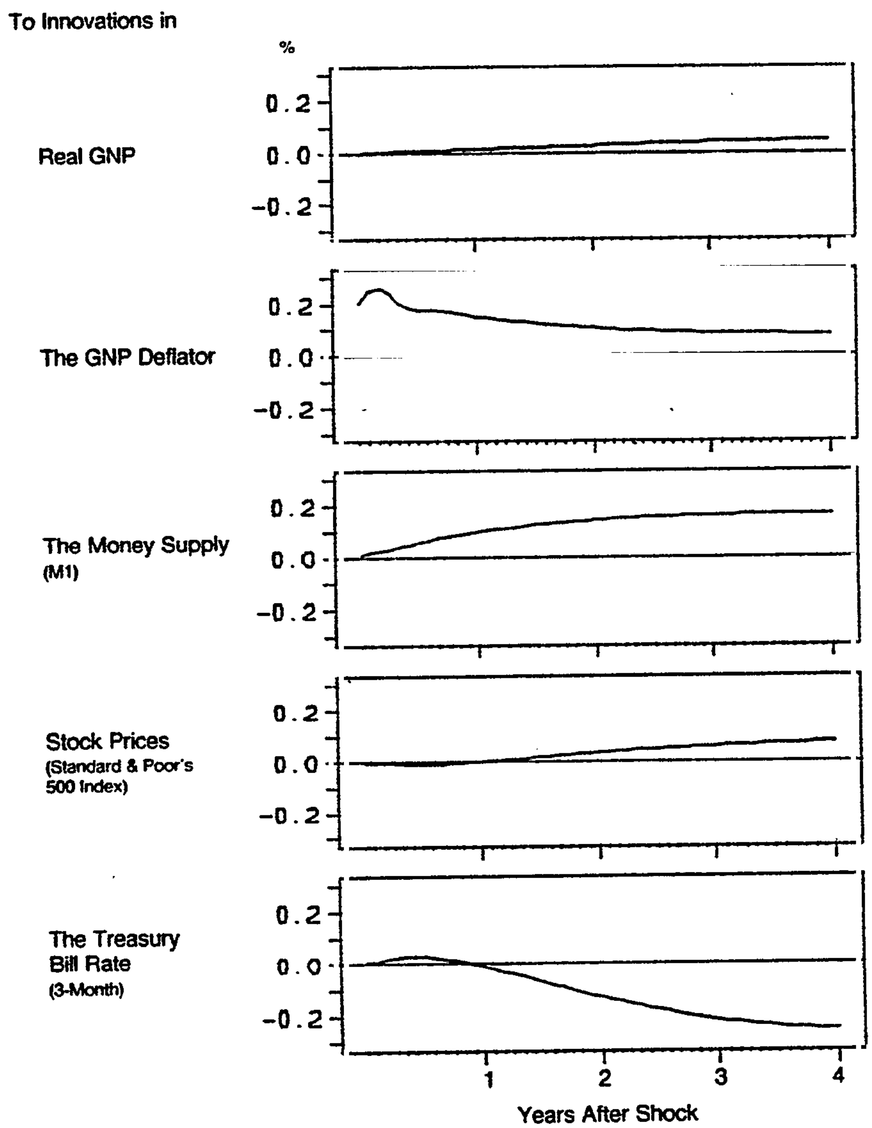


Chart 36. Responses of the GNP Deflator



(Continued)

Chart 36. Responses of the GNP Deflator

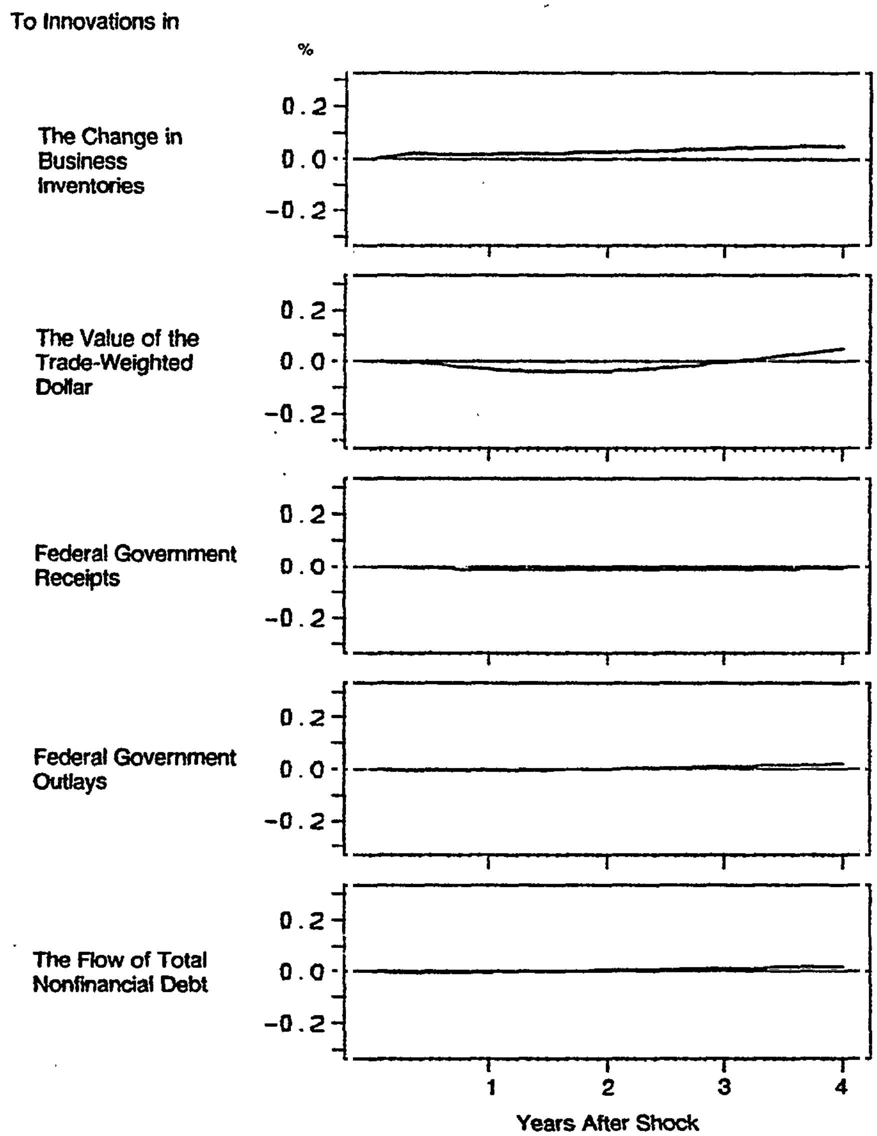
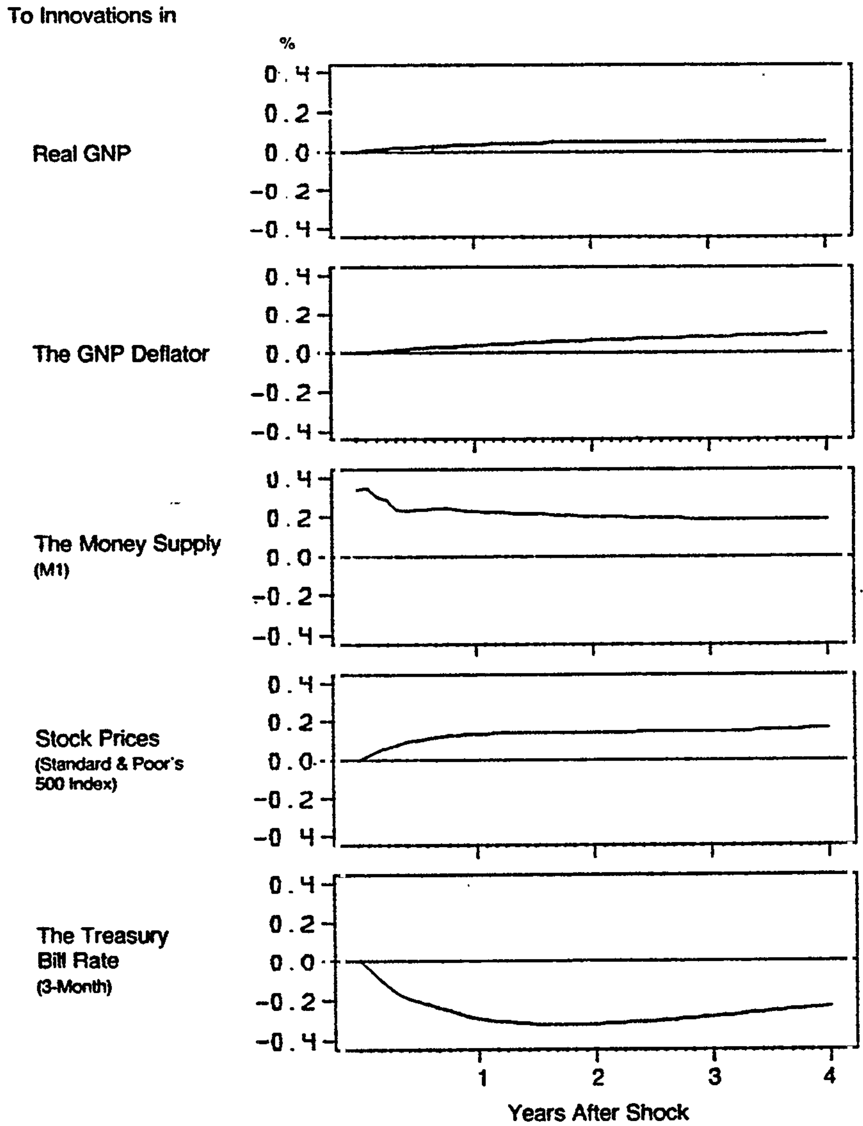


Chart 37. Responses of the Money Supply (M1)



(Continued)

Chart 37. Responses of the Money Supply (M1)

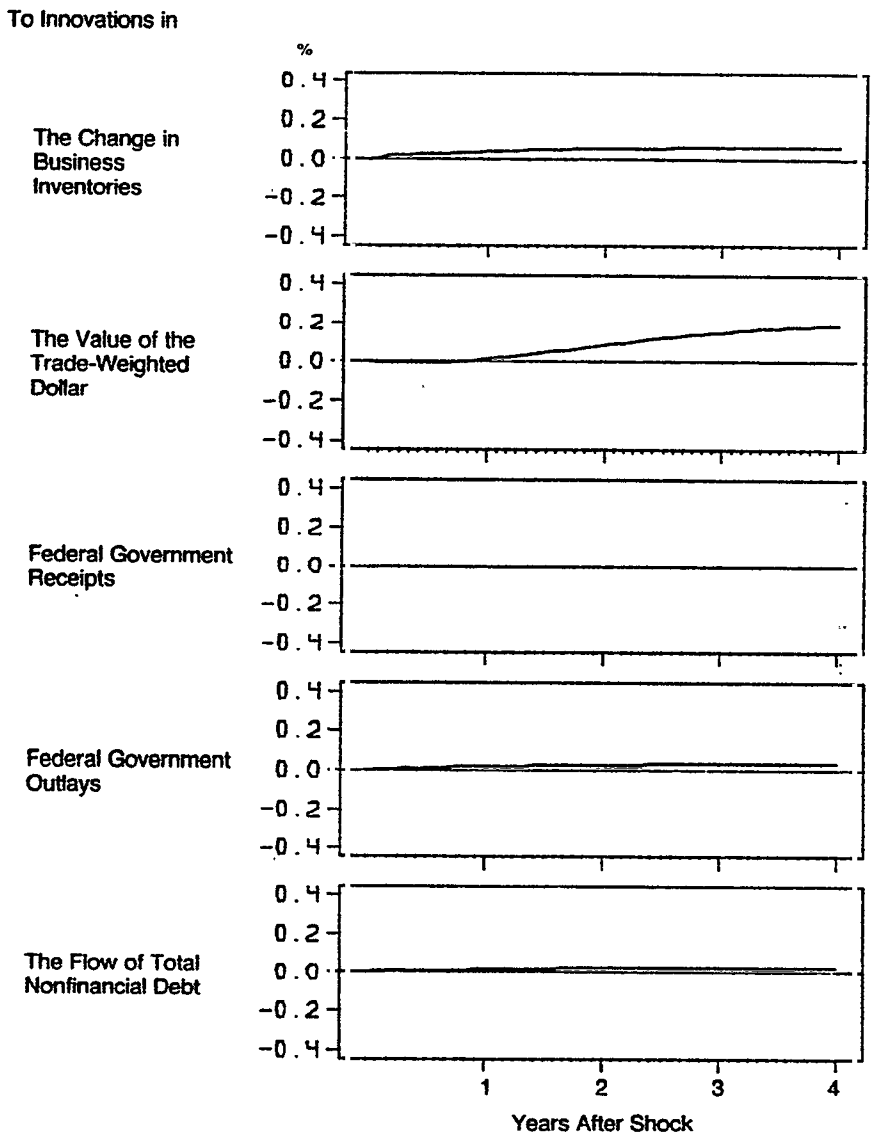
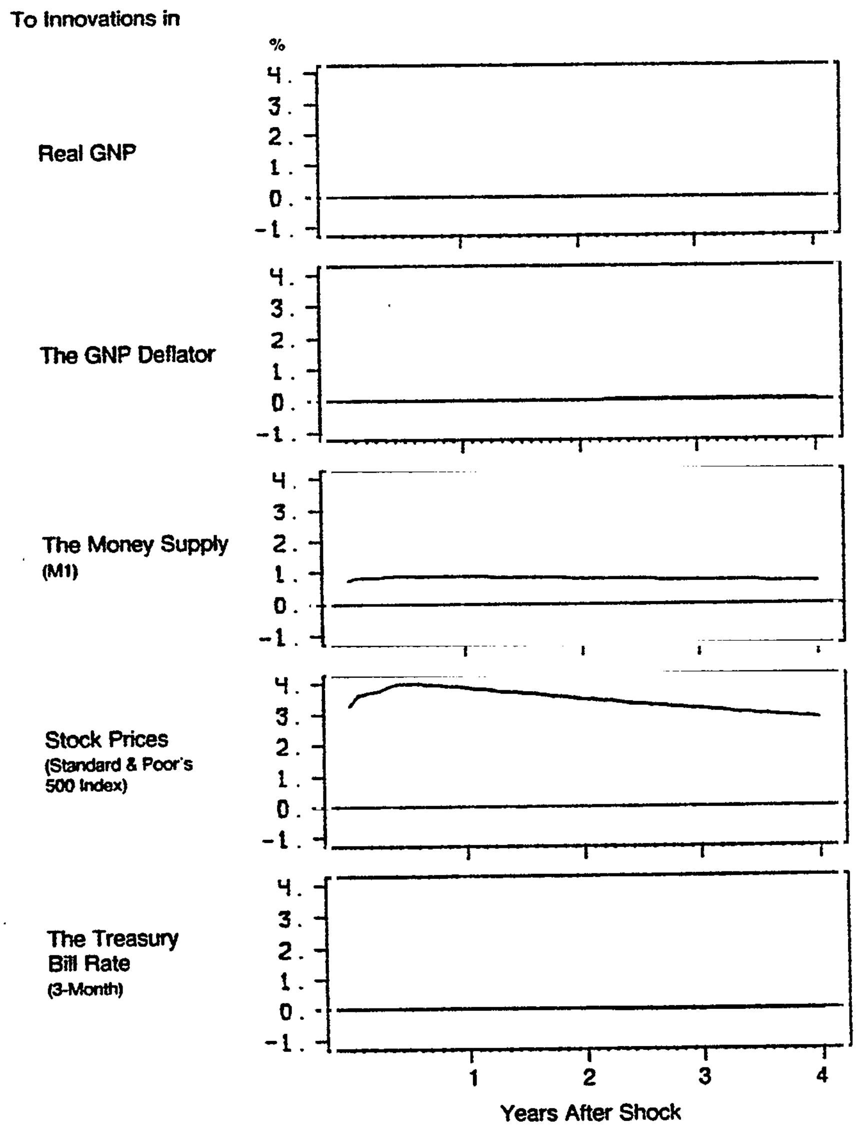


Chart 38. Responses of Stock Prices (Standard & Poor's 500 Index)



(Continued)

Chart 38. Responses of Stock Prices (Standard & Poor's 500 Index)

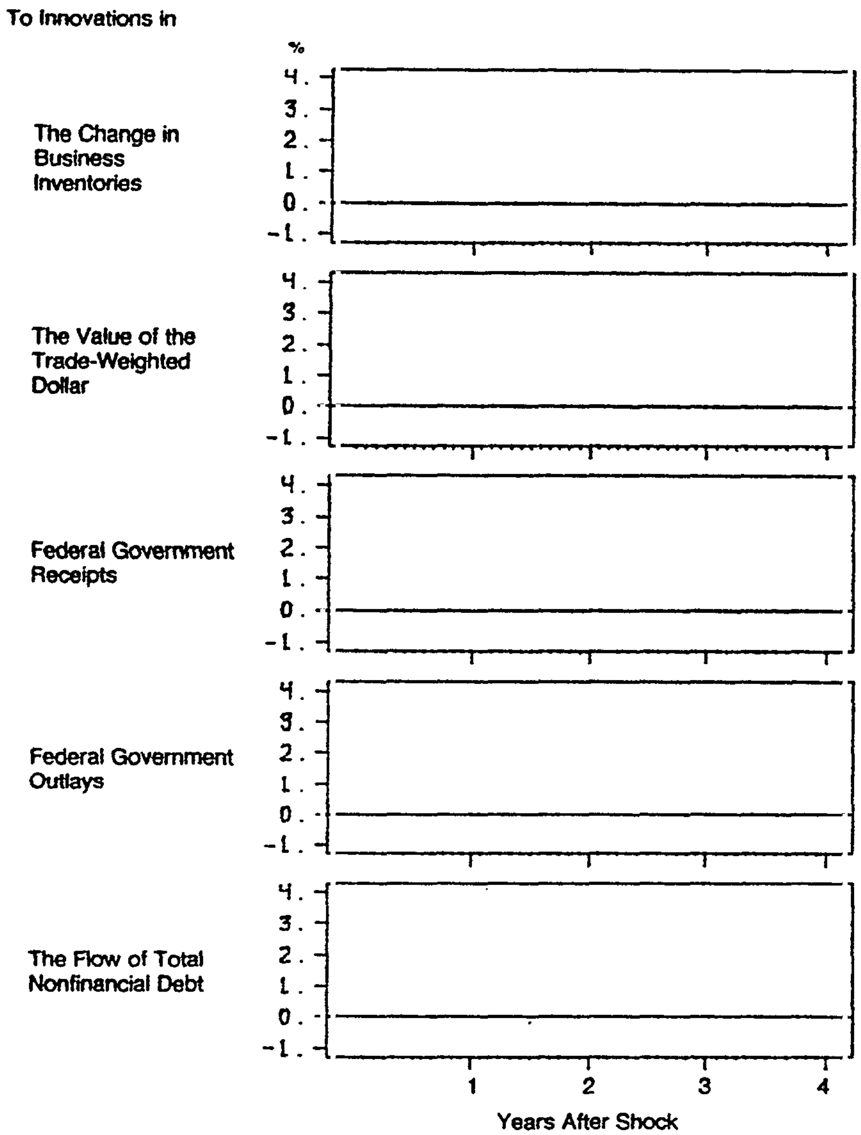
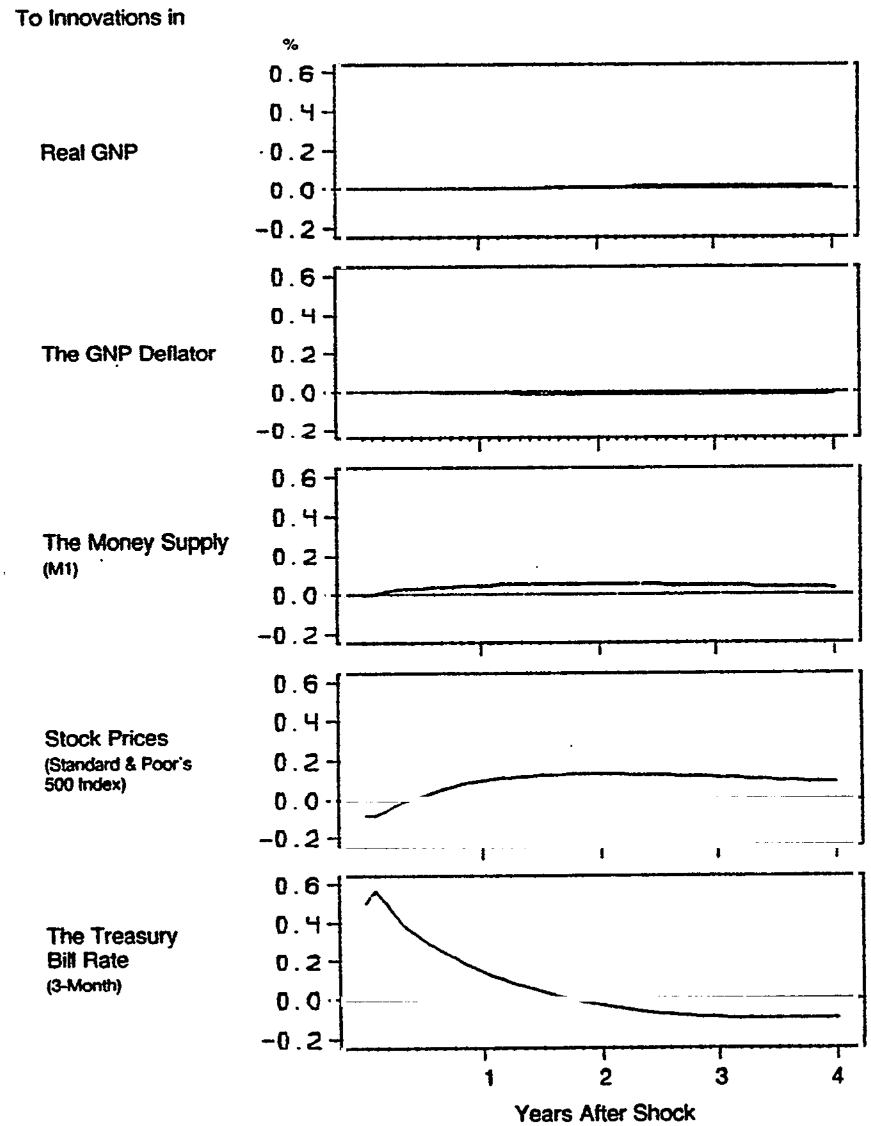


Chart 39. Responses of the Treasury Bill Rate (3-Month)



(Continued)

Chart 39. Responses of the Treasury Bill Rate (3-Month)

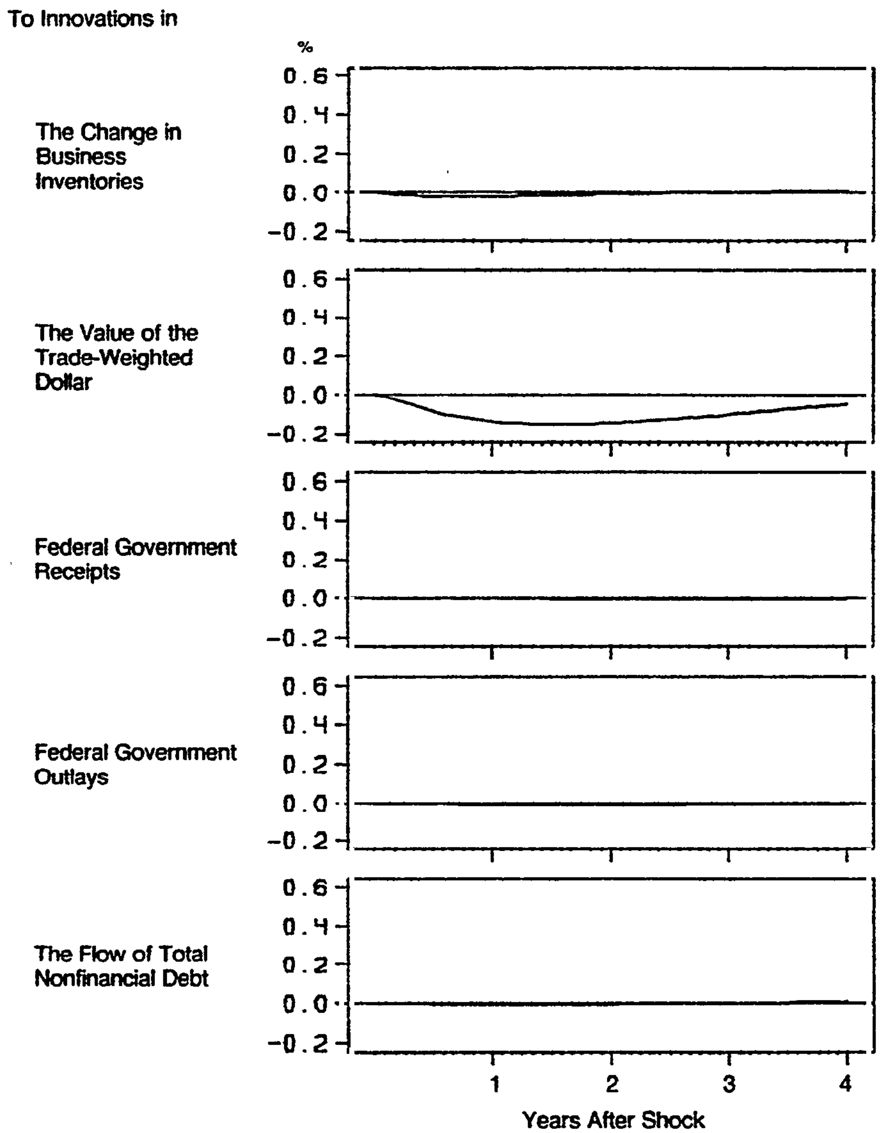


Chart 40. Responses of the Change in Business Inventories (1972 Dollars)

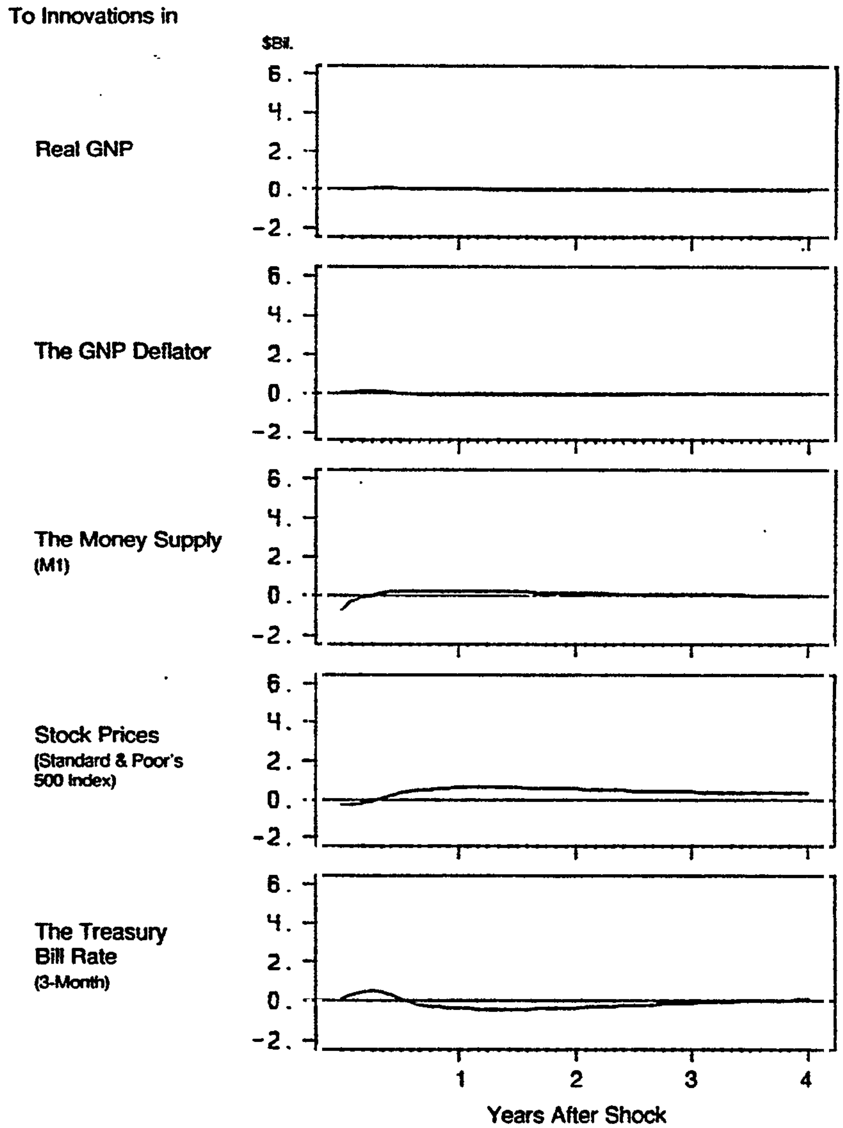


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Chart 40. Responses of the Change in Business Inventories (1972 Dollars)

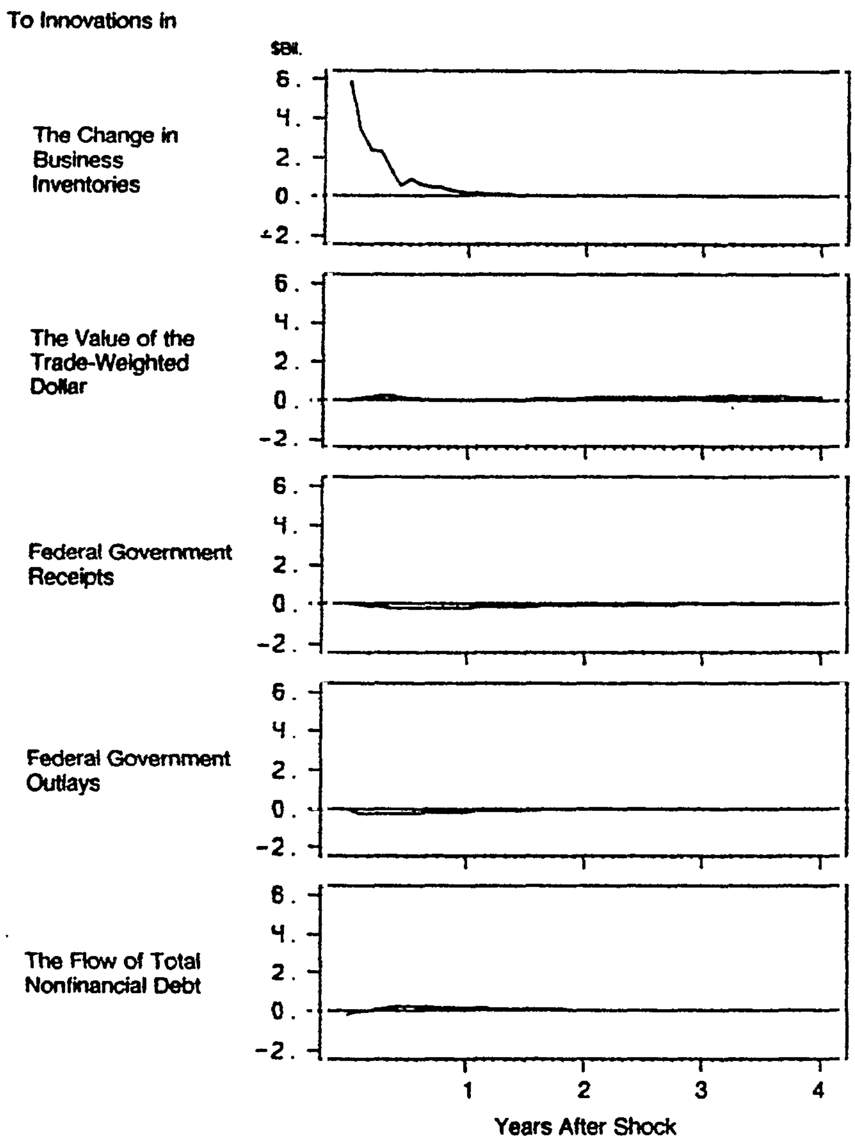

Chart 41. Responses of the Value of the Trade-Weighted Dollar

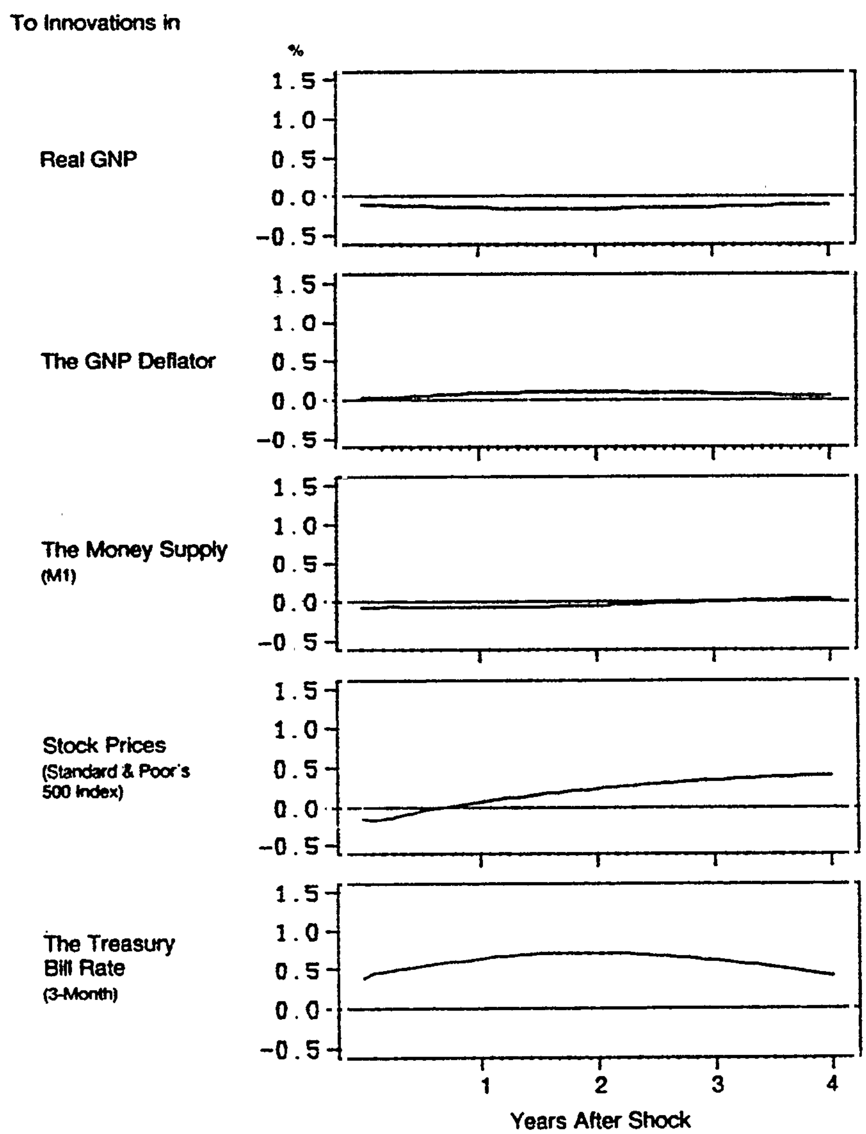

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Chart 41. Responses of the Value of the Trade-Weighted Dollar

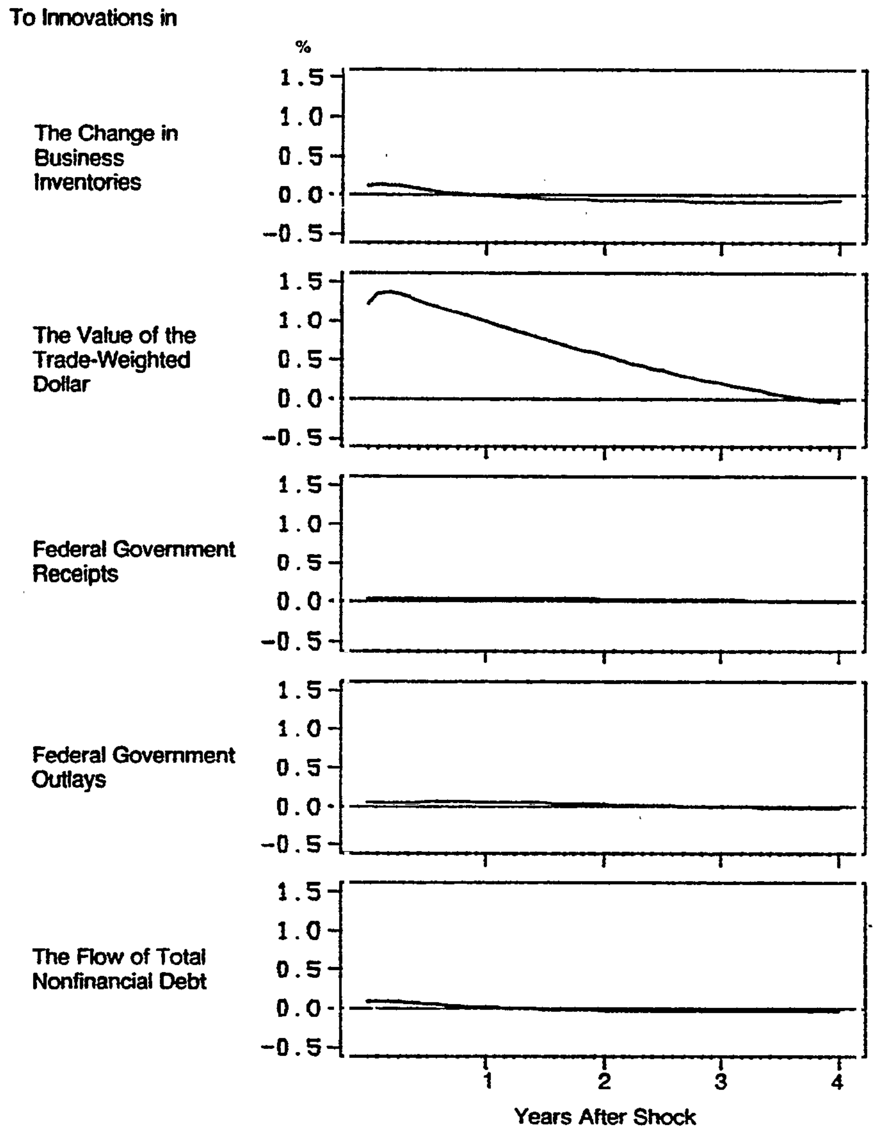

Chart 42. Responses of Federal Government Receipts

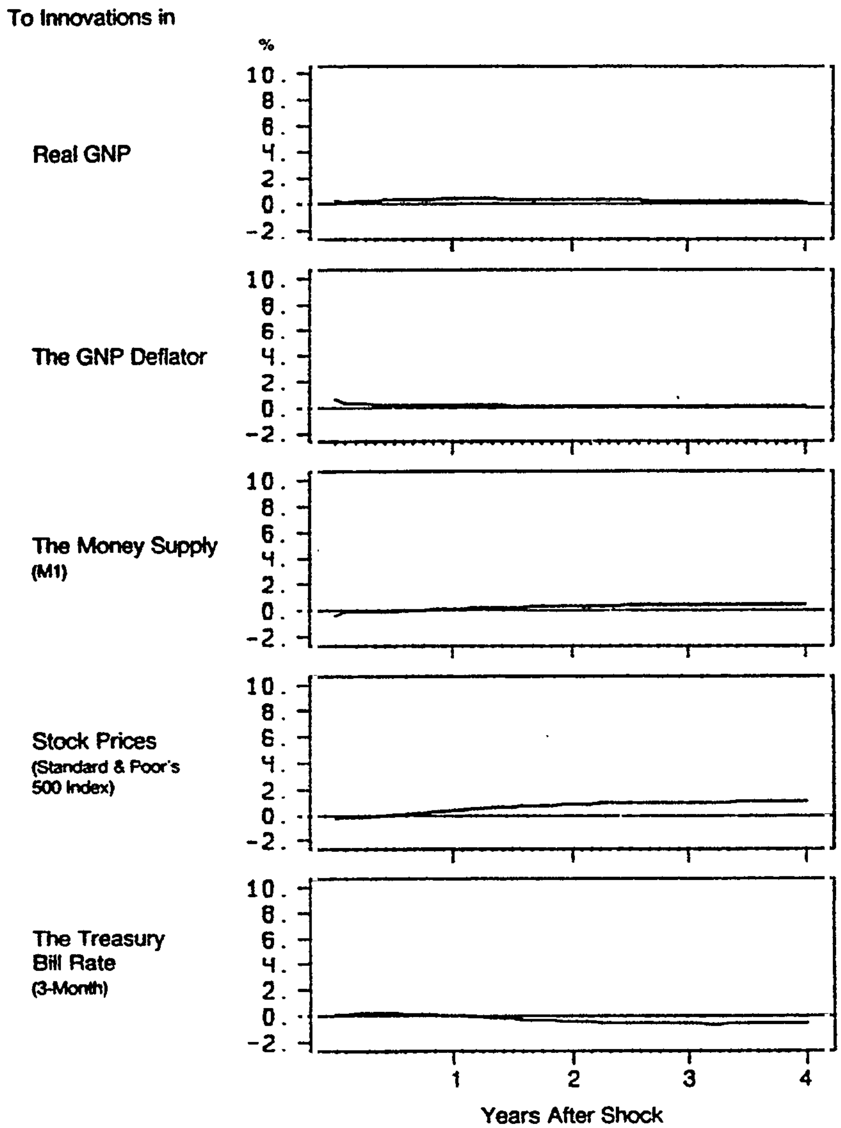

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Chart 42. Responses of Federal Government Receipts

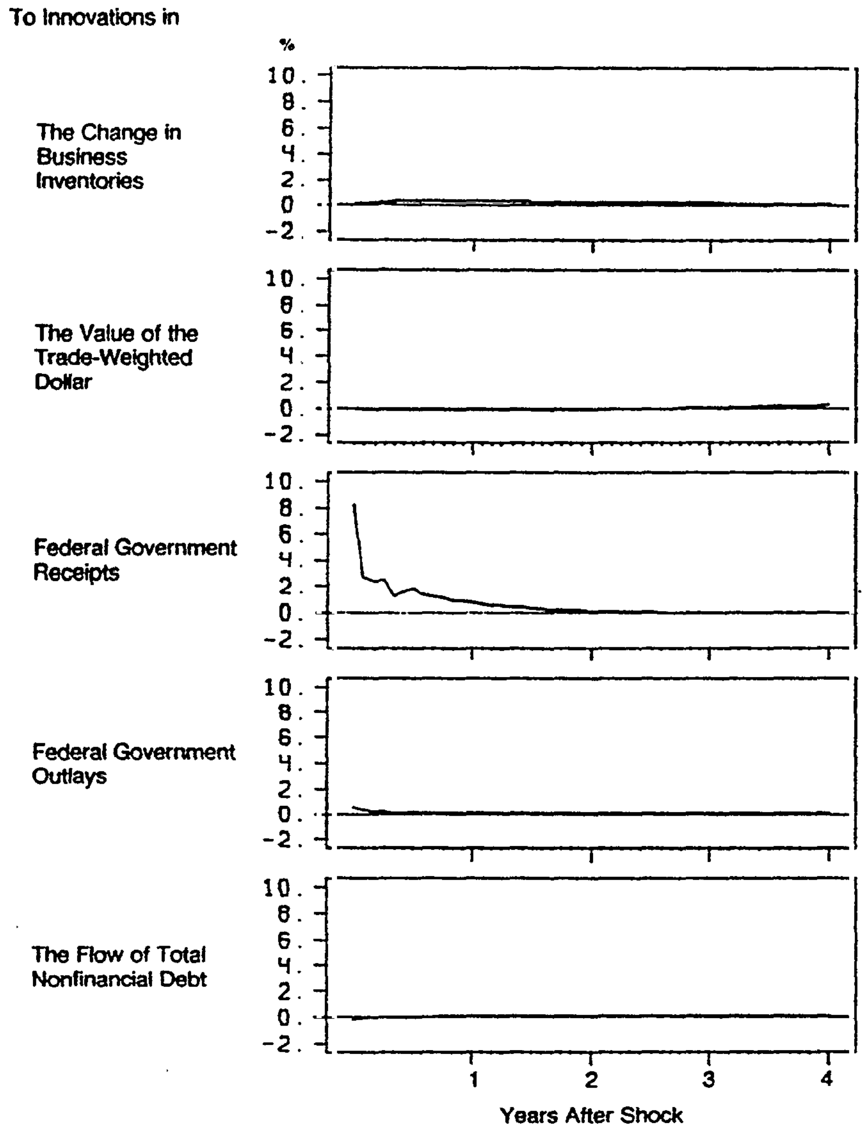

Chart 43. Responses of Federal Government Outlays

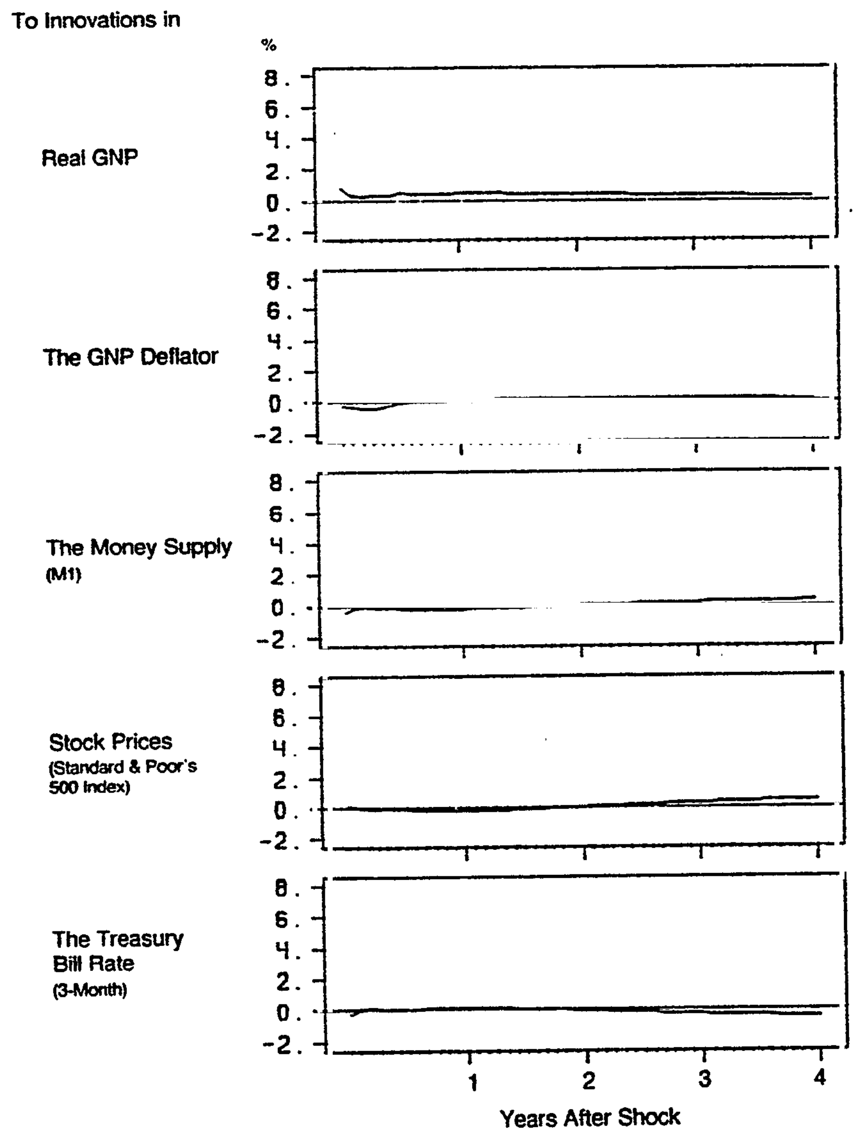

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Chart 43. Responses of Federal Government Outlays

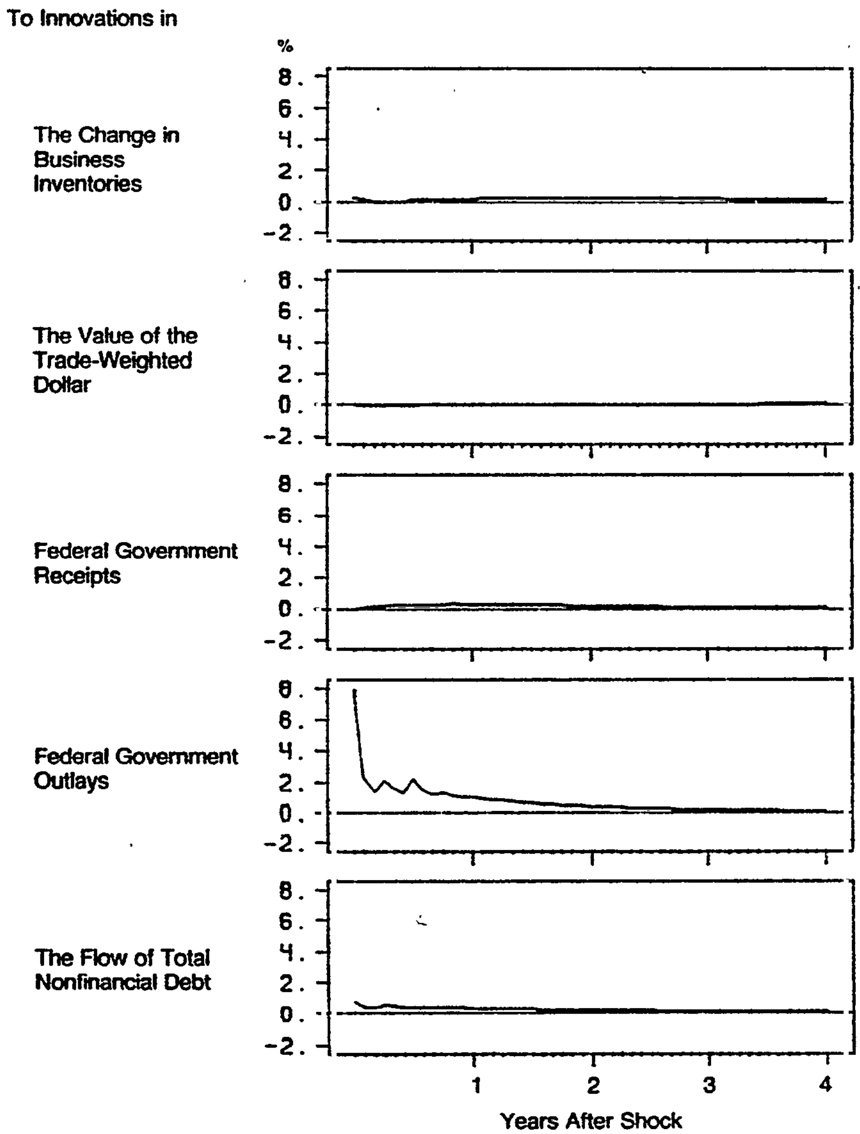

Chart 44. Responses of the Federal Government Deficit

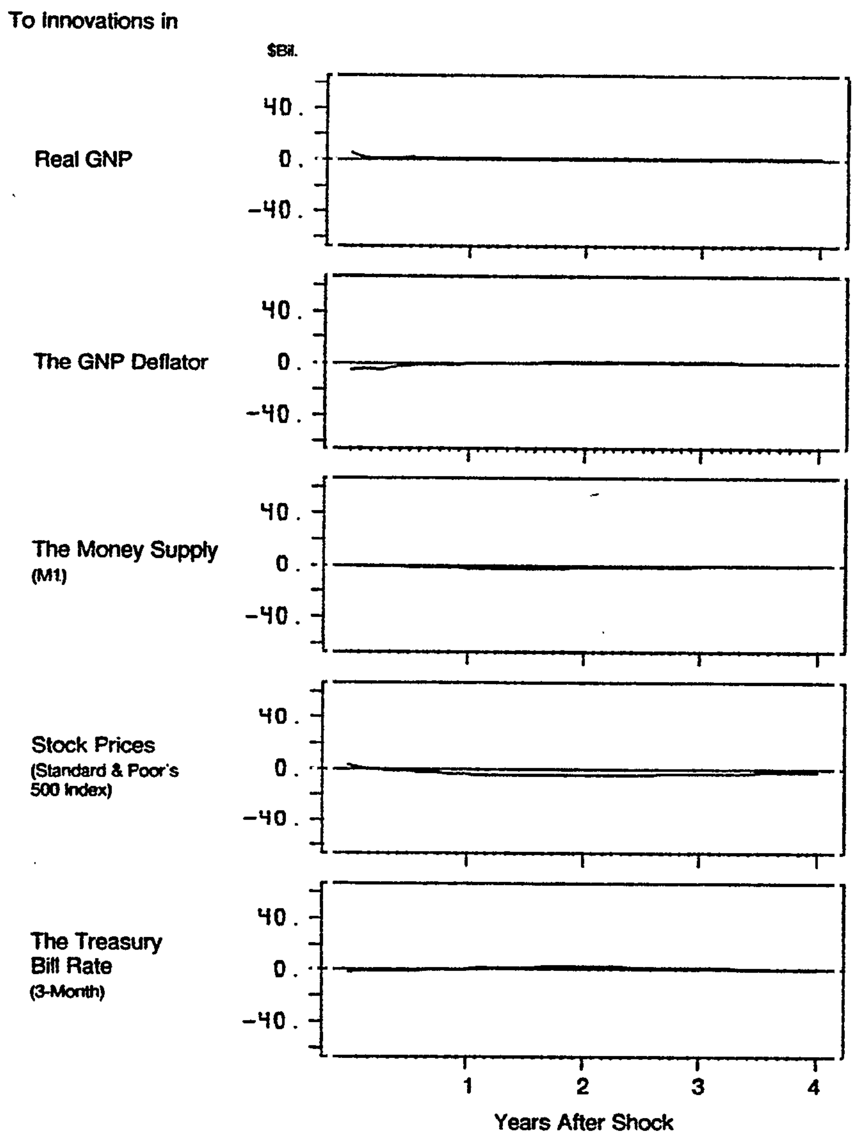

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Chart 44. Responses of the Federal Government Deficit

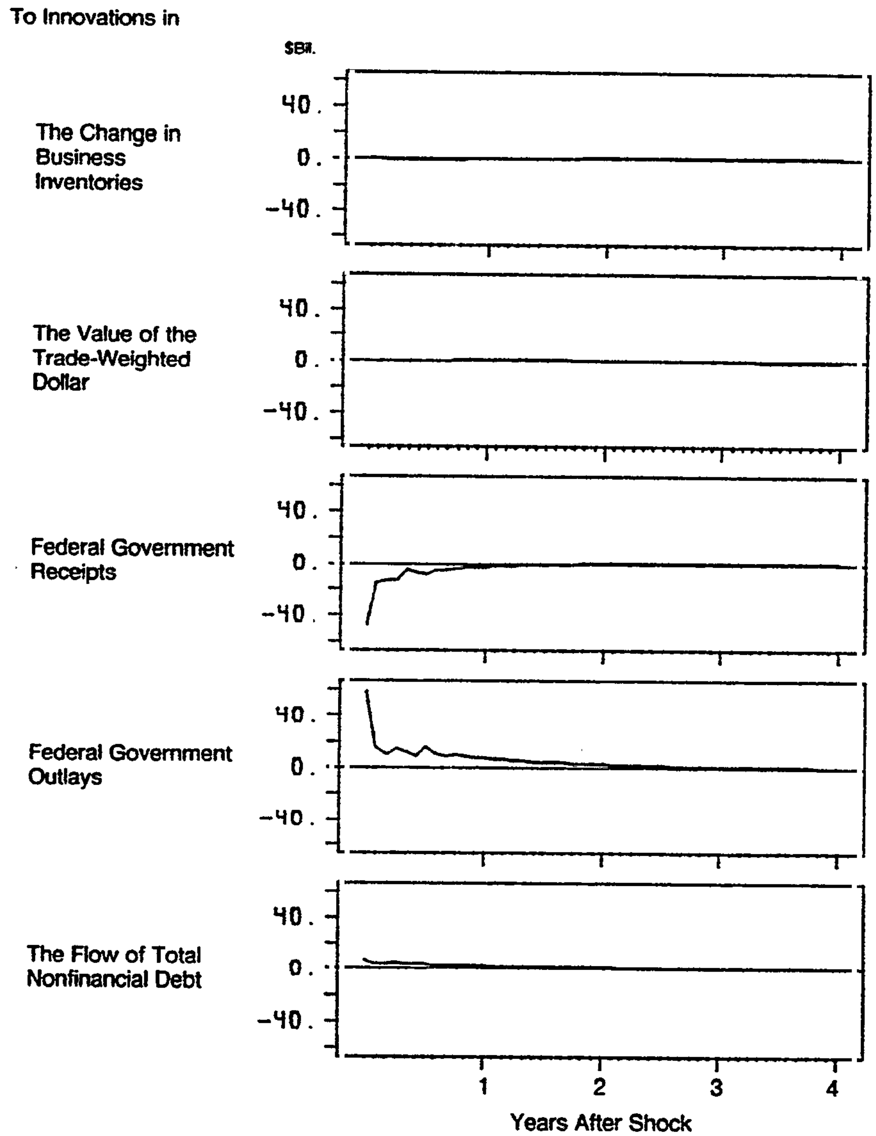

Chart 45. Responses of the Flow of Total Nonfinancial Debt

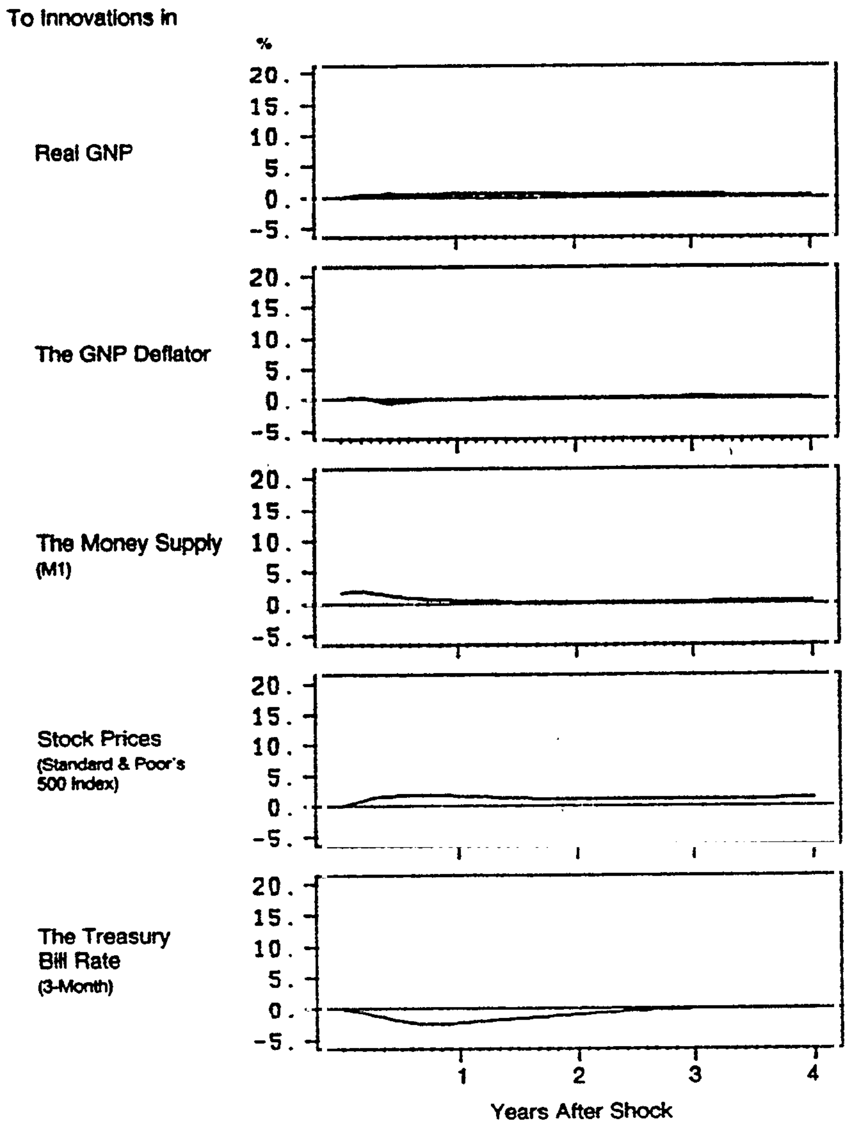

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Chart 45. Responses of the Flow of Total Nonfinancial Debt

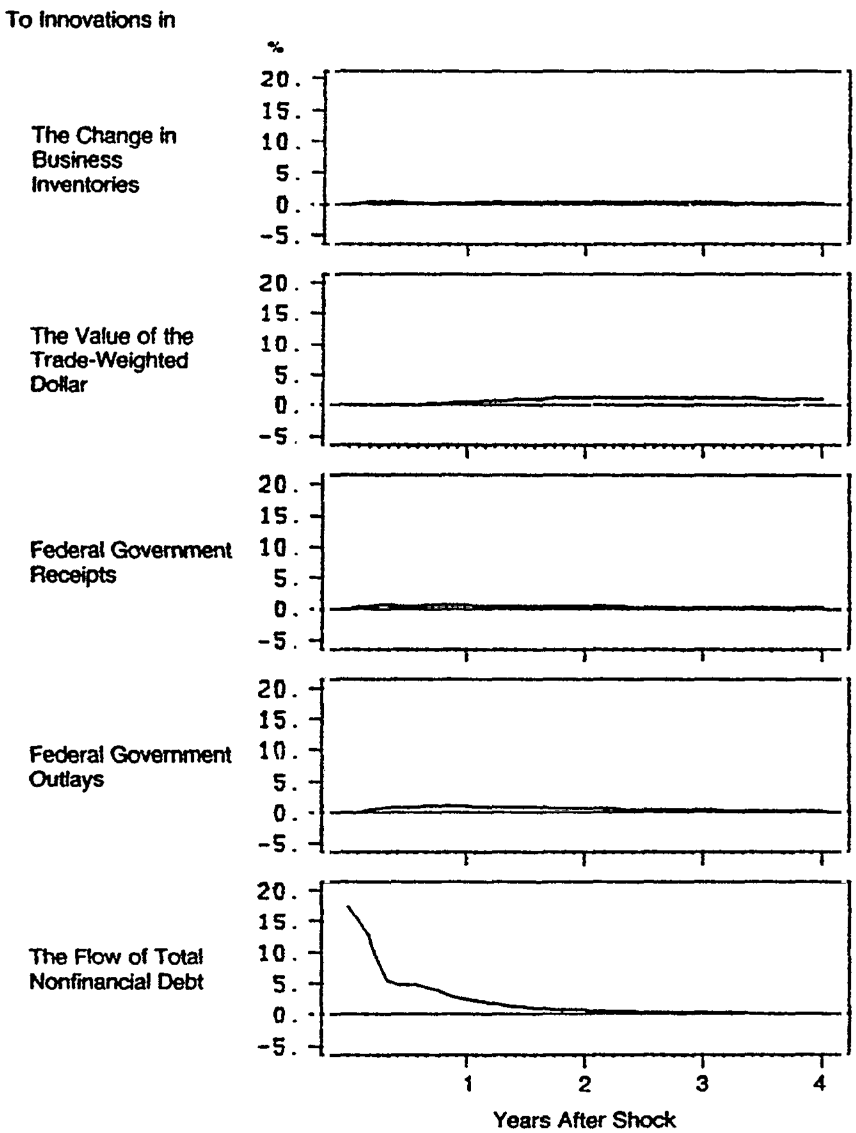
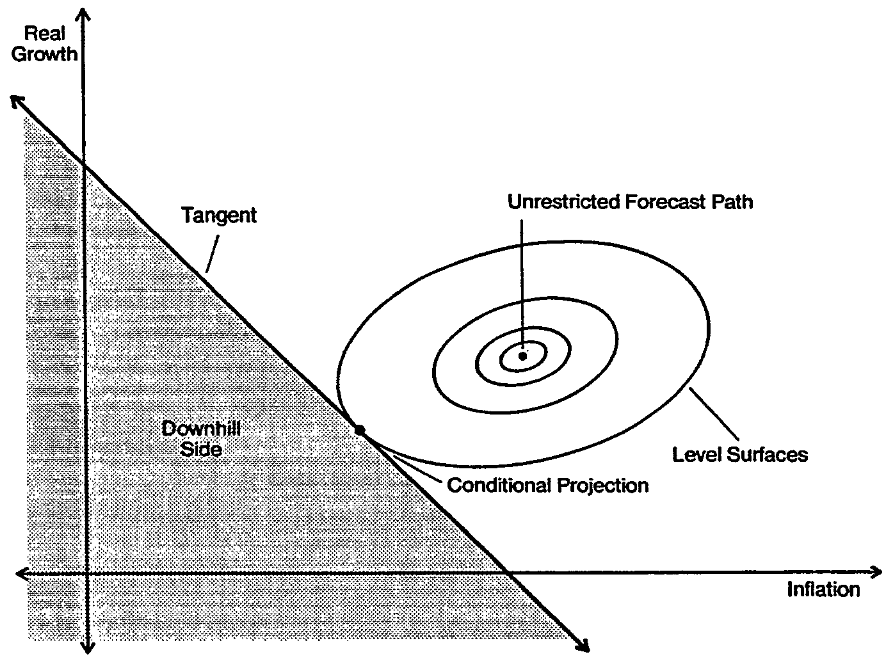


Chart 46

Construction of an Implausibility Index

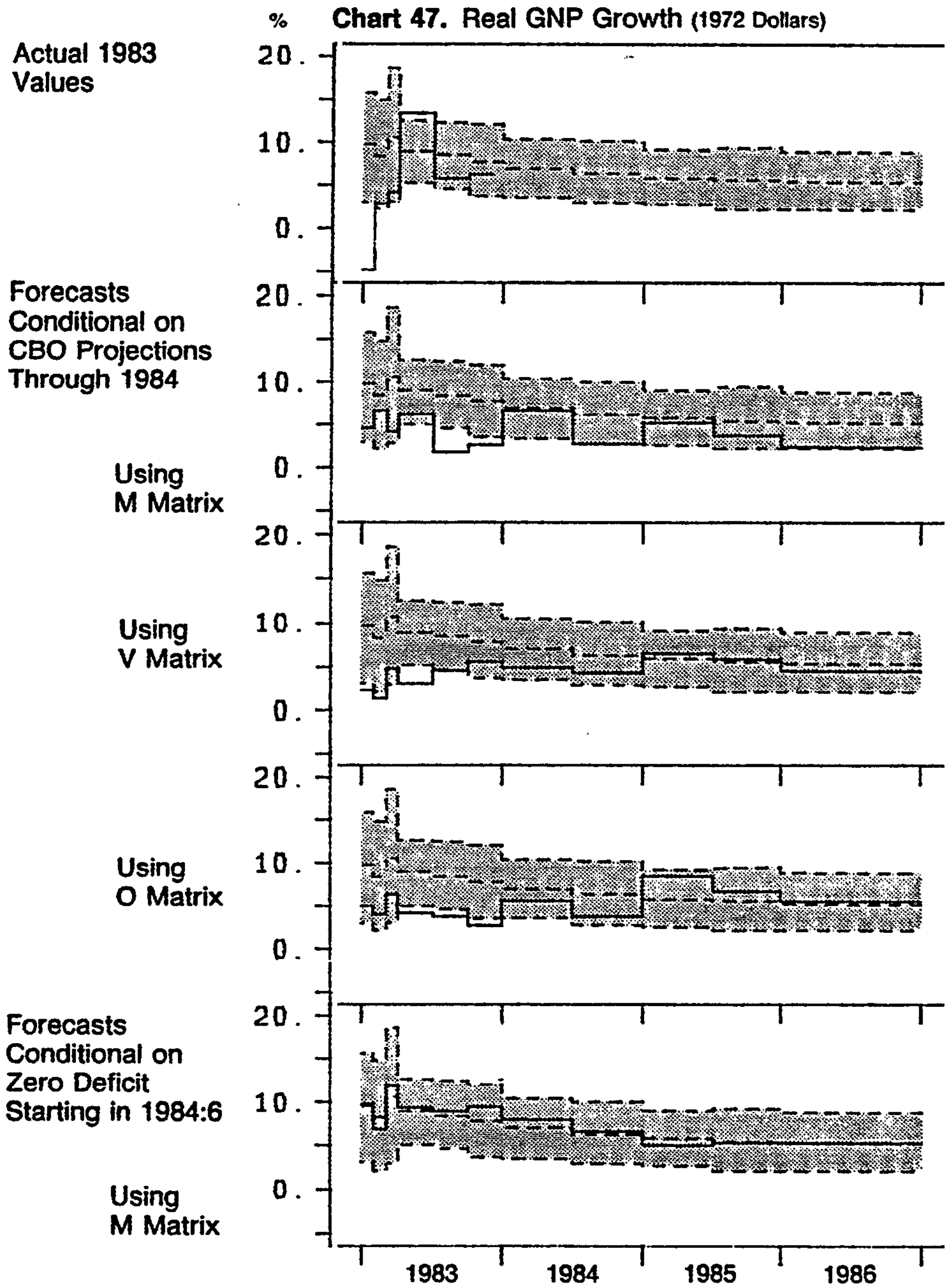
The *implausibility index* is a measure of the probability the model gives to outcomes on the downhill side of a tangent to the forecast's level curve at the point of a conditional projection.



Charts 47–57

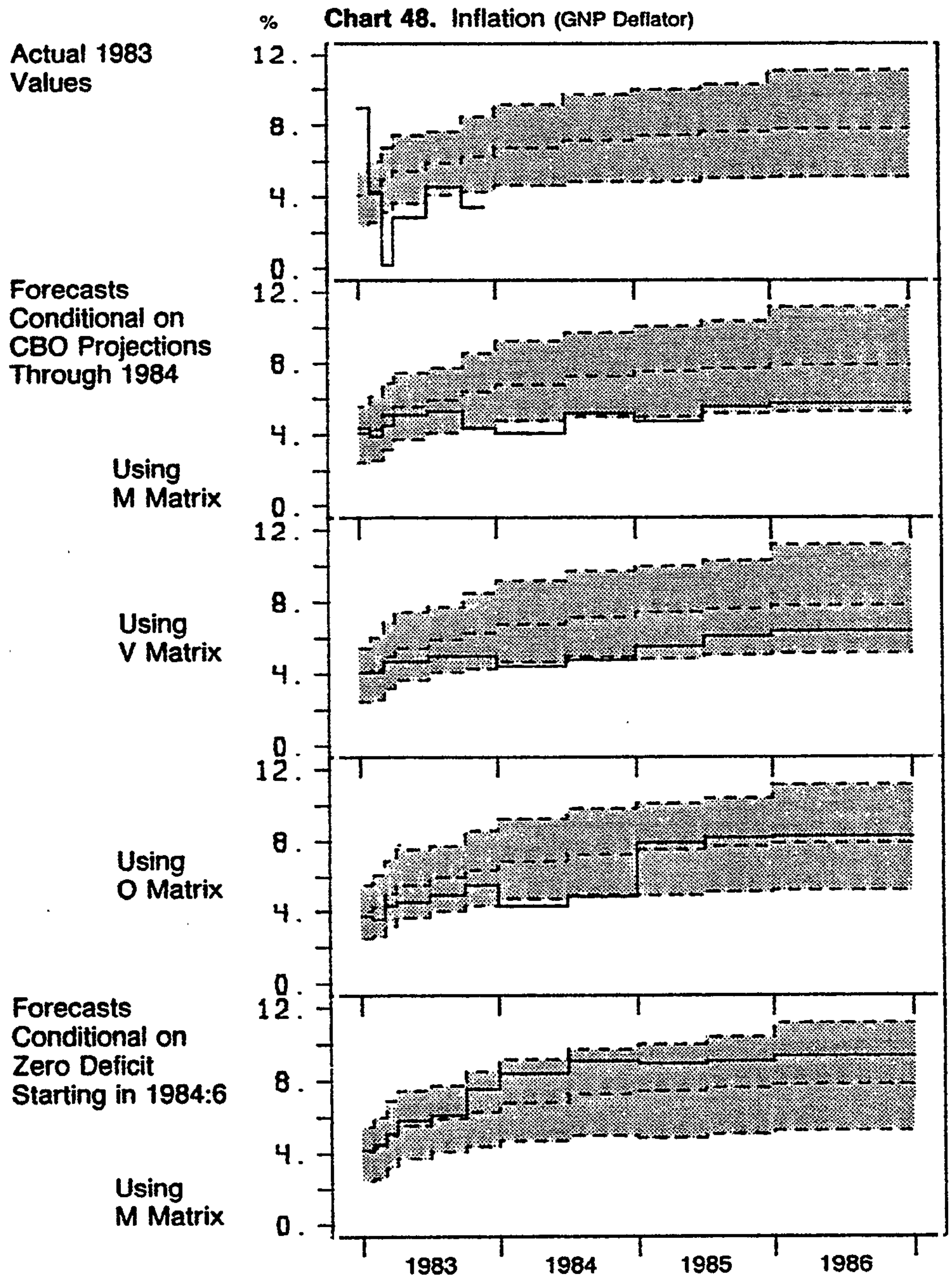
Forecasts and Conditional Projections

50% Probability Bands for Unconditional Forecasts Made in 1982:12



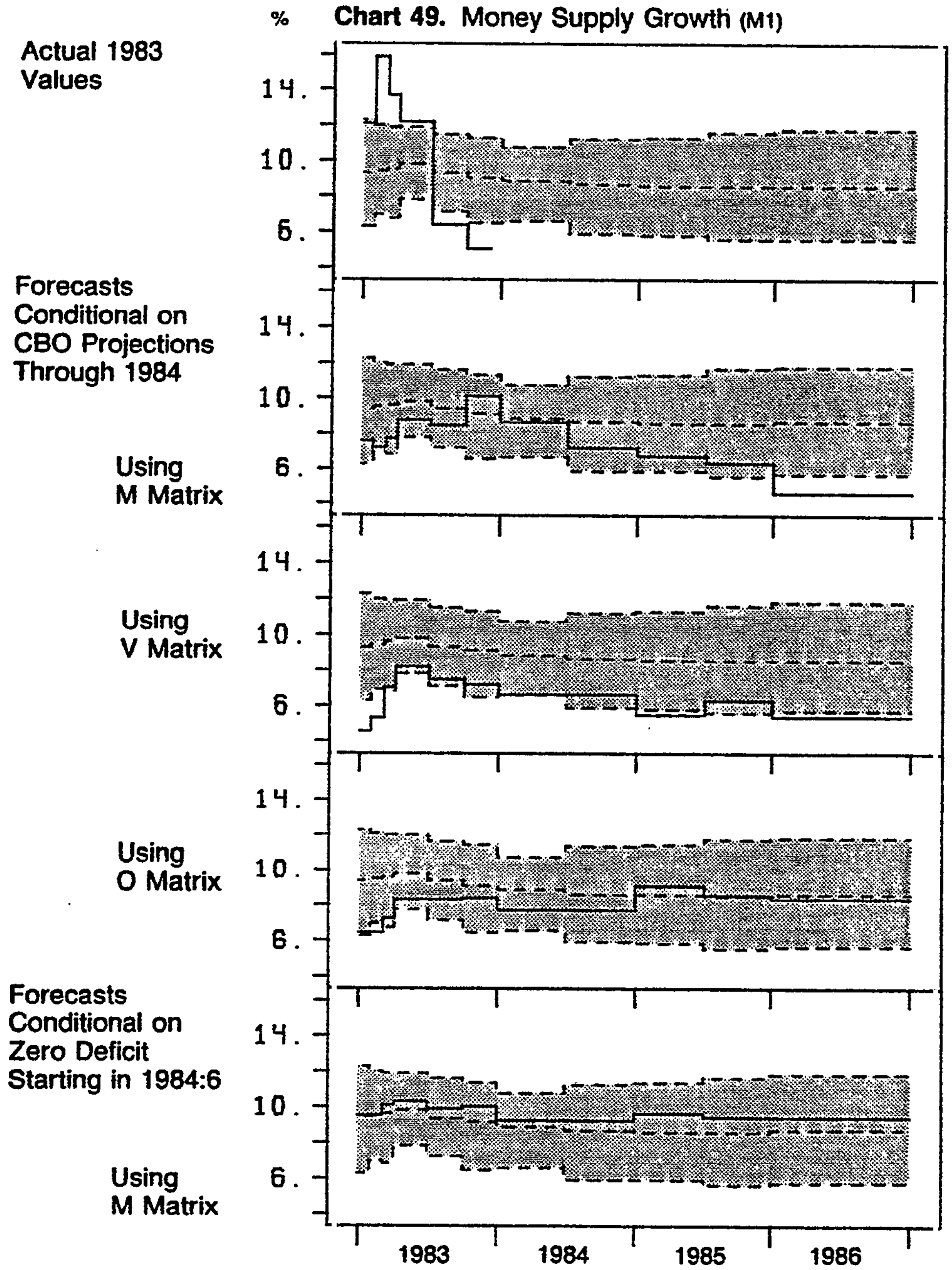
50% Probability Bands for Unconditional Forecasts Made in 1982:12

Chart 48. Inflation (GNP Deflator)



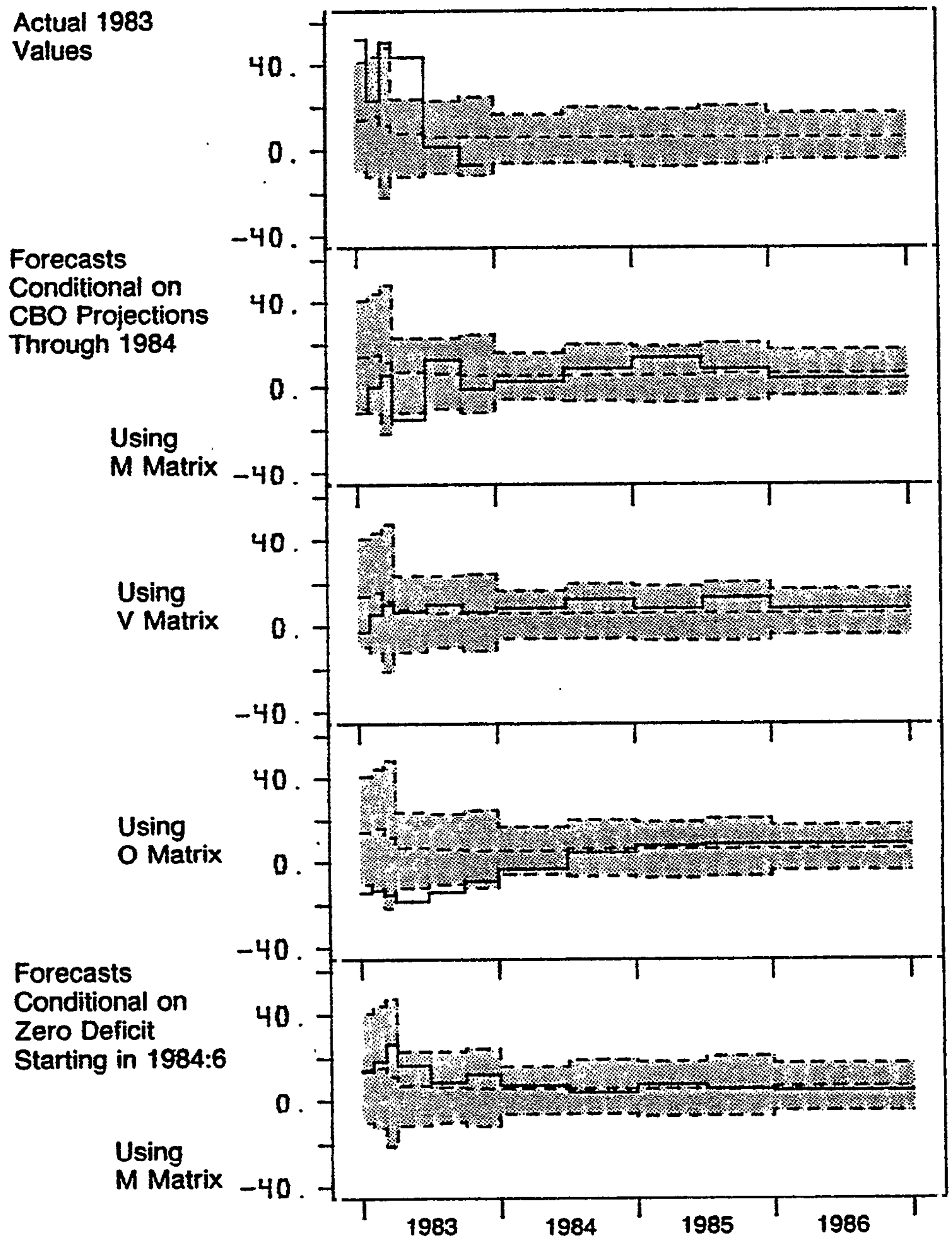
50% Probability Bands for Unconditional Forecasts Made in 1982:12

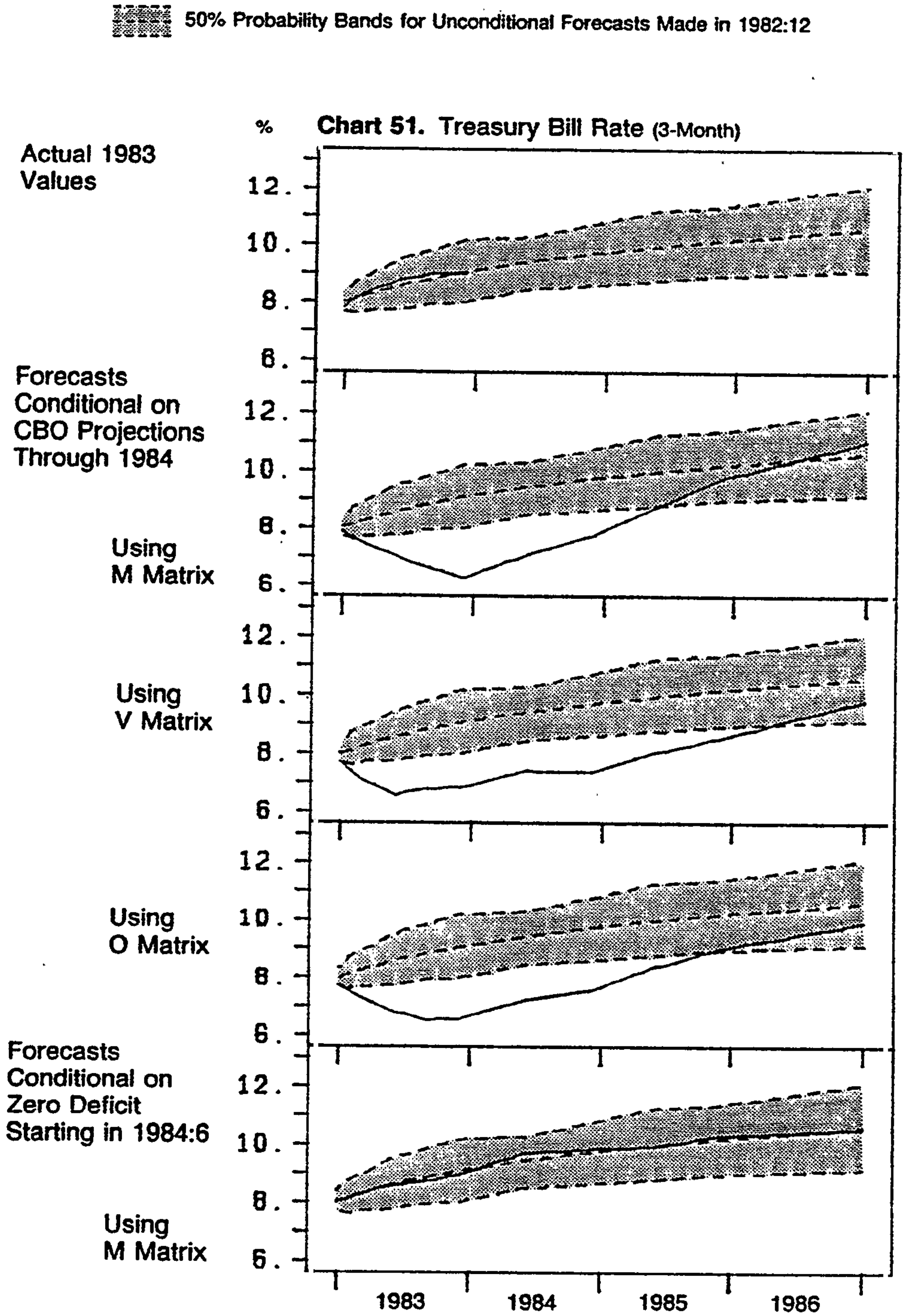
Chart 49. Money Supply Growth (M1)



50% Probability Bands for Unconditional Forecasts Made in 1982:12

Chart 50. Stock Price Growth
(Standard & Poor's 500 Index)




50% Probability Bands for Unconditional Forecasts Made in 1982:12
%
Chart 51. Treasury Bill Rate (3-Month)
Actual 1983 Values
Forecasts Conditional on CBO Projections Through 1984
Using M Matrix
Using V Matrix
Using O Matrix
Forecasts Conditional on Zero Deficit Starting in 1984:6
Using M Matrix
12.
10.
8.
6.
1983
1984
1985
1986

50% Probability Bands for Unconditional Forecasts Made in 1982:12

Chart 52. Change in Business Inventories
(1972 Dollars)

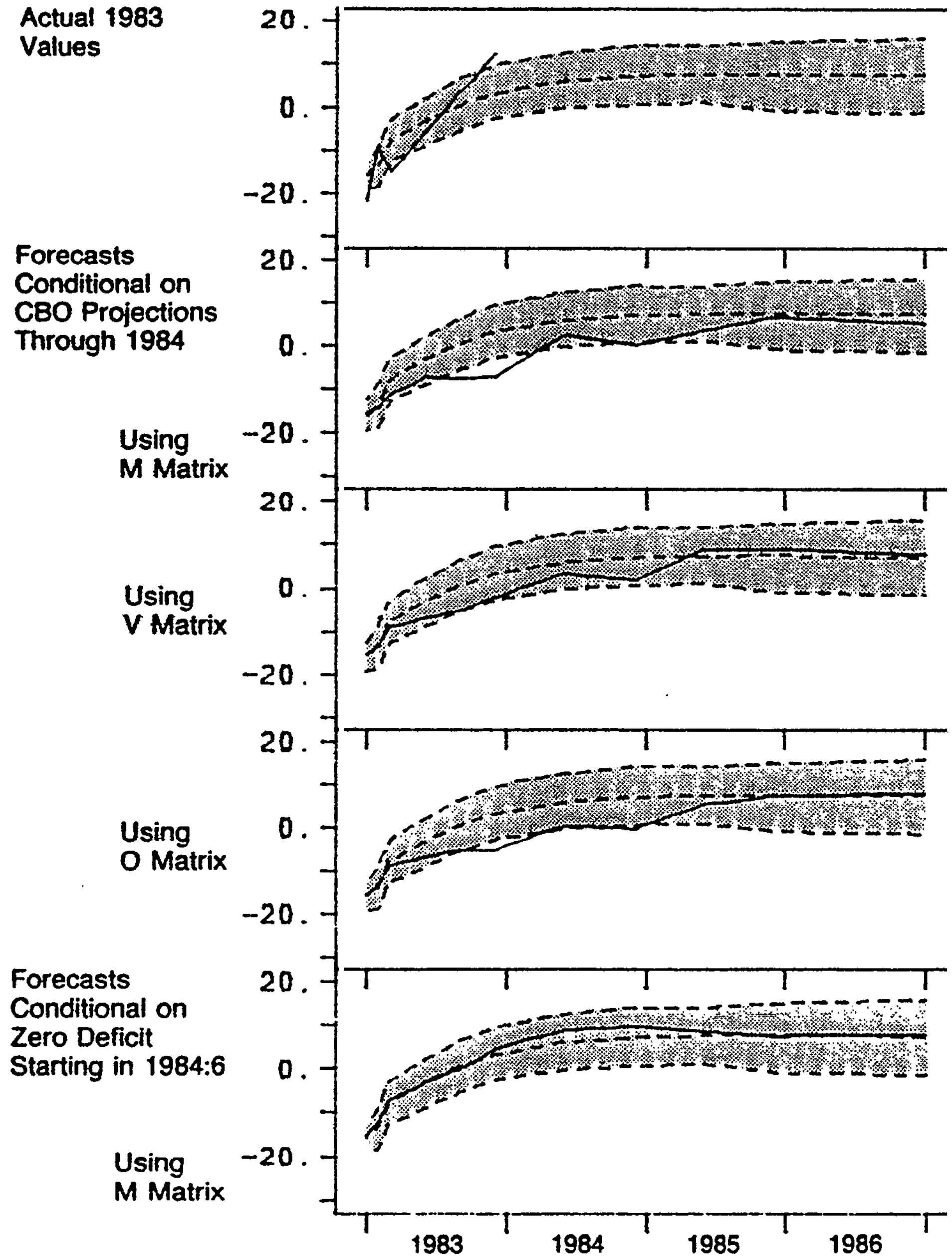
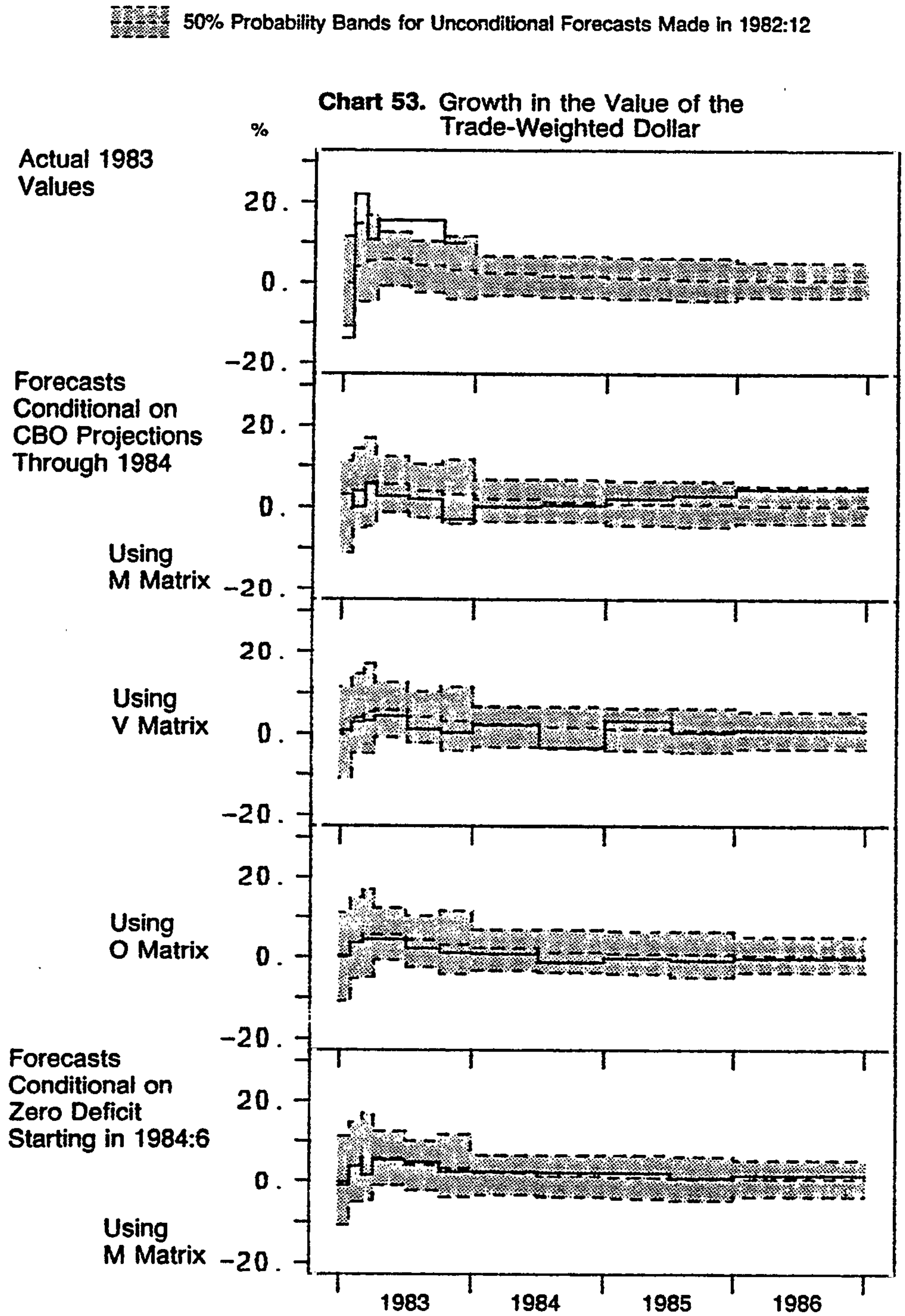
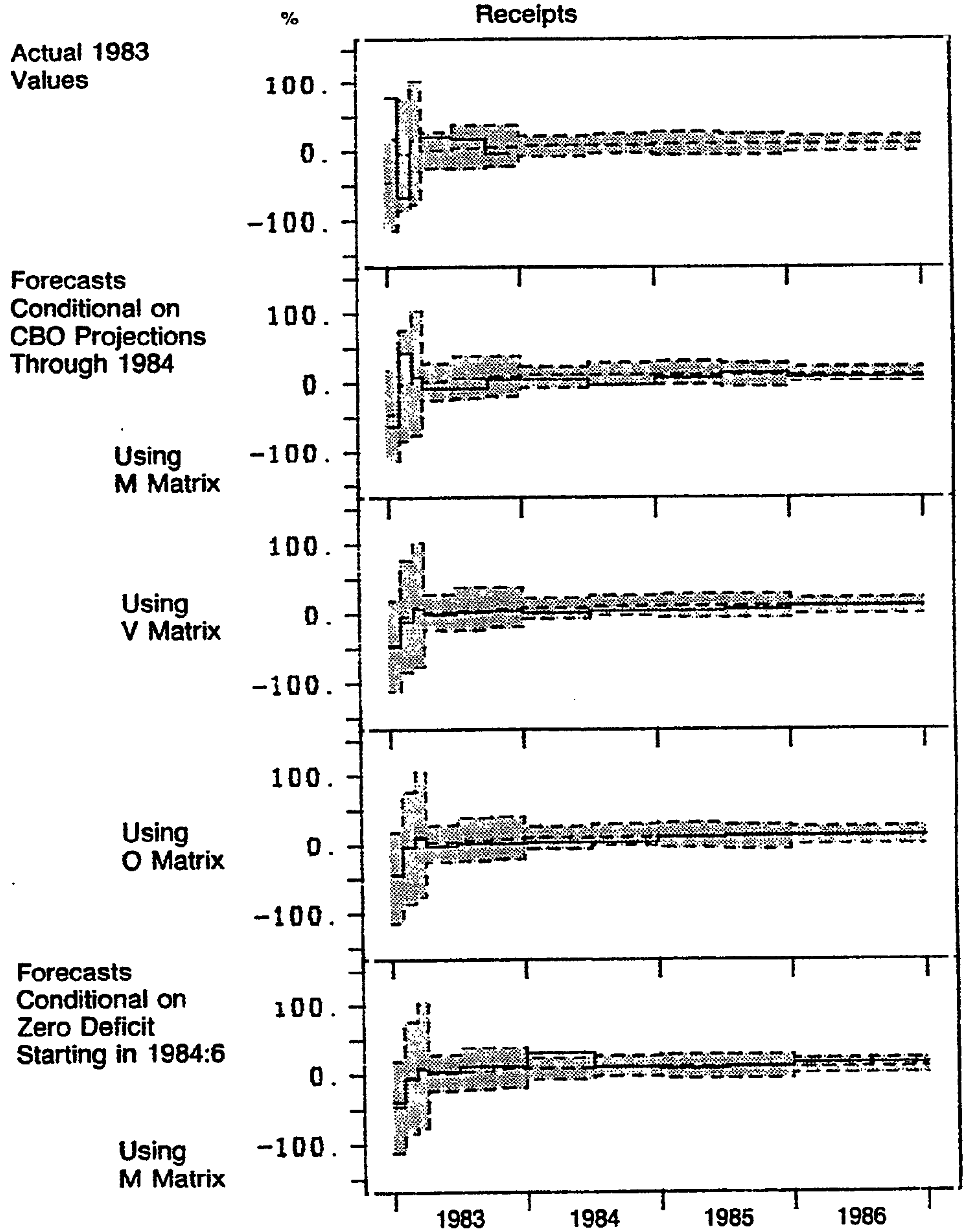


Chart 53. Growth in the Value of the Trade-Weighted Dollar



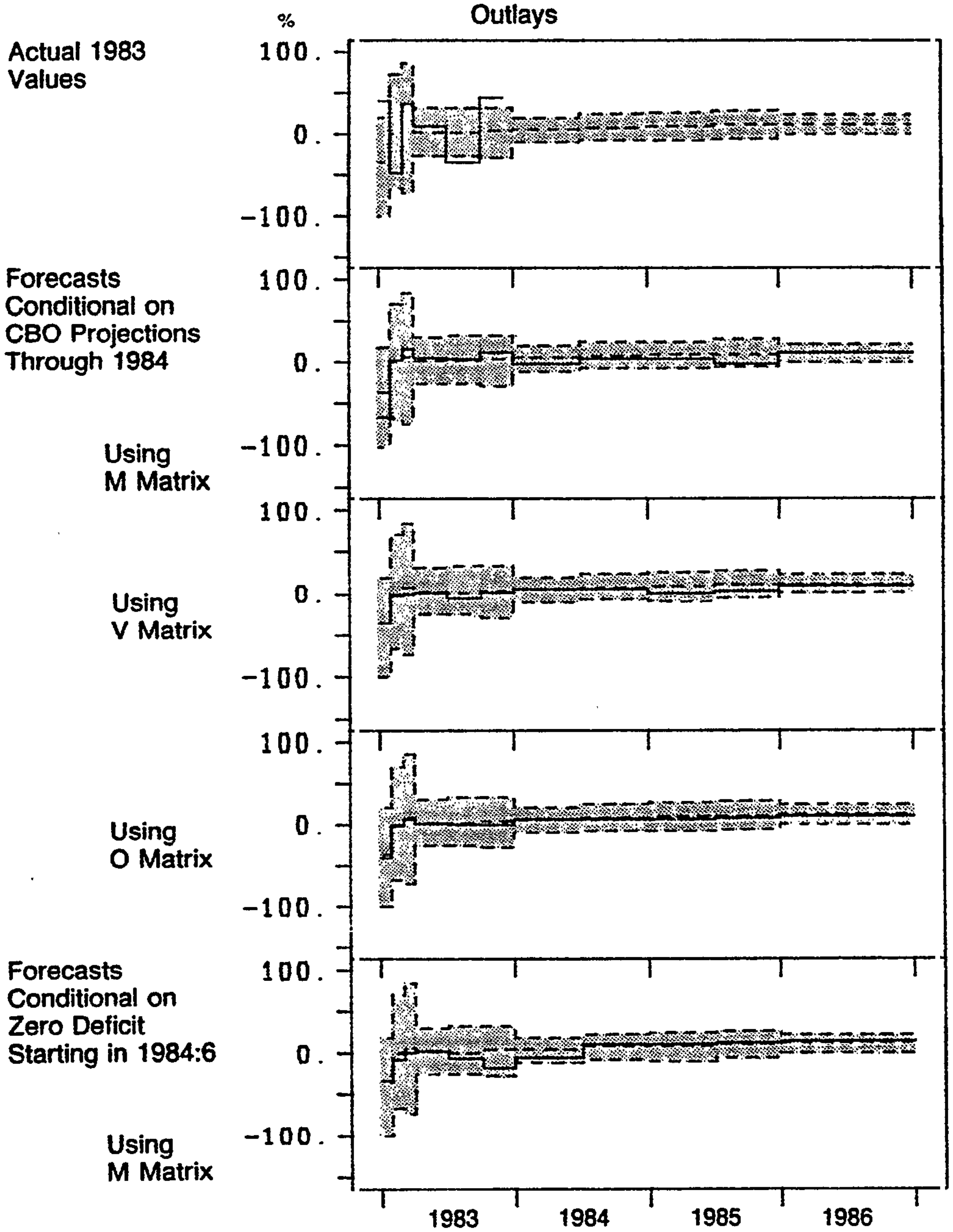
50% Probability Bands for Unconditional Forecasts Made in 1982:12

Chart 54. Growth in Federal Government Receipts



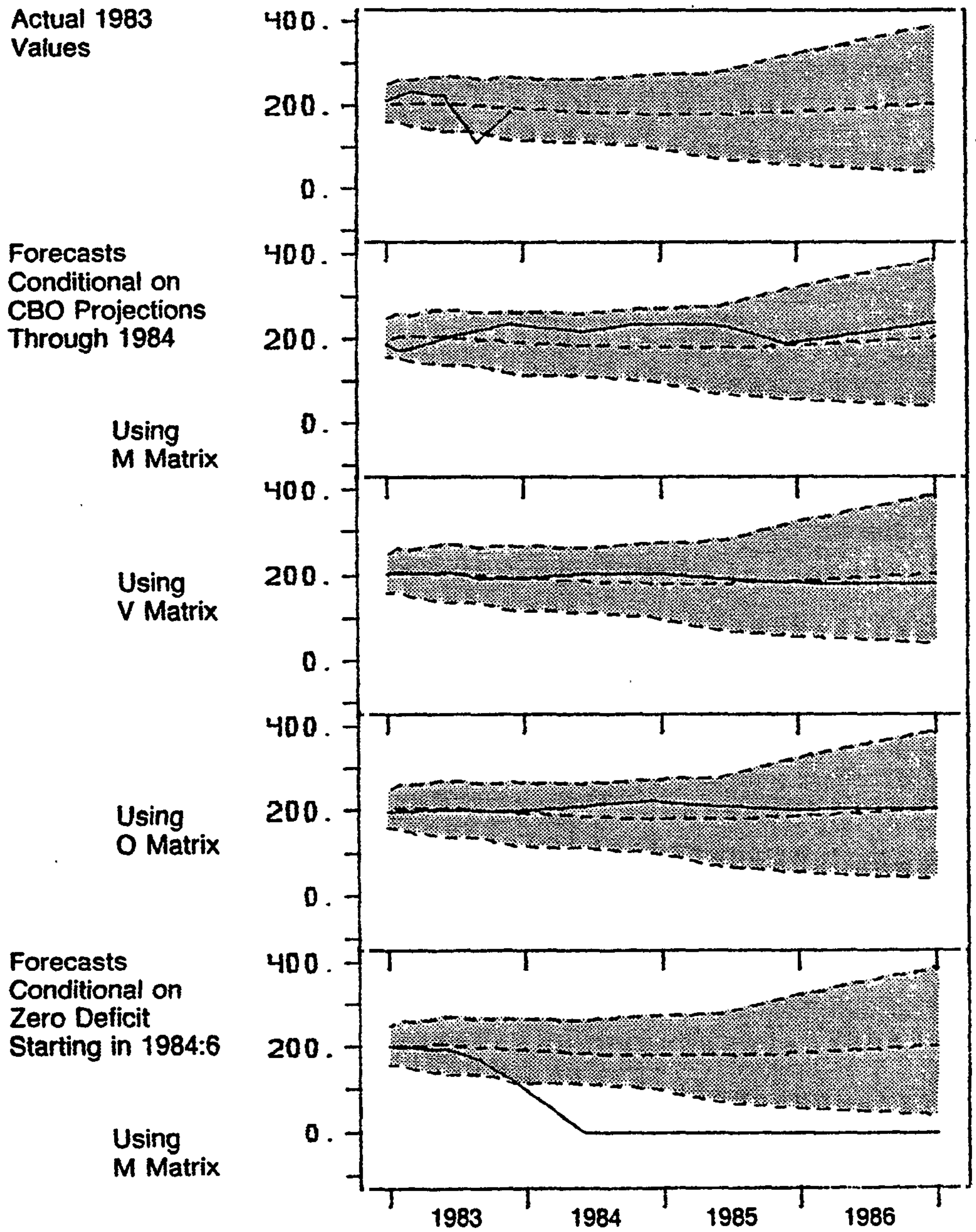
50% Probability Bands for Unconditional Forecasts Made in 1982:12

Chart 55. Growth in Federal Government Outlays



50% Probability Bands for Unconditional Forecasts Made in 1982:12

Chart 56. Federal Government Deficit



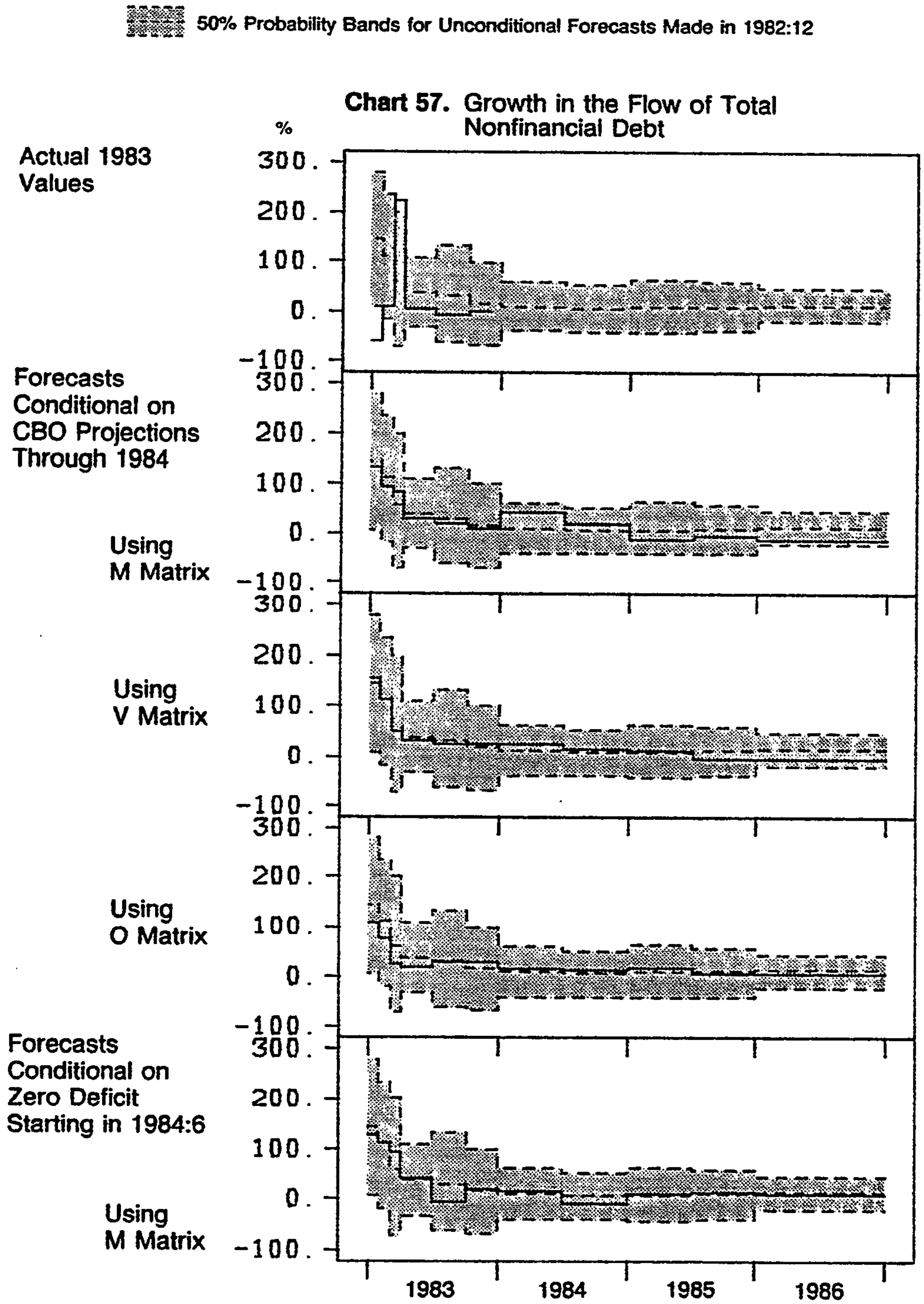


Chart 57. Growth in the Flow of Total Nonfinancial Debt