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DEMAND ANTICIPATION AND SPECULATION IN INVENTORIES: THE DECISIONS OF A PRICE SETTING FIRM

by

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The great bulk of the trading in our economy does not occur on the exchanges. In many markets sellers announce prices and buyers then decide how much to buy without bargaining. This paper examines the behavior of such price setting firms under demand uncertainty. Both price setting "perfect competitors" and monopolists are treated. The purpose is to derive important properties of optimal price, output, and speculative inventory stock decisions in these environments. In particular, the effects of demand anticipation and of unanticipated accumulation and depletions of inventories on these decisions are analyzed. These effects are important elements in the dynamics of those goods markets in which price does not continually clear the market.

In the usual perfect competition model under uncertainty, output and sales are determined by the firm which has a reservation price. The market price adjusts to equate supply and demand with the aid of the mythical caller. In a world of price setting firms and decentralized markets the applicability of the conventional model to short-run behavior is questionable. A second model of perfect competition which applies to such markets is used here. The firm sets price as well as maximal sales and output. A stochastic "market price" is assumed.

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If the firm's price is less than or equal to the "market price," the firm sells the predetermined desired amount at his predetermined price--not at the "market price." If the firm's price is larger than the market price, the firm sells nothing. In this model of perfect competition there is undesired inventory accumulation if the firm's price is above the market price. Moreover, if the firm's price is lower than the market price, there is undesired saving. This model has complete inflexibility of prices within a period, and the firm sells nothing in a period if overpriced. Therefore, the period under consideration must be of short duration.[1/]

The natural extension of this perfect competition model to monopoly has the monopolist also determine output, price, and maximal sales. When the stochastic demand curve is realized, the firm sells the amount demanded at the preset price up to the maximal sales level. If demand is unusually low, undesired inventory accumulation takes place. As long as the firm does not stock out (sell at maximal level) regularly, high realized demand can yield undesired inventory decumulation. If the firm does stock out, with demand at the preset price exceeding maximal sales, undesired savings occurs. Thus, this model too is applicable to the study of disequilibrium.

While the problem of such monopolists is analyzed in the literature, the demand schedule is assumed to be i.i.d.[2/] Realizations do not change firms' evaluation of future conditions, certainly a very unrealistic assumption. This paper extends the previous work on monopolists

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[1/] A period must be short enough that within period realized sales do not substantially affect firms' anticipations of "market price." For a single-period analysis of a market of such perfect competitors see [6].

[2/] See [15].

by allowing a wider class of error terms, but one in which the phrase "an increase in anticipated demand" can be given as unambiguous interpretation. Because firms do form anticipations of demand, speculation in inventories becomes an important consideration and is explicitly treated.

In the simple model examined there are three possible sources of uncertainty for the firm. Demand (or market price for perfect competitors) may be stochastic, cost of production may be stochastic, and the appropriate rate of discount of the future may be stochastic. Only stochastic demand is considered. Because the stochastic term in demand is not i.i.d., the firm forms anticipations of current and future demand, and there have to be state variables reflecting the distributions of the stochastic term present and future assumed by the firm. One could have the distributions of the stochastic term themselves be state variables. Besides other disadvantages, this approach does not make clear the meaning of an "increase in anticipated demand." To simplify and clarify the problem, the existence of a single real valued state variable which is a sufficient statistic for the current and future distributions of the stochastic term is assumed. Moreover, an increase in this variable, by shifting the density functions of the disturbance terms to the right, increases the probability of high demand and decreases the probability of low demand at all prices for the current and all future periods.[3/] This variable is taken as a measure of demand anticipation and is the most reasonable definition of "demand anticipation" in an infinite period problem which the author could devise.

[3/] Permanent-transitory change in the Gaussian framework, a simple form of Bayesian learning is an example of such a process. See [8]. In Box-Jenkins terms an IMA(1,1) is such a process.

The separate maximal sales decision of firms is important for two reasons. First, the firm will indeed use such an option if it is available. Second, the maximal sales decision allows a clear definition of speculation in inventories. If maximal sales are less than goods on hand (output plus initial inventory stock), the firm is determining a minimal inventory level, goods on hand minus maximal sales. This minimum inventory consists of goods which the firm is withholding from the market for future sales, inventories with which the firm is speculating.

The Models

Before turning to the individual models a few prefatory remarks are necessary. The firms maximize the discounted sum of the expected profit stream using an infinite horizon. A sufficient condition for the value of the firm to be a continuous function is that the discounted expected partial sums be continuous and converge uniformly. In the dynamic programming literature such uniform convergence is proven using contraction mappings on complete metric spaces.[4/] In order for the contraction mapping to guarantee uniform convergence, the metric of uniform convergence, the supremum metric (supremum of the absolute value of the difference between two functions), is used. However, the distance between the zero function and an unbounded function, as measured by the supremum metric, does not exist. Therefore, if contraction mappings are to be used, the space must be restricted to the set of bounded continuous functions. Therefore, much of the dynamic programming literature is directly applicable only to bounded return functions.

It is clear that in the perfect competition model the current return function is not a bounded function of initial inventory stock.

[4/] See [9].

Neither the probability of the firm selling nor the price at which he does sell is influenced by the amount he offers for sale. Therefore, the current return of the firm must be linear in initial inventory stock. Fortunately this unboundedness does not present a serious obstacle.

As can easily be shown, if the current return functions are uniformly bounded and a constant discount factor is used, the contraction mapping property follows. For the monopoly model below we assume the first two properties and the result that the valuation function is a unique continuous bounded function that can be approximated using either policy or value iteration.[5/] Because it is familiar, and to avoid repetition, the monopoly model is presented in a less formal manner.

I. Perfect Competition

A firm has to make its output (X), price (P), and minimum inventory (H^m) decisions before current market price is observed. If the chosen price is less than or equal to the realized market price, the firm sells all of the goods it has for sale, beginning of period inventory plus output minus minimum inventory, and ends the period with inventory equal to minimum inventory. If the chosen price is above the market price, the firm sells nothing and ends the period with inventory equal to beginning of period inventory plus output.

The assumptions are:

1. The firm is risk neutral and maximizes the discounted expected profit stream. Having an infinite horizon the firm uses a constant discount rate of β, $0 \leq \beta < 1$.

[5/] See [5] and [9].

2. A cost function of output, $c(X)$, where $c(X) > 0$, $c'(X) > 0$, $c''(X) > 0$. Positive and increasing marginal cost of output holds throughout.

3. K units of goods held from period t to period t+1 yield K units of goods in period t+1 where $0 \leq \gamma < 1$. This is a very simple form for the depreciation of inventories.

4. Market price in period t, P^m_t, is an element of a stochastic process such that $0 \leq P^m_t \leq 1$ for all t. The continuous conditional density functions

$$f_{P^m_{t+j}|P^m_{t-1},\, P^m_{t-2},\, \ldots}(P^m_{t+j}|P^m_{t-1},\, P^m_{t-2},\, \ldots)$$

j=1, 2, ... exist and are independent of t. There is an informational variable M_t, a member of a first-order Markov process with stationary transition probabilities, such that the conditional densities

$$f_{P^m_{t+j}|M_t}(P^m_{t+j}|M_t)$$

exist and equal

$$f_{P^m_{t+j}|P^m_{t-1},\, P^m_{t-2},\, \ldots}(P^m_{t+j}|P^m_{t-1},\, P^m_{t-2},\, \ldots)$$

j=1, 2, ... $M_{t+1} = h(M_t, P^m_t)$ where h is increasing in M_t and P^m_t and is differentiable. Further, $M''_t > M'_t$ implies

$$F_{P^m_{t+j}|M_t}(P^m_{t+j}|M''_t)$$

is stochastically larger than

$$F(P^m_{t+j} | M'_t)$$
$$P^m_{t+j} | M_t$$

(stochastic monotonicity property). For convenience assume $0 \leq M_t \leq 1$. The boundedness of P^m_t yields an easy proof that the firm's value is finite. The choice of 1 as an upper bound of P^m_t and M_t is arbitrary. M_t can be interpreted as a measure of the anticipated market price.

Firms have (or think they have) a great deal of information about the stochastic market price. The possible shifts in the perceived distribution of market price are summarized by a single parameter. An increase in this parameter implies that the density functions of market price are shifted to the right everywhere in the current and all future periods. This allows an increase in M to be unambiguously interpreted as an increase in anticipated price. For example, a simultaneous increase in the mean of the distribution and a reduction in the probability of very high market price is ruled out. If the parameter M is the mean of the distribution (as in the example in footnote 3), higher moments are constant.

Moreover, after market price is observed, anticipated demand, M, is recalculated using only the old anticipated demand and observed market price. The firm does not have to retain the whole sequence of past market prices, and we are dealing with a history forgetting process. When a firm observes a high market price, it raises its anticipation of future market prices, and when it observes a low market price, it lowers its anticipation. While this structure is very simple and restrictive, it allows for changes in anticipated market price which i.i.d. stochastic terms rule out.

First we must present the firm's problem and show that it is well defined.

There are two state variables, beginning of period inventory, H_t, and the sufficient statistic for P_t^m, M_t. Whenever confusion will not result, the time subscripts will be deleted and replaced by: $H_t = H$, $H_{t+1} = H'$, $M_t = M$, $M_{t+1} = M'$. It will also be understood that M' is a function of M and P^m. Subscripts will be deleted from probability density functions.

If S = sales, current profit = $r(X,P,H^m; H,M) = PS - c(X)$. Before demand is observed, expected profit is $E[r(X,P,H^m; H,M)] = PE(S) - c(X) = P\cdot(X+H-H^m)\cdot Pr(P^m \geq P) - c(X)$. $S = 0$ if $P^m < P$, and $S = X + H - H^m$ if $P^m \geq P$. Expected current profit is continuous in inventories and the sufficient statistic for market price, M. Several properties of the current profit function follow immediately.

Lemma 1: Current profit is bounded above for all X, P, H^m, and M given H.

Proof: Price is bounded above by 1. Further, it is never profitable for a firm to produce above a level $\overline{X}$ where $c'(\overline{X}) = 1$. Therefore, $r(X,P,H^m; H,M) \leq \overline{X} + H$.

Lemma 2: Expected current profit is nondecreasing in inventories.

Proof: P and $Pr\{P^m \geq P\}$ are bounded below by zero.

Lemma 3: Expected current profit is nondecreasing in M given P.

Proof: A higher M does not decrease $Pr[P_t^m \geq P]$.

The firm's maximization problem can be written:

$$\text{(I)} \qquad w(H,M) = \sup_{\substack{X(H,M) \\ P(H,M) \\ H^m(H,M)}} \sum_{t=0}^{\infty} \beta^t [P_t(X_t+H_t-H_t^m)Pr[P_t^m \geq P_t]-c(X_t)]$$

subject to

$$1/\gamma H_{t+1} = \begin{cases} H_t^m, & P_t^m \geq P_t \\ X_t + H_t, & P_t^m < P_t \end{cases}$$

$$H_t^m \leq X_t + H_t.$$

The next order of business is to show that the assumptions insure that the firm's maximization problem is well defined.

Theorem 1: w(H,M) exists.

Proof: $w(H,M) \leq H + \sum_{t=0}^{\infty} \beta^t \overline{X} = H + \overline{X}/(1-\beta)$ (see Lemma 1). w(H,M) is bounded below by

$$-\sum_{t=0}^{\infty} \beta^t c(0) = -c(0)/(1-\beta)$$

as $X_t = P_t = 0$ is a possible policy.

By the principle of optimality the "value" of the firm, w(H,M) can also equivalently be defined by the recursion relation:

$$\text{(II)} \qquad w(H,M) = \sup_{\substack{P,X,H^m \geq 0 \\ H^m \leq X+H}} E\{(r(X,P,H^m;\ H,M)+\beta w(H',M')\}$$

$$= \sup_{\substack{P,X,H^m \geq 0 \\ H^m \leq H+H}} \{P(X+H-H^m)\int_P^1 f(P^m|M)\,dP^m + \beta\int_P^1 w(\gamma H^m,M')f(P^m|M)\,dP^m$$

$$+ \beta\int_0^P w(\gamma(X+H),M')f(P^m|M)\,dP^m - c(X)\} = \Phi w(H,M).$$

Theorem 2: w[H,M] satisfying (II) is a unique continuous function.

Proof: $r(X,P,H^m;\ H,M)$ is unbounded above in H. Define $r_K = \min[r(X,P,H^m;\ H,M),K]$ for $K > 0$. Define the map Φ_K satisfying

$$\Phi_K v(H,M) = \sup_{\substack{P,X,H^m \geq 0 \\ H^m \leq X+H}} E\{r_K(X,P,H^m;\ X,H)+\beta v(H',M')\}$$

for v a continuous bounded function on RXR. Then Φ_K is a contraction mapping on the set of such continuous bounded functions. Therefore, Φ_K has a unique fixed point in that set of functions.[6/] Call this fixed point $w_K[H,M]$. This is the solution to a truncated version of the firm's decision problem: a fixed point of (II) must equal w_K for $H \leq K - \overline{X}$. Now all we need to show is that $w_K(H,M)$ converges uniformly to $w(H,M)$ over every compact subset of $[0,1]X[0,\infty)$ as $K \to \infty$. For if this is true, $w[H,M]$ satisfying (II) is unique and continuous over every compact subset and by the σ compactness of the real numbers is unique and continuous everywhere.

Choose any compact subset S of $[0,1]X[0,\infty)$ where the second argument is bounded above by $\overline{H}$, say. Then inventories j periods into the future are bounded above by $\overline{H} + j\overline{X}$. Let $K_j = \overline{H} + j\overline{X}$. In trying to achieve $w_{K_j}[H,M]$ the firm could have used the optimal decision functions for $w(H,M)$. If it did so, the return from the first j periods would be the same in the truncated and original problems. Therefore,

$$\sup_{(H,M)\ S} \{w(H,M)-w_{K_j}(H,M)\} \leq \beta^j w(\overline{H}+j\overline{X},1).$$

As $p \leq 1$, $w(\overline{H}+j\overline{X},1) \leq \overline{H} + j\overline{X} + \overline{X}/(1-\beta)$

and

$$\lim_{j\to\infty} \beta^j [\overline{H}+j\overline{X} + \overline{X}/(1-\beta)] = 0.$$

Therefore,

6/ See [9].

$$\lim_{j\to\infty} \sup_{(H,M)\in S} \{w(H,M)-w_{K_j}(H,M)\} = 0.$$

Theorem 3: w[H,M] is linear in M.

Proof: Suppose $v(H,M) = a_0(M) + a_1(M)H$, where a_0 and a_1 are nonnegative, nondecreasing continuous functions. Then it can easily be shown (see below) that the P and X which achieve $\Phi v(H,M)$ are independent of H, and that the H^m which achieves $\Phi v(H,M)$ is either 0 or X + H also depending only upon the value of M. This further implies that $\Phi v(H,M)$ is also of the form $a_0(M) + a_1(M)H$ (as can be seen by plugging in these decision functions). Moreover, $\lim_{n\to\infty} \Phi^n v(H,M) = w(H,M)$ must, then, be of this form as linear functions converge to linear functions.

Now let us examine the decision functions of the perfect competitor.

Because w(H,M) is known to be continuous in H, and because X, P, and H^m can be restricted to the compact set $[0,\overline{X}]X[0,1]X[0,\overline{X}+H]$ without altering the problem, "sup" can be replaced by "max" in the preceding expressions. Let $w(H,M) = a_0(M) + a_1(M)H$. The first-order conditions on P, X, and H^m required for maximization are:

$$P: \quad \int_P^1 f(P^m|M)dP^m - Pf(P|\overline{M}) + \beta\gamma a_1(\overline{M})f(P|M) \leq 0, = \text{ if } P > 0$$

$$X: \quad P\int_P^1 f(P^m|M)dP^m + \beta\gamma\int_0^P a_1(M')f(P^m|M)dP^m + \lambda \leq c'(X), = \text{ if } X > 0$$

$$H^m: \quad \beta\gamma\int_P^1 a_1(M')f(P^m|M)dP^m \leq P\int_P^1 f(P^m|M)dP^m + \lambda, = \text{ if } H^m > 0$$

$$\lambda: \quad X + H - H^m \geq 0, = \text{ if } \lambda > 0$$

where λ is the Langrangian multiplier and $\overline{M} = h(M,P)$.[7/]

[7/] The reader can verify that conditions sufficient for the saddlepoint theorem hold.

The first-order condition of price determines price depending only upon the function $f(P^m|M)$. As the first two terms on the LHS of the first-order condition of output are constants given $f(P^m|M)$, output too is determined independently of inventories. $\alpha_0 \equiv \beta\gamma\int_P^1 a_1(M')f(P^m|M)dPm$ and $\alpha_1 \equiv P\int_P^1 f(P^m|M)dP^m$ are constants given $f(P^m|M)$. If $\alpha_0 < \alpha_1$, then $H^m = 0$ and $\lambda = 0$. If $\alpha_0 > \alpha_1$, $\lambda = \alpha_0 - \alpha_1$ (assuming $X + H > 0$), $H^m = X + H - H^m$ and $c'(X) = \beta\gamma\int_0^1 a_1(M')f(P^m|M)dP^m$. All goods are withheld from sale, and marginal cost equals discounted expected marginal worth of inventories, which is constant given $f(P^m|M)$. If $\alpha_0 = \alpha_1$, H^m is indeterminate.

Let P_t^m be independent identically distributed so that M_t is constant. The first-order condition of P implies $P > \beta\gamma a_1$ and that together with the first-order condition of H^m implies that $H^m = 0$. If market prices are i.i.d., then there is no speculation in inventories.

The results of the perfect competition model are:

(a) Price depends only upon anticipated market price, the rate of discount, and the rate of depreciation of inventories.

(b) Output also is independent of inventory stock and depends upon these same variables.

(c) Minimum inventory is zero, equal to the stock of goods on hand (inventory plus output), or indeterminate, and the decision does not depend upon inventory stock.

(d) There is no speculation in inventories if market prices are i.i.d.

These results are all implied by the valuation function being linear in inventories.[8] This linearity is the result of infinitely elastic demand and a linear depreciation of inventories. A change in the assumptions that removes linearity will invalidate these results. In the perfect competition model this can only occur if inventory depreciation is nonlinear. The monopoly case, examined below, shows what happens if the valuation function is strictly concave.

II. Monopoly With Known Current Demand

Unlike the perfect competition case, the monopoly problem is not straightforward. However, if the firm is allowed to observe the current demand schedule (but not future ones!) before making its decisions, the problem is straightforward. Therefore, the simpler problem of known current demand is treated before moving to the model of monopoly with unknown current demand.

A firm makes its output, price, and minimum inventory decisions after observing its current (downward-sloping) demand curve, but without knowing future demand curves.

Assumption 4 is modified as follows:

(4)' Demand $= d(P_t,U_t)$

where U_t is an element of a stochastic process. d is twice continuously differentiable with $\partial d(P_t,U_t)/\partial P_t \equiv d' < 0$, $\partial d(P_t,U_t)/\partial U_t > 0$. $\partial^2 d(P_t,U_t)/\partial P_t^2 \equiv d'' \geq 0$ but $\partial^2 P_t d(P_t,U_t)/\partial P_t^2 = Pd'' + 2d' < 0$. Also, for large P, $d(P,U)$ has price elasticity greater than one. The demand curve is downward

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[8] Decisions being independent of inventories is an important property. The multi-period problem can be treated as a series of single-period problems as existing inventories do not act as a barrier to entry. See [6].

sloping and convex, but the revenue function is strictly concave in price and decreasing in price for large price. The stochastic process $\ldots, U_t, \ldots$ has the same properties as the $\ldots, P_t^m, \ldots$ stochastic process in the perfect competition case and bears the same relationship to the Markov process $\ldots, M_t, \ldots$.

There are three state variables, beginning of period inventories, H, the realized stochastic term in demand, U, and the sufficient statistic for future demand, M. Because current demand is known, there are two decision variables, output and price. $r(X,P;\ H,U,M) = PS - c(X) = Pd(P,U) - c(X)$; $P, X \geq 0$, $d(P,U) \leq X + H$. The firm does not sell less than demand, otherwise it could increase the price and increase revenue without decreasing next period inventory. Assumption (4)' guarantees that $r(X,P;\ H,U,M)$ is bounded above as $Pd(P,U)$ is bounded above.

Now let us examine the firm's problem.

The firm's maximization problem can be written:

$$\text{(I)}' \qquad w(H,U,M) = \sup_{\substack{X(H,U,M) \\ P(H,U,M)}} E\{\sum_{t=0}^{\infty} \beta^t [P_t d(P_t,U_t) - c(X_t)]\}$$

subject to

$$X_t,\ P_t \geq 0$$

$$1/\gamma H_{t+1} = X_t + H_t - d(P_t,U_t) \geq 0.$$

The current profit function is strictly concave, and the constraint functions are concave. Therefore, the decision functions are continuous in the state variables (single valued). The "value" of the firm can be defined by the recursion relation:

$$\text{(II)}' \qquad w(H,U,M) = \sup_{\substack{P,X\geq 0 \\ d(P,U)\leq X+H}} \{Pd(P,U)+\int_0^1\int_0^1 \beta w[\gamma[X+H-d(P,U)],U',M']f(U',M'|M)dU'dM'$$

$$-c(X)\} \equiv \Phi w(H,U,M).$$

Because the current return function is continuous and bounded and a constant rate of discount is used, $w(H,U,M)$ exists, is unique, and is a continuous bounded function. Further, $w(H,U,M) = \lim_{n\to\infty} \Phi^n v(H,U,M)$ for v any continuous bounded function on E^3.[9/] Φ maps continuous bounded functions concave and nondecreasing in H and nondecreasing in U and M into continuous bounded functions strictly concave and increasing in H and increasing in U and M. Therefore, $w(H,U,M)$ is concave in H and nondecreasing in H, U, and M. Therefore, $w(H,U,M) = \Phi w(H,U,M)$ is strictly concave in H and increasing in H, U, and M. $w(H,U,M)$, while continuous, need not be differentiable. However, $w(H,U,M)$ is treated as twice continuously differentiable.[10/]

We now analyze the optimal decision functions of these monopolists.

Define

$$\Phi^*(X,P)w(H,U,M) = r(X,P;\ H,U,M) + \beta E[w(H',U',M')]$$

so that

$$w(H,U,M) = \sup_{\substack{P,X\geq 0 \\ d(P,U)\leq X+H}} \Phi^*(X,P)w(H,U,M).$$

[9/] See [9].

[10/] Differentiability is a convenience only. The results hang on concavity not differentiability and can be worked through for finite changes. Moreover, it can be shown that the decision functions generated by a sequence of two smooth, concave functions converging to w themselves converge to the optimal decision functions uniformly over a compact subset of $[0,1]X[0,\infty)$. So replace "w[H,M]" by "$\Phi^n 0$" where 0 is the zero function.

The strict concavity of r and w imply the strict concavity of ϕ^*. Once again, as the reader can verify, X and P can be restricted to a compact set without changing the maximization problem, so that "sup" can be replaced by "max." The first-order conditions on P and X required for maximization are:

$$P:\quad Pd'(P,U) + d(P,U) - d'(P,U)\int_0^1\int_0^1 \beta\gamma\partial w/\partial H f(U',M'|M)dU'dM'$$

$$- \lambda d'(P,U) \leq 0, = \text{if } P > 0$$

$$X:\quad \int_0^1\int_0^1 \beta\gamma\partial w/\partial H' f(U',M'|M)dU'dM' - c'(X) + \lambda \leq 0, = \text{if } X > 0$$

$$\lambda:\quad X + H - d(P,U) \geq 0, = \text{if } \lambda > 0$$

where λ is the Langrangian multiplier.

The firm is speculating in inventories if X + H d(P,U). As the current demand is known, the firm is not withholding goods from sale when X + H > d(P,U), but he is purposefully setting price high enough so that they will not all sell. There is no unintended saving in this model as the firm never does stock out. Of course, there is no unintended accumulation or decumulation of inventories either. Substituting the first-order condition of output into that of price yields marginal revenue equals marginal cost. The first-order condition of price implies that marginal revenue equals or exceeds discounted marginal worth of inventories next period. You don't sell goods for less than they are worth to you as inventories. The first-order condition of P also shows when speculation will occur. If, at optimal P and X, the discounted expected marginal worth of inventories next period evaluated at zero inventory level exceeds marginal revenue (equals marginal cost), there will be speculation in inventories.

Φ^* is strictly concave. Therefore, to derive the effect of a state variable or interim parameter change on the firm's decisions, the first-order conditions can be totally differentiated and the changes solved for by Cramer's rule.

(A) Inventory Change

The impact of initial inventory stock upon output and price is the same as found by Edward Zabel, namely, $-1 < dX/dH < 0$, $dP/dH < 0$.[11/] Inventory carried into the next period, $X + H - d(P,U)$, is increasing in initial inventory stock. With marginal cost constant $dX/dH = -1$, $dP/dH = 0$. The second result is as sS policy with s and S coinciding because there are no fixed costs of purchasing ($c'(X)$ exists at zero).[12/]

(B) Change in Current Demand (Change in U)

The effects of a change in U on output and price are of ambiguous sign. The fact that an increase in U shifts the demand curve out is not sufficient to guarantee that higher U implies higher output and price. *Ceteris paribus* higher U implies higher output, but it also implies higher price *ceteris paribus*, which affects output negatively. It cannot be determined which effect dominates.

This indeterminancy is not, however, the result of the multiperiod structure or the introduction of inventories and uncertainty to the monopoly model. The problem is that with an increase in U, at the new optimal price and output, demand may be much more or much less elastic so that price or output may be at a reduced level. In order to rule this out, additional constraints must be placed on the demand function.

[11/] See [15].

[12/] See [1].

The demand function cannot be too convex in price, and the derivative of demand with respect to price cannot be too strongly increasing or decreasing in U.[13/] If these conditions hold, X and P are both increasing in U, and the amount of inventory carried into the next period, $X + H - d(P,U)$, is decreasing in U. For example, if demand is linear in price and additive in the stochastic term U, then $dX/dU,\ dP/dU > 0 > d(X+H-d(P,U))/dU$.

(C) Change in Demand Anticipation (Change in M)

The effect of a change in M on output and price depends on the sign of $\int_0^1\int_0^1 \beta\gamma \partial w/\partial H' \partial f(U',M'|M)/\partial M dU' dM'$. The fact that w is increasing in M is not sufficient for higher M to imply higher output and price. Higher M implies that the probability of high demand is increased and the probability of low demand decreased in all future periods (given prices in all future periods). Nevertheless, high anticipated demand does not necessarily imply higher output and price. Just as technological improvement does not necessarily imply that more capital will be used, higher anticipated demand does not necessarily imply that more goods will be produced.

Assume that the conditions on the demand function described in the previous section hold so that $\int_0^1\int_0^1 \beta\gamma \partial w/\partial H' \partial f(U',M'|M)/\partial M dU' dM' > 0$.[14/] Higher anticipated demand implies higher expected marginal value of inventories. Then $dX/dM,\ dP/dM > 0$.

[13/] The precise restrictions are complicated and unenlightening. They can be derived using the simple one-period monopoly problem, $\Phi 0$ where 0 is the zero function. Note that $\Phi^n 0 \to w$.

[14/] If $v(H,U,M)$ has this property, so does $\Phi v(H,U,M)$ given these conditions on the demand function.

The case where optimal X and P satisfy $X + H = d(P,U)$ within an open neighborhood of (H,U,M) is a one-period maximization problem which is not discussed.

III. Monopoly With Unknown Current Demand

The previous model is revised by making the current error term, U_t, unknown at the time decisions are made. M_t is a sufficient statistic for U_t, U_{t+1}, ... There are three decision variables, output, price, and minimum inventory, and two state variables, H and M. Otherwise the model is the same.

Let us examine the properties of the firm's problem.

The firm's maximization problem can be written:

$$(\mathrm{I})'' \qquad w(H,M) = \sup_{\substack{X(H,M)\\ P(H,M)\\ H^m(H,M)}} E\{\sum_{t=0}^{\infty} \beta^t [P_t \min[X_t+H_t-H_t^m, d(P_t,U_t)] + \beta w(H_{t+1},M_{t+1}) - c(X)]\}$$

subject to

$$H_t^m \leq X_t + H_t$$

$$M_{t+1} = h(M_t,U_t)$$

$$1/\gamma H_{t+1} = \max[X_t+H_t-d(P_t,U_t),H_t^m].$$

The function $\max[X_t+H_t-d(P_t,U_t),H_t^m]$ is not concave. Therefore, the firm is maximizing over a constraint set which is not convex. The decision functions need not be continuous (single valued), and the valuation function need not be concave in inventories. There is no hope of proving $w(H,M)$ concave in general. Edward Zabel proves concavity in the special case in which the U's are i.i.d. with the exponential distribution, demand is additive in the U's, and H^m is constrained to be zero.[15/]

Let $g(Z,P)$ be such that $d(P,g(Z,P)) = Z$. $F[g(X+H-H^m,P)|M] \equiv F(g)$ is the probability the firm does not stock out. The value of the firm can be defined by the recursion relation:

$$\text{(II)''} \quad w(H,M) = \sup_{\substack{X,P,H^m \geq 0 \\ H^m \leq X+H}} \{P\int_g^1 [P(X+H-H^m)+\beta w(\gamma H^m,M')]f(U|M)dU$$

$$+ \int_0^g [Pd(P,U)+\beta w(\gamma(X+H-d(P,U)),M')]f(U|M)dU-c(X) \quad = \Phi w(H,M).$$

Once again, it can be shown that Φ has a unique fixed point in the set of bounded continuous functions which solves (II)". The fixed point $w(H,M)$ is increasing in H and M. For the same reason as before, "sup" can be replaced by "max." Define Φ^* as before. While $w(H,M)$ is not necessarily concave in H, we assume Φ^* to be strictly concave in X, P, and H^m within an open neighborhood of (H,M). Further assume that the decision functions are continuous within this neighborhood. With these assumptions we can proceed as before and derive the effects of (small) changes of state variables and parameters upon output, price, and minimum inventory. In essence we are assuming that the uncertainty in the current period is not in some sense too large.

We turn now to the decision functions of these firms.

The first-order conditions required for maximization are:

$$P: \quad (X+H-H^m)\int_g^1 f(U|M)dU + \int_0^g (Pd'+d)f(U|M)dU$$

$$- \int_0^g \beta\gamma \partial w/\partial H' d' f(U|M)dU \leq 0, = \text{ if } P > 0$$

[15/] See [15].

$$X: \quad P\int_g^1 f(U|M)dU + \int_0^g \beta\gamma\partial w/\partial H' f(U|M)dU + \lambda \leq c'(X), = \text{ if } X > 0$$

$$H^m: \quad -P\int_g^1 f(U|M)dU + \int_g^1 \beta\gamma\partial w/\partial H' f(U|M)dU - \lambda \leq 0, = \text{ if } H^m > 0$$

$$\lambda: \quad X + H - H^m \geq 0, = \text{ if } \lambda > 0.$$

Notice the similarities with the first-order conditions of the perfect competitor. Adding the first-order conditions of X and H^m yields the result that marginal cost is greater than or equal to discounted expected marginal worth of inventories with equality if X, $H^m > 0$. Assume for the moment that d(P,U) is additive in U so that d' is independent of U. Then, for X, P > 0, substituting the first-order condition of output into that for price yields marginal cost greater than or equal to marginal revenue with equality if $X + H - H^m > 0$.

Suppose for the moment that the U_t's are independent and identically distributed so that M is constant. The first-order condition of P guarantees that

$$\int_0^g [P-\beta\gamma\partial\frac{w(\gamma(X+H-d),M')}{\partial H'}]f(U|M)dU > 0.$$

However, this does not imply that $H^m = 0$. The relevant term is

$$\int_g^1 [P-\beta\gamma\partial\frac{w(\gamma H^m,M')}{\partial H'}]f(U|M)dU$$

and w may be concave in inventories over a range so that this term may be negative. Therefore, in the monopolist case with current demand unknown and demand curves i.i.d. there may be speculation in inventories! If the marginal value of inventories is high at low levels of inventory, the firm may set minimum inventory above zero.

(A) Changes in Inventory Stock

Exactly as in the case (II) it can be shown that $dX/dH > -1$ whether $H^m = 0$ or $H^m > 0$. Further, if $w(H,M)$ is concave in inventories, $dX/dH < 0$. However, dP/dH has an ambiguous sign. With $H^m = 0$ the sign of dP/dH depends upon the sign of $\partial^2\phi^*/\partial X\partial P$ which is now ambiguous. A higher price does not necessarily imply a lower output ceteris paribus. With $H^m > 0$ the sign of dP/dH depends upon $\partial^2\phi^*/\partial X\partial H^m$ and $\partial^2\phi^*/\partial P\partial H^m$ as well. dH^m/dH depends upon these same factors.

(B) Changes in Demand Anticipation (M)

With current demand unknown it is not possible to get unambiguous signs for dX/dM and dP/dM. As in case (II), $\partial^2 d(P,U)/\partial P^2$ and $\partial^2 d(P,U)/\partial P\partial U$ must be restricted. In addition, assume that $w(H,M)$ is concave in inventories. With these additional assumptions it can be shown that dX/dM, $dP/dM > 0$. However, dH^m/dM is of ambiguous sign.

In the case (II) where current demand is known an increase in current demand has a negative impact upon minimum inventory and an increase in future demand has a positive impact. This suggests that we examine the case where there is an increase in anticipated future demand without an increase in anticipated current demand. This can be achieved in the framework of the current model by investigating the impact of an antonomous increase in the marginal value of inventories (assuming once again that increased anticipation does increase the marginal value of inventories). We find that an increase in anticipated future demand, anticipated current demand held fixed, increases output, price, and minimum inventory (if $H^m > 0$).

Conclusions

In the perfect competition model output and price are independent of initial inventory stock, while minimum inventory equals zero or output plus inventory stock and the decision is uninfluenced by inventory stock. In contrast, in the monopoly model with current demand known, output and price are decreasing in initial inventory stock, and inventory carried into the next period is increasing in initial inventory stock. Finally, in the monopoly model with demand unknown, output is decreasing in initial inventory stock, but the signs of the effects upon price and minimum inventory are ambiguous.

If market prices are i.i.d. and current market price is unknown, there is no speculation in inventories by the perfect competitor. This results from the linear valuation function of the perfect competitor. Even if demand curves are i.i.d. and current demand is unknown, the monopolist may still speculate in inventories by withholding goods from sale at a price he has set.

Output, price, and minimum inventory (speculative holdings) do not necessarily increase if future demand curves are expected to shift "outward." Just as technological improvement does not necessarily imply greater use of capital, higher anticipated future demand does not imply higher output, price, and speculative holdings, even though the value of the firm is unambiguously increased. However, if the demand curve is not highly convex and if its slope is not greatly affected by the stochastic term, the marginal value of inventories is increasing in anticipated demand. With these assumptions, output, price, and speculative holdings of inventories are increasing in anticipated future demand. The impact of higher anticipated current demand upon firm decisions also is ambiguous. If the above restrictions on the demand curve are imposed, output and price are increasing in current demand, but speculative holdings are decreasing in current demand given anticipated future demand.

It is possible to give an unambiguous interpretation to "an increase in anticipated demand (or market price)" in an infinite horizon framework with demand uncertainty. Moreover, firm decisions react in the expected way to such an increase in anticipated demand.

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