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OPTIMAL CONTROL OF THE MONEY SUPPLY

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ABSTRACT

Using optimal control theory and a vector autoregressive representation of the relationship between money and interest rates one can derive a feedback control procedure which defines the best possible tradeoff between interest rate volatility and money supply fluctuations and which could be used to reduce both from their current levels.

1. INTRODUCTION

The debate over the proper conduct of monetary policy has intensified in recent years as the Federal Reserve has focused its attention on reducing inflation by controlling the rate of growth of the money supply. Although most observers give the Federal Reserve credit for reducing the trend growth of money, many have criticized it for having increased the short run variability of money growth rates and the volatility of interest rates. The Federal Reserve is currently searching for procedures which will guarantee control over the trend growth of money, while at the same time reducing the short run fluctuations in both money and interest rates. This paper uses optimal control theory and a time series representation for money and interest rates to derive a feedback control procedure which defines the best possible tradeoff between interest rate volatility and money supply fluctuations and which could be used to reduce both from their current levels. The organization of the paper is as follows: section 2 reviews the control theory framework, section 3 describes the use of a time series model to represent the dynamic behavior of the system, section 4 presents the application to short-run control of the money supply, and a final section addresses the key issue of structural stability.

2. OPTIMAL CONTROL THEORY

Optimal Control Theory is a well developed set of mathmetical tools used primarily by engineers to solve problems involving a dynamical system which responds to exosenous inputs. These same tools are used here to generate a rule for targeting interest rates in order to balance optimally the competing goals of controlling the supply of money and reducing the volatility of interest rates.

In its usual form, optimal linear control theory specifies an algorithm for setting one or more inputs in order to minimize a quadratic loss function. This result, and other cited below, can be found in any standard control theory text, such as Kwakernak and Sivan (1972) or Chow (1976). The key elements in the optimal control problem are as follows:

The State: A vector of variables which contains all relevant information concerning the current state of the system. In particular, the State vector includes all variables which enter the loss function directly, and all other variables which help to predict their values.

The Laws of Motion: A difference equation which determines the state at time t as a function of the previous state, a vector of inputs called the control, and a disturbance vector. The dynamical system with its associated laws of motion is often referred to as "the plant."

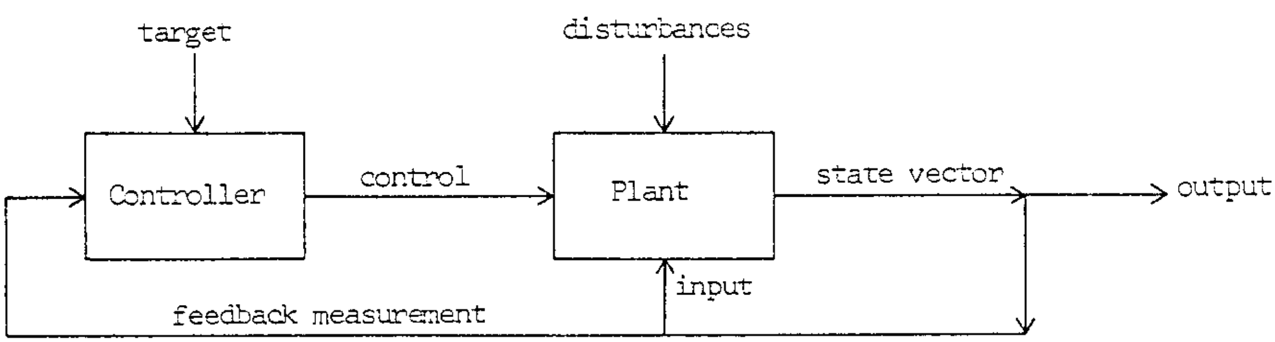
The Control: A vector of inputs which can be set by the controller in order to affect the future course of the state.

The Loss Function: A function specifying the criterion to be optimized in the setting of the control. Often the loss function includes a target or path of desired values for one or more components of the state.

Two types of control are differentiated by whether or not the control responds to the current state of the system. If the control is preset, the control is referred to as "open loop." If the control is adjusted each period in order to respond to the current state, then the system is said to be operating under "feedback control." Feedback control loops have several desirable properties relative to open loop control. A feedback controller compensates for disturbances, allowing the control to be much more effective. Futhermore, an unstable system can be stabilized by feedback control, a characteristic which cannot be obtained by open loop control. Finally, the effect of system parameter variation can be greatly reduced by continually updating the contol. This is of particular importance in the case of economic systems for which there is likely to be paramenter variation and a high degree of parameter uncertainty.

The control design is generally based on the following sequence of events. At time t-1 the state vector determines everything that is needed to predict the future course of the system. The controller observes this state and determines the optimal setting for the control which will impinge on the system at time t. The state of the system at time t is a function of the state at time t-1, the control at time t, and a disturbance vector which occurs at time t. A diagramatic representation makes this clear.

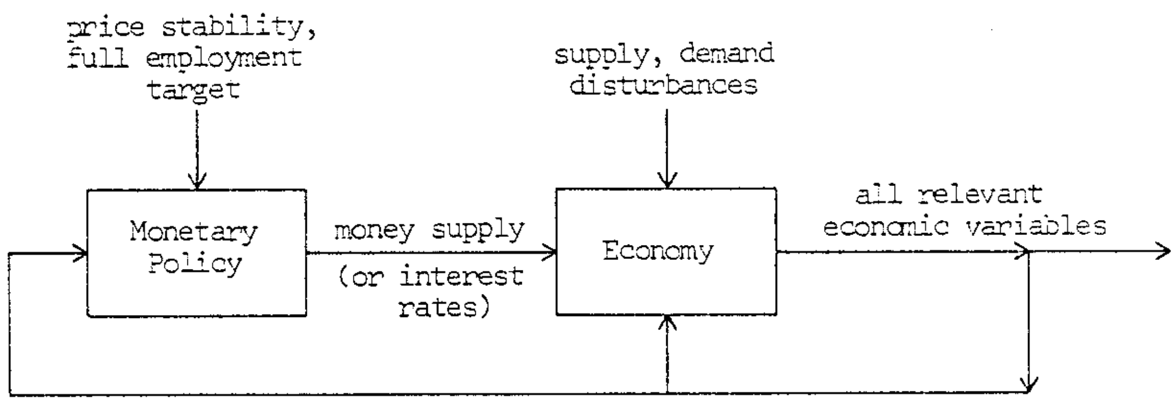
OPTIMAL CONTROL DESIGN



Finding the optimal setting of the control, given the laws of motion of the system and a particular loss function is generally a very difficult problem. However, for the particular case of a system whose laws of motion are linear and for which the loss function is quadratic, the problem has been solved. Under mild regularity conditions, a computational procedure known as "iterating on a matrix Riccati equation," leads to the optimal linear control rule. While the optimal control rule which solves the linear, quadratic problem may not be optimal relative to a more general formulation of the problem, in practice it is likely to be the best solution available.

The textbook application of control theory to monetary policy assumes that the Fed can control either money or interest rates perfectly.[1/] The question at issue is which variable the Fed should control, and how it should set that variable so as to achieve a full employment, stable price path for the economy. This standard application is not the problem which we are addressing. We bring it up here to illustrate a typical application of control theory and to contrast the framework adopted here with the usual approach.

USUAL APPLICATION TO MONETARY POLICY



The focus of this paper is more narrow than the usual textbook application in that no attempt is made to derive an optimal monetary policy. It is assumed here that the money target path is known. However, rather than taking as given the ability of the Fed to hit its money supply target, this

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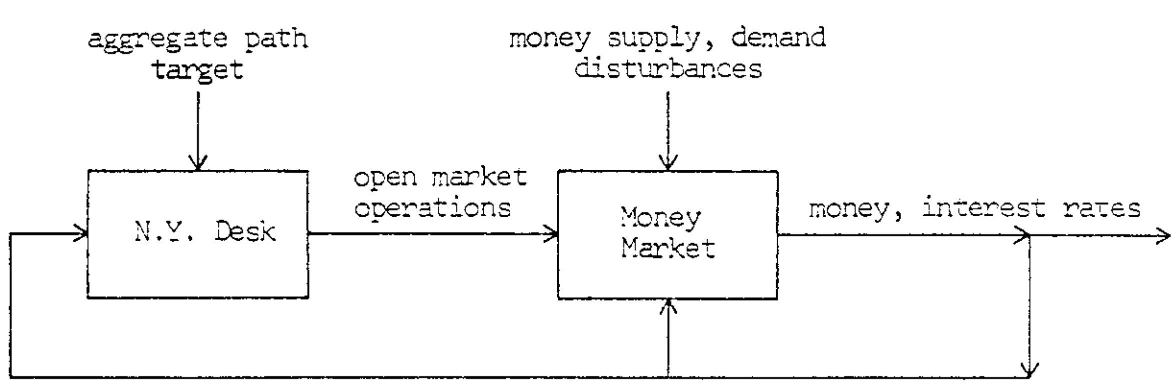
[1/] See, for example, Sargent (1979), Sargent and Wallace (1975), and Kalchbrenner and Tinsley (1976).

paper investigates the Fed's short run problem of attempting to control the money supply. For the purpose of this paper, the monetary target in a given week is that week's value for the level of M-1 which is on a long run trend growth path adopted by the Fed. The Fed is assumed to use open market operations to try to keep M-1 as close to the trend as feasible, on average. The open market operations, by increasing or decreasing the supply of reserves, cause the federal funds rate to go down or up, respectively. These movements in the federal funds rate will cause banks and other economic agents to adjust their portfolios, leading to predictable movements in the stock of money. We do not attempt to model the open market operations directly, instead, we focus on the levels of the federal funds rate which emerge each week and their effects on subsequent movements in money. In the control procedure modeled here the Fed receives, at the end of the week, the latest figures for M-1 (data for the week ending two weeks earlier), and decides on a new desired level for the funds rate for the following week. Other procedures and timing relationships could easily be modeled in a similar manner. In particular, we will later discuss, in turn, the applicability of this procedure to a funds rate target, in which the Fed can basically set its targeted funds rate, and to a reserves target, in which the Fed supplies reserves consistent with its chosen funds rate, but does not offset shocks which may cause significant deviations within a given week. A diagram of the short-run control application is shown below.

APPLICATION TO SHORT-RUN CONTROL OF MONETARY AGGREGATES



It is assumed that the Fed knows the dynamic response pattern of money and interest rates and uses this knowledge to set the funds rate so that the money supply will stay near its target path. In the next section we will address the question of estimating the necessary response patterns. Because the money supply is subject to random disturbances, the best the Fed can do is to cause the expected value of money to be on target each week. However, in order to achieve this level of accuracy with respect to money, the Fed might have to make large changes in the funds rate each week. The required changes might easily increase over time, leading to explosive oscillations in interest rates. This is the instrument instability problem suggested by Holbrook (1972). In fact, the Fed does not try to bring the expected value of the money supply onto its target path each week. Rather, it is assumed to recognize a short-run tradeoff between reducing expected deviations of money from its target path and reducing fluctuations in interest rates. In order to investigate the nature of that tradeoff we specify a loss function which has terms penalizing both money supply deviations from target and volatility of interest rates. These two objectives are assumed to capture the most important tradeoff in the current Fed operating procedures. However, the loss function could easily be generalized to include additional goals. It might be desirable, for example, to avoid large interventions in the market, in which case one could include a term representing a cost associated with the size of the control itself.

Optimal control is most often expressed in the context of a first order difference equation in the state vector. Let $x(t)$ be an nx1 state vector, $u(t)$ be the control, and $w(t)$ be an nx1 vector of disturbances. The laws of motion of the system are given by

$$x(t) = A\ x(t-1) + B\ u(t) + w(t) \tag{1}$$

where A is an nxn matrix and B is an nx1 vector. In order to fit the monetary control problem into this framework, $x(t)$ includes current and lagged values of M-1, $m(t)$, the federal funds rate, $r(t)$, possibly other informational variables, and a monetary target, $M^*(t)$. $u(t)$ is the Fed controlled shock to the funds rate. The matrix A includes two or more rows of estimated coefficients which define how M-1, the funds rate, and possibly other variables evolve through time. All but one of the other rows of A identify as their values in the previous state lags of m, r, and possibly other variables. The final row defines the target money supply path.

The quadratic loss function is written as

$$L = E\{\sum_{s=0}^{\infty} \beta^s[(M(t+s) - M^*(t+s))^2 + \frac{\lambda}{q}\sum_{k=1}^{q}(r(t+s)-r(t+s-k))^2]\} \qquad (2)$$

where $M^*(t)$ is the desired path for M-1. The cost associated with money deviations from target is balanced with interest rate volatility, measured as a weighted sum of expected squared changes in the federal funds rate over time. Different relative costs associated with deviations from the money target path and interest rate volatility can be represented by different values of λ. More terms in the sum measuring interest rate volatility, that is larger values of q, will lead to a smoother funds rate path. For example, a high λ, with q equal to one will avoid whipsawing the market--large movements in the funds rate in a given week--while still allowing significant movements over a period of time as short as say two or three months. A q of ten or twelve, on the other hand, would damp considerably these longer swings as well, leaving only very smooth changes in the funds rate over time. This form for the loss function is only one of many possibilities. It was chosen primarily because of its simplicity; the higher is q, the more it will respond to, that is penalize, low frequency variations in interest rates. A more sophisticated loss function in the linear-quadratic class could be constructed by making loss proportional to the square of particular linear combinations of expected future interest rates, the linear combinations being chosen specifically to respond to certain bands of frequencies of interest rate movements.

The loss function in (2) also includes a discount factor β, which allows the loss function to give relatively less weight to future losses than to current losses. For the purposes of this paper there is no reason to discount future losses, and the discount factor is taken to be 1. Although the expected loss is not finite when the discount factor is 1, there is a well defined feedback rule which is the limit as β goes to 1 of rules associated with β's less than 1 which do generate finite expected losses. In fact, it may be not particularly desirable to have a finite expected loss; this requires a discount factor less than 1, which is myopic in the sense that in a steady state the average stream of losses will be larger than need be. This occurs because the feedback rule does not look far enough ahead. For example, if movements in interest rates affect money with a lag, and if we heavily discount future losses, then we will be very reluctant to move interest rates in any given period and our average loss each period will become very large since money will deviate far from its target.

Given the environment described in (1) and the loss function (2), optimal control theory answers the following question, "What is the linear feedback rule for choosing u(t) which, on the basis of current information, minimizes the expected future loss?" The solution is a feedback matrix, F, and a rule

$$u(t) = F\,x(t-1) \tag{3}$$

which determines u(t) as a linear function of the past state and is optimal in the sense that this choice of F generates a smaller expected loss than any other choice.

3. TIME SERIES ANALYSIS

A critical element in optiml control theory is knowledge of the dynamic behavior of the system. Time series analysis, in particular the application of vector autoregression techniques, provides a reliable estimate of the laws of motion of the money market.

A special problem is encountered when optimal control theory is applied to economic systems. A key element in the optimal control framework, knowledge of the laws of motion of the system is either missing completely, or known only with a large degree of uncertainty. Engineering texts on optimal control spend little time considering this problem because it is usually assumed that the response functions can be measured directly to whatever degree of accuracy is needed. In economic systems it is impossible to perform controlled experiments in order to measure response functions. Instead, economists have come to rely on the laws of motion imbedded in econometric models.

Unfortunately, econometric models have generated a rather poor record with respect to forecasting the response of the economy to changes in policy. For example, when a key econometric relationship, the Phillips curve, was identified in the 1960's many economists[2/] claimed it could be used as the basis for attempting to trade off higher inflation for lower unemployment. After a decade of high inflation along with high unemployment, few would suggest such an approach today. The rational expectations critique of standard econometric models provides a reasonable explanation of why those models failed, and many economists have developed a cautious, if not skeptical, attitude toward the use of control theory based on this approach.

2/See, for example, Tobin's (1972) AEA presidential address.

At the same time that this dissatisfaction with traditional econometric models has been emerging, a number of economists including Christopher Sims, Thomas Sargent, and staff at the Federal Reserve Bank of Minneapolis have been developing alternative time series methods of forecasting economic variables.[3/] Not all economists would feel comfortable applying these models to the control framework, but a recent strong defense of a time series approach to policy analysis is given by Sims (1982). He argues that the normal business of policy formation is properly thought of as choice of shocks to the policy behavior equation, and he goes on to suggest the use of a vector autoregressive representation as the context in which this choice ought to be made.

We follow Sims advice here and construct a vector autoregression with M-1, the federal funds rate and other variables in order to represent the laws of motion of the money market. In constructing this representation, we have kept as a primary goal the desire to optimally forecast the movements of M-1. For this reason we have paid particular attention to a statistic measuring the out-of-sample forecasting performance of different models. We have also followed the Bayesian procedures suggested by Litterman (1981) for forecasting with vector autoregressions.

In searching through a variety of different variables, looking for those which help to predict weekly movements in seasonally adjusted M-1, the federal funds rate clearly stood out as the most important. This was followed at a considerable distance by the level of Commercial and Industrial Loans, the Standard and Poors Index of 500 stocks, Nonborrowed Reserves, Borrowed Reserves, and Total Reserves. The Business Week Index, a composite measure of

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[3/] Examples are in Sargent and Sims (1977), Anderson (1978), Sargent (1979), Sims (1980), and Litterman (1981).

real activity published by McGraw Hill, showed no explanatory power. Measures of stock market volume and the Discount rate did not help either. These results are based on experiments using systems with different sets of variables to forecast M-1. All systems were estimated using the same Bayesian prior, which is described in detail below. The results of some of these tests are given in Table 1 and displayed in Figures 1 and 2.

Table 1
Forecast Performance for M-1 and Federal Funds Rate

Included Variables	M-1 prediction error (Billions)	Funds Rate prediction error (Basis points)
(1 variable systems)		
M-1	1.1845	
Funds Rate		55.309
(2 variable system)		
M-1 and Funds Rate	1.1302	51.974
(3 variable systems M-1, funds rate, and:)		
C & I Loans	1.1129	52.079
S&P Index	1.1149	51.728
Borrowed Res.	1.1230	52.567
Nonborrowed Res.	1.1247	52.144
Total Reserves	1.1262	51.619
Business Wk Index	1.1354	52.208
Discount Rate	1.1388	50.023
NYSE Volume	1.1414	51.312

The prediction error in Table 1 is an out-of-sample statistic. It is based on residuals calculated by dropping, one at a time, each observation from the sample and using the estimator so obtained to generate the residual for that observation. The out-of-sample statistic is designed to distinguish variables which improve the fit only in-sample, and those which actually explain out-of-sample movements. The data are weekly observations from 1976:1 through 1982:12.

For the purpose of short run monetary control, the important aspect

Figure 1

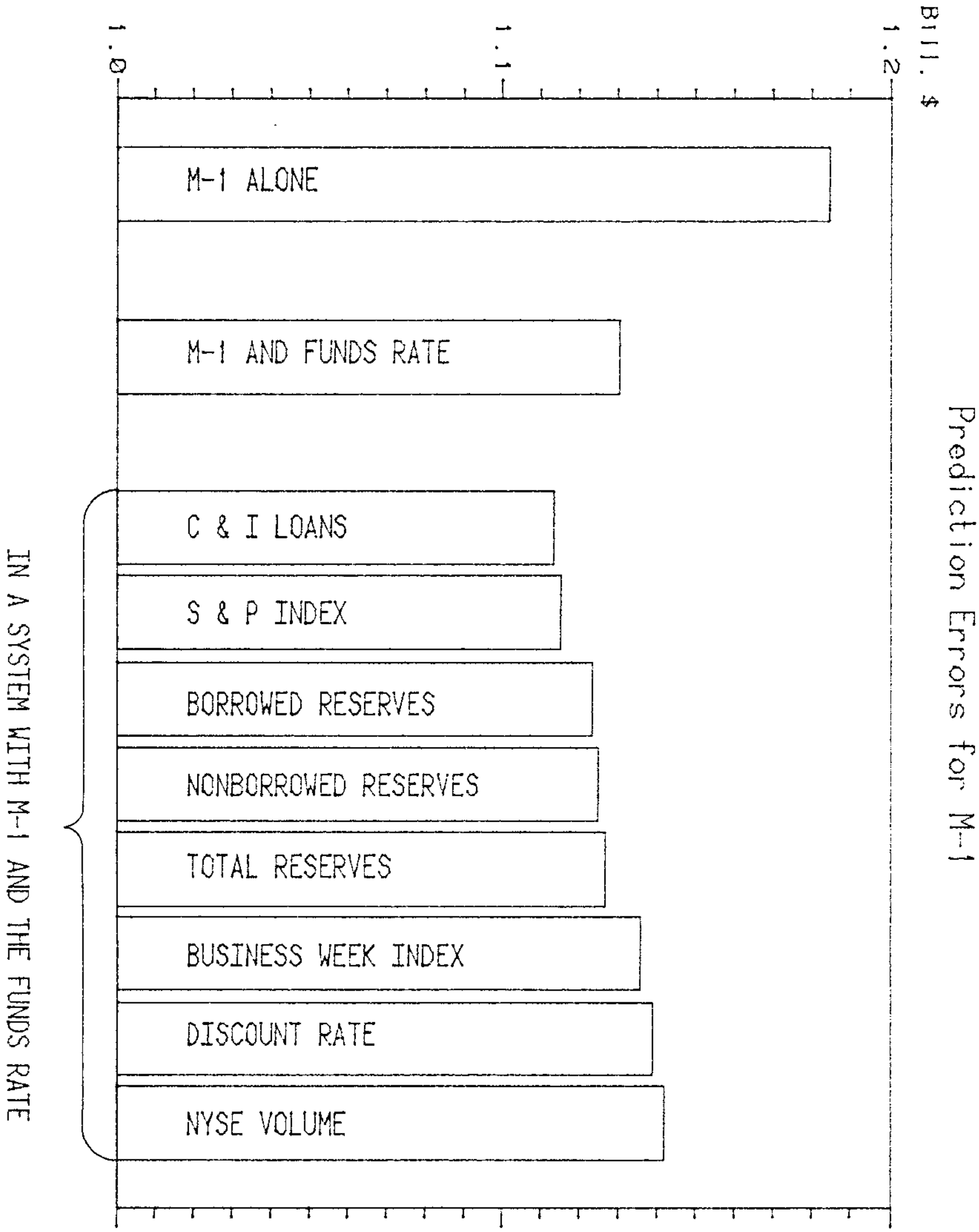
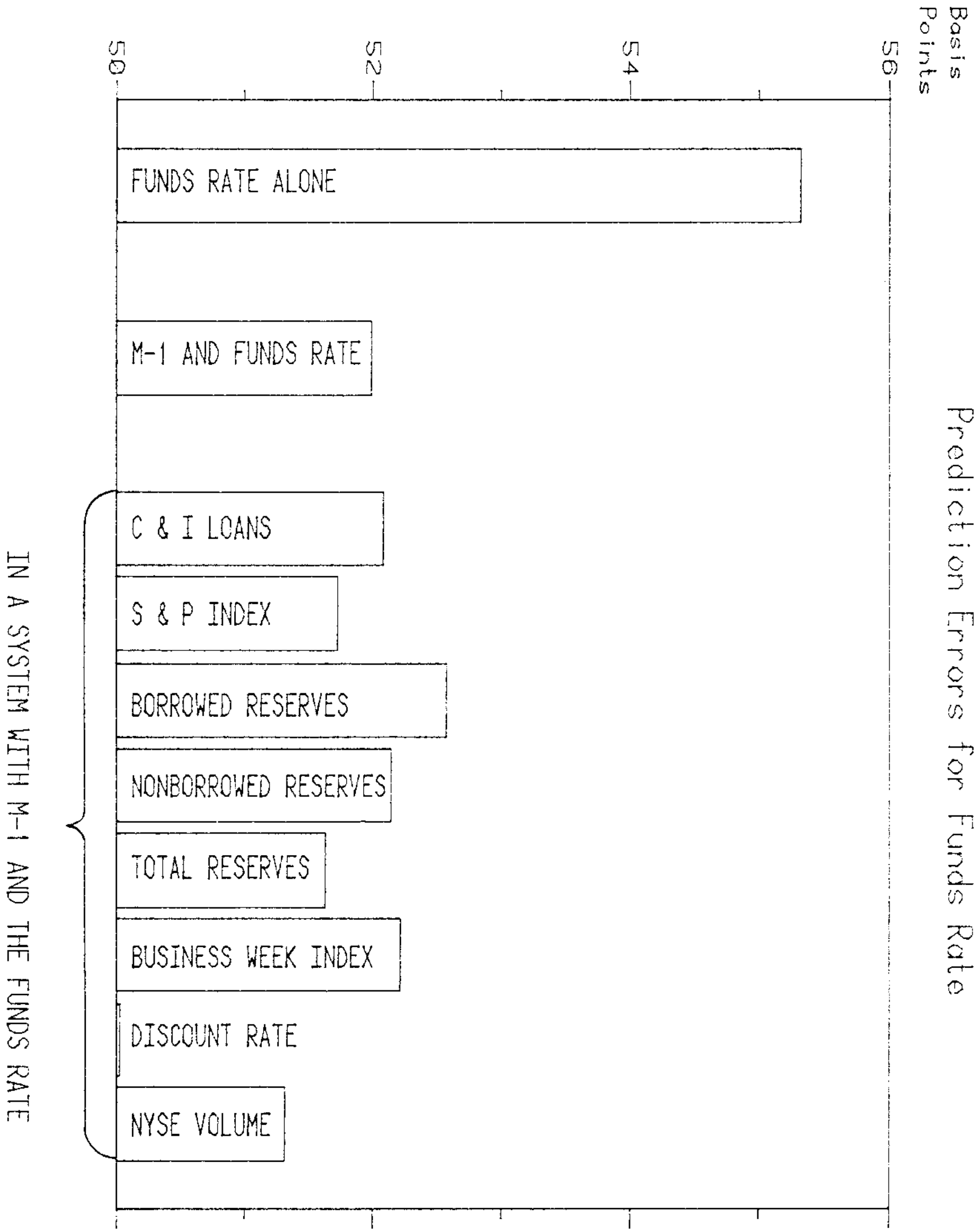

Prediction Errors for M-1
Bill. $
1.0
1.1
1.2
M-1 ALONE
M-1 AND FUNDS RATE
C & I LOANS
S & P INDEX
BORROWED RESERVES
NONBORROWED RESERVES
TOTAL RESERVES
BUSINESS WEEK INDEX
DISCOUNT RATE
NYSE VOLUME
IN A SYSTEM WITH M-1 AND THE FUNDS RATE

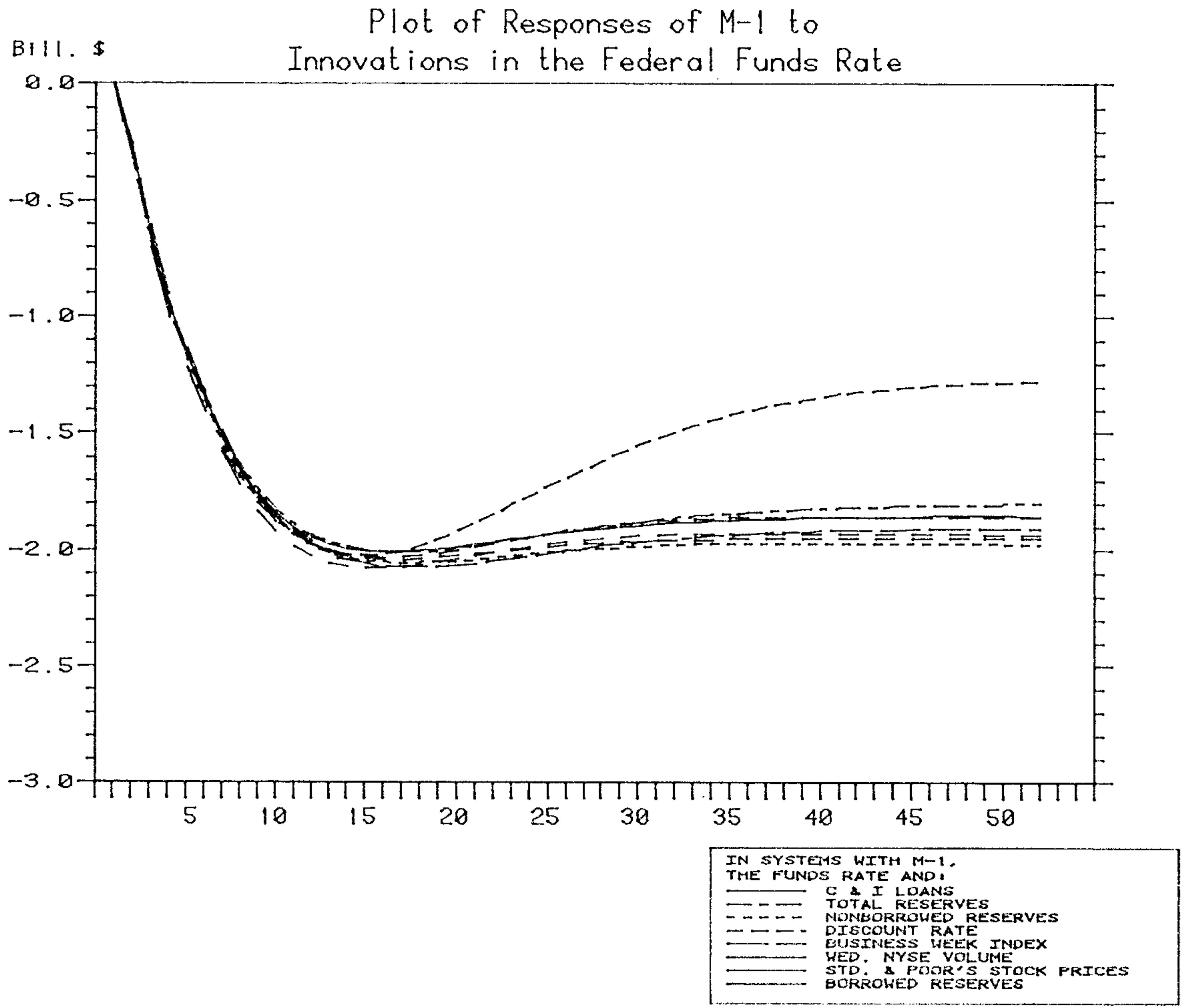
Figure 2


Prediction Errors for Funds Rate
Basis Points
50
52
54
56
FUNDS RATE ALONE
M-1 AND FUNDS RATE
C & I LOANS
S & P INDEX
BORROWED RESERVES
NONBORROWED RESERVES
TOTAL RESERVES
BUSINESS WEEK INDEX
DISCOUNT RATE
NYSE VOLUME
IN A SYSTEM WITH M-1 AND THE FUNDS RATE

of the estimated time series model is the response function of M-1 to shocks in the federal funds rate. It is this response upon which policy is based. Fortunately for our purposes, this response is relatively strong, stable across time periods, and insensitive to different specifications of the vector autoregressive representation. The response function we will use in computing an optimal control policy is shown below in Figures 3, 4, 5, as estimated under a variety of alternative specifications. Notice how little variation there is in the shape of this response in the different systems. Also notice how long lived is the response of money to the funds rate shock. Significant decreases in the money stock continue to occur two months after the initial shock.

Not only did the federal funds rate significantly improve the prediction of M-1, but it also explains a dramatically larger share of the variation of M-1 than any of the other variables considered. In the sets of three variable systems in Table 1, the percentage of the one-year-ahead forecast variance explained by innovations in the funds rate varied between 49 and 73 percent. The largest share received by any of the other variables considered was 1 percent, and in several cases it was less than 1 percent. In Figure 6 we show the response functions of money in a five variable system which adds commercial and industrial loans, borrowed reserves and nonborrowed reserves to M-1 and the funds rate. Notice how much larger is the response of money to the funds rate than to any of the other variables. Based on these results we have proceeded with a bivariate autoregression using only M-1 and the federal funds rate. All of the subsequent analysis could be generalized to include other variables in the state vector, but the results would probably not be materially affected.

The use of a time series representation as the basis for the



Plot of Responses of M-1 to
Innovations in the Federal Funds Rate
Bill. $
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
5
10
15
20
25
30
35
40
45
50
IN SYSTEMS WITH M-1,
THE FUNDS RATE AND:
C & I LOANS
TOTAL RESERVES
NONBORROWED RESERVES
DISCOUNT RATE
BUSINESS WEEK INDEX
WED. NYSE VOLUME
STD. & POOR'S STOCK PRICES
BORROWED RESERVES

Figure 3

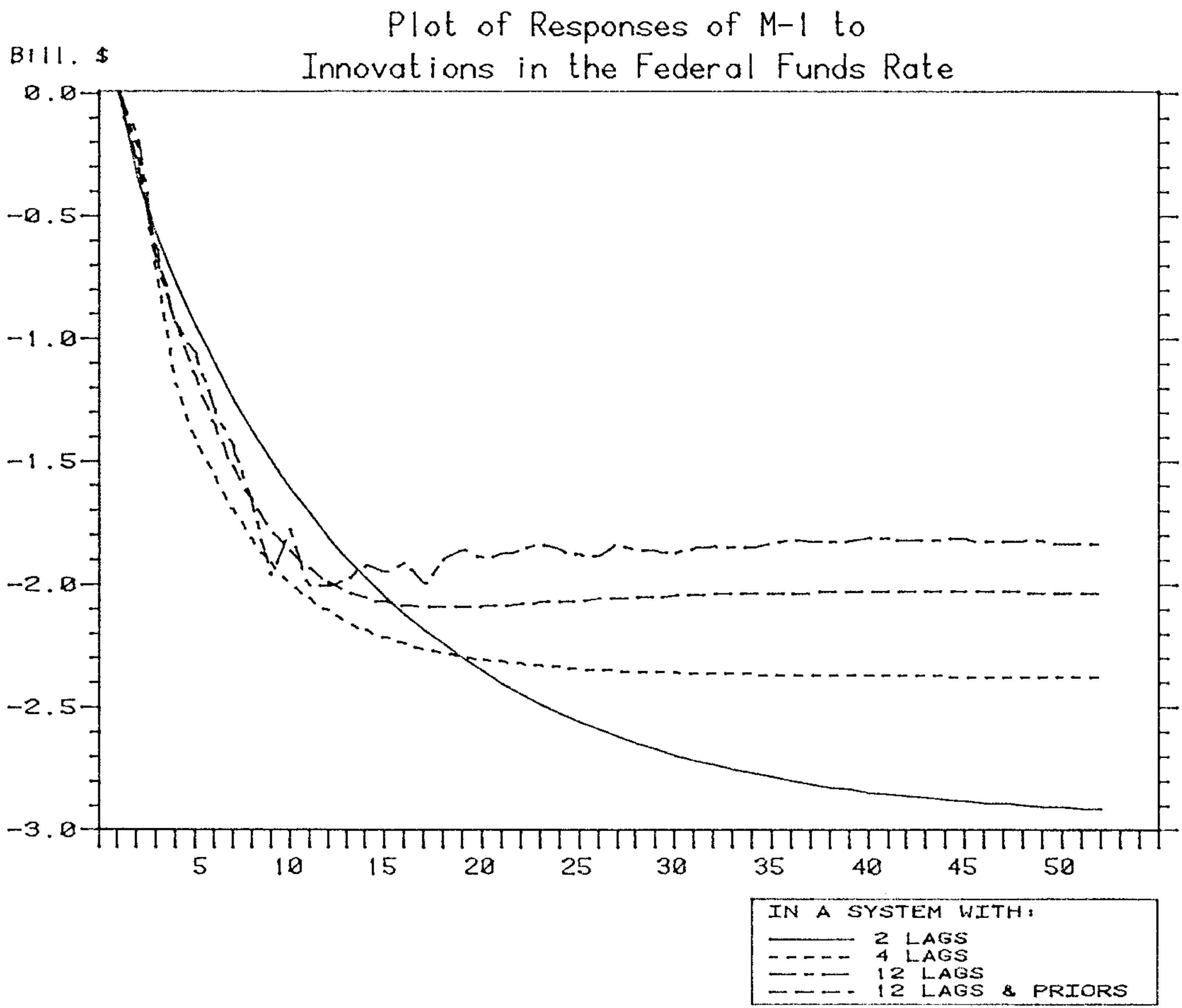

Plot of Responses of M-1 to
Innovations in the Federal Funds Rate
Bill. $
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
5
10
15
20
25
30
35
40
45
50
IN A SYSTEM WITH:
2 LAGS
4 LAGS
12 LAGS
12 LAGS & PRIORS

Figure 4

Plot of Responses of M-1 to
Innovations in the Federal Funds Rate

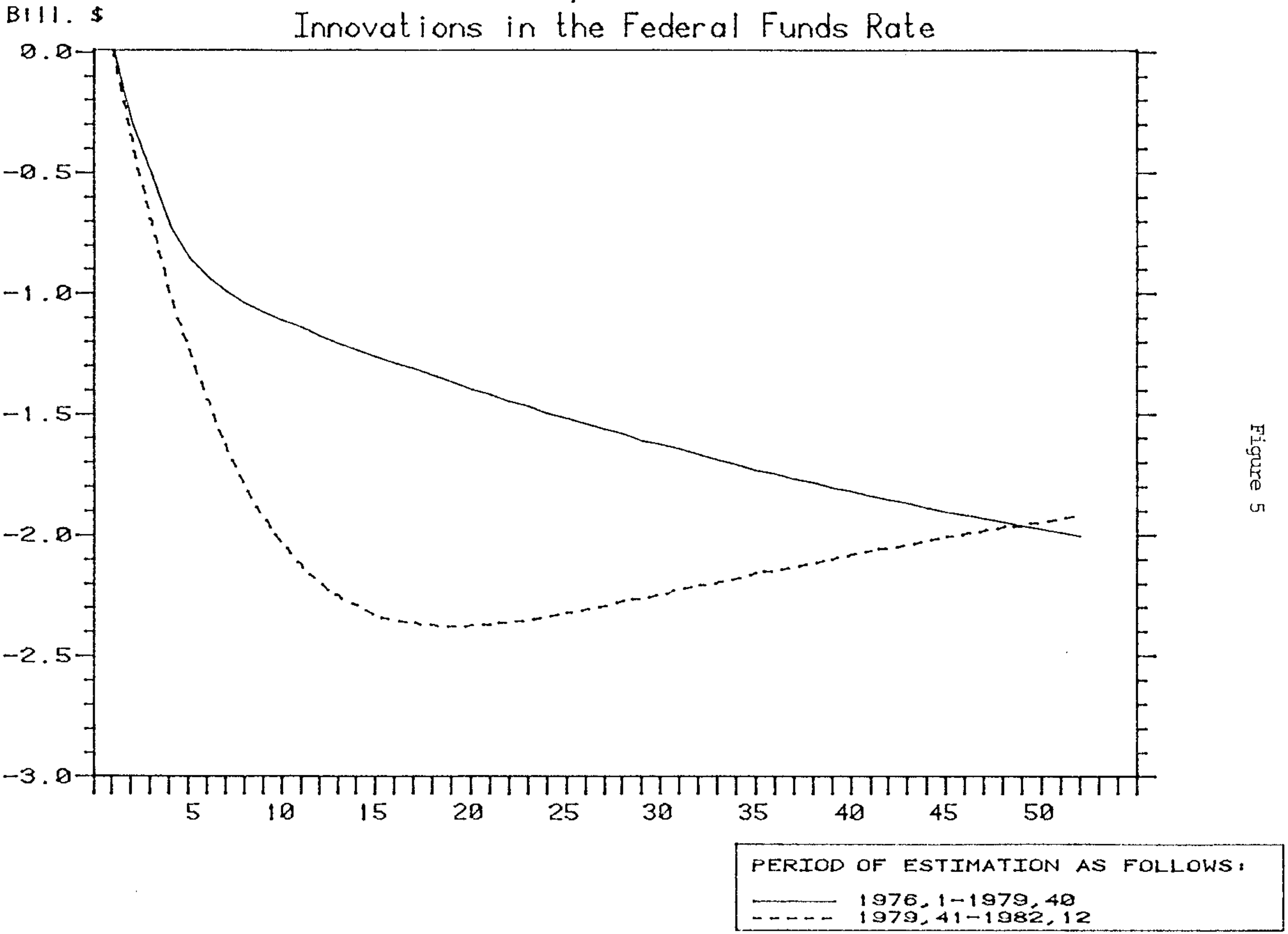


Figure 5

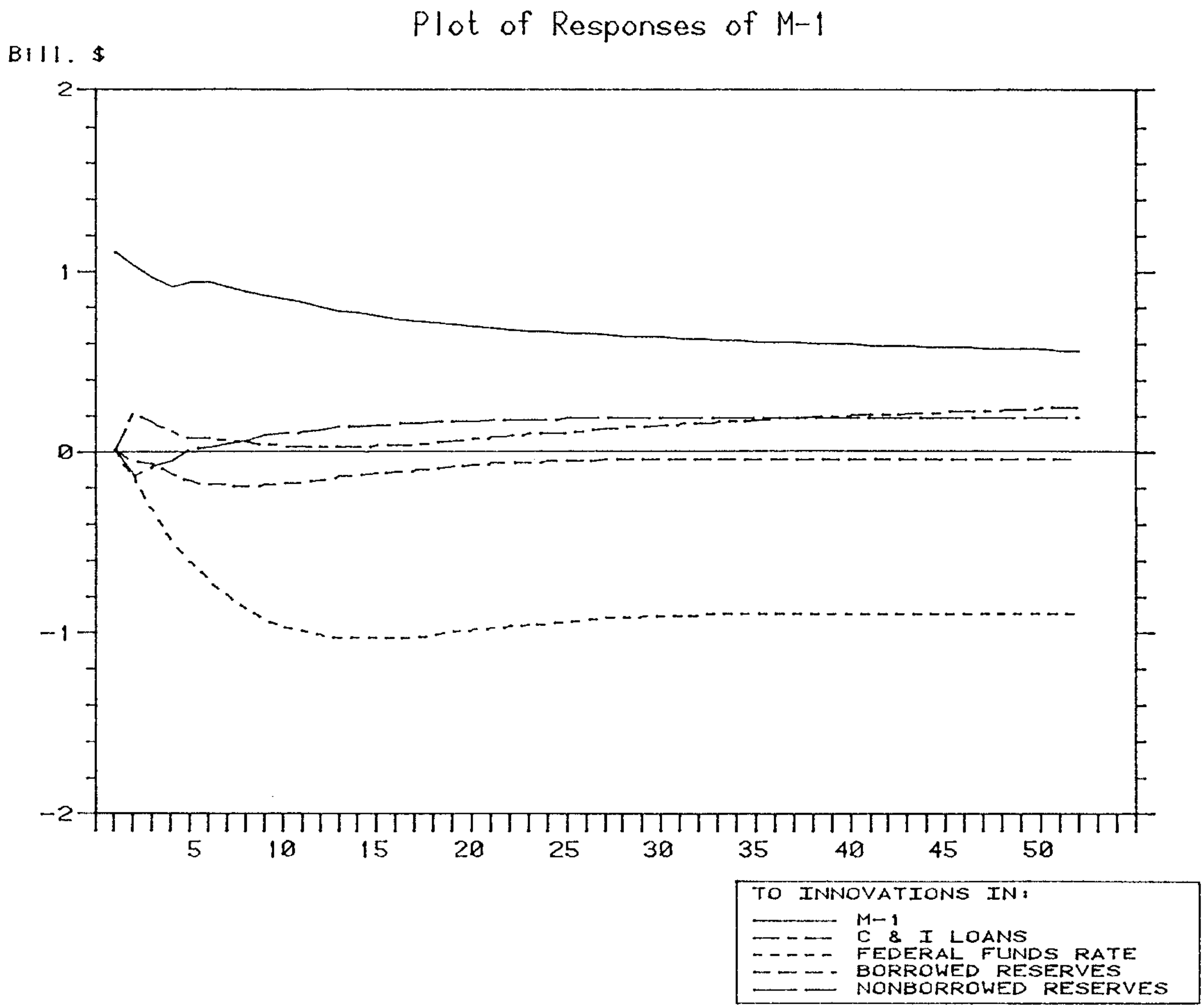

Plot of Responses of M-1
Bill. $
2
1
0
-1
-2
5
10
15
20
25
30
35
40
45
50
TO INNOVATIONS IN:
M-1
C & I LOANS
FEDERAL FUNDS RATE
BORROWED RESERVES
NONBORROWED RESERVES

Figure 6

dynamcial structure of a control exercise is a departure of this investigation from the standard econometric approach. Estimation of a structural model was rejected here because it would have greatly increased the cost and complexity of the exercise, and it probably would not lead to improved estimates. In fact, as stressed by Sims (1980), the usual identifying restrictions are likely to be false, and their application would probably lead to misspecification and therefore bias in the estimation of the crucial response function. Given the strength of the evidence in the data, as seen in the lack of sensitivity to alternative specifications, the results from using a reasonable structural model would presumably be similar to those obtained here. However, the risks of biasing results from imposing false restrictions and inappropriate specification of dynamic structures appear to far outweigh the expected benefit from a possible reduction in the variance of the estimates. Even if it would not improve the estimates, one might prefer a structural model if it would be more likely to remain valid in the face of interventions. Unfortunately, construction of such an invariant structural model is likely to be a difficult task. Moreover, the degree of inadequacy of the time series representation is not obvious. This issue is addressed at length in the final section of this paper.

4. IMPROVING SHORT-RUN CONTROL OF THE MONEY SUPPLY

Current Fed operating procedures do not apply optimal control techniques, even though the Fed appears to be trying to solve a problem of the type which optimal control theory is designed to handle. Therefore, the solution obtained by current procedures is likely to be suboptimal. It is possible, in fact, to estimate the tradeoff frontier which measures the obtainable combinations of interest rate volatility and expected deviations from monetary targets, and therefore, to measure the degree to which a change to an optimal control policy would likely improve operating characteristics. The tradeoffs which emerge suggest that the Federal Reserve could achieve a considerable smoothing of interest rates with little or no loss in terms of money supply control. There does not, however, appear to be much room for reducing the average size of money deviations from target. Moreover, such reductions would require large fluctuations in interest rates.

In order to discuss these tradeoffs, it is first necessary to motivate the model of short run monetary control suggested here. There are obvious differences between the earlier discussion of this model, in which the funds rate is the control, and the usual discussion of current Fed operating procedures, which stress reserve targets. Those differences, however, may be more apparent than real. Under current Fed policy there is an implicit role for the funds rate, and that role is the same as the one which it plays in the optimal control procedure. The main differences between current policy and the one suggested here is not the role of the funds rate, but rather that under current procedures the Fed does not minimize a loss function and does not optimally take into account the important lags in the response function of M-1 to shocks in the funds rate. Evidence of this behavior, and the suboptimal control it implies, is given below.

This interpretation of current Fed policy is based on the descriptions of operating procedures published in recent issues of the Federal Reserve Bank of New York (FRBNY) Quarterly Review, and the February 1981 Board of Governors Staff Study, "New Monetary Control Procedures." The following succinct summary by Richard G. Davis appeared in the Summer 1979 review in an article, "Broad Credit Measures as Targets for Monetary Policy."

> Fundamentally, there are two basic tactical approaches the Federal Reserve can use to attempt to control the behavior of the money supply or any other financial variable. One of these would be to attempt to project the path of bank reserves (or the monetary base) that seems most likely to be associated with the desired path of the aggregate. The success of this approach depends, in turn, on the stability and predictability of the 'multiplier' relationship between reserves and the aggregate in question. Even in the case of monetary definitions involving only currency and commercial bank deposits, there are significant problems with regard to the stability and predictability of the relevant multipliers. An alternative tactical approach open to the Federal Reserve in seeking to control the behavior of financial aggregates involves attempting to estimated the volume of the aggregate the public will want to hold under given conditions of aggregate demand and interest rates, then seeking to influence short-term money market rates accordingly. This approach also poses very real problems even in the case of a monetary aggregate because of difficulties in estimating what the public's demand for money will be under given conditions.

Shortly after this was written the Fed announced that it would change operating procedures from the second alternative, the funds targeting approach, to the first alternative, the reserves targeting approach. There are certainly important differences in these two approaches, but in one important respect they are similar. The similarity is that in both cases the control variable which directly affects the money supply is the federal funds rate. That this is true is not always obvious from Federal Reserve System descriptions, for instance, the one above. However, careful reading of the following passage from a staff report published as, "Monetary Policy and Open Market Operations in 1980," in the Summer 1981 FRBNY Quarterly Review makes this clear.

> As the Desk worked to achieve the average nonborrowed reserve path, borrowing at the discount window and money market rates tended to adjust whenever money growth deviated from the Committee's short-term aggregate objectives. When money growth was above these objectives, for example, as in the autumn of 1980, banks demand for total reserves exceeded the nonborrowed reserve path by more than the initial borrowing assumption. Hence, with the Desk supplying nonborrowed reserves in line with the path, interest rates tended to move higher as banks were forced to seek greater access to the discount window to meet their reserve requirements. These resulting changes in money market rates under the reserve approach, in turn, worked to encourage banks and the public to make the portfolio changes needed to return money growth in time back in line with the Committee's objectives.
>
> On occasion, as seemed appropriate, the nonborrowed reserve path was modified relative to the total reserve path in order to accelerate the adjustment process. These changes were intended to encourage an even sharper response in borrowing, and hence in reserve availability and interest rates, to monetary deviations so that the pressures for restoring money growth in line with the Committee's objectives were intensified.

Notice that the logic of the following description of the casual chain between the Fed's nonborrowed reserve target and the money supply gives a crucial role to the funds rate. The funds rate is never mentioned, but at a given discount rate, the funds rate is closely tied to the level of borrowings. The description is from another FRBNY Quarterly Review article, "The Monetary Base as an Intermediate Target for Monetary Policy," by Richard G. Davis in the Winter 1979/80 issue.

> In the short-run context, a critical point is that member bank excess reserves tend to average close to frictional minima over a period of weeks and to show little systematic sensitivity to interest rate movements. Consequently, movements in the total reserve component of the base tend largely to mirror movements in required reserves. And in the short period of a few weeks between FOMC meetings, required reserve movements tend to be only marginally responsive to the volume of nonborrowed reserves supplied. The volume of reserves supplied through open market operations, in the short run, mainly affects the extent to which member banks are forced to meet their reserve requirements through borrowings at the discount window. The effect on total reserves, nonborrowed plus borrowings, and on the total monetary base appears to be quite small over these short periods.

Clearly, then, whether the focus is directly on the funds rate, or on reserve targets, the fundamental link between the open market operations and their affect on the money supply is through their affect on the funds rate.

Moreover, the Open Market Committee and the Desk recognize that there is a fundamental tradeoff between the rapidity of reduction of short-run deviations in money and volatility of interest rates. Quoting again from the report in the Summer 1981 FRBNY Quarterly Review:

> [The Committee] tried to take into account lags in the effects of changes in financial market conditions on money growth. A more aggressive approach to setting short-term monetary targets--say, one that attempted close month-to-month control--risked the possibility of whipsawing the markets and ultimately destablizing money growth and interest rates over a longer period.

A recent staff study, Tinsley, von zur Muehlen, Trepeta, and Fries (1981), addressed the question of whether there exists

> "...a well-behaved tradeoff between the volatility of deviations of M-1A from long-run targets and volatility of short-term interest rates under current and alternative operating procedures that may be exploited by short-run monetary policy?"

The Tinsley, et al., study involved simulations of the Board's monthly money market model. Although their conclusions are similar to those reached here, their approach differs in that they did not adopt an explicit control theoretic framework, nor did they try to model the week-to-week dynamics of the money market.

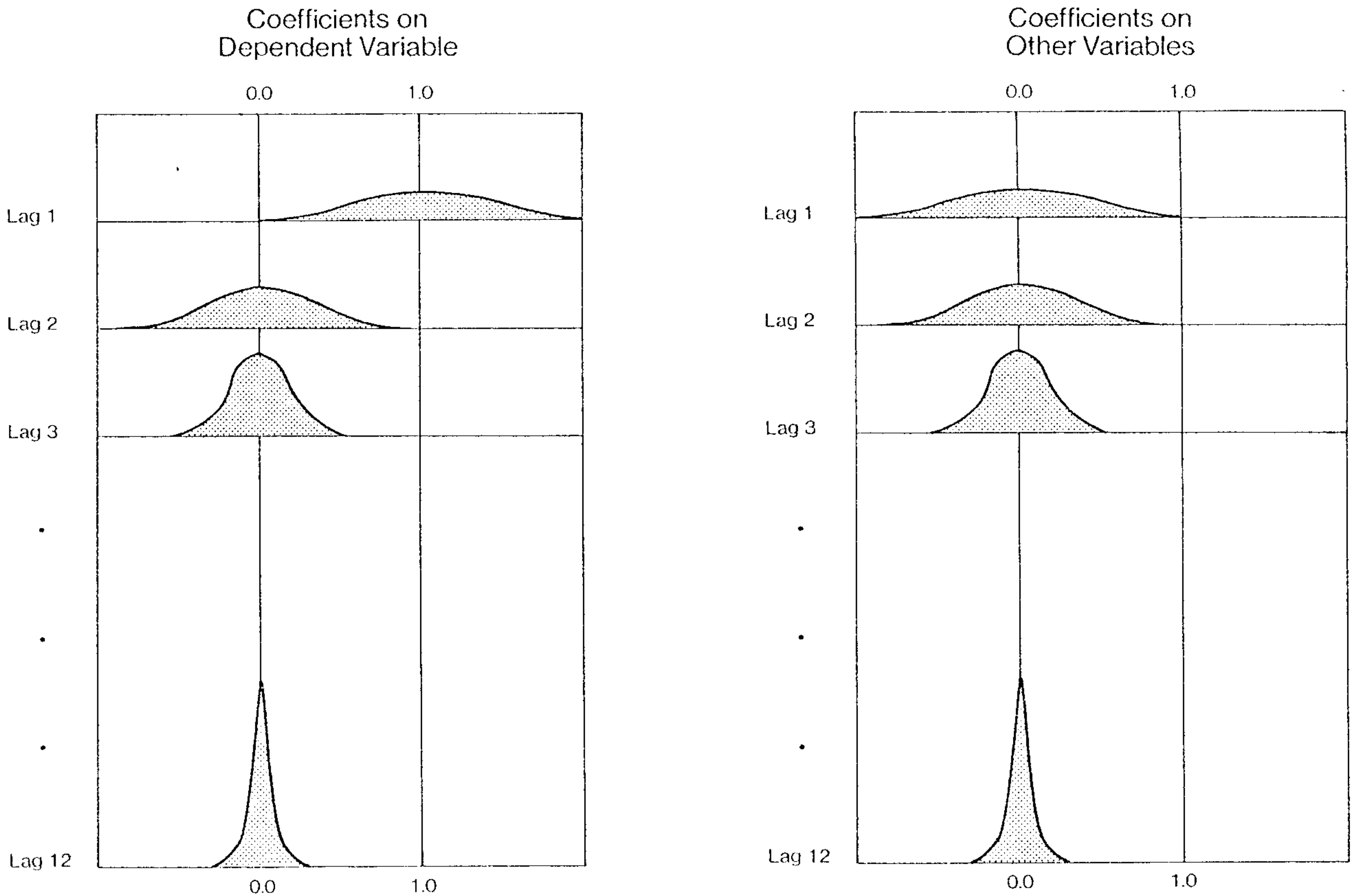
The optimal control approach to monetary control outlined above is an attempt to formalize the Fed's operating procedures and the implicit loss function which trades off short-run control for interest rate smoothness. Applying time series techniques to the estimation of the laws of motion of the M-1, federal funds process formalizes the Committee's attention to the lags inherent in the system. Because the Committee is, in effect, attempting to solve the same problem, but without the benefit of optimal control theory and time series analysis, its solution is likely to be suboptimal.

Although it may at first appear to be a funds rate targeting procedure, the control approach suggested here should not be thought of as either a funds rate or a reserves targeting procedure. As is stressed above, both of these operating procedures affect the level of the money supply through the funds rate. The feedback rule defined here is thus a necessary ingredient for either operating procedure to function optimally. As explained below, according to the model presented here the use of a funds targeting procedure is likely to reduce the losses incurred using an optimal control approach. Nevertheless, there are several reasons why the Fed might want to implement a control policy under a reserves targeting procedure, at least initially. First of all, if the Fed were obviously pegging the funds rate, it might become politically impossible for the Fed to set the rate at the levels determined to be optimal according to the control rule. Secondly, switching back to funds rate targeting procedure would be an obvious change in policy. The more smoothly a feedback rule such as this is implemented, the more likely it is that the money market will respond as it has in the past. This stability in response is a key issue which is discussed below. Another argument against switching back to a funds rate targeting approach is that it might send just the wrong signal concerning the Fed's intentions to control the money supply. Any possible signal that it has lost the ability or the desire to control the supply of money could raise inflation expectations, and consequently the level of interest rates. The operation of the feedback rule under a reserves targeting procedure would not be that much different from current procedures. Today, the FOMC picks target ranges for the funds rate and money growth rates, which the Board and the Desk translate into reserve path targets. Under an optimal control approach, the Board and the Desk could compute reserve targets on a week-by-week basis, consistent with the funds rate given by the feedback rule. As long as the Fed is willing to cause the federal funds rate to move as needed to control the money supply, the difference between a funds and a reserves targeting procedure is not sharp. A class of possible rules can be defined in terms of the frequency with which the reserves target is adjusted. The funds and reserves target procedures discussed here are two possible points in this class. In the limit as a reserves target is adjusted more and more often to reach a desired level for the funds rate, it becomes a funds targeting procedure.

The time series model that drives the analysis to follow is a bivariate autoregressive representation for M-1 and the funds rate. Twelve lags of each variable and a constant term are included in each equation. The model is estimated using weekly data from 1976:1 through 1982:12. The estimation procedure is Theil's (1971) mixed estimation procedure, applied equation by equation, that is, ordinary least squares with the data sets augmented to include a set of observations representing a Bayesian prior of the type described by Litterman (1981). A schematic representation of this prior is given below.

The estimation was carried out using the Regression Analysis of Time Series program of Doan and Litterman (1981). Using their notation, the prior is a Symmetric Random Walk with parameter 1. (each variable in each equation is treated symmetrically, the coefficient on the own first lag has a mean of 1., all other coefficients have a mean of 0.), and the lag decay is harmonic with parameter 2. (the prior for the coefficient on lag j is centered around 0. with a standard error $1/j^2$ times the standard error on the first lag). The overall tightness is .5 (the standard deviation of the prior distribution for the first lag of the dependent variable is .5) The prior standard deviations of other than the dependent variable in each equation are scaled by the

A Schematic Representation of the Prior



standard error of univariate equations in order to take account of the different units of the variables. The prior was chosen based on an informal search over the parameters of the prior for those which led to the best out-of-sample forecasts.

The coefficient estimates from this procedure can be viewed as an approximation of the posterior mean using this prior. These estimates are given in Table 2. It is not very enlightening to analyze the autoregressive representation directly, however, so we also present the moving average, or impulse response function, representation in Figure 7.

TABLE 2
Coefficient Estimates

EQUATION 1 M-1

OBSERVATIONS	318	DEGREES OF FREEDOM	317
R**2	0.99933269	RBAR**2	0.99933269
SSR	448.40716	SEE	1.1893417
DURBIN-WATSON	1.97448638		

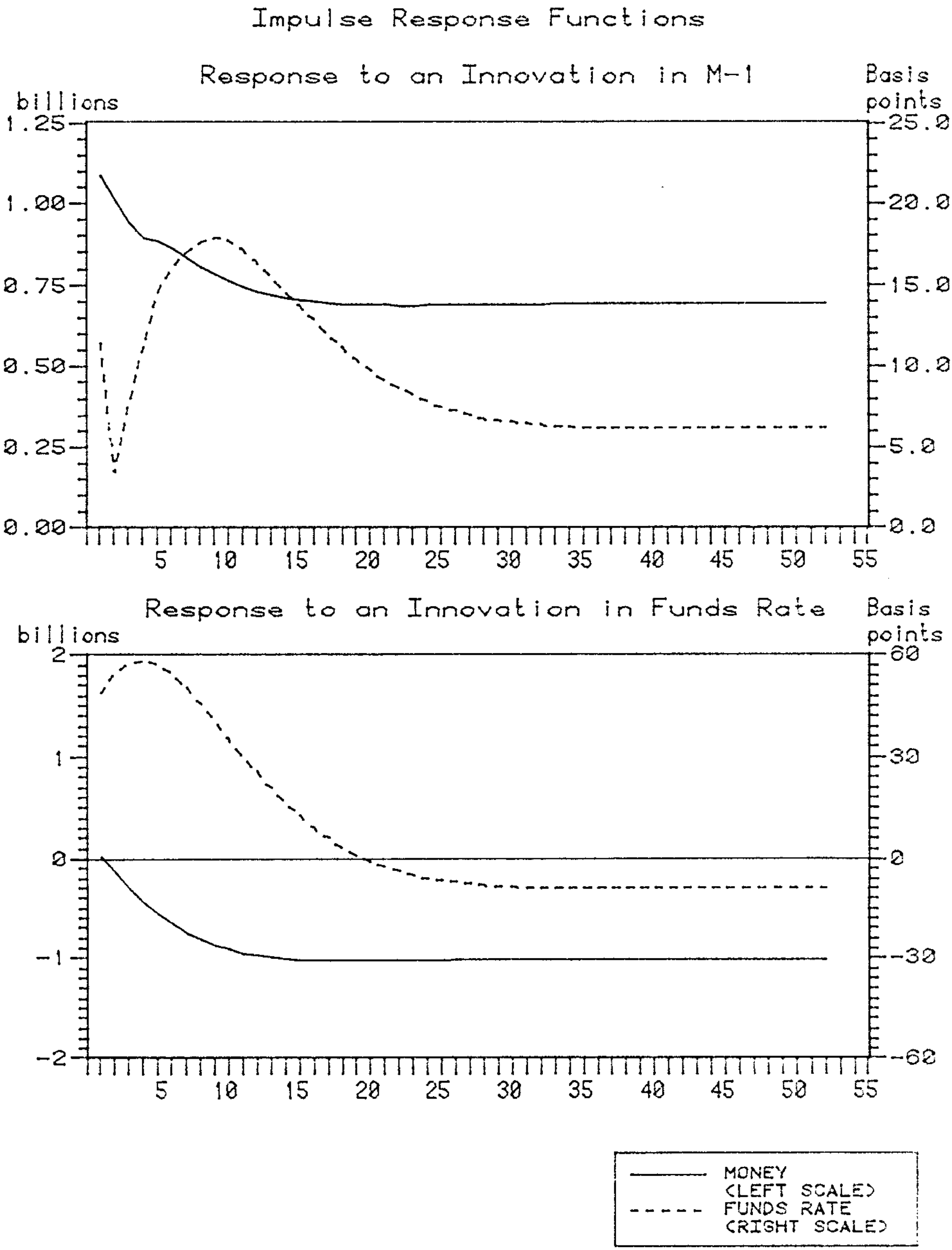
LABEL	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
*****	***	************	************	************	************
M-1	1	.9061776	.05330237	17.00070	.0000000
M-1	2	.0213272	.06180319	.34508	.7300314
M-1	3	.0096618	.04248938	.22739	.8201172
M-1	4	.0601479	.02784498	2.16009	.0076505
M-1	5	.0124172	.01914333	.64864	.5165686
M-1	6	-.0041191	.01375945	-.29937	.7646562
M-1	7	.0024200	.01027570	.23551	.8138094
M-1	8	.0039907	.00794580	.50225	.6154915
M-1	9	.0020050	.00632180	.31716	.7511174
M-1	10	-.0003031	.00514480	-.05892	.9530122
M-1	11	.0010058	.00426550	.23581	.8135741
M-1	12	.0010862	.00359225	.30239	.7623510
Funds Rt	1	-.2781385	.1170627	-2.37597	.0175024
Funds Rt	2	-.0838023	.1467635	-.57100	.5679978
Funds Rt	3	.0720601	.0946666	.76119	.4465383
Funds Rt	4	.0594890	.0605398	.98264	.3257835
Funds Rt	5	.0199801	.0409899	.48743	.6259468
Funds Rt	6	.0062840	.0293273	.21427	.8303347
Funds Rt	7	.0078717	.0219227	.35906	.7195457
Funds Rt	8	.0087101	.0169927	.51257	.6082456
Funds Rt	9	.0079570	.0135439	.58749	.5568700
Funds Rt	10	.0017977	.0110415	.16281	.8706652
Funds Rt	11	.0015909	.0091669	.17355	.8622129
Funds Rt	12	.0009123	.0077300	.11802	.9060515
Constant		-3.394371	1.040465	-3.26236	.0011048

EQUATION 2 Funds Rt

OBSERVATIONS	318	DEGREES OF FREEDOM	317
R**2	0.98782615	RBAR**2	0.98782615
SSR	78.194197	SEE	0.49665822
DURBIN-WATSON	1.93821134		

LABEL	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
*****	***	************	************	************	************
M-1	1	-.0852225	.022258	-3.828751	.0001287
M-1	2	.1214743	.025808	4.706767	.0000025
M-1	3	-.0125290	.017743	-.706130	.4801068
M-1	4	.0019431	.011627	.167115	.8672795
M-1	5	-.0093251	.007994	-1.166508	.2434091
M-1	6	-.0032406	.005745	-.563998	.5727552
M-1	7	-.0022331	.004291	-.520432	.6027623
M-1	8	-.0012125	.003318	-.365431	.7147893
M-1	9	-.0012527	.002639	-.474521	.6351284
M-1	10	-.0012026	.002148	-.559774	.5756330
M-1	11	-.0007926	.001781	-.445011	.6563117
M-1	12	-.0005638	.001500	-.375901	.7069903
Funds Rt	1	1.132931	.048884	23.17576	.0000000
Funds Rt	2	-.115426	.061287	-1.88337	.0596491
Funds Rt	3	-.034741	.039531	-.87881	.3795010
Funds Rt	4	-.013489	.025280	-.53357	.5936380
Funds Rt	5	-.013964	.017117	-.81579	.4146150
Funds Rt	6	-.592274	.012246	-.48361	.6286599
Funds Rt	7	-.006550	.009154	-.71554	.4742718
Funds Rt	8	-.003452	.007096	-.48650	.6266065
Funds Rt	9	-.001838	.005655	-.32502	.7451603
Funds Rt	10	-.001819	.004610	-.39453	.6931848
Funds Rt	11	-.001120	.003828	-.29275	.7697113
Funds Rt	12	-.000753	.003228	-.23337	.8154665
Constant		-1.469521	.434488	-3.38218	.0007191

Figure 7



Impulse Response Functions
Response to an Innovation in M-1
billions
Basis points
1.25
1.00
0.75
0.50
0.25
0.00
25.0
20.0
15.0
10.0
5.0
0.0
5
10
15
20
25
30
35
40
45
50
55
Response to an Innovation in Funds Rate
billions
Basis points
2
1
0
-1
-2
60
30
0
-30
-60
5
10
15
20
25
30
35
40
45
50
55
MONEY (LEFT SCALE)
FUNDS RATE (RIGHT SCALE)

The state vector for this exercise includes 12 lags of M-1, 12 lags of the federal funds rate, a constant, and a money target.

$$x(t) = [m(t), m(t-1), \ldots, m(t-12), r(t), r(t-1), \ldots, r(t-12), 1., M^{*}(t)]$$

The equation of motion is given by:

$$x(t) = A\ x(t-1) + B\ u(t) + w(t)$$

where $u(t) = F\ x(t-1)$

defines the control. The control, $u(t)$, is a scalar variable defined as a linear combination of the previous state vector by the feedback vector F. The vector F is generated by the solution of the matrix Riccati equation. B is a vector of zeros with a one as the 13th element, corresponding to the element $r(t)$ in the state vector. The vector $w(t)$ has zeros everywhere except in its first and 13th elements, which are white noise error terms with covariance matrix equal to the estimated covariance matrix of the residuals from the post-October 1979 data. This covariance matrix is as follows:

Table 3
Covariance Matrix of Innovations

	M-1	Funds Rt
M-1	1.4101	.1298
Funds Rt	.1298	.2459
Correlation	.2205	

The matrix A is given below:

a(1,1)	a(1,2)	...	a(1,12)	a(1,13)	...	1(1,24)	a(1,25)	0.
1.	0.	...	0.	0.	...	0.	0.	0.
0.	1.	...	0.	0.	...	0.	0.	0.
.	.		.	.		.	.	.
.	.		.	.		.	.	.
0.	0.	...	0.	0.		0.	0.	0.
a(2,1)	a(2,2)	...	a(2,12)	a(2,13)	...	a(2,24)	a(2,25)	0.
0.	0.	...	0.	1.	...	0.	0.	0.
0.	0.	...	0.	0.	...	0.	0.	0.
.	.		.	.		.	.	.
.	.		.	.		.	.	.
0.	0.	...	0.	0.	...	0.	0.	0.
0.	0.	...	0.	0.	...	0.	1.	0.
0.	0.	...	0.	0.	...	0.	0.	g

where g is the targeted growth rate of money (on a week to week basis). The a(i,j) are the coefficients from the time series model described above which determines m(t) and r(t) as a function of the lagged state.

This control model corresponds to a world in which the Fed at the beginning of the week picks a shock, u(t), which it does not modify as the week progresses. It is designed to model a reserves targeting procedure in which the level of nonborrowed reserves to supply during the week is chosen so as to cause an optimal movement in the funds rate. Because there are unforeseen shocks during the week, given by w(t), the funds rate has a stochastic element which is not under the Fed's control.

In order to model a funds rate targeting procedure, the state vector is augmented to include the next disturbance to the funds rate equation. The A matrix is augumented by a column which is zeros except for a one in the 13th row, corresponding to the funds rate equation. This inclusion is a device which allows the feedback rule to respond to the disturbance during the week in which it occurs. Responding to the disturbance is a way to model an operating procedure in which the funds rate is targeted each week and reserves are supplied or demanded by the Fed as necessary to keep the rate within a narrow band. In this approach the only difference between a reserves targeting procedure and a funds rate targeting procedure is that under the funds rate procedure the Fed can respond to the disturbance, whereas under the reserve procedure it cannot. Thus, using this approach implies that there will always be more noise under a reserves targeting procedure.

To this point, no mention has been made of the fact that the money supply is not observed contemporaneously with the funds rate. For the purpose of optimal control, there is an important separation of the problem of setting a control from the problem of observing the current state. For a statement of this result, see Bertsekas (1976). The implication of this result is that when one or more of the most recent observations of money are not available, the optimal strategy is to form the best linear prediction of these values of money, and then to proceed as if they had been observed. In practice, depending on the day of the week, the lag between the observation of the funds rate and M-1 varies between 7 and 12 business days. We model this as a two-week lag in the weekly data. Thus, we proceed in two steps. First we form the optimal linear forecast of the most recent two weeks of money data, then we proceed as above. The forecasting exercise is conditional on the two advanced observations on the funds rate. The optimal linear forecasting procedure in this case is described in Example 13.5 of Doan and Litterman (1981). The astute reader will have realized that the conditional forecast depends on the reduced form, which is a function of the feedback control rule; but the feedback control rule itself is a function of the conditional forecast. Thus, using this formulation with two unobserved values of money in the state vector, the problem of finding the optimal control rule requires a simultaneous solution with the problem of generating a conditional forecast. Actually, the problem is not all that serious. The method described below has worked quite well with a minimum additional computing expense.

The solution procedure is a simple iteration. The reduced form in the first step is derived by solving the matrix Riccati equation for a feedback control vector, F, and plugging it into the state equation.

$$x(t) = (A-BF) \quad x(t-1) + w(t) \tag{4}$$

The conditional forecast of $x(t-1)$ given a subvector of $x(t-1)$ can be written as

$$\hat{x}(t-1) = G\ x(t-1) \tag{5}$$

where G is a matrix which has zeros in the columns corresponding to the unobserved components of $x(t-1)$. Given G, the new reduced form, that is the reduced form for step two is

$$x(t) = (A-BFG)\ x(t-1) + w(t) \tag{6}$$

This reduced form implies a new G, and so on. Note that each iteration adds two lags to the state vector. Thus, in principle, the reduced form has an infinite autoregressive representation. In practice, within the relevant range of λ's iterating between these two equations quickly leads to convergence of G and the reduced form transition matrix, (A-BFG). Notice that this iterative procedure does not require repeated solution of the matrix Riccati equation, which determines F. We illustrate the reduced form response of money and the funds rate to an innovation in money in Figures 8 through 13 for funds rate and reserve targeting procedures and several values of λ and q. The response of the funds rate could be viewed as a Fed reaction function under an optimal control approach.

Several points should be noticed with respect to these graphs. First, there is a two week lag in the response of interest rates to the money innovation. This is due to the delay in the observation of the money innovation. Because of the differences in the definitions of the loss functions, the values of λ are not comparable between loss functions with different values of q. For q=12 and the funds targeting procedure graphs representing several values of λ are displayed. As λ gets larger more weight is given to smoothing interest rates; this causes a smaller interest rate response and a longer delay in returning money to the target path. The main difference between responses when q=1 and q=12 is that in the latter case responses are more of a discrete nature. When longer run smoothing is desired, the response to a given shock is to move the funds rate to a new level at which it is expected to stay, rather than the more gradual increase which is generated when only week-to-week changes are penalized.

Responses to an Innovation in Money
Federal Funds Targeting Procedure; q=12; lambda=.5

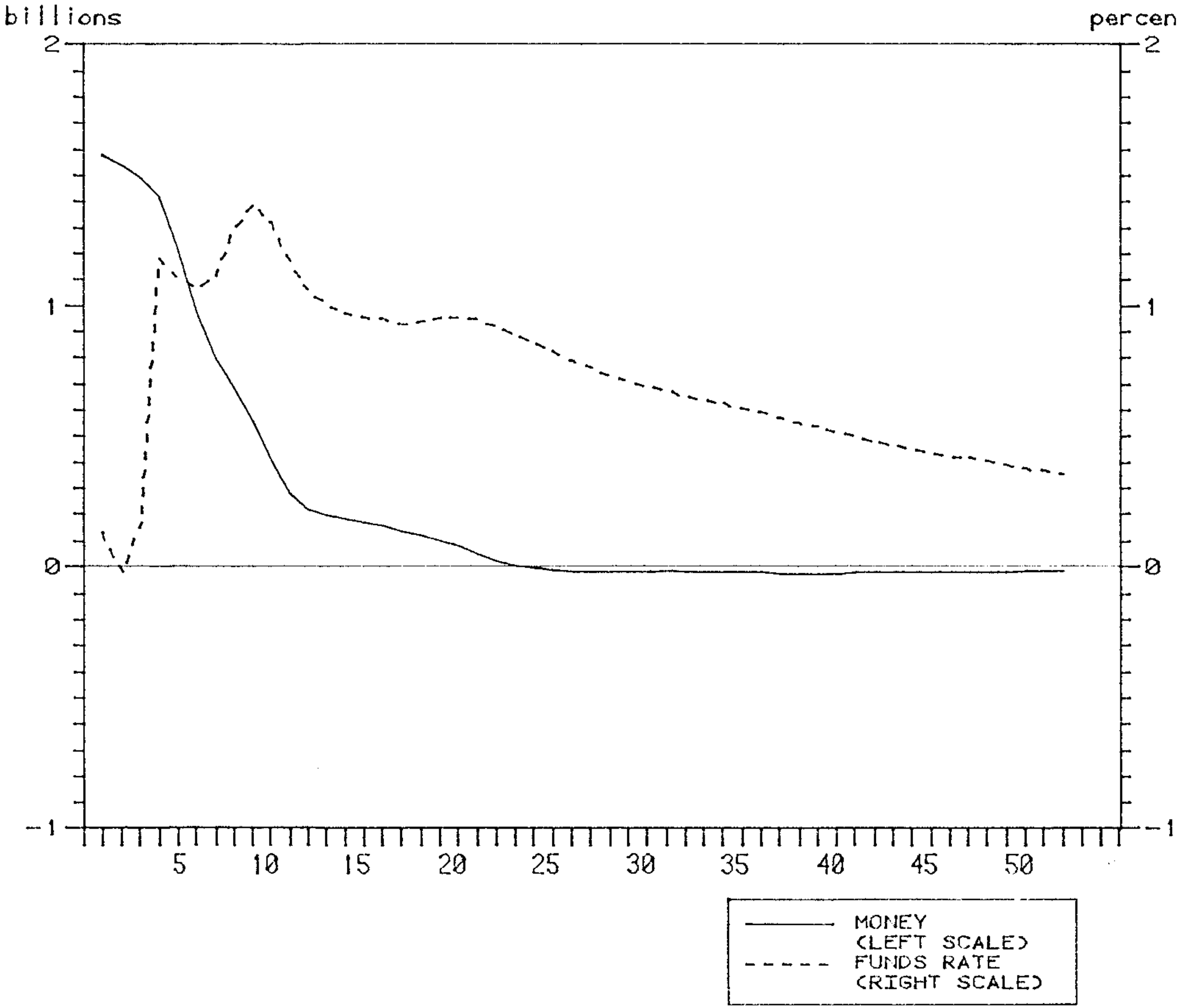


Figure 8

Responses to an Innovation in Money
Federal Funds Targeting Procedure; q=12; lambda=2

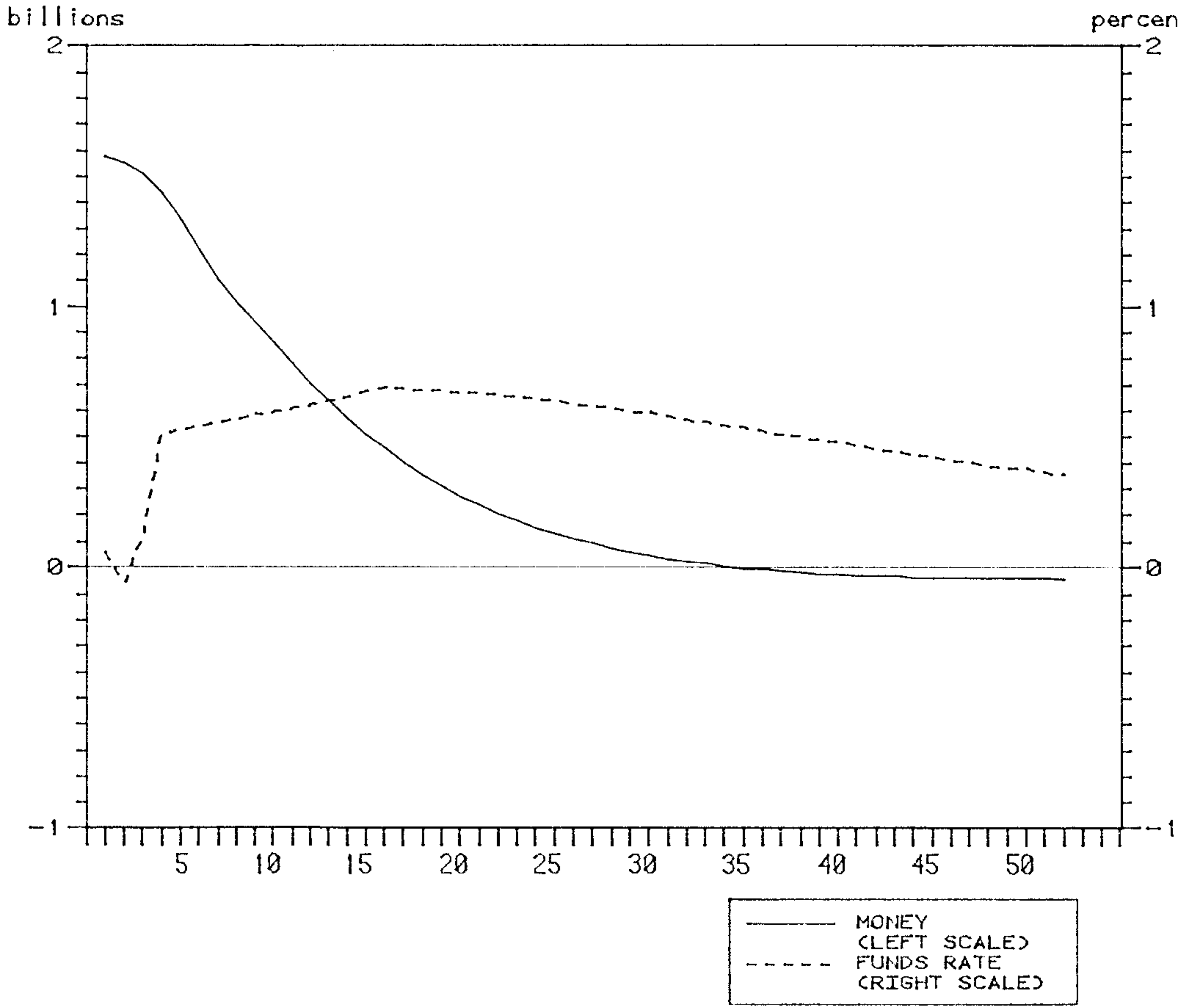


Figure 9

Responses to an Innovation in Money
Federal Funds Targeting Procedure; q=12; lambda=8

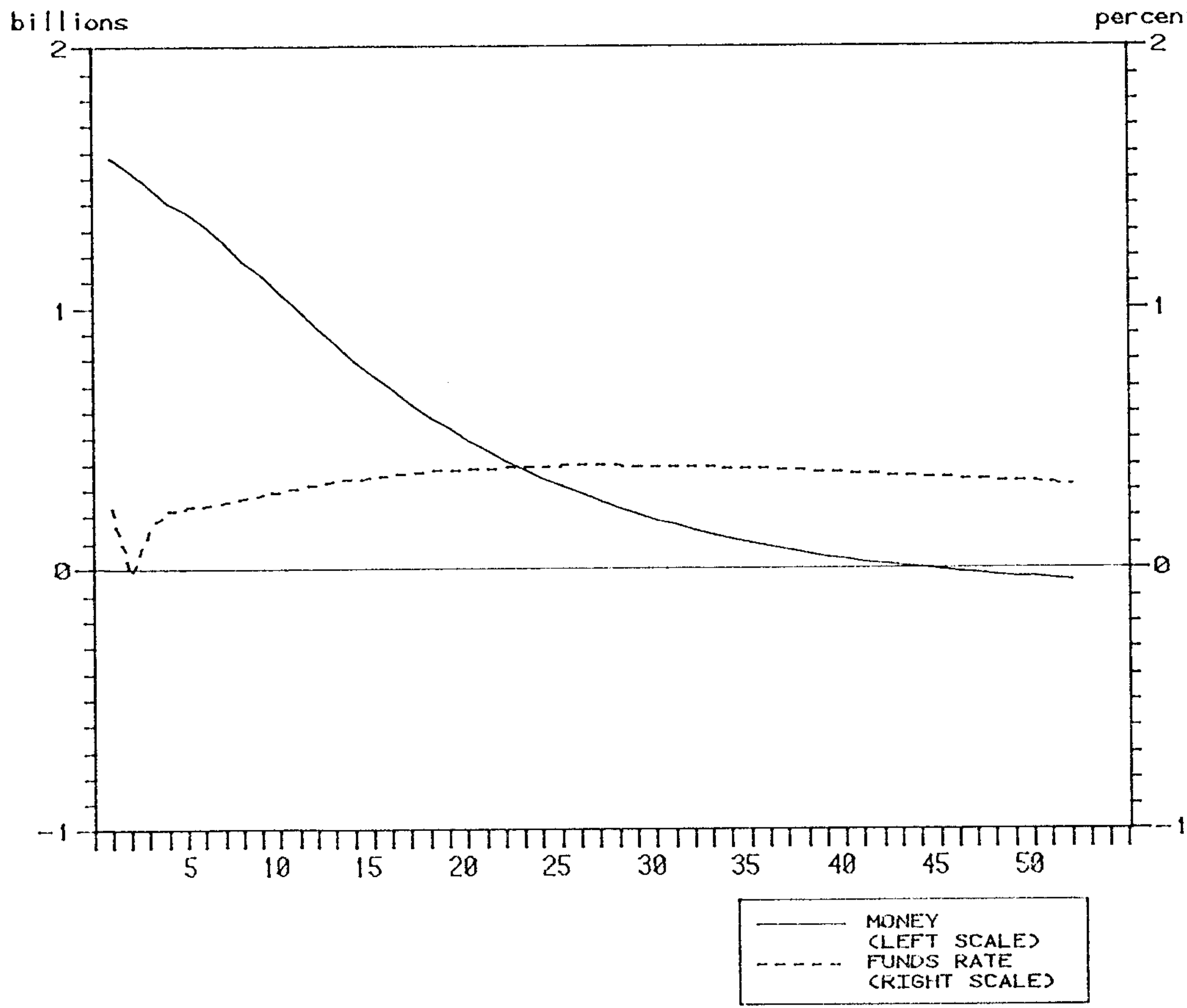


Figure 10

Responses to an Innovation in Money
Federal Funds Targeting Procedure; q=1; lambda=20

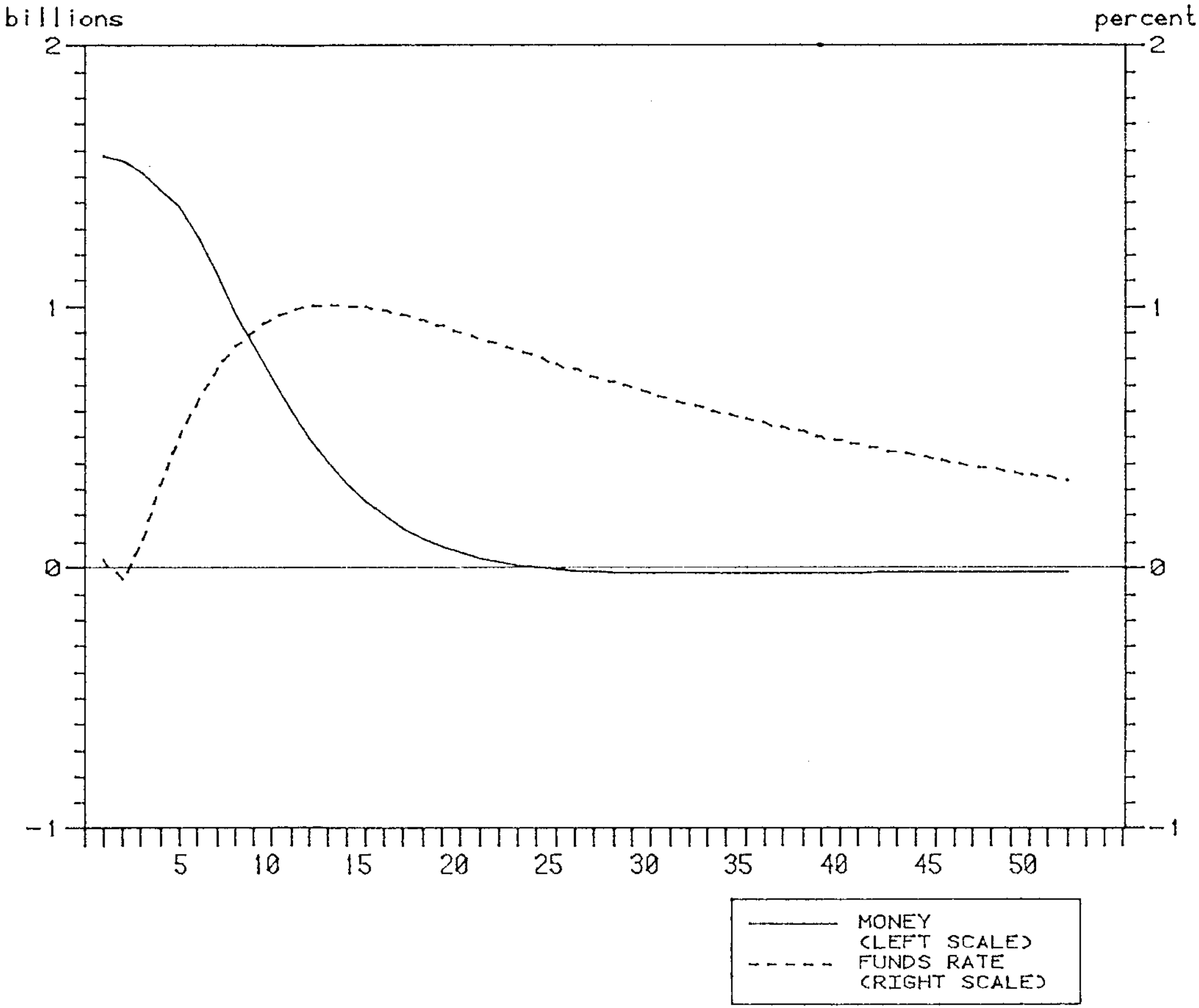


Figure 11

Responses to an Innovation in Money
Reserve Targeting Procedure; q=12; lambda=2

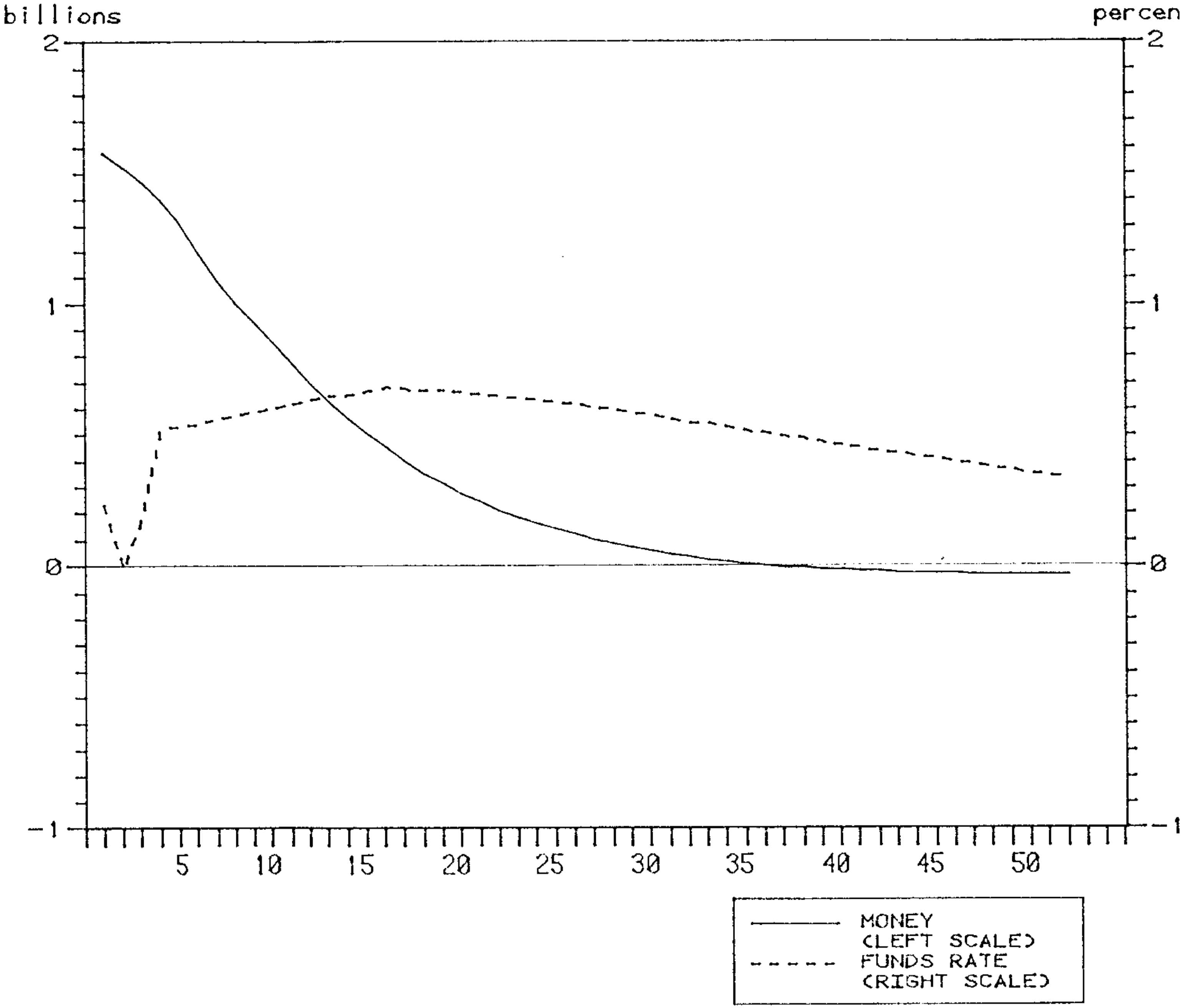


Figure 12

Responses to an Innovation in Money
Reserve Targeting Procedure; q=1; lambda=1

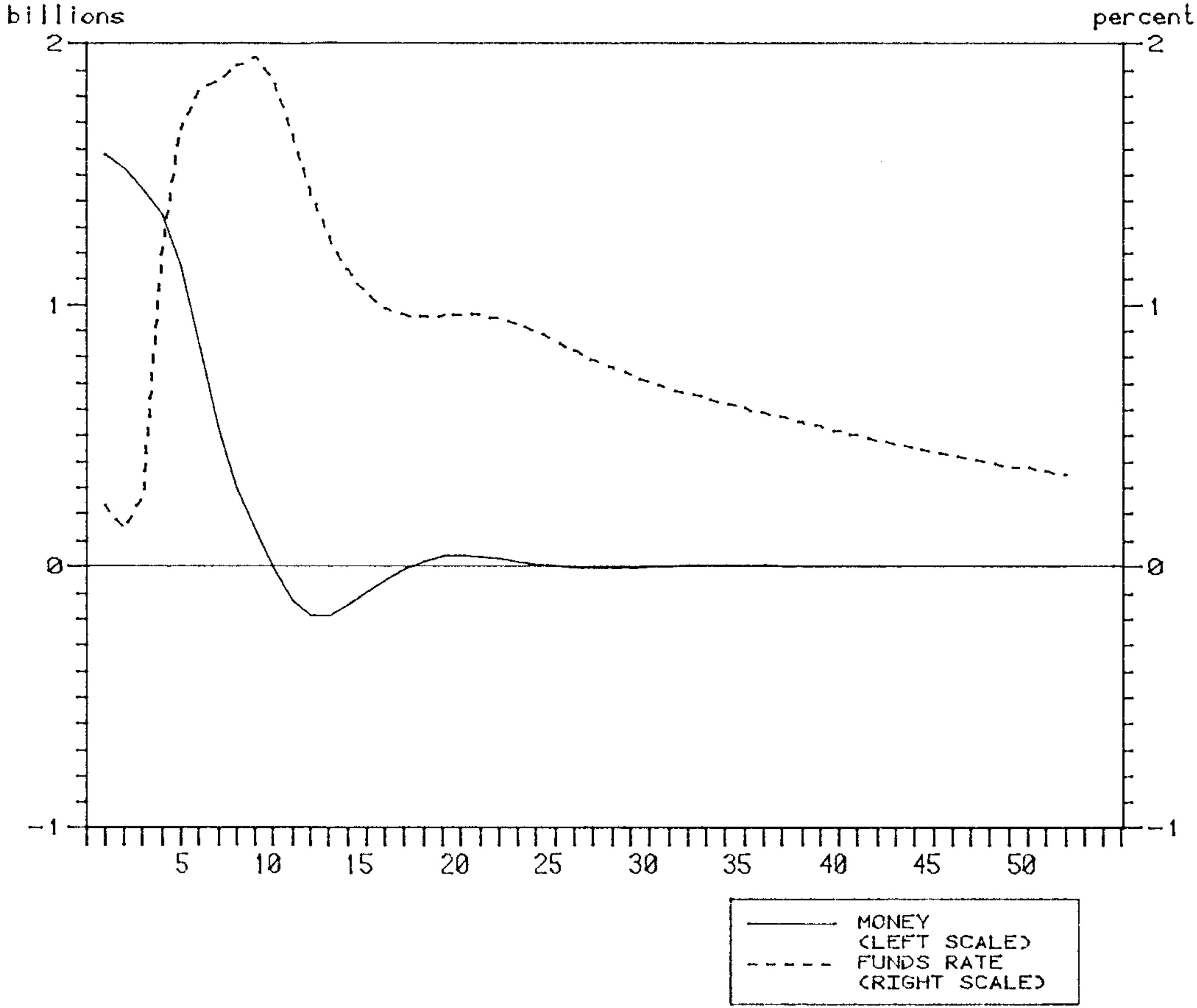


Figure 13

The shocks have been orthogonalized so that the shock we are calling a money innovation includes the component of interest rate innovations which are correlated with money innovations. A comparison of the response patterns for $q=12$, $\lambda=2$, between the funds rate and reserves targeting procedures shows that the difference between them is that under a funds rate target there is a contemporaneous offsetting of the funds rate movement associated with the money innovation. That offsetting response represents the degree to which all contemporaneous movements in the funds rate are offset; the controller does not yet recognize the movement as a money innovation. In fact, because the funds rate movement is equally likely to represent a funds rate innovation which lowers the expected money path, in the second week the funds rate is brought essentially back to, or even below, its previous path. It is not until the third week, when the money disturbance is seen by the controller, and recognized for what it is, that the reaction to it begins.

Once the optimal feedback rule has been calculated, taking into account the lagged observation of money, the probability laws of the controlled system are determined and thus we can calculate measures of expected interest rate volatility and money supply deviations. We can, that is, calculate the set of points, associated with different values of λ and q, which represent the best possible solutions to the problem of minimizing both money supply deviations and interest rate volatility. The tradeoff can be more easily understood by visualizing the costs associated with the 1976 to present period. These costs are illustrated in Figures 14 and 15. In the first, we show the money deviations from target. This target does not attempt to represent actual Fed policy, but rather is estimated as a long run trend. This assumes the Fed was always basically hitting its long run targets, which presumably understates the true situation, particularly prior to October-1979. For our purposes, which focus on short-run control, this is an adequate

Figure 14

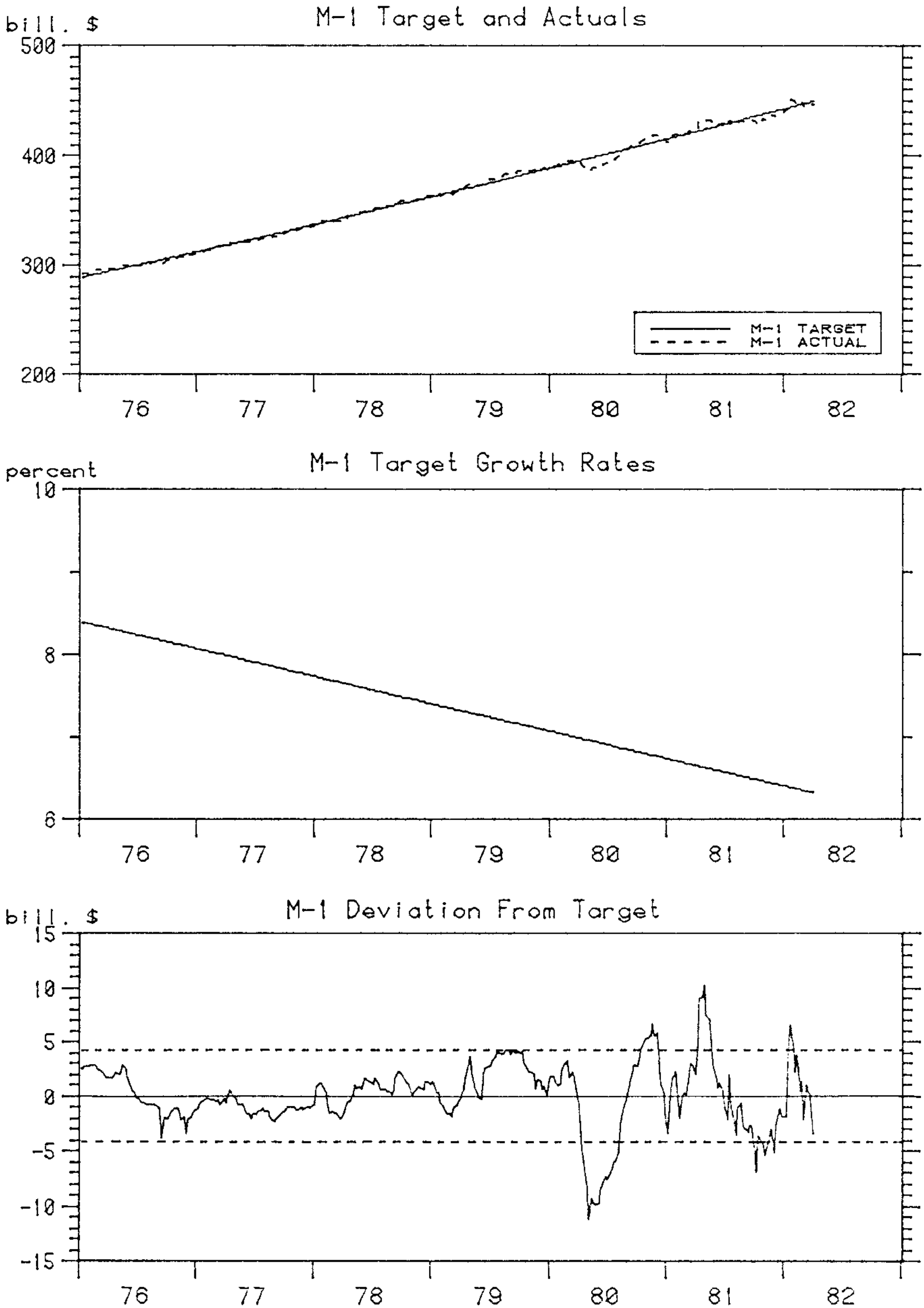
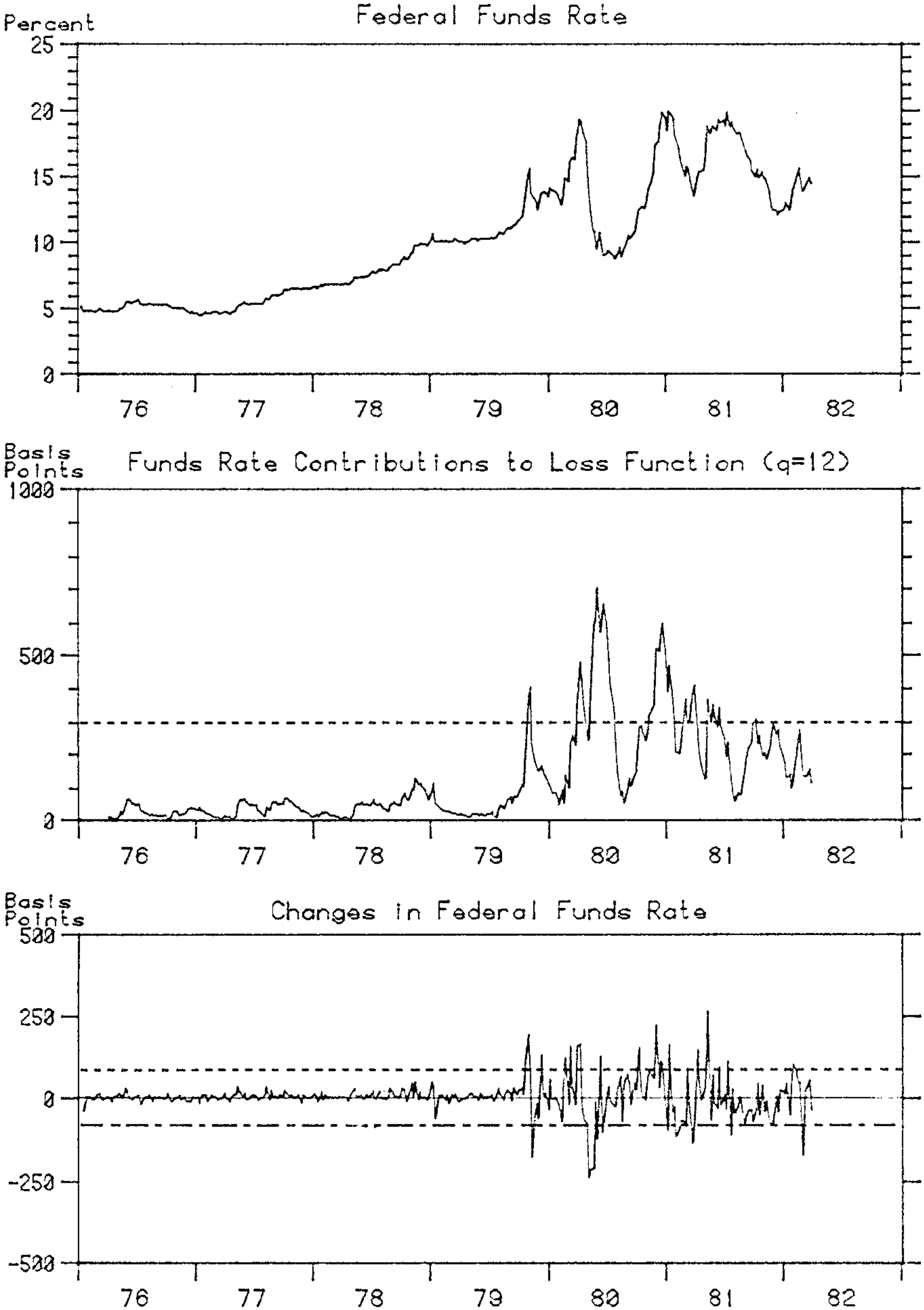

M-1 Target and Actuals
bill. $
500
400
300
200
M-1 TARGET
M-1 ACTUAL
76
77
78
79
80
81
82
M-1 Target Growth Rates
percent
10
8
6
76
77
78
79
80
81
82
M-1 Deviation From Target
bill. $
15
10
5
0
-5
-10
-15
76
77
78
79
80
81
82

Figure 15


Federal Funds Rate
Percent
25
20
15
10
5
0
76
77
78
79
80
81
82
Funds Rate Contributions to Loss Function (q=12)
Basis Points
1000
500
0
76
77
78
79
80
81
82
Changes in Federal Funds Rate
Basis Points
500
250
0
-250
-500
76
77
78
79
80
81
82

approximation. In fact, the target is estimated as the quadratic trend in the logged M-1 data, and the implied slowly declining growth rates, which are also shown in the figure, are quite consistent with the stated Federal Reserve intentions. Finally, in the plot showing the deviations from target we also show dotted lines at plus and minus the post-October-1979 root mean square deviation. The size of this mean square deviation is the measure of monetary control which enters the loss function.

In the next figure we show the federal funds rate path along with two plots illustrating how interest rate volatility is measured in the loss function. One plot shows week-to-week changes in the funds rate, with dotted lines at plus and minus the post-October-1979 root mean square change. This mean square change enters the loss function when q=1. In the other plot we show the square root of the average of squared changes from each of the twelve previous weeks. This value squared is the measure which enters the loss function when q=12. A dotted line shows its post-October-1979 average value. Notice that while they both have units of basis points, the levels of these measures of loss for q=1 and q=12 are not comparable.

Having now defined the appropriate measures of loss, we are able to present the optimal tradeoff as a curve in a graph with root mean square money supply deviations on the vertical axis and root mean square changes in interest rates (or root of averages of twelve squared changes, for q=12) on the horizontal axis. Two curves are shown in Figures 16 and 17, one for the model of a funds targeting procedure and one for the model of a reserves targeting procedure. The size of the shocks is based on the post-October-1979 experience. Notice that lowering the average money deviation from target below about 4.5 billion, which is close to the actual post-October-1979 level, begins to require very large increases in interest rate volatility.

The Steady State Tradeoff Between Short Run (q=1)
Interest Rate Volatility and Money Supply Deviations From Target

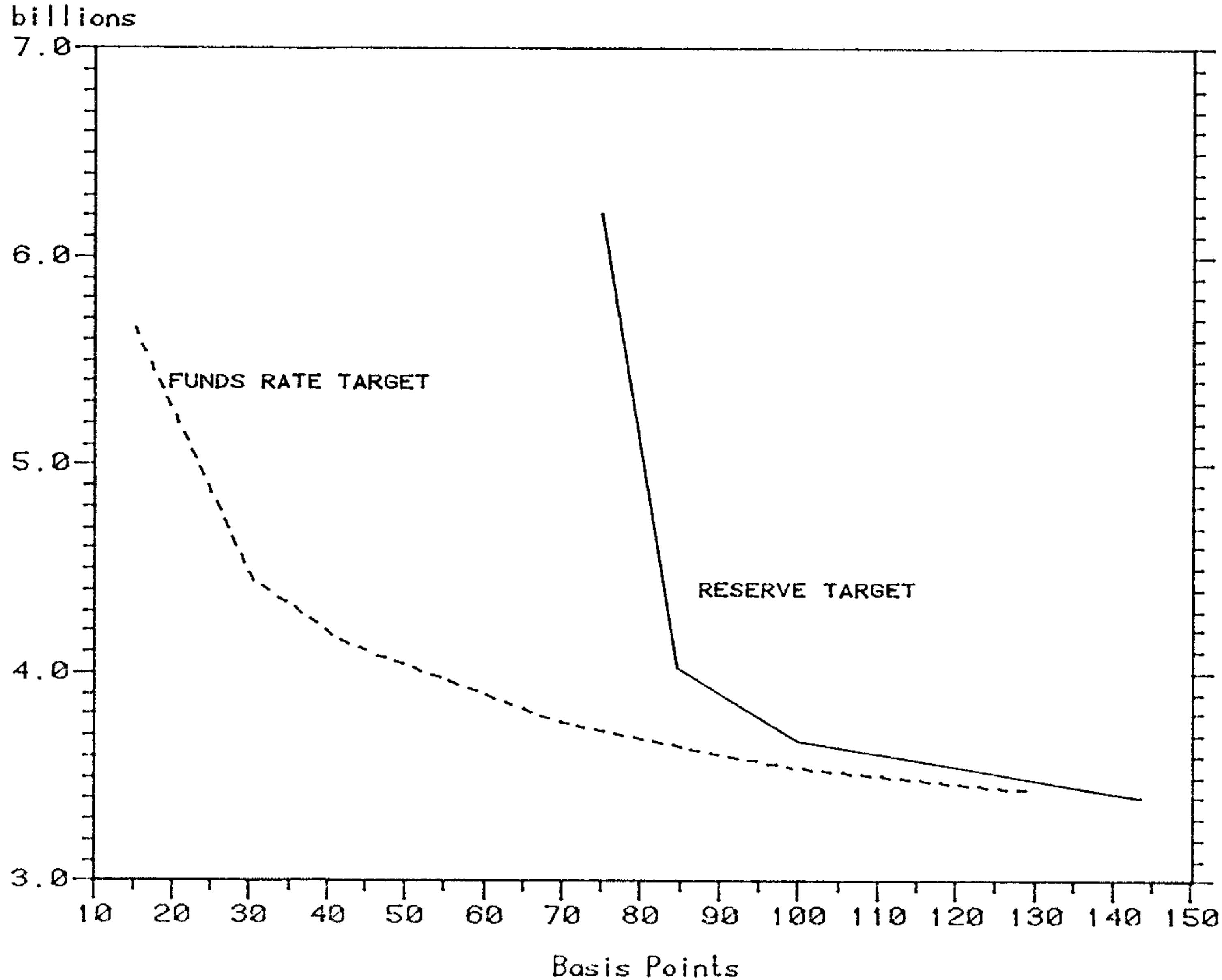


Figure 16

The Steady State Tradeoff Between Longer Run (q=12) Interest Rate Fluctuations and Money Supply Deviations From Target

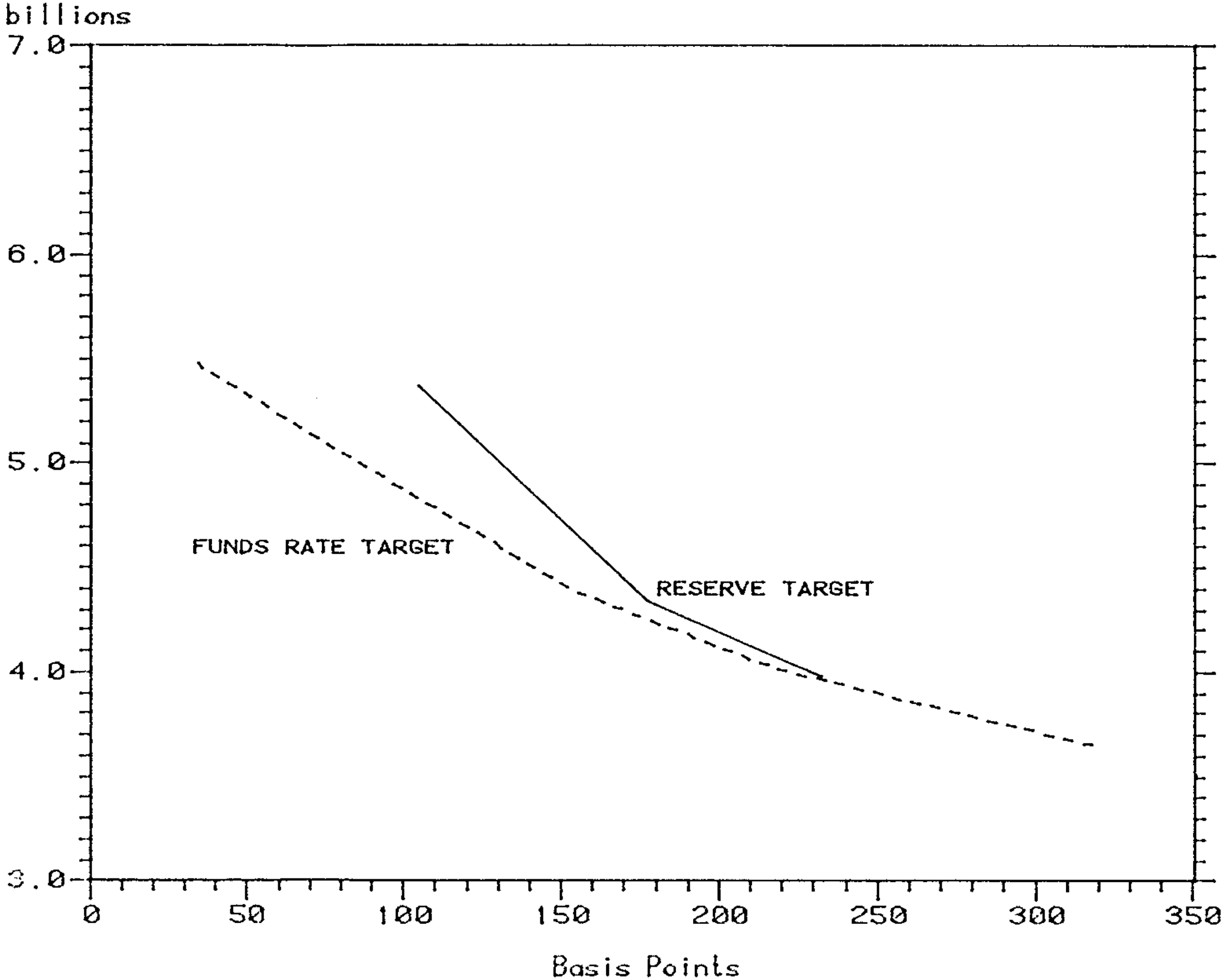


Figure 17

An interesting question is the following: where does the current policy, represented in this model by the time series representation with no control applied, leave us in this space? We can answer this question, but the curve described above, which represents an average expected performance in a steady state, is not the best point of departure. The problem is that the deviations from target of the uncontrolled money path accumulate over time so that there is no finite steady state expected money deviation. This aspect of the time series representation for current policy is not a characteristic about which we should be overly concerned. First of all, the relatively short segment of weekly data on which it is based does not contain much information about the long run properties of the system. In the second place, the random walk prior pulls the estimates toward a nonstationary representation. It is possible, however, to make a useful performance comparison in this space between the optimally controlled systems and the uncontrolled system. This can be accomplished by generating a kind of psuedo history as described below.

The vector autoregressive representation generates a set of one-step-ahead forecast errors, or shocks, for the period over which it is estimated. These shocks can be used to answer the question of how much better could the Fed have done in the past, had it been following an optimal control policy. First, we need to define a target path for the 1976 to present period. Since we are focusing on short-run control, we will take as the long-run target the downward trending growth path described above. Given the target, and taking the initial values at the beginning of 1976 as given, for any particular values of λ and q we can generate the paths the state variables would have taken if:

(a) the state had evolved according to the vector autoregressive representation,

(b) an optimal control policy had been in force, and

(c) the same set of shocks had hit the system.

The results of this type of experiment using different values of λ and q, lead to pseudo histories and tradeoff curves representing what would have occurred under different loss functions and an optimal control strategy. These tradeoff curves can be usefully compared with the actual performance over the same period. This is done in Figures 18 and 19 for the post-October-1979 period.

Four results stand out from this comparison.

1) With a loss function that focuses on high frequency volatility (i.e. q=1), there is not much loss associated with the current policy relative to an optimal reserve targeting procedure.

2) Second, there is very little possible improvement, under either procedure, in reducing short run money deviations from target without incurring large increases in interest rate volatility.

3) Third, there is a large gain possible with respect to reducing high frequency interest rate volatility by moving to a funds rate targeting procedure.

4) Finally, with respect to a loss function that focuses on smoothing both high and low frequency movements in interest rates (i.e. q=12), there is a large gain possible through optimal control of interest rates for either a reserve or a funds rate targeting procedure.

Another interesting comparison can be made by looking directly at the pseudo histories themselves. These are shown for several values of λ and q in Figures 20 to 25. Notice that comparing the actual movements in the funds rate with the movements generated by any of the optimal control procedures suggests that the Fed often responds to money deviations with too much of a delay, and then to react for too long of a period, leading to signficant overshooting of its M-1 targets. Furthermore, a comparison with the history generated with q equal to 12 and $\lambda=2$ suggests that a considerable degree of smoothing of the funds rate could have been achieved with no adverse effect on monetary control.

Post - October 1979 Possibility Frontier
Tradeoff Between the Short Run (q=1)
Interest Rate Volatility and Money Supply Deviations From Target

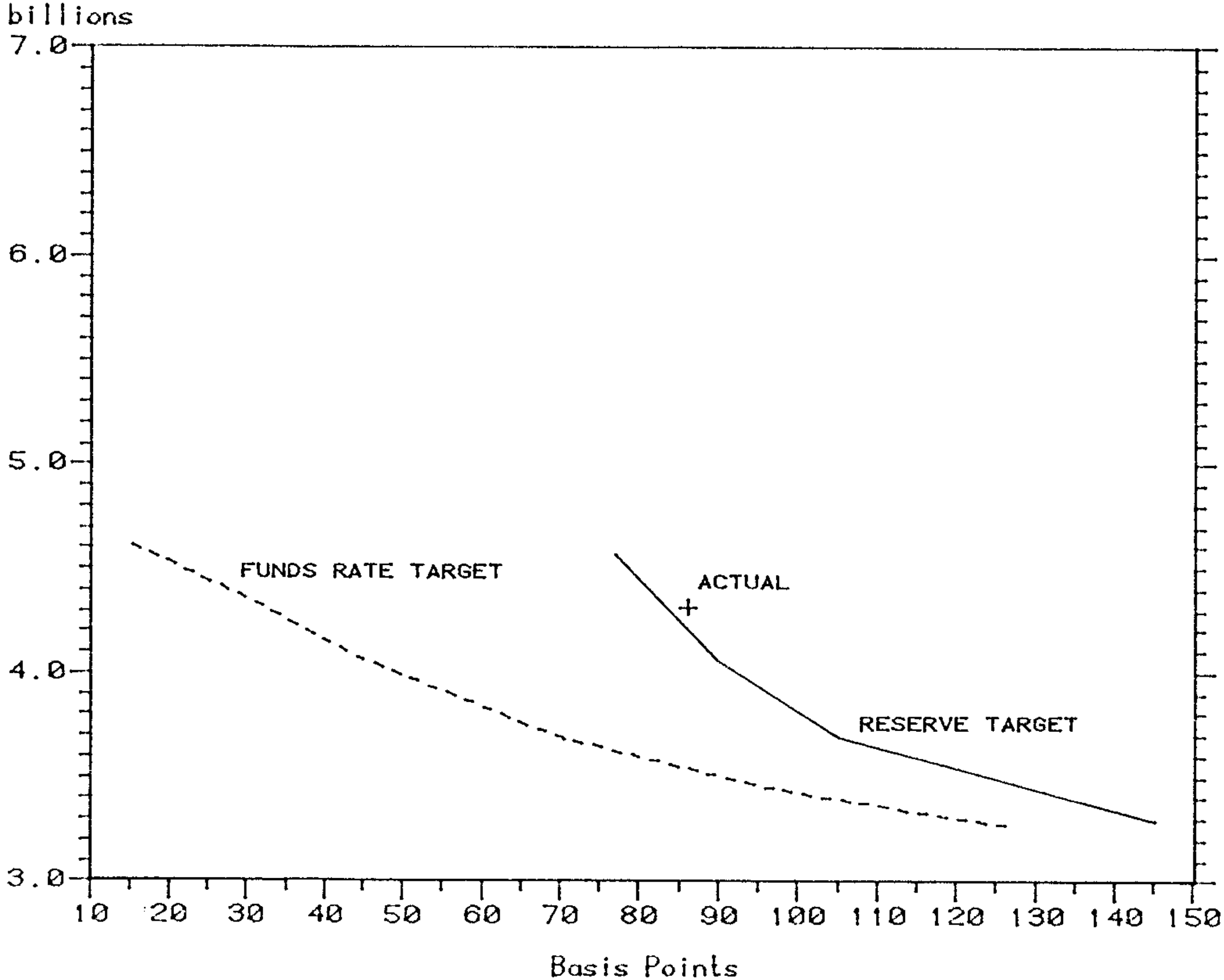


Figure 18

Post - October 1979 Possibility Frontier
Tradeoff Between the Longer Run (q=12)
Interest Rate Fluctuations and Money Supply Deviations From Target

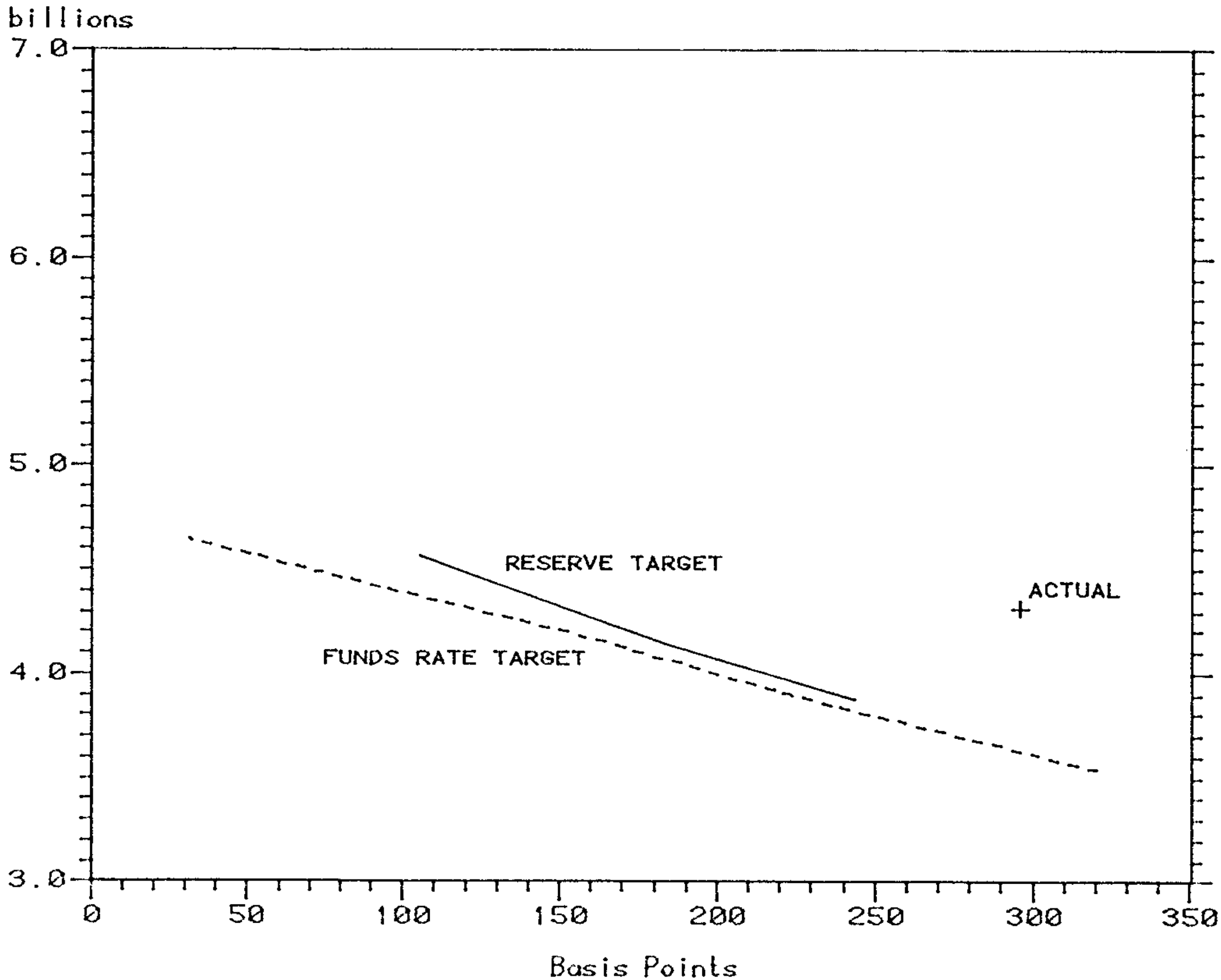


Figure 19

Figure 20

Pseudo History Comparing Actual With Controlled Federal Funds Targeting Procedure; q=12; lambda=.5

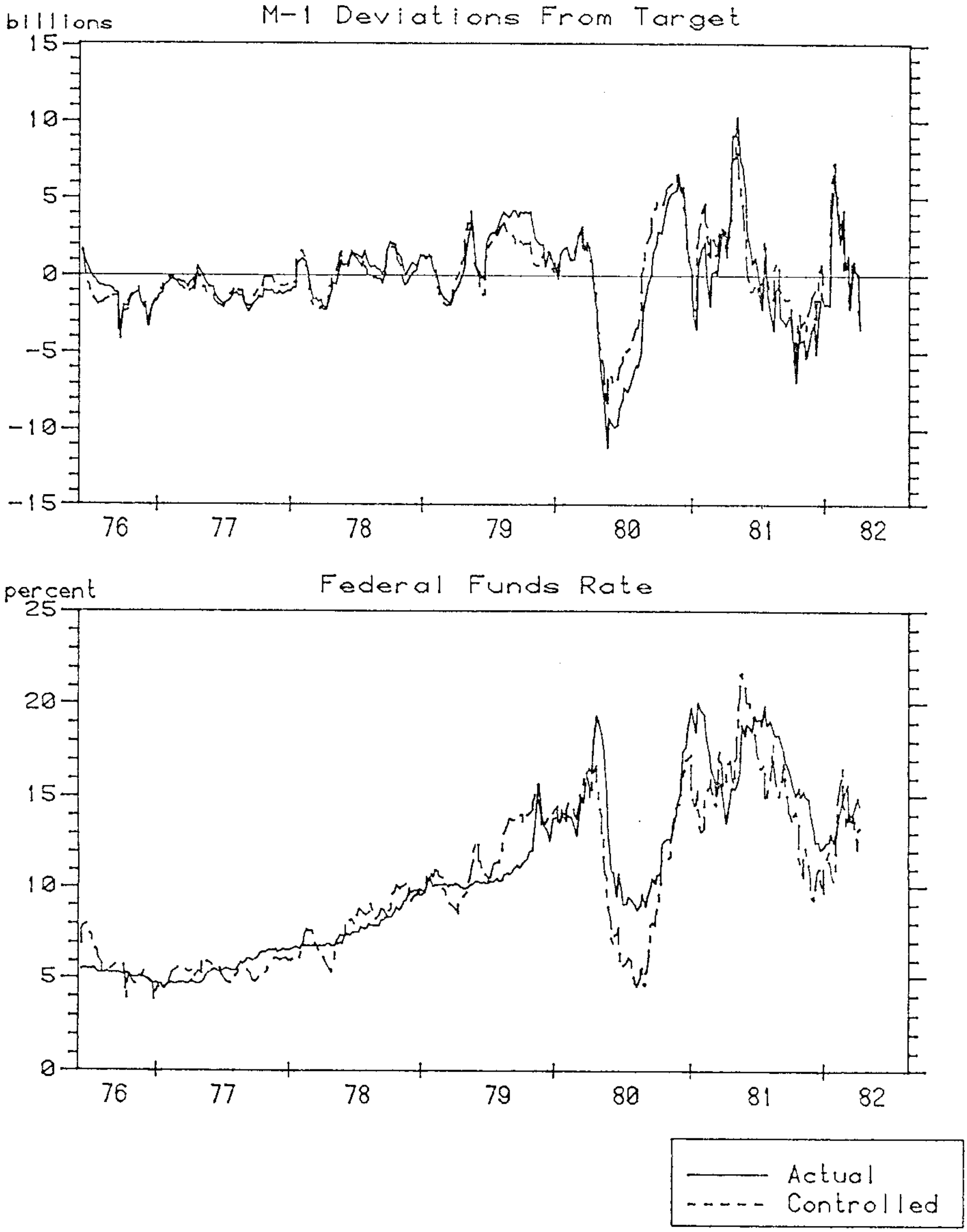


Figure 21

Pseudo History Comparing Actual With Controlled Federal Funds Targeting Procedure; q=12; lambda=2

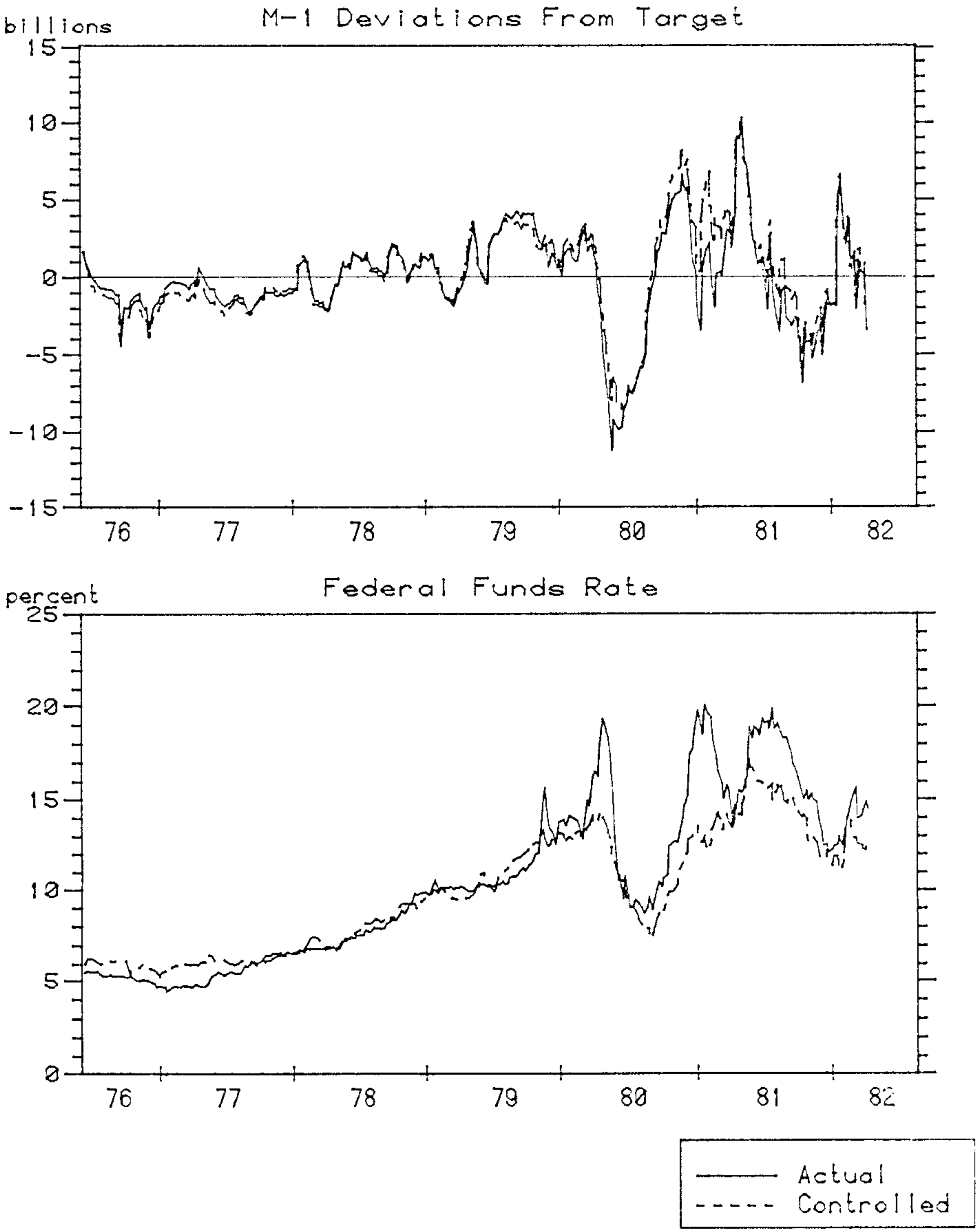


Figure 22

Pseudo History Comparing Actual With Controlled Federal Funds Targeting Procedure; q=12; lambda=8

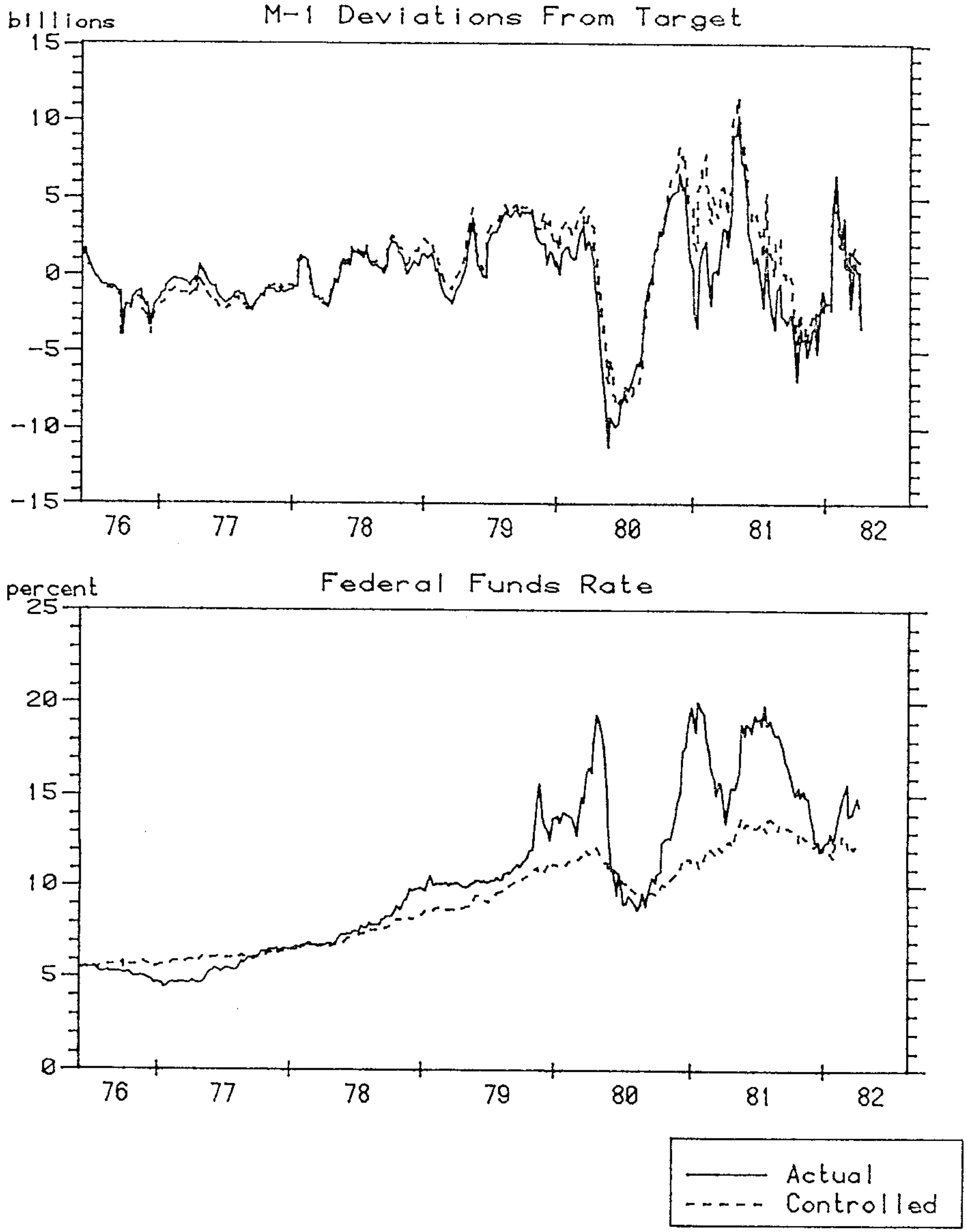


Figure 23

Pseudo History Comparing Actual With Controlled Federal Funds Targeting Procedure; q=1; lambda=20

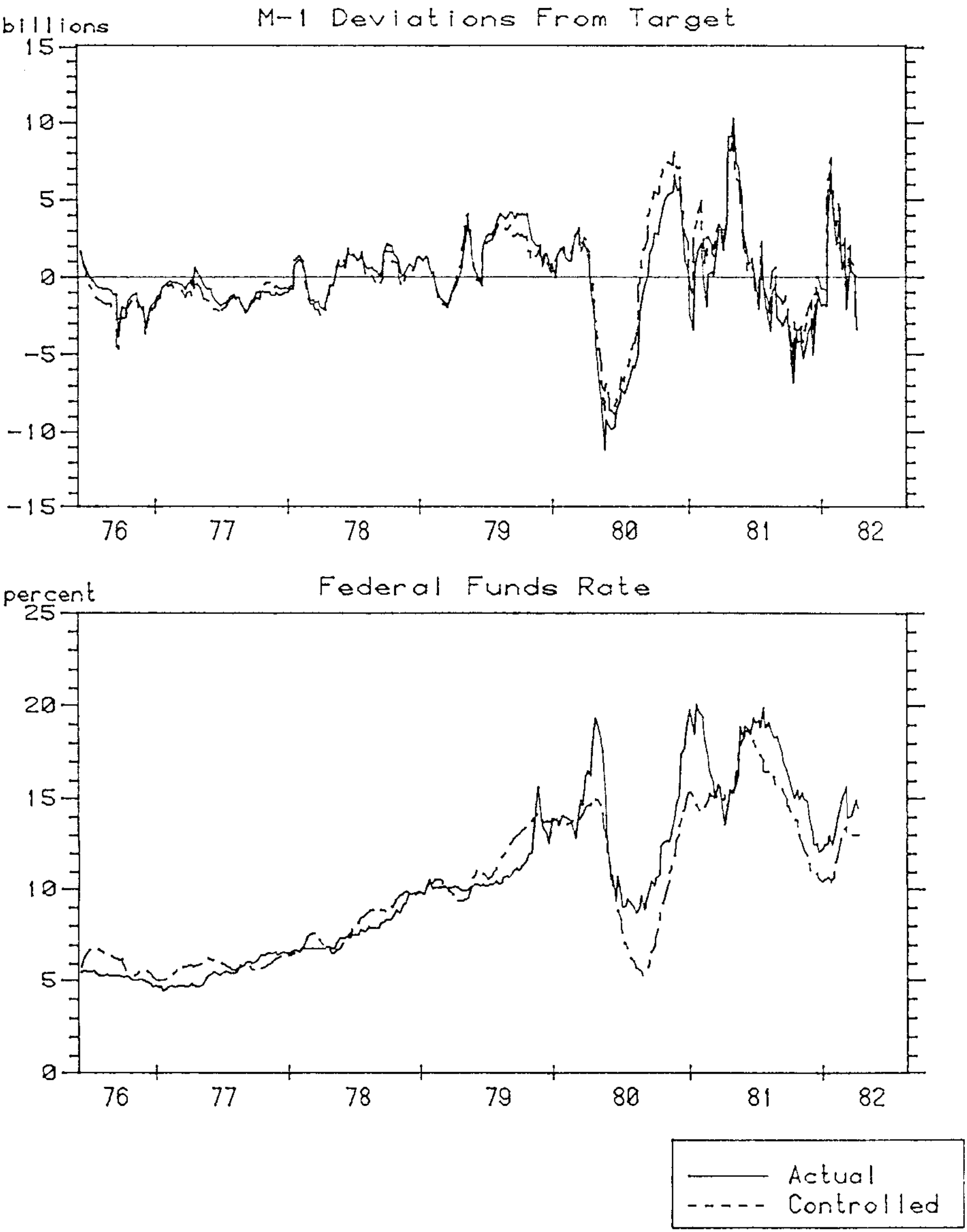


Figure 24

Pseudo History Comparing Actual With Controlled Reserve Targeting Procedure; q=12; lambda=2

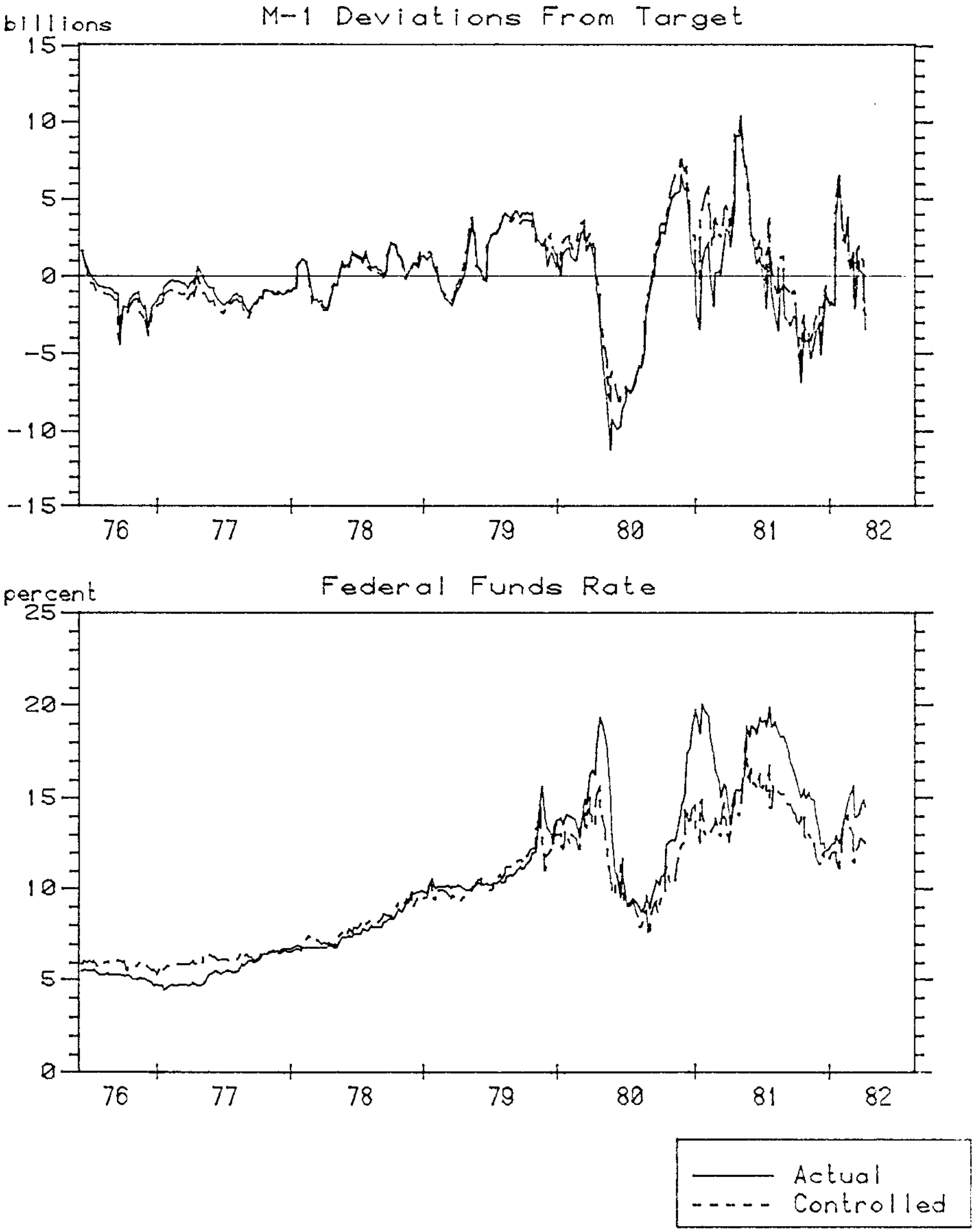
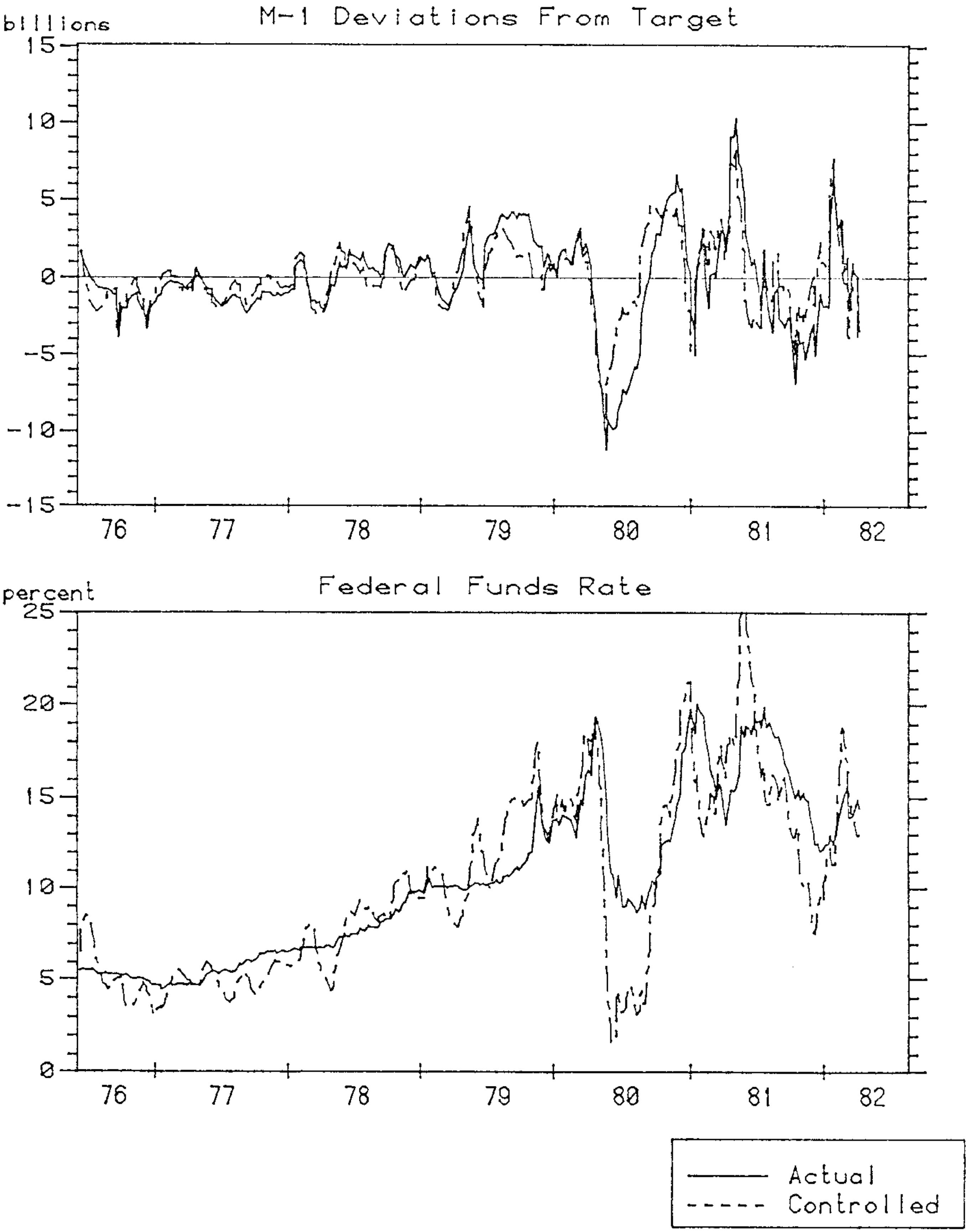


Figure 25

Pseudo History Comparing Actual With Controlled
Reserve Targeting Procedure; q=1; lambda=1



In practice, what the optimal control procedure gives the policy-maker each week is a suggestion for where the funds rate should be in the following week and a set of forecasts for values of the state variables conditional on the value of the funds rate. Examples of this type of output are given below in Figures 26, 27, and 28. Shown are actual values up the current time period, the projected paths of the variables if no control is applied, and the projected paths if the optimal control is applied this period.

Figure 26

Projections Showing the Effects of Adopting Control Procedure (q=12; lambda=2)

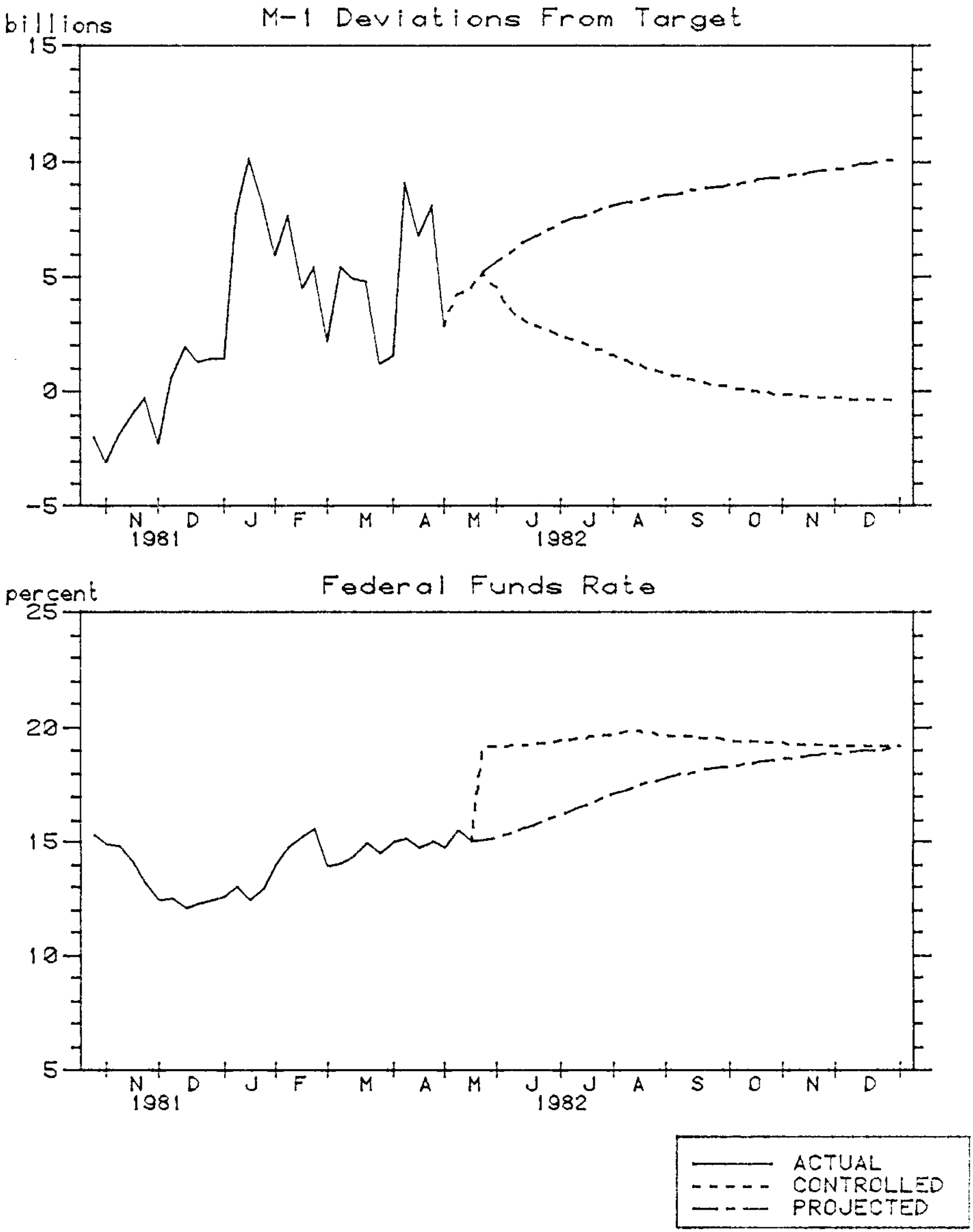
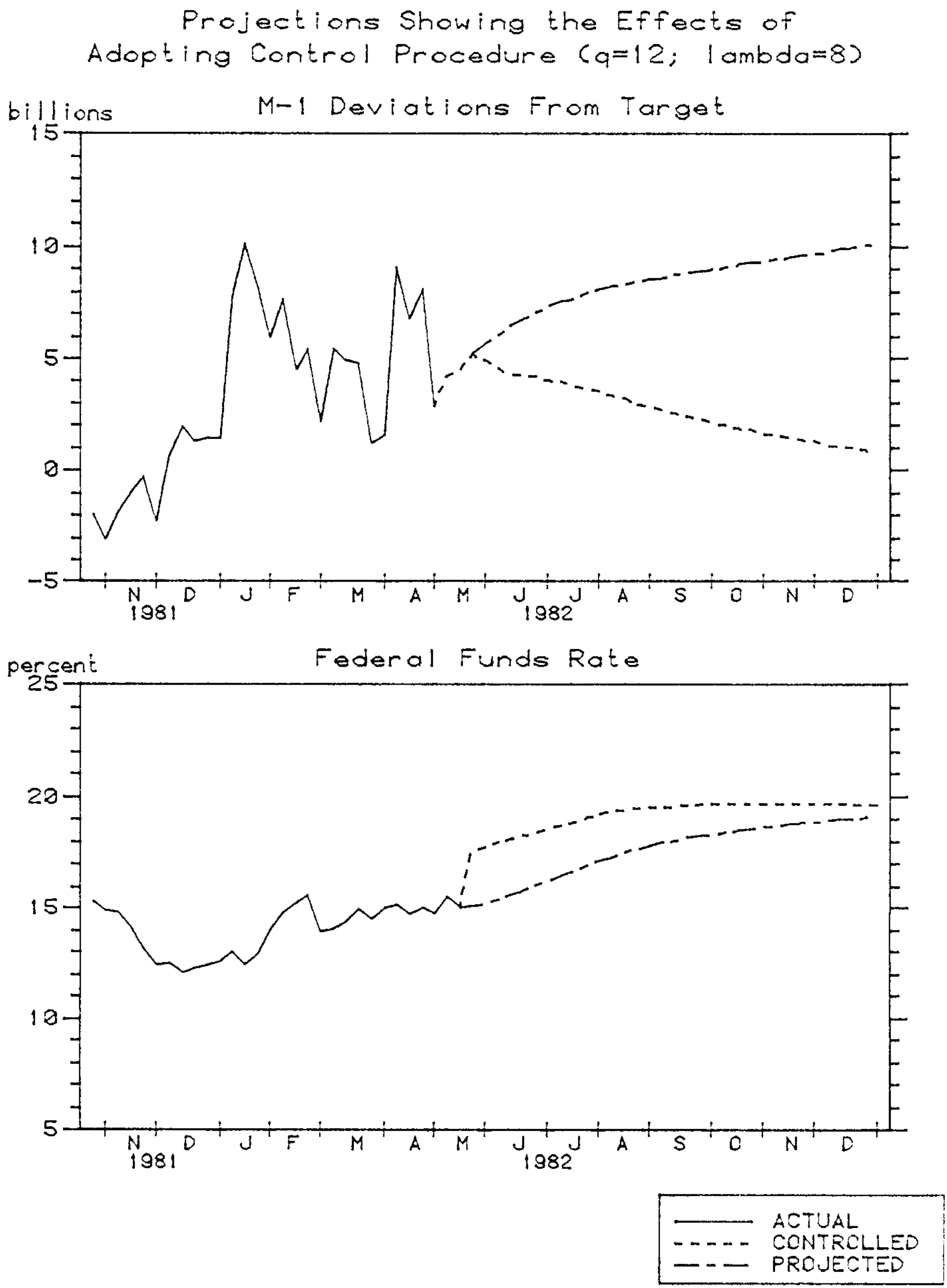


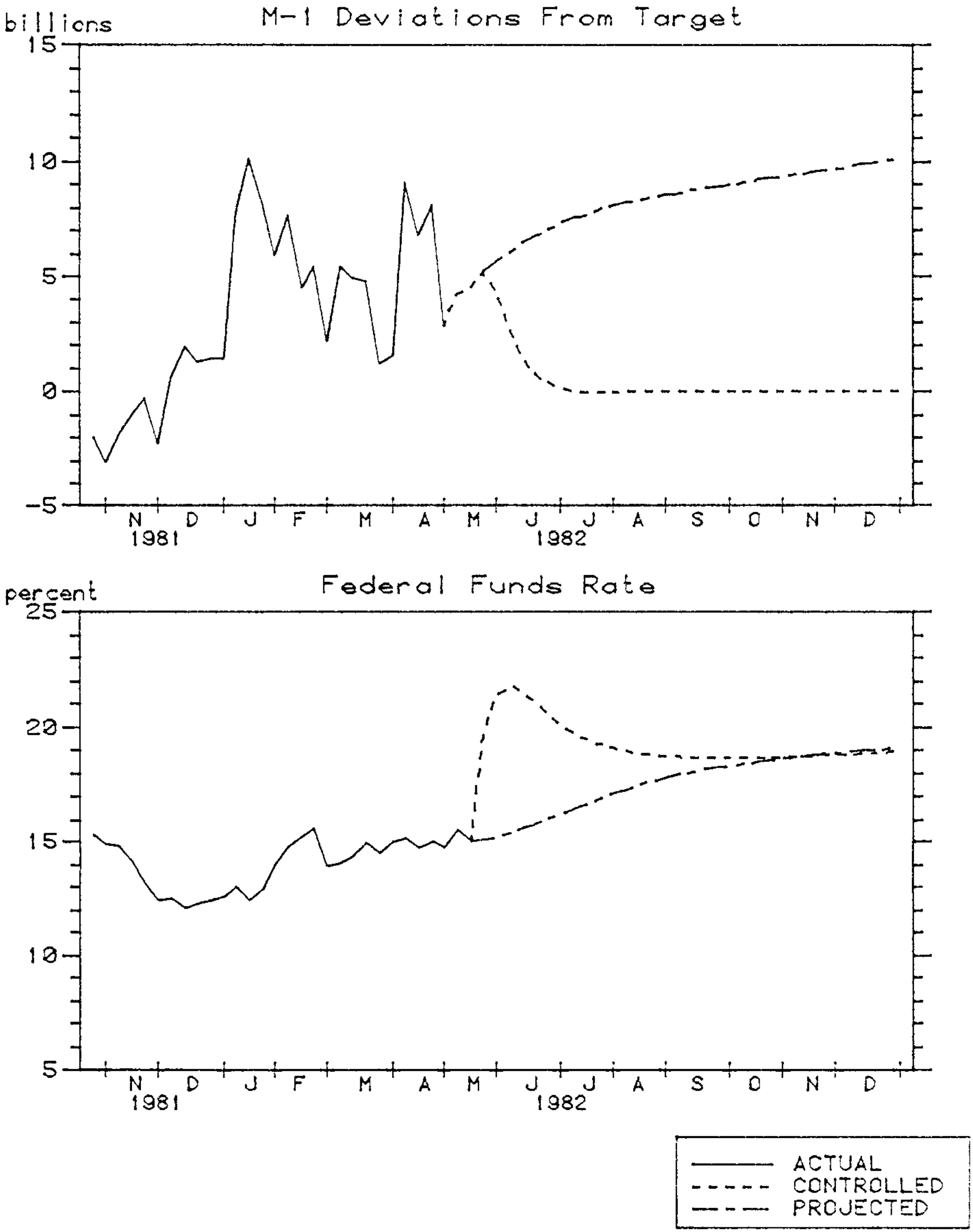
Figure 27



Projections Showing the Effects of
Adopting Control Procedure (q=12; lambda=8)
M-1 Deviations From Target
billions
15
10
5
0
-5
N D J F M A M J J A S O N D
1981
1982
Federal Funds Rate
percent
25
20
15
10
5
N D J F M A M J J A S O N D
1981
1982
ACTUAL
CONTROLLED
PROJECTED

Figure 28

Projections Showing the Effects of
Adopting Control Procedure (q=1; lambda=1)



5. EVIDENCE ON STRUCTURAL STABILITY

There is no guarantee that changes in the operating procedures of the Fed would leave unaffected the important dynamics of the money market on which this procedure depends. There is evidence which suggests that the impact would not be large. Moreover, there are reasons to think a structural model of the link between the funds rate and the money supply could be constructed in which that response would not be sensitive to the kinds of interventions we have been considering.

A key assumption of the above exercise is that the dynamics of the money market variables would not change too much as a result of the adoption of an optimal control rule. Whether this is likely to be true is a key question, it is, after all, the focus of the rational expectations criticism of traditional econometric exercises of this type. According to the rational expectations argument, changes in the policy rule of the government will lead to changes in the actions of agents in the economy and the new dynamic behavior of the economy is likely to be far different from before. For a forceful exposition of this viewpoint, see Lucas (1976).

The standard answer to the above question is that the dynamic behavior of the economy can be modeled structurally, and equations such as a consumption function, a money demand function, and so on, represent behavior of agents which will not change when the equation representing the policy rule is changed. However, if it is recognized that agents' behavior depend crucially on expectations of the future, which in turn depend on government policy, then unless expectations have been explicitly modeled, this defense breaks down. Since a time series representation is the reduced form implied by a structural model, the dynamics of the time series representation are subject to the Lucas critique.

In a draft of his forthcoming Brookings paper, Sims challenges the relevance of the Lucas critique, for policy choices of the type being considered here. Because it is a key issue, we quote at length.

> The normal business of policy formation is properly thought of as choice of shocks to the policy behavior equation, or equivalently as choice of values for policy variables, or again equivalently as implementation of an unchanged policy rule. It is an analytically nontrivial problem, given that the structure of the economy is subject to continual uncertain drift and that those with actual influence on policy are engaged in a complicated dynamic same with many players. It is fully as important as the problem of choice of policy rule. Though choice of rule has permanent consequences, while choice of the current level of policy variables has more short-lived consequences, choice of current levels is repeated very often, while choice of making the choice are therefore of comparable magnitude. Finally, statistical methods probably have more to contribute to policy choices which do not involve rule changes. This may seem to conflict with the recent flowering of econometric literature connected with rational expectations. But while choice of policy rule requires sophisticated probabilities modeling, and while econometric estimation of parameters structural under changes in rule is an intellectual challenge, it remains true that rule changes must be rare events. To make statistics yield conclusions over a dense, inevitably controversial scaffold of a priori theorizing. Since choices of shocks to policy equations have occured very often, the data can be expected to speak more directly about their consequences.
>
> Interesting and important as it may be to develop methods for optimally choosing policy rules in the face of the Lucas critique, it is a mistake to suppose that this should be the exclusive, or even the main focus of quantitatively oriented macroeconometric research. The normal business of making projections of the likely effects of various choices for the paths of policy variables is neither internally inconsistent, nor analytically trivial, nor inconsequential.

With respect to the money market, we are in the fortunate circumstance of having one bit of empirical evidence which may be of help in resolving this issue. In October 1979 the Federal Reserve made a change in operating procedures which arguably was a more striking change than would be the adoption of the optimal control techniques proposed here. If the dynamics of the system were not affected too much by that change, then there is good reason to hope that they would not be too sensitive to the change proposed here. Unfortunately, testing for structural change can be a tricky proposition. For example, it is obvious from the data that something changed in October 1979. The standard errors of innovations in M-1 and the funds rate are many times larger after that date. For our purpose, the question of interest, however, is whether there is evidence that the response function of M-1 to a shock in the funds rate changed. On this question, the evidence is comfortably unclear. Based on visual inspection of the response functions presented above, and a statistical test described here, there is no reason to believe that the response of money changed significantly when the operating procedures of the Fed were changed. This test is as follows: one-step-ahead forecasts of money were made separately based on the data before and after the change. The forecasts were made out-of-sample, in a sense to be made precise below. If there was a significant change in structure, then making forecasts based on using the full sample should lead to larger errors in both subsamples. In fact, using 12 lags, the forecasts of money in the first half of the sample improved only marginally after dropping the second half, and the forecasts in the second half actually improved using estimates based on the full sample. Using two lags, the forecasts based on the full sample were better in each subsample than the forecasts based on the subsample alone. The out-of-sample nature of the test is that for each period, the forecast of money for that period is based on an estimator using all observations in the relevant sample except that period's observation. The reason for this procedure is that if the test is done in-sample, then the subsample estimates must fit better. One version of the standard Chow test for structural stability is based on the asymptotic distribution of the size of this in-sample improvement. See, for example, Sims (1980). Asymptotically, our test will have the same distribution. The fact that there is little or no improvement in the two subsamples means that the change in structure, if it occurred at all, was not large.

TABLE 4
Stability Test Results

Prediction Error Based on Subsample Period	Prediction Error Based on Full Sample (Billions)	(Billions)
(Estimated Using 2 Lags)		
76,13 to 79,40	.58473	.57186
80,1 to 82,12	1.65326	1.64932
(Estimated Using 12 Lags)		
76,13 to 79,40	.59865	.60660
80,1 to 82,12	1.73695	1.60863

An important element in any argument of why the response of money to the funds rate would not be likely to change under a change in operating procedures should be based on an understanding of that response. Banks play a key role in the reaction of money to changes in the funds rate. That role is described in a recent paper, "A Critique of the Federal Reserve's New Operating Procedure" by Robert D. Laurent of the Federal Reserve Bank of Chicago.

> The money supply process is the means by which the monetary authority affects the purchase and sale of assets by banks and thereby the creation and destruction of deposits. It may be thought of as a two-step process. The first step is the action of the Fed. The second step is the reaction of the banks to the Fed's actions. The linchpin of the money supply process is the federal funds market. The federal funds market both resisters the actions of the Fed by setting the price of reserve credit and transmits its influence to every bank. The individual bank's response in terms of buying assets from, or selling assets to the public is what determines the change in deposits and money. To the individual bank, it is the federal funds rate and not reserves which determines how it changes its asset holdings and its impact on the aggregate level of deposits. The individual bank neither knows nor cares about the aggregate level of reserves in the banking system. Indeed, it can be argued that even its own level of reserves does not determine whether a bank buys or sells assets, creating or destroying deposits. Of course, a bank must have enough reserves to meet reserve requirements, but it can always obtain or dispose of reserves in the federal funds market. For example, even a bank deficient in reserves might still make loans and thereby increase deposits if the rate on loans were high relative to the federal funds rate. It would offset its loss of reserves resulting from the increase in loans by buying even more funds than otherwise in the federal funds market. The bank's response depends entirely upon what appears profitable, not upon

> the circumstances of the bank's reserve position. The effect of the federal funds rate on individual banks and the aggregate level of deposits is clear. Other things equal, the higher (lower) the federal funds rate, the lower (higher) will be the level of deposits. To an individual bank, the federal funds market can be either a source of, or an outlet for, funds. A bank compares the federal funds rate to the rates on assets available from the public. The lower the federal funds rate, the more attractive these other assets look. With a low federal funds rate, banks respond by increasing their holdings of assets obtained from the public, creating deposits and covering any reserve losses with federal funds purchases. Conversely, a high federal funds rate means that banks will reduce their holdings of assets obtained from the public, destroying deposits, and take the reserves acquired and sell them in the federal funds market....A bank actually compares (after adjusting for risk differential and transaction costs) the rate on an asset of a given maturity with the expected rate on one day federal funds rolled over for the same maturity. Thus, equilibrium is not necessarily where the rate on bank assest equals the federal funds rate. Policy affects the money stock through the impact of the current federal funds rate on expected future federal funds rates. The greater is the impact of a movement in the current federal funds rate on expected future funds rates, the greater is the impact on money.

If this understanding of the response is correct, then money should continue to react in essentially the same way it has in the past in response to a given movement in the funds rate. If any difference can be expected, it is likely to be that the response will become larger and quicker because given movements in the funds rate will carry more information about future movements in the funds rate than at present, particularly if people understand and believe the Fed's linear feedback rule. Such a change in structure would have the beneficial effect of causing the tradeoff curves defined above to shift down and toward the left. To make this argument precise would require a model of the equilibrium structure which would result from interaction of a Fed policy rule of the sort suggested here and banks' optimizing behavior subject to some costs of adjustment. Such an investigation would appear to be a good topic for future research.

CONCLUSION

This application of optimal control theory and time series analysis has identified an important tradeoff between degrees of short-run monetary control and interest rate volatility. Two principal conclusions emerge:

1) Application of optimal control theory would likely improve Federal Reserve operating procedures, and
2) Interest rate volatility can be reduced considerably from current levels without adversely affecting the degree of monetary control achieved.

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