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THE VARIABILITY OF INFLATION

by

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I. INTRODUCTION

Should the United States accept a higher rate of inflation as the price for more jobs? Our answer to that question depends on how many jobs we think the policy can provide and on how much weight we attach to the damage done by a rise in the rate of inflation. We are not yet able to measure at all accurately the costs and benefits of inflation, so our policy judgments are based on hunch or on incomplete evidence. In the face of so much uncertainty, one new piece of evidence on the cost of inflation is not likely to be decisive for setting national policy. But if new evidence suggests that the cost of inflation is higher than we had previously thought, that claim deserves attention. If it withstands scrutiny, it could cause some of us to change our opinion on the policy question, or at least change our degree of conviction.

Okun (1971) has suggested that we have overlooked one drawback that attaches to a more inflationary policy. Analysis of the tradeoff between inflation and jobs is typically based on an artificial model of the economy in which a more inflationary policy means a rise from one steady rate of inflation to a higher steady rate of inflation. The world may not be like that; a rise in the average rate of inflation may mean, inevitably, a rise in the variability of inflation. If so, there are significant costs.

First, as the rate of inflation becomes more variable, uncertainty about prices is increased. Uncertainty is bad in itself; that is, people typically are willing to pay to reduce it. In addition, as people change their habits in order to mitigate the effect of uncertainty, they use real resources which--in the absence of the uncertainty--could have been devoted to other ends.

A second cost comes from increased variability. When the rate of inflation changes, a lot of people are surprised. Their anticipations were wrong, and so many contracts will have been entered into on the basis of incorrect assumptions. As a result, unanticipated changes in the rate of inflation bring with them capricious changes in the distribution of income and wealth. These capricious changes are undesirable--especially when those who lose are poor to begin with. In a world in which the rate of inflation can be increased from a low steady rate to a high steady rate, economists usually assume that the accompanying redistribution of income and wealth will be a transitional problem. People's anticipations will eventually catch up with the change, and thereafter contracts will reflect the new reality. But if higher rates of inflation mean more variable rates, anticipations can never catch up; the average error in anticipations will increase and so must the average extent to which incomes and wealth are being redistributed by unanticipated changes in inflation.

Okun (1971) presented some evidence to suggest that countries with higher rates of inflation do experience more variable rates and suggested why it might be so; Gordon (1971) has called Okun's evidence into question. In section II of this paper I summarize Okun's and Gordon's results and report additional evidence that supports Okun's position. In section III I show that there is very likely to be a functional relation between the average rate of inflation and the variability of that rate; the function may or may not be positively sloped. However, observations on the two variables are inadequate to show the shape of the function, except in very special circumstances. If we consider the observed average rate of inflation to be deliberately chosen by government policy, in the light of its tastes and the alternatives open to it, there is an identification problem. If the different observations on average rates of inflation are accounted for by differences in "tastes" for inflation, all confronting the same environment, the observations can tell us something about the environment. On the other hand, if the different observations reflect different environments confronted by a stable pattern of tastes, the observations can tell us, at most, something about the pattern of tastes. More likely, in the cross-sectional observations summarized in section II, neither tastes nor environment are the same across observations, and so it would take considerable effort to decipher the meaning of the observed correlation.

One interesting conclusion comes from consideration of the formal decision problem, however: If a government is acting "rationally" so as to maximize expected utility, then over a certain region downward-sloping segments of the function are irrelevant; in that region efficient choice implies that increased variability will accompany an increase in the average rate of inflation.

II. EVIDENCE

Okun's and Gordon's Calculations

Okun chose as his sample a group of 17 industrial countries for which the OECD presented roughly comparable data for the period 1951-1968.[1/] Inflation was measured by the annual percentage increase in the GNP deflator. The average rate of inflation for each country was measured as the arithmetic average of the yearly figures (m), and variability was measured as the standard deviation of the annual rates of inflation about their mean (σ). The correlation coefficient between m and σ was 0.78, implying that $(0.78)^2$ or 61 percent of the variance in σ, among countries, can be associated with the differences in their mean rates of inflation.

Gordon observed that the aftermath of the Korean War made prices behave strangely in the 1950s. If the period 1951-1968 is broken into subperiods 1951-1960 and 1960-1968, the correlation coefficient between m and σ is 0.90 for the first and 0.40 for the second subperiod. Moreover, he suggests, five small countries in which fluctuations of farm prices and import prices loom large account for the small positive correlation that remains in the 1960s: Removing Denmark, Finland, Ireland, Italy, and the Netherlands from the list reduces the correlation to zero.

In the rest of this section I describe further evidence that supports Okun's original claim.

<u>An Alternative Measure of Variability</u>

If annual rates of inflation could be thought of as observations on a random variable, independently drawn from a stationary population with finite variance, the standard deviation about the mean (σ) would be a perfectly appropriate measure of variability. Any reasonable alternative measure, say S, would be a monotone function of σ; so at worst the particular measure of variability chosen might change a linear relation between m and S to a non-linear relation between m and σ and thereby reduce the linear correlation coefficient.

Annual rates of inflation are not independent random variables, however, and the systematic components of the year-to-year change are not the same for each country. In this situation there are two difficulties with σ, best illustrated by artificial example. Table 1 records the inflationary experience of three hypothetical countries, A, B, and C.

Table 1. Hypothetical Rates of Inflation and Measures of Variability (Inflation measured in % per year)

	Year:										Measures of Variability*	
Country	1	2	3	4	5	6	7	8	9	10	σ	S
A	1	2	1	2	1	2	1	2	1	2	0.50	1.00
B	1	2	3	4	5	6	7	8	9	10	2.87	1.00
C	1	6	2	7	3	8	4	9	5	10	2.87	4.55

*σ is the standard deviation around the mean, and S is the average absolute change from year to year. Where X_t is the rate of inflation in year t,

$$S = \sum |X_t - X_{t-1}|/9$$

with t running from 2 to 10.

The faster a country's rate of inflation is growing, the higher its standard deviation--even if the year-to-year change includes no random component but is strictly systematic. Thus for country B in the table, σ is over five times as great as for A. Since growth in the rate of inflation also tends to raise the average rate for the country, using σ as a measure of variability could bias the results in some cases, producing from the systematic components a positive correlation between m and σ. On the other hand, σ is independent of the serial pattern of the observations. As the examples of countries B and C show, a rearrangement of the observations so that a steady increase in the rate of inflation is transformed into violent year-to-year fluctuations has no effect on σ. For these two reasons, a measure that reflects year-to-year change rather than deviations about the mean seems more appropriate. I have chosen to use the average absolute change from year to year for computational convenience. Table 1 illustrates the effect of choosing this statistic. One might argue that I have not gone far enough--that, for example, country A's experience of fluctuation around a trendless average is somehow more variable than country B's experience of steady growth and that the measure selected should reflect that. One could make such an adjustment by subtracting out a time trend before differencing, but visual inspection of the data actually used, below, suggested that this refinement would not be worth the effort.[2/]

More International Comparisons

I have not been able to find GNP price deflators for most countries for the years since 1968. Because Gordon has suggested that the 1950s data are suspect, I thought it useful to extend the period of observation as close to the present as possible. I therefore switched to the consumer price index for each country as the basis for measuring inflation.

The first calculations are for a group of 23 advanced economies: these include all of Western Europe (except Luxembourg) plus Australia, Canada, Israel, Japan, New Zealand, and the United States. The countries added include all non-European countries with 1968 GNP per capita over $1,000, with the exception of Libya.[3/] On the other hand, three of the Western European countries included in the group do not meet the criterion of GNP per capita over $1,000: Greece ($858), Portugal ($529), and Spain ($773). The list differs from Okun's by including six countries that he excluded: Australia, Israel, and New Zealand were excluded by Okun because they are not in the OECD; Portugal and Spain are members, but GNP deflators were not available for the full period; and Iceland was excluded deliberately because it was an extreme observation (and extremely favorable to his hypothesis).

To test Gordon's suggestion that the 1950s are atypical, I computed year-by-year figures for the average rate of inflation and the average change in the rate of inflation for the 23 countries taken as a group. Table 2 shows these figures.

Table 2. Average Rate of Inflation and Average Absolute Change in Inflation, 23 countries,* 1950-1971

Year	$\overline{m}$**	$\overline{S}$**	Year	$\overline{m}$	$\overline{S}$
1951	12.3	---	1961	2.5	1.4
1952	7.4	7.4	1962	4.2	2.1
1953	2.9	5.6	1963	4.2	1.6
1954	2.6	2.6	1964	4.7	2.0
1955	2.1	2.8	1965	5.0	2.4
1956	4.1	2.5	1966	4.7	1.7
1957	4.0	1.9	1967	3.9	1.8
1958	3.9	2.2	1968	4.0	1.8
1959	1.9	2.4	1969	4.6	1.9
1960	2.1	1.9	1970	6.3	2.9
			1971	7.1	2.1

Source: Computed from consumer price index recorded in International Financial Statistics, July, 1972, and 1971 Supplement.

*The countries include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

**Where X_{it} represents the rate of inflation from year t-1 to t in country i, $\overline{m}_t = \frac{1}{23}\sum_i X_{it}$, $\overline{S}_t = \frac{1}{23}\sum_i |X_{it} - X_{i,t-1}|$.

The Korean War shows up clearly in the early 1950s, but these data do not suggest that there is much impact beyond 1953. In any event, I calculated two sets of data, one for the period 1954-1971, the other for the period 1961-1971.[4/]

Table 3 reports the resulting correlations; Table 4 and Figure 1 show the data on which the correlations are based.

Table 3. Simple Correlation Coefficients Between
Average Rate of Inflation and Variability in Rate of Inflation
23 Advanced Economies

	1954-1971	1961-1971
23 countries*	.94	.92
22 countries (excluding Iceland)	.87	.79
17 countries** (same as Okun's)	.84	.64

Source: Computed from consumer price index recorded in International Financial Statistics, July, 1972, and 1971 Supplement.

*The countries include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

**Excludes Australia, Iceland, Israel, New Zealand, Portugal, and Spain.

Table 4. Average Annual Rate of Inflation in Consumer Price Index (m), and Average Absolute Change in Annual Rate of Inflation (S), 23 Advanced Economies 1954-1971 and 1961-1971*

		1954-1971		1961-1971	
	Country	m	S	m	S
1.	Australia	2.7	1.7	2.8	1.2
2.	Austria	3.4	1.8	3.7	1.5
3.	Belgium	2.5	1.0	3.1	0.7
4.	Canada	2.3	0.7	2.7	0.5
5.	Denmark	4.6	2.1	5.9	2.1
6.	Finland	4.8	3.6	5.0	2.8
7.	France	4.3	2.2	4.3	1.0
8.	Germany	2.4	0.9	2.9	0.9
9.	Greece	3.1	2.3	2.2	1.8
10.	Iceland	9.0	5.6	11.3	6.7
11.	Ireland	4.0	1.9	5.2	1.8
12.	Israel	5.9	3.3	6.2	2.8
13.	Italy	3.3	1.7	4.0	1.6
14.	Japan	4.3	2.1	5.9	1.7
15.	Netherlands	3.8	2.4	4.5	2.2
16.	New Zealand	3.9	1.5	4.4	1.5
17.	Norway	3.9	2.5	4.7	2.5
18.	Portugal	4.2	2.4	6.1	3.1
19.	Spain	6.4	3.4	6.3	3.4
20.	Sweden	3.9	1.8	4.3	1.8
21.	Switzerland	2.7	1.1	3.6	1.1
22.	United Kingdom	3.9	1.5	4.5	1.6
23.	United States	2.4	0.9	2.9	0.7
	Average	4.0	2.1	4.6	2.0

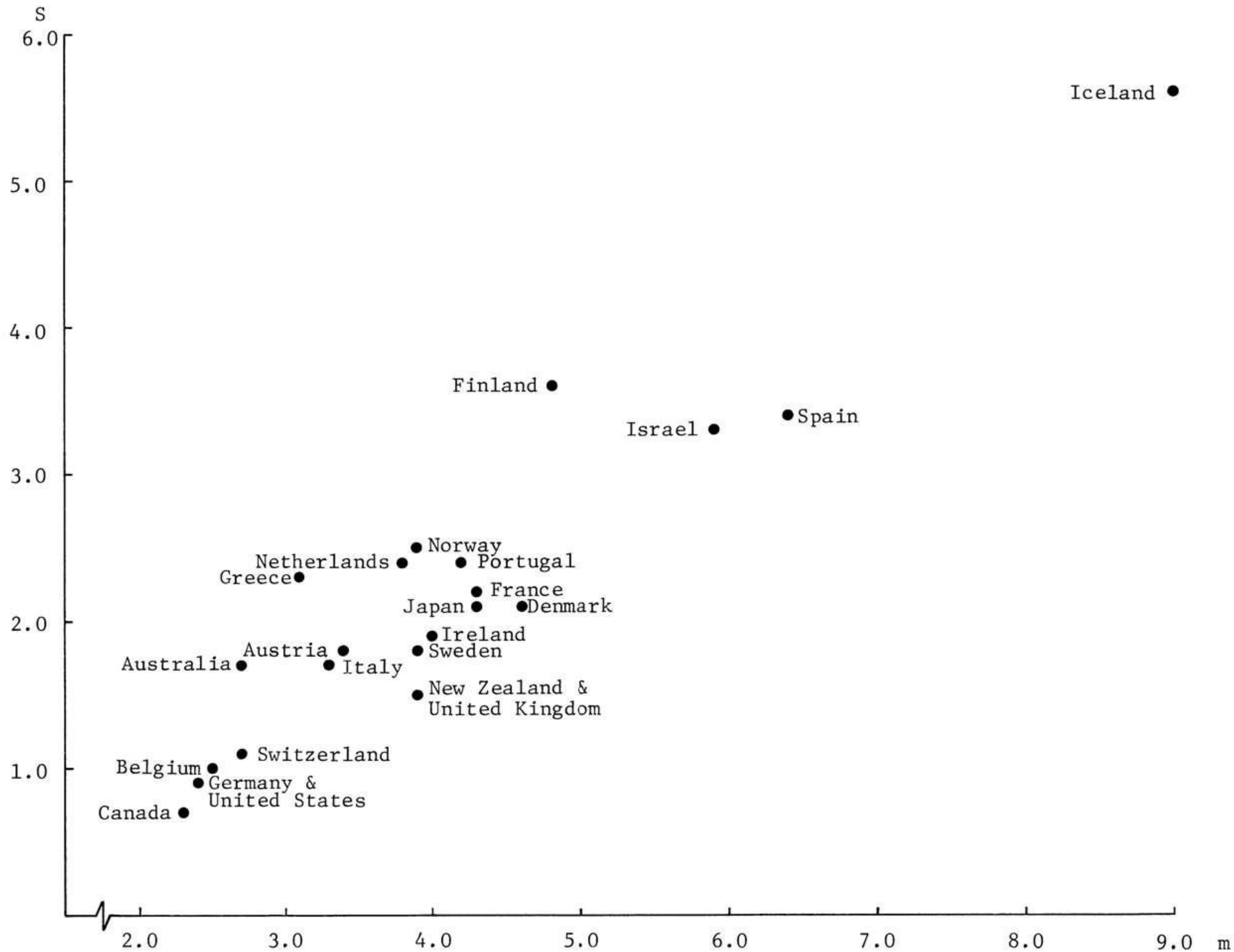
Source: Computed from consumer price index recorded in *International Financial Statistics*, July, 1972, and 1971 Supplement.

*Where X_t is the rate of inflation from year t-1 to t, and n is the number of years,

$$m = \frac{1}{n}\sum X_t, \text{ t running from 1 to n}$$

$$S = \frac{1}{n-1}\sum |X_t - X_{t-1}|, \text{ t running from 2 to n.}$$

Figure 1. Average Annual Rate of Inflation (m) and Average Absolute Change in Rate of Inflation (S), 23 Advanced Economies, 1954-1971



The variables being correlated are not normally distributed, so the usual test of significance on r is not strictly relevant.[5/] But all of the correlation coefficients pass those tests: In the weakest case (17 countries, 1961-1971) r^2 is significant at the .005 level. The evidence of these correlation coefficients does not completely answer the objections raised by Gordon. One of his concerns was that much of the positive correlation was contributed by small countries that are particularly sensitive to import prices, agricultural prices, or both. Visual inspection of Figure 1 may confirm that impression, depending on how one chooses to classify countries, but I prefer to separate this discussion of whether or not a relationship exists from later discussions of why it might exist; I have refrained from computing the correlation with the particular 12 countries selected by Gordon (excluding Denmark, Finland, Ireland, Italy, and the Netherlands from Okun's sample of 17 countries) both for this reason and because I cannot see what characteristic distinguishes these five countries from many of the others.

A second set of calculations has been made for a group of 17 Latin American economies.[6/] My purpose was to confirm that the phenomenon of correlation between the average rate of inflation and its variability is not confined to industrial countries. I chose Latin America rather than Africa or Asia for the test, because several years ago, Graeme Dorrance noticed the relationship while studying Latin American inflation, and I wanted to confirm his casual remark.[7/]

The resulting correlations (corresponding to those reported in Table 3) are 0.86 for 1954-1971, and 0.88 for 1961-1971. Table 5 and Figure 2 show the data on which these coefficients are based. The most cursory inspection shows that these high correlations are almost inevitable:

four of the countries maintain currencies at par with the dollar,[8/] four more have experienced average rates of inflation of 30 percent or more; such unstable economies "must" experience great fluctuations in the process, and we would expect strong correlations. Even if the four countries with currencies at par with the dollar are excluded, the correlation remains significant: 0.83 for 1954-1971, and 0.86 for 1961-1971; but there are other very stable economies in the sample that preserve the wide spread in inflationary experience.

Table 5. Average Annual Rate of Inflation in Consumer Price Index (m) and Average Absolute Change in Annual Rate of Inflation (S), 17 Latin American Economies 1954-1971 and 1961-1971*

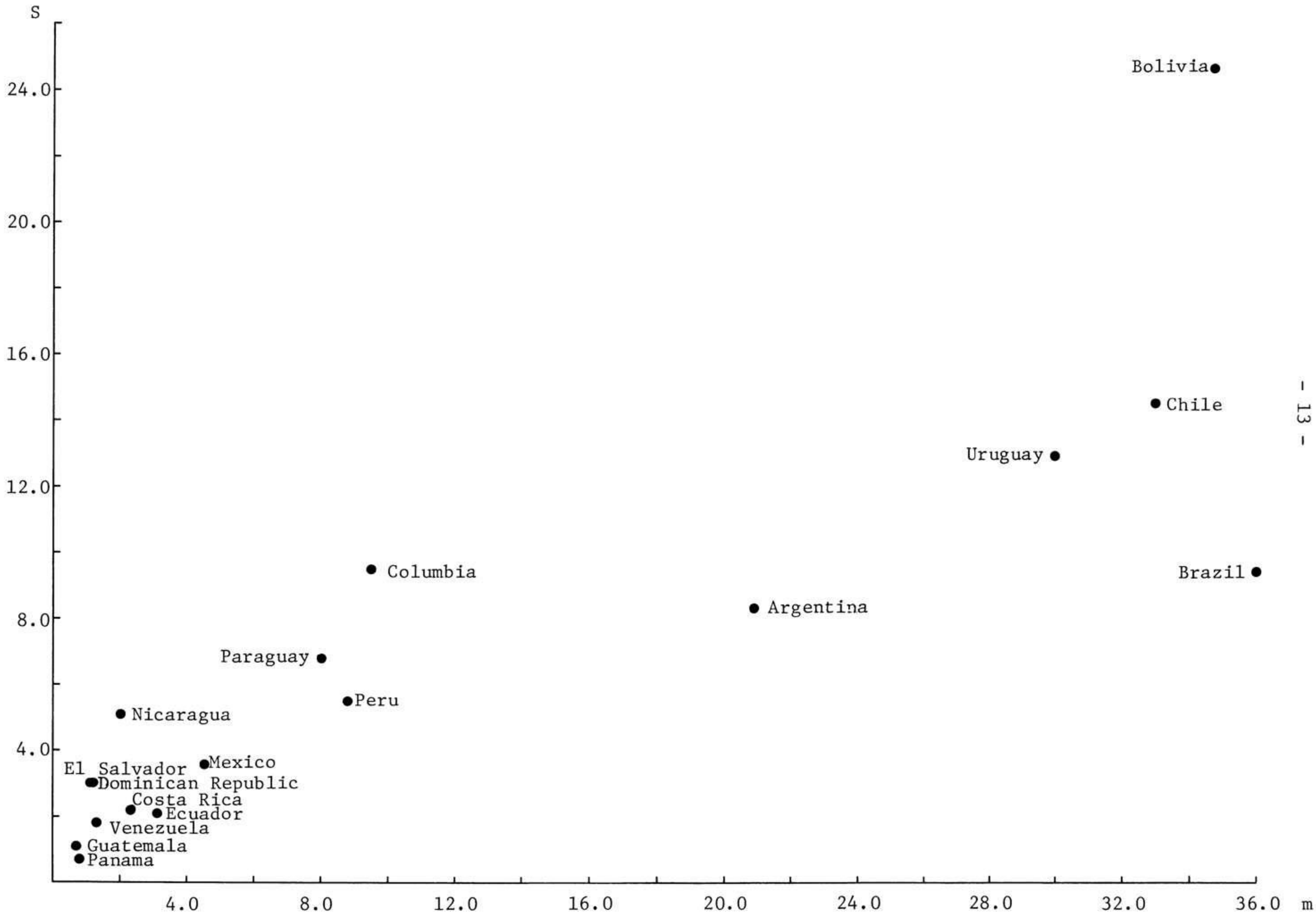
		1954-1971**		1961-1971**	
	Country	m	S	m	S
1.	Argentina	20.9	8.3	22.7	8.3
2.	Bolivia	34.8	24.6	5.6	4.9
3.	Brazil	36.0	9.4	43.5	11.1
4.	Chile	33.0	14.5	26.4	9.0
5.	Colombia	9.5	9.5	11.3	10.0
6.	Costa Rica	2.3	2.2	2.5	2.5
7.	Dominican Republic	1.2	3.0	2.1	3.0
8.	Ecuador	3.1	2.1	4.7	1.5
9.	El Salvador	1.1	3.0	0.7	1.9
10.	Guatemala	0.7	1.1	0.8	1.0
11.	Mexico	4.5	3.6	2.7	1.2
12.	Nicaragua	2.0	5.1	1.8	2.3
13.	Panama	0.8	0.7	1.3	0.8
14.	Paraguay	8.0	6.8	3.5	4.1
15.	Peru	8.8	5.5	9.6	7.4
16.	Uruguay	30.0	12.9	36.5	16.8
17.	Venezuela	1.3	1.8	1.1	1.5

Source: Computed from consumer price index recorded in International Financial Statistics, July, 1972, and 1971 Supplement.

*For definitions of m and S, see Table 4.

**1954-1970 and 1961-1970 for Bolivia, El Salvador and Mexico. 1954-1969 and 1961-1969 for Nicaragua.

Figure 2. Average Annual Rate of Inflation (m) and Average Absolute Change in Rate of Inflation (S), 17 Latin American Economies, 1954-1971



Intranational Comparisons

The cross-sectional evidence from international comparisons suggests that countries with higher rates of inflation have, on the average, experienced more variable rates of inflation; but it tells us nothing about cause and effect. It does not even tell us if we could expect to find the same sort of relationship within a single country between periods of low and high inflationary pressure. It could be that the factors which differ among countries so as to create the correlation we have reported would not differ among regimes of varying inflationary pressure within a single country, and so the correlation would not be observed.

Testing for the existence of such a relationship within a single country is difficult; the world will not stand still for long enough. So many things besides the rate of price increase changed between the 1930s and the 1960s, for example, that it seems pointless to use the experience of those decades as observations for a crude statistical analysis. Even between the 1950s and the 1960s, there was a worldwide stabilization in the rate of inflation, with an increase in its average rate, that would affect price behavior within any individual country.

However, the figures of Tables 4 and 5 do provide crude tests by decade: Of the 23 advanced economies reported in Table 4, m and S move in the same direction from 1954-1960 to 1961-1971 in only four countries--Greece, Iceland, Portugal, and the United Kingdom. For the Latin American economies, results are more favorable to the Okun hypothesis: m and S move in the same direction from the 1950s to the 1960s in 13 of the 17 countries reported in Table 5 (all except Argentina, the Dominican Republic, Ecuador, and Guatemala).

I supplemented these rather crude tests with others using monthly price statistics. The units of observation were then individual years for a given country, with observations on m and S calculated as monthly averages over the 12 months of a calendar year. No correlation between m and S can be detected in these monthly data.[9/]

While these negative results do not lend any support to the Okun hypothesis, neither can they be used to reject it. For example, if instability were due primarily to government actions geared to an annual budget cycle, month-to-month changes would not reflect the relationship. And if instability were created by a mechanism internal to the private economy that operates with substantial lags, month-to-month changes might not reflect the relationship; for example, a mechanism operating through the labor market, with its many annual wage contracts, might not be adequately reflected in unlagged monthly data.

III. INTERPRETATION

Recall the motivation for this discussion: A higher average rate of inflation may bring with it greater variability in the rate of inflation. If true, inflationary policies carry a cost which we have so far neglected; so it would be useful to know if such a causal relation exists. The crude evidence reported here from cross-sectional data is compatible with the hypothesis, but it proves little. The observed relation between m and S could result solely from the differences among countries in economic and governmental structure.

Suppose, for example, that two countries have the same low target rate of inflation but that forecasts of aggregate private demand in A are inherently more uncertain than in B. If prices are rigid downward, A would show higher values of both m and S; the same result could arise from simple ineptness of control in A compared to B. In either case, no single country would need to suffer more variability if it should opt for a higher rate of inflation. For m and S to be linked in a single country, the explanation must come from the internal workings of the economy.

Using a model that is too simple to be taken seriously, it is easy to show that a functional relation exists. It is not possible to show that this function is necessarily monotone increasing. I have not investigated a more realistic model, but I expect that the link would survive as long as the key relationships are preserved from the model presented here.

A Simple Model

The model[10/] consists of a short-run Phillips curve

(1) $\pi = f(U), \quad f' < 0,\ f'' > 0,$

a production function

(2) $Y = Pg(U), \quad g' < 0,\ g'' > 0,$

and an instrument for government policy

(3) $Y = \underline{a}G, \quad \underline{a} > 0.$

Here

π is the rate of inflation, $\pi = (P-P_{-1})/P_{-1}$

P is the current price level, and P_{-1} is last period's price level

U is the rate of unemployment

Y is nominal GNP

G is government spending (or, more generally, the total level of spending subject to government control, either directly or by means of fiscal and monetary policy)

$\underline{a}$ is the multiplier, assumed here to be a random variable with density function $\phi(\underline{a})$.

If we choose units so that $P_{-1} = 1$, then $P = 1 + \pi$, and equations (1)-(3) may be combined to give

$$(4) \qquad \underline{a}G = (1+\pi)g[f^{-1}(\pi)],$$

where f^{-1} is the inverse of f in (1). The relation in (4) is monotone, so it may be inverted to yield

$$(5) \qquad \pi = h(\underline{a}G), \quad h' > 0.$$

Within the framework of this one-period model, the average rate of inflation is appropriately measured by its expectation with respect to $\underline{a}$ (for given G)

$$(6) \qquad m = \int h(\underline{a}G)\phi(\underline{a})d\underline{a}.$$

The variability in the rate of inflation can be measured by the variance,

$$(7) \qquad \sigma^2 = \int [h(\underline{a}G)-m]^2\phi(\underline{a})d\underline{a}.$$

(6) and (7) define m and σ as functions of G. It is easily shown[11/] that m is a monotone-increasing function of G, so we may invert the function to express G as a function of m, then σ as a compound function of m, say $\sigma = \Omega(m)$. The slope of this function may then be evaluated by differentiating (6) and (7) with respect to G:

$$(8) \qquad \Omega' = (d\sigma^2/dG)/(2\sigma dm/dG).$$

I have already noted that dm/dG is positive; but it can be shown that $d\sigma^2/dG$ is equal to 2 $\text{cov}(\pi,\partial\pi/\partial G)$ and may be positive or negative. Equation (5) and its antecedents allow us to assess the possibilities. Since the covariance of π and $\partial\pi/\partial G$ is governed by the distribution of the random variable $\underline{a}$, we can see whether these two expressions are monotone functions of $\underline{a}$. If both π and $\partial\pi/\partial G$ rise with $\underline{a}$, for example, their covariance must be positive (this is of course a sufficient, but not necessary, condition for a positive covariance).

We have, from (5),

(9) $$\partial\pi/\partial\underline{a} = Gh'(\underline{a}G) > 0$$

(10) $$\partial^2\pi/\partial\underline{a}\partial G = h'(\underline{a}G) + \underline{a}Gh''(\underline{a}G).$$

The last expression is cumbersome but may, after some manipulation, be written

(11) $$\frac{\partial^2\pi}{\partial\underline{a}\partial G} = \frac{1}{D^3}\{g^2+\left(\frac{1+\pi}{f'}\right)^2[(g')^2+\frac{g}{f'}(gf''-fg'')]\},$$

where

(12) $$D = g + (1+\pi)g'/f' > 0.$$

Thus we see that $\partial^2\pi/\partial\underline{a}\partial G$ is necessarily positive if $f'' = g'' = 0$, but may become negative if $gf'' > fg''$. So $d\sigma/dm$ is definitely positive, in this model, when $\partial^2\pi/\partial\underline{a}\partial G > 0$, which is true unless $gf'' - fg''$ is negative and large in absolute value; on the other hand, $d\sigma/dm$ may still be positive even if $\sigma^2\pi/\partial\underline{a}\partial G < 0$ for some values of $\underline{a}$.

It would appear that the positive relation we seek between σ and m is substantially confirmed. However, minor changes in the model will weaken these results. For example, with an additive error term in either equation (1) or (3) and the multiplier $\underline{a}$ in (3) treated as constant, much stronger conditions are required to guarantee that $d\sigma/dm > 0$. If these conditions are not met, we might well observe an increase in the average rate of inflation, associated with a fall in variability. But to discuss this issue we need to discuss how a target rate of inflation is selected.

Government Policy and Observed Inflation

I now wish to move from the specific model just discussed to a somewhat more general class of models, characterized by

(a) a Phillips curve defining feasible combinations of π and U, $U = f(\pi)$,

(b) a functional relation between the mean and variability of the rate of inflation, $\sigma = \Omega(m)$ (I use σ to measure variability, but that is not essential to the argument),

(c) a government that attempts to achieve the highest possible value of a welfare function, where welfare, represented by $W(\pi,U)$, falls as either π or U rises.

Let us suppose, initially, that the government naively chooses the point on the Phillips curve that yields the highest welfare, say π_1, $U_1 = f(\pi_1)$, and sets its policy instruments--G in the model discussed above--so that the expected value of π is equal to the target value (this policy would be naive because it neglects the variability of observed outcomes). If the policy were maintained for long enough, we could estimate $m(\pi) = \pi_1$ and $\sigma(\pi_1)$ and, hence, estimate one point on the function $\sigma = \Omega(m)$. If, then, because of a change of heart or a change of government, the welfare function were to shift and remain steady at its new realization, we would have an opportunity for a second observation, say at π_2. As tastes change over time, or by observing several countries facing the same Ω function but with different welfare functions, we could trace out the shape of the Ω function. And it is such movements along the curve that tell us about the policy choices facing the government at any given moment. On the other hand, if the shift in the target rate of inflation from π_1 to π_2 were brought about by a shift in the Phillips curve rather than by a change in tastes, the Ω function would itself shift, and it would be difficult to make any inference about its shape. It would be possible to observe σ and m increasing together, even though the function slopes down, or it would be possible to observe the reverse. Such observed "slopes" tell us nothing about the nature of the alternatives available along a given curve.

But an extension of the model to consider more sophisticated choices by government leads to an interesting conclusion: There is then a presumption that efficient points will associate higher variance with higher average rates of inflation, at least over a wide range of inflation rates.

Suppose, following Brainard (1967), that the government wishes to maximize expected utility of the form[12/] $V(m,\sigma)$ (where $m = m(\pi)$, $\sigma = \sigma(\pi)$), with partial derivatives $V_1 > 0$ for $m < T$, $V_1 < 0$ for $m > T$, and $V_2 < 0$; T is to be interpreted as the target rate of inflation. In the (m,σ) plane, indifference curves slope up to the left of T and down to the right.[13/]

The curve $\sigma = \Omega(m)$ shows a feasibility locus of attainable points. Provided the optimal policy is to hold inflation below the target rate T, it must be at a point such as q in Figure 3, where the feasibility locus Ω slopes up. The only exception to this rule would come in a world where the optimal policy calls for an average rate of inflation higher than the target rate. In such a world, increases in inflation lower the variability by so much that it pays to overshoot, deliberately sustaining too high a rate in order to obtain increased steadiness.

Restricting attention to situations where the optimal value of m is below the target value, let us now consider what happens when policy shifts because of a change in the welfare function. Assume that (m_1, σ_1)

Figure 3

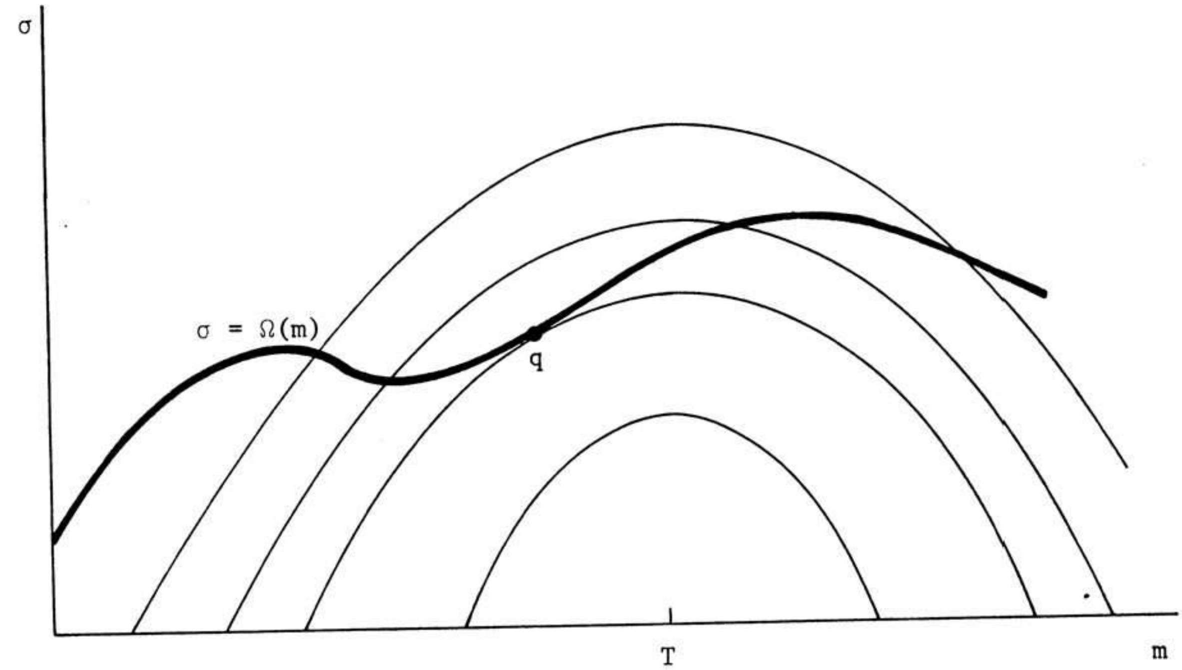
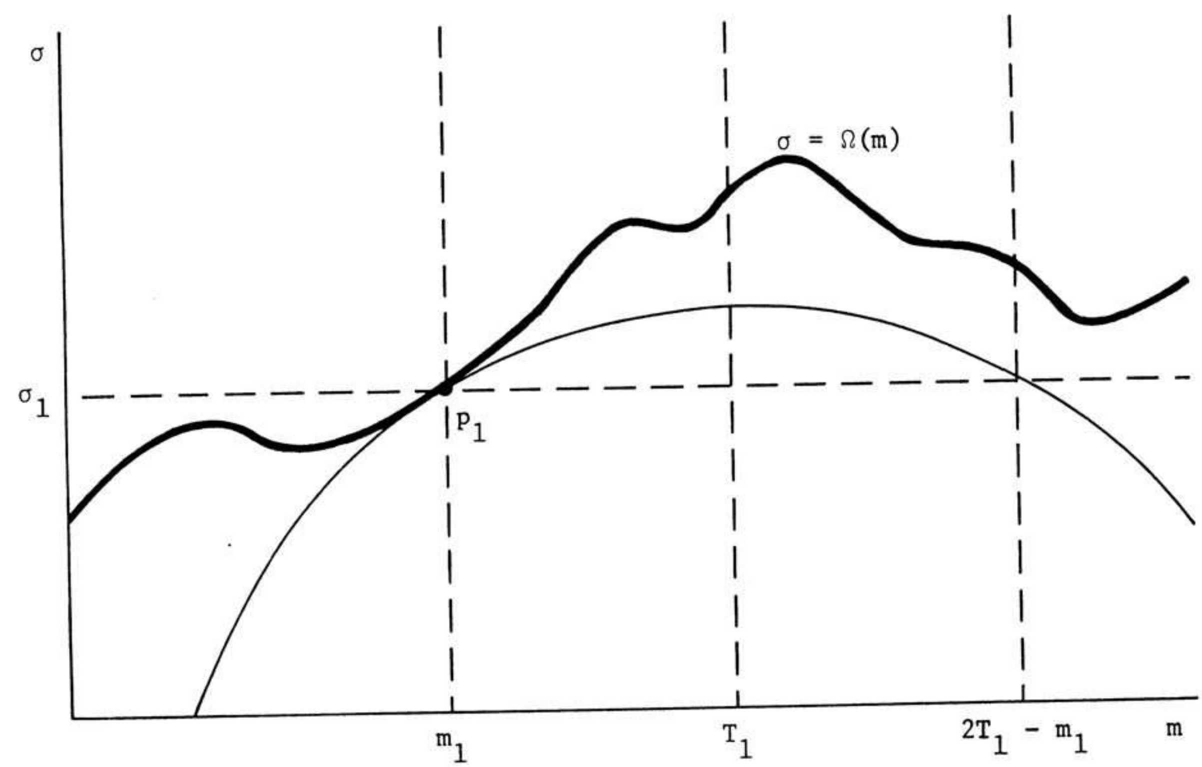


Figure 4



represents the optimal policy when the target rate of inflation is T_1 and that (m_2, σ_2) represents the optimal policy when the target rate of inflation is $T_2 > T_1$. Then $\sigma_2 > \sigma_1$ provided $m_1 < m_2 < T_1$ and $\sigma_2 < \sigma_1$ provided $m_2 < m_1$; in this restricted sense, an increase in m means an increase in σ. Figure 4 displays the reasoning. If $p_1 = (m_1, \sigma_1)$ is the optimal point associated with a target rate of inflation T_1, then no feasible point can lie to the southeast of p_1 in the box $m_1 < m < T_1$, $0 < \sigma < \sigma_1$. On the other hand, if $p_2 = (m_2, \sigma_2)$ is the optimal point associated with the higher target rate of inflation T_2, it cannot lie to the northwest of p_1 (because p_1 would then give higher utility). Hence there are three possibilities:

$m_2 < m_1$ and $\sigma_2 < \sigma_1$,

$m_1 < m_2 < T_1$ and $\sigma_1 < \sigma_2$,

or $m_2 > T_1$ with no restriction on σ_2.[14/]

<u>Summary</u>

We have shown that Okun may be right: If government shifts to a more inflationary policy (higher value of m) due to a change in its welfare function, it may automatically choose a more variable rate of inflation at the same time (higher value of σ). Within an extremely limited static model, we showed that a functional relation exists between m and σ and that it may be monotone increasing. There is no reason to think that the functional relation would disappear in a model of greater realism, although there is reason to suspect that the conditions required for monotonicity will change and may become more difficult to achieve. On the other hand, within a less limited model, it appears that government "should" choose among points for which more inflation means more variability, even though the function might include segments that are locally negatively sloped. This is true unless the opportunities for reducing variability by increasing the average rate of inflation are so overwhelmingly attractive that it pays to go beyond that target rate of inflation which would be chosen in the absence of all variability. In any event, evidence of the sort reported by Okun (or Gordon, or Foster) does not throw much light on the alternatives unless it can be shown that the feasible alternative values of m and σ are the same for each country in the sample.

FOOTNOTES

1/ The countries include Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, Norway, Sweden, Switzerland, the United Kingdom, and the United States.

2/ More generally, variability in the rate of inflation is important in terms of welfare because it makes for forecasting errors. If we knew what forecasting model people actually used, and if there were unanimity of opinion in constructing forecasts, an appropriate measure of variability would be the average absolute forecasting error. Put this way, my measure of variability S would be appropriate if all forecasts were naive forecasts of "no change in the rate of inflation from last year." The alternative suggested in the text would be appropriate if everyone could correctly forecast the time trend in rates of inflation.

3/ Taken from the United Nations Yearbook of National Account Statistics, 1969.

4/ The rate of inflation for 1954, X_{54}, is calculated as the year-over-year percentage change in the CPI from 1953 to 1954, and the measure of variability I use is $|X_{54} - X_{53}|$, so it depends on price data going back to 1952. But I believe that Table 2 justifies this starting date. Note that the second period chosen, starting in 1961 by my notation, begins with the 1960-1961 inflation; this conforms to Gordon's time period.

5/ If the underlying observations on annual rates of inflation were normally distributed, m would be normal, and S would be a sum of "folded normal" variables; I have not investigated the properties of this distribution.

6/ The countries include Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, the Dominican Republic, Ecuador, El Salvador, Guatemala, Mexico, Nicaragua, Panama, Paraguay, Peru, Uruguay, and Venezuela. Of the "Latin American" countries for which the IMF publishes country statistics, I rather arbitrarily excluded Guyana, Haiti, Jamaica, and Trinidad-Tobago for being adequately American, but insufficiently Latin.

7/ "If the current rate of price rise is 20 percent a year, the rate next year may almost equally well be approximately 10 percent or over 40 percent." (1969, p. 41)

8/ The Dominican Republic and Guatemala maintain their currencies strictly at par, Panama uses the U.S. dollar for currency (except for some coins), and El Salvador has maintained an exchange rate of 2.5 colones per dollar since 1934.

9/ Calculations were carried out for the United States (wholesale price index, not seasonally adjusted, 1953-1971 and consumer price index, 1953-1971), the United Kingdom (retail price index, not seasonally adjusted, 1953-1971), and France (wholesale price index, not seasonally adjusted, 1953-1970).

10/ Okun (1971, p. 495) suggested that a model of this general structure would yield the desired result, but he did not develop his argument formally.

11/ This and subsequent results require differentiation under the integral sign in (6) and (7) and, hence, assume a certain degree of smoothness for h and ϕ.

12/ My introductory remarks suggest that we do not know enough, in the real world, to write down the losses associated with a specified combination of inflation and unemployment. So even if we are willing to suspend disbelief and suppose that government chooses a policy in such a way that its actions could be tracked by a utility function, we must acknowledge that a specific policy does not give the government a known outcome; rather it gives them a ticket in a lottery in which the alternative outcomes are the true (but imperfectly observed) costs of inflation and unemployment, and in which the probabilities of alternative outcomes are, at best, subjectively determined. Needless to say, the specification in the text is an idealization.

13/ To fix ideas, suppose welfare is given by

$$W = -[\pi^2+\alpha U^2+\beta\sigma^2(\pi)].$$

$-W$ is a quadratic loss function in π and U, with an extra term to represent the costs assumed to arise directly from the variability of inflation (see the introduction). Suppose further that the Phillips curve is linear,

$$U = \gamma - \delta\pi.$$

Then

$$E(W) = -E[\pi^2+\alpha(\gamma-\delta\pi)^2] - \beta\sigma^2(\pi).$$

Dropping the constant term, this simplifies to

$$E(W) = -E[(1+\alpha\delta^2)(\pi-T)^2] - \beta\sigma^2(\pi)$$

where $T = \alpha\gamma\delta/(1+\alpha\delta^2)$; finally,

$$E(W) = V(m,\sigma) = -(1+\alpha\delta^2+\beta)\sigma^2(\pi) - (1+\alpha\delta^2)(m(\pi)-T)^2.$$

Indifference curves are half-ellipses about the point $m(\pi) = T$, $\sigma(\pi) = 0$, with semiaxes in proportion $[(1+\alpha\delta^2+\beta)/(1+\alpha\delta^2)]^{1/2}$:1. The "target rate of inflation," T, is the rate of inflation the government would choose if it neglected variability altogether and simply chose the point on the Phillips curve that maximizes the welfare function $-(\pi^2+\alpha U^2)$. Notice that if there are no direct costs arising from variability of inflation, $\beta = 0$, and the indifference curves are semicircles as in Brainard's paper.

14/ If we impose the further restriction on the utility function that it be symmetric about the target rate of inflation (i.e., that overshooting and undershooting are equally costly, as is true for the welfare functions of n. 13 and Figure 4), we can strengthen the result: Then $\sigma_2 > \sigma_1$ provided $m_1 < m_2 < 2T_1 - m_1$. This follows because in that case no feasible point can lie to the southeast of p_1 within the box $m_1 < m < 2T_1 - m$, $0 < \sigma < \sigma_1$.

On the other hand, the restriction that I have imposed on the indifference map, that each indifference curve reaches its maximum where $m = T$, enables me to state in the text a stronger result than would be true, in general, if some indifference curves were to reach their maxima at some value of $m < T$.