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A MONETARIST APPROACH TO FEDERAL BUDGET CONTROL

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I. Introduction and Summary of Results

Limiting the growth of the monetary base can effectively limit the size of federal deficits.[1/] By preventing monetization of federal debt above some level, a predetermined path for the monetary base puts federal debt issue on much the same footing as that of state and local governments. While debt still can be issued, above some level it now must be backed by higher government revenue in the future.

Suppose that monetary policy, as specified by a path for the monetary base, and fiscal policy, as specified by a path for deficits, are initially on consistent courses. Now, suppose fiscal policy makers do not recognize the limits to deficits imposed by monetary policy and attempt to raise the deficit path. The question is, what must give? Or more specifically, what developments might we expect to observe in an economy which attempted to follow such policies?

The paper seeks answers to these types of questions by extending the analyses of "Fiscal Policy in a Monetarist Model" and "Optimal Crowding Out in a Monetarist Model."[2/] Since only stationary solutions are examined in those papers, they cannot address the present question. The first paper assumes constant returns to capital in production, while the second assumes decreasing returns. In this paper nonstationary solutions are examined and a production technology is assumed which can display either decreasing or constant returns to capital.

The extension of the earlier models to allow nonstationary solutions yields some new insights. First, in the earlier papers it is claimed that under government restrictions which prohibit private borrowing and lending, the models exhibit many monetarist properties; a principal one being that the rate of inflation equals the growth rate of the monetary base.[3] But since only stationary solutions are examined, the rate of inflation also equals the growth rate of interest-bearing government debt, so that one cannot determine whether the source of inflation is monetary policy, fiscal policy, or some combination of the two. In the present paper the growth rate of bonds is allowed to differ from the growth rate of money and it is clear that the immediate source of inflation is solely monetary policy.

Second, in "Fiscal Policy" it is proved that once monetary and fiscal policies are on consistent courses, a higher deficit path cannot be financed by bond issue alone. This result assumes, however, that the only allowable policies are ones for which money and bonds grow at a common constant rate over time.

While in the present paper the growth rate over time of money still is assumed to be constant, that of bonds is allowed to be variable. It now follows that additional deficits can be financed by bond issue alone, but only to the extent that the initial real revenue from bonds is below the maximal steady-state level.[4] This result has three corollaries. One is that no additional deficits can be financed if policies are optimal initially. That is because optimality requires bonds to be issued in a quantity at least as great as the steady-state revenue maximizing amount. A second corollary is that no additional deficits can be financed when there are constant returns to capital and the real rate of interest is positive. That follows because when the real rate is positive, the maximal steady-state revenue from bonds is zero. A third corollary is that no additional deficits can be financed when there are decreasing returns to capital and the real rate of interest is above a critical, negative level. That is because in this case the maximal steady-state revenue from bonds occurs at a negative real interest rate. While an increase in bond issue at this interest rate has a positive quantity effect (increase in quantity at constant price), that effect is more than offset by a negative price effect (decrease in price at constant quantity). The critical real interest rate can be significantly negative depending on the production technology, initial endowments, and preferences.

Third, when fiscal policy becomes excessively expansionary relative to monetary policy, the government is forced into insolvency. The insolvency can come about in a number of ways. If, for example, the initial increase in the deficit exceeds the capacity of the government to raise real revenues through money and bond issues, it is immediately insolvent. But in other cases the insolvency takes time, and its course depends on the production technology and the permanency of the deficit increase. The key development in all these cases is a growing level of real interest payments on the debt, which together with the ongoing deficit, eventually exceeds the government's ability to raise real revenues from money and bond issue.

An increase in government borrowing always causes a decline in private capital investment. When there are decreasing returns to capital in production, the decline in investment is accompanied by a rise in the real interest rate. Thus, when fiscal policy becomes excessively expansionary and there are decreasing returns to capital, the growth in real interest payments on the debt is mirrored by a rising real interest rate and falling rate of private capital investment.

Finally, it is shown in "Fiscal Policy" that absent restrictions on private borrowing and lending, the government has only one debt instrument which must pay a rate of return at least as great as that on capital. Absent restrictions, monetary policy--the exchange of one government debt instrument for another--is irrelevant and deficits are more inflationary than when there are no restrictions.

If an economy with decreasing returns to capital initially is operating in a monetarist type setting--that is, government restrictions create separate demands for interest-bearing and noninterest bearing debt--an increase in deficits which is not accommodated by monetary policy can be expected to increase private incentives to circumvent those restrictions. Those incentives depend on the difference between the rates of return on bonds and money, and, as the present paper shows, that difference grows when deficits increase while the monetary base path remains invariant. Thus, not only is there the problem under a monetarist regime of higher deficits causing more inflation by requiring faster money growth in the future,[5/] there is also the problem of higher deficits causing more inflation by motivating the private sector to break down the restrictions between money and bonds and, thus, make bonds more liquid.

In the next section, the model is described in full, but the defense and interpretation of the set-up is left to the earlier papers. In the third section, the time paths of macro variables are examined under temporary and permanent increases in deficits coupled with a nonaccommodative monetary policy. Some numerical examples are presented. In the last section, the possibility is discussed that crowding out will spur private monetization of federal debt and make deficits more inflationary.

II. The Model

In any time period $t > 2$, the model is populated by agents born two periods ago $\{N(t-2)\}$--the "old"--, by agents born in the previous period $\{N(t-1)\}$--the "middle aged"--and by agents born in the current period $\{N(t)\}$--the "young." The number of individuals in each generation $\{N(t)\}$ is assumed constant, N.

Individuals can hold their wealth in three forms. First, they can invest in physical capital at time t.[6/] If they invest k(t) in terms of real goods, their investment is worth zero after one period and $Ak(t)^{\delta}$ after two periods, where $A > 0$ and $0 < \delta < 1$. The real gross rate of return over two periods on capital k(t) invested today, X(t), is, thus, given by

$$X(t) \equiv \frac{Ak(t)^{\delta}}{k(t)} = Ak(t)^{\delta-1}.$$

This technology implies constant returns to scale when $\delta=1$ and decreasing returns to scale when $\delta < 1$.

A second form of wealth is fiat money. If individuals purchase m(t) dollars at time t, they must give up p(t)m(t) in terms of goods. The variable p(t) is then the inverse of the price level. At time t+1 the money holdings are worth p(t+1)m(t) in real terms. The real gross rate of return over one period on money p(t)m(t) invested today, $R_1(t)$, is, thus, given by

$$R_1(t) \equiv \frac{p(t+1)m(t)}{p(t)m(t)} = \frac{p(t+1)}{p(t)}.$$

A third form of wealth is government-issued fiat bonds. One unit of bonds issued in period t can be redeemed for zero units of fiat money after one period and one unit of fiat money after two periods. Thus, if individuals purchase b(t) units of bonds in period t at a price in terms of fiat money of v(t), then in real terms they are making an investment of p(t)v(t)b(t). The real gross rate of return over two periods on bonds p(t)v(t)b(t) invested today, $R_2(t)$, is thus given by

$$R_2(t) \equiv \frac{p(t+2)b(t)}{p(t)v(t)b(t)} = \frac{p(t+2)}{p(t)v(t)}.$$

From these gross rates of return we define, respectively, the net two-period real rate of return on capital $\rho(t)$, the two-period rate of inflation $\Pi(t)$, and the two-period nominal interest rate r(t) by

$$\rho(t) \equiv X(t)-1,$$

$$\frac{1}{1+\Pi(t)} \equiv R_1(t)R_1(t+1) = \frac{p(t+2)}{p(t)}, \text{ and}$$

$$\frac{1}{1+r(t)} \equiv v(t)$$

For both fiat bonds and capital to be held, the condition $R_2(t) = X(t)$ must hold. In terms of the definitions above, this condition can be interpreted as a discrete-time Fisher effect,

$$\frac{1+r(t)}{1+\Pi(t)} = 1 + \rho(t), \text{ or } r(t) \approx \rho(t) + \Pi(t).$$

For money not to dominate bonds and capital as a two-period investment, the nominal rate of interest $r(t)$ must be nonnegative, that is

$$R_2(t) > R_1(t)R_1(t+1) \Rightarrow r(t) > 0.$$

The government consists of independent fiscal and monetary authorities. The fiscal authority consumes $G(t)$ of goods in period t and sells new bonds in the open market $\tilde{B}(t)$ to pay for its consumption and to retire maturing privately-held bonds $B(t-2)$:

$$G(t) + p(t)B(t-2) = p(t)v(t)\tilde{B}(t).$$

The monetary authority purchases government bonds in the open market by printing new money:

$$v(t)[\tilde{B}(t) - B(t)] = M(t) - M(t-1).$$

The consolidated government budget constraint, then, is

$$G(t) + p(t)B(t-2) = p(t)[v(t)B(t)+M(t)-M(t-1)].$$

In addition to consuming and issuing debt, the government is assumed to costlessly enforce a prohibition on private borrowing and lending. Such a prohibition is necessary to prevent private trading of government debt, which would eliminate by arbitrage the difference in returns on money and bonds.

The objective of each individual $h \varepsilon \{N(t)\}$ is to maximize a log-linear utility function subject to an endowment vector $< w_1^h(t), w_2^h(t), w_3^h(t) >$. The problem of the current old is to maximize $\gamma \ln c_3^h(0)$ with respect to $c_3^h(0)$ subject to

$$c_3^h(0) \leq w_3^h(0) = p(2)B(0)/N + X(0)K(0)/N,$$

where $\gamma > 0$, $B(0) > 0$ and $K(0) > 0$ are given, and capital letters denote economy-wide totals: N times individual holdings.

The problem of the current middle-aged is to maximize $\beta \ln c_2^h(1) + \gamma \ln c_3^h(1)$ with respect to $c_2^h(1)$ and $c_3^h(1)$ subject to

$$c_2^h(1) \leq w_2^h(1) = p(2)M(1)/N$$

$$c_3^h(1) \leq w_3^h(1) = p(3)B(1)/N + X(1)K(1)/N,$$

where $\beta > 0$ and $M(1)>0, B(1)>0, K(1)>0$ are given.

Finally, the problem of the current young and all future generations is to maximize $\ln c_1^h(t) + \beta \ln c_2^h(t) + \gamma \ln c_3^h(t)$ with respect to $c_1^h(t)$, $c_2^h(t)$, and $c_3^h(t)$ subject to

$$c_1^h(t) \leq w_1^h(t) - p(t)m^h(t) - p(t)v(t)b^h(t) - k^h(t)$$

$$c_2^h(t) \leq w_2^h(t) + p(t+1)m^h(t)$$

$$c_3^h(t) \leq w_3^h(t) + p(t+2)b^h(t) + X(t)k^h(t),$$

where $< w_1^h(t), w_2^h(t), w_3^h(t) > = < y,0,0 >$ and $y > 0$.

The three inequalities can be expressed as equalities and can be collapsed into the single constraint

$$c_1(t) + c_2(t)/R_1(t) + c_3(t)/R_2(t) = y,$$

where the h's are now being suppressed. This optimization problem generates the following individual demand functions for consumption and assets for $t > 2$:

$$\hat{c}_1(t) = \frac{y}{1+\beta+\gamma}$$

$$\hat{c}_2(t) = \frac{\beta\ R_1(t)y}{1+\beta+\gamma}$$

$$\hat{c}_3(t) = \frac{\gamma R_2(t)y}{1+\beta+\gamma}$$

$$\hat{m}^d(R_1(t),\ R_2(t)) \equiv p(t)\hat{m}(t) = \frac{\beta y}{1+\beta+\gamma}$$

$$\hat{b}^d(R_1(t),R_2(t)) \equiv p(t)v(t)\hat{b}(t) = \begin{cases} \dfrac{\gamma y}{1+\beta+\gamma} & \text{if } R_2(t) > X(t) \\ \dfrac{\alpha(t)\gamma y}{1+\beta+\gamma} & \text{if } R_2(t) = X(t) \end{cases}$$

where $\alpha(t)\varepsilon[0,1]$ (the proportion of third-period consumption financed by bonds) is arbitrary

$$\hat{k}^d(R_1(t),\ R_2(t)) = \frac{\gamma y}{1+\beta+\gamma} - \hat{b}^d(R_1(t),\ R_2(t))$$

We then have the following set of equilibrium conditions for $t > 2$:

1. $M(t) = N\hat{m}(t)$, money market equilibrium, which can be expressed

$$p(t)M(t) = \frac{N\beta y}{1+\beta+\gamma}$$

2. $B(t) = N\hat{b}(t)$, bond market equilibrium, which can be expressed

$$p(t)v(t)B(t) = \frac{\alpha(t)N\gamma y}{1+\beta+\gamma}$$

3. $$G(t) + p(t)B(t-2) = p(t)\ [v(t)B(t)+M(t)-M(t-1)],$$

goods market equilibrium or government budget constraint.

Since $R_2(t) > X(t)$ is required for the government to sell bonds, we also have

4. $$\frac{p(t+2)}{p(t)v(t)} \begin{cases} > X(t) \text{ and } \alpha(t) = 1 \text{ or} \\ = X(t) \text{ and } \alpha(t) \varepsilon [0,1] \end{cases}$$

where $X(t) \equiv A[(1-\alpha(t))(\frac{\gamma y}{1+\beta+\gamma})]^{\delta-1}$, $A > 0$, $0 < \delta < 1$.

With $G(t)$ and $M(t)$ given, equations 1-4 determine the time paths of $B(t)$, $p(t)$, $v(t)$, and $\alpha(t)$.

III. Limits to Deficits

Suppose monetary and fiscal policies are consistent and stationary initially in the sense that for all $t > 2$:

a. $G(t)^o + p(t)^o B(t-2)^o = p(t)^o\ [v(t)^o \tilde{B}(t)^o + M(t)^o - M(t-1)^o]$

(they imply budget balance at equilibrium prices and interest rates)

b. $\frac{M(t)^o}{M(t-1)^o} = 1+g^o$ (the growth rate of money is constant over time)

c. $G(t)^o = G^o$ (the rate of real government consumption is constant over time)

d. $\alpha(t) = \alpha^o$ (government's absorption of long-term investment funds is constant over time)

Condition a states that the policies are feasible. Condition b, in conjunction with the money market equilibrium condition, implies that inflation is constant: $(1 + \Pi^o) = (1+g^o)^2$. Conditions c and d, in conjunction with the bond market equilibrium condition, imply that the growth rate of bonds is equal to the growth rate of money and that the rate of interest $r(t)$ is constant over time, $r(t)^o = r^o$.[7]

In this section we examine how a new deficit path[8/] $G(t)$ coupled with the initial money path $M(t)^o$ determine new paths for $B(t)$, $p(t)$, $v(t)$ and $\alpha(t)$. These paths then determine the evolution of all net rates of return: $r(t)$, $\Pi(t)$, and $\rho(t)$.

The money market equilibrium condition indicates that the path of prices is invariant to a change in $G(t)$ as long as $M(t)$ is unchanged:

1. $$p(t)M(t)^o = \frac{N\beta y}{1+\beta+\gamma} \Rightarrow p(t) = p(t)^o \text{ for any } G(t)$$

Thus, the equilibrium conditions 2 - 4 form a system of equations which determine $B(t)$, $v(t)$, and $\alpha(t)$. It is convenient to write this system in terms of first differences ($\Delta Y(t) = Y(t) - Y(t)^o$ for any variable Y) and to substitute the linear approximation to 4 ($\ln(1+\varepsilon)\approx\varepsilon$). This transformed system is then:

2. $$p(t)^o\Delta[v(t)B(t)] = Z_2\Delta\alpha(t), \text{ where } Z_2 = \frac{N\gamma y}{1+\beta+\gamma}$$

3. $$\Delta G(t) + p(t)^o\Delta B(t-2) = p(t)^o\Delta[v(t)B(t)]$$

4. $$\Delta v(t) = \left[\frac{-(1-\delta)v^o}{1-\alpha^o}\right]\Delta\alpha(t)$$

This system is used to solve forward for $\Delta\alpha(t)$, $\Delta v(t)$ and $\Delta B(t)$ from t=2 given a path $\Delta G(t)$ and given all Δ's equal to zero for t = 0 or 1.

Solutions are considered for both temporary and permanent increases in G, when $\delta = 1$ and when $\delta < 1$. In all cases it is assumed that $\alpha^o < 1$, because if $\alpha^o = 1$ no additional real revenue can be raised by bond issue. Solutions to the system are examined at t = 2 and for arbitrary t > 2. The condition for insolvency is a solution value of $\alpha(t) > 1$ for some t > 2.

The new G paths are defined as follows. For a temporary increase in G we have

$$G(t) = \begin{cases} G(t)^o + \Delta G & t = 2,3 \quad \Delta G > o \\ G(t)^o & t > 3 \end{cases}$$

(Temporary is defined as 2 periods, because of the two-period maturity of bonds. This definition makes odd and even periods alike.)

For a permanent increase in G we have

$$G(t) = G(t)^{o} + \Delta G \quad \forall t \quad \Delta G > 0.$$

A. Temporary increase in G when $\delta = 1$

The solution to the system at $t = 2$ is given by

$$\Delta\alpha(2) = \frac{\Delta G}{Z_2}$$

$$\Delta v(2) = 0$$

$$\Delta B(2) = (v^{o}p(2)^{o})^{-1}(\Delta G)$$

The general solution of the system at even periods is given by

$$\Delta\alpha(2t) = R_2^{o(t-1)}\frac{\Delta G}{Z_2}$$

$$\Delta v(2t) = 0$$

$$\Delta B(2t) = (v^{ot}p(2)^{o})^{-1}\Delta G \quad , \text{ where } t=1,2,3....$$

The solution for odd periods is similar to the one for previous even periods:

$$\Delta\alpha(2t+1) = R_2^{o(t-1)}\frac{\Delta G}{Z_2}$$

$$\Delta v(2t+1) = 0$$

$$\Delta B(2t+1) = (v^{ot}p(3)^{o})^{-1}\Delta G \quad , \text{ where } t=1,2,3....$$

Thus, in periods 2 or 3 G can be financed only if

$$\alpha^{o} + \Delta\alpha(t) = \alpha^{o} + \frac{\Delta G}{Z_2} < 1 \iff \Delta G < (1-\alpha^{o})Z_2.$$

The right-hand side of this expression represents the maximal one-time increase in real government revenue from bond issue which can be achieved by increasing the absorption of investment funds from the initial proportion to the whole thing. If $\Delta G < (1-\alpha^o)Z_2$, the general solution indicates that $\alpha(t) <$ 1 for all t if, and only if, $R_2^o < 1$.

It follows from Proposition 4 of "Fiscal Policy" that no $\Delta G > 0$ can be financed if the initial policy is optimal. Proposition 4 states that when $R_2^o < 1$, any policy with $\alpha^o < 1$ can be Pareto dominated by a policy with $\alpha^1 > \alpha^o$.

B. Permanent Increase in G when $\delta = 1$.

The solution to the system at t = 2 and 3 is the same as for the temporary increase:

$$\Delta\alpha(2) = \Delta\alpha(3) = \frac{\Delta G}{Z_2}$$

$$\Delta v(2) = \Delta v(3) = 0$$

$$\Delta B(2) = \left(v^o p(2)^o\right)^{-1}(\Delta G)$$

$$\Delta B(3) = \left(v^o p(3)^o\right)^{-1}(\Delta G)$$

The general solution is given by the following for t = 2,3....

$$\Delta\alpha(2t) = \Delta\alpha(2t+1) = \left(\sum_{i=0}^{t-1} R_2^{o\,i}\right)\left(\frac{\Delta G}{Z_2}\right)$$

$$\Delta v(2t) = \Delta v(2t+1) = 0$$

$$\Delta B(2t) = \left(v^{ot} p(2)^o\right)^{-1}\left(\sum_{i=0}^{t-1} R_2^{o\,(-i)}\right)\Delta G$$

$$\Delta B(2t+1) = \left(v^{ot} p(3)^o\right)^{-1}\left(\sum_{i=0}^{t-1} R_2^{o(-i)}\right)\Delta G$$

In periods 2 and 3 we again have the condition that $\Delta G < (1-\alpha^o)Z_2$. This condition is not stringent enough to insure feasibility, however, because the general solution indicates that α must grow over time to finance the ongoing increase in the deficit and growing nominal interest payments. As in the earlier case, if $R_2^o > 1$ the higher deficit path cannot be financed. But now if $R_2^o < 1$, $\Delta\alpha(2t)$ approaches a finite limit:

$$\lim \Delta\alpha(2t) = \left(\frac{1}{1-R_2^o}\right)\frac{\Delta G}{Z_2}$$

Thus, for α to never exceed 1 requires

$$\alpha^o + \left(\frac{1}{1-R^o}\right)\frac{\Delta G}{Z_2} < 1 \iff \Delta G < (1-\alpha^o)(1-R_2^o)Z_2.$$

The right-hand side of this expression represents the maximal increase in steady-state real government revenue from bond issue, which can be achieved by increasing α from α^o to 1. To see this, the earlier paper shows that the steady-state real tax from bond issue is given by $(1-R_2)B^d(R_1,R_2) = (1-R_2)\alpha Z_2$. With the initial tax at $((1-R_2^o)\alpha^o Z_2)$, the maximal increase in the tax is $(1-R_2^o)Z_2 - (1-R_2^o)\alpha^o Z_2 = (1-\alpha^o)(1-R_2)Z_2$.

To summarize, a permanent increase in the deficit can be financed if, and only if, $R_2^o < 1$ and $\Delta G < (1-\alpha^o)(1-R_2^o)Z_2$. Again, by Proposition 4 of "Fiscal Policy" it follows that no increase in deficits can be financed if the initial policy is optimal.

C. Increases in G when $\delta < 1$

The solution to the system in general form could not be found. So in this section we first compare the general solution for $\Delta\alpha(4)$ at t=2 and t=4 under temporary and permanent increases in the deficit and under constant and decreasing returns to capital. The comparison indicates that a given increase in the deficit is more difficult to finance when the increase is permanent and when there are decreasing returns to capital. We next examine the solution to the system for arbitrary even t when there is a temporary increase in the deficit and decreasing returns to capital, and when the initial equilibrium value α^o is at the steady-state bond revenue maximizing point $\hat{\alpha}$. In "Crowding Out" it is shown that $\hat{\alpha}$ solves

$$\frac{1-\hat{\alpha}}{1-\hat{\alpha}\ \delta} = X \equiv A\ [(1-\hat{\alpha})\ (\frac{\gamma y}{1+\beta+\gamma})]^{\delta-1}$$

No increase in the deficit is found to be feasible at $\alpha^o = \hat{\alpha}$.

1. General solution for $\Delta\alpha(t)$ at t=2 and t=4

In all cases we have $\Delta\alpha(2) = \frac{\Delta G}{Z_2}$. The solution for $\Delta\alpha(4)$ in different cases is shown below.

$\Delta\alpha(4)$

Technology / Deficit Increase	$\delta = 1$	$\delta < 1$
Temporary	$R_2^o \frac{\Delta G}{Z_2}$	$(\frac{R_2^o}{1+h})\frac{\Delta G}{Z_2} - R_2^o\alpha^o(\frac{h}{1+h})$
Permanent	$(1+R_2^o)\frac{\Delta G}{Z_2}$	$(1 + \frac{R_2^o}{1+h})\frac{\Delta G}{Z_2} - R_2^o\ \alpha^o(\frac{h}{1+h})$

In the table $h \equiv \frac{-(1-\delta)}{1-\alpha^o}\frac{\Delta G}{Z_2}$. With $\delta < 1$ and with $\Delta G < (1-\alpha^o)Z_2$ required for feasibility it follows that $-1 < h < 0$. The table indicates that $\Delta\alpha(4)$ is larger for a given deficit increase when $\delta < 1$ than when $\delta = 1$. The reason is that the deficit increase causes a rise in the real interest rate when $\delta < 1$ and thus raises interest payments for a given amount of debt. It also follows that $\Delta\alpha(4)$ is larger for a given technology when the increase is permanent than when it is temporary. The reason obviously is that the present value of the permanent increase exceeds that of the temporary increase.

2. Temporary Increase in G when $\delta < 1$ and $\alpha^o = \hat{\alpha}$.

Since we want to show that any increase in G causes α to eventually exceed 1, it is necessary to examine the solution only for general even t's. Further, it follows that no permanent increases in G are feasible if no temporary increases are.

The solution at even periods is given by

$$\Delta\alpha(2t) = [\frac{1}{1+(t-1)h}][R_2^{o^{(t-1)}}\frac{\Delta G}{Z_2} - (\sum_{i=1}^{t-1} R_2^{oi})h\alpha^o],$$

$$\Delta v(2t) = \frac{hv^o}{1+(t-1)h}, \text{ and}$$

$$\Delta B(2t) = (\frac{1}{1+th})(\frac{1}{v^{ot}p(2)^o})[\Delta G - (\sum_{i=0}^{t-1} R_2^{o-i})h\alpha^o Z_2],$$

where $t=1,2,3\ldots$, $\sum_{i=1}^{0}(\cdot) \equiv 0$, and h is defined as before. Since $-1 < h < 0$, it follows there exists a t^* such that $B(2t^*)$ becomes arbitrarily large and subsequently $\alpha(2t^*+2)$ becomes arbitrarily large and $v(2t^*+2)$ becomes arbitrarily small (and negative).

D. Numerical Examples

In this section we examine the evolution of the system in different cases for particular parameter values. The actual equation 4 is used instead of the linear approximation. The initial steady-state equilibrium values are taken from an example in "Fiscal Policy."

<u>Initial Equilibrium</u>

Parameters:

$y = 1{,}000$

$\beta = .9$

$\gamma = .6$

$N = 100$

$G = 10{,}000$

$M(1) = 36{,}000$

$B(0) = 24{,}000$

$K(0) = K(1) = 0$

$R_2^o = .9$

$\alpha^o = .6$

Initial Equilibrium Values

$p(2)^o = .673$

$v^o = .645$

$r^o = .550$

$\Pi^o = .721$

$M(2)^o = 53{,}492$

$B(1)^o = 25{,}252$

$B(2)^o = 33{,}122$

and for general t

$$\frac{M(t+1)^o}{M(t)^o} = \frac{B(t+1)^o}{B(t)^o} = \frac{p(t)^o}{p(t+1)^o} = (1+\Pi^o)^{1/2}$$

In the numerical examples which follow parameters of the production function are chosen so that $X = .9$ in the initial equilibrium. Thus, with

$$X \equiv A\ [(1-\alpha^o)(\frac{\gamma y}{1+\beta+\gamma})]^{\delta-1},$$

different technologies are specified by choices of A and δ such that $A[96]^{\delta-1} = .9$.

Six examples are examined:

1. Temporary Increase in G = 100, Constant Returns: $A = .9$, $\delta = 1.0$
2. Temporary Increase in G = 100, Decreasing Returns: $A = 1.0$, $\delta = .977$
3. Permanent Increase in G = 100, Constant Returns: $A = .9$, $\delta = 1.0$
4. Permanent Increase in G = 100, Decreasing Returns: $A = 1.0$, $\delta = .977$

5. Permanent Increase in G = 1,000, Constant Returns: $A = .9$, $\delta = 1.0$

6. Permanent Increase in G = 1,000, Decreasing Returns: $A = 2.0$, $\delta = .825$

In the first four examples the deficit increase can be financed, while in the latter two it cannot. The solution values for these examples are displayed below:

Numerical Examples[1]

	1			2			3			4			5			6		
	Temporary $\Delta G = 100$ $\delta = 1.0$			Temporary $\Delta G = 100$ $\delta = .977$			Permanent $\Delta G = 100$ $\delta = 1.0$			Permanent $\Delta G = 100$ $\delta = .977$			Permanent $\Delta G = 1000$ $\delta = 1.0$			Permanent $\Delta G = 1000$ $\delta = .825$		
t	α	ρ	$\Delta g(B)$	α	ρ	$\Delta g(B)$	α	ρ	$\Delta g(B)$	α	ρ	$\Delta g(B)$	α	ρ	$\Delta g(B)$	α	ρ	$\Delta g(B)$
2	.604	-.100	1.0	.604	-.100	1.0	.604	-.100	1.0	.604	-.100	1.0	.642	-.100	9.6	.642	-.083	12.5
3	.604	-.100	1.2	.604	-.100	1.2	.604	-.100	1.2	.604	-.100	1.2	.642	-.100	12.0	.642	-.083	15.5
4	.604	-.100	-0.1	.604	-.100	-0.1	.608	-.100	1.1	.608	-.100	1.1	.679	-.100	10.1	.690	-.059	17.9
5	.604	-.100	0.1	.604	-.100	-0.1	.608	-.100	1.1	.608	-.100	1.1	.679	-.100	10.1	.690	-.059	17.9
⋮																		
11	.603	-.100	-0.1	.603	-.100	-0.1	.617	-.100	0.8	.618	-.099	0.9	.771	-.100	6.3	.983	.557	126.8
12	.602	-.100	-0.1	.603	-.100	-0.1	.620	-.100	0.7	.621	-.099	0.9	.795	-.100	5.5	(1.631)[2]		
⋮																		
50	.600	-.100	0.0	.601	-.100	0.0	.639	-.100	0.1	.652	-.097	0.2	.987	-.100	0.6			
⋮																		
61	.600	-.100	0.0	.601	-.100	0.0	.640	-.100	0.1	.656	-.097	0.2	.999	-.100	0.3			
62	.600	-.100	0.0	.600	-.100	0.0	.640	-.100	0.0	.657	-.097	0.2	(1.001)[2]					
⋮																		
75	.600	-.100	0.0	.600	-.100	0.0	.641	-.100	0.0	.660	-.097	0.1						
⋮																		
100	.600	-.100	0.0	.600	-.100	0.0	.641	-.100	0.0	.664	-.096	0.1						

[1] Recall in the initial equilibrium $\alpha^0 = .6$, $\rho = -.100$ and for $t \geq 4$ $g(B) = 72.1$ where $g(B)$ is the two-period percentage growth rate of bonds. 'Δ' is the difference from the initial equilibrium and ρ is calculated by: $\rho = \frac{1 + r}{1 + \pi} - 1$.

[2] The value of α calculated violates a feasibility condition and cannot be attained.

The examples show different responses of variables in equilibrium to deficit increases. In the first two examples a temporary increase in the deficit causes near-term deviations from initial equilibrium values of α and $g(B)$, the government's required share of investment funds and growth rate of bonds respectively, but the differences steadily dissipate over time. Because the degree of concavity of the production function is so slight in the second example the crowding out of capital does not result in an appreciable increase in the real interest rate.

In the next two examples a permanent increase in the deficit causes α to increase immediately and to gradually attain a new equilibrium value. The higher deficit causes the growth of bonds to accelerate at first, but then the growth rate steadily declines to the rate in the initial equilibrium. In the fourth example the crowding out of capital eventually leads to a slightly higher real interest rate.

In the last two examples a permanent increase in deficit spending causes α to rise and at some point exceed what is available. In the fifth example the infeasibility occurs, even though the growth rate of bonds is slowing over time. In the sixth example α, ρ, and $g(B)$ all follow explosive paths.

IV. Are Deficits Inflationary?

Monetarists commonly maintain that inflation is caused by excessive money growth and that there is no necessary link between deficits and money growth.[9/] Therefore, deficits need not be inflationary. This paper sheds some light on these propositions.

It is sometimes possible in this paper to run higher deficits and not affect the inflation rate, but only when the initial monetary-fiscal policy mix results in an overaccumulation of capital. In this case a larger budget deficit can be financed by bond issue alone leaving the paths of money and prices unaffected.

If the rate of capital accumulation is not excessive, however, this paper indicates that it is infeasible to finance higher deficits by bond issue alone. The growth of money and, hence, the inflation tax must be raised in addition or the government will be forced into insolvency. Thus, at times there is a necessary link between deficits and money growth.

But this paper suggests another channel through which deficits can lead to inflation. For government fiat bonds not to be directly inflationary requires that they cannot be directly spent. In this paper it is assumed that the government costlessly enforces restrictions on the use of bonds, so that only money is spent.[10/] Without these restrictions individuals would monetize the debt, perhaps by making the bonds a medium of exchange or by issuing private notes backed by the bonds. Without restrictions bonds and money would become perfect substitutes, and deficits would be directly inflationary.

While restrictions on the use of bonds make sense for society as a whole, individuals have the incentive to circumvent them. If possible, it is always profitable to buy bonds and issue private notes or deposits paying something less than the market rate of interest. The higher the rate of interest the greater is the profit potential.

When there are decreasing returns to capital, an increase in budget deficits raises the interest rate on bonds. If the new policy mix is infeasible, the interest rate rises through time without limit. As the interest rate rises, private inventives to circumvent the restrictions also rise. To the extent that individuals succeed in breaking down the restrictions, given deficit policies become more inflationary. In this way the private sector monetizes the debt, even though the central bank doesn't.

Interest rates have remained high over the last few years, and many financial innovations have occurred which make government bonds more liquid. Money market mutual funds are perhaps the most prominent example. Thus, deficits run in the future are likely to have a bigger inflationary impact than they had in the past.

FOOTNOTES

1/See, for example, Sargent-Wallace and Miller. On the other hand, if fiscal policy doesn't budge, government debt must be monetized in the future causing higher inflation.

2/The two papers will be referred to as "Fiscal Policy" and "Crowding Out," respectively.

3/Those restrictions are assumed throughout the present paper.

4/Initial policies are steady-state policies.

5/This assumes that deficits are not offset by higher revenues in the future.

6/Any capital investment requires a minimum of one individual to manage it each period. This assumption is necessary to prevent capital from being divided into infinitesimal units yielding arbitrarily high returns.

7/The existence of such equilibria given appropriate parameter values is demonstrated in "Fiscal Policy" and "Crowding Out." The rationale for confining attention to such equilibria is given in the former paper.

8/G can be considered as the government's real deficit net of interest payments.

9/See, for example, Friedman, Hein, and Weintraub.

10/The rationale for such restrictions is discussed in "Fiscal Policies."

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