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THE WELFARE EFFECT OF ENTRY
INDUCED BY ANTITRUST REGULATION

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Antitrust regulators often attempt to prevent proposed corporate market-extension mergers or acquisitions by arguing that doing so will result in the proposer entering the market as an additional, smaller, independent competitor. In cases where this so-called doctrine of probable future competition is valid, regulators still need guidance in ranking the priority of cases to pursue. This paper modifies the approach of Dansby and Willig to compute measures of the gross benefits arising from valid regulation. Such measures relate the change in consumer plus producer surplus caused by regulation, to measures of market concentration, firm conduct assumptions, small firm profits, and market demand data.

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## The Welfare Effect of Entry Induced by Antitrust Regulation

Antitrust regulators must often decide whether to permit a firm not presently in a market to merge with or acquire an oligopolist that is already in the market. Permitting such a market-extension will not change the number of separate competitors in the market. On numerous occasions, though, the FTC, Justice Department, Federal Deposit Insurance Corporation, and the Federal Reserve Board have decided that preventing a market-extension will result in the would-be acquirer entering the market as a smaller, independent competitor. The resultant increase in the number of competitors is presumed to improve the market's performance. This reasoning has been dubbed the doctrine of actual potential competition, or, synonomously, the doctrine of probable future competition. This paper attempts to measure the gross benefits which occur if regulatory prevention of a market-extension does indeed cause the independent entry of a firm. This represents the benefits which would occur, if any benefits do occur. Regulatory and private compliance costs incurred in these cases can be subtracted from this benefit to obtain an ex ante measure of the net benefits from effective regulation. 2/ Further subtraction of the regulatory and private compliance costs incurred in cases where market-extension preventions did not lead to independent entry, and in cases whose market-extensions were approved, yields an ex post measure of the benefits of actual regulation.

In a valuable contribution to antitrust analyses of oligopolistic markets, Dansby and Willig measured the largest marginal increase in consumer plus producer surplus obtainable by restricted, joint changes in oligopolists' outputs. They showed that this measure depends on the price elasticity of market demand, oligopoly conduct (i.e., behavioral hypothesis) and a measure of market concentration specific to the conduct assumption employed.

Unlike in Dansby and Willig, the benefits from market-extension prevention, if any, do not stem from optimal perturbations of the equilibrium outputs of the existing n oligopolists. Rather, they stem from the equilibrium n-vector of outputs changing to an equilibrium n + 1 vector of outputs, after the new firm enters. This paper modifies the approach of Dansby and Willig to accommodate this. First, a general expression for the change in consumer plus producer surplus induced by entry is derived. Then, in the spirit of Dansby and Willig, explicit measures of this change are computed for a wide variety of oligopoly conduct assumptions. While only the regulatory implications of these measures are discussed throughout, these measures will also be of value to those interested in the welfare effects of entry for other reasons.

### OLIGOPOLY MODELS AND THE PERFORMANCE CHANGE FROM ENTRY

Following Dansby and Willig, we always assume that, prior to regulatory action, there are n profit maximizing firms in the market producing a homogeneous product. In all of the models developed, each firm falls into one of two possible types. We refer to these two types as "large firms" and "small firms." For example, one model will assume that large firms form a cartel to maximize joint profits, while small firms behave competitively. Another model will assume that large firms follow the Cournot conduct assumption, while small firms behave competitively. We denote the number of large firms by m, so the number of small firms is n-m.

To permit the use of powerful comparative statics methods, it is essential to assume that all small firms are identical. While the assumption that large firms are identical with one another is not needed for deriving equilibrium conditions, it is useful in qualitative comparative statics and stability analyses. It is employed in the appendix for these purposes.

Denoting the sum of consumer and producer surplus by W, the change in performance resulting from independent entry of a small firm is thus:

(1) 
$$W(n+1) - W(n) = \int_{n}^{n+1} \frac{dW}{dn} dn$$
.

If, as is assumed here,  $\frac{dW}{dn}$  does not change sign in the interval from n to n + 1, then its sign is the same as that of (1). Thus, the sign of  $\frac{dW}{dn}$  indicates whether performance is improved or hurt by regulatory denial. Furthermore, assuming that  $\frac{dW}{dn}$  is "approximately constant" in this interval, (1) is approximately equal to  $\frac{dW}{dn}$ . Under these maintained assumptions, the magnitude of  $\frac{dW}{dn}$  indicates the size of the performance change resulting from independent entry of a small firm.

A general expression for  $\frac{dW}{dn}$ , to be used in each of the oligopoly models to follow, is now derived. To do so, some notation, which will continue in use throughout the paper, is needed. The n-m identical small firms each produce a single, homogeneous output  $q_s$  at a total cost of  $C_s(q_s)$ , and each earns equilibrium profits of  $\pi_s$ . Total small firm output is denoted  $Q_s = (n-m)q_s$ . Each of the m large firms produces its output  $q_i$  at cost  $C_i(q_i)$ ;  $i=1,\ldots,m$ . The profit of the  $i^{th}$  large firm, before any lump sum redistributions among them, is  $\pi_i$ ;  $i=1,\ldots,m$ . The total output of the large firms is denoted  $Q_k = \sum_{i=1}^m q_i$ . The inverse market demand curve for the homogeneous output is denoted by P = P(Q).

With this notation, the sum of consumer and producer surplus is

(2) 
$$W = \int_{P(Q(n))}^{\infty} Q^{-1}(P)dp + (n-m)\pi_{s} + \sum_{i=1}^{m} \pi_{i},$$

where Q(n) is the equilibrium output of the n-firm oligopoly. The first term

in (2) is the consumer surplus, and the sum of the other two terms is the producer surplus.

The evaluation of  $\frac{dW}{dn}$  proceeds termwise. First, use Leibniz's rule to differentiate the consumer surplus term, obtaining:

(3) 
$$\frac{d}{dn} \int_{P(Q(n))}^{\infty} Q^{-1}(P)dP = -\frac{dP}{dQ} \frac{dQ}{dn} Q.$$

Then substitute

(4) 
$$\pi_s = P(Q(n)) q_s(n) - C_s(q_s(n))$$

into the second term in (2) and differentiate to obtain

(5) 
$$\frac{d}{dn} (n-m)\pi_{s} = \pi_{s} + (n-m) \left[ \frac{dP}{dQ} \frac{dQ}{dn}q_{s} + P \frac{dq_{s}}{dn} - \frac{dC_{s}}{dq_{s}} \frac{dq_{s}}{dn} \right]$$

$$= \pi_{s} + \frac{dP}{dQ} \frac{dQ}{dn}Q_{s} + (n-m) \left( P - \frac{dC_{s}}{dq_{s}} \right) \frac{dq_{s}}{dn},$$

Finally, substitute

(6) 
$$\pi_i = P(Q(n))q_i(n) - C_i(q_i(n)); i = 1, ..., m$$

into the third term in (2), differentiate, sum and simplify to obtain

(7) 
$$\frac{d}{dn} \sum_{i=1}^{m} \pi_{i} = \frac{dP}{dQ} \frac{dQ}{dn} Q_{\ell} + \sum_{i=1}^{m} \left(P - \frac{dC_{i}}{dq_{i}}\right) \frac{dq_{i}}{dn},$$

Remembering that  $Q = Q_S + Q_\ell$ , sum (3), (5) and (7) to obtain:

(8) 
$$\frac{dW}{dn} = (n-m) \left(P - \frac{dC_s}{dq_s}\right) \frac{dq_s}{dn} + \sum_{i=1}^{m} \left(P - \frac{dC_i}{dq_i}\right) \frac{dq_i}{dn} + \pi_s.$$

(8) shows that  $\frac{dW}{dn}$  is the profit earned by a small firm plus a weighted average deviation of price from firm marginal costs. The weight for the i<sup>th</sup> firm's deviation is its equilibrium output response to entry. We see that entry induces more of its influence on performance through firms which have large markups from marginal cost, and whose output is increased the most by entry. As a consequence, entry into a long run, competitive equilibrium would not affect performance, because price equals each firm's marginal costs and profits are zero. The profit term in (8) is absent in the marginal measure of Dansby and Willig.

The deviations of price from firm marginal costs depend on both the structure (i.e., measure of concentration) and the conduct (i.e., behavioral assumptions) of the firms in the market. The exact dependence is examined for each of the oligopoly models detailed below.

#### Model 1: Large Firm Cartel/Small Firms Competitive

In this model, the m large firms are assumed to conduct themselves as if they had formed a profit maximizing cartel. They choose outputs  $\mathbf{q}_1$ , ...,  $\mathbf{q}_{m}$  to maximize joint profits:

(9) 
$$\max_{\substack{q_1,\dots,q_m \ i=1}} \sum_{i=1}^m P(Q_{\ell}+Q_s) q_i - C_i(q_i).$$

It is also assumed, although it is not necessary to do so, that the cartel thinks that its output decisions do not affect the total output of the small firms,  $Q_{\bf S}$ . As cited in Dansby and Willig, the first order conditions for a solution to (9) are equivalent to:

(10) 
$$P - \frac{dC_i}{dq_i} = \frac{CR_m}{E} P, i = 1, ..., m.$$

where  $E = \frac{-dQ}{dP} \frac{P}{Q}$  is the market elasticity of demand.  $CR_m$  denotes the m-firm

concentration ratio, which, denoting the ith firm's market share of output

(or, equivalently, sales revenue) by 
$$s_i$$
, is  $CR_m = \sum_{i=1}^m s_i$ .

The n-m profit maximizing small firms are assumed to behave competi-

tively, treating price and the output of all other firms as given. Each of

the identical small firms behaves as if it solved

(11) 
$$\max_{\mathbf{q}_{\mathbf{S}}} P\mathbf{q}_{\mathbf{S}} - C_{\mathbf{S}}(\mathbf{q}_{\mathbf{S}}).$$

The first order condition for (11) is, of course,

$$P - \frac{dC_s}{dq_s} = 0$$

An interior oligopoly equilibrium requires that there exists a vector of m + 1 positive output levels  $q_1$ , ...,  $q_m$ ,  $q_s$  solving the m + 1 equations (10) and (12), and that (10) and (12) are sufficient conditions for problems (9) and (11), respectively. Regularity conditions guaranteeing this are assumed to be met. $\frac{3}{}$ 

Substituting (10) and (12) into (8) yields the marginal performance change due to entry, for this model:

(13) 
$$\frac{dW}{dn} = \frac{CR_m}{E} P \frac{dQ_{\ell}}{dn} + \pi_s.$$

As might be expected by antitrust enthusiasts, the performance change from entry varies directly with market concentration. Note that the appropriate concentration index in this model, though, is the m-firm concen-

tration ratio. If the cartel of large firms had more (or less) than m members, a broader (or more restrictive) concentration ratio would have been appropriate.

Perhaps not as well understood by antitrust enthusiasts is the inverse relationship between performance change and the price elasticity of demand. Whatever the effect independent entry has on industry output Q, the impact on market price P is smaller when the price elasticity is larger. The change in consumer surplus is thus smaller when the price elasticity is larger. Market power is thus of little significance when its exertion dries up demand. In fact, (13) shows that independent entry could have the same performance impact in a market with a concentration ratio twice as high as in another, if the elasticity of demand in the former market was also twice as high. It is thought that the price elasticity of a good varies directly with the degree to which close substitutes are available. If, for example, this statement applies to the services offered by commercial banks, then bank antitrust regulators must seriously consider the availability of services from thrift and other competing financial institutions in every application to merge or acquire.

As mentioned earlier, profits have an explicit role to play in the performance change from independent entry. In (13), this is manifested by the additive contribution of the profit of a small firm. Thus, the higher the profits of small firms already in the market, the more benefit is derived from independent entry.

Finally, performance also varies directly with the change in cartel output,  $\frac{dQ_{\ell}}{dn_{dQ_{\ell}}}$ . Under reasonable regularity conditions derived in Lemma 1 of the appendix,  $\frac{dQ_{\ell}}{dn}$  has the same sign as  $\frac{dP}{dQ} + \frac{d^2P}{dQ^2}Q_{\ell}$ . Alternatively,  $\frac{dQ_{\ell}}{dn} \stackrel{\geq}{=} 0$  when  $-\frac{d^2P}{dQ^2}Q_{\ell}/\frac{dP}{dQ} \stackrel{\geq}{=} 1$ . The latter term is the inverse elasticity of the slope of

demand with respect to the output of large firms. It is nonpositive (nonnegative) when the inverse demand curve is concave (convex). This implies that  $\frac{dQ_{\ell}}{dn} < 0$  when demand is linear or concave. When demand is sufficiently convex, though,  $\frac{dQ_{\ell}}{dn} > 0$ . Thus, geometric properties of the demand curve other than E help determine the marginal performance change.

# Model 2: Large Firm Cartel/Small Firms Cournot

Rather than behaving competitively, each small firm behaves as if its output decision may affect the market price, yet will not affect the output decisions of other firms. The large firms still form a cartel and solve (9).

Each small firm solves

(14) 
$$\max_{q_s} P(q_s + (Q-q_s))q_s - C_s(q_s),$$

which yields the familiar Cournot first order necessary condition:

(15) 
$$\frac{dP}{dQ} q_s + P - \frac{dC_s}{dq_s} = 0.$$

Dansby and Willig show that (15) is equivalent to:

(16) 
$$P - \frac{dC_s}{dq_s} = \frac{P}{E} s$$
,

where s denotes the market share of a small firm.

An interior oligopoly equilibrium exists if a simultaneous solution to (10) and (16) exists, and if (10) and (16) are sufficient conditions for (9) and (14), respectively. As before, regularity conditions guaranteeing this are assumed to be met.

Noting that  $(n-m)s = 1 - CR_m$ , substitute (10) and (16) into (8) to obtain the marginal performance change from independent entry:

(17) 
$$\frac{dW}{dn} = \frac{CR_{m}}{E} P \frac{dQ_{\ell}}{dn} + \pi_{s} + \frac{(1-CR_{m})}{E} P \frac{dq_{s}}{dn}.$$

The noncompetitive behavior of the small firms results in an additional third term being added to those already in (13). One must additionally consider the change in output per small firm when entry occurs. Assuming that firms don't leave too many scale economies unexploited in equilibrium, both  $\frac{dq_s}{dn}$  and  $\frac{dQ_\ell}{dn}$  have the same sign as P' + P"q<sub>s</sub>, and P' + P"Q<sub>\ell</sub>, respectively. To be more precise, assume that equilibrium is stable, and that the large firms possess a common cost function, C<sub>ℓ</sub>. Lemma 2 of the appendix shows that  $\frac{dq_s}{dn}$  has the same sign as that of  $-(P'-C_s'')$  (P'+P"q<sub>g</sub>), while  $\frac{dQ_\ell}{dn}$  has the same sign as that of  $-(P'-C_s'')$  (P'+P"Q<sub>ℓ</sub>). With this one caveat, the implications for antitrust analysis stemming from (17) are similar to those discussed earlier.

## Model 3: Large Firms Cournot/Small Firms Competitive

The m large firms are assumed to follow the Cournot conduct hypothesis of Model 2 (see (18) below), while the small firms remain competitive as in Model 1. Interior oligopoly equilibrium requires that each large firm solve a problem like (14), while smaller firms solve (11). An interior oligopoly equilibrium exists if a simultaneous solution to (12) and (18) below exists,

(18) 
$$P - \frac{dC_i}{dq_i} = \frac{P}{E} s_i; i = 1, ..., m$$

and if (12) and (18) are sufficient conditions for their respective problems. As before, regularity conditions guaranteeing this are assumed to be satisfied. Substituting (12) and (18) into (8) yields the marginal performance change from entry:

(19) 
$$\frac{dW}{dn} = \frac{\sum_{i=1}^{m} s_i P \frac{dq_i}{dn}}{E} + \pi_s$$

By lacking a well-known concentration measure, (19) stands in contrast to both (17) and (13). Rather, the relevant concentration measure in this case is a weighted average of the large firms' market shares, where the weights are the large firms' respective output changes in response to entry.

## Model 4: All Firms Cournot and Identical

The comparative statics of this model has been recently treated by Seade. No further treatment of this topic will be given here. Equilibrium output per firm  $q_s$  solves (16), where  $s=\frac{1}{n}$  because all firms are identical. Substituting (16) into (8) yields the marginal performance gain:

(20) 
$$\frac{dW}{dn} = \frac{1}{E} P \frac{dq_s}{dn} + \pi_s.$$

This, of course, is also (17) when m = 0, and (19) when m = n and firms are identical.

Seade has shown that stability of equilibrium in industries with more than a few firms will likely lead to  $\frac{dq}{dn}$  being negative. However, it seems unlikely that the absolute value of the first term would exceed  $\pi_{\rm S}$ .

#### Antitrust Implications

The implications of these four models for regulators invoking the doctrine of probable future competition are threefold. First, the market performance improvement resulting from independent entry induced by merger or acquisition prevention varies inversely with the price elasticity of demand. Factors affecting the price elasticity, including the availability of close substitutes, exert as much influence on performance as does concentration. Other things equal, markets with high elasticity should receive lower priority for regulatory intervention than those with lower elasticity. Second, although concentration (i.e., the size distribution of firms) does affect the

size of the performance improvement resulting from independent entry, the correct measure of concentration depends on the conduct of the industry. The m-firm concentration ratio is appropriate when the industry has an m-firm cartel. In a noncooperative Cournot oligopoly, though, the appropriate concentration index is a weighted average of all firms' market shares. In the absence of reliable evidence about the form of industry conduct, regulators would do well to require that a variety of concentration indices be "high" before invoking the doctrine of probable future competition. Third, the performance gain from regulatory intervention varies directly with the preexisting level of profits per small firm, and the change in large firm output resulting from entry. Markets with high barriers to entry should have higher values for both factors. These markets are thus better candidates for regulatory intervention than are other markets. For example, banking markets in states which restrict branch banking are better candidates than markets in other states. To the extent that both of these factors reflect the absolute economic size of the market, large markets probably are better candidates for regulatory intervention than are small markets.

#### What if Applications are Permitted?

All of the above implications follow from calculations comparing the status quo with the market equilibrium which prevails when independent entry occurs subsequent to regulatory <u>prevention</u> of mergers or acquisitions. But suppose that the regulators <u>permit</u> mergers or acquisitions. Could the performance change resulting from permission exceed the performance change resulting from prevention?

Some argue that permission of mergers or acquisitions is unlikely to change performance at all. They reason that a market-extension acquisition of, or merger with, a firm leaves the number and size of competitors in the

market unchanged. Reasoning that the size and number of competitors is the primary determinant of performance, one might conclude that market extension mergers or acquisitions have no affect on performance. However, the latter statement implicitly depends on an assumption that no scale economies will be realized by merger or acquisition. For if such scale economies do exist, the costs incurred by the acquired firm may decline, and performance may improve, following the acquisition. For example, there have been numerous statistical studies attempting to determine whether, and to what degree, such scale economies exist in banking. In a survey of these studies, Tadesse states ". . . all of the studies reviewed indicated that the production of services in the commercial banking industry exhibits economies of scale. However, the magnitude and range of these scale economies remain unresolved." In the absence of good statistical evidence, regulators could examine the validity of cost projections submitted by the regulated firms. Market-extension mergers or acquisitions which, upon such examination, stand a reasonable chance of realizing significant scale economies should receive lower priority for prevention than those which don't.

## Footnotes

1/In the words of one of its proponents, Rhoades has summarized the doctrine as follows:

"The concept of probable future competition is applied in bank merger cases on grounds that a given bank, by filing application with the appropriate banking authority, has demonstrated an interest in entering that banking Since the bank may be viewed as a potential entrant, it is argued that denial of the merger will 'probably' cause the potential entrant, who has already expressed an interest in the market, to enter the market de novo at some time in the future. Such an outcome would change the structure of the market by increasing the number of competitors and possibly resulting in a decline in concentration. The economic rationale for this approach to merger analysis is straightforward. Specifically, economic theory strongly suggests that the degree of competition in a market is directly related to the number and size distribution of firms. words, it suggests that the structure of a market influences the competitive conduct or rivalry of firms in the market and ultimately their performance. Thus the emphasis on structure because of its effect on conduct and performance. Moreover, there is considerable empirical support for this general theoretical proposition."

2/This paper does <u>not</u> attempt to determine the conditions under which regulatory prevention of market-extensions <u>will</u> actually result in independent entry by either the acquiring firm, or by some other firm not presently in the market. Doing so would improve the standing of this doctrine in the eyes of both economic theorists and the courts. For an exhaustive case review and discussion of the doctrine, see Areeda and Turner. For a less lengthy presentation, see Kaplan.

3/Trivial modifications of the existence proof in Okuguchi (Ch. 1) would suffice to establish equilibrium in this model and models developed later in this paper.

 $\pm$ /See also Johnson and Helmberger, Cowling and Waterson, and Dansby and Willig.

## Appendix

To save space, total derivatives are symbolized by primes, and equilibrium arguments of functions are usually omitted.

## Lemma 1

Assume that an equilibrium in Model 1 is stable, in the sense to be defined below, and that the large firms possess a common cost function, denoted  $C_{\ell}$ . Then,  $\frac{dQ_{\ell}}{dn}$  has the same sign as that of P' + P" $Q_{\ell}$ .

## Proof

Because the large firms are identical,  $q_i = \frac{Q_k}{m}$ ; i = 1, ..., m. Then, the large firm cartel's problem (9) collapses to:

(i) 
$$\max_{Q_{\varrho}} \Pi_{\varrho} = P(Q_{\varrho} + Q_{g})Q_{\varrho} - mC_{\varrho}(\frac{Q_{\varrho}}{m})$$

with the first order condition for a maximum being:

(ii) 
$$\frac{\partial \Pi_{\ell}}{\partial Q_{\ell}} = p' Q_{\ell} + P - C_{\ell}' = 0.$$

The first order condition for the small, competitive firms is:

(iii) 
$$\frac{\partial \Pi_{s}}{\partial q_{s}} = P - C_{s}' = 0.$$

Remembering that  $Q_S = (n-m)q_S$ , the equilibrium conditions (iii) and (ii) are two equations in the two unknowns,  $Q_L$  and  $q_S$ . Totally differentiate (iii) and (ii) with respect to  $q_S$ ,  $Q_L$  and n to obtain the comparative statics matrix equation

(iv) 
$$\left(\frac{dq_s}{dn} \frac{dQ_{\ell}}{dn}\right)^T = -A^{-1}b$$

where A is a 2x2 matrix and b is a 2x1 vector, with elements:

$$a_{11} = (n-m) P' - C_s"; a_{12} = P'; a_{21} = (n-m)(P'+P''Q_l)$$
  
 $a_{22} = (P'+P''Q_l) + (P'-C_l''/m); b_1 = q_s P'; b_2 = q_s(P'+P''Q_l)$ 

Inverting A and solving for  $\frac{dQ_{k}}{dn}$  from (iv) yields, after simplifying:

$$(v) \qquad \frac{dQ_{L}}{dn} = (P'+P''Q_{\ell}) q_{s}C_{s}''/Det A.$$

Stability of equilibrium under the standard adjustment mechanism

(vi) 
$$q_{s}^{\bullet} = d_{1} \frac{\partial \Pi_{s}}{\partial q_{s}}; d_{1} > 0$$
$$Q_{\ell}^{\bullet} = d_{2} \frac{\partial \Pi_{\ell}}{\partial Q_{\ell}}; d_{2} > 0$$

requires that the eigenvalues of the Jacobian matrix of the right-hand side of (vi) have negative real parts for all positive speeds of adjustment  $d_1$  and  $d_2$ . In particular, this must be true for  $d_1 = d_2 = 1$ . In this case, the Jacobian of (vi) is just the Jacobian of the equilibrium conditions (iii) and (ii), which is simply A. A necessary and sufficient condition for the eigenvalues of any 2x2 matrix A to have negative real parts is that Trace A < 0 and Det A > 0. Trace A is always negative, because  $a_{22}$  is the second order sufficiency condition for (i) and is thus negative, while the second order condition for (11) is just  $C_s$ " > 0, so  $a_{11}$  < 0. Imposing stability as an additional condition thus only adds the information that Det A > 0. The lemma follows by inspection of (v).

### Lemma 2

Assume that an equilibrium in Model 2 is stable, in the same sense defined in Lemma 1, and that the large firms possess a common cost function, denoted  $C_{\ell}$ . Then,  $\frac{dQ_{\ell}}{dn}$  has the same sign as that of P' + P"Q $_{\ell}$ , and  $\frac{dq_{s}}{dn}$  has the same sign as that of -(P' -  $\frac{\ell}{m}$ ) (P'+P" $_{q}$ ).

### Proof

As in Lemma 1, the equilibrium condition for large firms is still:

(i) 
$$\frac{\partial \Pi_{S}}{\partial q_{S}} = P'Q_{\ell} + P - C'_{\ell} = 0.$$

The equilibrium conditions for the smaller firms is now also a Cournot condition:

(ii) 
$$\frac{\partial \Pi_{S}}{\partial q_{S}} = P'q_{S} + P - C'_{S} = 0$$

These conditions are formally equivalent to those of an asymmetric, Cournot duopoly.

As before, totally differentiate (ii) and (i) and solve the resulting comparative statics matrix equation to obtain:

(iii) 
$$\frac{dQ_{\ell}}{dn} = \frac{-(P'-C_{s}'')(P'+P''Q_{\ell})q_{s}}{Det A}$$

and

(iv) 
$$\frac{dq_s}{dn} = \frac{-(P' - \frac{C_{\ell''}}{m})(P' + P''q_s)q_s}{\text{Det A}}$$

where A is the Jacobian of (ii) and (i). Once again, stability of the adjustment mechanism defined in Lemma 1 requires that Det A > 0. Lemma 2 follows directly from (iii) and (iv).

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