Federal Reserve Bank of Minneapolis
Research Department Working Paper

UNEMPLOYMENT, SELF-SELECTION, AND EXTRANEOUS UNCERTAINTY: A SYNTHESIS

Bruce Smith

Working Paper 225
PACS File 3160

Revised January 1983

NOT FOR DISTRIBUTION
WITHOUT AUTHORS' APPROVAL

I have benefited in writing this paper from the comments of, and discussions with, V. V. Chari, Stanley Fischer, Edward Prescott, Donald Richter, Thomas Sargent, Sanjay Srivastava, Warren Weber, and the participants at a University of Iowa Seminar.

ABSTRACT

A model of a labor market is developed in which agents possess private information about their own productivities. This has the property that firms may use unemployment to create appropriate self-selection incentives. When this is the case, existence of an equilibrium may require that employment be stochastic. This is true even though all uncertainty is necessarily resolved prior to hiring. Even when existence is not at issue, it may be privately as well as socially desirable to randomize employment prospects. Finally, it is argued that this "adverse selection" approach is consistent with traditional "Keynesian" approaches to macroeconomics, but avoids some of the arbitrary features of several "Keynesian models."

Unemployment, Self-Selection, and
Extraneous Uncertainty: A Synthesis

Informational limitations in the presence of uncertainty have been used for some time as a rationale for the existence of unemployment, and as a theoretical underpinning for the Phillip's curve. However, existing literature has provided only incomplete suggestions regarding why arrangements do not arise to eliminate the sources of unemployment. Clearly any such suggestions require that these types of arrangements be either privately or socially disadvantageous, or that in short, some (set of) agents benefit from not creating such arrangements. This paper provides an argument to this effect. In particular, the paper considers a world in which prospective employers have information about workers' productivities if and only if market outcomes reveal such information. Borrowing from the logic of the adverse selection literature,[1/] it is demonstrated that in such an environment an equilibrium, if it exists, may require that some set of agents be "involuntarily" unemployed. This unemployment acts as a self-selection incentive. However, in some sense this outcome by itself is unsatisfactory. This is the case since in the model presented, any equilibrium has no randomness associated with it (all uncertainty is resolved in equilibrium). Thus, state dependent employment is not an implication of the analysis. Moreover, while it would be possible to allow there to be a set of spot markets with trading occurring after the realization of some random event, this would still leave agents with an incentive to (partially) insure themselves against fluctuations in employment.

Therefore, an alternate strategy is adopted. Retaining the version of the model with no randomness in preferences, technology, or endowments, it is demonstrated that nevertheless some uncertainty in employment prospects may be desirable. This is because such uncertainty can reduce the social cost of using unemployment as a self-selection incentive. More specifically, it will be demonstrated that the unemployed in an economy may prefer their employment prospects to be randomized. Incentives to fully insure against this randomness do not exist, because this would preclude the appropriate functioning of the self-selection mechanism.

It will be noted, then, that this line of argument suggests that there exist economies where random employment is desirable, both privately and socially, but where there is no underlying uncertainty in equilibrium. The phenomenon of randomness not directly related to random factors in the economy is termed "extraneous uncertainty."[2/] Our line of argument provides a rationale for this phenomenon which is more satisfying than existing ones, which have essentially to do with the possibility that a system of difference equations has more than one solution. The analysis below suggests that randomized employment is desirable in certain economies, not merely accidental.

Finally, our analysis suggests why randomness in employment is borne by a subset of agents. In particular, the conditions defining an equilibrium can be associated with a certain constrained optimization problem. This optimization problem has the feature that employment uncertainty is determined as if it was borne by all agents. However, self-selection incentives operate to insure that it is not.

As a by-product of the analysis, it is also easy to demonstrate that there is more than simply an efficiency role for random employment prospects. In fact, for some economies it is the case that stochastic employment is necessary to the existence of an equilibrium, despite the absence of underlying uncertainty. In short, state varying employment may be not only desirable, but for some economies it may be an essential aspect of any equilibrium.

It is interesting to contrast this with other explanations of unemployment based on the outcome of explicit optimization problems. In particular, the contrast between the results here and those obtained in the contracting literature[3/] is instructive. In the contracting literature, it is assumed that there are no, or only incomplete, opportunities to insure against income fluctuations. It is not made explicit in the models extant why the necessary insurance markets are not present. However, assuming that they are are not, unemployment is a by-product of certain types of employment-insurance contracts. Thus, in the contracting literature, unemployment arises out of attempts to reduce risk associated with random incomes. In contrast, in the model of this paper employers may attempt to increase the risk associated with labor income. Incentives to insure against this income uncertainty do not exist, as such insurance would undo the self-selection incentives provided by stochastic employment prospects.

There are also other interesting contrasts between the model of this paper and those of the contracting literature. One is that in the contracting literature, the assumption that labor is indivisible plays a crucial role.[4/] In the model presented here, it will be an equilibrium outcome that hours of employed agents do not fall when there is unemployment. In short, the model here delivers determinate hours levels in the presence of unemployment without introducing the device of labor indivisibilities.

Finally, it will be seen that the results of the paper can be derived in the context of an equilibrium concept which has several "Keynesian" features. In particular, firms unilaterally determine the employment levels of certain sets of workers. However, the rationing scheme which arises is endogenous. Thus the analysis of the paper contrasts with existing models of "quantity constrained equilibria," in that an arbitrary rationing scheme need not be imposed at the outset.

The scheme of the paper is as follows. Section 1 sets out the model, and the two alternate notions of equilibrium employed. Certain properties of these equilibria, which will be familiar from the adverse selection insurance literature, are also briefly reviewed. Section 2 demonstrates the possibility that stochastic employment is necessary to the existence of an equilibrium. Section 3 demonstrates that even when an equilibrium exists without randomness in employment, such randomization may result in strict Pareto improvements. Section 4 demonstrates that randomization is not always desirable. Section 5 concludes.

I. The Model

A. Description

In this section a description of the simplest possible economy of interest is presented. Since even for this simple economy no general existence results are obtainable, no real generality is lost through this simplification.

Consider an economy in which there are two types of agents, indexed by $i = 1, 2$. Each agent works (supplies labor), and consumes a single produced commodity. Denote the labor supply of a type i agent by L_i, and the consumption of that agent by C_i. Type i agents' preferences over nonnegative consumption-labor pairs are described by a function $U_i : R_+^2 \to R$, where the properties of U_i are standard:

(i) $U_i \varepsilon C^2$

(ii) $D_1U_i(C_i,L_i) > 0$

(iii) $D_2U_i(C_i,L_i) < 0$

(iv) U_i concave,

(D_j denotes the partial derivative of what follows with respect to its j^{th} argument). In addition, if C_i and/or L_i are random, each agent's preferences are additively separable over consumption-labor pairs in different states of nature. Finally, each agent is endowed with a single unit of labor, and nothing else.

Production is carried out entirely by firms, with there being free entry into the activity of goods production. For each unit of type i labor hired by any firm, π_i units of the commodity can be produced. The π_i are constant, and obey $\pi_1 > \pi_2$. In addition, the values of the πi are known by firms. However, an agent's index is private information, so that it is revealed to firms prior to the activity of production if and only if some observable characteristic of type 1 and 2 workers differs. We assume that the only such characteristic, prior to production, is the labor supply of each agent. Thus, an agent's index is identifiable by firms if, and only if, $L_1 \neq L_2$ in equilibrium. The wage rate received by each worker must be agreed upon prior to production, so different agents receive different wages if $L_1 \neq L_2$.

If market outcomes dictate that $L_1 \neq L_2$, then, each firm can distinguish agents' indices. Then agents of type i receive wage rate w_i, and firm profits for each type i agent employed are

$$(\pi_i - w_i)L_i,$$

where we have selected the produced good as numeraire. On the other hand, if market outcomes dictate $L_1 = L_2$, then firms cannot distinguish agents' indices, and there is only a single market wage rate, w. Since under such circumstances firms would be uncertain about any agent's index, this would be a random variable from the point of view of the firm, as would the agent's marginal product. Let θ denote the fraction of the population which has index i = 1. Then the mean of this random variable would be $\bar{\pi} \equiv \theta\pi_1 + (1-\theta)\pi_2$. In this case expected firm profits per capita would be

$$(\bar{\pi} - w)L.$$

Finally, all firms are expected profit maximizers given the information revealed to them by market outcomes.

B. Self-Selection

We wish to consider the possibility of a breakdown of self-selection incentives in a normal competitive equilibrium. However, we also wish to consider the possibility that some arrangements exist which can induce self-selection among agents, i.e., which induce $L_1 \neq L_2$. Therefore, we introduce the following assumption (which serves as an analog to the assumption that different agents have identical preferences in the insurance literature).

(v) For any $(C,L) \in [0,\pi_1] \times [0,1]$,

$$\left.\frac{\partial C}{\partial L}\right|_{U_1 \text{ constant}} \neq \left.\frac{\partial C}{\partial L}\right|_{U_2 \text{ constant}} .$$

In other words, we assume that a certain kind of correlation exists between productivity and preferences. In order to justify this assumption, one might think of the $U_i(C,L)$ as indirect utility functions derived from a model of home production.[5] Agents have similar preferences over goods, but productivities at home and in the workplace are correlated. Typically one would think these productivities to be positively related, and in fact we shall primarily focus on the case in which

(v') $$\left.\frac{\partial C}{\partial L}\right|_{U_1 \text{ constant}} > \left.\frac{\partial C}{\partial L}\right|_{U_2 \text{ constant}} .$$

In any case, it is the attempt to find self-selection incentives which exploit the correlation between preferences and productivity which is the focus of the following analysis.

C. Equilibrium

The equilibrium we envision is one in which firms compete against each other in order to attract certain kinds of workers. However, in doing so they face certain limitations imposed by the presence of private information. Therefore an equilibrium is characterized by two features. First, since there is free entry among firms, an equilibrium must display an absence of any rent opportunities. Second, firms may operate only on the basis of information they derive from the degree of self-selection among workers. Thus, a Nash equilibrium is one in which all firms offer wage-hours combinations to workers such that (a) there are no incentives to any firm to offer a deviant set of wage-hours packages, and (b) wage-hours combinations are structured in a way which is consistent with the self-selection of workers (if this occurs). Consistent with this approach, we adopt in this section a definition of equilibrium which is familiar from the adverse selection literature (Rothschild and Stiglitz (1976)).

Definition. A nonstochastic equilibrium is a nonnegative vector (w_1, w_2, L_1, L_2) satisfying

1) $$U_1(w_1L_1, L_1) \geq U_1(w_2L_2, L_2)$$

2) $$U_2(w_2L_2, L_2) \geq U_2(w_1L_1, L_1)$$

(if self-selection obtains it is incentive compatible; if $L_1 = L_2$, (1) and (2) hold trivially given (3))

3) $$w_i = \pi_i \text{ if } L_1 \neq L_2$$

$$w_i = \bar{\pi} \text{ ; } i = 1,2 \text{ otherwise.}$$

(zero expected profits)

4) L_1 maximizes $U_1(C_1,L_1)$ subject to (2) and

a) $C_1 \leq \pi_1 L_1$ if $L_1 \neq L_2$

b) $C_1 \leq \bar{\pi} L_1$ if $L_1 = L_2$.

(If $L_1 \neq L_2$, (C_1,L_1) is maximal for type 1 agents over the set of incentive compatible consumption-labor pairs earning nonnegative profits for their employers. If $L_1 = L_2$, (2) holds trivially and type 2 agents mimic type 1 agents. Then type 1 agents obtain the maximal (C,L) pair for them in the set $\{(C,L): C \leq \bar{\pi}L\}$.)

5) Taking L_1 as given, (C_2,L_2) is maximal for type 2 agents in the set $\{(C,L): C \leq \pi_2 L_2;\ L_2 \neq L_1\} \cup \{(\pi_1 L_1, L_1)\}$.

(If $L_2 \neq L_1$, (C_2,L_2) is maximal for type 2 agents subject to the firm's break-even constraint, and is weakly preferred to (C_1,L_1).}

6) There does not exist an alternate quadruple $(\hat{w}_1,\hat{w}_2,\hat{L}_1,\hat{L}_2)$ which

a) any workers prefer to (w_1,w_2,L_1,L_2) (i.e., which attracts any workers, taking the actions of other firms as given), and

b) earns nonnegative profits given the workers it attracts. (There is no incentive to disrupt an equilibrium.)

Our first result is that in any such equilibrium, there is no uncertainty regarding agents' types. This is

Proposition 1. Any equilibrium has $L_1 \neq L_2$.

Proof. Suppose to the contrary that there is an equilibrium with $L = L_1 = L_2$, and therefore with $w = \bar{\pi}$. Then by condition (6) there is no pair (w_1, L_1) preferred by type 1 agents, and which results in nonnegative profits for some firm. In particular, there is no pair of "small" numbers ε and δ such that $U_1(\bar{\pi}L + \varepsilon, L + \delta) > U_1(\bar{\pi}L, L)$, and such that firms earn non-negative profits by offering ε additional units of the good in exchange for δ additional units of labor. However, suppose that $\varepsilon, \delta > 0$, that

$$\left.\frac{\partial C}{\partial L}\right|_{\substack{U_1 \text{ constant} \\ (C,L) = (\bar{\pi}L, L)}} < \frac{\varepsilon}{\delta} < \left.\frac{\partial C}{\partial L}\right|_{\substack{U_2 \text{ constant} \\ (C,L) = (\bar{\pi}L, L)}},$$

and that some firm offers ε units of additional consumption in exchange for δ units of additional labor. It is readily verified that type 1 workers accept any such offer, and that type 2 workers do not. Since any firm making such an offer attracts only type 1 workers, for any sufficiently small ε and δ it must earn positive profits. Thus there exists an alternate quadruple which results in increases in type 1 utility, and profits for some firm, contradicting the initial hypothesis. An identical argument with $\varepsilon, \delta < 0$ applies if

$$\left.\frac{\partial C}{\partial L}\right|_{\substack{U_1 \text{ constant} \\ (C,L) = (\bar{\pi}L, L)}} > \left.\frac{\partial C}{\partial L}\right|_{\substack{U_2 \text{ constant} \\ (C,L) = (\bar{\pi}L, L)}}.$$

The resulting contradiction establishes the proposition.

The second result is that if an equilibrium exists, (1) holds with strict inequality for the equilibrium values. This is

Proposition 2. In an equilibrium, if one exists,

$$U_1(\pi_1 L_1, L_1) > U_1(\pi_2 L_2, L_2).$$

Proof. Suppose $U_1(\pi_2 L_2, L_2) = U_1(\pi_1 L_1, L_1)$ in equilibrium. It is easily verified that (1) can be binding only if (2) is binding in equilibrium. Therefore, under the supposition,

$$U_1(\pi_1 L_1, L_1) = U_1(\pi_2 L_2, L_2)$$
$$U_2(\pi_1 L_1, L_1) = U_2(\pi_2 L_2, L_2)$$

in equilibrium. But then suppose some firm offers all its employees (regardless of type) the (C,L) pair

$$(\hat{C}, \hat{L}) = [\lambda\pi_1 L_1 + (1-\lambda)\pi_2 L_2, \lambda L_1 + (1-\lambda)L_2];\ \lambda \varepsilon (0,1).$$

Then, by choosing

$$\hat{w}_i = \frac{\lambda\pi_1 L_1 + (1-\lambda)\pi_2 L_2}{\lambda L_1 + (1-\lambda)L_2}\ ;\ i=1,2,$$

which is the implicit wage associated with $(\hat{C}, \hat{L})$, and letting λ approach zero, we see that

$$\lim_{\lambda \to 0} \hat{w}_i = \pi_2\ ;\ i = 1,2.$$

Therefore, since $\hat{w}_i$ varies continuously with λ, for λ sufficiently small, $\hat{w}_1 = \hat{w}_2$ is arbitrarily near π_2. But then some firm could offer the quadruple $(\hat{w}_1, \hat{w}_2, L_1 = \hat{L}, L_2 = \hat{L})$ for some λ near zero, and if this attracts all workers, earn positive profits. But this offer will attract all workers, since by concavity of the U_i,

$$U_i[\lambda\pi L_1 + (1-\lambda)\pi_2 L_2, \lambda L_1 + (1-\lambda)L_2] \geqslant$$

$$U_i(\pi_1 L_1, L_1) = U_i(\pi_2 L_2, L_2).$$

However, this contradicts the assumption that the initial values constituted an equilibrium, and establishes the proposition.

The above propositions are important for two reasons. First, proposition 1 establishes that in any equilibrium, all uncertainty is resolved by market outcomes. Second, together the two propositions permit a simple characterization of a nonstochastic equilibrium. Since any candidate equilibrium has $L_1 \neq L_2$, any such (L_1, L_2) pair must be optimal among the set of all pairs satisfying the zero profit conditions and the self-selection constraint (2). (Constraint (1) is never binding in equilibrium.) If this were not the case, some firm could disrupt the candidate equilibrium while maintaining $L_1 \neq L_2$. Therefore, the only way in which an equilibrium could be disrupted is if some firm could offer a quadruple with $L_1 = L_2$ (and $w_1 = w_2$) which earns nonnegative profits given the workers it attracts. Moreover, any such quadruple could only be profitable if it attracted type 1 agents. But these agents can be attracted only if

$$\max_{\{L\}} U_1(\bar{\pi}L, L) \geqslant U_1(\pi_1 L_1, L_1)$$

for the equilibrium value of L_1. Therefore a nonstochastic equilibrium, if it exists, is a solution to the constrained maximization problem[6/]

(7) $\max_{\{L_1\}} U_1(\pi_1 L_1, L_1)$ subject to

(7a) $U_2(\pi_1 L_1, L_1) \leq U_2(\pi_2 L_2^*, L_2^*)$

(7b) $U_1(\pi_1 L_1, L_1) \geq \max_{\{L_1\}} U_1(\bar{\pi} L_1, L_1)$

(7c) $U_1\ (\pi_2 L_2^*, L_2^*) \leq U_1\ (\pi_1 L_1, L_1)$

(7d) $L_2 = L_2^*\ ,\ w_i = \pi_i,\ L_i \leq 1,$

where L_2^* is the maximizing value of $U_2\ (\pi_2 L_2, L_2)$.

Given this characterization of an equilibrium, it is now easy to see when an equilibrium will fail to exist. The problem (7) is simply the maximization of a continuous function over the compact set defined by (7a) - (7d). Therefore, a solution will exist unless (7a) - (7d) define an empty set. This is, in fact, a possibility for two reasons. The first is that there may be no value of L_1 simultaneously satisfying (7a) and (7b). Such a situation arises, for instance, in example 1 below. The second is that there may be no values of L_1 simultaneously satisfying (7a) and (7c). This problem will not arise, however, if

(8) $$\left.\frac{\partial C}{\partial L}\right|_{U_1 = \text{constant}} > \left.\frac{\partial C}{\partial L}\right|_{U_2 = \text{constant}}$$

for the equilibrium values of L_1 and L_2. It is easily checked that for all of the situations discussed below (8) is satisfied, and hence we largely ignore (7c) in subsequent discussion.

An Unemployment Equilibrium

The equilibrium notion advanced above is of interest because it is capable of generating unemployment of labor as an equilibrium outcome. In particular, let $L_1^* =$ argmax $[U_1(\pi_1 L_1, L_1)]$. Then L_1^* is commonly termed the notional supply of labor for type 1 agents. If $L_1 < L_1^*$ in equilibrium, this is the standard usage of the term unemployment. Then we have

Proposition 3. If (2) holds with equality in equilibrium and if

$$\left.\frac{\partial C}{\partial L}\right|_{U_1 \text{ constant}} < \pi_1$$

in equilibrium, there is unemployment of type 1 labor.

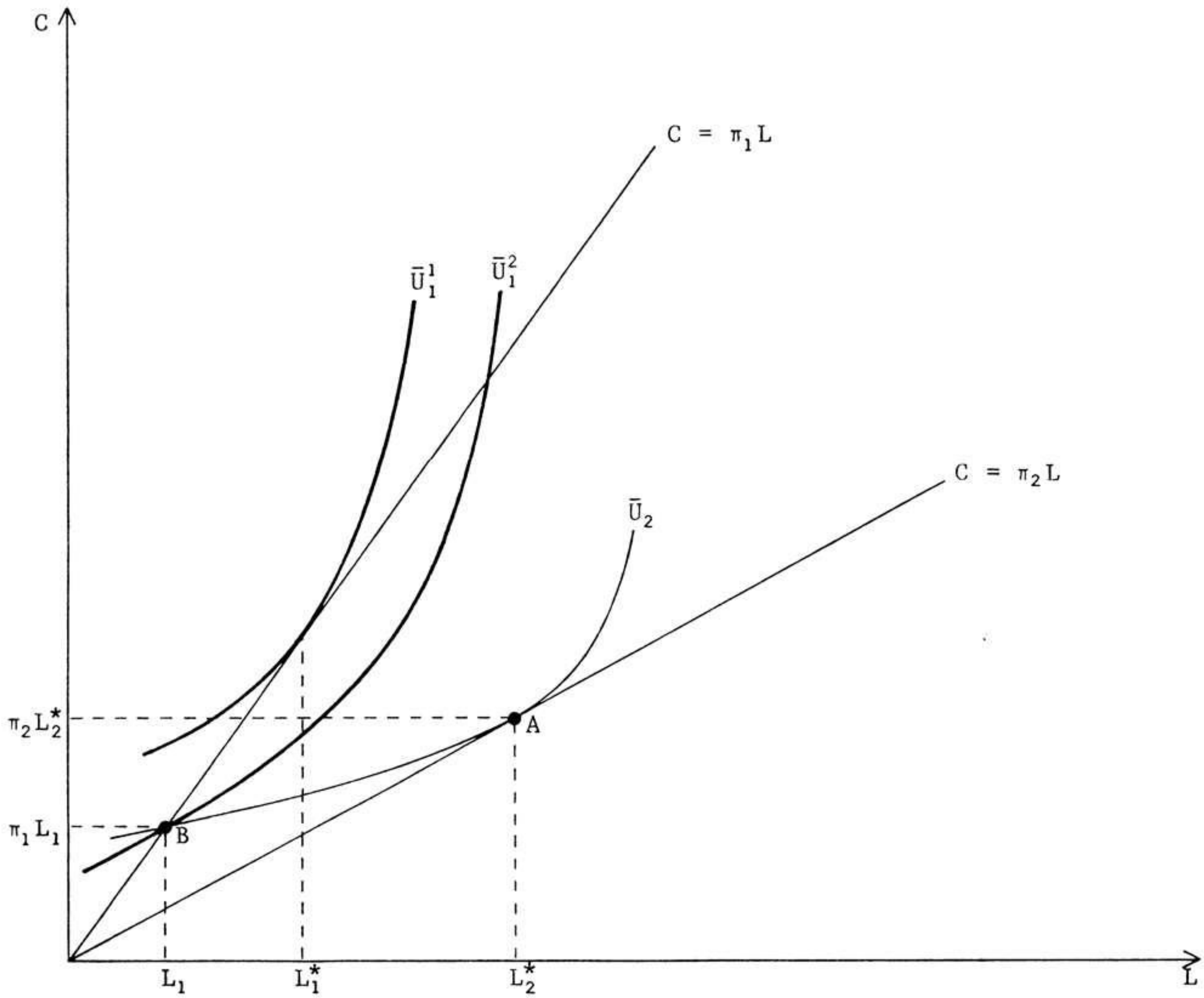
The proof of this proposition is trivial. An equilibrium in which $L_1 < L_1^*$ is depicted in Figure 1. In this figure, the loci labelled $\bar{U}_i$ are type i indifference curves, and the rays $C = \pi_i L$ are the zero profit loci. Point A is the equilibrium position for type 2 agents, which is merely the optimal (C,L) pair for these agents among those for which their employers "break even." Point B is the equilibrium position for type 1 agents. It is the maximal (C,L) pair for them among the set of pairs that break even for their employers, and that are not preferred to point A by type 2 agents. It will be noted that the equilibrium value of L_1 is less than L_1^*, which indicates that this is an equilibrium with unemployment, as claimed.

Definition. A nonstochastic unemployment equilibrium is a nonstochastic equilibrium with $L_1 < L_1^*$.

As something of a digression, it is useful to reflect briefly on the characteristics of the unemployment equilibrium

Figure 1

An Unemployment Equilibrium



depicted in Figure 1. In particular, it will be noted that firms simply select L_1 as the level of hours worked by type 1 agents. Inspection indicates that the preferences of type 1 agents play no role in this determination (although they are important for existence of equilibrium, as we shall see). Thus firms set employment levels more less unilaterally for this set of workers. This feature of the analysis is reminescent of standard "Keynesian" models, but with the difference that all prices here are flexible, and that quantity rationing schemes arise endogenously in equilibrium.

A second point of note is that in Figure 1 high productivity workers are unemployed. This is a general feature of an unemployment equilibrium and at first blush may seem a contrafactual implication of the model. However, consider the breakdown of data on average hours and average hourly earnings provided by the Economic Report of the President (1981, p. 273). Excluding agricultural workers and workers in wholesale and retail trades, we are left with production workers divided into two groups; manufacturing and construction. Workers in manufacturing typically earn 75 to 80 percent of the average hourly earnings of construction workers. In addition, the largest value for average weekly hours in construction in the postwar period is 38.9 The smallest such value in manufacturing is 39.1. Thus, this characteristic of the unemployment equilibrium, that workers with high wages work relatively few hours, is broadly consistent with U.S. data.

Stochastic Equilibria

It is the case, then, that the operation of self-selection mechanisms may require the presence of unemployment in a way which is consistent with actual experience. Yet, thus far, we have done nothing to indicate that such self-selection mechanisms may dictate random employment prospects. As a beginning in this direction we define a stochastic equilibrium. The idea behind this equilibrium concept is as follows. There is a finite and given set of states S, with typical element s. Associated with each element of S is a nonnegative quadruple $(w_1(s), w_2(s), L_1(s), L_2(s))$, which represents the wage-hours packages firms offer in state s. In addition, firms are permitted to choose a vector of probabilities that they will offer a particular set of wage and employment opportunities. Therefore, firms choose the vector p(s) of probabilities that a particular state occurs. In other words, we allow firms to employ mixed strategies, and to choose the probability of playing certain strategy.

Definition. A stochastic equilibrium is a mapping $[w_1(s), w_2(s), L_1(s), L_2(s), p(s)]: S \to R_+^4 \times [0,1]$ satisfying

$$(9) \qquad \sum_S p(s)\, U_1[w_1(s)L_1(s), L_1(s)] \geq \sum_S p(s)\, U_1[w_2(s)L_2(s), L_2(s)]$$

$$(10) \qquad \sum_S p(s)\, U_2[w_2(s)L_2(s), L_2(s)] \geq \sum_S p(s)\, U_2[w_1(s)L_1(s), L_1(s)]$$

(11) $w_i(s) = \pi_i \;\forall\; s \in S$ if $L_1(s) \neq L_2(s)$ for some s,

$w_i(s) = \bar{\pi} \;\forall\; s \in S$ if $L_1(s) \equiv L_2(s)$

(12) $p(s)$ and $L_1(s)$; $s \in S$ maximize

$$\sum_S p(s) U_1 [w_1(s) L_1(s), L_1(s)]$$

subject to (10), (11), and

$$\sum_S p(s) = 1.$$

(13) if $L_2(s) \neq L_1(s)$ for some $s \in S$, then $[C_2(s), L_2(s)]$ is maximal for type 2 agents over the set $\{[C_2(s), L_2(s)]: C_2(s) \leq \pi_2 L_2(s)\} \;\forall\; s \in S$.

(14) There does not exist an alternate mapping $[\hat{w}_1(s), \hat{w}_2(s), \hat{L}_1(s), \hat{L}_2(s), \hat{p}(s)]$ which

a) attracts any workers, and

b) given the workers it attracts, earns nonnegative profits.

Conditions (9) and (10) are the incentive compatibility restrictions, conditions (12) and (13) require that the $L_i(s)$ and $p(s)$ have certain optimality properties, (11) requires zero profits in each state, and (14) is the usual Nash equilibrium condition. The only one of these conditions which requires comment is (11). In a stochastic equilibrium, firms offer workers an employment-wage lottery. In order that firms not have an incentive to incorrectly report the outcome of this lottery, or to "fix" it in some way, it is necessary that they be indifferent concerning its outcome, i.e., that there be zero profits in each state (as opposed to zero expected profits). This, in turn, requires $w_i(s) = \pi_i$ (or $w_i(s) = \bar{\pi}$) $\forall$ $s \varepsilon S$.

The results obtained for a nonstochastic equilibrium carry over with only trivial modifications. In particular, any equilibrium has $L_1(s) \neq L_2(s)$ for some $s \varepsilon S$, if (10) holds with equality there can be unemployment of labor in some state, and (9) never holds with equality in equilibrium. It is also the case that for the same reasons as previously, an equilibrium (if it exists) is a solution to the problem

(15) $$\max \sum_S p(s)\, U_1[\pi_1 L_1(s), L_1(s)]$$

subject to

(15a) $$EU_2[\pi_1 L_1(s), L_1(s)] \leq U_2[\pi_2 L_2^*, L_2^*]$$

(15b) $$EU_1[\pi_1 L_1(s), L_1(s)] \geq U_1[\pi_2 L_2^*, L_2^*]$$

(15c) $$EU_1[\pi_1 L_1(s), L_1(s)] \geq [\max_{\{L_1\}} U_1(\bar{\pi} L_1, L_1)]$$

(15d) $$\sum_S p(s) = 1$$

(15e) $$L_2 = L_2^* \text{ , } w_i = \pi_i, \; L_i \leq 1,$$

by choice of $[L_1(s), p(s)]$; $s \varepsilon S$.

We have simplified the formulation of this problem by noting that since U_2 is concave, L_2 will never be random in equilibrium, and that any mapping which potentially disrupts the candidate equilibrium need not have L_1 random, since U_1 is concave. It will also be noted that the solution of (15), if it exists, has $L_1(s)$ constant $\forall$ s unless (15a) holds with equality. Thus, there will be no randomization of employment prospects unless (a) there is over or underemployment associated with the corresponding nonstochastic equilibrium, or (b) no nonstochastic equilibrium exists. Finally,

Definition. A stochastic unemployment equilibrium is an equilibrium where $L_1(s) < L_1^*(s)$ for some $s \varepsilon S$.

We are now prepared to examine the roles that randomized employment prospects can perform in this setting.

II. Stochastic Employment and Existence of Equilibrium

In this section it is established that randomized employment is required for an equilibrium to exist in some economies. In particular,

Proposition 4. There exist economies which have no nonstochastic equilibrium, but for which stochastic equilibria exist.

This proposition may be established by means of an example. The line of argument is to show that a particular economy has no nonstochastic equilibrium. It is then demonstrated that there exists some arrangement with random employment which cannot be disrupted by any allocation with $L_1(s) = L_2(s)$ $\forall$ s. This establishes the proposition.

Example 1. Preferences are given by $U_1(C,L) = qC - L$, and $U_2(C,L) = \phi C - (1/2)(L+1)^2$. Parameter values obey $\phi = 2$, $q = 2/3$, $\pi_1 = 3$, $\pi_2 = 1$. Then it is readily verified that in any arrangement with $L_1 \neq L_2$, (2) holds with equality. Therefore, (2) - (5) and the fact that the L_i must obey $L_i \leqslant 1$ imply that the candidate equilibrium values of L_1 and L_2 are $L_1 = 1/10$, $L_2 = 1$. The utility level for type 1 agents associated with this arrangement is $U_1 = 1/10$. However, suppose that $\theta = .3285$. Then this arrangement cannot be an equilibrium, since setting $w_1 = w_2 = \bar{\pi} = 1.657$, and $L_1 = L_2 = 1$ results in zero profits, and in $U_1 > 1/10$. (It also results in utility increases for type 2 agents.) But no equilibrium can have $L_1 = L_2$, so no nonstochastic equilibrium exists for this example.

Now suppose that some firm offers $L = 0$ with probability .895, and employment level $L = 1$ with probability .105. Then the expected utility level for a type 1 worker accepting this arrangement is $EU_1 = .105 > \max U_1(\bar{\pi}L,L) = .1047$. The expected utility level for a type 2 worker accepting this arrangement (and receiving wage rate $w_1 = 3$) would be $EU_2 = -.03 < \max U_2(\pi_2 L_2, L_2) = 0$. Thus, type 2 workers do not accept such an arrangement, while type 1 workers do. Therefore, this offer by a firm results in zero profits, and cannot be disrupted by any arrangement with $L_1 = L_2$. Since the equilibrium arrangement results in greater expected type 1 utility, this cannot be disrupted either. Thus, a stochastic equilibrium exists for this economy, establishing the proposition.

It may not be clear from Example 1 why randomization of employment can lead to the existence of an equilibrium. The usual role played by randomization is to "convexify" an economy. However, while there are nonconvexities associated with the incentive compatibility conditions in our economy, these are not the source of the nonexistence problem. Rather this problem lies with the fact that for some economies, such as that of example 1, (7a) and (7b) define a set of L_1 values which is empty. Randomization of L_1 produces existence as follows, then. For the economy of example 1, type 1 agents are risk neutral, and type 2 agents are risk averse. Moreover, the expected utility of type 1 agents increases with $EL_1(s)$. Therefore, if $EL_1(s)$ can be increased, while the $L_1(s)$ values are chosen "far enough apart," $EU_1[\pi_1 L_1(s), L_1(s)]$ can be increased without violating (15a). If it is increased sufficiently, (15a) and (15c) no longer define an empty set of values $\{L_1(s)\}$; $s \in S$. For the parameters of example 1, such choices of $L_1(s)$ are possible when # S = 2, so randomization of employment prospects permits an equilibrium to exist.

Note that for this economy, then, random employment prospects (with unemployment in some state) are a necessary feature of an equilibrium. This is true despite the fact that (a) in any equilibrium there is no uncertainty about any agent's index ($L_1(s) \neq L_2(s)$ for some s), and (b) there is no randomness of preferences, endowments, or technology. It is not the case, however, that randomization of employment always results in existence of equilibrium. No equilibrium at all need exist for the economy presented, as a suitable reinterpretation of Prescott and Townsend (1981) indicates.

It is also the case that randomization of employment prospects may be desirable in and of itself (i.e., even though it is not necessary for existence). This is the subject of the next section.

III. The Desirability of Random Employment

In this section it is established that for an economy in a nonstochastic unemployment equilibrium, randomization of employment may result in strict Pareto improvements. As a corrollary, then, the concept of a stochastic equilibrium suggests that any such economy is compelled to face random employment prospects in equilibrium. In particular, we establish,

Proposition 5. There exist economies which have a nonstochastic equilibrium, and a Pareto superior stochastic equilibrium.

The line of argument is to present a sample economy with a nonstochastic equilibrium. It is then demonstrated that randomization of employment results in strict Pareto improvement. By implication, then, there exists a Pareto superior stochastic equilibrium. We establish the proposition by example only, as it is demonstrated below that proposition 5 does not hold generally.

Example 2. The economy of this example is the same as that of example 1, with the exception that $\theta = 1/4$. It is clear that the candidate nonstochastic equilibrium pair is still $L_1 = 1/10$, $L_2 = 1$, and that this still results in $U_1 = 1/10$. However, it is now the case that $1/10 > \max U_1(\bar{\pi}L,L) = 0$, so that this candidate pair is, in fact, a nonstochastic (unemployment) equilibrium pair. It is not a stochastic equilibrium pair, however. To see this, suppose that some firm offered an employment lottery (paying wages equal to π_1) of $L = 1$ with probability .105, and $L = 0$ with probability .895. Then $EU_1 = .105 > 1/10 = U_1(\pi_1 L_1, L_1)$, and the expected utility to a type 2 agent accepting this offer would be $EU_2 = -.03 < 0 = \max U_2(\pi_2 L_2, L_2)$. Therefore, only type 1 agents would accept the proffered lottery, and it would earn zero profits in each state. Thus by condition (14), $L_1 = 1/10$, $L_2 = 1$ (with probability one) cannot be a stochastic equilibrium. The stochastic equilibrium for this economy, if it exists, must have random employment of type 1 labor. Finally, this equilibrium must exist, since it has $EU_1 > 1/10 > \max U_1(\bar{\pi} L, L)$. Note also that the equilibrium has $EU_2 = 0$, so that it results in levels of profits and type 2 utility identical to the nonstochastic equilibrium. It also results in an increase in type 1 utility, so that it Pareto dominates the nonstochastic equilibrium. This establishes the proposition.

Proposition 5 indicates that randomized employment prospects may be desirable, even when a nonstochastic equilibrium exists. Together propositions 4 and 5 indicate that "extraneous uncertainty" in employment prospects may be essential to the existence of an equilibrium, and that even when it is not, may be both socially desirable, and privately desirable from the point of view of every individual in an economy.

It remains to indicate what can be said about the form of the general solution to (15). This is the subject of section 4.

IV. The General Case

In this section we take up two issues. The first is why unemployment risk appears to be borne inequitably in the population. The second is why it is difficult to make general statements concerning when random employment will be an equilibrium outcome.

A. Employment Risk

Clearly any stochastic equilibrium where employment is state dependent has type 1 workers bearing the entire risk of unemployment. This is simply the analog in our economy of the standard externality imposed by one group on another in models of asymmetric information. However, there is more that can be said about the allocation of this employment risk. In particular, an examination of the optimization problem (15) indicates that in any equilibrium where stochastic employment is an outcome, (15a) is binding. Therefore, the level of employment in each state is set as if type 1 and 2 agents were both bearing employment risk. Moreover, the first order conditions associated with optimal selection of $L_1(s)$ are (for an interior optimum)

$$(16)\qquad \frac{D_L U_1[\pi_1 L_1(s), L_1(s)]}{D_L U_1[\pi_1 L_L(s'), L_1(s')]} = \frac{D_L U_2[\pi_1 L_1(s), L_1(s)]}{D_L U_2[\pi_1 L_1(s'), L_1(s')]}$$

$$\forall\ s,\ s' \varepsilon S.$$

Thus, at an interior optimum, the values of $L_1(s)$ are not only selected as if both sets of agents bore the risk of unemployment, but are selected in accordance with the standard conditions for optimal risk sharing among these groups. In other words, the fact that a subset of the population bears employment risk is not an indication that the amount of risk borne is determined independently of considerations of optimal risk sharing.

Unfortunately, however, it will not generally be the case that (15) has an interior optimum. In fact, it is easily verified that if the absolute risk aversion of type 1 agents is greater than that of type 2 agents, (16) defines a minimum. Thus, employment variability determined in accordance with the standard criteria for optimal risk sharing could also result in the worst possible outcome. We now take up the question of what can be said about the solution to (15) in the general case.

B. The Equilibrium Problem

Suppose that a solution to (15) does exist. What can be said regarding its properties? The answer is very little, for reasons which we now elaborate. To begin, define

$$B_2 = [C(s),L(s);s\varepsilon S \mid EU_2[C(s),L(s)] \geq U_2(\pi_2 L_2^*,L_2^*)\},$$

which is simply one upper contour set for type 2 agents. In addition, define

$$\Omega = (\mathop{X}_{j=1}^{S}[0,\pi_1]) \ X \ (\mathop{X}_{j=1}^{S}[0,1]),$$

and let $\Omega - B_2$ denote the complement of this upper contour set in agent 2's feasible set, Ω. Let $\overline{\Omega - B_2}$ denote the closure of the complement. Then the problem (15) (omitting constraints (15b) and (15c), which are assumed to be satisfied), is

$$\max \ EU_1[\pi_1 L_1(s),L_1(s)]$$

by choice of functions $L_1(s)$ and $p(s)$ such that $(\pi_1 L_1(s),L_1(s),p(s);\ s\varepsilon S)\ \varepsilon\ \overline{(\Omega - B_2)}\ X\ [0,1]\ X\cdots X\ [0,1]$. However, the constraint set is clearly not always convex, and therefore there is nothing in particular to be said about the form of this solution. An interior optimum may be possible for $L_1(s)$, but in general we can say little about the solution to (15) due to this nonconvexity of the constraint set.

In order to illustrate the difficulties which arise in any attempt to characterize the solution to (15), let us focus on a particular sequence of economies. For this purpose let Σ denote a specific economy, where an economy is a listing of preferences and parameter values, and let $\{\Sigma_n\}_{n=1}^{\infty}$ denote an infinite sequence of economies indexed by n. Then we present an example of a sequence of economies with the following features. For all n the Σ_n are identical, except that the marginal utility of consumption for type 2 agents varies with n. For all finite n, random employment is an equilibrium outcome. However, for the limit of the sequence, not only is random employment not an equilibrium outcome, but (15a) does not bind in equilibrium. This means that the limit economy behaves as if there were full public information regarding agents' indices.

Example 3. Preferences for Σ_n are given by

$$U_1(C,L) = (3/4)C_1-L_1; \; U_2(C,L;n) = \phi(n)C_2-(1/2)(L_2+1)^2,$$

and parameter values are $\pi_1 = 3/2$, $\pi_2 = 1$, $\theta = 1/6$. In addition, let $\phi(n) = \frac{n+1}{n}$. Thus, only D_1U_2 varies with n. It is straightforward to verify that for all finite n, the self-selection constraint (15a) binds, and that $L_2^* = 1/n$. Then the nonstochastic equilibrium value of L_1 for Σ_n, denoted $L_1(n)$, is determined by

$$(3/2)(\frac{n+1}{n})L_1(n) - (1/2)[L_1(n)+1]^2 =$$

$$(\frac{n+1}{n})(\frac{1}{n}) - (1/2)(\frac{1}{n}+1)^2 ,$$

if this equilibrium exists. Therefore,

$$L_1(n) = \frac{(1+\frac{3}{n}) - [(n+5)(n+1)n^{-2}]^{(1/2)}}{2}$$

(because the positive root for $L_1(n)$ results in $L_1(n) > 1 \; \forall \; n < \infty$). It is also readily verified that, in fact, an equilibrium does exist for these parameter values for all n.

We now demonstrate that for all finite n, randomization of L_1 is an equilibrium outcome. In order to do so, we need only show that there exist random values $L_1(s;n)$ and values $p(s)$; $s \in S$, such that these dominate the nonstochastic equilibrium.

Suppose, then, that $\#S = 2$, and that some firm offers $L_1(1) = 1$ with probability p for all finite n, and $L_1(2) = 0$ with probability 1-p for all finite n. Then the self-selection constraint (15a) will be satisfied if, for all $n < \infty$,

$$(17) \qquad (3/2)\ (\frac{n+1}{n})p - (1/2)p(2)^2 - (1/2)(1-p) \leq (1/2)(1-n^2)\ n^{-2},$$

and type 1 agents will prefer this randomization if $p > L_1(n)$ for all finite n. It is readily verified that (17) is equivalent to $p \leq \frac{1}{3n}$. In addition, it also the case that $\frac{1}{3n} > L_1(n)$ for all finite n. In order to see this, note that $L_1(n) < \frac{1}{3n}$ can be written as

$$\frac{n+3}{2n} - \frac{[(n+5)(n+1)]^{1/2}}{2n} < \frac{1}{3n}.$$

Multiplying both sides by n, we obtain

$$(18) \qquad (1/2)\{n+3 - [(n+5)(n+1)]^{1/2}\} < \frac{1}{3},$$

which holds for n=1. Now note that the left-hand side of (18) is a monotone decreasing function of n, so that for all $n > 1$, (18) holds as well. Therefore $\frac{1}{3n} > L_1(n)$ for all finite n, and thus random employment results in any equilibrium for all Σ_n; n finite.

Now consider

$$\Sigma_0 = \lim_{n \to \infty} \Sigma_n.$$

For Σ_0, $\phi(n) = 1$, and the equilibrium values of L_1 and L_2 are $L_1 = 1$, $L_2 = 0$. It is straightforward to check that these are the same values as would emerge under full information, so (15a) is not binding in the limit.

Example 3 demonstrates that there can exist two economies which are arbitrarily close, but where one has employment lotteries in equilibrium and one does not.7/ This is perhaps not surprising, since it is known that economies such as these have no continuity properties (Guesnerie and Seade (1982)). However, it serves to underline the fact that there is little to be said regarding the nature of the general solution to (15), when it does exist.

One might ask, then, what can be said regarding when an equilibrium will exist in any form, or when any type of solution to (15) will exist. The standard approach to this question is to note simply that for given preferences and values of π_i; i = 1, 2, this depends on θ (assuming

$$\left.\frac{\partial C}{\partial L}\right|_{U_1 = \text{constant}} > \left.\frac{\partial C}{\partial L}\right|_{U_2 = \text{constant}}$$

everywhere). If θ is sufficiently large, implying $\bar{\pi}$ is sufficiently large, the constraint set defined by (15a) - (15e) will be empty. However, none of the existing literature[8/] attempts to relate the critical value of θ to the shapes of preferences and to the π_i. In general, then, there has been little said on this question, and an attempt to expand on it is beyond the scope of this paper.[9/]

Prior to concluding this section, it is worthwhile to consider one additional case in which random employment is not an equilibrium outcome. While randomization did not emerge for the limit economy Σ_0 in example 3, it was also the case that Σ_0 had a full-employment equilibrium. We now establish that there exist economies with a nonstochastic unemployment equilibrium which is identical to the stochastic unemployment equilibrium. This is the point of

Example 4. Let $U_1(C,L) = qC - L$, and $U_2(C,L) = \phi C - L$. Parameter values are $q = 2$, $\phi = 3$, $\theta = 1/5$, $\pi_1 = 1$, $\pi_2 = 1/2$. It is readily verified that a solution to (7) exists, that it has $L_1 = 1/4$, $L_2 = 1$, and that this is an unemployment equilibrium. Now suppose that the solution to (15) differs from the solution to (7). Then this solution has $EU_1 > (q\pi_1-1)(1/4) = 1/4$, and $EU_2 \leq \max U_2(\pi_2 L_2, L_2) = 1/2$. However, the choice of parameter values is such that $U_2 = 2U_1$ for any particular consumption-leisure pair. Therefore, for (15) to have a different solution than (7), it is necessary that $EU_1 > 1/4 \geq EU_1$, a contradiction. Thus, this economy has a stochastic unemployment equilibrium where $L_1(s)$ is a trivial function of s, regardless of the number of states. This is the desired result.

Example 4, then, underscores the fact that there is little to be said regarding when random employment will result.

V. Conclusions

The preceding sections have presented a model in which unemployment, and employment uncertainty, are necessary features of some equilibria. In addition, when unemployment is an equilibrium outcome the model implies that employment uncertainty may be desirable. This is true despite the fact that there is no underlying uncertainty in equilibrium in the economy at hand. The model thus confronts the fact there appears to be unemployment, as well as employment uncertainty, which is not rooted in technological features (or features related to preferences) of observed economies. In particular, the model suggests that firms may tie employment to "extraneous" states of nature, or randomize agents' employment prospects.

It is useful to contrast this briefly with the existing literature on randomization in private information models. It is well-known, for instance, that in moral hazard economies randomization may be an equilibrium outcome. However, thus far, existing rationales for randomization have been derived in settings in which there is underlying randomness in preferences, endowments, or technology yet to be resolved at the time trading occurs. This is not the case in the model presented above. The state of nature has been realized when trading occurs, and yet employment lotteries may still be an equilibrium outcome. This result presents a contrast with existing discussions of adverse selection settings as well 10/, which suggest that random outcomes will not be a feature of an equilibrium.

It is also the case that the model presented can confront commonly observed phenomena other than those related to unemployment. As an example, consider virtually all existing estimates of the return to education. These suggest that the rate of return to schooling is substantially below other market rates of return. Why, then, do agents continue to invest in education? A suggestion provided by our model is that agents in low productivity jobs (ones with low rates of return) are induced to select themselves into such jobs by the levels of unemployment, and the degree of employment variability, associated with high productivity (high return) jobs. This is in accordance with the observation that jobs requiring high education typically involve less risk of unemployment than do other jobs. Thus, the model presented can also be used to explain other apparently anomolous features of labor markets.

Finally, we have argued that these results have been derived using an equilibrium concept similar in spirit to traditional "Keynesian" equilibrium notions. In particular, in an unemployment equilibrium firms set the hours of certain agents independently of their preferences for consumption and leisure. This is certainly consistent with Keynesian approaches, but avoids arbitrary specifications of how prices are set and of schemes for quantity rationing. Thus, the class of adverse selection economies examined would seem to provide a useful framework for studying unemployment, and the allocation of employment risk in labor markets. Moreover, they do so in a way which is consistent with a lengthy tradition in macroeconomics.

Footnotes

[1] E.g., Rothschild and Stiglitz (1976), Wilson (1977).

[2] Azariadis (1981).

[3] See Azariadis (1979), and the references he provides.

[4] This is discussed, for instance, by Akerlof and Miyazaki (1980). See also Chari (forthcoming).

[5] I am grateful to Ron Michener for suggesting this interpretation.

[6] That these equilibria are the solutions to such constrained optimization problems was pointed out by Spence (1978) and Miyazaki (1977).

[7] It is readily verified that for all sufficently large n, Σ_n and Σ_{n+1} have preferences for type 2 agents which are close in the sense of Kannai (1970). Since Σ_n and Σ_{n+1} are otherwise identical, these are clearly "nearby" economies.

[8] E.g., Rothschild and Stiglitz (1976), Wilson (1977), Prescott and Townsend (1981).

[9] There is likely to be no satisfactory answer to this question, as it is known that the more agent types there are, the less likely an equilibrium is to exist.

[10] See, e.g., Prescott and Townsend (1981).

References

Akerlof, S., and H. Miyazaki, "The Implicit Contract Theory of Unemployment Meets the Wage Bill Argument," Review of Economic Studies, 1980.

Azariadis, C., "Implicit Contracts and Related Topics: A Survey," unpublished, 1979.

Azariadis, C., "Self-Fulfilling Prophecies," Journal of Economic Theory, 1981.

Barro, R., "Long-Term Contracting, Sticky Prices, and Monetary Policy," Journal of Monetary Economics, 1977.

Chari, V.V., "Involuntary Unemployment and Implicit Contracts," Quarterly Journal of Economics, forthcoming.

Diamond, P., "Efficiency With Uncertain Supply," Review of Economic Studies, 1980.

Economic Report of the President, U.S. Government Printing Office, Washington, 1981.

Fischer, S., "Long-Term Contracting, Sticky Prices, and Monetary Policy: A Comment," Journal of Monetary Economics, 1977.

Guesnerie, R., and J. Seade, "Nonlinear Pricing in a Finite Economy," Journal of Public Economics, 1982.

Holmstrom, B., "Moral Hazard and Observability," Bell Journal of Economics,1979.

Kannai, Y., "Continuity Properties of the Core of a Market," Econometrica, 1970.

Miyazaki, H., "The Rat Race and Internal Labor Markets," Bell Journal of Economics, 1977.

Prescott, E., and R. Townsend, "Optima and Competitive Equilibria

With Adverse Selection and Moral Hazard," unpublished, 1981.

Rothschild, M., and J. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics, 1976.

Salop, J., and S. Salop, "Self-Selection and Turnover in the Labor Market," Quarterly Journal of Economics, 1976.

Spence, M., "Product Differentiation and Performance in Insurance Markets," Journal of Public Economics, 1977.

Wilson, C., "A Model of Insurance Markets With Incomplete Information," Journal of Economic Theory, 1977.