MONEY AS A MEDIUM OF TRANSACTION
IN THE OVERLAPPING GENERATIONS MODEL

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Abstract

A definition of a transactions medium is proposed. This is that a transactions medium permits the attainment of otherwise unattainable resource allocations. It is shown that by this definition money can be a transactions medium in a pure exchange, overlapping generations economy. It is also shown that money is a transaction medium only if there are informational asymmetries of a particular type. Finally, it is shown that the set of economies for which money is a transactions medium is not isolated, in a well-defined sense.

I. Introduction

Among the most widely used models of money are the class of over-lapping generations (OLG) models. However, these are also highly controversial as models of money due at least in part to the role for money implied by them. In particular, it is often claimed that OLG models provide no role for money as a "transactions medium." However, this claim clearly requires that some precise definition by given as to what it means for money to be a medium of transaction.

One such definition has been recently proposed by McCallum (1982). This is that money is not a medium of transaction unless it expands the set of aggregate consumption opportunities for society. While this serves as a useful starting point in defining a transactions medium, it is somewhat too strong for most purposes because it begs the question of how certain allocations of resources are to be attained. Therefore, in order to study the roles that OLG models can provide for money, a weaker related definition is proposed here. This is that money is not a transactions medium if monetary equilibrium allocations of resources can be reproduced without money. This definition has the advantage that it permits an analysis of transactions roles for money in exchange economies.

Using the above definition, the purpose of this paper is to demonstrate that the OLG setting is capable of providing a transactions role for money. Put otherwise, the paper demonstrates that there exist economies where the monetary equilibrium allocation cannot be realized without money. This captures the idea that a set of transactions (or exchanges) is not possible without money, although the resulting resource allocation is technologically feasible.

The important element in this demonstration is the presence of private information. In particular, two classes of economies with private information are embedded in an OLG setting. In the first, agents in the economy, including the government, are asymmetrically informed about the occurrence of a random state of nature. Two alternate mechanisms for reallocating resources are then considered. In the first, the government introduces a fixed stock of fiat money at some initial data, and any further reallocations occur through competitive exchange. (Thus, the value of the money introduced, and therefore the effect of the first type of intervention, is endogenously determined.) In the second, there is no exchange--all resource reallocations are accomplished through tax/transfer mechanisms. The question asked is whether allocations attained through the first arrangement are also attainable through the second. The answer is that they generally are not.

A suggested interpretation of this result is as follows. As noted at the outset of development of the OLG model by Samuelson (1958), there is a potential equivalence between monetary equilibrium allocations, and an appropriately structured social security system. If such an equivalence holds for an economy, then by our definition money is not a transactions medium. Thus throughout we ask two questions; whether tax/transfer schemes generally exist that are able to reproduce monetary equilibrium allocations, and if not, whether there is a generic equivalence between the two. The answer to both questions is negative.

A second class of economies which gives rise to nonequivalence is one where there is private information about agents' types (preferences, endowments, and information). In such an economy, we again ask whether a tax/transfer scheme exists which reproduces the monetary equilibrium allocation. Again, the answer is generally negative.

The two classes of economies considered are motivated by the follow-
ing train of thought. If money is inessential to the attainment of certain
resource reallocations (transactions between various private agents, or bet-
ween agents and the government), then the government must know when to make
certain state-contingent transfers, or what state has occurred (the first
economy), or it must know this as well as know which agents are to receive
which transfers (the second economy). Our analysis demonstrates that unless
the government possesses at the outset the best available information (in a
well-defined sense), it will often be unable to acquire the requisite informa-
tion.

As a by-product of this analysis, several results of independent
interest are obtained regarding the government's ability to generate informa-
tion while attempting to reproduce the monetary equilibrium allocation:

i) if the government has the best available information regarding the real-
ized state of nature, then it can always generate any necessary informa-
tion about agents' types.

ii) if, however, the government does not have the best possible initial infor-
mation regarding state realization, lack of knowledge of type can prevent
it from acquiring this information.

As a corollary to (i), money cannot be a transactions medium without private
information (to which the government is not privy). This accords well with
the notion that asymmetric information generates a necessary role for money.[1]

The preceding discussion has assumed the innate interest of the
issue of whether money is essential to certain resource reallocations. How-
ever, even were one to doubt the importance of this question, there is still a
reason for undertaking the investigation of this paper. Specifically, the

attainment of an allocation of resources corresponding to an equilibrium in which money has value <u>requires</u> some intervention by the government.[2/] In particular, at some initial date the government creates a stock of fiat currency, and essentially may be viewed as selling this to the initial young for goods, which are then redistributed to the initial old. No further intervention occurs. However, it is by no means obvious why this one-time intervention should be preferred to the continuing intervention associated with a social security scheme. The analysis here suggests a reason for this: allocations attainable under the one-time intervention need not be under continuing intervention.

It will be noted that thus far discussion has proceeded as if the government were a separate economic agent. This is not required for the results obtained, however. A view of government as a coalition among private agents can be accommodated while retaining the results metioned above. In particular, the notion of cooperative behavior introduced by Wilson (1978) in which agents collude without transmitting information can be incorporated into our framework. In this case the results may be interpreted as follows: when agents collude without transmitting information (so that the coarse core is being considered), the monetary equilibrium allocation will not generally be available as an option. On the other hand, if all information is transmitted when agents collude (the fine core), the monetary equilibrium allocation will be an option. Thus if government is viewed as a coalition of private agents, this implies that whether or not money is a transactions medium depends on the amount of information transmitted as agents collude.

Finally, the paper addresses the question of whether the introduction of (valued) fiat money can induce the revelation of information through prices. It will be seen that this is also a role that can by played by money.

The format of the paper is as follows. Section II describes the basic model, and defines its equilibria. Section III considers a class of economies where there is asymmetric information among agents regarding the state of nature, and shows that this may create a medium of transaction role for money. It is also shown that such a role is not restricted to an "isolated" set of economies. Section IV extends the possible scope of informational asymmetries to include "adverse selection"-like problems. It is shown that the presence of these may compound the informational problems of section III, but that they are never by themselves a source of a medium of transaction role for money. Sections V and VI relate the arguments of sections III and IV to other literature on money, asymmetric information, and taxation under asymmetric information. Section VII considers an alternate notion of equilibrium, and demonstrates that using this notion the introduction of valued fiat money can expand information. Section VIII concludes.

II. The Model

A. Description

We consider an economy in which time is discrete, and indexed by t = 0,1,.... The economy is peopled with two-period lived, overlapping generations at each date including t = 0, when there is an initial old generation, and an initial young generation. Within each generation there are I types of agents, indexed by i = 1, ..., I. Agent types are distinguished by preferences, endowment streams, and by information possessed. Finally, there is one agent—the government—which is infinitely lived. The government has the power to tax and make transfers, and is also endowed with an information structure.

To keep the analysis simple, we may focus on a setting in which there is only a single, nonstorable consumption good that agents desire directly, and money, which agents do not desire directly. Thus all trading will either be intragenerational borrowing and lending, or the trading of money for goods. Finally, at each date any of a finite set of states of nature (Ω) may occur. Let $\#\ \Omega = S$, $s = 1, \ldots, S$ index the elements of Ω, and let s' indicate an element of Ω which occurs one period in the future. For notational simplicity, the trading of state contigent claims will be ruled out. (This is inessential to any results.)

The preferences of type i agents for consumption when s is the realized state in their youth, $C_{1i}(s)$, and consumption when old and they have experienced states s and s', $C_{2i}(s,s')$, are denoted by $U_i[C_{1i}(s), C_{2i}(s,s')]$, and for simplicity are assumed state independent. (Again, this is inessential.) The functions $U_i(\quad)$ are assumed to be increasing in each argument, and concave. The endowment stream of a type i agent is determined entirely by the realization of s when young (again only for simplicity), and is denoted $[w_{1i}(s), w_{2i}(s)]$, where w_1 denotes the endowment when young, etc. Finally, type i agents are endowed with a certain amount of information concerning the economy in which they operate. As we will consider alternate structures of agents' information below, we will not completely describe agents' information here. However, part of the information of type i agents consists of a partition P_i of Ω which describes the set of states a type i agent can distinguish between. (P is a partition of Ω if its elements, say P^j, obey

$$P^j \cap P^k = \phi; \ j \neq k, \text{ and } \bigcup_j P^j = \Omega.)$$

The P_i are time invariant. In addition, it will be necessary to distinguish between ex-ante (pre-trade) partitions $\hat{P}_i$, and post-trade partitions P_i which incorporate information about prevailing states conveyed by prices.

Let $P_i(s)$ ($\hat{P}_i(s)$) be that element of P_i ($\hat{P}_i$) containing s. Then if s is the actual state, agent i knows only that $P_i(s)$ has occurred. For consistency, we require that

$$w_{ji}(s_1) = w_{ji}(s_2); \ j = 1, \ 2, \ \forall \ s_1, s_2 \text{ such that } \hat{P}_i(s_1) = \hat{P}_i(s_2).$$

Lastly, we denote the government's initial and ex-post partitions by $\hat{P}_g$ and P_g respectively.

As indicated previously, trading is either intertemporal, or trade of money for goods. We restrict consideration to economies with a fixed (for all time) stock of money, M. We choose the consumption good as numeraire, and money trades for goods at rate $Q(s)$ in state s. Throughout we focus on stationary states, so that prices and trades need not be dated.

At date t = 0, the initial old are endowed (in arbitrary fashion) with all of the money which circulates. No other agents have an initial endowment of money. Then denote the quantity of money acquired when young by a type i agent in state s by $M_i(s)$. $M_i(s)$ thus describes one component of type i agents' desired trades. The other component is the desired borrowing of type i agents. Let $x_i(s)$ be the youthful borrowing of type i agents in state s. This borrowing is not state contingent, so that the certain repayment is $R(s)x_i(s)$, where $R(s)$ is the gross rate of return in state s. $x_i(s) < 0$ denotes lending.

We focus throughout on economies where trading occurs after the current period state has been realized (although all agents need not know this realization). Thus an agent of type i has economic behavior described by the solution to the problem

max E U_i [$C_{1i}(s)$, $C_{2i}(s,s')$] by choice of $x_i(s)$, $M_i(s)$ subject to

(1) $C_{1i}(s) = w_{1i}(s) + x_{1i}(s) - Q(s)M_i(s)$

(2) $C_{2i}(s, s') = w_{2i}(s) - R(s)x_i(s) + Q(s')M_i(s)$

(3)
$$x_i(s_1) = x_i(s_2)$$
$$M_i(s_1) = M_i(s_2)$$
$\forall s_1, s_2$ such that $P_i(s_1) = P_i(s_2)$,

where $P_i(s_1) \neq P_i(s_2)$ iff $\hat{P}_i(s_1) \neq \hat{P}_i(s_2)$ or $[R(s_1), Q(s_1)] \neq [R(s_2), Q(s_2)]$.

Denote maximizing values for i by

$$[\widetilde{x}_i(s), \widetilde{M}_i(s)] = f_i[R(s), Q(s), \{Q(s')\}_{s=1}^{S}].$$

At this point, we have assumed competitive behavior on the parts of all agents, and that agents understand the (as yet unspecified) probability distribution of future prices of money. Below we will examine some consequences of allowing agents not to be perfect competitors (section VII).

We are now prepared to define a (competitive) equilibrium for this economy.

Definition. A stationary equilibrium is a mapping

$$[R(s), Q(s)] : \Omega \rightarrow R^2_+$$

which satisfies

i) $\sum_{i=1}^{I} n_i f_i[-] = (0, M) \forall s \epsilon \Omega$,

where n_i is the number of type i agents. (The n_i are time invariant for convenience.)

Definition. A valued fiat money equilibrium is a stationary equilibrium with $Q(s) > 0 \; \forall \; s \; \epsilon \; \Omega$.

B. Centralization of Equilibrium

It will be recalled that we wish to investigate whether or not any monetary equilibrium allocation for the economy above can be attained without money. To begin, note that any equilibrium in which money has value involves intergenerational resource reallocations. Without money, such reallocations cannot be attained as the result of voluntary exchange. If they can be effected at all, it must be through involuntary redistribution. It suffices to consider taxes and transfers as means of producing these.

The nature of our investigation will be of the following form: do there exist tax/transfer schemes which produce the monetary equilibrium allocation of resources, without violating any restrictions implied by the government's ability to acquire information? In other words, the focus is on whether the government can reproduce this allocation via a social security system which does not require more information than it possesses at any date.

To this end, let $T_{1i}(s)$ denote a net transfer to a young type i agent young in state s, and $T_{2i}(s,s')$ denote a net transfer to an old type i agent who has experienced states s and s'. For our purposes, it is easiest to think of the T_{ji} as lump-sum transfers, but they may also be thought of as being determined by arbitrary functions of agents' actions. (Thus one could think of agents trading in the background, and the T_{ji} as specifying their net receipts as a function of their actions, prices, etc.) We denote transfers meant to reproduce monetary equilibrium allocations by

(4) $\tilde{T}_{1i}(s) = \tilde{x}_i(s) - \tilde{Q}(s)\tilde{M}_i(s)$

    $\tilde{T}_{2i}(s,s') = \tilde{Q}(s')\tilde{M}_i(s) - \tilde{R}(s)\tilde{x}_i(s); \; \forall \; i, \; s, \; t \geqslant 0,$

where a "$\sim$" denotes an equilibrium value.

Definition. A monetary equilibrium allocation can be centralized if there exist values

$$\tilde{T}_{1i}(s) \; \text{and} \; \tilde{T}_{2i}(s,s'); \; i = 1, \; \ldots, \; I, \; j = 1, \; 2,$$

defined above which are consistent with information possessed by the government.

As we will consider different types of information which the government may possess, we postpone a more specific discussion of this information to subsequent sections. We now turn our attention to the question of what types of monetary equilibrium allocations can be centralized.

III. A "Moral Hazard" Economy

A. Description

    In this section, we consider an economy in which the only uncertainty concerns the prevailing (current period) state of nature. (In particular, each agent's type is public information.) In other words, some (or all) agents' partitions may not be fine enough to permit them to discover the prevailing state. If a valued fiat money equilibrium as defined by (i) exists for this setting, we wish to consider when it can be centralized. In order to discuss when centralization is possible, in turn, it is necessary to describe how the government acquires information.

    To this end, consider the determination of P_g. This is determined by the interaction of two forces, $\hat{P}_g$, and the government's observation of

agents' behavior under the prevailing tax/transfer system. More specifically, if the tax system is structured so that agents can be induced to reveal their knowledge of the state, the government can acquire relevant information. Consider then any s_1, s_2 such that $\hat{P}_g(s_1) = \hat{P}_g(s_2)$. If it is to attempt to reproduce the monetary equilibrium allocation, the government must make (for instance) the transfers $\tilde{T}_{1i}(s_1)$ and $\tilde{T}_{1i}(s_2)$ in the appropriate state. If $\tilde{T}_{1i}(s_1) \neq \tilde{T}_{1i}(s_2)$ for some i, then the government must be able to produce additional information in order to make the necessary transfers. This information production may be thought of in the following way. The government (since type is common knowledge) offers agent i his choice among the transfers $\tilde{T}_{1i}(s)$; $s = 1, \ldots S$. (Once $\tilde{T}_{1i}(s)$ is selected, this and the selection of future agents determines $\tilde{T}_{2i}(s,s')$.) If the values of the $\tilde{T}_{1i}(s)$ are such that agent i has no incentive to misrepresent his knowledge of the prevailing state, the government can acquire the information necessary to reproduce the monetary equilibrium allocation.

Formally, then, $s_1 \notin P_g(s_2)$ iff

(5) $\hat{P}_g(s_1) \neq \hat{P}_g(s_2)$, or

(6) $E_{s'} U_i[w_{1i}(s_1) + \tilde{T}_{1i}(s_1), w_{2i}(s_1) + \tilde{T}_{2i}(s_1, s')] > E_{s'}[w_{1i}(s_1) + \tilde{T}_{1i}(s_2),$

$w_{2i}(s_1) + \tilde{T}_{2i}(s_2, s')]$

for some i such that

(7) $\tilde{T}_{1i}(s_1) \neq \tilde{T}_{1i}(s_2)$
        $\hat{P}_i(s_1) \neq \hat{P}_i(s_2)$,

where $E_{s'}$ denotes the expectation over future states. In short, then, $s_1 \notin P_g(s_2)$ even if $\hat{P}_g(s_1) = \hat{P}_g(s_2)$ if (6) holds whenever (7) holds for some i.

Having described the information acquisition process, then, we are now prepared to discuss centralization of equilibrium for this moral hazard economy.

B. Nonequivalence of Monetary and Social Security Systems

For the economy of this section, we may restate the definition of when a monetary equilibrium allocation can be centralized.

Definition. The monetary equilibrium allocation of this moral hazard economy can be centralized iff the values $\tilde{T}_{1i}(s)$ and $\tilde{T}_{2i}(s, s')$ defined by (4) satisfy
$$\tilde{T}_{1i}(s_1) = \tilde{T}_{1i}(s_2) \; \forall \; i \; \forall \; s_1, s_2 \text{ such that } P_g(s_1) = P_g(s_2).$$
Given this definition, we have

Proposition 1. There exist monetary equilibria with allocations that cannot be centralized.

As there also exist monetary equilibrium allocations which can be centralized, the next section established proposition 1 by means of an example. Section D then establishes that there is not a generic equivalence between tax/transfer schemes and monetary equilibrium allocations.

C. An Example

In this section we demonstrate that there exist economies where social security systems and monetary equilibria cannot be equivalent. There is also a discussion of why this is the case.

Example 1. $I = 2$, $S = 2$, $n_1 = n_2$, $U_i(C_1, C_2) = \ln C_1 + \left(\frac{1}{10}\right) C_2$; $i = 1, 2$, $[w_{11}(1), w_{21}(1)] = [w_{11}(2), w_{21}(2)] = (10, 10)$, $[w_{12}(1), w_{22}(1)] = (15, 9)$ with probability $5/6 \; \forall \; t \geqslant 0$, $[w_{12}(2), w_{22}(2)] = (45, 9)$ with probability $1/6$ $\forall \; t \geqslant 0$,

$$\hat{P}_1 = [\{1,\ 2\}]$$

$$\hat{P}_2 = [\{1\},\ \{2\}]$$

$$\hat{P}_g = [\{1,\ 2\}].$$

It is straightforward to derive the equilibrium values of prices and consumption levels:

$$\tilde{Q}(1)M = 25/3$$

$$\tilde{Q}(2)M = 55/3$$

$$\tilde{R}(1) = 6/5,\ \tilde{R}(2) = 6/11$$

$$\tilde{C}_{12}(1) = 21.67,\ \tilde{C}_{22}(1,\ 1) = 15.67$$

$$\tilde{C}_{22}(1,\ 2) = 23.67$$

$$\tilde{C}_{12}(2) = 18.33$$

$$\tilde{C}_{22}(2,\ 1) = 21.88,\ C_{22}(2,\ 2) = 31.88,$$

and agent 1's consumption at each date is not required for the argument. Note that the values implied by (4) are

$$\tilde{T}_{12}(1) = -6.67$$

$$\tilde{T}_{22}(1,\ 1) = 6.67,\ T_{22}(1,\ 2) = 14.67$$

$$\tilde{T}_{12}(2) = -26.67$$

$$\tilde{T}_{22}(2,\ 1) = 12.88,\ \tilde{T}_{22}(2,\ 2) = 22.88.$$

Now suppose that these values are consistent with (6) and (7) (as clearly they are not possible on the basis of $\hat{P}_g$ alone). Then a young type 2 agent prefers to reveal the true state when s = 2. But

$$\ln[w_{12}(2) + \tilde{T}_{12}(2)] + (5/6)(1/10)[w_{22}(2) + \tilde{T}_{22}(2, 1)] + (1/6)(1/10)$$

$$[w_{22}(2) + \tilde{T}_{22}(2, 2)] = 5.27 < 5.35 = \ln[w_{12}(2) + \tilde{T}_{12}(1)] + (5/6)(1/10)$$

$$[w_{22}(2) + \tilde{T}_{22}(1, 1)] + (1/6)(1/10)[w_{22}(2) + \tilde{T}_{22}(1, 2)],$$

which contradicts (6). Therefore assuming the monetary equilibrium allocation can be centralized implies a contradiction, establishing the proposition.

Why is it the case, then, that the monetary equilibrium allocation cannot be centralized? At first blush it might appear that this is because market trading reveals information, and the government is not allowing such trading to occur. It might be conjectured, then, that if the government allowed trade to occur within each generation, but made transfers to reallocate resources between generations, the monetary equilibrium allocation might be centralized.

This view is not correct, however. In particular, as indicated previously, the $\tilde{T}_{ji}$ may depend in any arbitrary fashion on agents' actions (and they are then to be interpreted as net after-tax resource reallocations). So long as agents understand how the economy works, and understand the dependence of the $\tilde{T}_{ji}$ on their actions, they care only about final outcomes and will not reveal the state.

It may seem, though, that this argument gives agents a much better understanding of how the tax system works than of how the price system works. If agents understood how information was revealed by prices and acted accordingly, would the market and the tax/transfer mechanism then be equivalent?

The answer to this question is no. Before we indicate why, however, it is useful to point out that under the tax/transfer scheme agents view themselves as able only to declare the state incorrectly. Young agents in s = 2 take the allocation they receive if they declare s = 1 as given by the actions of type 1 agents. The set-up of the following example attempts to preserve this.

Example 2. This is the same as example 1, except that agents of type 2 who are young in s = 2 may attempt to prevent revelation of the true state. If they are successful in this, $\tilde{R}(1) = \tilde{R}(2) \equiv \tilde{R}$, and $\tilde{Q}(1) = \tilde{Q}(2) \equiv \tilde{Q}$. However to retain consistency with the remark above about the tax system, in order to prevent revelation of state type 2 agents must behave the same (have the same values of demand) when s = 2 as they would if s = 1 at the prevailing equilibrium prices. (In other words, agents act as if they can prevent revelation of the true state, but agents in the favorable state, s = 1, act competitively.) Then if type 2 agents young when s = 2 behave so as to prevent revelation, there are two cases to consider.

Case 1: $\tilde{Q} > 0$. Then in equilibrium, $\tilde{R} = 1$. Therefore $\tilde{x}_2(s) = 0 \; \forall \; s$, and $\tilde{Q}M = 5$. The realized level of expected utility for type 2 agents when the current state is s = 2 is $EU_2 = 5.09 < 5.27$, which is their level of expected utility under the equilibrium of example 1.

Case 2: $\tilde{Q} = 0$. Then $\tilde{R} = 4/5$, and $\tilde{x}_2(s) = 2 \; \forall \; s$. The level of expected utility attained is $EU_2 = 4.85$ under this arrangement.

In both cases, agents prefer the competitive equilibrium of example 1. Thus even if they understand how the price system reveals information, agents will prefer to allow it to reveal the state. The same is not true for the tax/transfer system. In short, the nonequivalence is not based on differential understanding of how information is transmitted under the two alternate arrangements.

Example 1, then, presents an economy where tax/transfer schemes cannot reproduce the monetary equilibrium allocation, i.e., in which money is (by our definition) a transactions medium. One would like to know, however, if this is in some sense an isolated phenomenon. This question is addressed in the following section.

D. Is There a Generic Equivalence Between Tax/Transfer Schemes and Money?

It would be of little interest to demonstrate proposition 1 if for almost all economies in the class considered, money were not a transactions medium. More specifically, if there were a generic equivalence between money and tax/transfer schemes, proposition 1 would be of little interest. As generic equivalence means that the set of exchange economies which displays an equivalence between tax/transfer schemes and money is open and dense, it will suffice to show that there exists a nonempty open set of economies where such equivalence breaks down. Here we prove this for a restricted class of economies.

To begin, we define an economy as follows:

Definition: An economy is

a) a list of agents' types $i = 1, \ldots, I$

b) a list of states $s = 1, \ldots, S$ with their evolution governed by a given probability law

c) for each type of agent a specification of a preference preordering $\succ_i$, an endowment pattern $[w_{1i}(s)], w_{2i}(s)]; s = 1, \ldots, S$, and a partition $\hat{P}_i$ of Ω,

d) a vector $n \equiv (n_1, \ldots, n_I)$ specifying the number of agents of each type.

Let E denote the set of economies obtained by fixing I, S, and the probability law governing the evolution of the prevailing state. Also, let

$w_i(s) \equiv [w_{1i}(s), w_{2i}(s)]$, $w_i \equiv [w_i(1), \ldots, w_i(s)]$, and $w \equiv (w_1, \ldots, w_I)$. Then a typical element of E is of the form $(\succ_1, \ldots, \succ_I, \hat{P}_1, \ldots, \hat{P}_I, w, n)$. The set E may also be restricted in various ways. We will restrict it in the following fairly severe manner. Following Hildenbrand (1972), denote by E_f the set of economies obtained by restricting preferences, partitions, and associated demand correspondences to be a fixed sequence $\{\succ_i\}_{i=1}^I$, $\{\hat{P}_i\}_{i=1}^I$, and $\{f_i\}_{i=1}^I$. If we restrict these sequences to be those arising from the economy of example 1, we denote the set of economies so obtained by E_f^*. Thus a typical element of E_f^* is a pair (w, n). Let $(w^*, n^*) \in E_f^*$ be the economy of example 1. Then we have

<u>Proposition 2.</u> There exists a nonempty, open set of economies in E_f^* for which money is a transactions medium.

Proof. A nonempty set exists from proposition 1. Suppose that there is not such an open set containing (w^*, n^*). Then there is a nearby economy $(\hat{w}, \hat{n})$ for which the monetary equilibrium allocation can be centralized. However, it is straightforward to establish that there exists an open neighborhood of (w^*, n^*) in E_f^* such that equilibrium values $[\tilde{R}(s), \tilde{Q}(s)] \in R_{++}^2$ are defined by a continuous function[3/]

$$[\tilde{R}(s), \tilde{Q}(s)] = W[w(s), w, n]; \quad s = 1, 2,$$

where $w(s)$ is the current state endowment vector. In addition, for any point in this neighborhood, it is easy to show that the state s net trade of a type 2 agent is a continuous function

$$[\tilde{C}_{12}(s)-w_{12}(s), \tilde{C}_{22}(s, s')-w_{22}(s)] = A_s[w_2(s), s', \tilde{R}(s), \tilde{Q}(s)];$$

$$s, s' = 1, 2.$$

Finally, define

$$H_s(w, n) \equiv E_{s'} U_2 \{ w_2(2) + A_s [w_2(s), s', W[w(s), w, n]] \}; \quad s = 1, 2,$$

which is the expected utility of a type 2 agent young in $s = 2$ who realizes the state s equilibrium net trade. Clearly the H_s are continuous in (w, n), as they are the compositions of continuous functions.

Now note that the fact that (w^*, n^*) has a monetary equilibrium allocation which cannot be centralized means that

$$(8) \qquad H_2(w^*, n^*) < H_1(w^*, n^*).$$

It has been further supposed that some arbitrarily close economy $(\hat{w}, \hat{n})$ has a monetary equilibrium that can be centralized. But then

$$(9) \qquad H_2(\hat{w}, \hat{n}) \geqslant H_1(\hat{w}, \hat{n}).$$

However, (8), (9), and the fact that we must be able to choose $(\hat{w}, \hat{n})$ arbitrarily close to (w^*, n^*) violate continuity of the H_s. This contradiction establishes that there exists an open neighborhood of (w^*, n^*) for which money is a transactions medium.

Thus, in the restricted class of economies considered, tax/transfer schemes are not generically able to produce the same allocation as arises from a monetary equilibrium.

IV. An "Adverse Selection" Economy

In the previous section, it was demonstrated that it might be impossible for the government to know _when_ to make certain transfers; i.e., the government might not be able to obtain sufficient information about the prevailing state to reproduce the monetary equilibrium allocation. A second problem that might arise concerns how the government can know to _whom_ transfers are to be made. In short, if an agent's type is private information, it

may be impossible for the government to reproduce the monetary equilibrium allocation because it may not be able to discover which agents should receive which transfers. This is the subject of this section.

The class of economies considered in this section retains all the features of the previous section, and in addition an agent's index is taken to be (at least initially) private information. Thus prior to the initiation of economic activity at each date agents' pre-trade information is again $\hat{P}_i$; $i = 1, \ldots, I$, plus each agent knows his own type (and each agent knows who the government is).

The equilibrium for this economy is the same as previously, since each agent's type is irrelevant in exchange.[4/] However, the conditions under which a monetary equilibrium allocation can be centralized are now different. In particular, the government must now be able to distinguish the current period state and each agent's type if it is to make the appropriate transfers. In order to have a means of thinking about information production, we may again think of the government offering agents their choice from among the array $\tilde{T}_{1i}(s)$ and $\tilde{T}_{2i}(s,s')$ defined by (4). If agents correctly select; i.e., select so as to reveal their knowledge of the state and their type, then the government will be able to acquire the information necessary to centralize the monetary equilibrium allocation. Formally

<u>Definition.</u> The monetary equilibrium allocation for this adverse selection economy can be centralized iff the values $\tilde{T}_{1i}(s)$ and $\tilde{T}_{2i}(s,s')$ defined by (4) obey

$$(10) \quad E_{s'}U_i[w_{1i}(s_1)+\tilde{T}_{1i}(s_1), \ w_{2i}(s_1)+\tilde{T}_{2i}(s_1, \ s')] \geqslant E_{s'}U_i[w_{1i}(s_1)+\tilde{T}_{1j}(s_2)$$

$$w_{2i}(s_1)+\tilde{T}_{2j}(s_2,s')] \ \forall \ s_1, \ s_2 \ \text{such that} \ \hat{P}_g(s_1) = \hat{P}_g(s_2), \ \forall \ i, \ j.$$

Condition (10) states that all agents do not wish to misrepresent their knowledge of the state, do not wish to misrepresent their type (select some other agent's transfer), or to misrepresent both. The remainder of this section is devoted to showing the kinds of additional problems that can arise in this setting, as clearly the moral hazard economy is a special case of the adverse selection economy of this section.

The fact that the moral hazard economy is a special case of the economy considered here means that propositions 1 and 2 continue to apply. However, a number of new problems arise when agents' types are private information. The first is detailed in proposition 3.

Proposition 3. There exist economies with the following property: if agents' types were public information, the monetary equilibrium allocation could be centralized. When indices are private information, this is not by possible. The proof is by example.

Example 3. The economy is the same as that of example 1 except that

$$[w_{12}(1), w_{22}(1)] = (1, 9) \text{ with probability } 1/4$$

$$[w_{12}(2), w_{22}(2)] = (21, 9) \text{ with probability } 3/4.$$

It is straightforward to compute the equilibrium values

$$\tilde{Q}(1)M = \frac{33}{13} , \tilde{Q}(2)M = \frac{93}{13}$$

$$\tilde{R}(1) = \frac{26}{11} , \tilde{R}(2) = \frac{31}{39}$$

$$\tilde{C}_{11}(1) = 4.33$$

$$\tilde{C}_{21}(1, 1) = 20.17 , \tilde{C}_{21}(1, 2) = 24.79$$

$$\tilde{C}_{12}(1) = 4.23$$

$$\tilde{c}_{22}(1, 1) = \tilde{c}_{22}(1, 2) = 1.36$$

$$\tilde{c}_{12}(2) = 11.92$$

$$\tilde{c}_{22}(2, 1) = 13.15 \ , \ \tilde{c}_{22}(2, 2) = 17.76.$$

Given these values, (4) implies that in order to reproduce the monetary equilibrium allocation we must have

$$\tilde{T}_{11}(1) = -4.33$$

$$\tilde{T}_{21}(1, 1) = 10.17, \ \tilde{T}_{21}(1, 2) = 14.79$$

$$\tilde{T}_{11}(2) = 1.94$$

$$\tilde{T}_{21}(2, 1) = \tilde{T}_{21}(2, 2) = -1.54$$

$$\tilde{T}_{12}(1) = 3.23$$

$$\tilde{T}_{22}(1, 1) = \tilde{T}_{22}(1, 2) = -7.64$$

$$\tilde{T}_{12}(2) = -9.08$$

$$\tilde{T}_{22}(2, 1) = 4.15 \ , \ \tilde{T}_{22}(2, 2) = 8.76.$$

First we demonstrate that if each agent's type were common knowledge, the government would be able to centralize the monetary equilibrium allocation. To see this, note that in s = 1, type 2 agents could not meet the tax obligation implied by declaring s = 2. Thus it is not feasible to misrepresent the state. In s = 2, we have

$$\ln[w_{12}(2) + \tilde{T}_{12}(2)] + (\tfrac{1}{4})(\tfrac{1}{10})[w_{22}(2) + \tilde{T}_{22}(2, 1)] + (\tfrac{3}{4})(\tfrac{1}{10})[w_{22}(2)$$

$$+ \tilde{T}_{22}(2, 2)] = 4.14 > \ln[w_{12}(2) + \tilde{T}_{12}(1)] + (\tfrac{1}{10})[w_{22}(2) + \tilde{T}_{22}(1, -)]$$

$$= 3.32,$$

so that agents opt for the appropriate transfer in each state, and each element of P_g is a singleton. Thus, in the moral hazard version of this economy, money is not a transactions medium.

Now consider the possibility that type 2 agents may misrepresent both their type <u>and</u> the prevailing state. If they are young in s = 2, note that

$$\ln[w_{12}(2)+\tilde{T}_{11}(1)] + (\tfrac{1}{4})(\tfrac{1}{10})[w_{22}(2)+\tilde{T}_{21}(1, 1)] + (\tfrac{3}{4})(\tfrac{1}{10})[w_{22}(2)$$

$$+\tilde{T}_{21}(1, 2)] = 4.40 > 4.14 = \ln[w_{12}(2)+\tilde{T}_{12}(2)] + (\tfrac{1}{40})[w_{22}(2)$$

$$+\tilde{T}_{22}(2, 1)] + (\tfrac{3}{40})[w_{22}(2)+\tilde{T}_{22}(2, 2)],$$

so that when the true state is s = 2, type 2 agents will identify themselves as young type 1 agents in s = 1. Moreover, for type 2 agents young in s = 1,

$$\ln[w_{12}(1)+\tilde{T}_{11}(2)] + (\tfrac{1}{10})[w_{22}(1)+\tilde{T}_{12}( )] = 1.82 > \ln[w_{12}(1)$$

$$+\tilde{T}_{12}(1)] + (\tfrac{1}{10})[w_{22}(1)+\tilde{T}_{22}( )] = 1.5.$$

Thus type 2 agents in s = 1 will claim to be type 1 agents in s = 2. In short, type 2 agents always misrepresent both state and type. Therefore, an inability to distinguish between types of agents <u>ex-ante</u> can prevent the government from being able to acquire information about the prevailing state, as claimed.

The main implication of proposition 3 is that some informational limitations can compound others. In particular, lack of <u>ex-ante</u> information regarding agents' types can prevent discovery of the prevailing state. The reverse is also the case. This is an implication of

<u>Proposition 4.</u> If the government initially possesses the "best available information" about the prevailing state, it can always discover agents' types. This proposition states that lack of knowledge of agents' indices is irrelevant, so long as the government has "full information" about the prevailing state.

Proof: Define "full information" on the part of the government by

$$\hat{P}_g = \overset{I}{\underset{i=1}{V}} \hat{P}_i,$$

where $\overset{I}{\underset{i=1}{V}} \hat{P}_i$ is the coarsest common refinement of the P_i. (A partition Φ is a refinement of another partition P if for every pair of elements Φ^j and P^j, either $\Phi^j \subseteq P^j$, or $P^j \cap \Phi^j = \phi$.) Then the claim is that

$$E_{s'}U_i[w_{1i}(s_1)+\tilde{T}_{1i}(s_1), \ w_{2i}(s_1)+\tilde{T}_{2i}(s_1, \ s')] \geqslant E_{s'}U_i[w_{1i}(s_1)$$

$$+\tilde{T}_{1j}(s_2), \ w_{2i}(s_1)+\tilde{T}_{2i}(s_2, \ s')] \ \forall \ s_1, \ s_2 \text{ such that } \hat{P}_g(s_1) = \hat{P}_g(s_2), \ \forall$$

$$i, \ j,$$

where as usual the $\tilde{T}_{ji}$ are defined by (4). Suppose that this is not true. There are then two cases to consider.

Case 1.

$$E_{s'}U_i[w_{1i}(s_1)+\tilde{T}_{1i}(s_1), \ w_{2i}(s_1)+\tilde{T}_{2i}(s_1, \ s')] < E_{s'}U_i[w_{1i}(s_1)+\tilde{T}_{1i}, \ (s_2),$$

$$w_{2i}(s_1)+\tilde{T}_{2i}(s_2, \ s')], \text{ and } \hat{P}_g(s_1) = \hat{P}_g(s_2),$$

or agents misrepresent s but not their index. But this clearly requires that

$\tilde{T}_{1i}(s_1) \neq \tilde{T}_{1i}(s_2)$ and/or $\tilde{T}_{2i}(s_1,s') \neq \tilde{T}_{2i}(s_2,s')$ for some s'. However,

since $\hat{P}_g = \overset{I}{\underset{i=1}{V}} \hat{P}_i$, <u>no</u> agent can distinguish s_1 and s_2 (i.e., $P_i(s_1) = P_i(s_2)$ $\forall$

i). This contradicts (3). Thus case 1 is impossible.

Case 2.

$$(11) \quad E_{s'}U_i[w_{1i}(s_1)+\widetilde{T}_{1i}(s_1), \ w_{2i}(s_1)+\widetilde{T}_{2i}(s_1, \ s')] < E_{s'}U_i[w_{1i}(s_1)$$

$$+\widetilde{T}_{1j}(s_2), \ w_{2i}(s_1)+\widetilde{T}_{2j}(s_2,s')]$$

for some i, j (i $\neq$ j), for some s_1, s_2 such that $\hat{P}_g(s_1) = \hat{P}_g(s_2)$. In other

words, agents misrepresent their type, and possibly the state as well. We

claim that this is impossible. Again, there are two cases to consider.

a) Equation (11) holds for $s_1 \neq s_2$. But we have already noted that (3) and

(4) imply that $\widetilde{T}_{1i}(s_1) = \widetilde{T}_{1i}(s_2)$, and $\widetilde{T}_{2i}(s_1, \ s') = \widetilde{T}_{2i}(s_2, \ s')$ $\forall$ s_2 ϵ

$\hat{P}_g(s_1)$ if $\hat{P}_g = \overset{I}{\underset{i=1}{V}} \hat{P}_i$. Therefore (11) must also hold for $s_1 = s_2$. This is

case b.

b) Equation (11) holds for $s_1 = s_2$. But then

$$(12) \quad E_{s'}U_i[w_{1i}(s_1)+\widetilde{T}_{1i}(s_1), \ w_{2i}(s_1)+\widetilde{T}_{2i}(s_1, \ s')] < E_{s'}U_i[w_{1i}(s_1)$$

$$+\widetilde{T}_{1j}(s_1), \ w_{2i}(s_1)+\widetilde{T}_{2j}(s_1, \ s')].$$

However, the definition of the $\widetilde{T}_{ji}$ indicates that they are constructed from

each agent's optimizing choices at equilibrium prices. Since all agents face

the same price, (12) would imply either that

i) $\widetilde{T}_{1j}(s_1)$ is not affordable for i. Thus i cannot select it (i.e., it is not

feasible for i). This contradicts the supposition that i would opt for

$\widetilde{T}_{1j}(s_1)$ in state s_1.

ii) $\widetilde{T}_{1j}(s_1)$ is affordable for i in state s_1. But this contradicts the maxi-mality of $\widetilde{x}_i(s_1)$ and $\widetilde{M}_i(s_1)$ in agent i's budget set. Thus i cannot opt for $\widetilde{T}_{1j}(s_1)$, contradicting (12).

In short, then, when the government has the best available informa-tion regarding the prevailing state, the monetary equilibrium allocation can always be centralized. This is true regardless of whether agents' types are private information. In other words, lack of knowledge of type creates no informational difficulties in centralizing the equilibrium allocation. These can occur only when the government does not have the best possible initial information regarding the prevailing state s.

V. Money as a Transactions Medium

The definition of a transactions medium used here is that a transac-tions medium is (an asset) something vital to the attainment of a particular allocation of resources. In other words, money is a transactions medium if and only if the monetary equilibrium allocation can not be centralized. A corollary to proposition 4 is

Proposition 5. Money is a transactions medium in an OLG pure exchange economy only if there exists an informational asymmetry regarding the prevailing state of nature.

In short, all agents (the government) cannot have the best available informa-tion if money is to be a transactions medium. This accords closely with the commonly advanced notion that the existence of money (as opposed to a pure credit economy) has to do with problems of private information.[5]

However, it will be noted that this is for a different reason than has been advanced elsewhere in the literature. In particular, Friedman (1960) argued that money might play an essential role in exchange for adverse selec-

tion/moral hazard reasons when agents might default on privately issued debt. This was formalized in Smith (1982). Here, though, the possibility of default has been excluded. Alternatively, Brunner and Meltzer (1971) and others have suggested that money might be explained by the existence of asymmetric information about the quality and attributes of various commodities (other than IOU's, as Friedman and Smith consider). This type of informational asymmetry is also excluded here. The only important kind of informational asymmetry considered here (as proposition 4 indicates) is asymmetric information about the prevailing state of nature. This type of informational asymmetry, embedded in an OLG model, is sufficient to generate a transactions medium role for money. Thus more complex informational frictions (such as those considered by Brunner and Meltzer) are not required for money to be a transactions medium.

When will money not be a transactions medium then? While proposition 5 answers this question in part, a more complete answer is that money can not be a transactions medium by the definition proposed here only if a complete set of lump-sum tax/transfers is available to the government. It is known from the optimal taxation literature that when private information exists, a complete set of lump-sum taxes will not typically be available to the government.[6/] In the economies of examples 1 and 3, the lump-sum transfers which would be required to reproduce the monetary equilibrium allocation are not available to the government for this reason. Thus private information creates a role for a transactions medium.

While it will be noted that the only kind of private information essential to this role concerns the prevailing state of nature, it was also seen that other kinds of private information (as about agents' types) could compound the government's problem of discovering the prevailing state. Thus

although the types of informational asymmetries discussed above are not essential to creating a transactions medium role for money, this is not to say that they might not be helpful in creating such a role.

Finally, we might ask whether the definition of a transactions medium offered here is really what is typically meant by a medium of transaction. While this term is generally not given a precise definition, a medium of transaction is always taken to be an asset which "lubricates" the process of exchange, or more generally, of the reallocation of resources. The definition advanced above requires that it be _impossible_ to attain certain reallocations of resources without a transactions medium. Thus the definition employed here is, if anything, stronger than the standard notion of a transactions medium. It has been seen that within the OLG framework there exist economies for which money satisfies this definition. By any standard notion, then, money "lubricates" resource transfers in these economies. There seems, therefore, to be little basis for the claim that OLG models are incapable of providing a role for money as a transactions medium.

VI. Some Comments on the Role of the "Government."

At least two comments seem to be in order concerning the role of the government in centralizing resource allocations. The first concerns the amount of information that the government has been assumed to possess in constructing economies where money is a transactions medium. In the construction of examples 1 and 3 (and the open set of economies of proposition 2), the government was given initial information which was in each case common knowledge in the sense of Aumann (1976).[7] A brief defense of this is now offered in order to indicate that *our results are not based on some "unusual" assumptions about the initial information of the government.*

Suppose that the government possessed more than common knowledge regarding the prevailing state. In an economy in which resource reallocations take place through exchange, if prices do not reveal more than common knowledge about the state then the partitions P_i impose constraints on the set of informationally feasible transactions (as in equation (3)). A subset of these constraints could be relaxed if the government were to simply announce its knowledge of the current state. However, then $\hat{P}_g$ would coincide with common knowledge.[8/] Thus this formulation seems to be the most reasonable one.

A second comment concerns the view taken here of the government as an independent agent in the economy. This view is not necessary to the analysis. In particular, we could consider an economy in which government is a collusive arrangement, but in which agents do not transmit information when they collude. This corresponds to the notion inherent in the definition of the "coarse" core proposed by Wilson (1978). Under this interpretation, the analysis above may be taken to imply that agents colluding in this manner will not generally face the monetary equilibrium allocation as one of their options. In short, it is not necessary to the argument to view the government as a deus ex machina in this setting.

VII. Money and Information.

As a final exercise, this section considers the incentives for agents to allow prices to reveal information in the presence, and in the absence of money. The economic set-up parallels that of example 2. To recap, type 2 agents in that example always know the prevailing state, and in the process of exchange, prices may reveal this information to type 1 agents. However, if the prevailing state is s = 2, young type 2 agents could obtain a better expected rate of return on their portfolios if they either could con-

vince type 1 agents that the true state were s = 1, or if they could prevent revelation of the state. Suppose these agents attempt to prevent revelation through prices. Then we need some assumption on the behavior of type 2 agents when s = 1. Because allowing agents to behave monopolistically presents problems in the treatment of money, we assume that young type 2 agents in s = 1 take R(1), Q(1), and Q(2) as parameters. Then in order to prevent state revelation, agents who are young when s = 2 must mimic the behavior of agents young in s = 1. We will call the resulting equilibrium a quasi-competitive equilibrium.

<u>Definition</u>. A quasi-competitive equilibrium for an element $(w, n) \in E_f^*$ is a pair of nonnegative values $\bar{R}$, $\bar{Q}$, and a set of values $\bar{x}_i$, $\bar{M}_i$; $i = 1, \ldots, I$ such that

(iii) $\bar{x}_i$, $\bar{M}_i$ are maximal for type i agents in the budget sets defined by $\bar{R}$ and $\bar{Q}$ when s = 1.

(iv) $\sum_{i=1}^{I} n_i \bar{x}_i = 0$

(v) $\sum_{i=1}^{I} n_i \bar{M}_i = M$

(vi) $E_s, U_2[w_{12}(2) + \bar{x}_2 - \bar{Q}\,\bar{M}_2, \; w_{22}(2) - \bar{R}\,\bar{x} + \bar{Q}\,\bar{M}_2] \geqslant E_s, U_2[w_{12}(2) + \tilde{x}_2(2)$

$-\tilde{Q}(2)\tilde{M}_2(2), \; w_{22}(2) - \tilde{R}(2)\tilde{x}_2(2) + \tilde{Q}(s)\tilde{M}_2(2)]$,

where a "$\sim$" denotes the competitive equilibrium value for $(w, n) \in E_f^*$. Thus a quasi-competitive equilibrium is one where equilibrium values clear markets at prices $\bar{R}$, $\bar{Q}$ in s = 1, type 2 agents in s = 2 act as if s = 1 in terms of their transactions, and they prefer this to allowing prices to reveal the state (condition (vi)).

Given this definition, we have

Proposition 6. There exist economies which have only a quasi-competitive equilibrium when $M \equiv 0$, and which have only a competitive equilibrium when $M > 0$ and money is valued.

The proof is by example.

Example 4. The economy is as in the previous examples, except that

$$[w_{12}(1), w_{22}(1)] = (11, 9) \text{ with probability } 1/2$$

$$[w_{12}(2), w_{22}(2)] = (12, 9) \text{ with probability } 1/2.$$

If $M = 0$, it is easily verified that the quasi-competitive equilibrium has $\overline{R} = \frac{20}{21}$, and $\overline{x} = \frac{1}{2}$.

The competitive equilibrium, on the other hand, would have (if it existed)

$$\widetilde{R}(1) = \overline{R} = \frac{20}{21}$$

$$\widetilde{x}(1) = \overline{x} = \frac{1}{2}$$

$$\widetilde{R}(2) = \frac{10}{11}, \quad \widetilde{x}(2) = -1.$$

Then for young type 2 agents in $s = 2$, the expected utility level if prices reveal the state is

$$\ln[w_{12}(2) + \widetilde{x}(2)] + (\frac{1}{10})[w_{22}(2) - \widetilde{R}(2)\widetilde{x}(2)] = 3.3888.$$

The expected utility from mimicing the behavior of young agents in $s = 1$ is

$$\ln[w_{12}(2) + \overline{x}] + (\frac{1}{10})[w_{22}(2) + \overline{R}\ \overline{x}] = 3.3899.$$

Thus (vi) is satisfied, and there is no incentive for young type 2 agents to allow prices to reveal the prevailing state.[9/] Then when $M = 0$, revelation does not occur, and there is not a full competitive equilibrium.

Suppose now that M > 0, and that money is valued. Then if type 2 agents do not allow the state to be revealed, $\bar{R} = 1$, $\bar{Q}M = 1$, and $\bar{x} = 0$. If these agents behave competitively, on the other hand (i.e., if they allow prices to reveal the state), equilibrium values are

$$\tilde{Q}(1)M = 1.465 \ , \ \tilde{R}(1) = 1.024$$

$$\tilde{Q}(2)M = 1.535 \ , \ \tilde{R}(2) = 0.977$$

$$\tilde{x}_2(1) = 0.234 \ , \ \tilde{x}_2(2) = -0.235.$$

The expected utility attained by young type 2 agents if they allow revelation in s = 2 is 3.399. If they do <u>not</u> allow state revelation, $EU_2 = 3.398$. Thus when M > 0 and money is valued, informed agents will allow revelation of the state.[10/] This establishes the proposition.

It is also possible to prove

<u>Proposition 7</u>. There exists a nonempty open set of economies in E_f* which have competitive equilibria when money is present and valued, but which have only quasi-competitive (nonrevealing) equilibria when M = 0.

The proof closely parallels the proof of proposition 2, and is omitted here. Proposition 7 states that the economy of example 4 is not isolated in any formal sense.

The significance of proposition 6 is that for the economies considered, the introduction of valued fiat money strictly expands the social opportunity set. In example 4, for instance, when M = 0 the constraint $x_i(1)$ = $x_i(2)$ was imposed on all potential exchanges. The introduction of valued fiat money relaxed a constraint on trade, and thereby expanded aggregate opportunities. In short, once a somewhat less than competitive equilibrium concept is allowed for, the introduction of money in a <u>pure exchange</u> OLG

economy can expand social opportunities. This in turn is in contrast to the claim of McCallum (1982), for instance. This is that money must appear in some objective function (specification of tastes or technology) for this to occur. In fact, it is fairly easy to construct a wide variety of OLG economies with various kinds of private information in which money expands trading opportunities in essentially the manner discussed here.

VIII. Conclusions.

As indicated previously, the act itself of introducing fiat money represents an intervention by some governmental authority. Given that some intervention inevitably takes place in any monetary economy, a natural question is whether this one-time intervention has any natural advantage over the complete centralization of resource reallocation. It has been seen that there is such an advantage; some resource allocations are attainable through a one-time intervention, but not through continuing intervention. Moreover, this provides, in a strong sense, a rationale for why money can be a medium of transaction in an OLG model. This is that money "lubricates" transfers of resources. In fact, certain transfers are impossible without the presence of some monetary asset.

In summary, then, the introduction of elements of private information in an OLG setting allow money to "lubricate" exchange without technologically based transactions frictions. Only certain types of uncertainty can accomplish this, however. In particular, there must be some underlying uncertainty regarding the prevailing state of nature for money to play this role if agents are not permitted to default on their obligations. It was also seen that other types of informational frictions could compound problems presented by private information regarding the current period state. Finally, it was

seen that when agents can use their understanding of the economy to conceal the state, the introduction of valued fiat money can result in an increase in the amount of information transmitted by prices. This, in turn, expands aggregate opportunities in these economies.

Footnotes

[1]/Brunner and Meltzer (1971), Friedman (1960).

[2]/The nature of this intervention is discussed in detail by Cass and Yaari (1965).

[3]/W maps the current period state, the set of possible future states, and n into positive pairs $\tilde{R}(s)$, $\tilde{Q}(s)$; i.e., we focus on valued fiat money equilibria.

[4]/Note in particular that agents are not allowed to default on loans in trade either with each other, or with the government.

[5]/See, e.g., Friedman (1960).

[6]/See Guesnerie and Seade (1982), or Stiglitz (1982).

[7]/See Milgrom (1981) for an axiomatic characterization of common knowledge.

[8]/In the economies examined here, the $P_i(s)$ were singletons. One might suggest, then, the government should be given post-trade common knowledge. It is easy to construct examples, though, where $\hat{P}_i(s) = P_i(s) \; \forall \; s \; \epsilon \; \Omega$, and where P_i is finer than P_j for some i, j. In other words, amending $\hat{P}_g$ in this way would not correct the informational asymmetry, and the problems discussed in the text could still arise.

[9]/Note that type 2 agents who are young in s = 1 are indifferent as to whether revelation occurs or not.

[10]/Type 2 agents who are young in s = 1 also prefer that revelation occur.

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