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LIMITED INFORMATION, CREDIT RATIONING, AND OPTIMAL GOVERNMENT LENDING POLICY

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A model of credit rationing based on asymmetrically informed borrowers and lenders is developed. In this context, sufficient conditions are derived for an appropriate government policy response to credit rationing to be a continuously open discount window. It is also demonstrated that such a policy can be deflationary, and that given a commitment to operate in this way, the monopoly issue of liabilities can Pareto dominate their competitive issuance.

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Limited Information, Credit Rationing, and
Optimal Government Lending Policy

Bruce Smith*

A great deal of attention has been devoted to the phenomenon of "credit rationing." This is in large part because a channel through which monetary policy can conceivably operate is the relaxation of this rationing. Surprisingly, however, the literature devoted to the microeconomic foundations of credit rationing[1/] has analyzed this phenomenon in models insufficiently rich to allow for policy analysis. This paper attempts an analysis, in a simple general equilibrium framework, of policy in the presence of credit rationing. The analysis proceeds in the context of an "informational" model of credit rationing along the lines of Jaffee and Russell (1976), but in a model rich enough to allow for money and an evaluation of government policies. The primary result which emerges is that, under fairly plausible circumstances, the government should lend as much as is demanded at some appropriately set rate of interest. At the level of abstraction of the paper, this may be interpreted as an argument that the government should operate a continuously open and unrestricted discount window. Moreover, while such an argument is reminiscent of the real bills doctrine, it is in fact stronger since it suggests unrestricted lending to "risky" borrowers. Finally, in contrast to textbook treatments of the use of the discount window, it will be seen that in the presence of credit rationing such a policy can be deflationary.

Under the policy suggested, the government lends enough to meet excess demand for credit at the prevailing market rate of interest. Nevertheless, it will be seen that under this policy there continues to be "rationing" in private credit markets. Moreover, the rate of interest which is charged in private credit markets is determined entirely by the rate charged by the government, and the elasticity of credit supply with respect to the interest rate. This focuses any discussion of the impact of further government actions on private market rates of interest and credit volume, since any such impact must occur through the alteration of this elasticity. Any policy, monetary or otherwise, which does not affect this elasticity will be irrelevant insofar as the amount of credit offered is concerned.

Finally, as something of a by-product, the analysis will be seen to permit statements (on a case-by-case basis) regarding the optimal structure of credit markets. In particular, the following possibility will be demonstrated: in a credit market with asymmetric information on the parts of borrowers and lenders the monopolistic issue of liabilities, along with an open discount window, may be Pareto superior to the competitive equilibrium. In short, the analysis offers a rationale for the restriction of entry into the activity of issuing liabilities (given that a particular policy will be followed by the government). Such restriction innately gives rise to credit rationing, since it tends to restrict the supply of liabilities relative to that which would arise in a competitive credit market. However, in contrast to the suggestions of Jaffee and Russell (and others), this rationing is not desirable in and of itself, and should be eliminated through use of the discount window.

Stated succinctly, the results are as follows:

(1) Under relatively unrestrictive conditions, it is welfare improving (except possibly for the initial old in the model) for the government to meet all excess demand for credit at prevailing market rates of interest.

(2) This open lending policy can be deflationary.

(3) Under such a policy, private market rates of interest depend only on the rate of interest charged by the government, and the elasticity of credit supply with respect to interest rates.

(4) Given that this policy is to be followed, monopoly issue of liabilities can be Pareto superior to competition in the issuance of liabilities.

The format of the paper is as follows. Section I discusses the Jaffee-Russell (1976) model of credit rationing as motivation for our definition of a credit rationed equilibrium. Section II discusses the model employed, and considers its competitive and credit rationed equilibria. Section III discusses government lending policies, and their welfare properties. It also demonstrates that under the policy suggested above, the equilibrium interest rate attained in the private credit market will depend only on supply elasticities. Finally, the possibility that an open discount window is deflationary, and that monopoly issue of liabilities is desirable is demonstrated. Section IV concludes.

I. Background

A large literature considers possible motivations for the existence of credit rationing. Here we focus on a subset of this literature which provides a motive for credit rationing based on limited, asymmetric information on the parts of borrowers and lenders. In this section we briefly discuss the model of this phenomenon produced by Jaffee and Russell (1976). This presentation will motivate the definition employed throughout of a credit rationed equilibrium.

In the Jaffee-Russell (J-R) model, there exist three broad groups of agents: lenders, safe borrowers, and risky borrowers. Safe borrowers never default on loans, whereas risky borrowers default with positive probability. To make matters simple, assume that this probability increases with the volume of lending. Lenders are assumed to be unable to distinguish *ex ante* between safe and risky borrowers unless their economic behavior differs. They are also expected profit maximizers who incur no transactions costs. Moreover, there is free entry into lending. Hence the condition defining credit supply is a zero profit condition.

It is an assumption of the J-R analysis that risky borrowers will wish to mimic safe borrowers so as to obtain more favorable rates of interest.[2] Under this assumption there is only a single credit market, and the supply curve of credit in the market is upward sloping. This is because as loan sizes increase, the probability of default *ex ante* increases for each loan (since safe and risky borrowers are indistinguishable *ex ante*), and hence interest rates must rise to make expected profits zero.

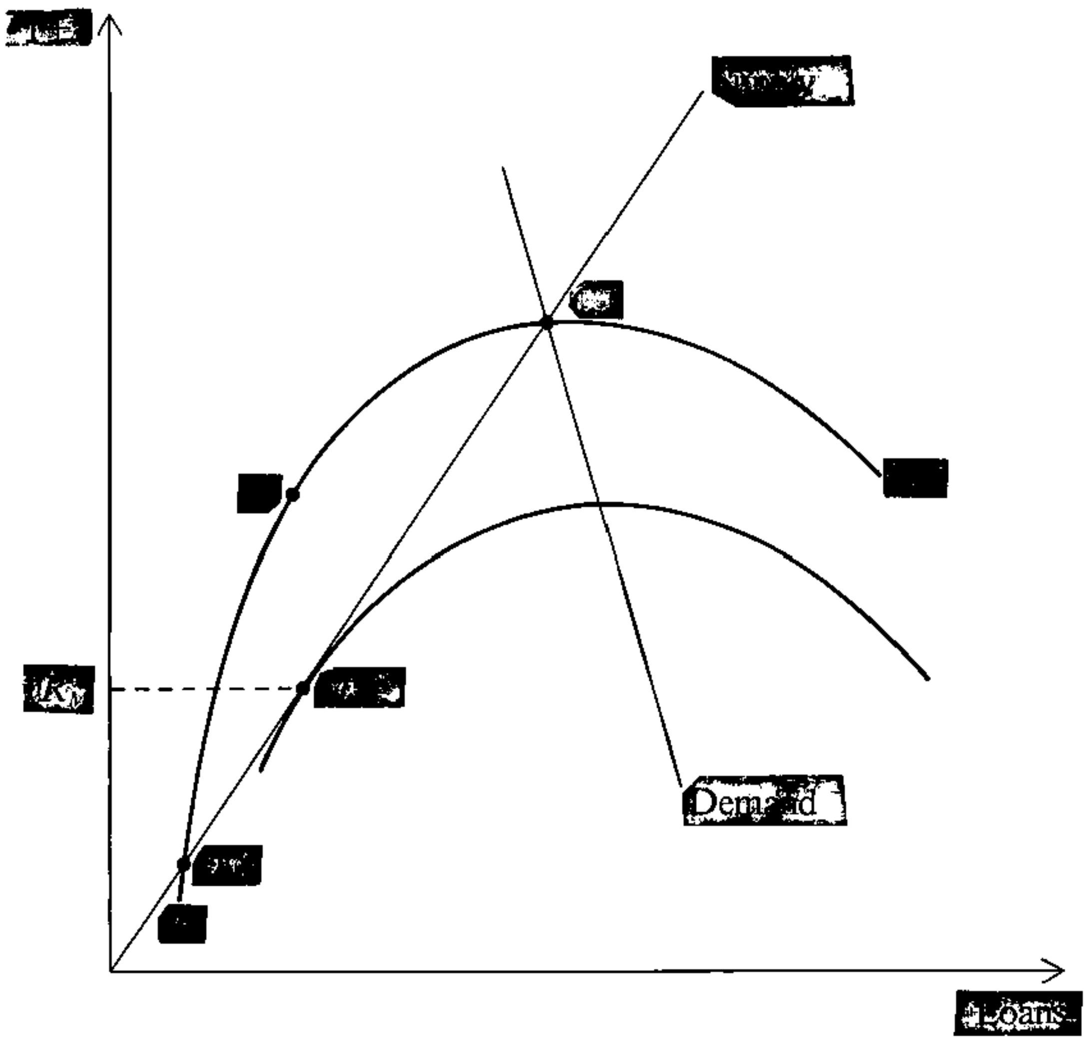
The situation in the credit market is depicted in Figure 1. The projection of the Lagrangian of safe borrowers into this space is the indifference curve II, which attains a maximum as a function of loan size where it intersects the loan demand schedule. Point C is the competitive equilibrium position in this market.

The motivation for credit rationing in this setting is to note that, given the implied departure from an Arrow-Debreu setting, the exercise of monopoly and/or monopsony power can be Pareto improving.[3/] To see this, consider point E in Figure 1. At point E safe borrowers are as well off as at the competitive equilibrium, and lenders make positive expected profit. In fact, any point in the region CED represents a joint improvement for safe borrowers and lenders. Under restrictions on the preferences of risky borrowers their welfare will also be improved by movement away from the competitive equilibrium.

With free entry into lending it is, of course, the case that a point like E cannot be an equilibrium. The J-R argument is then that at any point other than M, where an indifference curve is tangent to the supply curve, it is possible for some lenders to offer borrowers an interest rate-loan package which they prefer to the package offered by other lenders, and which makes positive expected profit. Thus, only M leaves no incentive for some lenders to offer a different interest rate-loan package, and hence M is the equilibrium of a particular game.

To motivate the definition we employ below of a credit rationed equilibrium, we now note two things. First, at the

Figure 1



interest rate R_M notional demand exceeds credit supply. Hence there is "credit rationing" in this equilibrium. Second, the claim made is that a credit rationed equilibrium occurs where safe borrowers' indifference curves are tangent to the offer curves of lenders. This is, of course, simply the monopsonistic equilibrium for this market. This will motivate the definition of a credit rationed equilibrium employed below.[4/]

Prior to concluding this section, it is of value to say something more about this notion of a credit rationed equilibrium. In particular, it could be argued with some validity that the coincidence of a credit rationed equilibrium with the monopsonistic equilibrium is something of an artifact of the J-R model. However, this is not to say that the monopsonistic equilibrium is not of interest. It is, in fact, of interest in its own right, since as will be seen, under appropriate government policies this equilibrium can be Pareto superior to the competitive equilibrium in a fairly general credit market. Moreover, the monopsonistic equilibrium features borrowers who would like to obtain more credit than they can at the market rate of interest; i.e., it generates credit rationing as an outcome. Thus, either as an extension of the J-R analysis, or as an equilibrium concept of interest in its own right, we focus below on monopsonistic equilibria as a source of credit rationing.

II. The Model and Its Equilibria

A. Specification

We wish to consider a model sufficiently rich to permit a welfare analysis of government policies in the presence of credit rationing. To this end we present a model in the spirit of Section I, but enhanced in several respects. The economy considered exists in a world where time is discrete, and indexed by $t = 0, 1, \ldots$. This economy is peopled by a sequence of two-period lived, overlapping generations, and an initial old generation at $t = 0$. There is a single nonstorable consumption good at each date, as well as a fixed (for all time) stock of money, M. Consumption loans are also permitted between agents. Throughout we confine our attention to equilibria in which fiat money has value.

With the possible exception of the initial old, each generation consists of three types of agents, indexed by $i = 1, 2, 3$. Given our assumptions on endowment patterns, type 1 agents lend (and hold money), type 2 agents borrow and never default, and type 3 agents borrow and default with positive probability. Type i agents have preferences described by the twice-continuously differentiable, strictly concave utility functions $U_i(C_1,C_2)$, with $U_i(\ )$ strictly increasing in each argument at every point in the consumption set, R_+^2. C_1 and C_2 are consumption when young and old respectively. We will not require a notation for the consumption of different agents, for the dating of consumption, or for consumption in alternate states of nature.

Endowment streams are as follows. Type 1 agents have endowment b of the consumption good when young, and zero when old. Type i agents (i=2,3) have zero endowments when young. Type 2 agents have endowment w_2 when old (with certainty). Type 3 agents have endowment $w_3 > 0$ when old with probability p, and endowment zero (so that they default on loans) with probability 1 - p. If agents receive positive endowments when old they do not default on loans. Agents have no endowments of either money or consumption-loans, except that the initial old are endowed at t = 0 with the entire money stock.

Consumption loans are made and repaid in the consumption good. This is also our choice of numeraire. Money trades for the consumption good at rate S_t at t. Throughout we restrict consideration to steady state equilibria, and hence often omit subscripts on S. If lenders can distinguish between safe and risky borrowers, then we denote loans to safe borrowers by x_1, and to risky borrowers by x_2. Safe borrowers repay amount R_1x_1 when old if they borrow x_1, and risky borrowers repay R_2x_2 if they do not default. If lenders cannot make this distinction *ex ante*, then we denote loans to both types of borrowers by x, and agents repay amount Rx if they do not default. Finally, for notational convenience, we assume that each generation (except possibly the initial old) consists of a single agent of each type.

B. Competitive Equilibrium

We are now ready to define a competitive equilibrium for the economy of Section I. In fact, we will define two types of competitive equilibrium due to the nature of the informational asymmetry in the model. Let us begin this section with a discussion of this asymmetry, and motivate in this way our two types of competitive equilibrium.

Note first that when young, type 2 and 3 agents have identical endowments, and in terms of currently realized states of nature appear identical prior to making any economic decisions. Hence, we assume that type 1 agents cannot distinguish between young type 2 and type 3 agents unless their economic decisions differ. (Of course, each agent knows his own future endowment as a function of future states of nature.) Since agents are distinguishable when young only through their behavior, agents with $i = 3$ may choose either to reveal or not to reveal their type. If they choose not to reveal they must mimic the behavior of type 2 agents. Hence there are two possibilities as regards competitive equilibria; these may be either revealing or nonrevealing as regards each agent's default probability.

We are now ready to define a full-information (revealing) and a limited information (nonrevealing) competitive equilibrium. As we confine our attention to steady states, we need not state our definitions in terms of sequences of values. Our definitions differ from standard definitions of competitive equilibrium, and from the Jaffee-Russell definition, for the following reason. In equilibrium type 3 agents either reveal their indices through their actions or not. If these agents choose to reveal themselves, or not to reveal themselves, we require this choice to be incentive compatible with actual outcomes. This adds an additional condition to our definitions of equilibrium.

Definition: A full-information (steady state) competitive equilibrium, or FIE, is a set of consumption pairs (C_1,C_2) for agents $i = 1, 2, 3$, a pair (R_1,R_2), a pair (x_1,x_2) with $x_1 \neq x_2$, and a value $S \geq 0$ such that

(i) for type 1 agents (C_1, C_2, x_1, x_2, M) solves

$$\max E\, U_1(C_1, C_2) \text{ subject to}$$

$$C_1 \leq b - x_1 - x_2 - SM$$

$$C_2 \leq R_1 x_1 + R_2 x_2 + SM \text{ in nondefault states}$$

$$C_2 \leq R_1 x_1 + SM \text{ in default states,}$$

where E is the expectations operator, with expectations taken with respect to the objective probability of default.

(ii) for type 2 agents (C_1, C_2, x_1) solves

$$\max U_2(C_1, C_2) \text{ subject to}$$

$$C_1 \leq x_1$$

$$C_2 \leq w_2 - R_1 x_1$$

(iii) for agents with $i = 3$ (C_1, C_2, x_2) solves

$$\max E\, U_3(C_1, C_2) \text{ subject to}$$

$$C_1 \leq x_2$$

$$C_2 \leq w_3 - R_2 x_2 \text{ in nondefault states}$$

$$C_2 = 0 \text{ in default states}$$

(iv) the values R_1, R_2, and S imply that loan and money markets clear.

(v) (incentive compatibility)

$$pU_3(x_2,w_3-R_2x_2) + (1-p)U_3(x_2,0)$$

$$\geq pU_3(x_1,w_3-R_1x_1) + (1-p)U_3(x_1,0).$$

This is simply the (parameter contingent) competitive equilibrium of Prescott and Townsend (1981) or Smith (1981).

Definition: A limited information (steady state) competitive equilibrium, or LIE, is a set of pairs (C_1,C_2) for each type of agent, a pair (R,x), and a value $S \geq 0$ such that

(vi) for agents with $i = 1$, (C_1,C_2,x,M) solves

$$\max E\, U_1(C_1,C_2) \text{ subject to}$$

$$C_1 \leq b - 2x - SM$$

$$C_2 \leq 2Rx + SM \text{ in nondefault states}$$

$$C_2 \leq Rx + SM \text{ in default states}$$

(vii) for agents with $i = 2$, (C_1,C_2,x) solves

$$\max U_2(C_1,C_2) \text{ subject to}$$

$$C_1 \leq x$$

$$C_2 \leq w_2 - Rx$$

(viii) $$p\, U_3(x,w_3-Rx) + (1-p)U_3(x,0)$$

$$\geq p\, U_3(x_2,w_3-R_2x_2) + (1-p)U_3(x_2,0)$$

(iv') the values of R and S imply loan and money market clearing.

Condition (viii) requires that if type 3 agents choose not to reveal their indices, then they must prefer the LIE outcome to the outcome which obtains if they do reveal their indices. If there are multiple values of R_2 and x_2 satisfying (i)-(iii) and (v), (viii) should be taken to hold for any of these values. This definition, then, is simply the J-R competitive equilibrium, with an incentive compatibility condition added.

Prior to proceeding, it may be of value to say a word regarding the relationship between these definitions of equilibrium, and the standard definitions of equilibrium employed (for instance) in the insurance literature. In keeping with the Rothschild-Stiglitz (1976) notion of equilibrium, we might think of some agents offering loan-interest rate pairs.[5/] If it is possible for lenders to distinguish safe from risky borrowers, they offer distinct interest rate-loan combinations. If not, these agents offer a single such pair. However, in contrast to the Rothschild-Stiglitz definition of equilibrium, we require any interest rate-loan pair offered to be market clearing at the stated interest rate. Relative to the Rothschild-Stiglitz definition (where any price-quantity pair can be offered), this makes it more difficult for a revealing equilibrium to exist, and less difficult for a nonrevealing equilibrium to exist. As we wish to consider nonrevealing equilibria (in keeping with the J-R analysis), some such modification of the equilibrium concept is required, with the definition above being perhaps the simplest one.

C. Existence of Equilibrium

In keeping with the J-R model of credit rationing, we confine attention to LIEs. Hence we require that equilibria satisfy, among other conditions, condition (viii). This requirement implies that a LIE (and in general, any competitive equilibrium) need not exist under normal regularity assumptions.[6/] Therefore, we establish that the concept of a LIE is nonvacuous by means of an example. (Moreover, the example also establishes that there exist economies with a LIE, and with no competing FIEs.)

Example 1: Let preference be given by

$$U_1 = C_1 + dC_2 - (\phi/2)(C_2)^2$$

$$U_2 = \ln C_1 + \ln(1+C_2)$$

$$U_3 = \ln C_1 + \ln(1+C_2),$$

with $d = 2$, $w_2 = 7$, $w_3 = 5$, $p = 1/2$, and $\phi = 1/10$. As long as it is chosen to guarantee feasibility, the value of b is immaterial.

We first establish that this economy has no FIE (steady state or otherwise. In fact, revelation of type is impossible at any date). Given the set of preferences above, condition (v) is

$$\ln\left(\frac{1+w_2}{2R_1}\right) + p \ln \left[1+w_3-\left(\frac{1+w_2}{2}\right)\right]$$

$$< \ln \left[\frac{1+w_3}{(1+p)R_2}\right] + p \ln \left[1+w_3-\left(\frac{1+w_3}{1+p}\right)\right].$$

For our parameter values, this condition reduces to $R_2 < R_1$. Since type 1 agents are risk averse, this is impossible in equilibrium. Hence this economy has no FIE. However, it is readily verified that a LIE does exist for this economy. This has R = 10/7, x = 2.8, and S_t = 4/M. For future reference, we note that the levels of expected utility attained in this equilibrium are $EU_1 = b + 5.2$, $EU_2 = 2.416$, and $EU_3 = 1.376$.

It is the case, then, that there exist economies with no FIEs, and for which the concept of a LIE is nonvacuous. Having established this, we now proceed in defining a credit rationed equilibrium.

D. Credit Rationed Equilibria

The definition of a credit rationed equilibrium is motivated by the recollection that the J-R credit rationed equilibrium is the limited information monopsonistic equilibrium for their economy. We define here only a limited information equilibrium for reasons discussed below.

For purposes of the definition, denote the optimal choice of x for type 1 agents at any given set of prices by $x_s^*\left(\frac{S_{t+1}}{S_t},R,b\right) = x_s(R,b)$, since our focus on steady states implies $S_{t+1} / S_t = 1$. Then

Definition: A limited information (steady state) monopsonistic equilibrium (LIME), or credit rationed equilibrium satisfies conditions (vi), (viii),

(ix) The 4-tuple (C_1,C_2,x,R) for type 2 agents solves

$$\max U_2(C_1,C_2) \text{ subject to}$$

$$C_1 \leq x$$

$$C_2 \leq w_2 - Rx$$

$$x = x_s(R,b),$$

(x) the value S clears the money market.

The reason for defining only a limited information version of this equilibrium is as follows. The J-R analysis is such that only the monopsony power of type 2 agents is relevant. We maintain a version of this by assuming that only type 2 agents can exercise monopsony power.[7/] Given this assumption, there is no full-information monopsonistic equilibrium, (or more precisely, any such equilibrium is a FIE). This is because, with full information, money holdings are a perfect substitute for loans to type 2 agents in the portfolios of type 1 agents. Hence safe borrowers face a perfectly elastic supply curve of credit and have no monopsony power. Therefore, the relevant interest rate in the incentive compatibility constraint is the full-information rate of interest R_2, which thus appears in (viii).

E. Existence of a LIME

In general, a LIME need not exist. There are two reasons for this. One is the conventional reason for nonexistence of monopolistic/monopsonistic equilibria: reaction correspondences need not be convex-valued. The second is that just as a competitive equilibrium need not exist as a result of imposing the incentive compatibility conditions, this may result in nonexistence of a monopsonistic equilibrium as well. Given that a LIME need not exist, the remainder of this section is devoted to demonstrating that the notion of a LIME is nonvacuous. This is done by means of an example.

Example 2: Preferences and parameter values are as for example 1. For convenience we note that

$$x_s(R,b) = \frac{(1+p)R-2}{(1-p)p\phi R^2}$$

for this example, and that

$$D_R\ x_s(R,b) = (1-p)p\phi R[4-(1+p)R][(1-p)p\phi R^2]^{-2},$$

where D_R denotes differentiation with respect to R. The supply curve of credit is upward sloping so long as $R < 4/(1+p)$, which is the relevant range for this example. It is also the case that over the range where the supply curve is upward sloping it is a strictly concave function of R, so that second order conditions for the problem (ix) are satisfied for this example.

It is tedious but straightforward to verify that the values of R and x satisfying (ix) are $R = 1.425 < (10/7)$, and $x = 2.709$. 10/7 is approximately 1.429, so for this example credit rationing results in about a 1 percent reduction in the interest rate (R-1). Finally, we must verify that the incentive compatibility condition (viii) is satisfied. For purposes of this we note that if $pR_2 = 1$, then loans to type 3 agents under full-information have equal expected return to the holding of money, or safe loans. Hence, since type 1 agents are risk averse, $(1/p)$ represents a lower bound for R_2. Then

$$p\ U_3(x_2, w_3 - R_2 x_2) + (1-p)U_3(x_2, 0)$$

$$\leq p\ U_3(x_2, w_3 - \frac{x_2}{p}) + (1-p)U_3(x_2, 0)$$

$$= \ln 4 - \ln 2 + (\tfrac{1}{2}) \ln 2 = 1.04$$

$$< 1.377 = \ln (2.709) + (\tfrac{1}{2}) \ln (6-3.86)$$

$$= \ln x + p \ln (1 + w_3 - Rx).$$

This verifies that type 3 agents do in fact prefer not revealing themselves to revelation, so the LIME is incentive compatible, and $S_t = (\frac{4.21}{M})\ \forall\ t \geq 0$, $R = 1.425$ is, in fact, a LIME.

For future reference, we note that the levels of expected utility attained under the LIME are $EU_1 = b + 5.186$, $EU_2 = 2.418$, and $EU_3 = 1.377$. Thus, it will be noted that the LIE and the LIME are not Pareto comparable.

Having established that the concept of a LIME is non-vacuous, we may now consider government policies for economies where a LIME exists.

III. Policy

In this section, it is demonstrated that under plausible circumstances, it is welfare improving for all agents (except possibly the initial old) if the government meets all excess demand for credit at prevailing market interest rates. This is true despite the fact that it involves operating an open "discount window" for risky borrowers. In addition, it is demonstrated that this policy has the effect of making free market interest rates (and credit volume) invariant to anything other than the elasticity of credit supply with respect to the rate of interest. Finally, it will be seen that this policy may be deflationary, and that in its presence, the monopoly issue of liabilities may be Pareto superior to their competitive issue.

The scheme of the section is as follows. Section A establishes sufficient conditions for the desirability of this policy under the assumption that a LIME exists before and after its implementation. Section B establishes the invariance argument for interest rates under the same assumption. Section C establishes that these results are nonvacuous, i.e., that an equilibrium can exist under this policy. It also establishes the possibility of deflation, and of desirable monopoly liability issue under the policy.

A. Government Lending

In this section, we establish that the following policy improves the welfare of all (but possibly the initial old) agents under fairly general circumstances. (a) The government lends as much as agents desire to borrow at the LIME interest rate R^*. (b) This policy is financed by appropriate lump-sum taxes-cum-transfers $T_i(s)$ on agent i in state s (s = 0 is the nondefault state, s = 1 the default state) which balance the government budget in each state and period.

This intervention by the government will alter the equilibrium which obtains. Therefore, we present the following

Definition: A post intervention LIME is an array of pairs (C_1,C_2), a pair $(\hat{R},\hat{x})$, and a value S satisfying

(xi) for type 1 agents the 4-tuple (C_1,C_2,x,SM) solves

$$\max E\, U_1(C_1,C_2) \text{ subject to}$$

$$C_1 \leq b + T_1 - 2x - SM$$

$$C_2 \leq 2Rx + SM;\ s = 0$$

$$C_2 \leq Rx + SM;\ \ s = 1$$

T_1 parametric.

Denote the solution for x by $x = x_s(R,b+T_1)$.

(xii) for type 2 agents the 4-tuple $(C_1,C_2,\hat{x},\hat{R})$ solves

$$\max E\, U_2(C_1,C_2) \text{ subject to}$$

$$C_1 \leq \hat{x}$$

$$C_2(s) \leq w_2 - R^*\hat{x} + (R^*-\hat{R})x_s$$

$$+ T_2(s);\ s = 0,\ 1$$

$$x_s = x_s(\hat{R},b+T_1)$$

$T_2(s)$ parametric, $s = 0,\ 1$.

(xiii) $$pU_3[\hat{x},w_3-R^*\hat{x}+(R^*-\hat{R})x_s+T_3]$$

$$+ (1-p)U_3(\hat{x},0) \geq p\, U_3(x_2,w_3-R_2x_2+T_3) + (1-p)U_3(x_2,0),$$

where R_2 and x_2 are to be understood as values satisfying (i)-(iii) and (v), given the values $T_i(s)$.

(xiv) the value S clears the money market.

In short, type 2 agents borrow as much as desired in private credit markets, exploiting ther monopsony power in the process. They then borrow the difference between this quantity and their notional demand from the government at the interest rate R*. Type 3 agents mimic the actions of type 2 agents in each case.

We are now prepared to present the primary result of this section. This is

Proposition 1. If

(a) $\hat{x} \geqslant x_s(R^*,b)$

and

(b) $x_s(R^*,b)$ is less than the notional demand of type 3 agents at the interest rate R*,

then except for the initial old, it is Pareto improving for the government to lend as much as demanded at the pre-intervention LIME interest rate.[8/]

To establish the proposition, and to derive the sufficiency of conditions (a) and (b), which we do simultaneously, we begin by defining $V_i(\frac{S_{t+1}}{S_t},R,y_1,y_2)$ to be the indirect utility function of type i agents who are young at t, where y_j is income in the j^{th} period of life of agent i. Since we focus on steady states, $S_{t+1}/S_t = 1$, and we omit this argument in our notation. In addition, as we construct the proof of the proposition each agent will have no endowment or income in one period of life. This argument is suppressed as well.

The proof of Proposition 1 proceeds as follows. Let R^* denote the LIME interest rate prevailing at the date of the initial policy implementation, which we take to be $t = 0$. $\hat{R}$ denotes the LIME interest rate in the presence of this policy. The policy consists of the government making all loans demanded at R^*, with the level of transfers implied taken as parametric by agents. The method of proof is to construct transfers in a way which satisfies the proposition. To do this, consider agents with $i = 1$. Since in both the pre- and post-intervention LIME type 1 agents are on their supply curves, the change in their utility from the program is

$$V_1(R^*,b) - V_1(\hat{R},b+T_1),$$

where transfers to type 1 agents are T_1, which are made when young and are <u>not</u> state dependent.[9/] Taking a first order approximation and equating this difference to zero, we have

$$D_R V_1(R^*,b)\ (R^*-\hat{R})$$

$$- D_y V_1(R^*,b)\ T_1 = 0.$$

For "small" levels of credit rationing this is a reasonable approximation. Solving for T_1, we obtain

$$T_1 = \frac{D_R V_1(R^*,b)(R^*-\hat{R})}{D_y V_1(R^*,b)} .$$

Noting that the intertemporal version of Roy's identity for this problem implies

$$\frac{D_R V_1(R^*,b)}{D_y V_1(R^*,b)} = \frac{2x_s(R^*,b)}{R^*} \quad ,$$

we have

$$T_1 = \frac{2x_s(R^*,b)(R^*-\hat{R})}{R^*} \ .$$

This transfer guarantees that all interest rate changes resulting from the program are income compensated from the point of view of type 1 agents.

Consider next agents with i = 3. Under the program the change in their utility is

$$p\ U_3[\hat{x},w_3-\hat{R}\hat{x}+(R^*-\hat{R})x_s(\hat{R}) + T_3(0)] + (1-p)U_3(\hat{x},0)$$

$$-p\ U_3(x^*,w_3-R^*x^*) - (1-p)U_3(x^*,0),$$

where $\hat{x}$ is the post-intervention LIME value of $x, x^* \equiv x_s(R^*,b)$, $T_3(0)$ are transfers to type 3 agents in state s = 0, and $T_3(1) \equiv 0$. For now, we simply state that

$$T_3(0) = -\ (R^*-\hat{R})x_s(\hat{R},b+T_1),$$

and return to a consideration of type 3 agents below.

In order for the government budget to balance, all remaining transfers must be made to type 2 agents. These will be state dependent transfers which type 2 agents receive when old. These are denoted $T_2(s)$, and by the budget balance condition, are defined by

(1) $$T_2(0) = 2(R^*-1)\ [\hat{x}-x_s(\hat{R},b+T_1)] - T_1 - T_3(0)$$

(2) $$T_2(1) = (R^*-2)\ [\hat{x}-x_s(\hat{R},b+T_1)] - T_1.$$

Equation (1) states that in nondefault states the government lends the difference between what agents wish to borrow at the interest rate R^*, and the amount offered in the marketplace at $\hat{R}$, the post-intervention LIME interest rate. In addition, two loans of the same amount are repaid. Finally, the government pays out T_1 to compensate type 1 agents, and collects T_3 from type 3 agents. The net profit to the government then accrues to type 2 agents (who are distinguishable _ex post_ when second period endowments have been realized). Equation (2) differs in that only one loan is repaid when $s = 1$, and $T_3(1) = 0$.

We now demonstrate conditions sufficient for the utility of type 2 agents to increase. First note that, since the problem (xii) is separable, the maximized value of (xii) is (with an obvious notation)

$$V_2[R^*,w_2+T_2(0)+(R^*-\hat{R})x_s,w_2+T_2(1)+(R^*-\hat{R})x_s],$$

where $x_s \equiv x_s(\hat{R},b+T_1)$. If this is greater than or equal to $V_2(R^*,w_2)$, the utility of type 2 agents does not worsen as a result of the program. Defining

$$\tilde{T}_2(s) = T_2(s) + (R^*-\hat{R})x_s(\hat{R},b+T_1)$$

and taking a first order approximation about $\tilde{T}_2(s) = 0$; $s = 0, 1$, gives

$$p\, D_y\, V_2(R^*,w_2)\, \tilde{T}_2(0) + (1-p)\, D_y\, V_2(R^*,w_2)\, \tilde{T}_2(1) \geq 0, \text{ or}$$
$$D_y\, V_2(R^*,w_2)\, E\, \tilde{T}_2(s) \geq 0$$

as the condition under which type 2 utility increases. This is equivalent to

(3) $$E\,\tilde{T}_2(s) = [(1+p)R^*-2]\,(\hat{x}-x_s) + (R^*-\hat{R})\,\left[(1+p)x_s - \frac{2x^*}{R^*}\right] \geq 0.$$

Now note that if

$$\frac{(1+p)R^*}{2} = 1,$$

the expected return on loans is unity. Since lenders are risk averse, $\frac{2}{1+p}$ constitutes a lower bound on R^*. Therefore, since $\hat{x} > x_s(\hat{R},b+T_1)$, 10/ the first term on the right-hand side of (3) is positive. It is also the case that $R^* > \hat{R}$. Then if $x_s \geq x^*$, it is immediate that $E\,\tilde{T}_2(s) \geq 0$, and the utility of type 2 agents is increased by the program. However, it will often be the case that $x_s(\hat{R},b+T_1) < x_s(R^*,b)$, so that the sign of (3) is ambiguous. We now investigate conditions under which (3) holds.

To begin, note that $\hat{R} > \frac{2}{1+p}$. Therefore, if the second term on the right-hand side of (3) is negative,

$$E\,\tilde{T}_2(s) > [(1+p)R^*-2]\,(\hat{x}-x_s) - \left(R^*-\frac{2}{1+p}\right)\left[\frac{2x_s(R^*,b)}{R^*} - (1+p)x_s\right].$$ 11/

Then $E\,\tilde{T}_2(s) > 0$ if

$$\left(R^*-\frac{2}{1+p}\right)\left\{(1+p)(\hat{x}-x_s) - \frac{2x_s(R^*,b)}{R^*} + (1+p)x_s\right\} = \left(R^*-\frac{2}{1+p}\right)\left[(1+p)\hat{x} - \frac{2x_s(R^*,b)}{R^*}\right] \geq 0.$$

Since $R^* > \frac{2}{1+p}$, this is equivalent to

$$(1+p)\hat{x} - \frac{2x_s(R^*,b)}{R^*} \geq 0.$$

Now note that since $R^* > \frac{2}{1+p}$,

$$(1+p)\hat{x} - \frac{2x_s(R^*,b)}{R^*} > (1+p)\hat{x} - (1+p)x_s(R^*,b).$$

Thus

(a) $\hat{x} \geq x_s(R^*,b)$

is sufficent for the utility of type 2 agents to increase as a result of the program. This condition is easily interpreted as type 2 utility increases if the changes in income for type 2 agents implied by the program would not result in notional demand being less than the original level of credit supplied in the LIME. In other words, $\tilde{T}_2(s)$ may not convert the situation of excess notional demand into excess supply at R^*.

Consider now the welfare of type 3 agents. Their welfare increases if

(4) $$E\ U_3[\hat{x},w_3(s)-R^*\hat{x}] > E\ U_3[(x^*,w_3(s)-R^*x_s(R^*,b)] \ ,$$

where x^* is the pre-intervention LIME value. To a first order approximation, (4) holds if

(5) $$[\hat{x}-x^*]\ E(D_{C_1}U_3) > R^*[\hat{x}-x^*]\ E(D_{C_2}U_3).$$

This is equivalent to

(b) $$\frac{E(D_{C_1}U_3)}{E(D_{C_2}U_3)} \gtrless R^* \text{ as } \hat{x} \gtrless x^*.$$

If

$$\frac{E(D_{C_1} U_3)}{E(D_{C_2} U_3)} > R^*,$$

then the notional demand of type 3 agents at R* exceeds x*. If (a) and (b) are satisfied, then the policy of Proposition 1 increases the welfare of both type 2 and 3 agents. Condition (b) has the plausible interpretation that if (a) is satisfied, risky lenders would borrow more than they actually do if their activity was not constrained by the actions of type 2 agents. In other words, if (a) is satisfied, (b) implies that type 3 agents, as do type 2 agents, face credit rationing in a LIME. This is certainly a plausible condition.

It remains to consider the financing of the program at t = 0. At all dates t > 0, the program is either self-financing, or financed by lump-sum taxes on old agents with i = 2. We assume that at t = 0 the program is financed by a lump-sum tax on the initial old. Sufficient conditions for their utility not to fall as a result amount to the price of money rising sufficiently to offset this tax. This, in turn, requires sufficiently large increases in money demand as $\hat{R}$ falls below R*. Such a condition will not typically be satisfied.[12] Thus, the initial old must generally be exempted from the proposition.

The notions behind Proposition 1 can be simply stated as follows. With reference to Figure 1, if safe borrowers are allowed to borrow according to their notional demand at R_M, they will clearly be made better off. If risky borrowers are also credit rationed at R_M, this represents a movement in a utility improving direction for them if it is not too large. Moreover, since incremental government lending reduces the use of private credit markets, the variability in second period consumption tends to be reduced for lenders.[13/] Counteracting this last tendency, however, is the fact that the interest rate received by private lenders is reduced by the policy. Proposition 1 establishes conditions which are sufficient to guarantee that, even if lenders are made worse off by an open discount window, borrowers gain sufficiently from the policy to compensate them. Moreover, it will be noted that the policy change is welfare improving under these conditions even though the government operates under the same informational limitations as private lenders.

The proposition has been established, then, under several assumptions. This first is that credit rationing is "small." This assumption is not strictly required for the proposition, as we have established much stronger conditions than required in the presence of small credit rationing. The second is that a post-intervention LIME exists. This is a nontrivial assumption even though, by hypothesis, a pre-intervention LIME exists. The problem essentially rests with the fact that, taking transfers as parametric, the corresponding full-information interest rate R_2 may change so as to increase the expected utility associated with revelation for type 3 agents by more than utility increases for these agents in a LIME. In other words, under the transfer scheme implied, a post-intervention LIME need not be incentive compatible even though the pre-intervention LIME was.

Thus, it remains to demonstrate that there exist economies for which the policy of Proposition 1 is feasible. This is demonstrated in Section C below.

B. The Post-Intervention LIME

In this section, we demonstrate that under the policy of Proposition 1, private market rates of interest and credit volume are invariant with respect to a large class of potential government actions. We also indicate what these market quantities do depend on. The result obtained is stated as

Proposition 2. Under the policy of Proposition 1, $\hat{R}$ depends only on R*, and the elasticity of credit supply with respect to R.

Proof: Consider the first order condition for $\hat{R}$ associated with the problem (xii). This is

$$p\, D_{C_2} U_2(\quad) \Big|_{s=0} [x_s-(R^*-\hat{R})D_R x_s] +$$

$$(1-p)D_{C_2} U_2(\quad) \Big|_{s=1} [x_s-(R^*-\hat{R})D_R x_s] = 0.$$

In order for this condition to be satisfied we require

$$x_s - (R^*-\hat{R})D_R x_s(\hat{R},b+T_1) = 0$$

or

$$\frac{\hat{R}D_R x_s(\hat{R},b+T_1)}{x_s} \equiv \eta_s(\hat{R}) = \frac{\hat{R}}{R^*-\hat{R}}, \tag{6}$$

where η_s is the elasticity of credit supply with respect to R. In short, (6) states that type 2 agents simply minimize expenditures by choice of $\hat{R}$, and establishes the proposition.

Proposition 2 demonstrates that while a wide class of policies <u>will</u>, in fact, affect private credit markets, a large class of policies will have no such effect. As an example, if we expanded our analysis to include safe bonds, open market operations would have no effect on private credit markets. Such irrelevance is, of course, not unique to this setting. However, the main point of Proposition 2 is that it demonstrates the exact focal point of any discussion of the impact of further policy actions on credit markets under government policies which are not clearly suboptimal. For such actions to be effective, it must be demonstrated that there is some impact of these policies on supply elasticities of credit.

C. An Example

In this section, we demonstrate that Propositions 1 and 2 are nonvacuous, or more specifically, provide a sample economy for which they hold (i.e., which has a post-intervention LIME). As a by-product of this demonstration, it will be seen that the policy of Proposition 1 is deflationary for this economy. It is also Pareto improving over both the LIME, <u>and the LIE</u> for the economy. This establishes the possibility that, given a commitment to run an open discount window at an appropriately set rate of interest, the monopoly issue of liabilities Pareto dominates their competitive issue. In other words, it is possible that restrictions on entry into the activity of issuing liabilities are desirable. This should not be taken as a generalizable result, however.

Example 3. The economy is as for examples 1 and 2. As a reminder,

$$T_1 = x_s(R^*,b)(R^*-\hat{R})$$
$$T_3(0) = -(R^*-\hat{R})x_s,\ T_3(1) = 0$$
$$T_2(0) = 2(R^*-1)(\hat{x}-x_s) - T_1 - T_3(0)$$
$$T_2(1) = (R^*-2)(\hat{x}-x_s) - T_1.$$

As is clear from the preferences of type 1 agents, the supply schedule of credit is unaffected by the presence of transfers. This supply curve, repeated for convenience, is

$$x_s(R) = \frac{(3/2)R-2}{(1/40)R^2} = \frac{60R-80}{R^2}$$

for the parameters of our example. The post-intervention LIME value of R, $\hat{R}$, maximizes

$$(R^*-\hat{R})x_s(\hat{R}) = (1.425-\hat{R})\ \left(\frac{60\hat{R}-80}{\hat{R}^2}\right).$$

The optimal $\hat{R}$, and associated level of credit supply are

$$\hat{R} = 1.378,\ 1.41 = x_s(\hat{R}).$$

It is tedious but straightforward to derive other equilibrium values. These are

$$\hat{x} = 2.86$$
$$T_2(0) = 1.34$$
$$T_2(1) = -1.01.$$

It is also possible to verify that the change in the steady state value of real balances between this equilibrium, and the equilibrium of example 2 is Δ SM = 2.88.

It remains to verify that the post-intervention LIME is incentive compatible. To see that it is, compute $EU_3 = 1.41$. It was established in example 2 that an upper bound for EU_3 under revelation was 1.04, so the post-intervention LIME is incentive compatible. This completes the demonstration of nonvacuousness.

It will also be noted that the example establishes that this "liberal use of the discount window" can be deflationary (real balances rise with the stock of money constant). Hence it is <u>not</u> the case in the presence of credit rationing that liberal use of the discount window generally constitutes an inflationary policy action.[14/]

Finally, we use the example to demonstrate that this policy can be strictly welfare improving. Note first that

$$\Delta\ SM = 2.88 > \hat{x} = 2.86 > \hat{x} - x_s(\hat{R}),$$

so that the real balances of the initial old rise by more than enough to compensate them for the tax imposed to finance initial government lending.

Secondly, we have already seen that the utility of type 3 agents is $EU_3 = 1.41$ in the post-intervention LIME. It is also easy to compute $EU_1 = b + 5.23$, and $EU_2 = 2.46$. Thus the policy results not only in a Pareto improvement over the LIME, but over the LIE as well.[15/] This establishes that, in the presence of the suggested policy, restriction of competition in the issue of liabilities can produce Pareto improvements.

IV. Conclusions

For some time, "credit rationing" was thought to be the channel through which monetary policy operated. Considerable attention has been focused for this reason on the "micro foundations" of credit rationing. Among the reasons deduced for its existence is asymmetric information. The analysis here has adopted this approach in a framework sufficiently rich to allow for the presence of money and a discussion of the welfare implications of alternate policies. It was seen that when money is present, under extremely plausible circumstances (except for possibly the initial old) it is optimal to make liberal use of the "discount window," or of government lending. Moreover, for some economies a policy where the government lends as much as desired at above market rates of interest is both Pareto improving and deflationary. Finally, in the presence of such a policy, only a limited set of factors under the control of the government influence private market rates of interest and credit volume.

It should be noted that these results are not deducible in the absence of money. Consideration of a steady state monetary equilibrium alters the analysis since the presence of money bounds below the rate of return on private debt. This is a forceful argument that if "credit rationing" is to provide a role for monetary policy, this policy must be analyzed in the context of the model generating the credit rationing. This paper seems to be the first to attempt such an analysis.

Moreover, it is also not the case that these results are deducible in the absence of asymmetric information. In particular, one could consider a world with full information where risky borrowers exercise monopsony power in credit markets. It is not the case that such an economy behaves like the one examined here. Specifically, it can be verified by considering the economy of examples 1-3 that the policy suggested is not welfare improving in a full-information setting. Thus the informational asymmetry is also essential to the analysis.

Finally, it should be noted that we have <u>not</u> provided an irrelevance argument for further government policy actions. As an example, suppose that the rule governing the evolution of the money stock is amended to

$$M_t = (2/3)\ M_{t-1}.$$

Then the steady state deflation rate becomes $S_{t+1}/S_t = 3/2$, and $\hat{R}$ rises for the example in the text.

In contrast to the discussion over the "neutrality of money," then, policy analysis is fundamentally different in the presence of valued fiat money. Thus if credit rationing operates, and this is to provide a role for monetary policy, money must be present in the model. If this is the case the result is an emphasis on discount window and government lending policies. It remains to be demonstrated that in the presence of such policies other monetary policies are effective, or desirable.

Footnotes

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1/E.g., Barro (1976), Fried and Howitt (1980), Jaffee and Russell (1976), Smith (1972), Stiglitz and Weiss (1981).

2/In fact, of course, this represents a restriction on preferences.

3//See, e.g., Diamond (1980).

4/Note that the J-R argument is that banks do not lend as much as borrowers demand at prevailing interest rates so as to replicate the monopsonistic equilibrium outcome in credit markets. While it is difficult to accept this as an explanation of real world credit rationing, this is a commonly accepted rationale for its existence. Hence we proceed along these lines.

5/For a more formal definition of FIEs and LIEs along these lines, see Smith (1981).

6/See Smith (1981).

7/Even in the J-R analysis this is an implicit assumption. Otherwise there would be no need for limited information as a rationale for credit rationing. Moreover, the full information monopsonistic equilibrium is of no particular interest. This is discussed further below.

8/In fact, the statement holds for any pre-intervention interest rate which results in credit rationing, or in other words, holds for a wide variety of equilibrium concepts other than that of a LIME.

9/We require in all cases transfers to be received only in the period (and states) in which the agent in question has a positive endowment. Otherwise we are allowing government redistribution of income which is not feasible for agents, and the proposition is trivial. In addition, it is this requirement which makes government transfers informationally feasible. It is an (implicit) assumption that the receipt of each agent's second period endowment is public information. Thus, while type-dependent transfers to type 2 and 3 agents are not possible when these agents are young (they are indistinguishable), they are possible when these agents are old and the second period endowment has been realized (so long as $w_2 \neq w_3$).

10/This follows from the fact that the economy was initially in a LIME, and from type 2 agents' maximization problem in the presence of government policy.

11/Where strict inequality holds because $\hat{R} > \frac{2}{1+p}$. This is easily seen from the conditions in section 3b for the optimal value of $\hat{R}$.

12/Although below we will see that it can be.

13/This is irrelevant in the J-R analysis because lenders are risk neutral. It also does not appear in the demonstration of Proposition 1 because of the focus on "small" credit rationing.

14/Note that the government lends goods in the setting described in Section A. However, it does not matter if the government lends fiat currency instead, so long as loans must be repaid in money, and tax-transfers are payable only in money.

[15]Type 1 utility increases because of the reduction in variance accompanying the policy. This was omitted in the approximation used for Proposition 1, which explains why type 1 agents experience a utility increase as opposed to the constant utility predicted by the proposition. The disparity arises because credit rationing in this example is not "small."

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