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National Debt

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Nontransferable Interest-Bearing National Debt

by John Bryant

In earlier papers, Bryant and Wallace (3) and (4), the inefficiency of interest-bearing national debt is studied. In those papers, examples are given in which bonds do not dominate money because bonds are issued in too large denominations. A costly technology for subdividing debt is assumed. This paper extends the previous analysis by considering bonds which do not dominate money because the bonds are nontransferable. It also differs from the previous papers in that bonds have a role to play in the economy. It is assumed that transferable assets are subject to an uninsurable stochastic loss, while the nontransferable bonds are not. As a result, the optimal ratio of bonds to money plus bonds is not zero. However, at the optimal ratio of bonds to money plus bonds, the rate of interest on bonds is zero. This result that the rate of interest on bonds should be zero can be overturned by assuming that the cost to government of servicing money and bonds are not the same.

The Model

The model employed in our analysis is a version of the Samuelson (5) pure consumption-loans model.^{1/} N individuals are born each period, and they live three periods. In their first period of life individuals are endowed with $k > 0$ units of the single transferable but nonstorable consumption good, while in their second two periods of life individuals are endowed with nothing. Their common increasing, concave utility functions have as arguments the individual's consumption of the consumption good in his three periods of life. There exist M units of fiat money which the young get from the middle-aged or old in exchange

^{1/} For a defense of the use of the pure consumption-loans model as the model of fiat money, see Bryant (2) and Wallace (6).

for the consumption good. In addition, there are B units of bonds which the young get from the government in exchange for money. The government costlessly services money and bond holdings. A unit of bonds is a certain claim to a unit of money two periods hence.

Money is subject to a random loss. Each period half the middle-aged individuals lose $1 > \delta > 0$ percent of their individual money holdings, while the other half share the losses equally as lump-sum transfers. Similarly, each period one-half of the old lose δ percent of their money holdings, while the other half of the old share those losses equally. Each individual is equally likely to win or lose, but one-half of the middle-aged winners become old-aged winners. Therefore, each individual at birth faces four possible, equally likely outcomes. Moreover, the losses are not insurable.^{2/} The losses occur before individuals acquire the consumption good in the second and third periods of life. Bonds are not transferable and are not subject to this random loss. However, when the government pays off on a bond that money is included in the stock of money of the recipient and is subject to the same percentage loss.

Let U be the individual's utility function, C_1, C_2, C_3 , his consumption of the consumption good in his three periods of life, m his money holdings at the beginning of his second period, m' his money holdings at the beginning of his third period, k his bond holdings, i the state of his random drawing on money in his middle age (1 "loser," 2 "winner"), and i' the state of his random drawing on money in his old age (1 "loser," 2 "winner"). Let P be the goods price of money and S be the money price of bonds. For simplicity we will consider only the stationary solution where P and S are constant through time (one can impose utility functions that ensure that this is the only monetary equilibrium, see

^{2/} Because, for example, it's impossible to prove which half of the population the individual belongs to, the finders or the losers. This device appears in Bryant and Wallace (4).

Bryant (1)). To ensure internal solutions we assume that $U_1(0, C_2, C_3) = U_2(C_1, 0, C_3) = U_3(C_1, C_2, 0) = \infty$.

The young individual's problem can be displayed as:

$$\max_{m, b, m'(i)} EU[C_1, C_2(i), C_3(i, i')]$$

subject to

$$C_1 = k - Pm - PSb \geq 0$$

$$C_2(1) = P(1-\delta)m - Pm'(1) \geq 0$$

$$C_2(2) = P(m+\delta\bar{m}) - Pm'(2) \geq 0$$

$$C_3(i, 1) = P(1-\delta)(m'(i)+b) \geq 0$$

$$C_3(i, 2) = P[m'(i)+b] + P\delta[\bar{m}'+\bar{b}] \geq 0$$

$$m \geq 0$$

$$b \geq 0$$

$m'(i)$ is a function with domain $\{1, 2\}$ and range $(0, \infty)$

where \bar{m} , \bar{m}' , \bar{b} are the per capita money and bond holdings of the other half of the population, which the individual treats as given. However, to generate the proposition we are interested in we need not work with this formulation.

We are now ready for our central proposition concerning the issuance of money and bonds. Let z be the ratio of bonds to money plus bonds after the young have made their purchases of bonds.

Theorem 1: Considering only the current young and future generations, the unique optimum ratio of bonds to money plus bonds, z^* , is characterized by interest rate on bonds equal to zero, $S = 1$, and is the only ratio of bonds to money plus bonds with $S = 1$.

Proof: Suppose that, unlike the above problem, individuals can costlessly convert money to bonds or the reverse in their first period of existence. Then in equilibrium $S = 1$ for $1 > z \geq 0$, for if $S < 1$, individuals demand only bonds and for $S > 1$, individuals demand only money. Because of the convexity of the individual's problem, he will create a unique ratio of bonds to money plus bonds at $S = 1$, \bar{z} , say. Consider now our original model with individuals constrained not to convert bonds to money or the reverse. If the government imposes the same ratio of bonds to bonds plus money in the aggregate as created by the individual in the unconstrained case, \bar{z} , at $S = 1$ the constraint is not binding and there is no excess supply or demand for money and bonds. Therefore, $S = 1$ is the equilibrium at $z = \bar{z}$. Moreover, as that equilibrium $S = 1$, $z = \bar{z}$ is optimal in the unconstrained case, and as welfare in the constrained case is at best no better than welfare in the unconstrained case, then $S = 1$, $z = \bar{z}$ is also optimal in the constrained problem. By the convexity of the individual's problem in the constrained case, there is a unique ratio of bonds to money plus bonds that yields $S = 1$ and a unique optimum ratio of bonds to money plus bonds. Therefore, at $S = 1$ the unique optimum ratio $z^* = \bar{z}$ is achieved. Q.E.D.

It is not necessarily true that moving from $z = z' \neq z^*$ to $z = z^*$ is welfare improving. While the current young and future generations are made better off, the current middle-aged and old may be worse off if the current young's demand for money falls. For the move to $z = z^*$, $S = 1$ to be welfare improving, it is sufficient that the improved return on saving for the young causes the young to save more, that current and future consumption are gross substitutes.

Embellishments

The technology of money and bond holding in the above model is easily modified to make it more "realistic." We could assume, for example, that bonds can be converted to money in either the second or third period but that conversion of any amount entails a fixed or a proportional conversion cost. For the option of converting in the second period to be meaningful, it would be necessary to assume that the individual could convert after observing his loss of money and/or that bond-converted money is not subject to random loss. The fixed conversion cost can be viewed as a "trips to the bank" technology. One could also assume that the private sector has costly technologies for converting (risky or riskless) transferable assets into nontransferable assets and the reverse. While such modifications would change the optimal z , it would not affect the above theorem that the optimal z is characterized by $S = 1$. Our model differs from the previous Bryant and Wallace, (3) and (4), models in that a positive interest rate on bonds does not reflect a real resource cost. However, addition of the above costly technologies does add this real resource cost element to the model.

What would alter our conclusion that optimal z implies zero interest on bonds? The proof of the above theorem indicates how this can be done. In the proof optimal z is determined by examining the problem in which the individual has access to the costless technology of the government of exchanging bonds for money. In the original restricted problem this was replicated at $S = 1$. Suppose now that the government has costly technologies of servicing money and bonds. Similarly, the optimal z can be determined by including the marginal rate of transformation faced by the government into the individual's problem, if the cost functions are nicely shaped. If the optimal z is internal, $0 < z^* < 1$, this is replicated in the restricted problem by setting aggregate z such that the

resulting $1/S$ equals the resulting ratio of the government's marginal cost of servicing money to its marginal cost of servicing bonds. The optimal equilibrium rate of interest is positive if the implied marginal cost of bond servicing is less than the marginal cost of money servicing. The costliness of government servicing of money and bonds, particularly when coupled with private technologies of transformation, do raise the possibility that $z = 0$ or $z = 1$ is optimal.

Conclusion

In this model of nontransferable bonds at the optimum distribution of the government debt between money and bonds, the ratio of the face value of a bond to its market price equals the government's marginal rate of transformation between money and bonds. Naturally, there is good reason to suppose that this is a general result.

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