Historical

Costly Information and the Stock Market

John Bryant

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Costly Information and the Stock Market

by John Bryant

In a recent article, Sanford Grossman [4, p. 94] states, "It is clear that if information is costly, then no competitive equilibrium exists which reveals information . . . . It is very important to note that each trader assumes that the equilibrium price random variable will not be affected by his decision to buy information . . . ."

The seeming paradox can be described as follows. If traders gather costly information, then the stock market reflects that information, and there is no return to be made on the information. If traders do not gather information, then the stock market does not reveal the information, and there may be a profit to be made in gathering information. Therefore, there may be no Nash equilibrium. This seems to imply that costly information involves the well-known problem of an everywhere-decreasing marginal cost technology: marginal cost pricing does not recoup costs. Gathering information is costly, but using it is not. This further implies that costly information provides the stock market with a severe pricing problem.

However, in reading Grossman's statement, it is unclear whether costly information implies a problem for the stock market or for the equilibrium concept used. We contend, in this note, that the problem lies with the equilibrium concept, or more accurately, in the description of the strategy space. The relevant solution concept is, indeed, Nash equilibrium. But the trader should not take the price vector as given, but rather the strategies of the other traders, that is to say, the strategy of the auctioneer.[1/] This is argued in the context of a simple, coherent, general equilibrium model. To peek ahead, the

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[1/] This is analogous to the result that there is no pure strategy Nash equilibrium for price setting competitive firms. See Bryant [2].

reason that the decreasing marginal cost problem does not arise is that individuals do not collect costly information for the purpose of investing in the stock market. Uninformed traders are a Nash equilibrium.

The Model

Now we turn to our simple model. It differs from standard models of the Finance literature in being a coherent general equilibrium model.

There are a continuum of individuals indexed by $x \in [0,1]$. They live two periods. Each individual is equally endowed with leisure, $L(x) = L$, in his first period of life. They are endowed with nothing in their second period of life. There exists, for each individual, a technology for transforming work this period into the single transferable consumption good next period. These technologies work as follows. With probability .5, the individual gets two units of goods for each unit of work; and with probability .5, the individual gets one unit. These individual drawings are independent. Each individual can also transform leisure into the consumption good in her first period one-for-one. The individual's utility is a function of her consumption of the consumption good in her two periods of life. All individuals have the same utility function, and it is strictly concave, differentiable, and additively separable.

$$U(C_1(x),C_2(x)) = U_1(C_1(x)) + U_2(C_2(x))$$

where $C_1(x)$, $C_2(x)$ are first- and second-period consumption of the individual. Moreover, $U_1'(0) = U_2'(0) = \infty$. Henceforth, we will deal with the representative individual and drop the index x.

Now let us add the stock market. The risk of individual technologies can be perfectly diversified against. Claims against individual technologies are costlessly bought and sold using the first-period consumption good in exchange, and each individual ends up with the safe portfolio. The market is run by an auctioneer who settles on the prices at which supply and demand are equated

for each x. It is this strategy of eliminating nonzero excess demand which the individual takes as given, not a particular price decision of the auctioneer.

Let W be the individual's amount of work, S his holding of the safe portfolio, and P the goods price of the safe portfolio. The price of the safe portfolio is, of course, the price of claims on individual technologies. The individual's problem can be written:

$$\underset{W,S}{\text{Max}} \quad U_1(C_1) + U_2(C_2)$$

$$\text{s.t.} \quad C_1 = L - W + P(W-S)$$

$$C_2 = 1.5\ S$$

with equilibrium condition W = S. It follows that P = 1.

Now suppose that after the stock market opens, individuals learn the outcome of their own technology. They will choose to buy their portfolio before they learn this so they can diversify. As soon as individuals discover their outcome, they either try to buy the claims on their own technology, or to sell them short. The market clearing price for a technology is then P(x) = 4/3 if the technology is good, or P(x) = 2/3 if it is bad. Moreover, no trades take place. To see this in another light, suppose, for the moment, that there is no auctioneer. It is clear from the symmetry of the problem that a core solution implies that economy can be divided into two groups containing equally well-off people--those learning of a good technology, and those learning of a bad one. But these groups must be equally well off, too, because no trade is always an option. We conclude that the outcome is no trade following the disclosure of the information. Does the market reflect the information? As in practice, prices are only registered when a trade occurs, it seems a reasonable interpretation that prices do not reflect the information.

Naturally, if free information is not reflected in the stock market prices, costly information is not either. Clearly, then, costly information is not gathered, for if free information is not exploitable, neither is costly information. Suppose a deviant individual does gather information. As he is the only person desiring to trade, his supply or demand is market excess supply or demand. His information is revealed, and no trades take place. He gets no return on his incurred cost. No one wants to trade with an informed person, and only the informed want to trade.

But ours is the limiting version of a "thin" market, no trades at all. We now make the market "thick," and show how this influences our results. To look ahead, in this circumstance prices reflect free information, but not costly information which is still not gathered.

First, let us generate a "thick" stock market. Suppose after individuals make their work decisions and buy their portfolios of stocks, $\alpha \cdot 100$ percent of them learn they are going to die between periods.[2/] This knowledge comes only to the affected individual, and is not verifiable. The "early diers'" utility function becomes $U_1(C_1)$. They sell their portfolio of stocks for goods to healthy individuals. Let P' be the price that the "early diers" get for their portfolio, and S' be the purchase of additional stock by the healthy individuals. The individual's problem now can be written

$$\max_{W,S} \{(1-\alpha)\{\max_{S'}[U_1(L-W+P(W-S)-P'S')+U_2[1.5(S+S')]]$$

$$+ \alpha U_1[L-W+P(W-S)+P'S]\}$$

with equilibrium condition $W = S = \frac{(1-\alpha)}{\alpha}S'$. It is easily seen that this implies $P = P' = 1$. The "early diers" make neither capital gain nor loss. Because being

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[2/] This device was introduced in Bryant [3].

an early dier is not verifiable, the market cannot share the risk of being an "early dier," however.[3/]

Now let us suppose that, once again, individuals learn the outcome of their own technology, and at the same time that they learn whether or not they will die early. Once again, the market clearing prices settled on by the auctioneer reveal the information. The sales by the "early diers" are known beforehand, and there is an equal demand for their diversified portfolios. Only their identity is not known. The information alone impacts on prices. The early diers are no better or worse off because of the information, as they hold diversified portfolios. However, as the early diers enter sales in all stocks, the observed prices do reflect the information. Moreover, if information is costly, it is not collected, as any informed deviant impacts price completely. Therefore, costly information is not reflected in observed prices.

Now let us suppose that the market does not act as if there is an auctioneer, and costly information is available. Irreversible sales are made before supplies and demand are equated (no recontracting). Then one can well suppose that, depending on the queueing of orders, the informed can buy and sell at unchanged prices until the normal value of "early diers" transactions are met. Then prices change to reflect the information. The informed make some profit, and the early diers take some capital loss. Further, one could suppose that the number of information gatherers would rise until the value of the capital gain from information just equals the cost of information.

The suppositions of the previous paragraph are wrong, however. Previously, in the "thin" market, we argued that a group of individuals could

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[3/] This requires that early death is not verifiable before second-period consumption. Otherwise a liar could be penalized in second-period consumption.

guarantee itself no capital losses by refusing to trade when information became available. This is not possible for the "early diers," however, they must trade. Nevertheless, there still is a strategy which the uninformed "early diers" can use to protect themselves. They hold a diversified portfolio of stocks. Rather than selling the stocks individually, they can refuse to disbundle, and just sell shares of this portfolio.[4/] If they do so, the portfolios sell at unchanged prices, and information gatherers cannot use their information. Therefore, the information is not collected.

We conclude that costly information is not gathered. In the model, the information has no social value. The market does not, then, produce the distortion of resources wasted on the gathering of socially useless information. But what about useful information? There appears to be no reason that useful information cannot be sold to those needing it like any other input to the production process. There is no reason why such information should be transmitted via the stock market.

[4/] Are these demand deposits? See Bryant [3].

References

[1] Bryant, J. "Nontransferable Interest-Bearing National Debt," Journal of Finance (forthcoming).

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[3] _____. "A Model of Reserves, Bank Runs, and Deposit Insurance," Working Paper #126, Federal Reserve Bank of Minneapolis, 1979.

[4] Grossman, S. "Further Results on the Informational Efficiency of Competitive Stock Markets," Journal of Economic Theory 18, 1978, 81-101.