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THE ATTAINMENT OF MINNESOTA'S INDEXING OBJECTIVES VIA ALTERNATIVE TAX INDEXING TECHNIQUES

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The Attainment of Minnesota's Indexing Objectives via Alternative Tax Indexing Techniques

Since 1978, at least nine states, including Minnesota, have adopted income tax indexing measures intended to inflation proof their tax systems. In addition, indexing measures have been introduced in many other state legislatures. Most of these measures attempt to inflation proof income taxes by a technique of widening the tax brackets and/or increasing the level of deductions and credits permitted. The amount of change depends, at least in part, on an index of inflation, which is typically the consumer price index. Therefore, these measures are referred to as <u>tax indexing</u>.

But whether or not these measures really do inflation proof an income tax system depends on what is meant by the term of "inflation proofing", and on how to measure progress towards it. In the absence of indexing, inflation teams up with a progressive state income tax schedule to push people into higher and higher tax brackets--a pheonomena referred to as "bracket creep". This results in unlegislated changes in both the tax burden borne by some income classes, and in the income tax revenue collected by state government. To some, inflation proofing means the prevention of the former unlegislated change. To others, it means the prevention of the latter. But progress towards the prevention of either or both of these unlegislated changes can't really be measured until some quantitative measures of progress are stated. Once such quantitative measures, which in this paper are termed <u>objectives</u>, are stated, tax planners can attempt to determine just how effective proposed indexing techniques will be.

One commonly stated inflation proofing objective, which I dub <u>objective C</u>, is that a representative taxpayer with constant real income should have her/his real tax liability held constant as well. The ACIR succinctly states this objective in a recent paper. 1/

> "Under an indexed tax system, if an employee receives a 10 percent wage increase to offset a 10 percent inflation rate, he would only pay 10 percent more in income taxes."

An employee in the stated circumstance has an unchanged real income. In an unindexed, progressive tax system, this employee would, due to bracket creep, pay more than a 10 percent increase in income taxes. In the absence of indexing or tax rate cuts, both the employee's tax burden and the state's revenue would rise in real terms. The attainment of objective C would help inflation proof the tax system on both fronts. Another inflation proofing objective, which I dub objective F, motivated recent changes in Minnesota tax law. It is that a representative taxpayer's real tax liability should never fall at a faster rate than her/his real income does. Another way of stating this is that a representative taxpayer's nominal tax liability should rise at least as fast as her/his nominal income does, when the latter does not rise as fast as inflation does. The indexing technique adopted in 1979 by Minnesota resulted in the violation of this objective. Taxpayer real income in Minnesota declined, and the indexing technique caused nominal tax liability to grow slower than nominal income did. The resulting slow revenue growth was unanticipated and hence, in a sense, "unlegislated". The attainment of objective F would help inflation proof the tax system against this "unlegislated" change in tax revenue. In 1981, a new, rather complicated indexing technique was adopted in Minnesota which does attain objective F in addition to objective C. [2] But how does it work? And are there other, simpler, indexing technques which can also attain these objectives?

This paper will show how a different indexing technique can be easily used to attain both objective C and F, even in complex tax systems. When compared to the commonly used bracket expansion technique, tax planners should find this indexing technique easier to use in designing quantitative objectives like C and F into indexed tax systems. The technique is also compared to a third possible indexing technique, which, we will see, cannot attain objectives C and F in complex tax systems.

To show which objectives, if any, these three different indexing techniques can attain in simple tax systems, assume, for the moment, an unindexed, progressive [3/] tax system not complicated by the permissibility of federal income tax deductions nor the existence of other deductions or credits. These assumption will be relaxed in turn. For example, consider the simple tax system [4/] whose brackets are shown in Table I and whose tax schedule is graphed in Figure 1.

Figure I shows that a representative taxpayer earning \$15,000, which in this simple tax system equals her/his taxable income, pays 14 percent more taxes when her/his income rises 10 percent to \$16,500. The (arc) income elasticity of the tax schedule at the \$15,000 income level is thus 14/10 = 1.4. For this tax system, the income elasticity varies somewhat from income level to income level. Different tax schedules in which the income elasticity does not vary from income level to income level are said to have constant elasticity. To illustrate the difference, the constant elasticity tax schedule which best approximates the tax schedule of Figure 1[5/] is graphed in Figure 2. In addition, while the elasticity may vary, it always exceeds one in progressive tax schedules[6/], like this one.

If the rate of inflation was also 10 percent, the representative taxpayer in Figure 1 would have no real income change. But, the representative taxpayer's real tax liability would rise by 14 - 10 = 4 percent. Thus, at least at the \$15,000 income level, objective C is violated. In fact, this violation of objective C will occur at any income level, because the elasticity of the progressive tax schedule exceeds one at all income levels. How can this violation be remedied?

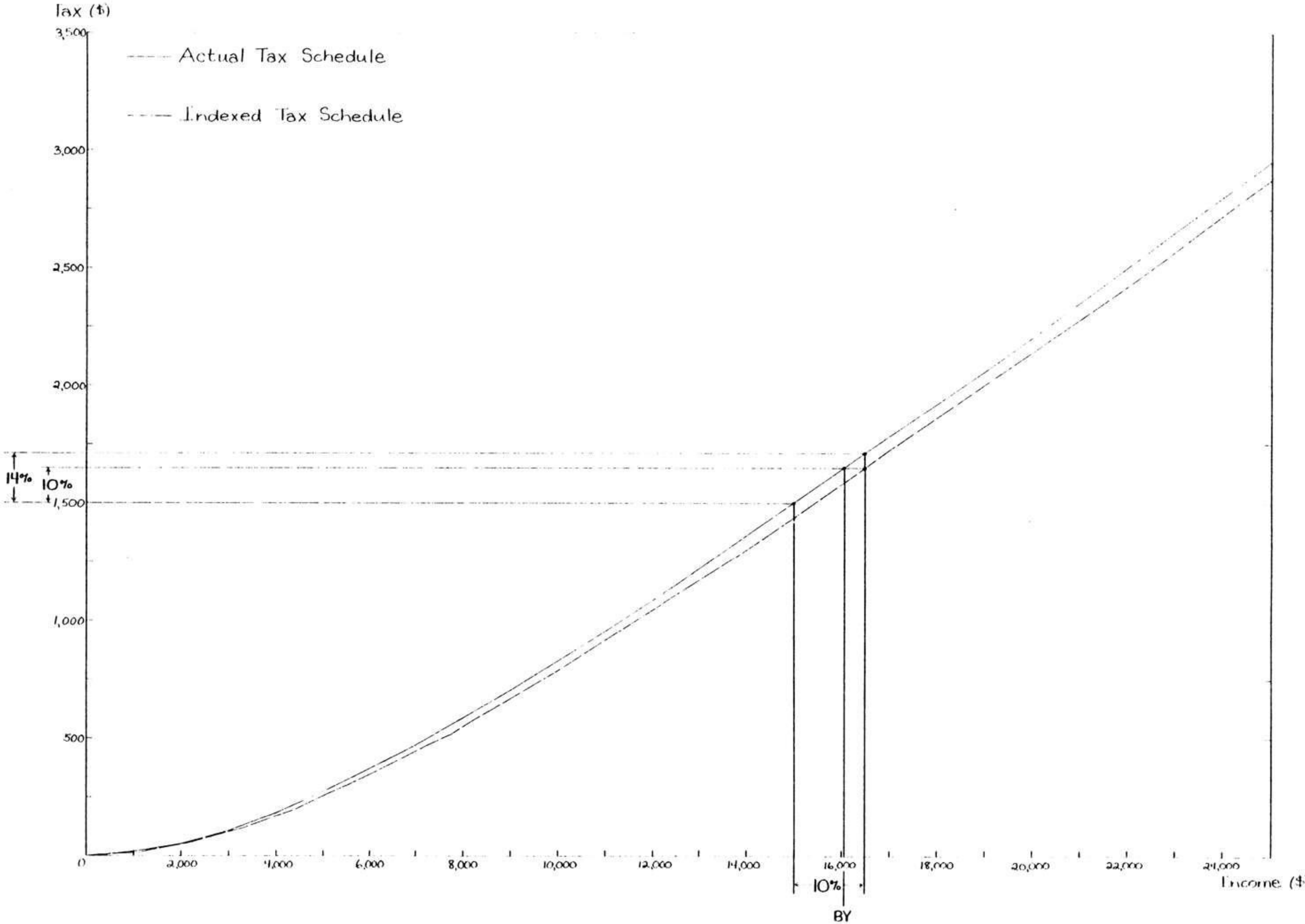

Figure 1

A single taxpayer with $15,000 income would pay 14% more in taxes if her/his income rose at the 10% inflation rate. Indexing the schedule by widening the brackets by the 10% inflation rate lowers the tax schedule to the dashed line. Widening the brackets by more than 10% would lower the tax schedule even more.

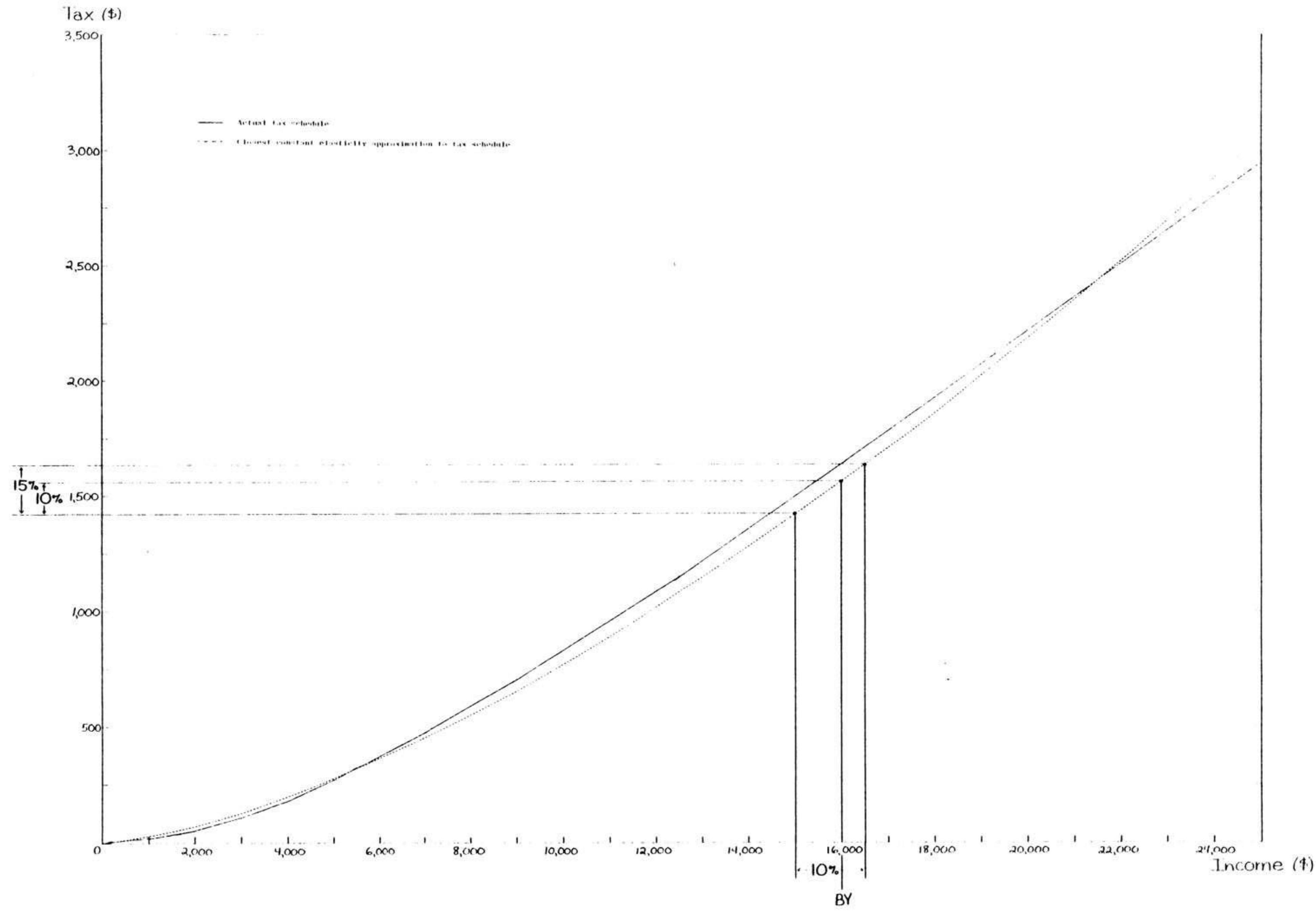

Figure 2 The tax schedule and its closest constant elasticity approximation

In the simple tax system, one indexing technique which will attain objective C is to widen the tax bracket by 100 percent of the rate of inflation. In our example, the brackets would be expanded by 10 percent, as shown in Table 1a. The resulting indexed tax schedule is also graphed in Figure 1. There, it is seen that the bracket widening does indeed limit the tax increase for the representative taxpayer to 10 percent, thus attaining objective C for taxpayers whose income level is $15,000. In fact, objective C will be attained for a representative taxpayer with any income, because any taxpayer with a 10 percent income increase will not leave the tax bracket that she/he started in. Bracket creep is thus eliminated. But this says nothing about the attainment of objective F.

Even in the simple tax system, bracket widening by 100 percent of the rate of inflation will violate objective F. To prove this, continue to suppose that the income of a representative taxpayer in Figure 1 rose by 10 percent, but that the inflation rate had been greater than 10 percent--say, 12 percent. So, the taxpayer's real income declined by 2 percent. Then, widening the brackets by 12 percent would have lowered the indexed tax schedule further than is represented in Figure 1. The representative taxpayer's liability would thus increase by less than 10 percent, i.e. by less than her/his nominal income growth. This violates objective F.

However, bracket widening by 100 percent of the rate of nominal income growth, rather than by 100 percent of the rate of inflation, will attain objective F without sacrificing objective C. In the previous example, that suppose the brackets had been widened by only the nominal income growth rate of 10 percent. Then, the indexed tax schedule would not have been lowered further than is represented in Figure 1, and the taxpayer's liability would have risen by 10 percent, i.e. by the rate of nominal income growth.

Thus, objective F would have been attained. Objective C would also be attained, for if the rate of inflation has actually been 10 percent, both the representative taxpayer's real income and real tax liability would have remained unchanged. Of course, these arguments apply equally well to any representative taxpayer with an income level other than $15,000.

Thus, at least in the simple tax system unfettered by deductions and/or credits, the indexing technique of expanding the brackets by 100 percent of the rate of inflation when the representative taxpayer's real income is constant or rising, and by 100 percent of the representative taxpayer's rate of nominal income growth when her/his real income is falling, will attain objectives C and F. Of course, there may be alternative indexing techniques which also attain objectives C and F. In addition, there may be alternative objectives which may be attained by some indexing techniques but not by others [7/]. Thus, it is interesting to explore alternative indexing techniques.

One alternative indexing technique can, at least in this simple tax system, attain objectives C and F. The technique, which I'll call the <u>income adjustment technique</u>, is to adjust a taxpayer's taxable income downward by a multiplier termed "B", which is computed from data for a representative taxpayer. A taxpayer's adjusted taxable income would then be entered onto the unindexed tax schedule to compute her/his tax liability. An example of this is illustrated in Figure 1. There B is computed so that a representative taxpayer with $15,000 taxable income will only pay 10 percent more taxes when her/his income rises 10 percent. If the inflation rate were 10 percent, and the representative taxpayer made $15,000, objective C would be attained. To prove that some value of B can always be found so that this technique can attain objectives C and F in more general circumstances, it helps to introduce a mathematical framework. This framework will also be of use in analyzing the third indexing technique discussed later.

Denote the price level by P and a representative taxpayer's real income by Q. Then a representative taxpayer's nominal income is given by

(1) $Y = PQ$.

In this simple tax system, any taxpayer's liability will depend solely on her/his adjusted taxable income. For the representative taxpayer with adjusted income BY, this is represented by:

(2) $T = T(BY)$

where the function T is the unindexed tax schedule. In the absence of indexing, setting B=1 yields the unindexed tax liability. We denote percentage changes in the values P, Q, B, Y and T by their lower cases p, q, b, y and t, respectively. Making use of the facts that the percentage change in real tax liability T/P is t - p and that the percentage change in nominal income Y is $y=p+q$ 8/, objective C is written:

[C] if $q = o$, then $t - p = o$, or, alternatively, $t = p$ when $q = o$.

Objective F is written

[F] if $q < o$, then $t - p > q$, or, alternatively, $t > p + q$ when $q < o$.

Now we are ready to see how objectives C and F can be attained by this indexing technique, i.e. how to find an appropriate value of B. As before, the attainment of objective C is treated first. Thus, it is shown how to additionally attain objective F.

To find a value of B which will lead to the attainment of objective C, we first logarithmically differentiate (2) to obtain 9/.

(3) $t = E(BY)[b+p+q]$,

where B=1 in the initial, unindexed situation, and where E(BY) denotes the elasticity of the unindexed tax schedule at the representative taxpayer's adjusted income of BY. Note that objective C requires that if q = o, then b must equal p in (3). Then, a little rearranging of terms in (3) shows that be must be set by the following formula to accomplish objective C:

(4) $b=\left[\frac{1}{E(BY)} - 1\right] p$, with B=1 initially.

For example, for the representative taxpayer with $15,000 income in Figure 1, the (arc) income elasticity is 1.4. Also as in Figure 1, suppose that the rate of inflation is 10 percent. Then, B must be changed to 1 + b to accomplish objective C, where $b=\left\{\frac{1}{1.4} - 1\right\}.10 = -.029$. Thus, in Figure 1, B equal to .971 would be used to adjust the taxpayer's inflated income of 16,500 downward to $16,021.50. The latter figure is then entered onto the unindexed tax schedule to compute the liability. All taxpayers would have their income adjusted downward by B=.971 as well. The following year, the procedure would be repeated starting with B = .971. This technique will attain objective C. But will it attain objective F as well?

While the use of (4) will violate objective F, a simple replacement of y for p in (4) will rectify the situation. To prove this, substitute (4) into (3) and simplify to find

(5) $t - p = E(BY)q$

Because the unindexed schedule is progressive, $E(BY)>1$. Therefore, when (4) is used, $t - p < q$ when $q < o$, which violates [F]. When $q < o$, suppose we replace p in (4) with y, obtaining a new formula (6) used when $q < o$.

(6) $$b = \left\{\frac{1}{E(BY)} - 1\right\}y, \text{ with } B=1 \text{ initially.}$$

Substituting (6) into (3) find:

(7) $$t = p + q = y.$$

Using (6) when $q < o$ causes the tax liability to grow at the rate of nominal income growth, which attains [F]. Thus, applying (4) when $q \geq 0$, and applying (6) when $q < o$, will attain both objectives C and F.

The third, and final, indexing technique examined herein will, at least in simple tax systems, attain [C] but not [F]. The technique is to deflate the taxpayer's taxable income, then enter the resulting real taxable income onto the unindexed tax schedule, and finally reflate the resulting tax figure to compute the taxpayer's liability. 10/ Mathematically, this technique is denoted by (8) below:

(8) $$T = PT\left(\frac{Y}{P}\right) = PT(Q).$$

To see that [C] is attained by (8) but that [F] is not, logarithmically differentiate (8) to obtain:

(9) $$t = p + E(Q)q.$$

If $q = o$, (9) shows that $t = p$, thus attaining [C]. But because $E(Q)$ exceeds 1, $t - p < q$ when $q < o$. This violates [F].

In summary, we have established that all three indexing techniques can be easily used to attain objective C in simple tax systems devoid of deductions or credits. However, only the bracket widening and income adjustment techniques could also be used to attain objective F in these simple tax systems. We now examine whether any of these techniques can be easily used to attain either or both of these objectives in the several states, like Minnesota, which permit the deduction of federal income tax liabilities in computing taxable income.

When deductions of the unindexed federal income tax are permitted, it may be possible to attain objectives C and F via a bracket widening technique, but it is certainly not easy to see how to do so. For example, unlike the case in simple tax systems, bracket widening by 100 percent of either the rate of inflation or the rate of the representative taxpayer's income growth will not attain objective C. To see this, denote the unindexed federal tax liability T_F of the representative taxpayer by

$$(10) \qquad T_F = T_F(Y),$$

where $T_F(Y)$ is the unindexed, progressive, federal tax schedule. The representative taxpayer's taxable income is then $Y - T_F$, so she/he has a state tax liability in the absence of state tax indexing of

$$(11) \qquad T = T(Y-T_F(Y))$$

when Y increases by a given percentage 100y, T_F will increase by more than that percentage, due to the progressivity of the unindexed federal tax schedule (10). Taxable income $Y - T_F$ will thus rise by <u>less</u> than 100y. For example, in Figure 1, the representative taxpayer's <u>taxable</u> income would rise by less than the 10 percent inflation rate assumed there. But then widening the brackets by the full 10 percent rate of inflation will result in an increase in the representative taxpayer's liability of less than 10 percent. If the representative taxpayer's real income remained constant, [C] would have thus been violated, as $t < p$. It might be possible to attain objective C by widening the brackets by less than 100 percent of the rate of inflation, but by how much less? Minnesota later found that bracket widening by 85 percent of the rate of inflation, as prescribed in its initial 1979 indexing measure, was insufficient. It is just not easy to see how much the brackets should be widened to attain objective C, let alone objective F. Fortunately, it is still easy to attain objectives C and F with the income adjustment technique. As in the case of simple tax systems, we first show how to attain objective C.

To attain objective C via the income adjustment technique, represent the tax liability incurred under it by:

$$(12) \qquad T = T(B(Y-T_F(Y))), \text{ with } B=1 \text{ initially.}$$

Logarithmically differentiate (12) to obtain:

$$(13) \qquad t = E(B(Y-T_F)) \left[b + \frac{Y}{Y-T_F} (Y-E_F(Y)T_F)\right],$$

where $E_F(Y)$ is the elasticity of the federal income tax at the representative taxpayer's nominal income Y. Remembering that $y = p + q$, substituting p for t, setting $q = o$, and solving for b in (13) yields the adjustment required to attain objective C:

$$(14) \qquad b = \left[\frac{1}{E(B(Y-T_F)} - \left(\frac{Y-E_F(Y)T_F}{Y-T_F}\right)\right] p, \text{ with } B = 1 \text{ initially.}$$

Federal tax progressivity implies that $E_F(Y) > 1$, which means that the second term in (10) is less than 1. Comparing (14) with the adjustment (4) would give for a taxpayer with income Y, and assuming that E does not change too much with Y, we then see that a smaller indexing adjustment is required when progressive federal taxes are deducted. Thus, the inappropriate use of (4) when federal income taxes are deducted would lead to too much indexing. For example, suppose a representative taxpayer with Y = \$15,000 and T_F = \$2,472 faced a federal elasticity of about 1.8. Suppose the state elasticity at a taxable income of \$12,528 is still 1.4. Then, the initial percentage indexing adjustment is to adjust gross income by $b = \frac{1}{1.4}\left(\frac{15{,}000-1.8(2{,}472)}{15{,}000-2{,}742}\right) = -.146$ of the inflation rate p, rather than by $\frac{1}{1.4} - 1 = -.286$ of p. With p = .10 the correct B = 1 + b = 1 - (.146 x .10) = .985, rather than the value of .971 computed earlier from (4).

Formula (14) may or may not result in the attainment of objective F, though, depending on the size of the average federal tax rate paid by the representative taxpayer. To prove this, substitute (14) into (13) and re-arrange to obtain:

$$(15) \qquad t - p = \left[E(B(Y-T_F))\,\frac{(Y-E_F(Y)T_F)}{Y-T_F}\right] q.$$

[F] will be violated if the bracketed term exceeds one. However, it is not clear that it does exceed one, because while progressivity implies that $E(B(Y-T_F)) > 1$, it also implies that $\frac{Y-E_F(Y)T_F}{Y-T_F} < 1$. The bracketed term, being the product of the two, could possibly be either greater or less than one. Algebraic manipulation demonstrates that the bracketed term will exceed one, so that [F] will be violated, if and only if the following condition holds:

$$(16) \qquad \frac{T_F}{Y} < \frac{E(B(Y-T_F))-1}{E_F(Y)E(B(Y-T_F))-1}\ .$$

With the data set used before, $E(B(Y-T_F)) = 1.4$ and $E_F(Y) = 1.8$, so the representative taxpayers' average federal tax rate $\frac{T_F}{Y}$ must be less than .26

for (RT) to be violated. In the data set used above $\frac{T_F}{Y} = .16$, so (F) is indeed violated. Of course, it is possible that a different tax system, or a different choice of representative taxpayer in the same tax system, would produce the opposite conclusion.

To insure that [F] will always be attained along with [C], one can adopt a two-part formula analogous to the one derived for simple tax systems. To do so, one need merely change p to y in (14) when $q < 0$, which will cause $t = y$ when $q < 0$, and thereby attain [F]. This can be seen by substituting

$$(17) \qquad b = \left[\frac{1}{E(B(Y-T_F))} - \left(\frac{Y-E_F(Y)T_F}{Y-T_F}\right)\right] y$$

into (13) when $q < o$ and simplifying the result to find that $t = y$ when $q < o$ when (17) is used. (14) is used when $q \geqslant 0$. We now turn to an examination of objective attainment by our third possible indexing technique.

Our third indexing technique, which attained [C] but not [F] is simple tax systems, will no longer even attain the former when unindexed federal taxes are deductable. To prove this, denote the tax liability incurred under it by:

$$(18) \qquad T = PT\left\{\frac{Y-T_F(Y)}{P}\right\}.$$

If $q = o$, then $y = p$. But the unindexed, progressive, federal income tax liability T_F will rise by a percentage greater than 100p. Then, taxable income $Y-T_F(Y)$ will rise at a percentage rate less than 100p. 11/ But this causes real taxable income $\frac{Y-T_F(Y)}{P}$ to fall. Then, after reflating the now smaller $T\left(\frac{Y-T_F(Y)}{P}\right)$ by the rate of inflation p, the taxpayer's indexed liability will rise by less than the rate of inflation, i.e. $t < p$, when $q = 0$. This, of course, violates [C].

However, in 1985, when the federal tax is supposed to be indexed [12] the use of (18) or bracket widening by 100 percent of the rate of inflation will attain [C]. In both cases, the arguments which showed that [C] would not be attained depended solely on the fact that the rate of growth of taxable income $Y-T_F$ was less than the rate of growth of Y, [13] when q = 0 and the progressive federal tax schedule is not indexed. When the federal tax is indexed, though, these growth rates will be identical when q = 0. A glance at the rest of the respective arguments shows that both (18) and bracket widening by 100 percent of the rate of inflation will then attain [C].

In the meantime, tax planners in states having tax systems which permit the deduction of the federal tax must take this fact into account, perhaps by using the income adjustment technique. Also, most of these tax systems also permit other deductions and/or credits as well. Of course, bracket widening and our third technique will be of no more use in these states, like Minnesota. We now turn to a discussion of how the income adjustment technique can attain objectives C and F in states having these more complicated tax systems.

A more complicated tax system permits other deductions and/or credits in addition to the federal tax deduction. These deductions and/or credits grow over time, due to natural factors or tax law changes. For a representative taxpayer, denote the level of other deductions and the level of credits by D, and C, respectively. Their respective rates of change are thus denoted d and c. Denote the taxable income $Y-T_F(Y)-D$ by $NI(Y,D)$, and its rate of change by ni. Finally, denote the indexed tax liability, including tax credits, by T_c, and its rate of change by t_c. Then, the indexed tax liability of the representative taxpayer is:

(19) $T_c = T(B(NI(Y,D)))+C$, with B=1 in the initial, unindexed, situation.

To find a level of B which attains [C] in this more complicated tax system, logarithmically different (19) to obtain

$$t_c = E(B(NI))\left[b+ni\right]\frac{T}{T_c} + \frac{cC}{T_c}$$

or

$$(20) \qquad t_c = E(B(NI))\ \left[b+y\frac{Y}{NI} - E_F(Y)\ y\frac{T_F}{NI} - d\ \frac{D}{NI}\ \right]\frac{T}{T_c} + c\ \frac{C}{T_c}\ .$$

Remembering that $y = p + q$, multiplying (20) out and requiring [C] allows us to solve (20) for the value of b such that $B = 1 + b$ will attain [C]. It is given by the following rather long formula, where E(B(NI)) has been shortened to just E to save space:

$$(21) \qquad b = \left[\frac{T_c}{ET} - \left(\frac{Y-E_F T_F}{NI}\right)\right] p - \frac{D}{NI}\ d + \frac{C}{ET}\ c.$$

For example, assume as before, that $E = 1.4$, $E_F = 1.8$, $Y = \$15,000$, and $T_F = \$2,472$. Also assume, not unrealistically, that $D = \$1,000$, $C = \$50$, and $p = c = d = .10$. So, $NI = \$15,000 - \$2,472 - \$1,000 = \$11,528$. Then, if the unindexed tax schedule is the one graphed in Figure 1, $T = T(NI) = T(11,528) = \$1030$, and $T_c = T + C = \$1,080$. Substituting this data into (21) yields $b = -.022$, so the value of B which will initially attain [C] is $B = 1 + b = .978$. As it should be, (21) is equivalent to (14) when $D = C = 0$, because T_c is then equal to T and NI is then equal to $Y - T_F$.

As was true in the less complicated tax systems considered earlier, substituting y for p in the income adjustment formula will result in the additional attainment of [F]. To prove this, when $q < o$ substitute $y = t_c$ in (20) and solve for b to obtain:

$$(22) \qquad b = \left[\frac{T_c}{ET} - \left\{\frac{Y-E_F T_F}{NI}\right\}\right] y - \frac{D}{NI}\ d + \frac{C}{ET}\ c.$$

Thus, using (21) to adjust income when $q > o$, and using (22) when $q < o$, will attain both [C] and [F]. This technique contrasts sharply with the bracket widening technique, which could possibly attain [C] and [F], although its hard to see how. But perhaps it would be easier to see how a hybrid technique, containing elements of both the bracket widening and the income adjustment technique, could attain [C] and [F].

The state of Minnesota adopted just such a hybrid indexing technique in 1981. It is carried out in two steps. The first step consists of an upward adjustment to taxable income NI. It is followed by a second step consisting of a bracket widening by 100 percent of either the rate of inflation (the CPI), or the rate of nominal income growth, whichever is smaller. The representative taxpayer used to compute the income adjustment and the rate of nominal income growth is taken to be a hypothetical taxpayer who "pays" the statewide total for the data employed. Salomone (198~~0~~(b)) shows that this rather complicated hybrid technique does indeed attain [C] and [F].

Compared to the hybrid technique, using (21) when $q > 0$ and (22) when $q < o$ is simpler to administer and comprehend. It only requires a single calculation, which makes it simpler to administer. Furthermore, the taxpayer would more readily comprehend the effects of a downward adjustment to her/his taxable income, then she/he would comprehend the combined effects of an upward adjustment to taxable income followed by a bracket widening. If tax planners in Minnesota had no other objectives besides [C] and [F], they may wish to adopt the simpler technique instead.

However, even if tax planners do indeed have other objectives in mind, it is hoped that the income adjustment technique described herein can be used to attain them. Once tax planners precisely specify a set of objectives, the methods of proof used herein should help to prove whether or not the income adjustment technique can attain them.

FOOTNOTES

1/ See Shannon, J. and Lucke, R. (1980). This paper has been relied upon for much of the material used up to this point.

2/ See Salomone (1981a) for a statement of, and motivation for, this objective.

3/ Denoting income by Y and the tax schedule by T(Y), a progressive tax schedule is defined to be one whose average tax rate $\frac{T(Y)}{Y}$ rises with income.

4/ This was the 1977 tax schedule Minnesota single taxpayers entered their state taxable income (income less their federal income tax and their other deductions) onto.

5/ The constant elasticity function $T(Y) = KY^E$, where K and E are parameters, was fitted in log form to the unindexed tax schedule by ordinary least squares. Point estimates are k = .0008 and E = 1.4449

6/ Differentiate $\frac{T(Y)}{Y}$ to obtain $\{\frac{dT}{dY}Y-T\}/Y^2 > 0$. Multiplying both sides by Y^2 and dividing by T yields the conclusion that E (Y) > 1. Reversing the operations yields the converse proposition.

7/ Economists, for example, usually discuss the desirability of changes to existing tax systems in the framework of the so-called optimal tax theory (see Atkinson and Stiglitz (1980) for an exposition of optimal tax theory). In this framework, a desirable indexing objective would be to reduce deadweight loss. Because the discussion of state tax indexing objectives has not taken place in this framework, it is unclear whether or not objectives C and F are consistent with the reduction of deadweight loss. Thus, we make no claim that objectives C and F are desirable, and merely examine different ways of attaining them.

8/ In fact, for any two variables X and Z, the percentage change in the quotient X/Z is approximately equal to x-z, and the percentage change in the product XZ is approximately $x+z$. They are approximate rules only because linearization, i.e. calculus, is used to derive them. To derive the latter, rule, for example, totally differentiate XZ to obtain $d(XZ)=(dX)Z+X(dZ)$. Divide both sides by XZ and note that $\frac{dZ}{Z} = z$ to obtain the rule. The rule is, of course, more accurate the smaller dX and dZ are.

9/ The general rule being used here is that if F(Z) is any function of a variable Z, then $f = E(Z)z$, where $E(\cdot)$ is the elasticity of F with respect to Z. If Z is itself the product or quotient of 2 or more other variables, one just applies the simple rules discussed in footnote 8 above to compute z.

10/ This technique was suggested by my fellow staffer, Dick Todd.

11/ Logarithmically differentiate $Y-T_F$, and find that its percentage change is $(p+q)\ [\frac{Y-E_F(Y)T_F}{Y-T_F}]$ which is less than p when $q = o$.

12/ The Economic Recovery Tax Act of 1981 specifies that the federal income tax be indexed, starting in 1985.

13/ Actually, when $q = 0$, taxable income could possibly grow at a rate different than the inflation rate even in our simple tax system. This is because it may be true in some economies that real income 0 depends on the rate of the price level P, i.e. Q(P). then $Y = PQ(P)$, so $\frac{Y}{P} = Q(P)$ is not independent of changes in P. If this is true, (8) would not attain [C] in simple tax systems, either. The author is indebted to Dick Todd for this point.

References

Atkinson, A. and Stiglitz, J. "Lectures in Public Economics," McGraw Hill, 1980.

Salomone, Daniel A.

(a) "Individual Income Tax Indexing in Minnesota," unpublished mimeo, Minnesota Department of Revenue, June 15, 1981.

(b) "Indexing Proposal," unpublished memorandum to Commissioner of Revenue, Minnesota Department of Revenue, April 10, 1981.

Shannan, J. and Lucke, R. "State Experimentation With Indexed Income Taxes - Early Results," Advisory Commission on Intergovernmental Relations, Washington, D.C., 1980.