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VARIABLE RATE SUBSIDIES: THE INEFFICIENCY OF IN-KIND TRANSFERS REVISITED

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Variable Rate Subsidies: The Inefficiency of In-Kind Transfers Revisited

The inefficiency of fixed rate price subsidies for consumer goods is one of the best-known propositions in welfare economics. Among others, Aaron and Von Furstenberg and Smolensky have used this proposition to examine the inefficiency of housing subsidies when compared to cash transfers. Haskell, Thurow, and Wilde have also used it to show that recipient governments will prefer intergovernmental aid with no strings attached to equally costly, fixed rate matching grants. Finally, Friedman has used the proposition to promote the negative income tax as a replacement for food stamps and other in-kind transfers.

In addition, these studies often cite another drawback of fixed rate price subsidies. In the absence of a good estimate for the price elasticity of the subsidized good, the sponsoring government cannot determine the total amount of aid to be distributed. While the sponsoring government can place a "cap" on the total amount of aid to be distributed, doing so introduces a kink into the budget constraint of the recipient. This kink complicates the problem of predicting the recipient response to changes in the subsidy rate.[1/]

In this paper, I show that a broad class of variable rate price subsidies dominates fixed rate price subsidies on both counts. Recipients always like them better than equally costly fixed rate subsidies, and the total amount of aid can be fixed in advance without introducing troublesome kinks into the budget constraint.

The variable rate price subsidies examined herein have an additional advantage. The rate of subsidy can be designed to vary among recipients, depending on their socioeconomic characteristics. For example, a special case of the variable rate subsidy is found in the federal government's three-factor General Revenue Sharing (FGRS) formula. Through a tax effort factor incorporated into it, FGRS lowers the price of government services provided by recipient state and local governments. The rate of price reduction depends on the recipient governments' population and personal income.

After a brief review of the theory of fixed rate subsidies, I define the class of variable rate subsidies to be studied and demonstrate their dominance in the two areas cited above. The demonstration assumes that all recipients are identical, an assumption which is dropped in the final section. There, as an application of the general theory, I estimate the differences between FGRS in 1972 and a hypothetical, fixed rate subsidy calculated to provide each state government with their actual 1972 FGRS allotment.

A Brief Review

A brief review of the standard inefficiency argument is in order. There are N recipients of a price subsidy, the i^{th} of which is assumed to allocate its disposable income $(1-t)M_i$ by maximizing a utility U^i of the subsidized good G_i and a composite unsubsidized good C_i. Units are chosen so that prices are initially equal to one. In addition, the analyst splits spending on G_i into two components: spending from disposable income T_i, and the dollar amount of subsidy R_i, so that:

(1) $$G_i = T_i + R_i.$$

A fixed rate price subsidy is calculated as a fixed rate r_i of T_i, i.e.

(2) $$R_i = r_i T_i.$$

Consumption of the unsubsidized composite good is then given by:

(3) $$C_i = (1-t)M_i - T_i.$$

The problem faced by the i^{th} recipient is to maximize $U^i(C_i,G_i)$ subject to (1), (2), and (3). Solving (3) for T_i and substituting into (2), find

(4) $$R_i = r_i[(1-t)M_i-C_i]$$

Substituting for T_i and R_i in (1) and simplifying yields the budget constraint:

(5) $$C_i + \frac{1}{1+r_i} G_i = (1-t)M_i$$

which clearly shows that (2) is a fixed rate price subsidy of G. The i^{th} recipient is thus assumed to:

(6) $$\max U^i (C_i,G_i)$$

$$\text{s.t. } C_i + \frac{1}{1+r_i} G_i = (1-t)M_i.$$

Assuming an interior solution (CMG,GMG) exists, it is characterized by:

(7) $$\frac{\partial U^i}{\partial C_i} / \frac{\partial U^i}{\partial C_i} (CMG,GMG) = 1+r_i$$

The solution occurs at the tangency MG depicted in figure 1. Kay, Rosen, and Stutzer (1982) measure recipient i's deadweight loss with an equivalent variation-based measure, which for a subsidy is:

(8) $$W_i = R_i - EV_i,$$

where EV_i is the equivalent variation in income needed to produce the same utility U_o as did the fixed rate subsidy. W_i is the largest amount of money recipient i would be willing to forgo in order to obtain a lump sum subsidy rather than a fixed rate subsidy. Thus, W_i measures the savings the sponsoring government could attain by replacing a fixed rate price subsidy with an equal utility producing lump sum subsidy, EV_i.

Finally, it is clear that the total amount of subsidies, denoted Q, cannot be fixed in advance without detailed knowledge of each recipient's problem (6), because:

$$(9) \qquad Q = \sum_{i=1}^{N} R_i = \sum r_i \, TMG_i$$

Variable Rate Subsidies

A broad class of variable rate subsidies for both consumer and intergovernmental aid can be created by generalizing the so-called three-factor or "Senate" formula used, in part, to distribute Federal General Revenue Sharing funds to the states.[2/] The formula is:

$$(10) \qquad R_i = I_i(T_i)Q = \frac{w_i T_i}{\sum_{j=1}^{51} w_j T_j} Q \; ; \; i=1, \ldots, 51$$

where R_i is the aid to state i.[3/] Note that $\sum_{i=1}^{51} R_i \equiv Q$, so that Q can be fixed in advance. $I_i(T_i)$ is thus a function giving the fraction of Q distributed to state i. It depends on the level of taxes T_i levied within state i, and a weight w_i reflecting the socioeconomic characteristics of state i. The weight w_i equals the square of the reciprocal of state i's per capita income.[4/] The good being subsidized is state government spending $G_i = T_i + R_i$, as:

(11) $$\frac{\partial G_i}{\partial T_i} = 1 + \frac{\partial R_i}{\partial T_i} = 1 + \frac{\partial I_i}{\partial T_i} Q = 1 + \frac{w_i \sum_{j \neq i} w_j T_j}{(\sum_j w_j T_j)^2} Q > 1 \;,$$

because an increase in state taxes increases its revenue sharing grant, ceteris paribus. Also,note that the marginal rate of subsidy $\partial I_i/\partial T_i$ to state i declines as it spends more of its own revenue T_i, because:

(12) $$\frac{\partial^2 I_i}{\partial T_i^{\,2}} = \frac{-2w_i^{\,2} \sum_{j \neq 1} w_j}{(\sum_j w_j T_j)^3} < 0$$

Thus, unlike the fixed rate subsidy (2), (10) is nonlinear in T_i. It is increasing and concave in T_i. Assuming that state i's share of the fund Q is less than 50 percent, its marginal rate of subsidy also declines when some other state increases its taxes, because:

(13) $$\frac{\partial^2 I_i}{\partial T_i \partial T_k} = \frac{w_i w_k}{(\sum_j w_j T_j)^3} \left[I_i - \sum_{j \neq i} I_j\right] < 0$$

Similar formulae have been used in some states to distribute state revenue to local governments. More generally, for both consumer and inter-governmental programs, one could define a system:

(14) $$R_i = I_i(T_i)Q = \frac{w_i f_i(T_i)}{\sum_j w_j f_j(T_j)} Q, \quad i = 1, \ldots, N$$

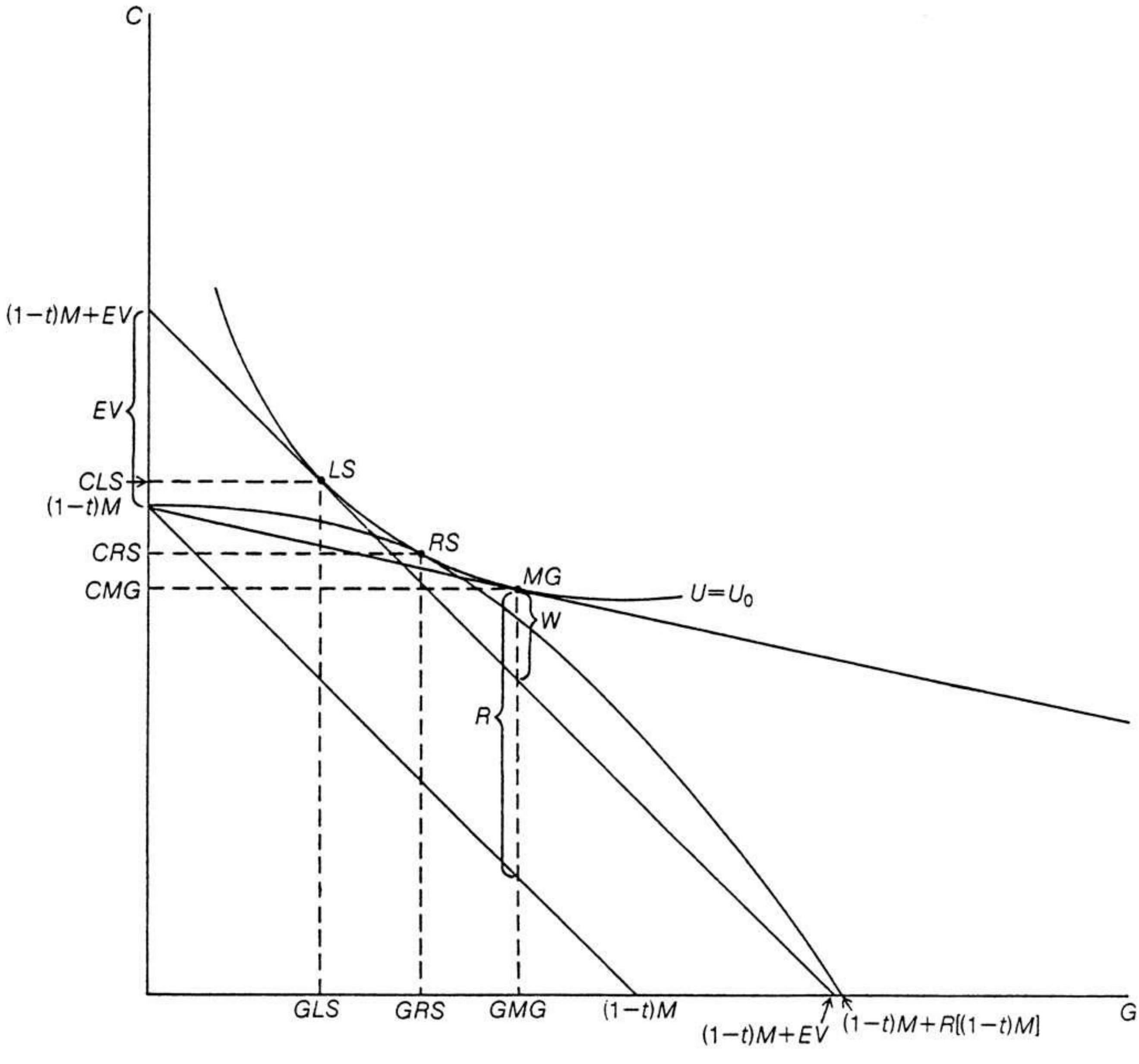

Figure 1: Equal Utility Programs

where f_i is concave and increasing in T_i, the recipient's own-source expenditure on the subsidized good.

Theory of Variable Rate Subsidies

By substituting (10) for (2), a model of recipient response to variable rate subsidies can be created. Following the same steps as done earlier, the i^{th} recipient is assumed to solve:5/

$$(15) \quad \max_{C_i,G_i} U^i(C_i,G_i) \qquad i = 1, \ldots, N$$

$$\text{s.t. } C_i + G_i = (1-t)M_i - I_i((1-t)M_i-C_i)Q$$

$$= (1-t)M_i - \frac{w_i\big((1-t)M_i-C_i\big)}{\sum_{j=1}^{N} w_j\big((1-t)M_j-C_j\big)} Q$$

treating C_j, $j \neq i$ parametrically.

Its solution is characterized by:

$$(16) \quad \frac{\partial U^i}{\partial C_i}\Big/\frac{\partial U^i}{\partial G_i}(C_i,G_i) = 1+\frac{\partial I_i}{\partial T_i}Q = 1+\frac{w_i \sum_{j\neq 1} w_j\big((1-t)M_j-C_j\big)}{\big(\sum_j w_j\big((1-t)M_j-C_j\big)\big)^2}Q; \; i=1, \ldots, N.$$

Once the budget constraint from (15) is solved for G_i and substituted into (16), there results N simultaneous equations in N unknowns $C_1, \ldots, C_N$. Because the i^{th} recipient is assumed to treat C_j, $j \neq i$ parametrically, a simultaneous solution (CRS_i,GRS_i), $i = 1, \ldots, N$ to (16) is a Nash equilibrium for the noncooperative game described by (15).6/ Given values CRS_j, $j \neq i$ in a Nash equilibrium, the i^{th} recipient's budget constraint in (15) is represented in Figure 1 as the concave curve tangent to U_0 at RS. Its concavity follows from the concavity of I_i in T_i. In the figure, the parameters Q and $w_1, \ldots, w_N$ have been set so that the utility U_0 attained is the same as that attained under a fixed rate subsidy (2). At least in this figure, we see that:

(a) The deadweight loss of the variable rate subsidy (which is the analogous vertical distance below RS) is smaller than for an equal utility fixed rate subsidy.

(b) Variable rate subsidies stimulate less spending on the subsidized good G than do equal utility fixed rate subsidies.

In Chapter 3 of Stutzer (1981), both of these properties are shown to hold for general formulae $I_i(T_i)$ which are concave and increasing in T_i, like (14) is. Furthermore, under the additional mild assumption that $\partial^2 U_i/\partial C_i \partial G_i > 0$, it is also rigorously proven there that properties (a) and (b) hold for equal cost, rather than equal utility, subsidies.[7/] How <u>much</u> less the deadweight loss and the spending on the subsidized good is depends on all the utility functions U^i and all the parameters r_i, w_i, $(1-t)M_i$, and Q.

<u>Quantitative Comparison of Fixed and Variable Rate Subsidies</u>

In order to isolate the inherent differences between fixed and variable rate subsidies, one must first control for variations in the utility functions and parameters. To do so, I follow Aaron and Von Furstenberg's study of fixed rate subsidies in assuming that recipients possess identical utility functions, have identical disposable incomes, and face the same fixed rate subsidy r. Also, following Fisher (1981), I assume the weights w_i in (10) have a common value w.[8/] These assumptions will be relaxed in the empirical application which follows this section, though. Because recipients are, for the moment, assumed to be identical, one can drop the subscript i and sum the right-hand side of (16) to obtain the Nash equilibrium condition for the variable rate subsidy (10):

$$(17) \qquad \frac{\partial U}{\partial C}/\frac{\partial U}{\partial G}(CRS,GRS) = 1 + \frac{(N-1)Q}{N^2T} .$$

Making use of the budget constraint in (15) and the fact that N recipients with identical weights w and tax levels T will each obtain $R = \frac{Q}{N}$, the following nonlinear equation in CRS results:

$$(18) \qquad \frac{\partial U}{\partial C}/\frac{\partial U}{\partial G}(CRS,(1-t)M+Q/N-CRS) = 1 + \frac{(N-1)Q}{N^2\big((1-t)M-CRS\big)}$$

Similarly, a fixed rate subsidy r would result in a common level of C, denoted CMG, solving:

$$(19) \qquad \frac{\partial U}{\partial C}/\frac{\partial U}{\partial G}\big(CMG,(1+r)\big((1-t)M-CMG)\big) = 1 + r$$

Also following Aaron and Von Furstenberg, I assume a CES utility function

(20) $U(C,G) = \big(aC^v+(1-a)G^v\big)^{1/v}$, where $v = 1 - \frac{1}{\sigma}$, and σ is the constant elasticity of substitution of C for G.

Denote the share of income the recipient allocates to G, in the absence of any subsidy (r=0), by b. Then, it is easy to show from (19) that:

$$(21) \qquad a = \big(1+(\tfrac{1-b}{b})^{-1/\sigma}\big)^{-1}$$

Then, for any levels of (1-t)M and b, the solution of (19) depends solely on r and σ. Thus, given levels of these four parameters, one can compute CMG from

(19), the cost of the subsidy per recipient from (4), the consumption of the subsidized good GMG = (1+r)((1-t)M-CMG), and the deadweight loss W per recipient from (8). For homothetic utility functions such as (20), Stutzer (1982) has shown that W is strictly proportional to disposable income, so it is convenient to define:

$$(22) \qquad WMG = \frac{W}{(1-t)M}$$

as the fixed rate deadweight loss per recipient dollar of disposable income. This number will, of course, be much smaller than had the ratio been expressed as a fraction of the subsidy cost. But a decrease in welfare may not always be signalled by a decrease in the latter fraction, so (22) is adopted instead.

To compare a fixed rate subsidy r with an equal cost variable rate subsidy for various σ and b, we set Q in (18) equal to (4) times N, and solve to obtain CRS. From this, we compute GRS from the budget constraint in (15) and the deadweight loss per recipient from (8). To compare with (22), the variable rate deadweight loss is also expressed per recipient dollar of income, and is denoted WRS.

Finally, in keeping with convention from other studies, we represent the fixed rate subsidy r as a percentage price reduction in G. Noting from (5) that the price of G is $\frac{1}{1+r}$, the percentage price reduction S is:

$$(23) \qquad S = \frac{r}{1+r}$$

The comparisons for N = 50 recipients with $10,000 disposable income who spend b = .25 of the income on G in the absence of a subsidy are shown in Figure 2 below.[9/] There, note that spending on G increases with the price reduction S and the elasticity of substitution σ. As was claimed earlier, GRS < GMG, and WRS < WMG. While the quantitative differences per recipient

S \ SIGMA	.50	.75	1.50	2.00	4.00
.05	2581.285784	2606.350978	2682.521162	2734.107997	2946.793796
.10	2669.478593	2723.264632	2888.966971	3003.003003	3486.385664
.20	2870.856056	2996.083191	3393.712849	3676.470588	4929.022082
.30	3115.284130	3337.943323	4070.027169	4608.294931	7040.766035
.40	3420.218269	3780.439828	5014.356090	5952.380952	10113.268608
.50	3814.871397	4378.646672	6407.544820	8000.000000	14545.454545
.60	4352.809872	5238.563897	8628.524963	11363.636364	20973.154362
.70	5146.236293	6595.957391	12611.130834	17543.859649	30835.646007
.80	6486.595378	9114.031648	21352.549156	31250.000000	48828.125000
.90	9535.767394	15785.722755	51316.701949	76923.076923	99700.897308

GMG

S \ SIGMA	.50	.75	1.50	2.00	4.00
.05	2580.341845	2604.939288	2679.721536	2730.394690	2939.502088
.10	2667.578424	2720.431187	2883.388607	2995.632286	3472.052906
.20	2867.011430	2990.382110	3382.608957	3661.855568	4900.851058
.30	3109.465186	3329.356311	4053.383790	4586.384986	6999.305972
.40	3412.420744	3768.973045	4992.080392	5922.979500	10060.949076
.50	3805.136575	4364.345194	6379.446177	7962.822117	14488.480754
.60	4341.262615	5221.538316	8594.286176	11318.459435	20921.993974
.70	5133.164913	6576.439115	12570.268904	17490.970872	30801.234215
.80	6472.647958	9092.512553	21304.397904	31191.670536	48813.749612
.90	9522.604386	15763.465112	51260.854003	76869.321028	99698.669786

GRS

S \ SIGMA	.50	.75	1.50	2.00	4.00
.05	.000125	.000188	.000381	.000513	.001059
.10	.000534	.000808	.001661	.002252	.004798
.20	.002465	.003772	.007987	.011029	.024740
.30	.006499	.010080	.022104	.031106	.071911
.40	.013811	.021778	.049785	.071429	.164361
.50	.026484	.042655	.102547	.150000	.326253
.60	.048564	.080445	.206034	.306818	.588534
.70	.089017	.153394	.427293	.644737	1.003396
.80	.172498	.316025	.994678	1.500000	1.731448
.90	.403325	.831078	3.245149	4.673077	3.667182

WMG

S \ SIGMA	.50	.75	1.50	2.00	4.00
.05	.000120	.000181	.000367	.000494	.001023
.10	.000515	.000780	.001604	.002177	.004652
.20	.002384	.003653	.007753	.010721	.024138
.30	.006311	.009800	.021558	.030382	.070501
.40	.013463	.021263	.048764	.070063	.161778
.50	.025920	.041818	.100843	.147685	.322283
.60	.047726	.079189	.203348	.303104	.583441
.70	.087856	.151610	.423140	.638895	.998245
.80	.170998	.313596	.988094	1.490703	1.727830
.90	.401603	.827896	3.233251	4.657435	3.665931

WRS

FIGURE 2: N=50, (1-t)M=10000, b=.25

SIGMA	.50	.75	1.50	2.00	4.00
S					
.05	2581.285784	2606.350978	2682.521162	2734.107997	2946.793796
.10	2669.478593	2723.264632	2888.966971	3003.003003	3486.385664
.20	2870.856056	2996.083191	3393.712849	3676.470588	4929.022082
.30	3115.284130	3337.943323	4070.027169	4608.294931	7040.766035
.40	3420.218269	3780.439828	5014.356090	5952.380952	10113.268608
.50	3814.871397	4378.646672	6407.544820	8000.000000	14545.454545
.60	4352.809872	5238.563897	8628.524963	11363.636364	20973.154362
.70	5146.236293	6595.957391	12611.130834	17543.859649	30835.646007
.80	6486.595378	9114.031648	21352.549156	31250.000000	48828.125000
.90	9535.767394	15785.722755	51316.701949	76923.076923	99700.897308

GMG

SIGMA	.50	.75	1.50	2.00	4.00
S					
.05	2581.281068	2606.343926	2682.507180	2734.089456	2946.757410
.10	2669.469106	2723.250488	2888.939139	3002.966240	3486.314238
.20	2870.836887	2996.054775	3393.657541	3676.397815	4928.881940
.30	3115.255154	3337.900577	4069.944368	4608.185960	7040.560088
.40	3420.179483	3780.382805	5014.245363	5952.234844	10113.009228
.50	3814.823018	4378.575606	6407.405230	7999.815378	14545.173004
.60	4352.752522	5238.479332	8628.354941	11363.412196	20972.902925
.70	5146.171389	6595.860451	12610.928001	17543.597520	30835.478288
.80	6486.526099	9113.924726	21352.310277	31249.711523	48828.055646
.90	9535.701920	15785.612041	51316.425279	76922.812326	99700.886656

GRS

SIGMA	.50	.75	1.50	2.00	4.00
S					
.05	.000125	.000188	.000381	.000513	.001059
.10	.000534	.000808	.001661	.002252	.004798
.20	.002465	.003772	.007987	.011029	.024740
.30	.006499	.010080	.022104	.031106	.071911
.40	.013811	.021778	.049785	.071429	.164361
.50	.026484	.042655	.102547	.150000	.326253
.60	.048564	.080445	.206034	.306818	.588534
.70	.089017	.153394	.427293	.644737	1.003396
.80	.172498	.316025	.994678	1.500000	1.731448
.90	.403325	.831078	3.245149	4.673077	3.667182

WMG

SIGMA	.50	.75	1.50	2.00	4.00
S					
.05	.000125	.000188	.000381	.000513	.001059
.10	.000534	.000808	.001660	.002252	.004798
.20	.002464	.003772	.007985	.011028	.024737
.30	.006498	.010078	.022102	.031102	.071904
.40	.013809	.021775	.049780	.071422	.164348
.50	.026481	.042651	.102539	.149988	.326233
.60	.048559	.080439	.206021	.306800	.588509
.70	.089011	.153385	.427272	.644708	1.003370
.80	.172491	.316013	.994646	1.499954	1.731430
.90	.403317	.831063	3.245090	4.673000	3.667176

WRS

FIGURE 3: N=10000, (1-t)M=10000, b=.25

are small, they will be magnified in the aggregate. Also, the deadweight losses usually increase with σ, with exceptions occuring only for very high levels of σ. The latter must eventually occur, for as $\sigma \to \infty$, C and G become perfect substitutes, in which case there is no difference between cash and in-kind transfers.[10/] Finally, note that the deadweight loss index can exceed unity for both subsidy schemes when both S and σ, and, hence, the subsidy costs, are very high. Thus, for the very costly programs, the deadweight loss per recipient can actually exceed the $10,000 disposable income per recipient.

In Figure 3, the same comparisons are made for N = 10,000. The gaps between GRS and GMG and between WRS and WMG have narrowed substantially. To examine this further, a relative inefficiency index RI = WMG/WRS is tabled in figure 4. There, note that the relative inefficiency falls when N is increased to 10,000. Fixed rate subsidies, which were at most 3.9 percent more inefficient when N = 50, are always less than .1 percent more inefficient when N = 10,000. These computations suggest that there would be no differences between equal cost fixed and variable rate subsidies (10) in the limit as $N \to \infty$, at least when the recipients are identical. In the appendix, I present a simple proof of this claim, which is valid for any utility function.[11/] This result suggests that the welfare and spending differences between fixed and variable rate subsidies may be more important for high cost programs with a small number of recipients, such as federal aid to states, than for consumer welfare programs like food stamps or housing assistance.

An Application: General Revenue Sharing Vs. Fixed Rate Matching Grants

In this section, state-by-state and aggregate impacts of Federal General Revenue Sharing in 1972 are simulated and contrasted with a system of hypothetical, equally costly, fixed rate matching grants.

S \ SIGMA	.50	.75	1.50	2.00	4.00
.05	1.039399	1.039106	1.038266	1.037737	1.035836
.10	1.037542	1.036978	1.035435	1.034516	1.031508
.20	1.033756	1.032722	1.030153	1.028780	1.024922
.30	1.029868	1.028471	1.025334	1.023825	1.020000
.40	1.025872	1.024237	1.020931	1.019497	1.015965
.50	1.021765	1.020031	1.016903	1.015674	1.012318
.60	1.017543	1.015867	1.013210	1.012254	1.008730
.70	1.013208	1.011764	1.009813	1.009144	1.005160
.80	1.008773	1.007746	1.006664	1.006237	1.002094
.90	1.004289	1.003844	1.003680	1.003358	1.000341

N=50

S \ SIGMA	.50	.75	1.50	2.00	4.00
.05	1.000191	1.000190	1.000186	1.000183	1.000174
.10	1.000182	1.000180	1.000172	1.000168	1.000153
.20	1.000164	1.000159	1.000147	1.000140	1.000122
.30	1.000145	1.000139	1.000124	1.000116	1.000098
.40	1.000126	1.000118	1.000102	1.000095	1.000078
.50	1.000106	1.000098	1.000083	1.000077	1.000060
.60	1.000086	1.000078	1.000065	1.000060	1.000043
.70	1.000065	1.000058	1.000048	1.000045	1.000025
.80	1.000043	1.000038	1.000033	1.000031	1.000010
.90	1.000021	1.000019	1.000018	1.000017	1.000002

N=10000

Figure 4: RI=WMG/WRS

In the presence of FGRS in any year, assume that state, i, would behave as if it solved problem (15), with w_1, ..., w_{51}, and Q given by data and obtained for that year, and with U^i given by the CES form:

$$(24) \qquad U^i(C_i,G_i) = \left(a_i C_i^{\,v}+(1-a_i)G_i^{\,v}\right)^{1/v};\ i = 1,\ \ldots,\ 51$$

Thus, the distribution parameter a_i is permitted to vary across recipients, while the elasticity of substitution $\sigma = \frac{1}{1-v}$ is not. For each state i, the distribution parameter a_i must be estimated.

To estimate a_i for 1972, assume that the advent of FGRS was not anticipated prior to recipient government budgeting for 1972. Then, in 1972, state i acted as if it maximized (24) subject to $C_i + G_i = (1-t)M_i$. For any $\sigma = \frac{1}{1-v}$, the first order conditions for this problem can be solved for a_i in terms of the observed C_i and G_i in 1972:

$$(25) \qquad a_i = \frac{G_i^{\,v-1}}{G_i^{\,v-1}+C_i^{\,v-1}}$$

Thus, for any assumed common elasticity of substitution σ in 1972, one can obtain the a_i from 1972 NIPA data on C_i and G_i.

After calibrating the model by this method, (16) is solved simultaneously to obtain the Nash equilibrium (CRS_i, GRS_i), i = 1, ..., 51. The equilibrium FGRS allocation to state i is then $RS_i = I_i\left((1-t)M_i - CRS_i\right)Q$ given in (15). The deadweight loss W_i for each state i is calculated by subtracting a computed equivalent variation EV_i from RS_i, and is summed to obtain a total deadweight loss estimate for FGRS in 1972, denoted W.

To obtain the comparison between the revenue sharing equilibrium and the fixed rate matching grants, compute an equally costly matching grant rate r_i for each state i by solving the following two equations in the unknowns r_i and TMG_i:

(26) $$\frac{\partial U^i}{\partial C_i}/\frac{\partial U^i}{\partial G_i}\left((1-t)M_i-TMG_i,(1+r_i)TMG_i\right) = 1 + r_i \; ; \qquad i = 1, \ldots, 51$$

$$r_i \; TMG_i = RS_i.$$

Then, numerically compute an equivalent variation in income for (26) and subtract from RS_i to obtain the deadweight loss resulting from state i receiving an equally costly, fixed rate matching grant at the rate r_i. This is denoted WM_i in figure 5, and is summed to obtain a total fixed rate deadweight loss WM.

Examining figure 5, we see that the 1972 deadweight loss from the \$5.3 billion FGRS program would have been W = 242.8 million, which is 4.6 percent of the program cost, had the common elasticity of substitution σ = 2. An equally costly system of fixed rate subsidies would have generated a larger deadweight loss WM = 258.8 million, which is 6.6 percent larger than the loss due to FGRS. Both of these figures would have been lower had σ = .67, thus confirming the evidence from the identical recipients case results of figure 2. However, the relative inefficiency of 1.066 is somewhat larger than one would have inferred from the identical recipients evidence of figure 4.

A Qualification

It has been assumed to this point that the total cost Q of either subsidy program is not financed by the recipients of the program, i.e., that t is independent of Q. This may be a valid assumption for consumer welfare programs, but is surely not as valid for intergovernmental aid programs. Reinterpreting M to be recipient disposable income gross of its contribution to finance Q, its contribution to a fully funded program must be:

	σ=2			σ=.67		
STATE	W	WM	S	W	WM	S
AL	7.5609	8.2480	.0893	2.8314	2.9279	.0950
AK	.1576	.1582	.0367	.0694	.0696	.0391
AZ	2.7529	2.8054	.0581	1.0037	1.0236	.0618
AR	5.3997	5.5066	.0978	1.8993	1.9373	.1039
CA	13.9177	16.7925	.0386	5.1372	6.2612	.0413
CO	2.6354	2.6892	.0520	.9641	.9846	.0554
CT	1.6594	1.6942	.0332	.6155	.6291	.0354
DE	.5106	.5132	.0439	.1880	.1889	.0467
DC	.3760	.3779	.0303	.1397	.1404	.0323
FL	7.4791	7.9076	.0549	2.7266	2.8915	.0585
GA	7.1769	7.4753	.0685	2.5899	2.7010	.0730
HI	.7274	.7328	.0405	.2659	.2710	.0432
ID	1.4527	1.4641	.0708	.5243	.5285	.0753
IL	7.6002	8.5181	.0395	2.8783	3.1571	.0422
IN	5.5597	5.8349	.0529	2.0426	2.1361	.0564
IA	4.0397	4.1554	.0576	1.4731	1.5170	.0613
KS	2.7374	2.7915	.0547	.9983	1.0188	.0583
KY	6.7456	6.9641	.0800	2.4132	2.4932	.0851
LA	9.5218	9.9584	.0834	3.4124	3.5727	.0887
ME	2.1496	2.1744	.0726	.7754	.7845	.0772
MD	3.2072	3.3121	.0415	1.1829	1.2234	.0442
MA	4.9155	5.1707	.0418	1.8135	1.9123	.0446
MI	8.0510	8.7002	.0454	2.9609	3.2113	.0484
MN	4.9602	5.1790	.0530	1.8240	1.9000	.0564
MS	10.0366	10.3554	.1155	3.5038	3.6153	.1226
MO	5.1935	5.3881	.0561	1.8917	1.9652	.0597
MT	1.3125	1.3225	.0667	.4751	.4788	.0709
NE	2.1716	2.2033	.0584	.7903	.8023	.0622
NV	.4054	.4072	.0386	.1500	.1506	.0411
NH	.9203	.9260	.0572	.3351	.3372	.0609
NJ	4.4756	4.7126	.0370	1.6546	1.7467	.0394
NM	2.4860	2.5182	.0784	.8930	.9041	.0834
NY	12.5005	14.9960	.0362	4.6222	5.6011	.0386
NC	9.6744	10.1732	.0754	3.4760	3.6624	.0803
ND	1.7364	1.7504	.0808	.6218	.6270	.0859
OH	8.5176	9.1681	.0485	3.1194	3.3684	.0518
OK	3.9147	4.0012	.0680	1.4120	1.4442	.0723
OR	2.3988	2.4424	.0528	.8768	.8934	.0562
PA	12.0207	13.3521	.0510	4.4043	4.9149	.0544
RI	1.0954	1.1049	.0506	.4013	.4049	.0539
SC	6.2147	6.3798	.0875	2.2089	2.2687	.0930
SD	1.8251	1.8412	.0791	.6553	.6612	.0841
TN	7.5140	7.7919	.0785	2.6905	2.7926	.0836
TX	13.7036	15.0567	.0615	4.9794	5.4901	.0657
UT	2.0725	2.0962	.0700	.7482	.7570	.0749
VT	.8709	.8757	.0634	.3169	.3187	.0675
VA	5.3652	5.5725	.0564	1.9545	2.0330	.0601
WA	3.2768	3.3768	.0460	1.2059	1.2436	.0490
WV	4.2340	4.3142	.0839	1.5104	1.5396	.0892
WI	6.3486	6.6745	.0544	2.3267	2.4513	.0580
WY	.5630	.5651	.0597	.2051	.2059	.0633
	242.8478	258.5201		88.1654	94.1593	

Figure 5

$$(27) \qquad tM = \frac{Q}{N}$$

One could argue that this contribution is treated as a lump sum tax by the recipient in (6) or (15), in which case the recipients will still prefer equal cost variable rate subsidies to fixed rate ones. However, as has been noticed by both Teeples and Fisher (1981), funding fixed rate subsidies by recipient contributions introduces another distortion, for in this case:

$$(28) \qquad tM = \frac{Q}{N} = \frac{r \sum_{j=1}^{N} \left((1-t)M_j - C_j\right)}{N}$$

which results in the maximization condition (19) being modified to

$$(29) \qquad \frac{\partial U}{\partial C} / \frac{\partial U}{\partial G}\left(CMG, (1+r)\left((1-t)M - CMG\right)\right) = \frac{1+r}{1+r/N} .$$

Clearly, as $N \to \infty$, there is no difference between (29) and (19). Because of this, the limiting equivalence of fixed and variable rate subsidies proven in the appendix is still valid when the program is fully funded by its recipients.12/

Michael J. Stutzer
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Footnotes

[1] See Waldauer for further discussion of this problem.

[2] Federal General Revenue Sharing is a far more complicated system than just the "Senate" formula. See Nathan, Manvel, and Calkins for a more detailed description.

[3] Puerto Rico is counted as the 51st state.

[4] See Johnson (1975) for details.

[5] Also see Johnson (1977) and Fisher (1977).

[6] For a formal existence proof, see Stutzer (1981), Chapter 4.

[7] See Johnson (1975) for a graphical illustration of these propositions.

[8] These assumptions are not necessary for actual applications. In Chapter 4 of Stutzer (1981), large scale simulation with recipients differing in their weights and incomes is shown to be a practical technique.

[9] A computer program calculating the comparisions for arbitrary parameter values is available from the author.

[10] The author is indebted to Henry Aaron for pointing this out.

[11] For a more lengthy proof of this, valid only for Cobb-Douglas utilities, see Fisher (1981).

[12] Fisher (1981) has argued that this distortion brings variable rate subsidies closer to fixed rate subsidies for finite N as well.

Appendix

Denote the "demand" function solving (18) by CRS(N), substitute the equal cost condition $Q = Nr((1-t)M-CMG)$ into (18), and take the limit as $N \to \infty$ in (18) to obtain:

$$\text{(i)} \qquad \frac{\partial U}{\partial C} / \frac{\partial U}{\partial G} \left(CRS(\infty),\ (1-t)M+r((1-t)M-CMG)-CRS(\infty)\right)$$

$$= 1 + r\,\frac{((1-t)M-CMG)}{((1-t)M-CRS(\infty))}$$

where CMG solves the fixed rate subsidy maximization condition (19). A simple substitution verifies that $CRS(\infty) = CMG$ is the unique solution to (i).

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