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OPTIMAL CROWDING OUT IN A MONETARIST MODEL

Preston J. Miller

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Optimal Crowding Out in a Monetarist Model

I. Introduction and Summary of Results

In a monetarist model private demands for fiat money and fiat bonds are distinct. Money is used for spending, and bonds are used for saving. Increased reliance on bond issue to finance government deficits, then, crowds out private capital holdings as a form of savings. If there are decreasing returns to capital, a reduction in capital holdings causes the real rate of interest to rise.

This paper examines this crowding out process in a modified version of "Fiscal Policy in a Monetarist Model." While the earlier paper assumed constant returns to capital in production, the present paper assumes decreasing returns. This modification yields some new insights into the macro and welfare effects of alternative policies to finance given real deficits. In both papers such policies consist of time paths of bonds and money which imply constant interest and inflation rates and allow the resources the government expends to match the resources it acquires.

In the earlier paper it is found that the macro and welfare effects of alternative policies depend on the sign of the real rate of interest. With a zero or negative rate, greater reliance on bond financing is found to reduce inflation and increase welfare. With a positive rate, greater reliance on bond financing is found to increase inflation, increase the welfare of the current old and middle-aged, but decrease the welfare of the current young and all future generations. It follows that as long as the real rate of return on capital is non-positive, bonds should be issued in sufficient quantity to drive out all capital. It also follows that a money-bond financing strategy which produces the minimum inflation rate feasible under a given deficit policy cannot be Pareto dominated by any other strategy.

In the present paper, in contrast, it is found that the inflationary consequence from greater reliance on bond issue depends on whether additions to the number of bonds sold increase or decrease real government revenues. In even sharper contrast, it is found that the minimum inflation policy is Pareto dominated by other higher inflation policies.

If additions to bond sales raise real government revenues, the same real deficit can be financed with a lower inflation tax. When there are constant returns to capital, it follows that the effect of additional bond sales on real revenues turns solely on the sign of the real rate of interest. They increase real revenues when the rate is negative and decrease real revenues when it is positive. With decreasing returns to capital, however, an increase in bond issue has both a "quantity" and a "price" effect: an increase in the quantity of bonds at the initial price less a decrease in the price at the initial quantity. In the current paper, the change in real government revenues from an increase in bond issue always becomes negative at some critical, negative real rate of interest which is reached before all capital is driven out. If bond issue is below this critical level, an increased reliance on bond financing lowers inflation. If it is at or above this critical level, however, an increased reliance on bond financing raises inflation. This critical level of bond issue then permits the lowest inflation rate for the given real deficit policy.

The minimum inflation policy is not an optimal policy. It results in too low of a real rate of return on capital. By issuing more bonds the government can effect a favorable trade for all by raising the real rate of return on capital at the expense of a lower real rate of return on money; that is, at the expense of more inflation.

The model is described in the next section. In the following section, the relationship between the critical level of bond issue and inflation is derived, and the relationship of the critical level to various parameters is examined. The welfare implications of alternative policies are explored, and the principal result that the minimum inflation policy is suboptimal is proved. The paper concludes with some brief comments about the role of the inflation tax in the optimal tax structure.

II. Basic Relationships

The model is exactly the same as in "Fiscal Policy in a Monetarist Model" except for the assumption about the storage technology. In both papers it is assumed that if $k(t)$ units of goods are stored at time t, the investment is worth zero at time $t + 1$ and $X(t)k(t)$ at time $t + 2$. In the earlier paper it is assumed that X is independent of k, while in the present paper it is assumed that $X(t) = Ak(t)^{\delta-1}$ where $A > 0$. With $\delta = 1$ we have constant returns to capital as assumed in the previous paper, while with $\delta < 1$ we have decreasing returns to capital as assumed in this paper. In order to prevent capital from being divided into arbitrarily small units to generate arbitrarily large returns when $\delta < 1$, it also is assumed that a minimum of one period of labor each period must be used as an input with any amount of capital stored.

Since except for the specification of $X(t)$ the current model is identical to the earlier one, the definitions and relationships required for analysis simply will be listed. Readers interested in derivations and explanations can refer to the earlier paper.

Time is discrete and runs from $t = 2, \ldots.$ N individuals are born each period, and they live for 3 periods. The current old $h \in \{N(0)\}$ consume their endowments at $t = 2$

$C_3^h(0) = W_3^h(0) \equiv p(2)B(0)/N + X(0)k(0)/N$, where

$C_i^h(t)$ is the real consumption of individual h in generation t in the i^{th} period of life;

$W_i^h(t)$ is the real endowment of individual h in generation t in the i^{th} period of life;

$B(t)$ is the total number of bonds issued at time t which are held by individuals. One bond can be purchased at a dollar price of $v(t)$. The bond pays nothing if held one period and one dollar if held two periods. In any period the bonds $B(t)$ are held in equal amounts by the N young, $Nb^h(t) = B(t)$;

$K(t)$ is the total capital stock in real terms held by the young at time t, $Nk^h(t) = K(t)$;

$p(t)$ is the amount of goods which exchanges for one dollar at time t; the inverse of the price level; and

$B(0) \geqq 0$, $K(0) \geqq 0$ are given.

The current middle-aged $h \in \{N(1)\}$ consume their endowments at t = 2 and t = 3

$$C_2^h(1) = W_2^h(1) \equiv p(2)M(1)/N$$

$C_3^h(1) = W_3^h(1) \equiv p(3)B(1)/N + X(1)K(1)/N$, where $M(t)$ is the total stock of money held by the young at time t, $Nm^h(t) = M(t)$; $K(1) \geqq 0$ is given; and $B(1) \geqq 0$ is a policy parameter.

The current young and all future generations $h \in \{N(t)\}$ $t \geqq 2$ maximize

$$\ln C_1^h(t) + \beta \ln C_2^h(t) + \gamma \ln C_3^h(t) \text{ subject to}$$

$$c_1^h(t) \leqq w_1^h(t) - p(t)m^h(t) - p(t)v(t)b^h(t) - k^h(t)$$

$$c_2^h(t) \leqq w_2^h(t) + p(t+1)m^h(t)$$

$$c_3^h(t) \leqq w_3^h(t) + p(t+2)b^h(t) + X(t)k^h(t), \text{ where}$$

$$\langle w_1^h(t),\ w_2^h(t),\ w_3^h(t)\rangle = \langle y,\ 0,\ 0\rangle, \text{ with } y > 0$$

and $\beta > 0$, $\gamma > 0$ are given.

This maximization problem generates the following individual demands for consumption and assets in a steady state:

$$\hat{c}_1 = \frac{y}{1+\beta+\gamma}$$

$$\hat{c}_2 = \frac{\beta y R_1}{1+\beta+\gamma}$$

$$\hat{c}_3 = \frac{\gamma y R_2}{1+\beta+\gamma}$$

$$\hat{m}^d(R_1,R_2) \equiv p(t)\hat{m}^h(t) = \frac{\beta y}{1+\beta+\gamma}$$

$$\hat{b}^d(R_1,R_2) \equiv p(t)v(t)\hat{b}^h(t) = \frac{\alpha\gamma y}{1+\beta+\gamma}, \text{ where } \alpha \in [0,1) \text{ is arbitrary}$$

$$\hat{k}^d(R_1,R_2) \equiv \hat{k}^h(t) = \frac{(1-\alpha)\gamma y}{1+\beta+\gamma}.$$

In the formulas above the t's and h's have generally been suppressed.

The gross rates of return R_1 and R_2 are defined by

$R_1 \equiv \frac{p(t+1)}{p(t)}$ and $R_2 = \frac{p(t+2)}{p(t)v(t)}$, which are independent of time in a stationary equilibrium. Since bonds and capital are perfect substitutes in individual portfolios, both will be held only if $R_2 = X$. Individuals will be indifferent to the proportions of bonds and capital in their portfolios at these rates.

The inflation rate Π, the nominal interest rate r, and the net real rate of return on capital ρ are all defined over two periods in terms of the two rates R_1 and R_2:

$$\frac{1}{1+\Pi} \equiv \frac{p(t+2)}{p(t)} \text{ or } \Pi = \frac{1}{R_1^{\,2}} - 1,$$

$$\frac{1}{1+r} \equiv v(t) \text{ or } r = \frac{R_2}{R_1^{\,2}} - 1, \text{ and}$$

$$\rho \equiv X(t) - 1 \text{ or } \rho = R_2 - 1.$$

A stationary equilibrium consists of sequences of prices $p(2)^*$, $p(3)^*$, ... and $v(2)^*$, $v(3)^*$, ..., such that:

a. the inflation rate Π and the nominal interest rate r are constant over time,

b. the aggregate demands for money, bonds, and goods equal the respective aggregate supplies:

i. $N\hat{m}^h(t) = M(t)$, $t = 2, \ldots$

ii. $N\hat{b}^h(t) = B(t)$, $t = 2, \ldots$

iii. $N\hat{C}_3(t-2) + N\hat{C}_2(t-1) + N\hat{C}_1(t) + K(t) + G(t) =$
$Ny + X(t)K(t-2)$, $t = 2, \ldots$, and

c. each individual maximizes utility subject to given endowments and prices $p(2)^*$, $p(3)^*$ ..., and $v(2)^*$, $v(3)^*$, ...

III. Alternative Feasible Stationary Policies with $\delta < 1$

In this section we define "feasible stationary policies," show that they can be characterized by the gross rates of return, R_1 and R_2, which they imply, and then examine how alternative policies affect macro variables and individual welfare. The analysis assumes decreasing returns to scale in production and parallels the analysis for constant returns to scale in "Fiscal Policy in a Monetarist Model."

By a "stationary policy" we mean a sequence of policy variables $<G(t), M(t), B(t)>_{t=0} \cdots$ such that $G(t) = G$ for all t and the resulting gross rates of return $R_1(t)$, $R_2(t)$ are constant for $t \geqq 2$. Under a stationary policy, the government's budget constraint for $t \geqq 4$ can be written:

$$G = (1-R_1)M^d(R_1,R_2) + (1-R_2)B^d(R_1,R_2),$$

where $M^d(R_1,R_2) \equiv Np(t)\hat{m}(t)$ and $B^d(R_1,R_2) \equiv Nvp(t)\hat{b}(t)$ are, respectively, the aggregate real demands for money and bonds. Stationary policies are feasible if they imply R_1 and R_2 such that $R_2 = X$, $R_2 \geqslant R_1{}^2 > 0$, and the rates satisfy the government's budget constraint.

We will choose to identify a stationary policy by a pair $<R_1,\alpha>$. When $\delta < 1$, a constant R_2 can be associated with a unique constant $\alpha \in [0,1)$, since

$$R_2 = R_2(t) = X(t) = A[1-\alpha(t))(\frac{\gamma y}{1+\beta+\gamma})]^{\delta-1}.$$

We then have the following proposition, which is proven in the previous paper: Define

$$S(G) = \{(R_1,\alpha) \mid (1-R_1)M^d(R_1,R_2) + (1-R_2)B^d(R_1,R_2) = G,$$
$$R_2 = X \geqslant R_1{}^2 > 0,\ B^d(R_1,R_2) = \frac{\alpha N\hat{C}_3}{R_2},\ \alpha \in [0,\ 1),$$
$$<M^d(R_1,R_2),\ B^d(R_1,R_2)> \ > \ <0,0>\}.$$

<u>Proposition 1</u>

If

(a) $(R_1,\alpha) \in S(G)$ and

(b) $B(0) + M(1) > 0$,

then an equilibrium is given by the $\{p(t)\}$, $\{v(t)\}$, $\{M(t)\}$, and $\{B(t)\}$ solutions to

(i) $\frac{p(t+1)}{p(t)} = R_1, \; t \geqq 2$

(ii) $v(t) = R_1^2/R_2, \; t \geqq 2$

(iii) $p(t)M(t) = M^d(R_1,R_2), \; t \geqq 2$

(iv) $v(t)p(t)B(t) = B^d(R_1,R_2), \; t \geqq 2$

with initial conditions

(v) $$p(2) = \frac{B^d(R_1,R_2) + M^d(R_1,R_2) - G}{B(0) + M(1)}$$

(vi) $$B(1) = \frac{[B(0) + M(1)]\,[B^d(R_1,R_2) + (1-R_1)M^d(R_1,R_2)-G]}{R_1[B^d(R_1,R_2) + M^d(R_1,R_2)-G]}$$

Note that for feasible stationary policies, the initial endowments $B(0)$ and $M(1)$ can be any positive quantities. Together, they determine the initial price level $p(2)$. The endowment $B(1)$, however, must be the unique quantity which guarantees that $\frac{p(3)}{p(2)} = R_1$.

In order to examine the effects of alternative policies, we can simplify notation and focus on a subset of equilibrium conditions. For our special log-linear model, the aggregate demand functions for money and bonds take very simple forms. Let $Z_1 = \frac{N\beta y}{1+\beta+\gamma}$ and $Z_2 = \frac{N\gamma y}{1+\beta+\gamma}$, both positive constants. Then $M^d(R_1,R_2) = Z_1$ and $B^d(R_1,R_2) = \alpha Z_2$. We can write $S(G)$ and condition (v) of Proposition 1 as,

(a) $(1-R_1)Z_1 + TR(\alpha) - G = 0$ and

(b) $$p(2) = \frac{Z_1 + \alpha Z_2 - G}{B(0) + M(1)},$$

where $TR(\alpha) \equiv [1-X(\alpha)]\alpha Z_2$ is total real government revenue from bond financing and $X(\alpha) \equiv A[(1-\alpha)Z_2/N]^{\delta-1}$. We now can examine how the macro variables (initial price level, rate of inflation, and real interest rate) change and how individual welfare changes when the index of policy α changes. Changes in α then can be related to changes in the sequences M and B, using the other conditions of Proposition 1.

From (b) we have

$\frac{dp(2)}{d\alpha} = \frac{Z_2}{B(0) + M(1)} > 0$, so that increased reliance on bond financing always raises the initial value of money, or alternatively, always lowers the initial price level.

From the definition of the gross rate of return on capital, we have

$$\frac{dX(\alpha)}{d\alpha} = \frac{(1-\delta)A[(1-\alpha)Z_2/N]^{\delta-1}}{1-\alpha} \equiv \frac{(1-\delta)X}{1-\alpha} > 0 \quad \text{for } 0 < \delta < 1 \text{ and } 0 \leqq \alpha < 1.$$

Thus, when there are decreasing returns to scale, an increased reliance on bond financing crowds out capital and raises the real rate of interest. From (a) we have $\frac{dR_1}{d\alpha} = \frac{1}{Z_1}\frac{dTR(\alpha)}{d\alpha}$. Thus, if an increase in α raises total real revenue from bond financing, R_1 must rise. With $R_1{}^2 = \frac{1}{1+\Pi}$, the rate of inflation then must fall.

Total revenue from bond financing has an internal maximum with respect to α. Call the maximizing value $\hat{\alpha}$. We then have

$$\frac{dTR(\alpha)}{d\alpha} \begin{cases} > 0 \text{ if } \alpha < \hat{\alpha} \\ = 0 \text{ if } \alpha = \hat{\alpha} \\ < 0 \text{ if } \alpha > \hat{\alpha} \end{cases}$$

By determining how the parameters A, δ, Z_2, and N affect $\hat{\alpha}$, we can determine how they affect $\frac{dTR(\alpha)}{d\alpha}$ and, consequently, $\frac{d\Pi}{d\alpha}$.

The first order condition for $\hat{\alpha}$ to be a maximizer of TR(α) is

$$\frac{dTR(\alpha)}{d\alpha} = (1-X(\hat{\alpha}))Z_2 - \hat{\alpha}Z_2 \frac{dX(\alpha)}{d\alpha} = 0.$$

At $\hat{\alpha}$ we have $\frac{d^2TR(\alpha)}{d\alpha^2} < 0$. Also, $\lim_{\alpha \to 0} TR(\alpha) = 0$, $\lim_{\alpha \to 1} TR(\alpha) = -\infty$, and $\left.\frac{dTR(\alpha)}{d\alpha}\right|_{\alpha = 0} = (1 - X(0))Z_2$.

Thus, if $X(0) < 1$, $\hat{\alpha} \in (0,1)$. If $X(0) \geqq 1$, $\hat{\alpha} = 0$.

Let us suppose $X(0) < 1$, which states that under zero bond issue, the net real rate of return on capital is negative. This condition is required for the issue of bonds to raise government's real revenue in a stationary equilibrium. We can write the first-order condition for $\hat{\alpha}$ to be an internal maximizer as

$$\hat{\alpha} + (1-\hat{\alpha}\delta)X(\hat{\alpha};\gamma) - 1 = 0, \text{ where}$$

$$\gamma \equiv \langle A, \delta, Z_2, N\rangle \text{ and}$$

$$X(\alpha;\gamma) \equiv A[(1-\alpha)Z_2/N]^{\delta-1}.$$

Then, we have

$$\frac{d\hat{\alpha}}{d\gamma_i} - \delta X(\hat{\alpha};\gamma)\frac{d\hat{\alpha}}{d\gamma_i} + (1-\hat{\alpha}\delta)\left[\frac{\partial X}{\partial \hat{\alpha}}\frac{d\hat{\alpha}}{d\gamma_i} + \frac{\partial X}{\partial \gamma_i}\right] = 0$$

for i = 1, 3, 4

$$\frac{d\hat{\alpha}}{d\delta} - \delta X(\hat{\alpha};\partial)\frac{d\hat{\alpha}}{d\delta} - \hat{\alpha}X(\hat{\alpha};\gamma) + (1-\hat{\alpha}\delta)\left[\frac{\partial X}{\partial \hat{\alpha}}\frac{d\hat{\alpha}}{d\delta} + \frac{\partial X}{\partial \delta}\right] = 0$$

These equations can be solved for $\frac{d\hat{\alpha}}{d\gamma_i}$ to yield

$$\frac{d\hat{\alpha}}{d\gamma_i} = \frac{-(1-\hat{\alpha}\delta)\frac{\partial X}{\partial \gamma_i}}{1 - \delta X(\hat{\alpha};\,\gamma) + (1-\hat{\alpha}\delta)\frac{\partial X}{\partial \hat{\alpha}}} \qquad i = 1,\ 3,\ 4$$

$$\frac{d\hat{\alpha}}{d\delta} = \frac{\hat{\alpha}X(\hat{\alpha};\,\gamma) - (1-\hat{\alpha}\delta)\frac{\partial X}{\partial \delta}}{1 - \delta X(\hat{\alpha};\,\gamma) + (1-\hat{\alpha}\delta)\frac{\partial X}{\partial \hat{\alpha}}}$$

The denominator D of each of the expressions above can be written

$$D = 1 - \delta X + \frac{(1-\alpha\delta)(1-X)}{\alpha}$$

So $D \equiv (\frac{1}{\alpha})\alpha D = \frac{1}{\alpha}[1-X+\alpha(1-\delta)] > 0$ for

$0 < \alpha < 1$, $0 < \delta < 1$, and $0 < X < 1$. 4/

So to determine the signs of $\frac{d\hat{\alpha}}{d\gamma_i}$, we need expressions for $\frac{\partial X}{\partial \gamma_i}$. They are given as follows;

$$\frac{\partial X}{\partial A} = [(1-\hat{\alpha})Z_2/N]^{\delta-1} > 0$$

$$\frac{\partial X}{\partial \delta} = A[(1-\hat{\alpha})Z_2/N]^{\delta-1}\ln[(1-\hat{\alpha})Z_2/N]$$

Writing $X = Ak^{\delta-1}$, we have

$$\frac{\partial X}{\partial \delta} \begin{cases} < 0 \text{ for } k < 1 \\ = 0 \text{ for } k = 1 \\ > 0 \text{ for } k > 1 \end{cases}$$

Also,

$$\frac{\partial X}{\partial Z_2} = (\delta-1)A[(1-\hat{\alpha})Z_2/N]^{\delta-2}((1-\hat{\alpha})/N) < 0$$

4/ Note $X(\alpha) < 1$ is necessary for $\alpha > 0$ to be a maximizer.

$$\frac{\partial X}{\partial N} = (\delta-1)A[(1-\hat{\alpha})Z_2/N]^{\delta-2}[-(1-\hat{\alpha})Z_2/N] > 0.$$

Thus, the gross rate of return on capital increases with

(a) an increase in the scale parameter A

(b) an increase in the exponential parameter δ when $k > 1$

(c) a decrease in the exponential parameter δ when $k < 1$

(d) a decrease in the amount of capital invested measured by either a rise in Z_2 for constant N or a fall in N for a constant Z_2.

Substituting the expressions for $\frac{\partial X}{\partial \gamma_i}$ into those for $\frac{d\hat{\alpha}}{d\gamma_i}$ and using the condition $D > 0$, we have

$$\frac{d\hat{\alpha}}{dA} < 0, \frac{d\hat{\alpha}}{dZ_2} > 0, \frac{d\hat{\alpha}}{dN} < 0 \text{ and}$$

$$\frac{d\hat{\alpha}}{d\delta} > 0 \text{ for } (1-\hat{\alpha})Z_2/N \leqq \xi \text{ where } \xi > 1 \text{ and}$$

$$\frac{d\hat{\alpha}}{d\delta} < 0 \text{ for } (1-\hat{\alpha})Z_2/N > \xi > 1.$$

We can summarize these results as follows:

$$\frac{dp(2)}{d\alpha} > 0 \text{ for } \alpha \in [0,1)$$

$$\frac{dX}{d\alpha} = \frac{dR_2}{d\alpha} > 0$$

$$\frac{dR_1}{d\alpha} \begin{cases} > 0 \text{ if } \alpha < \hat{\alpha} \\ = 0 \text{ if } \alpha = \hat{\alpha} \\ < 0 \text{ if } \alpha > \hat{\alpha} \end{cases}$$

where $\hat{\alpha} = \hat{\alpha}\,(A, \delta, Z_2, N)$ and the derivatives of $\hat{\alpha}$ with respect to each argument are given above. In addition, with $R_1 = (1+\Pi)^{-1/2}$, the sign of $\frac{d\Pi}{d\alpha}$ is the reverse of the sign of $\frac{dR_1}{d\alpha}$. And since the nominal interest rate r is essentially the sum of the real interest rate ρ and the rate of inflation Π, it follows that

$$\frac{dr}{d\alpha} \begin{cases} > 0 \text{ for } \alpha > \alpha' \\ = 0 \text{ for } \alpha = \alpha' \\ < 0 \text{ for } \alpha < \alpha', \text{ where } \alpha' < \hat{\alpha}. \end{cases}$$

When $\alpha \geqq \hat{\alpha}$, both the real interest rate and the rate of inflation increase as α increases. When $\alpha < \hat{\alpha}$, an increase in α causes the real interest rate to increase and the rate of inflation to fall. At some α', the two effects exactly cancel, and below α' the reduction in inflation exceeds the increase in the real interest rate.

Using the relationships in Proposition 1, changes in policy indexed by α can be translated into changes in the stocks of money and bonds and in their growth rates over time. Thus, $\frac{dM(2)}{d\alpha} = \frac{-M(2)}{p(2)} \frac{dp(2)}{d\alpha} < 0$, so that an increase in α always implies a decrease in the initial stock of money. The changes in B(1) and B(2) with respect to an increase in α can be either positive, zero, or negative, depending on initial parameter values. Finally, since the growth rates of money and bonds are equal to the rate of inflation, we have

$$\frac{d[g(M)]}{d\alpha} = \frac{d[g(B)]}{d\alpha} = \frac{d\Pi}{d\alpha} \begin{cases} < 0 \text{ for } \alpha < \hat{\alpha} \\ = 0 \text{ for } \alpha = \hat{\alpha}, \\ > 0 \text{ for } \alpha > \hat{\alpha} \end{cases}$$

where $g(f) \equiv \frac{f(t+2)}{f(t)} - 1$ and f = M or B.

The effects of a change in α on individual welfare can be determined by computing the changes in consumption which result from changes in the initial price level and in the gross rates of return. Thus,

$$\frac{d\hat{C}_3(0)}{d\alpha} = \frac{B(0)}{N} \frac{dp(2)}{d\alpha} = \frac{B(0)}{N} \left(\frac{Z_2}{B(0)+M(1)}\right) > 0,$$

$$\frac{d\hat{C}_2\ (1)}{d\alpha} = \frac{M(1)}{N} \frac{dp(2)}{d\alpha} = \frac{M(1)}{N} \left(\frac{Z_2}{B(0)+M(1)}\right) > 0,$$

$$\frac{d\hat{C}_3(1)}{d\alpha} = \frac{1}{N}\,\frac{d(p(3)B(1))}{d\alpha} = \frac{1}{N}\,\frac{d(R_1 p(2)B(1))}{d\alpha}$$

$$= \frac{1}{N}\,XZ_2\left(\frac{1-\alpha\delta}{1-\alpha}\right) > 0 \text{ for } \alpha,\ \delta\ \varepsilon\ (0,1),$$

and for $t \geqq 2$

$$\frac{d\hat{C}_1(t)}{d\alpha} = 0$$

$$\frac{d\hat{C}_2(t)}{d\alpha} = \frac{Z_1}{N}\,\frac{dR_1}{d\alpha} = \frac{Z_1}{N}\,\frac{Z_2}{Z_1}\left[1 - R_2\left(\frac{1-\alpha\delta}{1-\alpha}\right)\right]$$

so that
$$\frac{d\hat{C}_2(t)}{d\alpha} \begin{cases} > 0 & \alpha < \hat{\alpha} \\ = 0 & \alpha = \hat{\alpha}\text{, and} \\ < 0 & \alpha > \hat{\alpha} \end{cases}$$

$$\frac{d\hat{C}_3(t)}{d\alpha} = \frac{Z_2}{N}\,\frac{dR_2}{d\alpha} = \frac{Z_2}{N}\left(\frac{1-\delta}{1-\alpha}\right)R_2 > 0$$

For $t \geqq 2$ we then have

$$\frac{dU}{d\alpha} = \frac{\beta}{\hat{C}_2}\,\frac{d\hat{C}_2}{d\alpha} + \frac{\gamma}{\hat{C}_3}\,\frac{d\hat{C}_3}{d\alpha} = \frac{\beta}{R_1}\,\frac{dR_1}{d\alpha} + \frac{\gamma}{R_2}\,\frac{dR_2}{d\alpha}$$

$$= \gamma\ \left\{\frac{1-\delta}{1-\alpha} + \frac{1}{R_1}\left[1 - R_2\left(\frac{1-\alpha\delta}{1-\alpha}\right)\right]\right\}$$

Gathering the results from this section, we have the following two propositions.

<u>Proposition 2</u>. Suppose $\alpha < \hat{\alpha}$. Then for a small increase in α such that $\alpha + \Delta\alpha < \hat{\alpha}$:

(1) the initial price level, $\frac{1}{p(2)}$, falls,

(2) the inflation rate, Π, falls,

(3) the real rate of return on bonds and capital, R_2 and X, respectively, rises, and

(4) the welfare of the old, middle-aged, young and all future generations rises.

Proposition 3. Suppose $\alpha \geqq \hat{\alpha}$. Then for an increase in α

(1) the initial price level, $\frac{1}{p(2)}$, falls,

(2) the inflation rate, Π, rises,

(3) the real rate of return on bonds and capital, R_2 and X, respectively, rises, and

(4) the welfare of the old and middle-aged increases while the welfare of the young and future generations can change in either direction.

Given the continuity of $\frac{dU}{d\alpha}$ with respect to α and the fact that $\frac{dU}{d\alpha}\Big|_{\alpha = \hat{\alpha}} > 0$, the next proposition follows.

Proposition 4. Welfare for all generations can be raised by increasing the inflation rate above the minimal inflation rate. More precisely, for sufficiently small $\Delta\alpha > 0$, the policy indexed by $\hat{\alpha} + \Delta\alpha$ Pareto dominates the policy indexed by $\hat{\alpha}$.

Bond issue generates real revenues in a steady state only when $R_2 < 1$. At the revenue-maximizing $\hat{\alpha}$ we have $R_2 < 1$, but Proposition 4 suggests that α should be pushed above $\hat{\alpha}$. A question is whether α should be pushed so high that $R_2 \geqq 1$; that is, so that bond issue does not generate real revenues. Proposition 5 states that can be the case.

Proposition 5. Let α^* be such that $R_2 = X = 1$, so that $R_1 = 1 - \frac{G}{Z_1}$. Then a policy should have $\alpha \geqq \alpha^*$ if and only if $R_1 \geqq \alpha^*$.

Note that the condition $R_1 \geqq \alpha^*$ depends on parameters of the utility function, parameters of the production function, initial endowments, population number, and the level of the real government deficit. Since an increase in α always increases the welfare of the current old and middle-aged, the condition is found by setting $\frac{dU}{d\alpha} \geqq 0$ for the current young and all future generations at $\alpha = \alpha^*$.

IV. Concluding Remarks

Inflation is a distorting tax. If the government has available non-distorting taxes, it should use those and not use the inflation tax at all. In fact, arguments can be made for using a negative inflation tax.[1/]

In this paper it is assumed, however, that the government has no non-distorting taxes at its disposal. The government has only the inflation tax and the tax on bond issue to finance ongoing deficits. If it minimizes the inflation tax, it maximizes the revenue it raises through bond issue. When there are decreasing returns to capital, the maximum tax from bond issue occurs at an inadequate real rate of return from society's point of view. The welfare of all can be improved by opting for slightly higher rates of inflation and real return.

[1/] See, for example, Drazen.

References

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