OUTPUT VARIABILITY IN AN OPEN-ECONOMY MACRO MODEL
WITH VARIANCE-DEPENDENT PARAMETERS

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Abstract_____

This paper analyzes the variability of output under money supply and exchange
rate rules in an open economy in which the slope of the aggregate supply curve
depends on the variances of aggregate demand and market-specific innovations.
It demonstrates that results regarding the dominance of one rule over the
other when the slope of the aggregate supply curve is constant are reversed
when the slope of the aggregate supply curve depends on the variances of
innovations and these variances are sufficiently large.

One of the innovations in the seminal paper by Robert Lucas (1973)
is that the parameters of the aggregate supply curve depend on the variances
of the stochastic disturbances in the economy. Specifically, in his model,
the slope of the aggregate supply curve is an increasing function of the ratio
of the variance of market-specific disturbances to the variance of aggregate
demand disturbances. The purpose of this paper is to analyze the variability
of output under money supply and exchange rate rules in an open economy which
has an aggregate supply curve with Lucas' innovation.

Recently, there have been two different approaches to incorporating
in open-economy macro models the same type of dependence of the aggregate
supply curve on variances of stochastic disturbances. One approach, presented
in the papers by Robert Flood and Nancy Marion (1982) and Gary Fethke (un-
dated), adapts the optimal wage indexation models of Jo Anna Gray (1976) and
Stanley Fischer (1977) to an open economy. In these models, the nominal wage
is indexed to the price level, with the degree of indexation increasing with
the ratio of the variance of supply disturbances to the variance of demand
disturbances. Further, since the variance of demand disturbances depends on
whether the economy is operating under a money supply rule (flexible exchange
rates) or an exchange rate rule (fixed or managed exchange rates), the slope
of the aggregate supply curve, in general, differs under these different
policy regimes.

The other approach, which is presented in the papers by Michael
Parkin, Brian Bentley, and Christina Fader (1981) and Parkin (undated), re-
tains Lucas' specification in which the suppliers of output exist in a larger
number of separated, competitive markets with aggregate demand in each period
being unevenly distributed over these markets. This approach generalizes the
Lucas specification to an open economy by having supply depend on the price in

the isolated market relative to a price index which includes both the domestic and the foreign price level. This approach obtains the Lucas result that the slope of the aggregate supply curve is an increasing function of the ratio of the variance of the market-specific disturbances to the variance of aggregate demand disturbances. It also finds that the way in which the variances of the stochastic disturbances affect the slope of the aggregate supply curve depends on whether the economy is operating under fixed or flexible exchange rates.

This paper adopts the second approach to obtaining an open-economy macro model in which the parameters of the aggregate supply curve depend on the variances of stochastic shocks to the economy. However, it differs from that approach in three important respects. First, it allows domestic and foreign goods to be imperfect substitutes instead of adopting the stochastic purchasing power parity specification used in the papers by Parkin, Bentley, and Fader and by Parkin. Second, it uses an IS-LM approach to modeling the aggregate demand side of the economy instead of the quantity theory approach used both in those two papers and in Lucas'. Using IS-LM lets me distinguish between money demand shocks and real demand shocks throughout the analysis. Third, this paper adopts general formulations for money supply and exchange rate rules instead of random walk money supply rules or fixed exchange rates.[1]

The paper proceeds as follows. First I present the model and its reduced-form solutions for real output under the alternative policy rules. Then I compare the variability of output under the two alternative policy rules and describe some implications. A summary concludes the paper.

The Model

The basic model underlying the analysis is a generalization of the model used in my 1981 paper. Specifically, I model a country which is small enough in the world capital market that it cannot affect the interest rate it faces. Thus, capital mobility insures the equality of foreign and domestic interest rates after adjustment for expected exchange rate appreciation or depreciation. Also, in this model, domestic goods are imperfect substitutes for foreign goods, so the domestic price level can differ from the foreign price level by more than the current exchange rate. Further, the country is small enough that its demands for imports do not affect the price of imports, and the price of foreign goods is arbitrarily set equal to unity. The generalization of my earlier model is that here aggregate supply is modeled using Lucas' separated, competitive suppliers.

The following discussion of the model will use this notation:

z = an index of markets and also the stochastic disturbance in market z.

$y_t(z)$ = the log of domestic output in market z at time t.

$P_t(z)$ = the log of the price of domestic output in market z at time t.

y_t = $\int y_t(z)\, dz$ = aggregate domestic output at time t.

P_t = $\int P_t(z)\, dz$ = the domestic price level at time t.

m_t = the domestic nominal money supply at time t.

i_t = one plus the domestic nominal interest rate at time t.

e_t = the exchange rate (the domestic currency price of foreign currency) at time t.

I_{t-1} = the information available to domestic agents at time $t - 1$.

$I_t(z)$ = $\{P_t(z), I_{t-1}\}$ = the information available to domestic agents in market z at time t.

v_t, n_t = white noise stochastic disturbance terms at time t assumed to be distributed independently of each other and independently of z.

An asterisk (*) will denote a foreign variable.

The operator σ_x^2 will denote the variance of the random variable x. The operator E_{t-1} will denote the rationally formed expectation based on information at time t − 1, and the operator E_t will denote the rationally formed expectation based on information at time t available in market z. For example, $E_{t-1}P_t = E[P_t | I_{t-1}]$ and $E_t P_t = E[P_t | I_t(z)]$. All rational expectations will be calculated as linear projections.

From these definitions follow several others:

$\varepsilon_t = m_t - E_{t-1}m_t =$ the innovation in the domestic nominal money supply at time t.

$\psi_t = i_t^* - E_{t-1}i_t^* =$ the innovation in the foreign interest rate at time t.

$\omega_t = e_t - E_{t-1}e_t =$ the innovation in the exchange rate at time t.

Aggregate Supply

Following Lucas, I assume that suppliers in this economy are located in a large number of separated, competitive markets. Aggregate demand in each period is assumed to be unevenly distributed among these markets, giving rise to the possibility that the price of output varies among markets. The price faced by suppliers in any period can change due to changes in the distribution of demand across markets and due to changes in the level of aggregate demand. The supply in market z is assumed to be an increasing function of the perceived relative price of output in that market:[2]

$$y_t(z) = a_1 [P_t(z) - E_t P_t] \tag{1}$$

for all $a_1 > 0$. Equation (1) will be called the _market supply curve_.[3]

Also following Lucas, I assume that the price in market z is determined according to

$$P_t(z) = P_t + z. \tag{2}$$

The stochastic disturbance z is assumed to be a mean zero, white noise variable with variance σ_z^2. The justification for (2) is that z reflects the random distribution of aggregate demand in market z. Aggregate supply is obtained by substituting (2) into (1) and integrating over z:

$$y_t = a_1 [P_t - \int E_t P_t \, dz]. \tag{3}$$

Using linear projection theory, $\int E_t P_t \, dz$ can be expressed in terms of expectations about the price level in period t. By definition,

$$E_t P_t = E[P_t | I_t(z)] = E[P_t | I_{t-1}, P_t(z)].$$

Using recursive projections,

$$E_t P_t = E[P_t | I_{t-1}] + E[(P_t - E[P_t | I_{t-1}]) | (P_t(z) - E[P_t(z) | I_{t-1}])].$$

From the definition of a linear projection,

$$E[(P_t - E[P_t | I_{t-1}]) | (P_t(z) - E[P_t(z) | I_{t-1}])]$$
$$= \theta(P_t(z) - E[P_t(z) | I_{t-1}]) \tag{4}$$

where θ is a parameter described below, so that

$$E_t P_t = E_{t-1} P_t + \theta(P_t(z) - E[P_t(z) | I_{t-1}]).$$

Consequently,

$$\int E_t P_t \, dz = E_{t-1} P_t + \theta(P_t - E_{t-1} P_t). \tag{5}$$

Substituting (5) in (3) yields the aggregate supply curve

$$y_t = a_1(1 - \theta)(P_t - E_{t-1} P_t). \tag{6}$$

Since (4) is a linear projection, the parameter θ will depend on the variances of the stochastic disturbances in the model. (The exact specification of θ will be discussed later.) Thus, (6) is an aggregate supply curve with parameters that depend on the variances of the stochastic disturbances in the economy.

Aggregate Demand

The aggregate demand side of the economy is identical to that in my 1981 paper, with one exception noted below. The demand side consists of three equations:

IS Curve

$$y_t = b_1[i_t - E_{t-1}(P_{t+1} - P_t)] + b_2(e_t - P_t) + v_t \tag{7}$$

IM Curve

$$m_t = c_0 P_t + c_1 y_t + c_2 i_t + c_3 e_t + n_t \tag{8}$$

Interest Rate Parity Condition

$$i_t = i_t^* + E_{t-1}(e_{t+1} - e_t). \tag{9}$$

The IS curve (7) is that of conventional theory with the same two modifications made in my earlier paper. They are the use of the real rather than the nominal interest rate and the presence of the relative price of foreign to domestic goods. (The logarithm of the foreign price level is omitted since it is zero by assumption.) Since higher real interest rates should depress the demand for domestic output and since an increase in the price of foreign goods relative to domestic goods should increase the demand for domestic goods, I expect that $b_1 \leqslant 0$ and $b_2 \geqslant 0$. The real demand disturbance v_t can be thought of as arising due to innovations in either investment demand or the foreign price level.

The IM curve (8) is the conventional IM curve with the exception that the exchange rate is included in the determination of real balances. That is because individuals in the economy can purchase both domestic and foreign goods and so may deflate their holdings of nominal money by a combination of both domestic and foreign prices. I expect that c_0, $c_3 > 0$ and $c_0 + c_3 = 1$. Further, the conventional demand-for-money view is that $c_1 > 0$ and $c_2 < 0$. The random variable η_t allows for innovations to money demand.

Equation (9) is the interest rate parity condition. It results from the assumptions that the country is relatively small in world capital markets and that foreign and domestic assets are perfect substitutes for investors. Those assumptions imply that capital mobility will force equality of expected nominal returns on domestic and foreign bonds expressed in units of the same currency. The difference between the specification above and that in my previous paper is that here i_t^* is allowed to be a white noise variate with mean i* and variance σ_i^2. [4]

Solutions

Now I solve the model given by equations (6) through (9), first under a money supply rule and then under an exchange rate rule.

Money Supply Rule

In an economy in which the monetary authority follows a rule for the determination of the money supply, the exchange rate is an endogenous variable; that is, the country operates under flexible exchange rates. When e_t is endogenous, solving equations (6) through (8) for the price level yields[5]

$$P_t = \beta^{-1}\{(c_3 + b_2 c_1)[a_1(1 - \theta)E_{t-1}P_t - a_2 y_{t-1}]$$

$$+ c_3 b_1 [i_t - E_{t-1}(P_{t+1} - P_t)] + b_2(m_t - c_2 i_t) + c_3 \nu_t - b_2 \eta_t\} \tag{10}$$

where $\beta = [b_2 + a_1(1 - \theta)(c_3 + b_2 c_1)]$. Applying the E_{t-1} operator to both sides of (10) and subtracting the result from (10) yields

$$P_t - E_{t-1}P_t = \beta^{-1}x_t \tag{11}$$

where

$$x_t = c_3 \nu_t - b_2 \eta_t + b_2 \varepsilon_t + (b_1 c_3 - b_2 c_2)\psi_t$$

which is the aggregate demand curve innovation under a money rule. Thus, the reduced-form solution for real output, obtained by substituting (11) into (6), is

$$y_t = a_1(1 - \theta_m)\beta^{-1}x_t \tag{12}$$

where the subscript m has been added to θ to denote its value under a money supply rule.[6/]

The reduced-form solution for output has the familiar monetary neutrality result for rational expectations models which incorporate the natural rate hypothesis in the aggregate supply curve. Further, since $0 < \theta_m < 1$ (for reasons to be discussed below) and $\beta > 0$, real output will respond nonnegatively to innovations to real demand (ν_t) and the nominal money stock (ε_t) and nonpositively to innovations to money demand (η_t). The effects on output of innovations to the foreign interest rate (ψ_t) are indeterminate.

In order to determine output variability under a money rule, I must solve for θ_m in terms of the variances of the innovations in the model. From (2),

$$P_t(z) - E_{t-1}P_t(z) = P_t - E_{t-1}P_t + z. \tag{13}$$

From (11), (13), and the definition of a linear projection,

$$\theta_m = \sigma_x^2/(\sigma_x^2 + \beta^2\sigma_z^2). \tag{14}$$

Since both the numerator and the denominator of (14) are nonnegative, $0 \leqslant \theta_m \leqslant$ 1.

Substituting (14) into (12) yields relationships between the magnitudes of the variances of the stochastic disturbances and the response of aggregate supply to aggregate demand innovations. These relationships are similar to those obtained by Lucas. Specifically, in my model, as $\sigma_x^2 \to \infty$ (or $\sigma_z^2 \to 0$), $\theta_m \to 1$ and $\partial y_t/\partial x_t \to 0$. Thus, when all changes in output prices are due to changes in aggregate demand, aggregate supply is constant, since no relative price changes occur. Alternatively, as $\sigma_x^2 \to 0$ (or $\sigma_z^2 \to \infty$), $\theta_m = 0$ and $\partial y_t/\partial x_t = a_1[b_2 + a_1(c_3 + b_2c_1)]^{-1}$. Thus, when all changes in output prices are due to relative price changes, the model gives the standard results.

Exchange Rate Rule

Next I consider an economy in which the monetary authority follows an exchange rate rule rather than a money supply rule. A monetary authority doing this sets a target value for the exchange rate $(\bar{e}_t)$ according to some known rule. The rule could be that $\bar{e}_t = \bar{e}$ for all t (a fixed exchange rate) or that e_t appreciates or depreciates at a given rate over time. The rule could also be of the linear feedback type. The particular form of the exchange rate rule is not important, however. All that is necessary is that the rule used to determine the exchange rate be known and constant over time and be used by the agents in the economy to form their expectations of e_t.

Of course, the monetary authority cannot establish $\bar{e}_t$ by fiat. Under an exchange rate rule, the monetary authority must accomplish its objectives by buying or selling foreign or domestic reserves until the target exchange rate is achieved. However, since both actions affect the domestic money supply, it becomes an endogenous variable under an exchange rate rule.[7]

When m_t is endogenous, solving (6) through (9) for the price level and substituting (5) in the result yields

$$P_t = \gamma^{-1}\{b_1[i^* + E_{t-1}(e_{t+1} - e_t) - E_{t-1}(P_{t+1} - P_t)]$$
$$+ b_2 e_t + \nu_t + a_1 E_{t-1} P_t\} \tag{15}$$

where $\gamma = b_2 + a_1(1 - \theta)$. Applying the E_{t-1} operator to both sides of (15) and subtracting the result from (15) yields

$$P_t - E_{t-1} P_t = \gamma^{-1} w_t \tag{16}$$

where $w_t = \nu_t + b_1 \psi_t + b_2 \omega_t$, the aggregate demand curve innovation under an exchange rate rule. Substituting (16) into (6) yields the reduced-form solution for real output:

$$y_t = a_1(1 - \theta_e)\gamma^{-1} w_t \tag{17}$$

where the subscript e has been added to θ to denote its value under an exchange rate rule.

Once again, the reduced-form solution for output has the familiar monetary neutrality result for rational expectations models which incorporate the natural rate hypothesis in the aggregate supply curve. Since $0 \leqslant \theta_e \leqslant 1$, real output depends nonnegatively on innovations to real demand (ν_t) and innovations to the exchange rate (ω_t) and nonpositively on innovations to the foreign interest rate (ψ_t). Innovations to money demand (η_t) do not affect output under an exchange rate rule.[8]

The parameters of the aggregate supply curve once again depend on the variances of the stochastic disturbances through θ_e. Specifically, using (16), (13), and the definition of a linear projection,

$$
\begin{aligned}
\theta_e &= \gamma^{-2}\sigma_w^2/(\gamma^{-2}\sigma_w^2 + \sigma_z^2) \\
&= \sigma_w^2/(\sigma_w^2 + \gamma^2\sigma_z^2).
\end{aligned}
\tag{18}
$$

An examination of (17) and (18) reveals that, as was true under a money rule, when all changes in prices are due to changes in aggregate demand ($\sigma_w^2 \to \infty$ or $\sigma_z^2 \to 0$), aggregate supply is constant. When all changes in prices are due to relative price changes ($\sigma_w^2 \to 0$ or $\sigma_z^2 \to \infty$), the results are standard.

Implications

In this section, I compare the variance of real output under a money supply rule,

$$
\sigma_y^2\big|_m = \beta^{-2}[a_1(1 - \theta_m)]^2\sigma_x^2
\tag{19}
$$

with the variance of real output under an exchange rate rule,

$$
\sigma_y^2\big|_e = \gamma^{-2}[a_1(1 - \theta_e)]^2\sigma_w^2.
\tag{20}
$$

This is done by examining the magnitude of the ratio of these variances,

$$
\begin{aligned}
\lambda &= \sigma_y^2\big|_m/\sigma_y^2\big|_e \\
&= [\theta_m(1 - \theta_m)]/[\theta_e(1 - \theta_e)]
\end{aligned}
\tag{21}
$$

focusing primarily on how that ratio is affected by changes in the variances of the innovations in the model. First I examine the effects on λ of changes in the variances of the demand-side innovations η_t, ν_t, and ψ_t, assuming that

$\sigma_z^2 \neq 0$. Then I examine the effects on λ of changes in the variance of the distribution of aggregate demand, assuming that σ_x^2, $\sigma_w^2 \neq 0$.

Demand-Side Innovations

The three demand-side innovations in the model are innovations to money demand (η_t), real demand (ν_t), and the foreign interest rate (ψ_t). I consider each by assuming that it is the only innovation (other than z) in the economy.

When the only demand-side innovation is an innovation to money demand, $x_t = -b_2\eta_t$ and $w_t = 0$. Since $w_t = 0$, $\sigma_w^2 = \theta_e = \lambda = 0$. Consequently, when the economy is subject only to money demand innovations, the variance of output is always smaller under an exchange rate rule than under a money rule. That is because output is determined independently of the money market under an exchange rate rule. This result is standard in the literature. (See Weber 1981.)

When the only demand-side innovation is an innovation to real demand (ν_t), $x_t = c_3\nu_t$ and $w_t = \nu_t$. Since $0 \leqslant \theta_e$, $\theta_m \leqslant 1$, λ is a continuous function of σ_ν^2 when $\sigma_\nu^2 > 0$. Further, if $\sigma_z^2 \neq 0$, then

$$\lim_{\sigma_\nu^2 \to 0} \lambda = c_3^2\{(b_2 + a_1)/[b_2 + a_1(c_1b_2 + c_3)]\}^2 = \lambda_0 \qquad (22).$$

and

$$\lim_{\sigma_\nu^2 \to \infty} \lambda = c_3^{-2}. \qquad (23)$$

Since $0 \leqslant c_3 \leqslant 1$ and under that presumption c_3 is small, $\lambda_0 < 1$.[9/] Thus, λ and σ_ν^2 are related as shown in Figure 1.[10/]

Figure 1 indicates that a money rule yields a smaller variance of output against real demand disturbances when $\sigma_\nu^2 < \bar{\sigma}_\nu^2$. This dominance of a

money rule over an exchange rate rule is the standard result when only real demand innovations affect the economy. It occurs because the nominal interest rate is fixed by the interest parity condition (9) under an exchange rate rule. Thus, disturbances to the IS curve must be cleared solely by price level and output adjustments. However, under a money rule, part of the market equilibration can occur through changes in the interest rate, so less need occur through changes in output.

Figure 1 also indicates that an exchange rate rule yields a smaller variance of output against real demand disturbances when $\sigma_\nu^2 > \bar{\sigma}_\nu^2$. This reversal occurs because of the relative steepness of the aggregate supply curve under the exchange rate and money supply rules as σ_ν^2 increases. Under both regimes, this aggregate supply curve becomes steeper as σ_ν^2 increases. However, letting δ equal the ratio of the slope of the aggregate supply curve under the two regimes produces, from (6),

$$\delta = (1 - \theta_e)/(1 - \theta_m). \tag{24}$$

Since

$$\lim_{\sigma_\nu^2 \to \infty} \delta = c_3^2 < 1,$$

the slope of the aggregate supply curve becomes infinite more rapidly under an exchange rate rule than under a money rule. Therefore, once σ_ν^2 exceeds $\bar{\sigma}_\nu^2$, the steeper slope of the aggregate supply curve under an exchange rate rule causes that rule to dominate a money rule.

The final demand-side disturbance is an innovation to the foreign interest rate (ψ_t). Letting foreign interest rate innovations be the only demand-side innovations results in $x_t = (b_1 c_3 - b_2 c_2)\psi_t$ and $w_t = b_1 \psi_t$. Given the restrictions on θ_m and θ_e, λ is again a continuous function of σ_ψ^2. Further, if $\sigma_z^2 \neq 0$, then

$$\lim_{\sigma_\psi^2 \to 0} \lambda = h^2 \{(b_2 + a_1)/[b_2 + a_1(c_1 b_2 + c_3)]\}^2 = \lambda_1 \qquad (25)$$

and

$$\lim_{\sigma_\psi^2 \to \infty} \lambda = h^{-2} \qquad (26)$$

where $h = (b_1 c_3 - b_2 c_2)/b_1$.

A comparison of (25) and (26) with (22) and (23) reveals that the analysis of whether an exchange rate rule or a money supply rule dominates with foreign interest rate innovations is similar to the analysis of what happens with real demand curve innovations. Specifically, when $|h| < 1$ and $\lambda_1 < 1$, σ_ψ^2 and λ are related in a way similar to that depicted in Figure 1. Therefore, a money rule provides less output variability than an exchange rate rule in the face of foreign interest rate disturbances only as long as the variance of these innovations is below some critical value. Once the variance becomes large, an exchange rate rule dominates. Since

$$\lim_{\sigma_\psi^2 \to \infty} \delta = h^2,$$

the reasons for these results are the same as those for the results with real demand innovations.

Alternatively, when $|h| > 1$ and $\lambda_1 < 1$, the curve in Figure 1 becomes downward-sloping and the results are reversed. When the variance of foreign interest rate innovations is relatively small, an exchange rate rule dominates, and when the variance of foreign interest innovations is relatively large, a money rule dominates.[11]

Distribution of Aggregate Demand

Finally, I consider the effects on λ of changes in the variance of the distribution of aggregate demand across markets. The analysis closely follows the last two analyses.

Using those arguments here, λ is a continuous function of σ_z^2, and if σ_w^2, $\sigma_x^2 \neq 0$, then

$$\lim_{\sigma_z^2 \to 0} \lambda = \sigma_w^2/\sigma_x^2 \tag{27}$$

and

$$\lim_{\sigma_z^2 \to \infty} \lambda = (\sigma_x^2/\sigma_w^2)\{(b_2 + a_1)/[b_2 + a_1(c_1 b_2 + c_3)]\}^2 = \lambda_2. \tag{28}$$

Suppose that $c_1 b_2 + c_3 \doteq 1$ (the aggregate demand curve has approximately the same slope under the two types of rules) and $\sigma_x^2 < \sigma_w^2$. A plot of λ against σ_z^2 is shown in Figure 2. This figure and equations (27) and (28) illustrate that when $\sigma_z^2 > \bar{\sigma}_z^2$, a money rule dominates an exchange rate rule. When σ_z^2 is large, the aggregate supply curves under the regimes have approximately the same slope, so the regime with the smaller variance of aggregate demand innovations dominates. Since by assumption $\sigma_x^2 < \sigma_w^2$, the variance of the aggregate demand innovations is smaller under a money rule, and that rule dominates an exchange rate rule. However, when $\sigma_z^2 < \bar{\sigma}_z^2$, an exchange rule dominates. That, once again, is because of the relative rates at which the slope of the aggregate supply curve becomes infinite as $\sigma_z^2 \to 0$. Since

$$\lim_{\sigma_z^2 \to 0} \delta = \sigma_x^2/\sigma_w^2,$$

the slope of the aggregate supply curve becomes infinite faster under an exchange rate rule when $\sigma_x^2 < \sigma_w^2$. The opposite implications hold when $\sigma_w^2 < \sigma_x^2$.

Summary

In this paper I analyze the variability of output under money supply and exchange rate rules in an open economy in which the slope of the aggregate supply curve depends on the variances of aggregate demand and market-specific innovations. The results of this analysis can be summarized and contrasted to those for an economy in which the aggregate supply curve has a constant slope:

- In general, I have not been able to determine that one type of rule always dominates the other. This result agrees with that for the constant-slope economy.

- When only one type of aggregate demand innovation has a nonzero variance, one type of rule dominates in a constant-slope economy. The same rule dominates in this economy only when the variance of the innovation is sufficiently small. When that variance exceeds some critical value, the constant-slope results are reversed.

Footnotes

1/Nigel Duck (1984) also adopts a Lucas-type approach to modeling aggregate supply. The major differences between this paper and his are that this analysis includes capital flows whereas his does not and that I use an IS-LM approach to modeling aggregate demand whereas he uses the approach of Robert Barro (1976).

2/The formulation could also include lagged output to take account of the costs of adjusting output. However, that would complicate the analysis without changing any results.

3/In this market supply curve, the supply in market z depends on the price of output in market z relative to the perceived domestic price level. This formulation differs from that in the papers by Parkin, Bentley, and Fader and by Parkin, where the supply in market z depends on the price in market z relative to the perceived index of domestic and world prices. The reason for the difference in formulations is that the other analyses assume domestic and foreign goods are perfect substitutes whereas mine does not.

4/As formulated in (7)-(9), the demand side of my model may be asymmetric with respect to its information assumptions. Expectations about the rates of change of the price level and the exchange rate are assumed to be conditioned on information available in period t - 1, whereas period t information on the price level, the nominal interest rate, and the exchange rate is assumed to be used by agents in their output and money demand decisions. In the Appendix, I analyze the symmetric formulation in which agents also use period t information to form expectations. This change in the timing of the demand-side expectations does not change any of the qualitative conclusions of the analysis; it merely complicates the exposition of the results.

5/Since the system is recursive in i_t, I solve it initially treating i_t as exogenous.

6/Michael Dotsey and Robert King (1983) find that the variance of real output depends on the type of money rule when the money rule depends on contemporaneously observed variables. Their result does not hold in my model because I condition expectations of future prices and exchange rates on I_{t-1}, whereas they condition expectations on $(I_{t-1}, P_t(z))$. This also explains why in my model the variance of real output under an exchange rate rule [equation (17) below] does not depend on the actual exchange rate rule chosen.

7/Given the structure of the economy, a given exchange rate rule implies the time path of the domestic money supply and the monetary author-ity's holdings of international reserves. Consequently, certain exchange rate rules might require the monetary authority to sell a quantity of international reserves exceeding its initial holdings. Such exchange rate rules would obviously not be feasible and are implicitly ruled out of the analysis. However, since the variance of real output under an exchange rate rule is independent of the actual rule chosen, the infeasibility of some exchange rate rules does not affect my results.

8/For both types of policy rules, the signs of effects of innova-tions on real output from my model essentially agree with those obtained by Parkin, Bentley, and Fader if their changes in domestic nominal demand are equated with ε_t innovations and their foreign price surprises are equated with ν_t innovations. One difference between the two analyses is that expected changes in the terms of trade affect real output in their analysis but not in mine. This difference is caused by the different formulations of the aggre-gate supply curve. (See footnote 2.)

These results about the effects of innovations on output also agree with those of Flood and Marion, who find that foreign innovations affect domestic real output under both fixed and flexible exchange rate regimes when purchasing power parity (PPP) is not assumed to hold. When PPP is assumed to hold, however, Flood and Marion find that domestic output is insulated from all foreign disturbances. In my analysis, when PPP is assumed to hold, the IS curve becomes $P_t - e_t + \nu_t$ and foreign price level innovations still affect domestic real output through ν_t, even though foreign interest rate innovations (ψ_t) no longer have an effect. Our results differ because the slope of my aggregate supply curve differs from that which would obtain with optimal indexation.

9/More specifically, $\lambda_0 < 1$ if and only if

$$\{(b_2 + a_1)/[b_2 + a_1(c_1 b_2 + c_3)]\}^2 < c_3^{-2}. \tag{†}$$

Obviously, there is some c_3 small enough to satisfy this condition. Of course, if the condition is not satisfied, then a money rule always dominates an exchange rule for real demand innovations.

The question of whether or not $\lambda_0 < 1$ can be approached in another way. Under an exchange rate rule, the slope of the aggregate demand curve is

$$(\partial P_t/\partial y_t)e = -1/b_2$$

whereas under a money rule that curve's slope is

$$(\partial P_t/\partial y_t)m = -(b_2 c_1 + c_3)/b_2.$$

Thus, for a given c_3, equation (†) is satisfied as long as the aggregate demand curve is not too much less steeply sloped under a money rule than under an exchange rate rule.

10/Figure 1 assumes that λ is a monotone increasing function of σ_ν^2. I have not been able to determine whether or not this assumption is correct. The important point in Figure 1, however, is that there is at least one point for which $\lambda = 1$.

11/Of course, the discussion of footnotes 8 and 9, properly adapted, also holds for foreign interest rate innovations. Thus, the slopes of the aggregate demand curves may allow an exchange rate rule or a money rule to dominate for all values of σ_ψ^2.

Appendix

Here I solve the model with a symmetric formulation of information, one in which information in period t is assumed to be available to agents in making their commodity and money demand decisions. This model has four equations:

Aggregate Supply Curve

$$y_t = a_1(1 - \theta)(P_t - E_{t-1}P_t) \tag{A1}$$

IS Curve

$$y_t = b_1[i_t - (E_tP_{t+1} - P_t)] + b_2(e_t - P_t) + v_t \tag{A2}$$

IM Curve

$$m_t = c_0P_t + c_1y_t + c_2i_t + c_3e_t + n_t \tag{A3}$$

Interest Rate Parity Condition

$$i_t = i_t^* + E_te_{t+1} - e_t. \tag{A4}$$

Here the operator E_t denotes the rationally formed expectation based on information at t; for example, $E_tP_{t+1} = E[P_{t+1}|I_t]$.

Money Supply Rule

First I consider an economy with a money supply rule. When e_t is endogenous, $b_2^* = b_2 - b_1$, and $c_3^* = c_3 - c_2$, solving (A1) through (A4) for P_t results in

$$\beta^*P_t = [a_1(1 - \theta)(c_3 + b_2^*c_1)]E_{t-1}P_t + c_3^*b_1[i_t^* + E_t(e_{t+1} - P_{t+1})]$$

$$+ b_2^*m_t - b_2^*c_2(i_t^* + E_te_{t+1}) + c_3^*v_t - b_2^*n_t \tag{A5}$$

where $\beta^* = b_2^*(1 - c_2) + a_1(1 - \theta)(c_3^* + c_1b_2^*)$.

Since (A1) must hold for all periods, it must hold for period $t + j$, where $j > 0$. Thus,

$$y_{t+j} = a_1(1 - \theta)(P_{t+j} - E_{t+j-1}P_{t+j}). \tag{A6}$$

Applying the operator E_t to both sides of (A6) yields

$$E_t y_{t+j} = 0 \tag{A7}$$

for all $j > 0$.

Equation (A2) must also hold for all periods. Applying the E_t operator to both sides of (A2) for period $t + j$ and using (A4) and (A7) yields

$$0 = b_1[i^* + E_t(e_{t+j+1} - P_{t+j+1})] + b_2^* E_t(e_{t+j} - P_{t+j}). \tag{A8}$$

The first-order difference equation in $E_t(e_{t+j} - P_{t+j})$ can be solved forward, yielding

$$E_t(e_{t+j} - P_{t+j}) = -b_1 i^*/b_2^* \tag{A9}$$

for all $j > 0$.

Equation (A9) requires the assumption that

$$\lim_{n \to \infty} -(b_1/b_2^*)^n E_t(e_{t+n} - P_{t+n}) \to 0.$$

The imposition of this terminal condition rules out the possibility of expected changes in future relative prices which would lead individuals to demand infinite quantities of either foreign or domestic goods in the present period. This terminal condition is similar to that of Thomas Sargent and Neil Wallace (1975, p. 248). Equation (A9) also requires that $\{E_t m_{t+j}\}$ not be too explosive and c_2 not be too large.

Equation (A3) must also hold for all periods. Applying the E_t operator to both sides of (A3) for period $t + j$ and using (A4) and (A7) yields

$$E_t m_{t+j} = -c_0 E_t(e_{t+j} - P_{t+j}) + c_2 i^* + c_2 E_t e_{t+j+1}$$
$$+ (1 - c_2) E_t e_{t+j}. \qquad (A10)$$

Equation (A10) can be solved forward, and using (A7) results in

$$E_t e_{t+j} = -(c_2 + c_0 b_1/b_2^*) i^*$$
$$+ (1 - c_2)^{-1} \sum_{i=0}^{\infty} [-c_2/(1 - c_2)]^i E_t m_{t+j+1}. \qquad (A11)$$

In (A5), after substituting for $E_t(e_{t+1} - P_{t+1})$ using (A9) and $E_t(e_{t+1})$ using (A11), I apply the E_{t-1} operator to both sides of (A5) and subtract the result from (A5) to obtain

$$\beta^*(P_t - E_{t-1}P_t) + x_t^* \qquad (A12)$$

where $x_t^* = c_3^* v_t + b_2^*(\varepsilon_t - \eta_t) + (c_3^* b_1 - b_2^* c_2)\psi_t$. Substituting (A12) into (A1) yields the solution for real output:

$$y_t = a_1(1 - \theta_m)x_t^*/\beta^*. \qquad (A13)$$

And from (A13) comes the variance of output under a money rule:

$$\sigma_{y|m}^2 = [a_1(1 - \theta_m)/\beta^*]^2 \sigma_{x^*}^2. \qquad (A14)$$

Further,

$$\theta_m = \sigma_{x}^2*/[\sigma_{x}^2* + (\beta^*)^2 \sigma_{x}^2*]. \qquad (A15)$$

A comparison of (A14) and (A15) with (19) and (14), respectively, shows that the change in the timing of expectations does not change any of the qualitative results for the economy with a money rule.

Exchange Rate Rule

Next I consider an economy with an exchange rate rule. When m_t is endogenous, solving (A1) through (A4) for P_t results in

$$\gamma^* P_t = a_1 (1 - \theta) E_{t-1} P_t + b_1 [i_t^* + E_t(e_{t+1} - P_{t+1})] + b_2^* e_t + \nu_t \quad (A16)$$

where $\gamma^* = b_2^* + (1 - \theta)a_1$. Following the same procedures used to obtain (A12) gives

$$\gamma^* (P_t - E_{t-1} P_t) = w_t^* \qquad \qquad (A17)$$

where $w_t^* = \nu_t + b_1 \psi_t + b_2^* \omega_t$. Substituting (A17) into (A1) yields

$$y_t = a_1 (1 - \theta_e) w_t / \gamma^*. \qquad (A18)$$

Therefore, the variance of output under an exchange rate rule is

$$\sigma_y^2 \big|_e = [a_1 (1 - \theta_e)/\gamma^*]^2 \sigma_w^2{}^*. \qquad (A19)$$

Further,

$$\theta_e = \sigma_w^2{}^* / [\sigma_w^2{}^* + (\gamma^*)^2 \sigma_z^2]. \qquad (A20)$$

A comparison of (A19) and (A20) with (20) and (18), respectively, shows that the change in the timing of expectations also does not change any of the qualitative results for the economy with an exchange rate rule.

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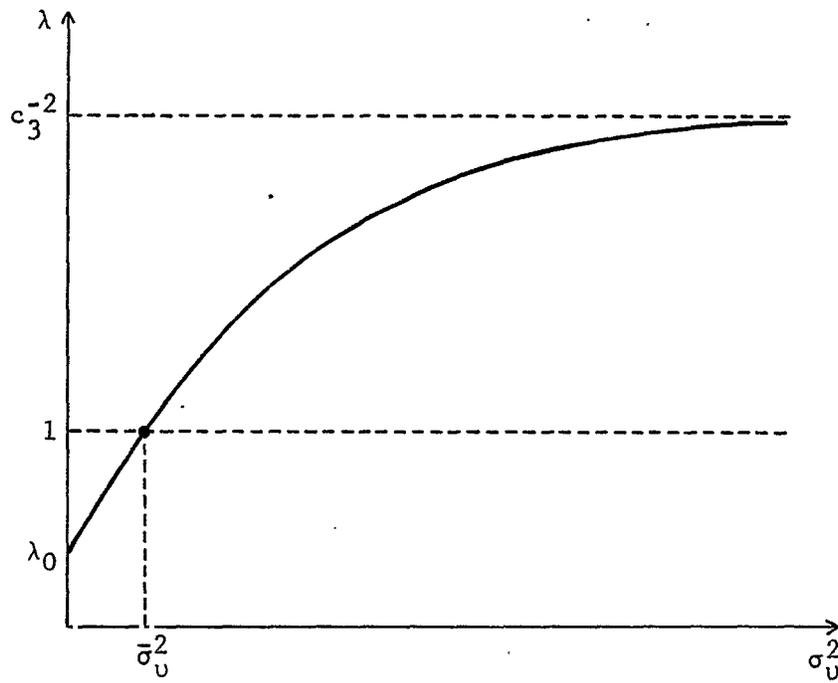
Figure 1

Figure 2