Historical

Money, Real Interest Rates, and Output

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Interest rates have long occupied a central role in theories of income determination. Traditional Keynesian theory has emphasized the role of interest rates for determining the level of aggregate demand, particularly for new capital goods. More recent equilibrium theories have emphasized the role of interest rates on the desired intertemporal allocation of labor supply. In the theories of Lucas (1972) and Barro (1976, 1980), fluctuations are portrayed as stemming from the response of labor supply to perceived temporary abnormal rates of return. In the model presented in Grossman-Weiss (1980) interest rates affect output because they help to distinguish relative from aggregate productivity shocks which influence each agent's desired level of investment.

Another common theme of these disparate theories is the close connection between monetary phenomena and interest rates. In each theory the principal channel by which changes in the demand or supply of money affects real variables is by altering perceptions of intertemporal terms of trade. The Keynesian "liquidity preference" theory posits that the link between money and interest rates is direct and causal; the nominal interest rate must adjust to equilibrate the demand for money with the exogenously determined supply. Under the additional assumption that the path of nominal prices is given exogenously, nominal interest rate changes are changes in the real rate relevant for a firm's investment decision and hence output. In the newer equilibrium theories the connection between money and perceived real interest rates is more subtle. A key assumption of the newer theories is that there are barriers to information flows and agents use observed nominal price signals as imperfect summaries from the rest of the world. These theories imply that monetary phenomena may effect perceptions of the real rate only to the extent that such disturbances are not directly perceived as such, but are confused with real changes.

The empirical relationship between money, nominal interest rates, and output has recently been studied by Sims (1979, 1980). Using a four variable autoregressive system (money, prices, output, and a short-term interest rate), Sims found that upward innovations in interest rates were followed by a deline in production after a lag of about six months, reaching a minimum about 18 months later for postwar U.S. data. Equally striking is the fact that the inclusion of interest rates leads to the rejection that the money stock is strongly Granger-causally prior for income. When interest rates are omitted, monetary innovations explain 37 percent of the 48-month forecast error variance for industrial production; when interest rates are added the proportion falls to 4 percent. Sims concludes (p. 253), "some of the observed comovements of industrial production and money stock are attributed to common responses to surprise changes in the interest rate." Although the magnitudes and timing of the response differ among the several samples studied, the relationship appears in both prewar and postwar U.S. data and postwar French, U.K., and German data.

In this paper, we attempt to reexamine the empirical relationship between money, interest rates, and output in postwar monthly U.S. data emphasizing the distinction between movements in expected (ex ante) real interest rate movements and movements in expected inflation rates. As Fama (1975), and Fama and Gibbons (1980) have shown, a substantial part of the movements in short-term interest rates, at least over the postwar U.S. experience, can be attributed to changes in expected inflation.[1/] In light of this finding, it is surprising that interest rate innovations explain so much of the variance of forecasts of future output, since most theoretical models of income determination argue that expected real rates are pertinent for current period supply and demand decisions.

The paper is organized as follows: In Section I we review the basic results of Sims' four variable vector autoregressions on postwar monthly U.S. data. In Section II we construct proxies for the ex ante real rate which allow us to separate the effects of changes in the expected real rate from changes in expected inflation. In one set of vector autoregressions we treat the projection of future prices on currently available data using the Kalman filter technique to be the price expectations held by a representative agent. In another set of vector autoregressions we attribute to agents' perfect one-period-ahead price predictions. We cannot unambiguously attribute the effects of nominal interest rate innovations documented in the first section to either expected inflation innovations alone or to real rate innovations alone. However, there is strong evidence to suggest that upward innovations in expected inflation have a depressing effect on future output which is not readily explained by demand driven models of output. In light of these descriptive statistical results, we go on to test a number of specific hypotheses concerning money, real rates, and output which are suggested by particular structural models. We first examine the influence of output, money, and prices on the ex ante real short rate. Specifically, we test whether current and past money and prices have any additional predictive content for future expected real rates given current real rates. We cannot reject the hypothesis that the real rate is exogenous. This finding casts strong doubt on the money, real interest link implied by either Keynesian IS-LM analysis and the informationally based equilibrium models. Because the real rate is unobserved, the hypothesis that the real rate is exogenous to money and prices takes the form of nonlinear cross-equation restriction on a vector autoregressive system which is estimated by a maximum likelihood method. We then go on to test a number of hypotheses concerning the relationship between interest rates and output. These tests are designed to pass on the validity of alternative theories of the transmission mechanism between financial variables and real variables. In particular, we are interested in reexamining the evidence that changes in money supply have real effects on output. Our most interesting finding of this section is that we cannot reject the hypothesis that income is governed only by its own history and the history of nominal interest rate innovations. Since nominal interest rate innovations are, by definition, orthogonal to past money innovations, this finding implies that the data is consistent with structural models in which there is no feedback from past money to output, but is generally inconsistent with both IS-LM analysis and most intertemporal versions of the equilibrium models. In Section IV an alternative structural model is outlined which is consistent with the data. This model has the central feature that changes in money supply do not affect output. The fifth section is the conclusion.

I. Review of Earlier Work

Using a multivariate, linear time series model, Sims (1979, 1980) showed that nominal interest innovations explain a substantial fraction of variance in industrial production. Furthermore, the inclusion of interest rates decreases significantly the fraction of variance in industrial production attributed to innovations in nominal money supply.

Table I shows the decomposition of variance of industrial production in both a three variable (industrial production, IP; money stock, FM1B; and consumer prices less shelter, PU106; and a four variable (plus nominal interest on 3-month Treasury bills, FYGM3) vector autoregression at various time horizons. In both systems the data is in logs and is monthly for the period 1949:1 to 1980:9. Twelve lags of each variable were estimated.

In the three variable system, the test of the hypothesis that all twelve lags of money have zero coefficients in the output equation may be rejected at the conventional 5 percent level (the F-test has a marginal significance level of .023), but this is not true in the four variable system (marginal significance of .098). As can be seen in Table I, the dominance of interest rate innovations over money innovations becomes stronger as the time horizon for predicting output lengthens. This accords with Sims' finding that the response of output to interest rate innovations is essentially flat for about six months, followed by a smooth decline reaching a minimum of about 18 months later.

As a further check of the robustness of this nominal interest rate--output link,[2/] the four variable system was reestimated separately for the two periods 1955:1 to 1971:12 and 1972:1 to 1980:9. For this comparison only three lags of each variable were included. This further restriction is motivated by the desire to perform hypothesis tests described in the next section in which long lag lengths are computationally unwieldy. Although a likelihood ratio test of the hypothesis that lags four through six of all variables have zero coefficients in each of the four equations is rejected (marginal significance level is .028), in out-of-sample forecasting (reestimating each period from 1971:12 to 1980:8), the three lag system actually performs better for most choices, of variables and time horizons. In a likelihood ratio test the hypothesis of six lags versus 12 lags is not rejected (marginal significance .0505). The three lag system shows most clearly that the interest rate output relationship is remarkably stable over time. In Figure I, the moving average response of each of the four variables to an innovation in nominal interest rates is presented for each period. In Figure II, the response of industrial production to an innovation in each of the four variables is shown. In both periods, income declines in response to interest rate innovations. This response is much quicker in the more recent period; there is no discernable lag and the response is strongest at the 12-month horizon. In the earlier period, a six-month lag is evident and the maximum impact is at the 24-month horizon. In both periods, interest rate innovations are followed by increased inflation and a decrease in nominal balances. In both periods, both interest rates and output respond to their own innovations with a characteristic "hump shape" moving average.

Although a Chow test of structural stability of coefficients is overwhelmingly rejected (Chi-Square test with 52 degrees of freedom = 112.65, marginal significance $< 10^{-5}$), the qualitative properties of the impulse responses look remarkably similar. This similarity should give pause to those who argue that the preponderance of "supply shocks" in the more recent period has radically altered the dynamic interactions between money, prices, interest rates, and output.

II. Interest Rates and Output--Real or Nominal?

The response of output to innovations in nominal interest rates is surprising if most variations in nominal interest rates stem from changes in anticipated inflation. In this section we attempt to formulate proxies for the ex ante real rate to get a better idea of whether the nominal interest rate innovations isolated in the preceding section represent innovations in the real rate, or innovations in expected inflation.

The first proxy for the expected real rate was derived by projecting the log of prices at t+1 on a constant and current and two lags of data at time t using a constant coefficient Kalman filter technique. This procedure is equivalent to reestimating an OLS regression each period. The resulting monthly expected inflation, $E(P_{t+1} | M_{t-s}, P_{t-s}, IP_{t-s}, R_{t-s}, s=0,1,2) - P_t$ is presented in Figure VII.

For the nominal rate, we use a series, BILLS1, constructed as the first-of-the-month yield of Treasury bills with one-month maturity.[3/] By subtracting the expected inflation from this series we generate an ex ante real rate, which is presented as in Figure VIII.

This ex ante real rate series was employed in several four variable, monthly autoregressive systems using three lags and a constant term in each equation. The major difficulty in interpreting these systems arises from the strong contemporaneous correlation between innovations in expected inflation and innovations in the expected real rate. Table II reports the variance-covariance matrix of the innovations arising from a system which includes industrial production, nominal rates, expected inflation, and money.

As can be seen in Table II, there is a strongly negative (-.94) correlation between innovations in expected inflation and innovations in the real rate. Also striking is the fact that the variance of both expected inflation innovations and real rate innovations is about eight times larger than that of nominal rate innovations. Apparently, most of the innovations to expected inflation are negatively correlated with innovations to real rates so as to leave the nominal rate largely unaltered. These results imply that, contrary to the results of Fama, a one unit innovation in the nominal rate can be decomposed into a .373 innovation in expected inflation and a .627 innovation in the expected real rate. However, most of the variability of these innovations is not explained by nominal rate innovations; the R^2 of a regression of innovations in expected inflation on nominal interest rate innovations is about .02, the corresponding regression with expected real rates as the dependent variable is about .05.

The high negative correlation between real rate innovations and expected inflation innovations implies that, unlike the system examined in

Section I, the qualitative properties of the moving average response graphs and the decomposition of variance might be expected to depend on the particular orthogonalization chosen. This is confirmed in Table III, which reports the variance decomposition of output in three alternative systems which all lead to equivalent predictions of future values.

The linearity of the vector autoregression system and the identity, real rate ≡ nominal rate - expected inflation, implies that given one of these variables, the predictive content for output is identical whichever of the other two variables is included. Thus, in the first vector autoregression examined in Table III, the first variable is expected inflation and the second variable can be interpreted alternatively as either nominal rate innovations or real rate innovations.

The proportion of variance attributed to the real rate and expected inflation varies dramatically, depending on the ordering chosen. These results are consistent with the view that the nominal interest rate contains most of the information for predicting output. Expected inflation contains a large amount of high frequency variation, which when subtracted from the nominal rate reduces the information content of that series. When real rates are first in the ordering, they account for only 3 percent of the variance of industrial production at the 48-month horizon. Whichever of the nominal rate or expected inflation is put second captures more than half of the variance of output at the same horizon. When the real rate follows expected inflation in the ordering, however, it captures more than 40 percent of the variance at the 48-month horizon. This change can occur because real rates are highly correlated with expected inflation, and coming second in the ordering they allow the nominal rate to be recovered.

Nevertheless, the moving average response graph (Figure III) shows that there is a qualitatively different response on output arising from a real rate innovation than from an expected inflation innovation. Figure (IIIa) shows that when it is first in the ordering, an expected inflation innovation has an immediate and unambiguous negative effect on output. Figure (IIIb) shows that when real rates are first they exert a positive response throughout, and a nominal interest rate innovation (equivalently expected inflation innovation) has a depressing effect past the five-month horizon. Table IV is designed to test the significance of this decomposition for predicting output. Specifically it calculates the standard error and marginal significance of the moving average responses of a system with nominal rates first and real rates (or equivalently expected inflation) second. This chart shows clearly that nominal rates have a significantly negative effect from months six through 48, while real rates have a significantly positive effect over the same horizon.

The qualitative properties of inflation innovations and real rate innovations in a system which attributes to agents perfect one-period-ahead price predictions may be seen in Table V and Figure IV, which report the decomposition of variance of industrial production and the moving average responses of industrial production. In these ex post systems, the real rate is qualitatively more important than in the ex ante system and has a positive effect on output for either orthogonalization.

The results in Figure V and Table VI show that both the ex ante and ex post real rates are largely exogenous. The real rate responds mostly to itself, dampening very quickly. At the 48-month horizon, 78 percent of its own variance is explained by its own innovations for the ex ante case and 97 percent for the ex post case. The real rate does not feed into other series. We can easily accept the hypothesis that all three coefficients on the ex ante real rate in the money equation are zero. The ex ante real rate never explains more than .8 percent of the variance of future money. These results cast doubt on any interpretations linking ex ante real rates with anticipations of future policy-induced changes.

The results in Table II can be taken as weak evidence against a Lucas-Barro intertemporal substitution model, even though our proxy for the ex ante rate ignores the barriers to current period information flows which are essential to their theories. There is insignificant correlation between innovations in nominal money and innovations in the real rate. This finding implies that, contrary to the Lucas-Barro models, the expectations held by a representative trader about the real rate would not be materially affected by information about current period monetary innovations. Table VI confirms the finding that monetary innovations do not strongly feed into the real rate series, explaining only 8.63 percent of the 48-month horizon forecast variance of ex ante real rates and only .36 percent of the forecast variance of ex post real rates.

III. Tests of Specific Hypotheses

The preceding descriptive empirical findings cast doubt on the money-interest-output link suggested by Keynesian analysis. Not only did the real rate fail to reflect any systematic influence from money or prices, but output appeared to respond to changes in expected inflation in a way not consistent with most demand-driven models of output. In this section we propose to test specific restrictions to examine more closely these anomolous results.

A. Is the Real Rate Exogenous?

A central feature of Keynesian IS-LM analysis is the idea that changes in the demand or supply of nominal balances can change the real interest rate. Keynesian theory achieves this connection by invoking sluggish nominal price adjustments in nonfinancial markets, particularly the labor market. We propose to test this money-real interest rate structural link by testing the alternative hypothesis that the real rate is governed only by its own past history with no separate influence coming from money, prices, or income. Although this hypothesis is not an implication of any particular alternative to the Keynesian theory, it is incompatible with models of this sort except for some very restrictive and economically uninteresting special cases.

Consider the following IS-LM model

$$\text{IS} \quad y_t = -\beta_1 r_t + \varepsilon_t, \ \beta_1 > 0$$

$$\text{LM} \quad \frac{M_t}{P_t} = \alpha_1 y_t - \alpha_2(r_t + \Pi^e_t) + \phi_\tau \ \ \alpha_1 > 0, \ \alpha_2 > 0 \tag{1}$$

where r_t is the real rate and Π^e_t is expected inflation, ε_t represents all exogenous spending (including government spending and variations in desired investment not related to interest rate movements), ϕ_t represents random influences on real money demand (the state of "liquidity preference"). The reduced form equations for the endogenous variables r_t, y_t are given by

$$r_t = \gamma_1 \varepsilon_t + \gamma_2(m_t - \phi_t) + \gamma_3 \Pi^e_t$$

$$y_t = \gamma_4 \varepsilon_t + \gamma_5(m_t - \phi_t)\gamma_6 \Pi^e_t \tag{2}$$

where

$$\gamma_1 = \frac{\alpha_1}{\alpha_2+\alpha_1\beta_1},\ \gamma_2 = \frac{-1}{\alpha_2+\alpha_1\beta_1},\ \gamma_3 = \frac{-\alpha_2}{\alpha_2+\alpha_1\beta_1}$$

$$\gamma_4 = \frac{\alpha_2}{\alpha_2+\alpha_1\beta_1},\ \gamma_5 = \frac{\beta_1}{\alpha_2+\alpha_1\beta_1},\ \text{and } \gamma_6 = \frac{\beta_1\alpha_2}{\alpha_2+\alpha_1\beta_1},\ m_t = \frac{M_t}{P_t}.$$

Under what auxiliary hypotheses can equation (1) be compatible with the finding that

$$E(r_{t+1}|r_{t-s},s\geq 0) = E(r_{t+1}|r_{t-s},M_{t-s},P_{t-s},Y_{t-s},S\geq 0)?$$

One possibility is that, over the observed sample, it was the deliberate objective of Fed policy to set expected real rates in such a way that the two hypotheses are observationally equivalent. This might arise, for example, if the policy objective was to minimize $E(Y_t-\overline{y})^2$, by setting $r_t = \frac{-1}{\beta_1}(\overline{y}-\varepsilon_t)$. If ε_t followed a univariate autoregressive process, then so would r_t. Although we cannot reject this possibility a priori, it is unlikely that desired interest rate targets could be expressed in terms of any single factor, let alone the past history on interest rates. It certainly appears as if policy has aimed for both price and output stability. Since prices and output exhibit some independent variations, it is implausible to take the finding that the real rate is exogenous as indicative of a particular policy reaction function.

Another possibility which could explain the lack of any influence from past money, prices, and output on current ex ante real rates is that the IS curve is horizontal. This would be true if β_1, the interest sensitivity of demand, were infinite, so that variations in money supply or demand affected only output without a measurable impact on interest rates. This possibility is both highly implausible in a monthly system and is easily rejected by subsequent findings.

Still a third possibility, less easily dismissable, is that over the sample period most variations in money supply, m_t, were passive responses to ϕ_t, money demand shocks. Under this hypothesis there would be no added explanatory power from past money to future real rates. Again, this hypothesis denies any deliberate attempt on behalf of the Fed for controlling real rates, except insofar as interest rate targets depend only on lagged values. A more prosaic interpretation would argue that policy-induced interest rate variations have been sufficiently small compared with exogenous money demand shifts so that our procedure cannot distinguish this possibility from errors due to sample variation.

These possibilities, while being neither mutually exclusive nor exhaustive, seem sufficiently restrictive, so that failure to reject the hypothesis that the real rate is exogenous casts doubt on the Keynesian notion that monetary policy can affect the real rate.

The ex ante real rate is unobservable. To test the hypothesis that the real rate is exogenous requires an auxiliary hypothesis of how agents forecast future prices. Keynesian theory is noticeably vague about the process of expectations formation. We propose to identify the projection of future prices on current and lagged endogenous variables with the expectations held by a representative agent. If we have successfully captured the available information set, and we assume structural stability, then such projections are equivalent to "rational expectations" of the type first proposed by Muth. The assumption that all current period aggregate outcomes are available to agents when forming their expectations implies that this test cannot pass on the empirical validity of models which explicitly posit barriers to information flows.

The hypothesis that the ex ante real rate of interest, r_t, is a function of only its own lagged values, a constant term, and an uncorrelated random error, can be written as follows:

$$r(t) = \sum_{j=1}^{M} b(j)r(t-j) + c + u(t). \tag{3}$$

The assumption of rational expectations implies

$$r(t) = R(t) - {}_{t}\Pi(t+1) \tag{4}$$

where R(t) is the observed nominal interest rate and ${}_{t}\Pi(t+1)$ is the projection of the annualized growth rate of the price level from t to t+1 on information available at time t.

Substitution of (4) into (3) leads to the following expression for the nominal interest rate:

$$R(t) = \sum_{j=1}^{M} b(j)R(t-j) + {}_{t}\Pi(t-1)$$
$$- \sum_{j=1}^{M} b(j)\,{}_{t-j}\Pi(t-j+1) + c + u(t). \tag{5}$$

This equation imposes testable restrictions across the autoregressive representation for R(t), Π(t), and the other variables, Z(t), in the information set individuals use in projecting future values of Π.

Suppose a finite order autoregressive representation exists for the k-vector

$$X'(t) = [R(t)\Pi(t)Z'(t)].$$

$$X(t) = \sum_{\ell=1}^{L} A(\ell)X(t-\ell) + c + \eta(t). \tag{6}$$

The i^{th} equation of this representation has the scalar form

$$x_i(t) = \sum_{j=1}^{k} \sum_{\ell=1}^{L} a_{i,j}(\ell)x_j(t-\ell) + c_i + \eta_i(t). \tag{7}$$

Thus, for example, the projection of inflation during period t on observables at time t-1 is given by

$$_{t-1}\Pi(t) = \sum_{\ell=1}^{L} a_{2,1}(\ell)R(t-\ell) + \sum_{\ell=1}^{L} a_{2,2}(\ell)\Pi(t-\ell)$$

$$+ \sum_{j=3}^{k} \sum_{\ell=1}^{L} a_{2,j}(\ell)Z_{j-2}(t-\ell) + c_2. \tag{8}$$

The restrictions implied by (3) are generated by projecting the quantities on the right-hand side of (5) on information available at time t-1. By advancing the time subscript in (8) and substituting for $_t\Pi(t+1)$ in (5) we have

$$(1-a_{2,1}(1))R(t) = \sum_{j=1}^{M} (b(j)+a_{2,1}(j+1))R(t-j)$$

$$+ \sum_{\ell=m+2}^{L} a_{2,1}(\ell)R(t-\ell+1) + a_{2,2}(1)\Pi(t)$$

$$+ \sum_{\ell=2}^{L} a_{2,2}(\ell)\Pi(t-\ell+1)$$

$$+ \sum_{j=3}^{K} [a_{2,j}(1)Z_{j-2}(t) + \sum_{\ell=2}^{L} a_{2,j}(\ell)Z_{j-2}(t-\ell+1)] + c + c_2. \qquad (9)$$

Replacement of the term $\Pi(t)$ with the projection (8) and similar expressions for the $Z_j(t)$'s leads to a reduced form projection for R(t) on observables at t-1, the coefficients of which are nonlinear functions of the coefficients in the autoregressive representation for Π and Z.

In the system reported below in Table VII, we have M=1, L=3, and k=4 with $Z' = [Y\ M1]$ where Y is industrial production and M1 is the money stock, M1B.

The constant in the first equation, the b(j)'s and all coefficients in the other equations are the free parameters in the maximum likelihood estimation.

The result of this hypothesis test is consistent with our interpretation of the evidence in the impulse response functions reported in Section II. A strong (because it involves only one lag) version of the test of exogeneity of the real rate leads to the hypothesis not being rejected.

It is revealing to compare the response of the real rate to innovations in each of the four variables in both the unrestricted system and the first restricted system (Figure VI). Basically, only the response to output innovations and the contemporaneous correlation of nominal interest rate innovations with the real rate are altered by the restriction that the real rate is exogenous.

Notice that since both inflation and the real rate have persistent components, the strong negative contemporaneous correlation between real rates and innovations in inflation is enough to explain the negative correlation between inflation levels and future real rates observed by Summers (1980) and Mishkin (1980): Our interpretation of this phenomenon differs, however, from that of Summers who argues that money illusion is "the most plausible explanation for the nonresponse of interest rates to inflation." Since the result is compatible with an economic structure in which there is no feedback from prices and money to real interest rates, we conclude that short-run changes in both real rates and inflation can be attributed to the same, as yet unidentified, random factor.

B. What Causes Output

An important question addressed by most business cycle theories is the transmission mechanism between financial variables and real variables. The importance of money for predicting output was demonstrated by Sims in 1972 in a bivariate system and was generally taken as evidence of real effects of purely monetary phenomena. Explaining this relationship has occupied a central role of recent theoretical developments. Modern theories share with Keynesian theory the idea that monetary phenomena affect real variables by altering perceptions of real interest rates. Given the descriptive results in Section II in which real rate innovations appear significantly less useful for predicting output than nominal interest rate innovations, we are led to question the usefulness of both Keynesian and equilibrium theories for explaining the observed correlations between nominal and real variables.

We first confirm Sims' finding that industrial production is not completely exogenous in a four variable vector autoregression with nominal rates, money, and prices. We then go on to test whether these observed effects from nominal quantities can be explained through three lags of the real rate. In light of the previously accepted hypothesis that the real rate is exogenous, acceptance of these implied restrictions would indicate that real rates and output are block exogenous. This restriction again involves an unobservable and thus is imposed through a set of nonlinear cross-equation restrictions. In light of the Section II results, it is not surprising that this hypothesis may be rejected.

The next two hypotheses examined are not implications of any completely articulating theory, but are designed to confirm the descriptive results presented in the previous section. We first test whether all lagged financial effects can be filtered through three lagged levels of nominal interest rates. This hypothesis cannot be rejected. However, since money and prices have predictive content for nominal interest rates, we can reject our fifth hypothesis that income and nominal rates are block exogenous in a four variable system. Thus, we cannot conclude from these tests alone that money has no predictive content for output past a one-period forecast horizon.

Our sixth hypothesis is designed to assess the importance of money for predicting output at any forecast horizon. Specifically, we test whether three lags of nominal interest rate innovations are sufficient to capture all lagged financial effects. Since nominal interest rate innovations are, by construction, orthogonal to all past variables, this is a test of block exogeneity of output and nominal interest rate innovations in a four variable autoregressive system. We cannot reject this hypothesis. In the next section we present a structural model which is consistent with these findings.

<u>TEST PROCEDURES</u>

The test of the hypothesis that industrial production is exogenous is the easiest to implement because it does not involve any unobservable quantities. In fact, the likelihood ratio test of the equation

$$Y(t) = \sum_{\ell=1}^{L} b(\ell)Y(t-\ell) + c + u(t) \tag{10}$$

relative to the corresponding equation in an unrestricted vector autoregressive system is equivalent to the likelihood ratio test of the entire system with and without the output equation so restricted. This result is shown by Doan (1980) using results in Revankar (1974).

The hypothesis that Y is a function only of lagged Y's, three lags of the real rate, a constant, and an uncorrelated random error leads to nonlinear, cross-equation restrictions. This hypothesis can be written as

$$Y(t) = \sum_{\ell=1}^{L} b(\ell)Y(t-\ell) + \sum_{\ell=1}^{L} d(\ell)r(t-\ell) + c + u_t. \qquad (11)$$

Replacing r(t) with $R(t) - {}_t\Pi(t+1)$ and substituting for ${}_t\Pi(t+1)$ leads to[4/]

$$\begin{aligned} Y(t) = & \sum_{\ell=1}^{L} b(\ell)Y(t-\ell) + c + \sum_{\ell=1}^{L} d(\ell)R(t-\ell) \\ & - \sum_{\ell=1}^{L} d(\ell)\{\sum_{m=1}^{L} [a_{3,2}(m)Y(t-\ell-m)+a_{3,3}(m)R(t-\ell-m) \\ & + \sum_{j=4}^{k} a_{3,j}(m)Z_{j-3}(t-\ell-m)]\} + u(t). \end{aligned} \qquad (12)$$

In the system reported in Table VII we have L=3, k=4, and Z=[M1]. In this system the constant in the first equation, the b(j)'s, the d(j)'s, and all coefficients in the other equations are the free parameters in the maximum likelihood estimation.

The next hypothesis test, in which a nominal interest rate replaces the ex ante real rate, involves only observable quantities, and therefore does not require the use of nonlinear restrictions. The restricted equation,

$$Y(t) = \sum_{\ell=1}^{L} b(\ell)Y(t-\ell) + \sum_{\ell=1}^{L} d(\ell)R(t-\ell) + c + u(t) \qquad (13)$$

is compared with the corresponding equation in the unrestricted vector autoregression.

In all of the systems with nonlinear constraints initial estimates were obtained by estimating an unrestricted vector autoregression, generating the implied expected inflation and ex ante real rate, and estimating the restricted equation on the basis of these observations. This procedure leads to consistent estimates of the parameters and is the one followed by Barro (1977). It is not fully efficient, however, and we prefer the full-information-maximum-likelihood (FIML) estimates obtained by minimizing the log of the determinant of the variance-covariance matrix of residuals in the constrained system. A FORTRAN program utilizing analytic gradient and Hessian was written by Litterman for this purpose.

The hypotheses are tested using the likelihood ratio statistic formed by taking

$$(T\text{-}dfc) \quad \log[\det \textstyle\sum^{c}/\det \sum^{u}] \tag{14}$$

where T is the number of observations in each equation, dfc is a degrees of freedom correction suggested by Sims (1980b) equal to the number of parameters in each equation of the unrestricted system,[5/] $\sum^{c}$ is the determinant of the covariance matrix of residuals in the constrained system, and $\sum^{u}$ is the covariance matrix of residuals in the unrestricted system.

V. A Possible Explanation

A central result of this paper is that there is information in the level of nominal interest rates for predicting future output which is not contained in history of past output or past and future expected real interest rates. A natural question is what types of structural relationships could give rise to this finding.

Two approaches seem possible. One of these points to the numerous institutional features of the American economy which imply perfectly foreseen inflation can have real and depressing output effects. Among the leading examples cited in support of this view are the nonindexation of the tax system, the nonindexation of some administered prices, the effects of nominal interest rate ceilings, and the distortionary effects of taxation of liquidity services. However, earlier theoretical arguments, particularly the Tobin and Mundell effects, would lead to opposite conclusions. More troublesome is that this explanation leaves unanswered the cause of inflation responsible for income movements. The original four variable system shows clearly that nominal interest rate innovations (and implicitly expected inflation innovations) have a negative impact on the future level of money. This response was replicated in an alternative system where high-powered money replaced M1B. These results show that movements in the short-run inflation rate cannot be ascribed to short-term movements in (rationally) anticipated money.

The other possibility which could account for a predictive content of rationally anticipated inflation for future output argues that this correlation is spurious. To see how this could arise, consider the following model in which output is independent of the money supply process

$$Y_{t+1} = Y_t + Z_t + u_{t+1} \tag{15}$$

$$M_t - P_t = m_1 U_t - m_2 R_t \tag{16}$$

$$R_t \equiv P^e_{t+1} - P_t + r_t \tag{17}$$

$$r_t = \lambda r_{t-1} + \varepsilon_t. \tag{18}$$

The crucial feature of this model is that there is some information summarized in Z_t which is knowable to agents in the economy and is useful for predicting future output, but is not directly observable to the econometric investigator. If the model is closed by specifying a money supply process

$$M_t \equiv 0 \tag{19}$$

given that ε_t, Z_t, u_t are serially independent, it is straightforward to show that the solution for nominal interest rates is

$$R_t = (\frac{-m_1}{1+m_2})Z_t + (\frac{1}{1+m_2(1-\lambda)})r_t, \tag{20}$$

and the solution for nominal interest rate innovations, $R_t - ER_t = \hat{R}_t$ is given by

$$\hat{R}_t = (\frac{-m_1}{1+m_2})Z_t + (\frac{1}{1+m_2(1-\lambda)})\varepsilon_t. \tag{21}$$

This model shows clearly that nominal interest rate innovations or expected inflation innovations will be correlated with "Z" innovations, and thereby will be useful for predicting output when Z_t is not observed directly. This occurs despite the lack of any structural feedback from past, current, or future money or prices to output. An argument similar to this has been suggested by Fama (cf. 1980) to explain both the postwar U.S. inflation experience and the observed negative correlation between real stock returns with both anticipated and unanticipated inflation.

It would not be too difficult to append this model to explain other characteristic features of the data. Changing the money supply process to

$M_t = M_{t-1} - \lambda_1(P^e_{t+1} - P_t)$ could account for the predictive content of nominal money in a bivariate system in a way which is consistent with the block exogeneity of income and interest rate innovations in the context of a larger system. A "Phillips Curve" relationship--a positive correlation between ex post inflation and ex post output growth--could be made compatible with this economic structure if money supply expanded with unexpected output shocks (i.e., $M_t = M_{t-1} + \lambda_2 u_t$).

These two hypotheses have different implications for the desirability and feasibility of policy-induced attempts to lower inflation by changing the path of nominal money supply. The first hypothesis, emphasizing the structural nonneutralities of inflation, would imply that such a policy is desirable, but admits that other, as yet unidentified, nonmonetary factors have been dominant for causing changes in short-run inflationary expectations. The second hypothesis, which argues that the relationship is essentially spurious, would imply that such a policy is feasible but would not necessarily have any real benefits.

Unfortunately, the evidence to distinguish between these hypotheses is ambiguous. Favoring the view that this is a spurious relationship is our inability to reject Hypothesis VI, which implies that output and interest rate innovations are block exogenous. Since we know that lagged money and prices have some additional predictive content for future inflation, this test would seen to deny a structural link between inflation levels and output. Against this view is the result of Hypothesis VIII, which tests whether output and expected inflation innovations are block exogenous. This hypothesis is rejected at the 5 percent significance level. Evidence that the level of expected inflation affects output (as opposed to the unexpected change in the level of expected inflation) comes from comparing the results from Hypothesis

IV (output is governed by its own history and lagged nominal interest rate levels) with Hypothesis VI (output is governed by its own history and lagged nominal interest rate innovations) and similarly comparing the results of Hypothesis VII with Hypotheses VIII, which replaces the level and innovations of nominal rates with the level and innovations of expected inflation, respectively. In both sets of results it can be seen that there is more predictive content in the three lagged levels of either nominal interest rates or expected inflation than in three lagged values of the innovations in these variables. However, these tests' results might be misleading if the information going into current nominal interest rates and inflationary expectations concerns changes in output at horizons larger than three months. In this case levels of these variables, which depend on past innovations, might have more explanatory power in systems where the lag length is shorter than agents' informational horizons.

The tests which utilize lagged innovations as explanatory variables follow essentially the same estimation procedure as for those with ex ante real rates. Innovations in nominal interest rates, for example, are defined by

$$\tilde{R}(t) = R(t) - E[R(t) \mid Y(t-s), R(t-s), M(t-s), \Pi(t-s), s=1,2,3]. \qquad (22)$$

The expectation is determined by the autoregressive representation for R, and the $\tilde{R}$'s can be substituted out in a manner similar to that in the earlier tests.

Innovations in expected inflation are defined by

$$\tilde{\hat{\Pi}}(t) = E[\Pi(t+1) \mid Y(t-s), R(t-s), \Pi(t-s), M(t-s), s=0,1,2]$$

$$- E[\Pi(t+1)\,|\,Y(t-s),R(t-s),\Pi(t-s),M(t-s),s=1,2,3]. \qquad (23)$$

Upon substituting the expectations implied by the autoregressive representation it is easily seen that

$$\tilde{\tilde{\Pi}}(t) = a_{3,1}(1)\tilde{Y}(t-1) + a_{3,2}(1)\tilde{R}(t-1)$$

$$+ a_{3,3}(1)\tilde{\Pi}(t-1) + a_{3,4}(1)\tilde{M}(t-1). \qquad (24)$$

Conclusion

This paper has examined the empirical support for a number of hypotheses about the money-interest-output link. Because the relevant real rate is unobservable, an appropriate empirical counterpart suggested by a particular class of structural models was formulated. This class might be considered "dynamic IS-LM" with rational expectations. Although this class does not include those models which explicitly posit barriers to information flows, some of our results bear on their empirical validity.

The first set of tests sought to identify the determinants of the real interest rate. Specifically, we could not reject the hypothesis that this variable is governed only by its own past history, with no separate influence coming from money or prices. Although this hypothesis is not an implication of any particular alternative to the Keynesian theory, it is incompatible with models of this sort, except for some very restrictive and economically uninteresting special cases. Taken literally, our results imply that monetary policy has not discernably affected the real rate, although it has causally influenced nominal interest rates. Our results also show a strongly negative correlation between expected real rates and inflation innovations. Since both inflation and expected real rates have some persistent component, this can explain the well-documented negative correlation between the level of current period inflation and real rates, even in the absence of any structural link between past inflation and future real rates.

The inability to filter the observed influence of money and nominal interest rates on output through expected real rates casts doubt on the Keynesian transmission mechanism between money and output. The lack of any significant correlation between money innovations and the real rate innovations vitiates the case that unanticipated (and unperceived) monetary injections cause business cycles by leading to misperceptions of the real rate, a key element of the Lucas-Barro theory.

A central new result of this paper is that there is information in the level of nominal interest rate for predicting future output which is not contained in the history of past output or past and future expected real interest rates. Two explanations for the apparent predictive content of expected inflation on output have been advanced. The structuralist interpretation focuses on nonneutralities of various (nonoptimal) institutional nominal rigidities, but leaves unanswered the causes of changes in inflation. The other hypothesis argues that output is structurally exogenous to money and prices, but that new information is first reflected in expected inflation and interest rates.

Appendix A

DATA

The four data series used in this study are monthly, seasonally adjusted, observations from 1948:1 to 1980:9. The first three series were taken from the Citibank Data Base in February 1981. These series are the Industrial Production Index, IP, the Consumer Price Index Less Shelter, PU106, and the money stock, FM1B. This last series is available only starting in 1959:1. In order to extend it back to 1948:1 FM1B was regressed on the old M1 series and a constant. This specification was then used to extend FM1B from 1948:1 to 1958:12. The fourth series, BILLS1, is the rate on Treasury bills with one-month maturity described in footnote 3.

All series were logged and the series for Hypothesis I were defined by

$$R(t) = \log[1. + BILLS1(t)/100.] * 100.$$

$$\Pi(t) = [\log PU106(t) - \log PU106(t-1)] * 1200.$$

$$Y(t) = [\log IP(t) - \log IP(t-1)] * 1200.$$

$$M1(t) = [\log FM1B(t) - \log FM1B(t-1)] * 1200.$$

The series for the other hypotheses are the same except that Y(t) = log IP (t) and M1(t) = log FM1B(t) rather than the first-differenced forms used above.

Interpretation of Impulse Response Graphs

The graphs of impulse response functions in this paper are the responses to orthogonalized innovations. In some cases we plot the responses of all variables in the system to a particular innovation, in other cases we plot the response of a particular variable to all innovations. In both cases the ordering of the orthogonalization is given by the alphabetical ordering of the responses in the graph. Given the relatively small contemporaneous correlation in these monthly data, the observed impulse responses are probably not too sensitive to the ordering. All graphs have normalized the responses in terms of standard deviations of the variable's innovations. The scale for the graph is given at the top. The monthly responses are displayed over the four-year horizon.

The response functions are based on the moving average representation of a vector time series. A more extended discussion and further application of this technique may be found in Sargent (1978). The exact procedure is given in Doan and Litterman (1980). The orthogonal decomposition of variance or "innovation accounting" used in Sections I and II is based on the same moving average representation. The details are given in the above-mentioned references.

Footnotes

1/ Fama and Gibbons report that "The ratio of the variance of changes in the monthly expected real return, to the variance of the change in the monthly expected inflation rate is about .2, while the corresponding quarterly ratio is .25."

2/ We also estimated a number of larger systems including (not all at one time) inventories, retail sales, real wages, wage settlements, M1A, the monetary base, a stock price index, the unemployment rate, 10-year bond yields, and a trade-weighted index of the value of the dollar. The qualitative behavior of the output response to interest rate innovations described above appeared in every system estimated.

3/ The data are from Salomon Brothers. However, prior to 1964 Salomon Brothers reports midmonth yields, and after 1977 the data are taken from the U.S. Treasury Bulletin, which reports yields on the last trading day of the previous month.

4/ The subscripts here refer to an autoregressive representation as in (6) where X_t' is now $[Y_t R_t \Pi_t Z_t']$.

5/ The correction made little difference in the results reported here since T = 389 and dfc = 13.

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Table I
Decomposition of Variance
(1949:1 to 1980:9)

	Four Variable				Three Variable		
Month	FM1B	PU106	IP	FYGM3	FM1B	PU106	IP
1	1.5	.3	98.2	0	1.4	.2	98.5
13	9.7	5.4	69.2	15.7	28.6	8.4	63.6
25	6.3	9.9	49.3	34.4	34.2	18.9	46.9
37	5.2	9.2	46.7	39.0	34.1	19.1	46.7
49	4.4	9.9	46.5	39.2	33.7	17.0	49.2

Table II
Variance-Covariance Matrix of Innovations
(1955:1 to 1980:9)

	FYGM3	RLRATE	$INFL^e$	IP	FM1B
FYGM3	.25322	.15875	.09440	.000600	-.0002600
RLRATE	.22100	2.03000	-1.87640	.001050	-.0000930
$INFL^e$	.13372	-.93693	1.97400	-.000457	-.0001680
IP	.12821	.12821	-.12821	.000086	.0000024
FM1B	-.15996	-.02000	-.03691	.079500	.0000106

Numbers beneath diagonal are correlations.

Table III
Decomposition of the Variance of
Industrial Production at Various Forecast
Horizons Using Ex Ante Inflation and Ex Ante Real Rates
(1955:1 to 1980:9)

Month	$INFL^e$	RR^e FYGM3	IP	FM1B
12	11.18	9.10	78.35	1.37
24	15.43	26.28	53.69	4.59
36	16.33	35.49	41.91	6.26
48	16.61	40.40	35.93	7.06
	FYGM3	RR^e $INFL^e$	IP	FM1B
12	11.60	8.65	78.35	1.37
24	31.23	10.49	53.69	4.59
36	41.35	10.47	41.91	6.26
48	46.99	10.32	35.93	7.06
	RR^e	$INFL^e$ FYGM3	IP	FM1B
12	4.78	15.49	78.35	1.37
24	4.07	37.65	53.69	4.59
36	3.35	48.47	41.91	6.26
48	2.95	54.05	35.93	7.06

Table IV
Standard Errors and Significance Levels
For Industrial Production Response Functions

Responses to Nominal Rate Innovations

Step	Mean	Standard Error	Significance*
1	.123	.059	.02
2	.151	.103	.07
3	.046	.136	.33
4	-.102	.160	.25
5	-.239	.180	.08
6	-.365	.199	.03
12	-.955	.296	.00
24	-1.408	.450	.00
36	-1.506	.573	.00
48	-1.493	.682	.00

Response to Real Rate Innovations

Step	Mean	Standard Error	Significance
1	.045	.061	.22
2	.121	.103	.12
3	.200	.136	.06
4	.310	.161	.03
5	.404	.185	.01
6	.477	.202	.01
12	.619	.229	.00
24	.617	.262	.01
36	.592	.298	.01
48	.565	.334	.01

*Significance is used here in a Bayesian sense to refer to the integral, given a noninformative prior, of the posterior distribution of the response function less than zero, or greater than zero, whichever is less.

Table V
Decomposition of the Variance of Industrial Production at Various Forecast Horizons Using Ex Post Real Rates and Ex Post Inflation

	INFL	FYGM3 RLRATE	IP	FM1B
12	12.54	6.39	80.75	.30
24	21.42	21.16	55.87	1.55
36	24.72	30.00	42.96	2.31
48	26.15	34.94	36.29	2.61

	FYGM3	RLRATE INFL	IP	FM1B
12	6.66	12.27	80.75	0.30
24	21.84	20.73	55.87	1.55
36	30.88	27.87	42.96	2.31
48	35.92	25.17	36.29	2.61

Month	RLRATE	FYGM3 INFL	IP	FM1B
12	9.44	9.49	80.75	0.30
24	14.16	28.42	55.87	1.55
36	15.52	39.20	42.96	2.31
48	16.00	45.09	36.29	2.61

Table VI
Decomposition of Variance
at the 48-Month Horizon
(1954:4 to 1980:9)

Variable Explained → By Innovations in ↓	Ex Ante Real Rate	Nominal Rate	IP	FM1B
Ex Ante Real Rate	78.26	2.64	2.93	.80
Nominal Rate	4.02	69.76	54.39	83.91
IP	9.08	16.04	35.58	2.38
FM1B	8.63	11.55	7.10	12.92

	Ex Post Real Rates	Nominal Rate	IP	FM1B
Ex Post Real Rate	96.92	5.89	16.00	7.34
Nominal Rate	2.54	67.42	45.09	71.88
IP	.17	18.62	36.29	2.42
FM1B	.36	8.06	2.61	18.27

Table VII

Test Results
Real Rate Exogeneity
Hypothesis I:

$$r_t = b_0 r_{t-1} + c + u_t$$

Estimate of Restricted Equation (standard errors in parentheses)

$$r_t = \underset{(.040)}{.651}\, r_{t-1} + \underset{(.025)}{.029} + u_t$$

Likelihood Ratio Test Statistic = 17.14
Chi-squared with 11 degrees of freedom
(marginal significance .104)

Output Exogeneity
Hypothesis II:

$$y_t = b_1 y_{t-1} + b_2 Y_{t-2} + b_3 y_{t-3} + c + u_t$$

Estimate of Restricted Equation

$$y_t = \underset{(.051)}{1.434}\, y_{t-1} - \underset{(.087)}{.361}\, y_{t-2} - \underset{(.051)}{.0738}\, y_{t-3} + \underset{(.0061)}{.0078} + u_t$$

Likelihood Ratio Test Statistic = 19.48
Chi-squared with 9 degrees of freedom
(marginal significance .021)

Output and Real Rates
Hypothesis III:

$$y_t = b_1 y_{t-1} + b_2 y_{t-2} + b_3 Y_{t-3} + b_4 r_{t-1} + b_5 r_{t-2} + b_6 r_{t-3} + c + u_t$$

Estimate of Restricted Equation

$$y_t = \underset{(.037)}{1.434}\, y_{t-1} - \underset{(.064)}{.363}\, y_{t-2} - \underset{(.038)}{.0721}\, y_{t-3} - \underset{(.00035)}{.000228}\, r_{t-1}$$

$$- \underset{(.00045)}{.000006}\, r_{t-2} + \underset{(.00032)}{.000231}\, r_{t-3} + \underset{(.0046)}{.00708}$$

Likelihood Ratio Test Statistic = 25.08
Chi-squared with 14 degrees of freedom
(marginal significance .034)

Output and Nominal Rates
Hypothesis IV:

$$y_t = b_1 y_{t-1} + b_2 y_{t-2} + b_3 y_{t-3} + b_1 R_{t-1} + b_5 R_{t-2} + b_6 R_{t-3} + c + u_t$$

Estimate of Restricted Equation

$$y_t = \underset{(.051)}{1.406}\, y_{t-1} - \underset{(.085)}{.342}\, y_{t-2} - \underset{(.051)}{.0565}\, y_{t-3} - \underset{(.0011)}{.000266}\, R_{t-1}$$

$$- \underset{(.0015)}{.002072}\, R_{t-2} + \underset{(.0011)}{.000569}\, R_{t-3} - \underset{(.0115)}{.0243}$$

Likelihood Ratio Test Statistic = 7.47
Chi-squared with 6 degrees of freedom
(marginal significance .279)

Output, Nominal Rate Block Exogeneity
Hypothesis V:

$$y_t = b_{11}y_{t-1} + b_{12}y_{t-2} + b_{13}y_{t-3}$$

$$+ b_{14}R_{t-1} + b_{15}R_{t-2} + b_{16}R_{t-3} + c + u_t$$

$$R_t = b_{21}y_{t-1} + b_{22}y_{t-2} + b_{23}y_{t-3} + b_{24}R_{t-1}$$

$$+ b_{25}R_{t-2} + b_{26}R_{t-3} + c + u_t$$

Likelihood Ratio Test Statistic = 31.29
Chi-squared with 12 degrees of freedom
(marginal significance .001)

Output and Nominal Rate Innovations
Hypothesis VI:

$$y_t = b_1 y_{t-1} + b_2 y_{t-2} + b_2 y_{t-3} + b_4 \tilde{R}_{t-1} + b_5 \tilde{R}_{t-2} + b_6 \tilde{R}_{t-3} + c + u_t$$

where

$$\tilde{R}_t = R_t - E[R_t | Y_{t-s}, R_{t-s}, M_{t-s}, \Pi_{t-s}, s=1,2,3]$$

Estimate of Restricted Equation

$$Y_t = \underset{(.036)}{1.423}\, Y_{t-1} - \underset{(.062)}{.3358}\, Y_{t-2} - \underset{(.036)}{.0890}\, Y_{t-3} - \underset{(.00085)}{.000735}\, \tilde{R}_{t-1}$$

$$- \underset{(.00085)}{.003302}\, \tilde{R}_{t-2} - \underset{(.00089)}{.000692}\, \tilde{R}_{t-3} + \underset{(.0043)}{.0076}$$

Likelihood Ratio Test Statistic = 28.05
Chi-squared with 18 degrees of freedom
(marginal significance .098)

Output and Expected Inflation
Hypothesis VII:

$$Y_t = b_1 Y_{t-1} + b_2 Y_{t-2} + b_3 Y_{t-3} + b_4 \hat{\Pi}_{t-1}$$

$$+ b_5 \hat{\Pi}_{t-1} + b_6 \hat{\Pi}_{t-2} + c + u_t$$

where

$$\hat{\Pi}_t = E[\Pi_t | Y_{t-s}, R_{t-s}, \Pi_{t-s}, M_{t-s}, s=1,2,3]$$

Estimate of Restricted Equation

$$Y_t = \underset{(.036)}{1.425}\, Y_{t-1} - \underset{(.064)}{.355}\, Y_{t-2} - \underset{(.038)}{.067}\, Y_{t-3} + \underset{(.00037)}{.000133}\, \hat{\Pi}_t$$

$$- \underset{(.00048)}{.000427}\, \hat{\Pi}_{t-1} - \underset{(.00034)}{.000209}\, \hat{\Pi}_{t-2} - \underset{(.0021)}{.0051}$$

Likelihood Ratio Test Statistic = 22.15
Chi-squared with 14 degrees of freedom
(marginal significance .075)

Output and Innovations in Expected Inflation
Hypothesis VIII:

$$Y_t = b_1 Y_{t-1} + b_2 Y_{t-2} + b_3 Y_{t-3} + b_4 \tilde{\hat{\Pi}}_{t-1} + b_5 \tilde{\hat{\Pi}}_{t-2} + b_6 \tilde{\hat{\Pi}}_{t-3} + c + u_t$$

where

$$\tilde{\hat{\Pi}}_t = \hat{\Pi}_{t+1} - E[\Pi_{t+1} | Y_{t-s}, R_{t-s}, M_{t-s}, \Pi_{t-s}, s=1,2,3]$$

Estimate of Restricted Equation

$$Y_t = \underset{(.036)}{1.430}\, Y_{t-1} - \underset{(.062)}{.351}\, Y_{t-2} - \underset{(.036)}{.081}\, Y_{t-3} + \underset{(.00038)}{.000243}\, \tilde{\hat{\Pi}}_{t-1}$$

$$- \underset{(.00041)}{.000190}\, \tilde{\hat{\Pi}}_{t-2} + \underset{(.00038)}{.000381}\, \tilde{\hat{\Pi}}_{t-3} + \underset{(.0044)}{.00727}$$

Likelihood Ratio Test Statistic = 32.75
Chi-squared with 18 degrees of freedom
(marginal significance .018)

Figure I. Responses to Nominal Interest Rate Innovations:

KEY: Response of
A: Nominal Interest Rate
B: Price Level
C: Industrial Production
D: Money Stock

Figure Ia. Estimation Period: 1955:1–1971:12

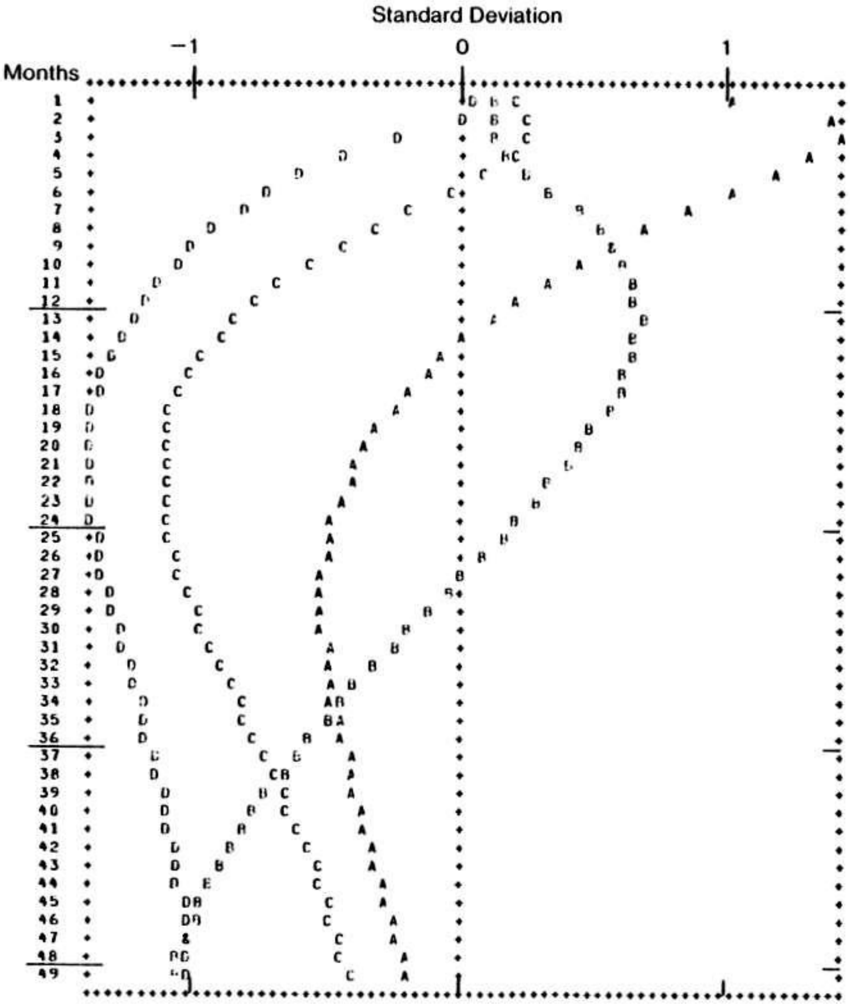

Figure Ib. Estimation Period: 1972:1–1980:9

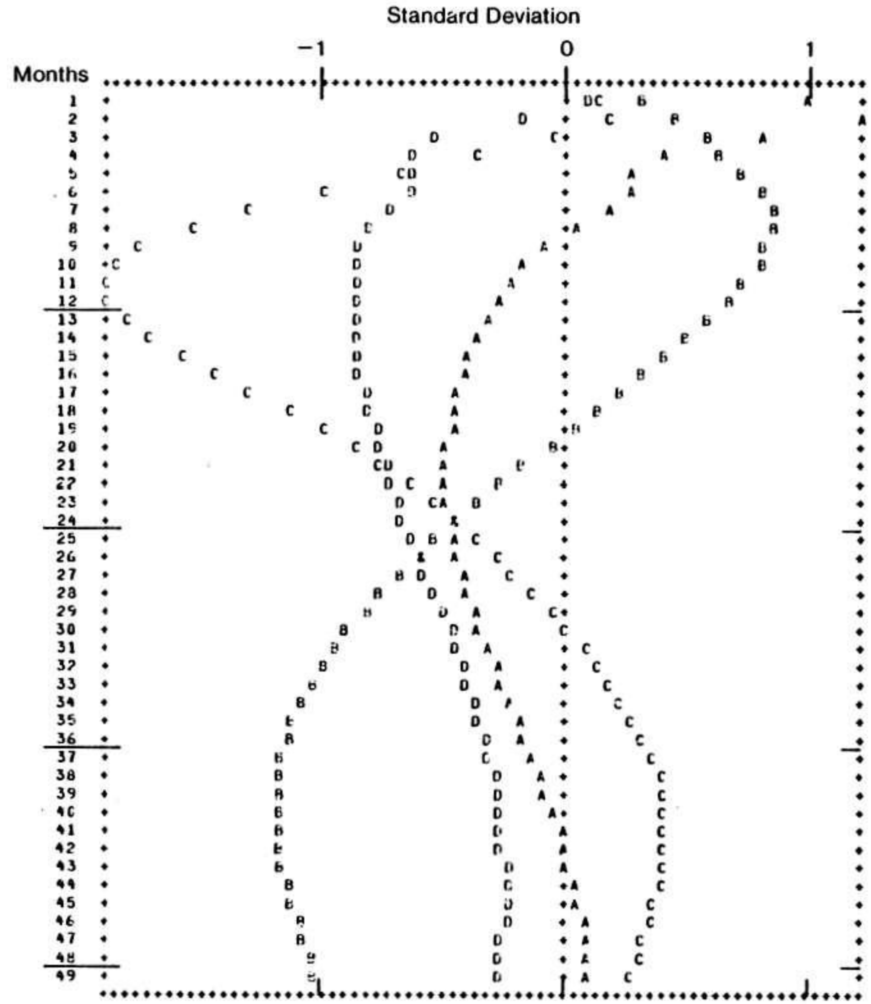

Figure II. Responses of Industrial Production

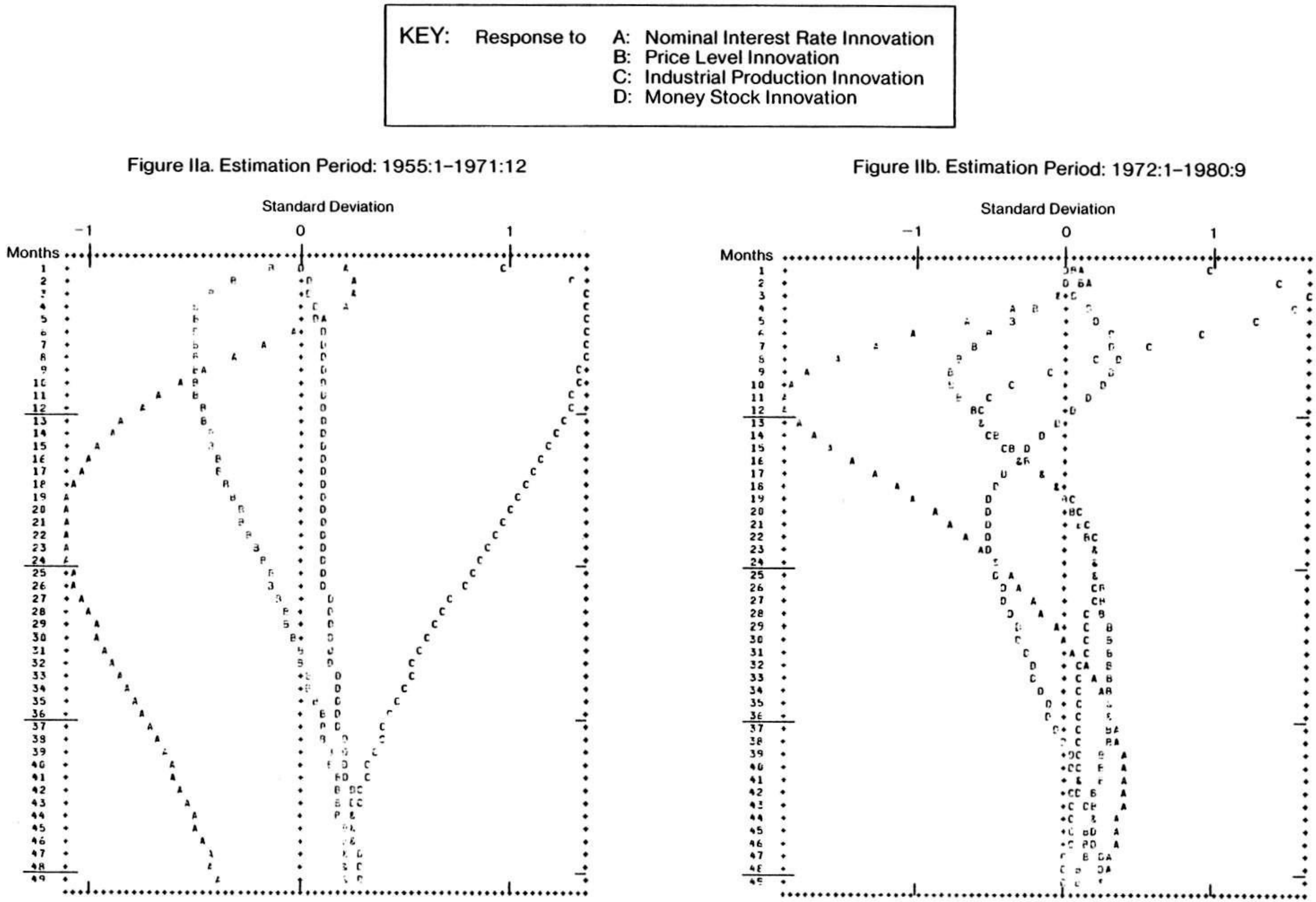

Figure III. Responses of Industrial Production—Ex-ante Expectations

Estimation Period: 1954:4–1980:9

Figure IIIa. Including Expected Inflation

KEY: Response to
A: Ex-ante Inflation Innovation
B: Nominal Rate Innovation
C: Industrial Production Innovation
D: Money Innovation

Figure IIIb. Including Expected Real Interest Rates

KEY: Response to
A: Ex-ante Real Rate Innovation
B: Nominal Rate Innovation
C: Industrial Production Innovation
D: Money Innovation

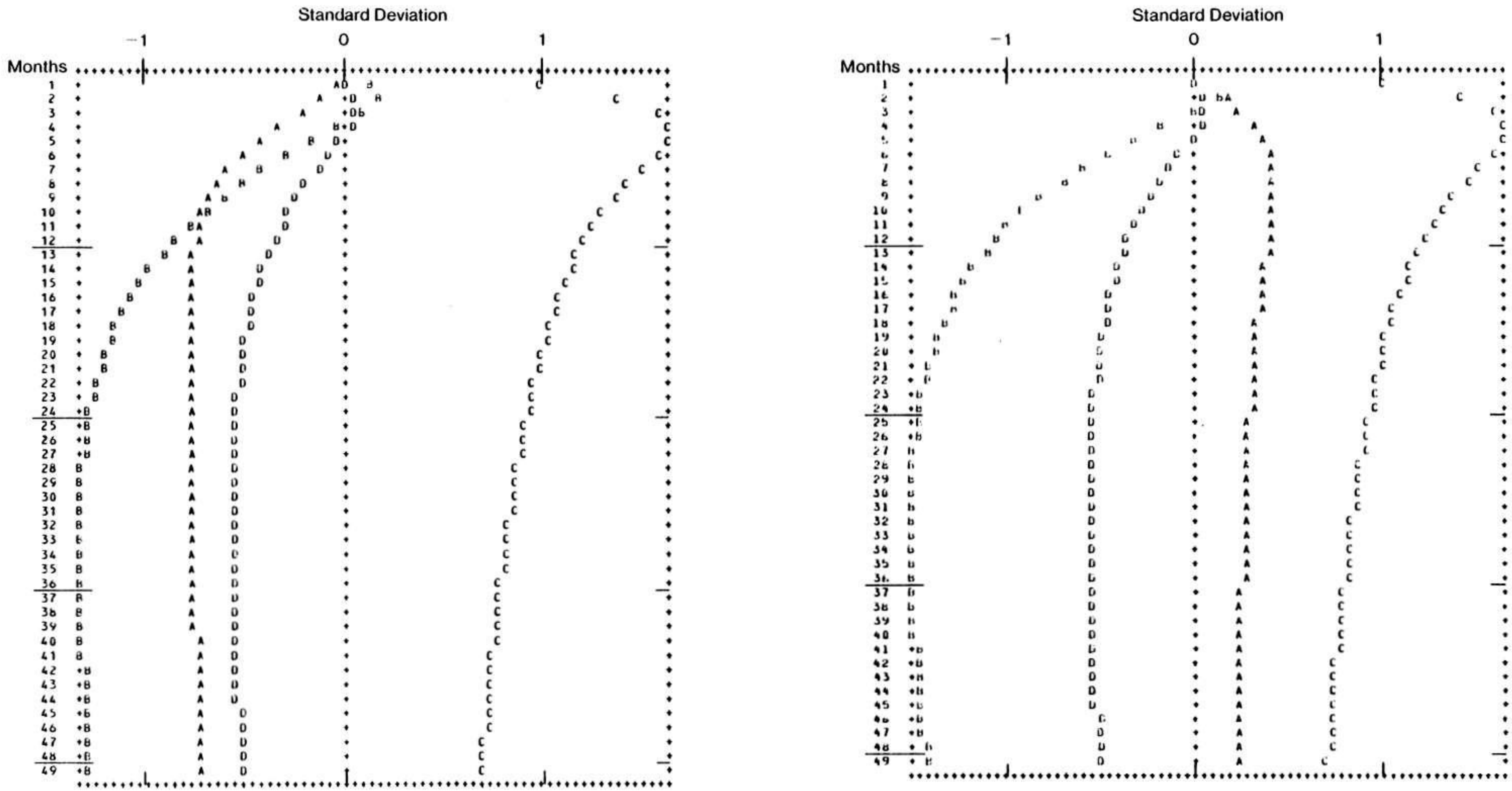

Figure IV. Responses of Industrial Production—Ex-post Expectations

Estimation Period: 1954:4–1980:9

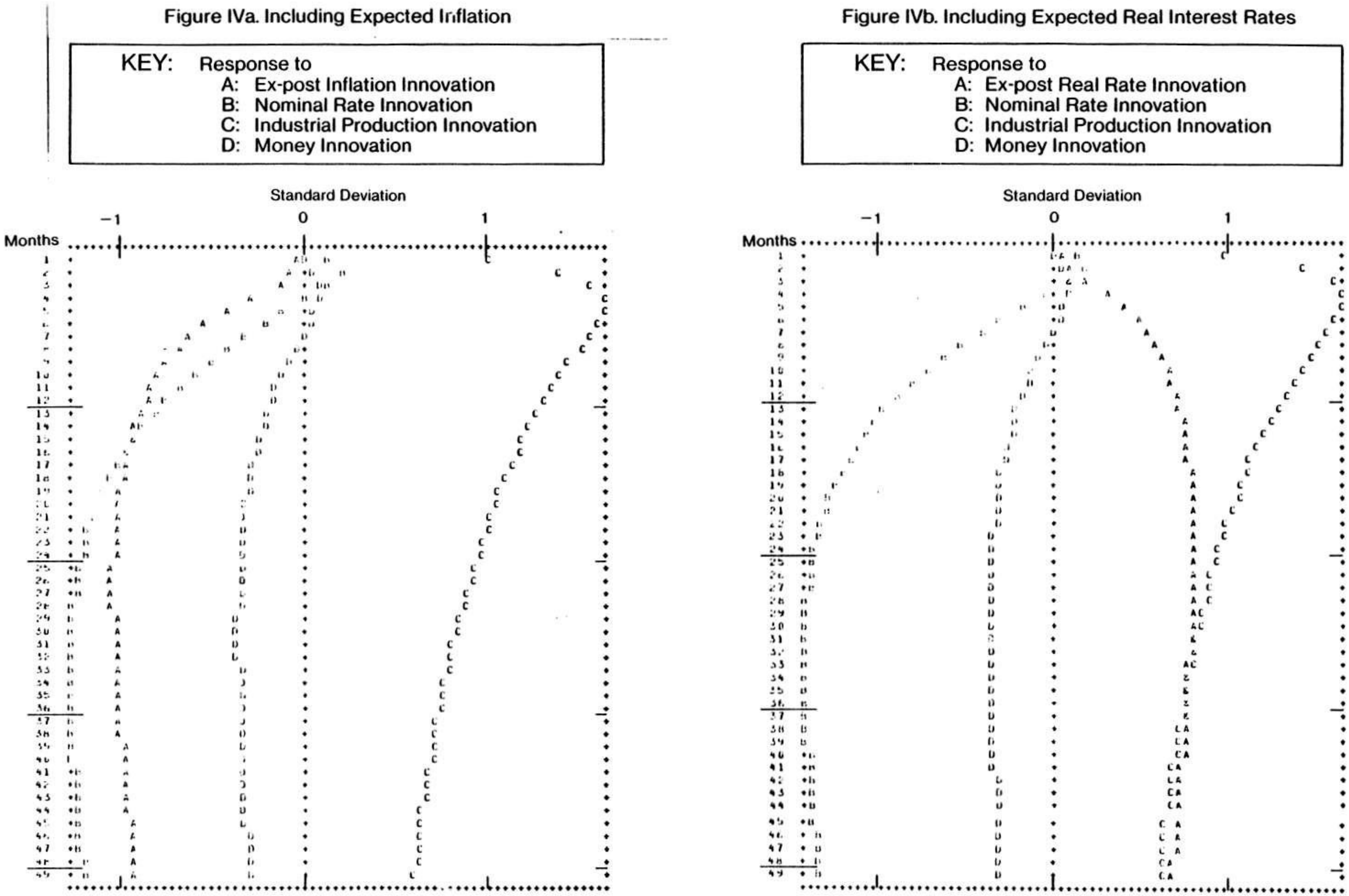

Figure V. Responses of the Ex-ante Real Interest Rate

KEY: Response to
A: Real Interest Rate Innovation
B: Nominal Interest Rate Innovation
C: Industrial Production Innovation
D: Money Stock Innovation

Figure Va. Estimation Period: 1955:1–1971:12

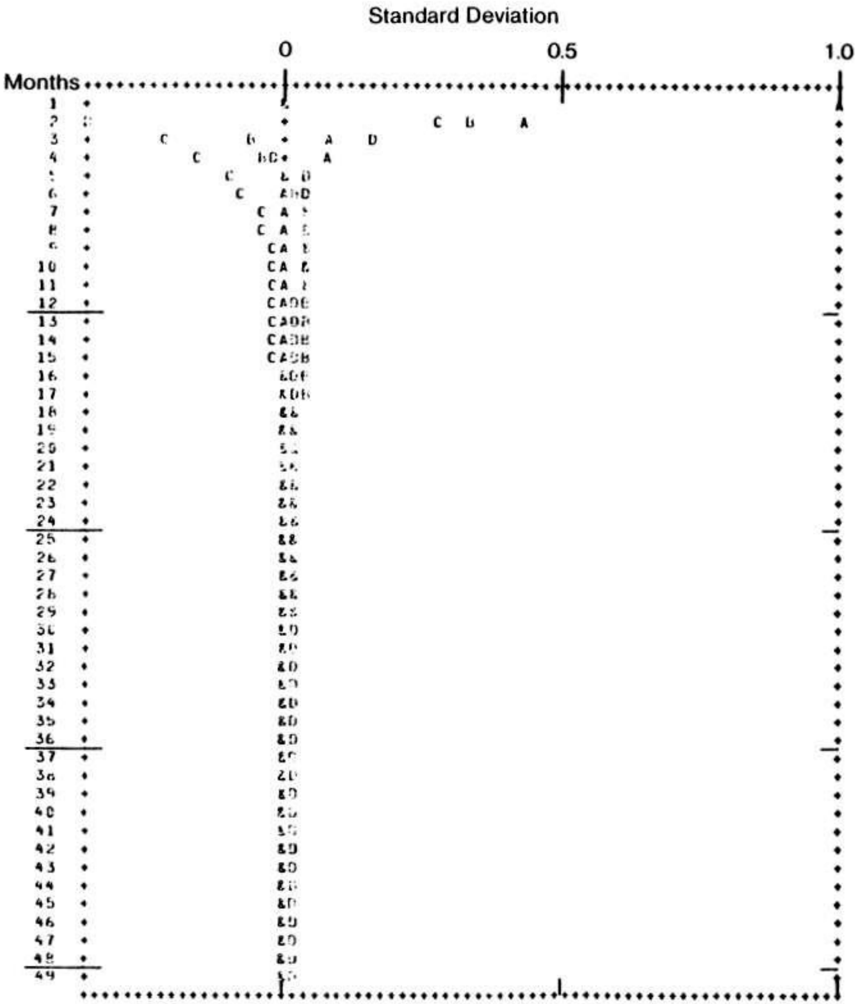

Figure Vb. Estimation Period: 1972:1–1980:9

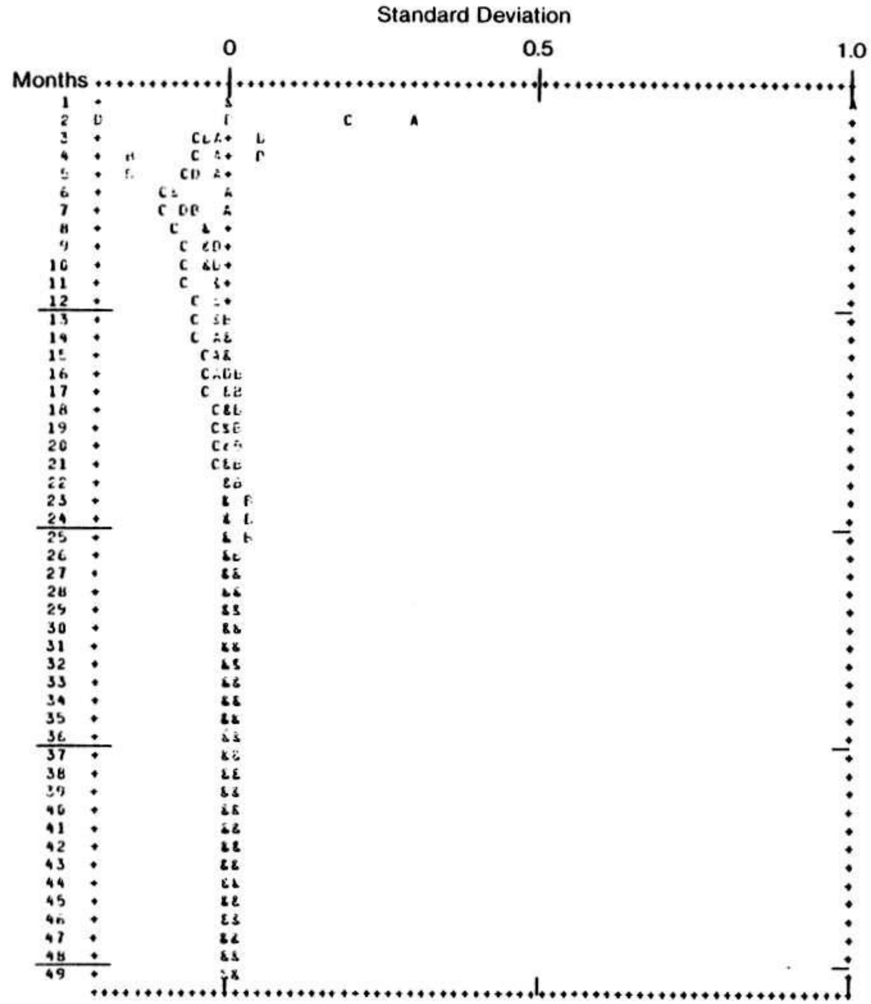

Figure VI. Responses of the Ex-ante Real Interest Rate

Estimation Period: 1948:4–1980:9

KEY: Response to
- A: Nominal Interest Rate Innovation
- B: Inflation Innovation
- C: Industrial Production Growth Innovation
- D: Money Stock Growth Innovation

Figure VIa. Unrestricted Vector Autoregression

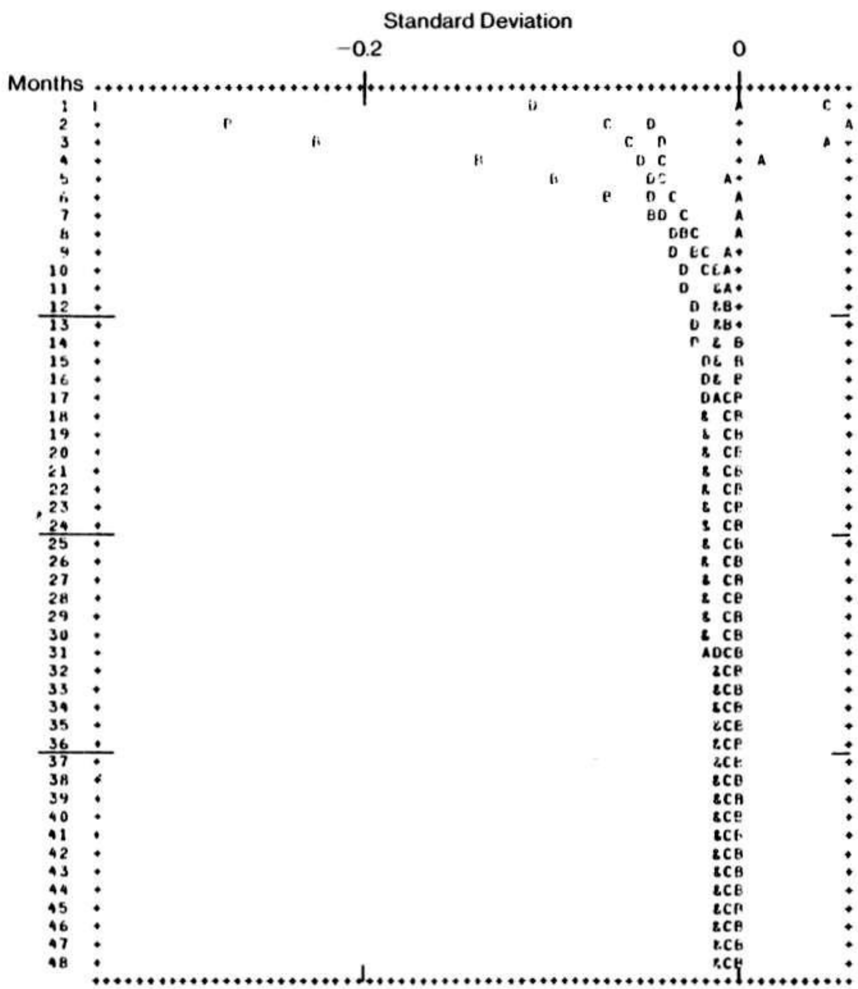

Figure VIb. Estimated Subject to Hypothesis I

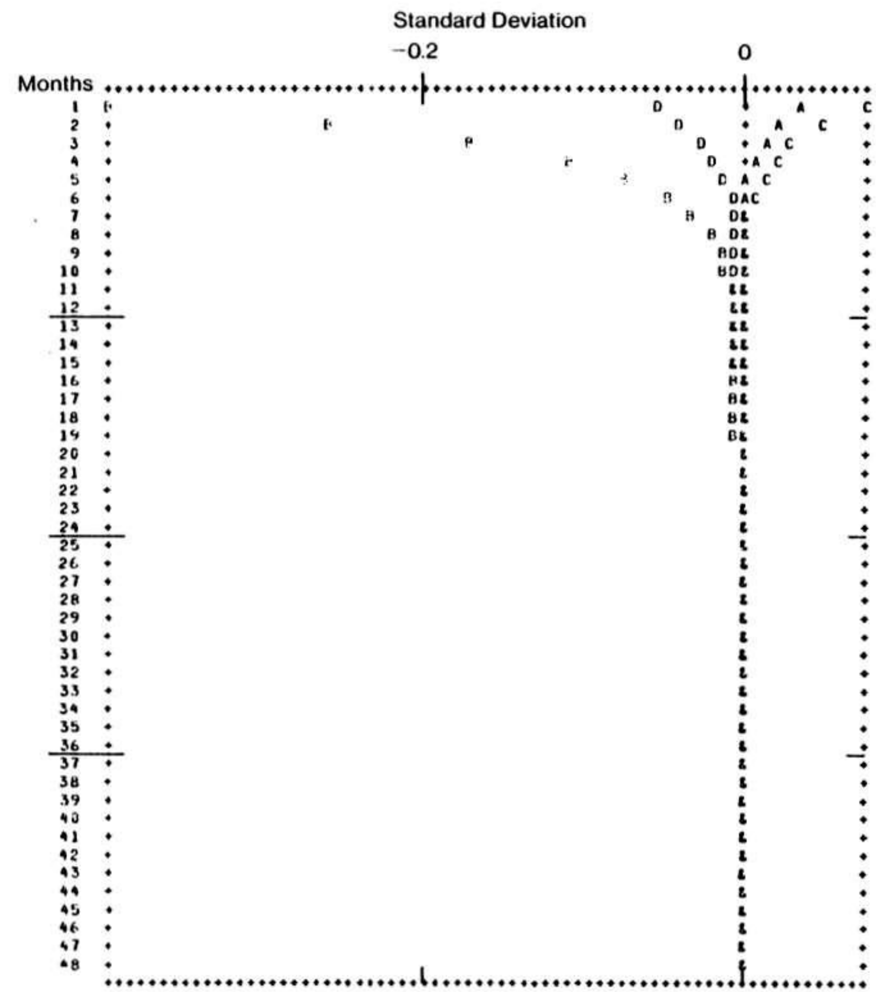

Figure VII. Inflation Rates, 1954:1–1980:9

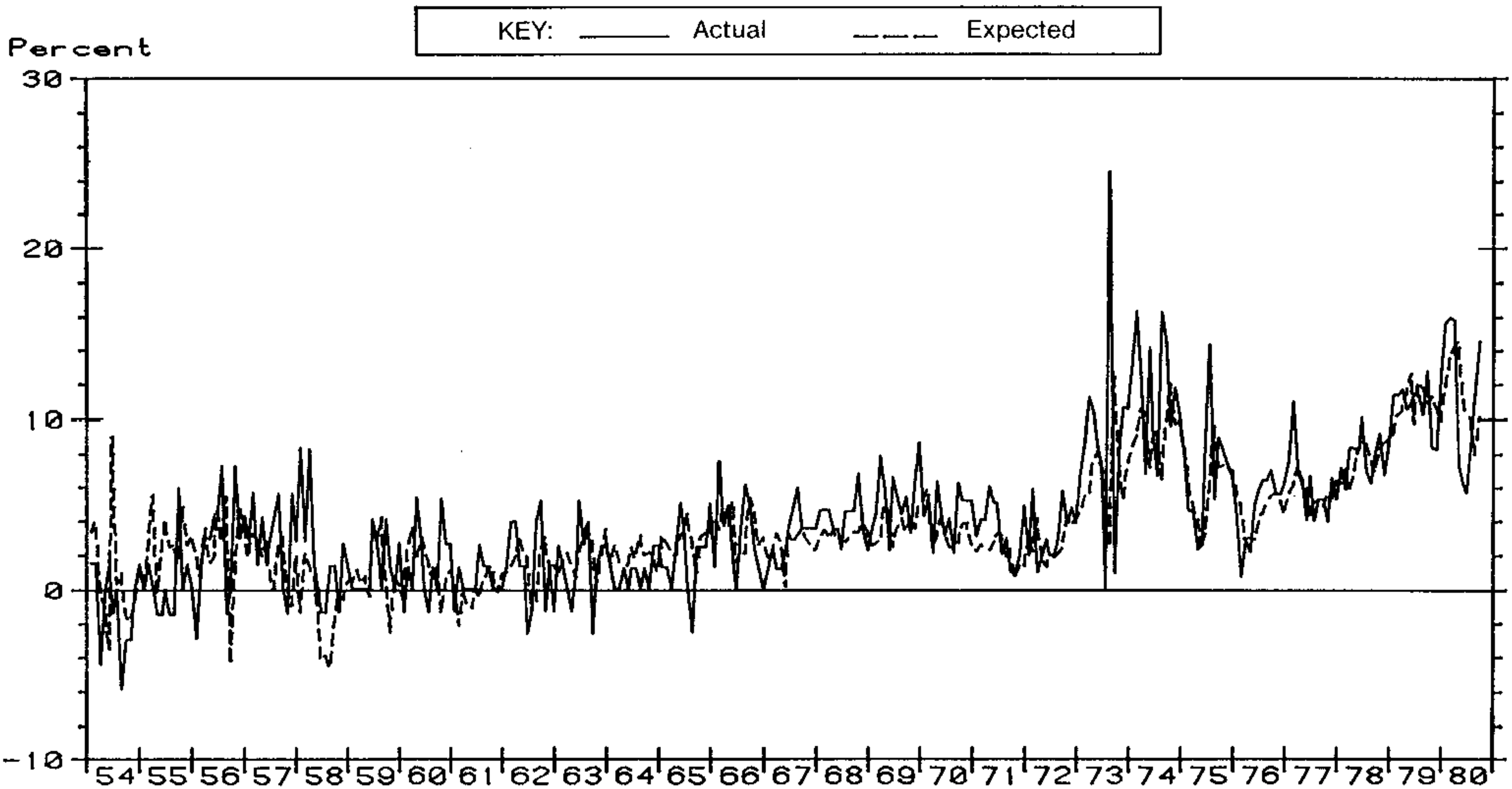

Figure VIII. Real Interest Rates, 1954:1–1980:9