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THE QUANTITY THEORY FAVORABLY RECONSIDERED: A COMMENT

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I have benefitted in writing this from conversations with Thomas Sargent, who is not responsible for any errors.

Abstract

This paper comments on "The Real Bills Doctrine vs. the Quantity Theory: a Reconsideration" by T. Sargent and N. Wallace. It argues that there exists a class of models similar to theirs that is (a) favorable to the quantity theory view of price stability, (b) supports the imposition of 100 percent reserve requirements, and (c) explains a long history of legal credit restrictions. In particular, lending restrictions stabilize price levels and result in Pareto improvements.

A long standing debate focuses on the regulation of borrowing and lending, and whether or not a cogent economic rationale can be established for such regulation. While at one time such regulation was concerned with the borrowing and lending of a large number of agents in the economy,[1] the modern focus has been on whether or not "banks" should face limitations on lending. With regard to this question, one school of thought has held that banks should face 100 percent reserve requirements, or simply be money warehouses.[2] While the economic benefit to this restriction is at best vaguely specified, one outcome is clearly meant to be price level stability.

This view has received little support. At one time this was because it was viewed as "unrealistic."[3] More recently, however, it has been argued that such a scheme is suboptimal relative to laissez-faire. In particular, Sargent and Wallace (1981, 1982) have constructed a model in which a subset of agents are prohibited from making loans. The portfolios of these agents consist entirely of stored fiat money. It is then shown that, for a special economy, this arrangement is not Pareto optimal. A laissez-faire arrangement is. Moreover, this is true despite the fact that restrictions on lending stabilize the price level in the Sargent-Wallace setting. Thus it is argued that restrictions on lending, which they refer to as a "Quantity Theory regime," are undesirable.

The Sargent-Wallace analysis is carried out for economies that "satisfy the quantity theory claims about the degree of price level fluctuations and even price-level determinacy that prevails with and without restrictions. Yet, despite all this, the examples do not support the quantity theory position."[4] However, this analysis is carried out in an economy where there is no economic rationale for stabilizing the price level. It is less than surprising, then, that the "quantity theory intervention" of imposing 100 percent reserve requirements is not Pareto optimal.

The purpose of this note is to demonstrate that there exist worlds much like the Sargent-Wallace one where the quantity theory view is supported. More specifically, an economy is presented in which preventing some agents from lending (a) stabilizes the price level, (b) is Pareto improving, and (c) (somewhat redundantly) is deflationary. The only substantive feature of this economy which differs from that of Sargent and Wallace is the presence of random endowments. In addition, the model is set up in such a way that insurance against random endowments is precluded. Therefore, the laissez-faire regime need not result in a first-best Pareto optimal competitive equilibrium. It is shown that in some such cases, lending restrictions of the "quantity theory" type proposed by Sargent and Wallace make some agents sufficiently better off that all losers from regulation can be compensated. Thus the quantity theory regime is Pareto superior to the laissez-faire arrangement. Moreover, this is true even though the economy is specifically structured so that agents who face any randomness in the return on their portfolio are risk neutral with respect to this randomness. Thus, at the date of his/her birth, no agent cares about the randomness he/she faces in future periods.

It will be noted that the feature of risk-neutrality with regard to portfolio returns is meant to be "favorable" to laissez-faire regimes; i.e., it implies that agents do not "care" about future price level fluctuations. Nevertheless, the quantity theory regime is Pareto superior to laissez-faire. It will also be noted that Sargent and Wallace state that (p. 24), "some may argue that our model is rigged against the quantity theory because it abstracts from uncertainty. . . . We doubt that merely complicating the model to deal with additional phenomena would change its basic message." However, when uncertainty cannot be insured against, government intervention of the quantity theory type can correct (partially) for some market incompleteness.

I. The Model

The model is a slight variant of the overlapping-generations model presented by Sargent and Wallace (1981). Time is discrete, and indexed by $t = 0, 1, \ldots$. At $t = 0$ there is a set of initial old agents who are in the last period of their life. Let C_1 (scalar) be consumption (of any agent) when young, and C_2 be consumption when old. Then the utility of these agents is simply C_2.

There are also a set of initial young agents at $t = 0$. These agents are of three types, α, β, and γ. They then become old at $t = 1$, their last period of life. At $t = 1$ there appears a new young generation, etc. Each generation has (large) equal numbers of each type of agent, and within each generation there are equal numbers of α, β, and γ agents.

At the time each agent appears, he/she realizes a lifetime endowment stream, which for some agents is random. Let there be two states of nature, indexed by $s = 1, 2$, each occurring with probability (1/2). Then the state of nature is drawn prior to trading (perhaps prior to an agent's birth), so that in each period trading occurs poststate.

There are two kinds of commodities that an agent could (potentially) trade. The first is fiat money, and the second is consumption loans. We choose the single consumption good as numeraire. Money trades for the good at the rate $Q(s)$ in state s (we focus on steady states, and hence time arguments) at each date, and circulates in fixed amount M forever. One unit of the consumption good lent in state s returns $R(s)$ with certainty one period hence, so $R(s)$ is the gross real rate of interest. There is no market in state contingent claims, which does not affect any results, but which does economize on notation.

The preferences of agents are as follows:

$$U_\alpha(C_1, C_2) = \ln C_1 + C_2$$

$$U_\beta(C_1, C_2) = \ln C_1 + 10C_2$$

$$U_\gamma(C_1, C_2) = \ln C_1 + \ln C_2.$$

We use a very specific example as we want merely to establish the possible desirability of restrictions on lending. The endowments of agents are as follows. Type α agents have the certain endowment $\alpha = 13$ when young, zero when old. Type γ agents have the certain endowment zero when young, $\gamma = 100$ when old. Type β agents have endowment $\beta(s)$ when young, zero when old. $\beta(1) = 30$, $\beta(2) = 50$.

Finally, let $\ell_i(s)$ and $M_i(s)$ be young type i agents' lending and money holding in state s (i.e., when s is the realized young state). $\ell_i(s) < 0$ implies that agents of type i are borrowers.

II. Equilibrium under Laissez-Faire

Following Sargent and Wallace, we wish to compare the rational expectations competitive equilibrium which emerges under unrestricted borrowing and lending with that emerging under restrictions preventing certain types of agents from making loans. This section considers the laissez-faire (LF, or unrestricted) competitive equilibrium.

Under rational expectations and LF, agents' behavior is described by the solutions to the following set of constrained maximization problems. Type α agents solve (α) $\max \ \ln[13-\ell_\alpha(s)-Q(s)M_\alpha(s)] + (1/2)Q(1)M_\alpha(s) + (1/2)Q(2)M_\alpha(s) + R(s)\ell_\alpha(s)$

by choice of $\ell_\alpha(s)$ and $M_\alpha(s)$ subject to $M_\alpha(s) \geq 0$. Type β agents young in state s solve

(β) $\max \ \ln[\beta(s)-\ell_\beta(s)-Q(s)M_\beta(s)] + 10[(1/2)Q(1)M_\beta(s)+(1/2)Q(2)M_\beta(s)+R(s)\ell_\beta(s)];\ M_\beta(s) \geq 0,$

and type γ agents solve

(γ) $\max \ \ln[-\ell_\gamma(s)] + \ln[\gamma+R(s)\ell_\gamma(s)].$

If we restrict our attention to equilibria in which fiat money has value ($Q(s) > 0 \ \forall\ s$), then an equilibrium for this economy will have at least one agent holding both loans and money in positive amounts $\forall$ s. This requires

(1) $$R(s) = \frac{EQ(s)}{Q(s)} \ \forall\ s.$$

Then in equilibrium, it will be the case that the $\ell_i(s)$ and $M_i(s)$; $i = \alpha, \beta, \gamma$, obey

(2) $$\ell_\alpha(s) + Q(s)M_\alpha(s) = 13 - \frac{1}{R(s)} .$$

(3) $$\ell_\beta(s) + Q(s)M_\beta(s) = \beta(s) - \frac{1}{10R(s)} .$$

(4) $$\ell_\gamma(s) = \frac{-50}{R(s)};\ M_\gamma(s) = 0.$$

Definition. A rational expectations competitive equilibrium (with valued fiat money) is a positive vector $[R(1), R(2), Q(1), Q(2)]$ such that $\forall$ s

(a) $\sum_i \ell_i(s) = 0$

(b) $\sum_i M_i(s) = M.$

It is easy to compute equilibrium values for this economy. These are $R(1) = 1.233$, $R(2) = .841$, $Q(1)M = 1.542$, $Q(2)M = 2.258$. For future reference, it will also be convenient to compute the levels of expected utility realized by young agents in alternate states under this equilibrium. As a shorthand, let $EU_i(LF,s)$ denote agent i's expected utility under LF when the realized state in his youth is s. Then

$$EU_\alpha(LF, 1) = 14.820$$

$$EU_\alpha(LF, 2) = 10.106$$

$$EU_\beta(LF, 1) = 366.389$$

$$EU_\beta(LF, 2) = 417.371$$

$$EU_\gamma(LF, 1) = 7.615$$

$$EU_\gamma(LF, 2) = 7.997$$

III. Equilibrium under a "Quantity Theory" Regime

Following Sargent and Wallace, a quantity theory (QT) regime prohibits certain agents from lending. This is meant to be an analog to restricting banks from lending (100 percent reserve requirements). In order to accomplish this, a legal minimum is imposed on the real value any agent can lend. Denote this minimum by v. Then, under the QT regime, type i agents' behavior is described by the solution to the relevant problem, (α), (β), or (γ), along with

(5) $\ell_i(s) \geq v$ if $\ell_i(s) > 0.$

For our purposes, it suffices to set $v = \alpha$.

Under these circumstances, clearly type α agents are restricted to storing money when young. Then their desired portfolio, given (5), is described by

$$(6) \qquad Q(s)M_{\alpha}(s) = \alpha - \frac{Q(s)}{EQ(s)}$$

$$\ell_{\alpha}(s) = 0.$$

It is also the case that given (5), the resulting equilibrium for this economy has type β agents holding only loans. Their desired portfolio is then described by

$$(7) \qquad \ell_{\beta}(s) = \beta(s) - \frac{1}{10R(s)}$$

$$M_{\beta}(s) = 0.$$

The desired portfolio of γ agents is unaffected by (5).

An equilibrium for this economy is defined as before: a set of prices such that

$$\sum_i M_i(s) = M \;\forall\; s$$

$$\sum_i \ell_i(s) = 0 \;\forall\; s.$$

Then it is easily verified that the rational expectations, competitive equilibrium under QT has $Q(1)M = Q(2)M \equiv QM = 12$, $R(1) = 1.67$, and $R(2) = 1.002$.

We can also compute the consumption levels of various agents in alternate states of nature. For type α agents if $s = 1$, or $s = 2$, then $C_1 = 1$, $C_2 = 12$. For young type β agents, if $s = 1$, then $C_1 = .0599$ and $C_2 = 50$. If $s = 2$, then $C_1 = .0998$ and $C_2 = 50$. For type γ agents, $C_2 = 50$ in all states. In $s = 1$, $C_1 = 29.94$, and in $s = 2$, $C_1 = 49.900$.[5]

IV. A Welfare Comparison of Regimes

In this section a welfare comparison of the LF and QT regimes is undertaken on the basis of the following criterion. Before any young agents are born, a decision must be made regarding which regime is to prevail. The expected utility of each agent under the two regimes is to determine which is preferred. However, on the basis of this criterion alone, the two regimes will not be comparable. Thus we will say that the QT regime is preferred if those who gain under it relative to LF (in terms of expected utility) can compensate those who lose relative to LF.

From data in section I, we can compute

$$EU_{\alpha}(LF) = (1/2)(14.820) + (1/2)(10.106) = 12.463$$

$$EU_{\beta}(LF) = 391.88$$

$$EU_{\gamma}(LF) = 7.806.$$

Now consider the following compensation arrangement. For $t \geq 1$, old type β agents transfer 19.96 units of the good to young type γ agents if $s = 1$. They transfer nothing to these agents in state 2. In addition, they transfer .463 units of the good to old type β agents in each state. Then letting $EU_i(QT+T)$ denote the expected utility of type i agents under QT when transfers are made, we have

$$EU_{\alpha}(QT+T) = 12.463,$$

$$EU_{\beta}(QT+T) = (1/2)\ln(.0599) + (1/2)\ln(.0998) + (1/2)(10)(50-19.96-.463)$$

$$+ (1/2)(10)(50-.463) = 393.010,$$

and

$$EU_\gamma(QT+T) = (1/2)\ln(29.94+19.96) + (1/2)\ln(49.9) + \ln(50) = \ln(49.9) + \ln(50) = 7.822.$$

Thus for $t \geq 1$, gainers under QT can compensate losers.

Consider $t = 0$, then. Suppose the initial old compensate young γ agents by transferring 10.612 units of the good if $s = 1$, and 9.553 units if $s = 2$. This yields

$$EU_\gamma(QT+T) = (1/2)\ln(29.94+10.612) + (1/2)\ln(49.9+9.553) + \ln(50) = 7.806.$$

Thus all initial young agents can be compensated. The initial old are also gainers, as under LF $EQ(s)M = 1.9$, while under QT, $QM = 12$. Then for the initial old, expected consumption rises by .0175 units, even after compensation has been made.

We have established the following results, then. First gainers in expected utility terms can compensate losers if a quantity theory regime is adopted in preference to a laissez-faire regime. In short, the competitive equilibrium under LF is not a first-best Pareto optimum. This is the contrast between this model and that of Sargent and Wallace.

Second, as in the Sargent-Wallace setting, the quantity theory regime stabilizes the price level. Third, the QT regime results in a higher level of real balances in <u>each</u> state, so it is deflationary.

V. Conclusions

Restrictions on the ability of various agents to lend have a long history. In 15th century England, for instance, there were restrictions against lending by various private (i.e., nonbank) agents.[6/] The model of the previous sections captures this quite closely. In the modern period, these restrictions have typically been against banks. The model above can be viewed as an analog to a world in which some agents (bankers) face prohibitions on lending (100 percent reserve requirements). This type of world corresponds to that suggested by Friedman (1960).

The "frequent anticredit measures of the Crown"[7/] in the 15th century have been viewed by contemporary economists as simply growing out of bullionist sentiment. Similarly, the quantity theory restrictions discussed here have been viewed as being based on a misplaced emphasis on price level stability. In fact, however, in a world where markets are incomplete, welfare justifications for these types of restrictions are easy to construct.

As a final comment, consider the role of price level stabilization in the argument above. Under LF, in equilibrium $R(s) = \frac{EQ(s)}{Q(s)}$ held for each s. It is easily verified that for all agents in the model, indirect utility functions are convex in R(s), and are strictly convex for all young agents. Thus agents prefer interest rate instability. However, if $\frac{EQ(s)}{Q(s)}$ is substituted for R(s) in these indirect utility functions, it will be clear that agents dislike price instability *ex ante*. In short, the quantity theory view that interest rates should be allowed to fluctuate while prices should be stabilized has some merit in the model above. Moreover, this is true even though once young endowments are realized, agents are risk neutral as regards future price level movements.

Footnotes

1/Postan (1927).

2/See Friedman, pp. 65-75.

3/Friedman, p. 75.

4/Sargent-Wallace (1981), p. 4.

5/These numbers do not sum to the total quantity of resources because of rounding.

6/See the discussion by Postan, pp. 240-243.

7/Postan, p. 241.

References

Friedman, M., A Program for Monetary Stability, Fordham University Press, 1960.

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