

Federal Reserve Bank of Minneapolis
Research Department Working Paper

A BUSINESS CYCLE MODEL
WITH PRIVATE INFORMATION

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Working Paper 232

Revised July 1985

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An earlier version of this paper was presented at the Second Annual NBER Summer Macro Conference (Cambridge, July 1984) and at the Summer Econometric Society Meetings (Stanford, June 1984) under the title "Reconciling Individual and Aggregate Differences in Labor Markets: An Adverse Selection Approach." The ideas contained in this paper developed over the course of several conversations with Edward Prescott. Comments from and conversations with Russell Cooper, Lars Hansen, Finn Kydland, Michael Rothschild, Thomas Sargent, Kenneth Singleton, and Sanjay Srivastava were also helpful. None of these persons is responsible for any errors of commission or omission, however.

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Abstract

A model of a "real" business cycle is produced in which labor market participants possess private information. A class of economies is considered in which interesting cycles cannot arise without private information. A methodology adapted from Kydland and Prescott (1982) is then employed to show that models based on private information can empirically confront salient features of postwar U.S. business cycles. Moreover, this can be done in a way which is consistent with existing microeconomic evidence on wages and labor supply. Finally, it is shown that the important features of the model related to private information are fairly general.

Introduction

One of the most important recent developments in macroeconomic theory has been the use of models with private information to explain the appearance of unemployed labor, and the apparent failure of relative prices to adjust to the presence of "excess supplies" of labor.^{1/} While a great deal of effort has been expended on refinements of the theory of labor contracting under private information, there has to date been little attempt to show that models of contracting with asymmetrically informed agents are capable of confronting observed cyclical features of labor markets, however. This paper is a preliminary attempt to do so. In particular, the paper develops a simple model of a labor market in which an adverse selection problem arises. This problem is resolved in a standard manner: imperfectly competitive firms offer workers contracts consisting of (state contingent) wage-hours pairs which, in equilibrium, induce self-selection of workers by contracts accepted. As in other adverse selection models,^{2/} self-selection is attained through a form of quantity rationing in labor markets. An attempt is then made to match the implications of the model with observed features of post-war U.S. business cycles.

The method of generating cycles in the model is as follows. The economy consists of a sequence of (overlapping) generations. The technology available at each date for converting labor into a single consumption good is subject to random (and possibly persistent) disturbances. These shocks to technology are the only source of aggregate uncertainty here. Hence our exercise is in the spirit of exercises performed for competitive economies by Kydland and Prescott (1982) or Long and Plosser (1983) which produce purely "real business cycles." Our strategy is then to see whether a real business cycle model based on private information is empirically adequate.

This of course raises the issue of how such a model is to be empirically implemented. Since no estimation theory has been developed for models where agents face incentive constraints as well as more standard constraints, the only course of action open is to proceed in a manner analogous to that of Kydland and Prescott (1982). In particular, a fully specified model economy is described, and an equilibrium of the model is computed numerically. In addition, moments of the equilibrium processes for variables in the model can be computed, and then both levels and relevant moments of equilibrium values generated by the model can be matched with post-war time series. Of course, this simply amounts to working out more or less elaborate numerical examples for our model. However, parameter choices for these examples are not left entirely free, but are governed in part by observations on long-run (trend) behavior, and in part by post-war time series observations on wage and hours behavior that is listed in detail in Section III. These sets of observations severely limit the set of admissible parameter values for the model. Nevertheless, it will be seen that the model performs well in accounting for a wide range of observed cyclical features of labor markets.

As already indicated, the economy employed consists of a set of overlapping generations. Within each generation there is a set of firms, who have access to a technology for converting labor into a single produced commodity, and a heterogeneous workforce. Workers vary in their marginal products in production, and in their attitudes towards consumption-leisure streams. All workers know their current (although not necessarily their future) marginal products. However, these are private information, ex ante. Moreover, firms are imperfect Nash competitors in labor markets. Then in equilibrium firms must (as is well-known) create incentives for workers to reveal their marginal products. This is done by offering (one-period) con-

tracts consisting of wage-hours pairs, which may involve some workers being quantity constrained at prevailing wage rates. In addition, there is stochastic technological variation. This variation induces co-movements in hours and real wages (as well as output). This, then, is the business cycle in our model.

In order to check that the economies at hand are consistent with observed features of business cycles, we require the equilibria of these economies to be consistent with observations at both aggregated and disaggregated levels. However, since our model does not contain capital, we do not focus on issues related to persistence. Hence, we would like our economies to generate equilibria with the following features.

- (i) Labor is (sometimes) unemployed (underemployed).
- (ii) The average productivity of labor is procyclical, as is the average real wage.^{3/}
- (iii) In aggregate data there is a strong positive co-variation between hours worked (per capita) and real wage rates.^{4/}
- (iv) Workers, who earn high real wages on average work low levels of hours.^{5/}
- (v) Despite (iii), trends in real wages in the twentieth century are not associated with trends in hours.
- (vi) Sectoral shifts in (relative) employment are an important feature of recent business cycles.^{6/}
- (vii) Wage dispersions across workers are reduced at cyclical peaks.^{7/}
- (viii) Relative wages across occupations seem to be an important "determinant" of labor market behavior.^{8/}

It will be seen that our economies are capable of reproducing, in an empirically plausible manner, all of these features.

There is a second natural question to be raised here. In particular, since the competitive, representative agent model of Kydland and Pre-

scott (1982) is consistent with all of the above features (that are not related to heterogeneity of the labor force), is there a reason to prefer the private information model to the competitive model? The answer is yes, and is related to the widely discussed inconsistency of micro-evidence on labor supply elasticities with the kind of co-movements required in hours and real wages for consistency with aggregate data. As stated by Ashenfelter (1983, p. 29),

The average labor supply elasticity must apparently be quite large to square up these hours and wage rate movements, while the available estimates of its slope that I have surveyed are, in fact, very small. The basic empirical problem seems to be that within the life-cycle, the person-specific correlation between hours and wages is simply too small to explain the time series movements in average hours relative to the time-series movements in average wage rates. The intertemporal substitution hypothesis originally advanced by Lucas and Rapping was, of course, precisely the suspicion that this was not the case.

In fact, of course, existing competitive models of the business cycle do rely on an intertemporal substitution mechanism. However, the empirical evidence against such a mechanism is quite strong.^{9/}

In the model presented here, however, person-specific correlations between wages and hours can be quite small while aggregate correlations are quite large. In particular, the following possibility will be demonstrated. In a panel data set every agent in the economy displays either no variation of wages and hours, or a nonpositive correlation between wage and hours movements. Nevertheless, in the aggregate hours and real wage covariation can be quite large.

The scheme of the paper is as follows. Section I sets out the model, and Section II defines an equilibrium. It also develops some properties of this equilibrium. Section III contains some examples illustrating

the results already mentioned. These are fairly extended examples with parameter values selected so as to imply consistency with a number of observations in the manner of Kydland and Prescott (1982). Section IV indicates that most of the important features of these examples hold generally within a certain class of economies. Section V concludes.

I. The Model

As indicated above, we wish to construct an economy which generates a panel data set for each of its members, as well as aggregate time series data. Here we construct the simplest possible model with these features, with heterogeneity in the population, and with private information. Time is discrete, and indexed by $t = 0, 1, \dots, \infty$. The population consists of a sequence of two-period lived, overlapping generations. Within each generation there are three kinds of agents. One of these we term entrepreneurs. These agents are endowed with access to a technology, to be described below, for converting labor into a single produced consumption good. The other two kinds of agents we call workers, who are of two types. We will refer to these as type 1 and 2 workers, with i indexing types. Finally, there are two states of nature which could occur in any period indexed by $s = 1, 2$. In order to fix ideas, we may think of there as being a countable number of entrepreneurs, and a continuum of workers. Within each young generation a fraction $\theta \in (0,1)$ of workers is of type 1.

Each worker is endowed with a single unit of "time" in each period, which is to be allocated between labor and leisure, and nothing else. A type i agent in his j th period of life may exchange labor for the single produced good at rate $w_{ji}(s)$ in state s , so $w_{ji}(s)$ is the state s real wage. (This will be independent of t . We will also later let the wage depend on one additional characteristic: an agent's labor market "history.") The differ-

ence between type 1 and 2 agents in production is as follows. For each unit of type i labor employed in state s , $\pi_i(s)$ units of the consumption good can be produced. The values $\pi_i(s)$ obey $\pi_1(s) > \pi_2(s) \forall s$, and are scalar constants. At the beginning of each period each agent knows his own type. This is private information, ex ante. The prevailing state is common knowledge. Firms will then offer wage-hours packages to attempt to induce agents to self-select by the type of package chosen. This is discussed below.

While within a period each agent's type is fixed and known to him, we do permit workers' types to change over time (workers do not become entrepreneurs, however). Suppose a current young worker is of type i . Then the objective probability, known to all, of becoming a type 1 worker when old is q_i , and the probability of becoming a type 2 worker is $1 - q_i$. Let $L_{1i}(s)$ denote the hours worked in the first period of life by a type i worker in state s , and let $C_{1i}(s)$ similarly denote consumption when young. For hours worked when old and old age consumption, a more elaborate notation is required. This is because an agent's type last period may matter in the type of package he obtains in equilibrium. Thus, let $L_{2i}^j(s,s')$ be hours worked by an old type i agent who was type j when young, if the sequence of states s, s' has been realized in his lifetime. Similarly, $C_{2i}^j(s,s')$ is old age consumption.

At any date, a type i agent has a period utility function $U_i(C,L)$, where $\forall i$

$$(i) \quad U_i \in C^2$$

$$(ii) \quad D_1 U_i(C,L) > 0$$

$$(iii) \quad D_2 U_i(C,L) < 0$$

(iv) U_i concave.

It will be convenient to restrict consideration to the case where

$$U_i(C,L) = U(C) + \phi_i V(L); i = 1, 2,$$

with ϕ_i a positive scalar constant. This choice of utility function is made to guarantee that Nash equilibria for this economy do not involve firms offering consumption-hours lotteries.^{10/}

The fact that an agent's type may change between periods means that when young both future productivity and a parameter of the utility function are random variables. Thus, for a type i worker there may be at least two motivations for saving when young. One is to reallocate consumption intertemporally, and one is as a form of insurance. (We assume for simplicity the absence of any insurance markets.) Or conversely, agents may borrow to reallocate consumption intertemporally, and as a means of "selling" insurance. Borrowing and lending (saving) can be undertaken at the gross interest rate $R(s)$ in state s (where below we consider only steady states, so that this is independent of t). Then let the savings (positive or negative) of a type i agent young in state s be denoted by $\phi_i [w_{1i}(s), R(s), s]$.^{11/}

In order to complete our description of consumer behavior, we must discuss the nature of transitions between current and future states of nature. To this end, let p_s be the probability that $s' = 1$ if s is the current period state, where " ' " indicates "next period's state" throughout. Then when old, a type i worker's period utility is just

$$(1) \quad W_i^j(s,s') = U\{w_{2i}^j(s')L_{2i}^j(s,s') + R(s)\phi_j [w_{1j}(s), R(s), s]\} + \phi_i V[L_{2i}^j(s,s')],$$

where the superscript j denotes last period's type. When young, a type i agent's expected lifetime utility in state s is

$$(2) \quad U\{w_{1i}(s)L_{1i}(s) - \phi_i[w_{1i}(s), R(s), s]\} + \phi_i V[L_i(s)] + \\ (\beta_i)\{q_i p_s W_1^i(s, 1) + q_i(1-p_s)W_1^i(s, 2) + (1-q_i)p_s W_2^i(s, 1) + \\ (1-q_i)(1-p_s)W_2^i(s, 2)\} \equiv \Psi_i(s),$$

where β_i is the discount rate of a young type i agent. Then our description of savings is just that $\phi_i(\)$ maximizes (2) taking $w_{1i}(s)$, $L_{1i}(s)$, and $R(s)$ as given.^{12/}

On the other side of the labor market are firms. As indicated above, we assume that these are owned by some countable number of entrepreneurs, all of whom have access to the same technology for goods production. Also, our assumptions on population imply that no workers' individual contribution to output is directly observable. Thus, in light of the simple nature of the technology here, the only interesting aspect of firm behavior is the attempt by firms to discover workers' types. In particular, we have assumed that type is unobservable, ex ante. However, the observable attributes of each individual may signal his type if these differ across agents. We assume here that the only such attributes for young agents are hours worked and savings. If a young agent is identified as being of type i , this identification is taken to be common knowledge when old. Thus, the previous period's identified type and current employment are the relevant observable attributes for old workers.^{13/} In short, then, agents' types are distinguishable when young in state s iff $L_{11}(s) \neq L_{12}(s)$, and/or $\phi_1(-) \neq \phi_2(-)$. When old, agents of common previous (identified) type are distinguishable in s' iff $L_{21}^j(s, s') \neq L_{22}^j(s, s')$. Thus firms can distinguish between workers by offering distinct

wage-hours packages. If any package is selected only by workers of a given type, then hours worked signal type, and we say self-selection has occurred.

To complete our description of the economy we make a final assumption on preferences. In particular, we assume that

$$(v) \quad \phi_1 > \phi_2.$$

Since for both young (by the envelope theorem) and old agents the slope of an indifference curve in income-labor space is given by (where y is income)

$$\left. \frac{\partial y}{\partial L} \right|_{dU_i = 0} = \frac{-\phi_i V'(L)}{U'(-)},$$

(v) implies that at any common income-hours pair, type 1 agents require more compensation for an incremental unit of leisure foregone than do type 2 agents along an indifference curve. This assumption implies that any equilibrium where agents are quantity constrained will be associated with unemployment.

(v) is an appealing assumption for this reason, and also because it has the following plausible interpretation. Let $U_i(C, L)$ be an indirect utility function derived from a model of home production. Then (v) is that workers who are more productive in the labor market are also more productive at home.^{14/} We do not insist on this interpretation, however.

II. Equilibrium

As indicated above, our analysis envisions firms offering contracts consisting of alternate wage-hours combinations, which workers either do or do not accept. Thus, the model is one of firms, who operate in an imperfectly competitive labor market, competing against each other for workers. Due to this competition, an equilibrium is characterized by an absence of rents in this setting. In short, in equilibrium firms offer wage-hours packages such that there are no potential gains to any firm which offers some alternate

package. This imposes several conditions which equilibrium contracts must satisfy. We elaborate on these below. It will also be recalled that firms operate subject to an informational asymmetry regarding the productivities of individual workers. Hence, each firm must decide in announcing its contracts whether it wishes to induce self-selection of employees by contract selected, or whether it wishes to offer contracts which "pool" type 1 and 2 workers. If firms wish to induce self-selection, this requires that its announced contracts are consistent with the occurrence of self-selection, or are incentive compatible. For old workers incentive compatibility requires

$$(3) \quad W_1^j(\tilde{s}, s) \geq U\{w_{22}^j(s)L_{22}^j(\tilde{s}, s) + R(\tilde{s})\phi_j(-)\} + \phi_1 V[L_{22}^j(\tilde{s}, s)]; \tilde{s}, s, j = 1, 2.$$

$$(4) \quad W_2^j(\tilde{s}, s) \geq U\{w_{21}^j(s)L_{21}^j(\tilde{s}, s) + R(\tilde{s})\phi_j(-)\} + \phi_2 V[L_{21}^j(\tilde{s}, s)]; \tilde{s}, s, j = 1, 2,$$

where (3) implies that type 1 workers prefer their contract to that accepted by type 2 workers, and where (4) is the converse. Notice that we assume that workers are not tied to firms for two periods, but rather are mobile when old. Hence, incentive constraints must apply period by period for each worker. Incentive compatibility for young workers then requires

$$(5) \quad \Psi_1(s) \geq U\{w_{12}(s)L_{12}(s) - \Phi_2(-)\} + \phi_1 V[L_{12}(s)] + \beta_1 EW_i^2(s, s') \quad \forall s,$$

$$(6) \quad \Psi_2(s) \geq U\{w_{11}(s)L_{11}(s) - \Phi_1(-)\} + \phi_2 V[L_{11}(s)] + \beta_2 EW_i^1(s, s') \quad \forall s,$$

where an obvious abbreviation of notation has been employed. Notice that if a firm does not attempt to induce self-selection, i.e., offers all workers of the same age identical contracts, then the incentive constraints are satisfied trivially.

In addition to satisfying conditions (3)-(6), firm announcements of wage-hours pairs must satisfy a number of "no surplus" conditions. First, in

light of competition among firms in the model, each firm must earn zero-profits in equilibrium. For simplicity we impose a stronger condition on announced contracts drawn from Rothschild and Stiglitz (1976) or Wilson (1977). In particular, we require that each wage-hours package offered at least breaks even given the workers accepting it. This zero-profit condition, together with the constant returns to scale technology assumed here, implies that if (for young workers) $L_{11}(s) \neq L_{12}(s)$ for some s ,

$$(7a) \quad w_{1i}(s) \leq \pi_i(s)$$

for that s . Similarly, if (for old workers) $L_{21}^j(s,s') \neq L_{22}^j(s,s')$ for some s' ,

$$(7b) \quad w_{2i}^j(s') \leq \pi_i(s')$$

for that s' . If young workers do not self-select (so $L_{11}(s) = L_{12}(s)$) for some s , then they accept contracts according to their population proportions. Then, since proportion θ of young agents are of type 1, the mean productivity of agents accepting the contract is $\bar{\pi}^1(s) = \theta\pi_1(s) + (1-\theta)\pi_2(s)$, and we require

$$(7c) \quad w_{11}(s) = w_{12}(s) \leq \bar{\pi}^1(s)$$

for that s . If old workers fail to self-select for some s' , but self-selection did occur when young, then we require that

$$(7d) \quad w_{21}^j(s') = w_{22}^j(s') \leq \bar{\pi}^{2j}(s') \equiv q_j\pi_1(s') + (1-q_j)\pi_2(s'); \quad j = 1, 2.$$

Finally, if self-selection also failed to occur when these workers were young, we impose

$$(7e) \quad w_{21}^1(s') = w_{22}^1(s') = w_{21}^2(s') = w_{22}^2(s') \leq \bar{\pi}^2(s') \equiv$$

$$[\theta q_1 + (1-\theta)q_2]\pi_1(s') + [\theta(1-q_1) + (1-\theta)(1-q_2)]\pi_2(s').$$

In light of the requirements imposed in (7a)-(7e), an absence of rent opportunities for firms requires that there fails to exist for any worker type a wage-hours package preferred to the equilibrium package, and which (at least) breaks even for the firm offering it. We now define a standard Nash equilibrium for our economy.

Definition. A stationary Nash equilibrium for this economy is a set of values $\{R(s)\}_{s=1,2}$, a set of announced contracts for young workers $[w_{1i}(s), L_{1i}(s)]$; $i = 1, 2$; $s = 1, 2$, and a set of announced contracts for old workers $[w_{2i}^j(s'), L_{2i}^j(s, s')]$; $i = 1, 2$; $s, s' = 1, 2$; $j = 1, 2$, such that

- (a) all announcements satisfy (3)-(6) and (7a)-(7e).
- (b) no firm has an incentive to offer a different set of contracts (satisfying (3)-(7)), given the announcements of other firms.
- (c) the loan market clears, i.e.,

$$(8) \quad \theta\phi_1(-) + (1-\theta)\phi_2(-) = 0; s = 1, 2.$$

It will also be useful to define an equilibrium in which borrowing and lending are ruled out. This is identical to the above, but with (c) replaced by $\phi_i \equiv 0$; $i = 1, 2$.

This model and these equilibrium definitions are a straightforward translation to a labor market setting of the adverse selection constructs considered by Rothschild and Stiglitz (1976) and Wilson (1977).^{15/} Notice that, as in those cases, we have focused only on pure strategy equilibria. The properties of the Nash equilibria here are similar in their qualitative features to the properties of the Rothschild-Stiglitz (1976) equilibrium, or

to Wilson's (1977) E1 equilibrium. In particular, no equilibrium in pure strategies need exist here. However, in the sequel parameter values are always chosen so that an equilibrium in pure strategies does exist.

Secondly, as in Rothschild and Stiglitz (1976), every equilibrium has the property that, in every state and for every past labor market history, self-selection occurs in equilibrium, i.e., $L_{11}(s) \neq L_{12}(s) \forall s$, and $L_{21}^j(s,s') \neq L_{22}^j(s,s') \forall (s,s'), \forall j = 1, 2$. The proof of this proceeds in two steps, which are just briefly outlined here. First note that, when workers are old, the argument given by Rothschild and Stiglitz for why self-selection occurs applies. Hence, old workers self-select. Given this fact, the Rothschild-Stiglitz argument then applies to young workers as well, so self-selection always occurs.

Given that self-selection always occurs, and given that parameter values are chosen to guarantee existence, it is easy to characterize the equilibrium that emerges here. First, obviously (7a) and (7b) hold with equality in equilibrium. Second, $L_{22}^j(s,s')$ solves $\max W_{22}^j(s,s')$ subject to (7b); $j = 1, 2, \forall (s,s')$. If this were not the case, since type is observable in equilibrium, there would exist a wage rate $\hat{w}_{22}^j(s')$ and a value $\hat{L}_{22}^j(s,s')$ such that

$$U_2[\hat{w}_{22}^j(s')\hat{L}_{22}^j(s,s') + R(s)\phi_j(-), \hat{L}_{22}^j(s,s')] >$$

$$U_2[\pi_2(s')L_{22}^j(s,s') + R(s)\phi_j(-), L_{22}^j(s,s)],$$

and such that $\hat{w}_{22}^j(s') < \pi_2(s')$. Hence, any firm offering the wage-hours pair $[\hat{w}_{22}^j(s'), \hat{L}_{22}^j(s,s')]$ would attract all old type 2 workers (who were type j when young), and earn a profit. This, of course, contradicts that $w_{22}^j(s') = \pi_2(s')$, $L_{22}^j(s,s')$ is an equilibrium. A similar line of argument establishes that $L_{12}(s)$ solves $\max \Psi_2(s)$ subject to (7a) for all s . Hence, each type 2

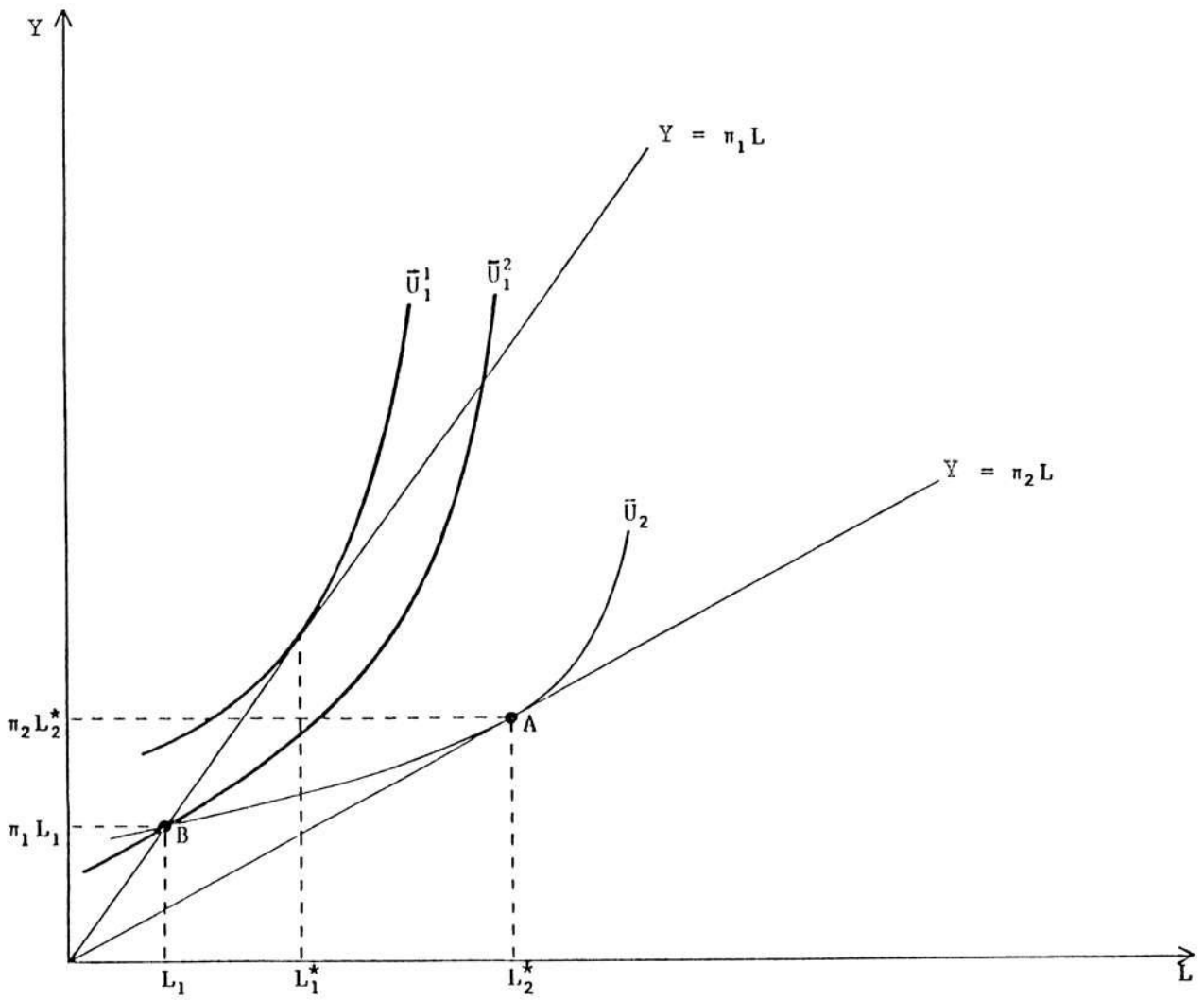
agent obtains his maximal hours level in each state consistent with the employers of type 2 agents breaking even (again in each state).

The determination of type 1 hours is more complicated, for the following reason. Since $w_{12}(s) = \pi_2(s) < \pi_1(s) = w_{11}(s)$ (and $w_{22}^j(s) = \pi_2(s) < \pi_1(s) = w_{21}^j(s)$), and in light of assumption (v), type 1 agents never wish to accept the wage-hours package offered to type 2 workers. Thus, the latter packages are not constrained by considerations of inducing self-selection. However, $L_{11}(s)$ and $L_{21}^j(s, s')$ may be so constrained. Nevertheless, it is still the case that type 1 agents must work the maximal hours level for them consistent with zero-profits and self-selection. As before, if this were not the case some firm could offer a wage-hours package preferred by all type 1 agents to their equilibrium package, and could do so in a manner consistent with self-selection, and in a manner which would permit them to earn a profit. Hence, $L_{21}^j(s, s')$ solves $\max W_1^j(s, s')$ subject to (7b) and (4), while $L_{11}(s)$ solves $\max \Psi_1(s)$ subject to (7a) and (6).

Determination of equilibrium values is depicted in Figure 1, where the self-selection conditions hold with equality. Hours worked is depicted on the horizontal axis, and income (per period) appears along the vertical axis. The loci labelled \bar{U}_i are type i indifference curves in this space, and the rays $y = \pi_i L$ are zero-profit loci for firms employing type i agents. The figure may be viewed as determination of either first- or second-period hours, as both are qualitatively similar. Assumption (v) implies that the type 1 indifference curve through each point is steeper than the type 2 indifference curve through the same point.

As argued above, the income-hours pair received by type 2 agents occurs at a tangency of a type 2 indifference curve with $y = \pi_2 L$. The income-hours combination received by type 1 agents must be maximal for them among the

Figure 1
An Unemployment Equilibrium



set of such combinations earning nonnegative profits (on or below $y = \pi_1 L$), and that are consistent with self-selection (on or below \bar{U}_2). Our assumptions on preferences imply that the maximal such income-hours combination occurs at point B in the figure, where \bar{U}_2 intersects $y = \pi_1 L$. Notice that the notional labor supply of type 1 agents at the real wage π_1 is L_1^* , so that for the preference maps depicted this economy generates unemployment (underemployment) of labor. We now turn our attention to the task of showing that it can do so in an empirically plausible way consistent with the cyclical features listed in the introduction.

III. Two Examples

In order to simplify the computation of equilibrium, we consider versions of our economy in which savings is ruled out ($\phi_i \equiv 0$; $i = 1, 2$) ^{16/} Under these circumstances it is easy to verify that there is nothing in the model to "tie" young and old hours determination together, or more specifically, each period may be treated separately. This should be clear, as the economic circumstances of old agents do not depend on their type last period if $\phi_i(-) = 0$, and as with saving ruled out old and young agents do not face different economic circumstances. Hence, by our prior arguments

$$(9) \quad L_{12}(s) = L_{22}^j(\hat{s}, s) = \operatorname{argmax} U_2[\pi_2(s)L(s), L(s)]; \forall s, \hat{s}, j = 1, 2,$$

where \hat{s} denotes "last period's state." Similarly, by the same arguments, $L_{11}(s)$ and $L_{21}^j(\hat{s}, s)$ solve

$$(10) \quad \max U_1[\pi_1(s)L(s), L(s)]$$

subject to

$$(11) \quad U_2[\pi_1(s)L(s), L(s)] \leq U_2[\pi_2(s)L_2(s), L_2(s)]; s, \hat{s}, j = 1, 2,$$

with an obvious abbreviation of notation. If (11) holds with equality this reduces to (as in Figure 1)

$$(12) \quad L_{11}(s) = L_{21}^j(\hat{s}, s) = \min \{L:U_2[\pi_1(s)L(s), L(s)] = \\ U_2[\pi_2(s)L_2(s), L_2(s)]\},$$

due to assumption (v) ¹⁷. Given these facts, we may now proceed with our computations under the assumption that $\phi_i(-) \equiv 0$; $i = 1, 2$. In Section IV it is shown that the important features of the following two examples generalize when saving is permitted.

As discussed in the introduction, the empirical methodology employed here is similar to that of Kydland and Prescott (1982). In particular, our construction of examples is guided by a set of observations which strongly restricts the admissible choice set for parameter values. These observations are set out below. Also, both of our examples use logarithmic utility functions. This is for three reasons. First, this is the simplest special case of the preference specification employed in related empirical research for competitive economies (e.g., Altug (1983), Eichenbaum, Hansen, and Singleton (1982), Kydland and Prescott (1982)). Second, Eichenbaum, Hansen, and Singleton (1982) and Altug (1983) present evidence favoring logarithmic utility. Third, the observation that relative wages across occupations are an important "determinant" of labor market behavior suggests such a specification, as will be seen below.

Given our specification of preferences, choices of parameter values are governed by the following sets of observations.

- a) Labor force participants work roughly a third of available time.
- b) Postwar U.S. unemployment rates range between 4 and 10 percent.

- c) Hodrick and Prescott (1981) report a 2 percent standard deviation about trend of average hours per capita, and a 1 percent standard deviation about trend of average productivity.
- d) Lucas and Rapping (1969) report one of the largest short-run elasticities of aggregate hours worked with respect to real wage movements; 1.4. When their calculations are converted into levels of hours and real wages their estimated elasticity becomes 2.12.^{18/}
- e) The percentage standard deviation of postwar U.S. GNP about trend is 1.8 percent.^{19/}

Finally, since our model contains two types of workers, we require some guidance on the choice of relative productivity parameters. For simplicity, we have followed the Economic Report of the President (1981) and divided nonagricultural, nonmilitary employees not engaged in wholesale or retail trade into two categories: manufacturing and construction.

- f) In 1947 average hourly earnings in manufacturing divided by average hourly earnings in construction was .79. In 1980 this number was .73. This ratio is largely in this range for the postwar period.^{20/}

A final point to note from the same source is that average weekly hours in manufacturing are always greater than are average weekly hours in construction. In conjunction with (f) this indicates that workers earning relatively high wages work relatively few hours.

A. Example 1

The above observations largely govern our choices of parameter values in computing our examples. For the first example, period preferences for type 2 agents are given by

$$U_2(C,L) = \ln C + \ln(1-L)$$

and preferences for type 1 agents are

$$U_1(C,L) = \ln C + 2\ln(1-L).$$

We assume that population demographics are the same at each date within each generation. We also set $q_1 = q_2$. Together these imply $\theta = q_1 = q_2 \equiv q$. We set $\theta = q = .5$, so at each date half the economy is of each type. Finally, let $p_s = .5$; $s = 1, 2$, and let $\pi_1(1) = 8.5$, $\pi_1(2) = 8.6$, $\pi_2(1) = 7$, and $\pi_2(2) = 6.8$. Notice that, since in equilibrium the values $\pi_i(s)$ are type i real wages, relative wages across occupations are

$$\frac{w_2(1)}{w_1(1)} = \frac{\pi_2(1)}{\pi_1(1)} = .81$$

$$\frac{w_2(2)}{w_1(2)} = \frac{\pi_2(2)}{\pi_1(2)} = .79.$$

These relative wages are pretty much in the range implied by observation (f) above.

It is useful to begin by considering the behavior of this economy under full information. Clearly $L_1 = 1/3$ and $L_2 = 1/2$ (where L_i is the common youth and old age value of L for type i workers) independently of which state prevails. Any given agent, of course, may change type in his lifetime. If a young type 1 agent becomes an old type 2 agent, his hours rise and his wages fall (regardless of s and s'). Agents who do not change type do not experience hours changes. Thus, across individuals we would observe either no responsiveness of hours worked to real wage changes, or among agents who change type, we would observe negative correlations between wage and hours movement.

At any aggregate level, however, since the number of agents is large type changes net out. Thus, in the aggregate, all changes in average wages per person are due to technology shocks. Clearly aggregate hours are not responsive at all to these movements, since total population hours of employment are constant across time and across states of nature. In short, with

full information this model is completely incapable of explaining cyclical variation in hours, or any co-movements in the average per capita levels of hours and real wages.

Now consider the version of the model with private information. From our previous discussion clearly $L_2(s) = 1/2$; $s = 1, 2$. This is because type 1 agents will not claim to be of type 2, so type 2 agents are not constrained by considerations of self-selection. It is readily verified that self-selection constraints do bind in the determination of the $L_1(s)$, $s = 1, 2$, however. Therefore, $L_1(s)$ solves

$$(13) \quad \ln \pi_1(s) L_1(s) + \ln [1 - L_1(s)] = \ln \left[\frac{\pi_2(s)}{2} \right] + \ln(1/2),$$

which is the maximal value of $L_1(s)$ for type 1 agents given that $w_1(s) = \pi_1(s)$, and subject to the constraint that type 2 agents may not strictly prefer $[w_1(s), L_1(s)]$ to $[w_2(s), L_2(s)]$.

The solution to (13) is^{21/}

$$L_1(s) = .5 - .5 \left[1 - \frac{\pi_2(s)}{\pi_1(s)} \right]^{.5} = .5 - .5 \left[1 - \frac{w_2(s)}{w_1(s)} \right]^{.5}.$$

We now note three things about our example. The first is that type 2 preferences here are equivalent^{22/} to $U_2(C, L) = C^\alpha L^{1-\alpha}$, with $\alpha = 1/2$. The choice of $\alpha = 1/2$ is made so that (13) is equivalent to a quadratic, i.e., is motivated solely by a desire to easily obtain a closed form solution for $L_1(s)$. Other choices of α do not permit this. The second is that $L_1(s)$ is determined entirely by the values $\pi_1(s)$ and by type 2 preferences, as is clear from (13). Thus, although type 1 preferences are equivalent to $C^\gamma L^{1-\gamma}$, there is only one preference parameter here that affects any equilibrium values other than the level of unemployment.

The third point to note is that $L_1(s)$ depends solely on the relative wage across occupations in any date-event. This is consistent with the observation made by Dunlop (1948) and Keynes (1964) that relative wages are important determinants of labor market behavior. More strongly, this is in line with the suggestion of Solow (1981) that macroeconomic theories be constructed consistent with the Dunlop-Keynes observation. Note, though, that we need not assume the importance of relative wages, in contrast to the approach outlined by Solow.

The first piece of evidence in support of the model is its consistency with the Dunlop-Keynes observation. Now we compute equilibrium values. These are $L_1(1) = .29$, and $L_1(2) = .27$. It will be noted that type 1 workers if unconstrained would select $\hat{L}_1 = 1/3$ at any real wage. Since \hat{L}_1 corresponds to the usual definition of the notional supply of labor for type 1 agents, clearly the model gives rise to unemployment. The unemployment rate in state s , $u(s)$, is given by

$$100 \frac{[.33 - L_1(s)]\theta}{(.33)\theta + (.5)(1-\theta)} = u(s),$$

so $u(1) = 4.8$ percent and $u(2) = 7.1$ percent for our parameter values. Thus, the range of values for unemployment is consistent with most of the post-war U.S. experience. Also note that average per capita hours worked in state s , denoted $\hat{L}(s)$, is given by $\hat{L}_1(s) = \theta L_1(s) + (1-\theta)L_2(s)$. Therefore $\hat{L}(1) = .395$ and $\hat{L}(2) = .385$. This is not grossly at variance with the casual observation that typically people work about one-third of available time.

Now consider what one would observe in cross-sectional or panel data in this economy. At the cross-sectional level, high wage individuals always work fewer hours than low wage individuals, i.e., $L_1(s) \leq .29$ while $L_2(s) = .5$

* s. Thus, cross-sectionally we observe negative correlations between wages and hours. In panel data, we would observe several "types" of agents. Some would display no change in either wages or hours (those who did not change type, and who lived during periods such that $s = s'$). For those who were type 2 in both periods and who experienced $s \neq s'$, hours would not appear to respond at all to wage movements. For those who change type, regardless of s and s' , there is a negative relation between wage and hours changes even stronger than that for the full-information version of the economy. And finally, for those who are type 1 when young and old, and who experience $s \neq s'$, we observe the following. $s = 1$ is the low wage state, but the high hours state. Thus, wage and hours movements again are negatively related.

In short, then, every individual in the economy displays either no change in both hours and wages, no change in hours, or has hours falling as wages rise. Thus, all individuals display nonpositive correlations between hours and real wage movements. Moreover, these correlations are always algebraically smaller than for the full-information version of the economy.

Now consider aggregate per capita wages and hours, denoted $\hat{w}(s)$ and $\hat{L}(s)$ respectively. We have already computed $\hat{L}(1) = .395$, $\hat{L}(2) = .385$. Average per capita wages, weighted by hours worked are

$$\hat{w}(1) = \left[\frac{\theta L_1(1)}{\hat{L}(1)} \right] (8.5) + 7 \left[\frac{(1-\theta)L_2(1)}{\hat{L}(1)} \right] = 7.551$$

$$\hat{w}(2) = 8.6 \left[\frac{\theta L_1(2)}{\hat{L}(2)} \right] + 6.8 \left[\frac{(1-\theta)L_2(2)}{\hat{L}(2)} \right] = 7.431.$$

Then at an aggregate, or "representative agent" level, we would observe two elasticities of hours worked with respect to real wages; one in moving from state $s = 1$ to $s' = 2$, and one in the reverse case. These elasticities are

$$\left[\frac{\hat{L}(2) - \hat{L}(1)}{\hat{L}(1)} \right] \left[\frac{\hat{w}(1)}{\hat{w}(1) - \hat{w}(2)} \right] = 1.61$$

$$\left[\frac{\hat{L}(1) - \hat{L}(2)}{\hat{L}(2)} \right] \left[\frac{\hat{w}(2)}{\hat{w}(2) - \hat{w}(1)} \right] = 1.57.$$

These are very much in line with the short-run elasticities of Lucas and Rapping (1969) (observation (d) above).

Clearly, then, the model is consistent with very small (even negative) observed responses by all individuals to changes in their real wages, and with the relatively large aggregate elasticities reported by Lucas and Rapping (1969) and Kydland and Prescott (1982). Notice also that the effect of the adverse selection setting relative to full information is to reduce individual responsiveness to real wage movements while increasing this responsiveness at the aggregate level.

We have seen, then, that this example is consistent with magnitudes of observed hours employed, and with the general range of experienced unemployment levels. It is also consistent with the relevant Lucas-Rapping elasticities, and with several other observations on the cyclical behavior of labor markets. In particular, it is the case that in $s = 1$, the low unemployment state,

$$\frac{w_2(1)}{w_1(1)} = .81,$$

and in the high unemployment state,

$$\frac{w_2(2)}{w_1(2)} = .79.$$

Thus wage dispersions decline at cyclical "peaks." This is in accordance with general evidence on wage differentials (Reder (1962), Freeman (1973)). Also, since the $\hat{w}(s)$ are just hours-weighted average productivities of labor, it is

clear that the productivity of labor is procyclical, as we observe. And finally, it will be noted that the percentage standard deviations of aggregate per capita hours and average productivity, respectively, are $\sigma_L = 1.79$ percent and $\sigma_w = 1.13$ percent for this example. These are extremely close to the standard deviations of hours and productivity of 2 percent and 1 percent (respectively) reported by Hodrick and Prescott (1981) for U.S. aggregates. Also, the percentage standard deviation of per capita GNP about trend is 2.1 percent. This is quite similar to the actual value reported by Prescott (1983) of 1.8 percent. Thus, the example reproduces quite closely several observed features of U.S. business cycles.

Note that this is true despite the small number of free parameters in the model. In particular, given our focus on stationary environments with $q_1 = q_2$ and $p_1 = p_2$, there are only eight free parameters in the model. These are the two preferences parameters discussed above, $\theta = q_1 = q_2$, $p_1 = p_2$, and the four values of marginal products. Thus, the choice of these parameters is almost entirely governed by attempts to reproduce broad features of aggregates.

Finally, there are other noncyclical features of labor markets consistent with the model. Perhaps the primary one of these is that over time there has been a significant upward trend in real wages, with no such trend in average hours. The example presented is capable of incorporating trends in productivity and hence real wages. Specifically, we might let $\pi_1(s,t) = (1+n)^t \pi_1(s)$, where n is a trend rate of growth. Since $L_2(s) = 1/2$ independently of $w_2(s)$, and since $L_1(s)$ depends solely on

$$\frac{\pi_2(s,t)}{\pi_1(s,t)} = \frac{\pi_2(s)}{\pi_1(s)},$$

the example is consistent with this observation.

At this point two observations should be made. First, the model implies unemployment among high productivity agents. This is, of course, not a satisfactory implication of the model. However, when the hours implications of the model are examined these are fairly reasonable. In particular, the model implies that workers who earn high wages work relatively few hours. We have seen that this is the case for workers in manufacturing and construction. Also, unemployment rates in construction are relatively high. Hence, when interpreted in this way, the implications of the model are not entirely counterfactual.

An obvious objection to this argument might be the following. Construction workers and workers in manufacturing may not be intrinsically different, as workers of different types are in the model here. Rather unionization in construction might raise wages in that sector and increase unemployment, with the higher wages "compensating" construction workers for the unemployment they experience. However, this argument is incomplete, since Abowd and Ashenfelter (1981) present estimates of compensating wage differentials which never exceed 14 percent, and generally are more in the range of 6-8 percent. Wage differentials between manufacturing and construction are much larger than this, so that even accepting the logic above suggests there is still much to be explained in the differences between workers in these two sectors.

Our second observation is that the model implies that cyclical variation in output will be accompanied by shifts in the relative shares of various sectors in average employment. This is a feature of recent cyclical variation in the U.S., as documented by Lilien (1982). Hence, our model is also consistent with this aspect of the business cycle.

B. Example 2

Clearly example 1 is somewhat extreme in its implication that all individuals display nonpositive elasticities of hours worked with respect to real wages. It also produces an aggregate elasticity near the upper end of (most) estimated elasticities. Thus, we now briefly present a less extreme example. This has the same preferences as example 1, with parameter values $\theta = q_1 = q_2 = .4$, $\pi_1(1) = 8.5$, $\pi_2(1) = 7$, $\pi_1(2) = 8$, $\pi_2(2) = 6$. These values avoid the feature of the previous example that π_1 and π_2 move in opposite directions as the state changes from $s = 1$ to $s = 2$.

Equilibrium values are computed as before. (Clearly the unconstrained value of $L_1(s)$, $\hat{L}_1(s) = 1/3 \forall s$.) Then $L_1(1) = .29$, $L_1(2) = .25$, $u(1) = 5$ percent, $u(2) = 10$ percent, $\hat{L}(1) = .416$, $\hat{L}(2) = .4$, $\hat{w}(1) = 7.42$, $\hat{w}(2) = 6.5$. Relative wages in the two states of nature are $w_2(1)/w_1(1) = .82$ and $w_2(2)/w_1(2) = .75$. Thus unemployment rates are again within the range suggested by recent U.S. experience, hours worked are not substantially different from a third of available time, the average productivity of labor is procyclical, and wage dispersions decline at cyclical "peaks." It is also the case that all agents who are type 2 in both periods display no responsiveness of hours to real wages, and that all agents who change types display a negative correlation between wages and hours. For our parameter values, 36 percent of all agents are type 2 in both periods and 48 percent change types. Thus, for any two consecutive periods in which $s \neq s'$, 84 percent of all agents display nonpositive elasticities of hours worked with respect to real wages. For the remaining workers, the 16 percent who are type 1 in both periods display a positive response of hours to wages, since $s = 1$ is both the high wage and the high hours state. In fact, agents who are of type 1 in both periods, and experience the sequence of states $s = 2$, $s' = 1$, display an elasticity of hours with respect to real wages of

$$\left[\frac{L_{21}(1) - L_{11}(2)}{L_{11}(2)} \right] \left[\frac{\pi_1(2)}{\pi_1(1) - \pi_1(2)} \right] = 2.56.$$

Thus, a segment of the population displays a strong positive relationship between employment and hours. This is in accordance with evidence presented by Heckman and Macurdy (1980) that some subsets of the population do display quite large elasticities in panel data.

At the aggregate level, as one observes movements from $s = 2$ to $s' = 1$,

$$\left(\frac{\Delta L}{L} \right) \left(\frac{w}{\Delta w} \right) = .28$$

and as movements from $s = 1$ to $s' = 2$ are observed

$$\left(\frac{\Delta L}{L} \right) \left(\frac{w}{\Delta w} \right) = .31.$$

These values are generally in line with a more conservative elasticity obtained by Hall (1980, p. 20) of .46. Thus, changes in two parameters bring the example into line with less extreme outcomes.

C. A Remark

Two natural questions arise regarding the analysis performed above. The first concerns the importance of the role played by type changes in the example and the second the use of an overlapping generations framework to analyze aggregate cyclical variation. With respect to the question of type changes, these play two roles. The first is simply to prevent an agent's type from being public information in his second period of life, i.e., type changes prevent the determination of old hours from being uninteresting. The second role is that type changes serve to make the examples more dramatic in the following sense. At an aggregate level, the model with type changes is indistinguishable from a model in which an agent's type last period is not

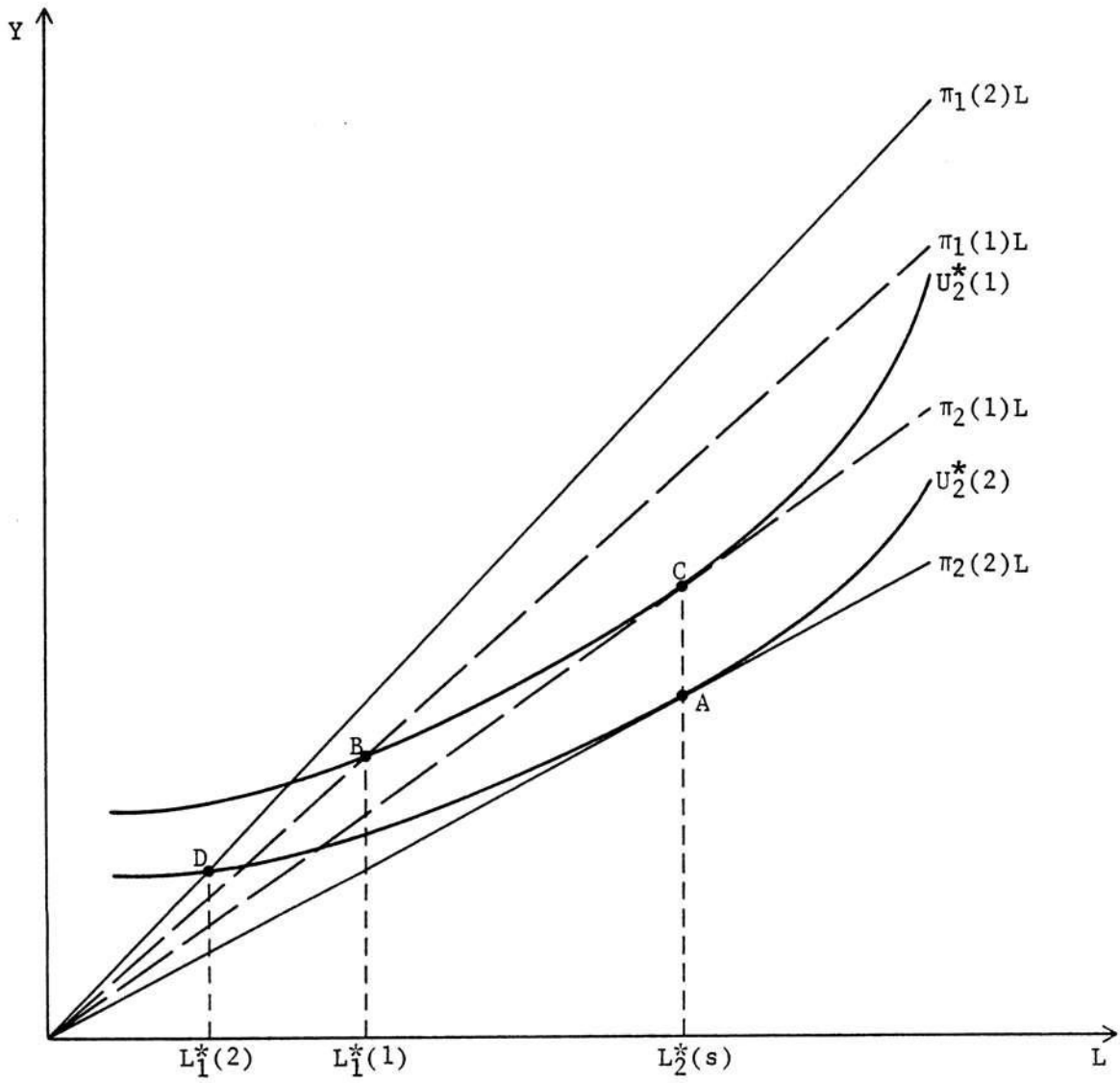
publicly known, and in which $q_1 = 1$, $q_2 = 0$ (so long as the economy is large). However, since type changes induce negative correlations between hours worked and real wages, they imply that a larger proportion of the population displays such negative correlations than would be the case in the absence of type changes. Thus, type changes are inessential to any results at an aggregate level. They serve merely to enhance the range of possible outcomes in the model.

The second remark to be made concerns the use of an overlapping generations setting to study cyclical variation. At first glance this may seem strange, since if we take the model literally it means that our model generates only one observation each fifteen to twenty years. However, it will be noted that in the analysis attention has been confined to levels of relevant variables and to their variances. In a stationary environment (with a large sample) frequency of sampling does not affect measured variances. Hence the use of the overlapping generations model here does not create any real problems in matching the statistics generated by the model to those observed in practice.

D. A Diagrammatic Exposition

While the two examples are quite simple, they may not provide ample intuition regarding how all individuals could display nonpositive correlations between hours and real wage movements at the same time that aggregate correlations are positive. Thus, we present a diagrammatic exposition of the model. To this end consider Figure 2. In the absence of a loan market this may be viewed as a depiction of the determination of the $L_{ji}(s)$; $i, j = 1, 2$. If borrowing and lending are allowed, typically $L_{2i}(\hat{s}, s) \neq L_{1i}(s)$. However, determination of young and old hours is qualitatively similar, and in line with the following discussion, which assumes no savings.

Figure 2



In the figure, L denotes hours and y denotes income. The rays $y = \pi_i(s)L$ are those values of y and L which result in zero profits for firms employing type i agents in state s , and the loci labelled $U_2^*(s)$ are the equilibrium indifference curves for type 2 agents in state s . Points A and C denote the equilibrium hours-income combinations for type 2 agents. These are simply the maximal income-hours pairs for these agents subject to the constraint that firms employing them break even. Note that if firms offered type 2 agents some alternate income-hours combination earning nonnegative profits, a deviant firm could offer these agents a preferred, profitable income-hours combination. Thus A and C are equilibrium combinations as claimed. (θ and the q_i can always be chosen such that an equilibrium exists as depicted.)

Now consider type 1 agents. If self-selection constraints bind, then the values $(y_1(s), L_1(s))$ must lie on or below the rays $y = \pi_1(s)L$, and on the indifference curve $U_2^*(s)$. In other words, since in equilibrium self-selection must occur, the equilibrium values $(y_1(s), L_1(s))$ must not be preferred by type 2 agents to $(y_2(s), L_2(s))$. Thus, if self-selection constraints bind, income-hours levels for type 1 agents occur at B in $s = 1$, and D in $s = 2$ (since type 1 agents have steeper indifference curves in this space than do type 2 agents).

Under the assumption of, e.g., Cobb-Douglas preferences, $L_2(1) = L_2(2)$ as shown. Also, consistent with example 1, $\pi_1(1) < \pi_1(2)$, but θ and the q_i can be chosen (consistent with existence of an equilibrium) such that $\hat{\pi}(1) > \hat{\pi}(2)$. Thus, we have the following phenomena. First, cross-sectionally $w_1(s) > w_2(s)$ but clearly type 1 hours are always less than type 2 hours. Thus, cross-sectionally one observes negative relations between wages and hours. At a panel data level, workers who are type 2 in both periods show no responsiveness of hours to real wages. Clearly workers who change types

display negative relations between wage and hours changes. And finally, $s = 2$ is the high wage state for type 1 agents, but clearly $L_1(1) > L_1(2)$. Thus, agents who are type 1 show negative responses of hours to own wage changes. Nevertheless, since $\hat{\pi}(1) > \hat{\pi}(2)$, clearly aggregate hours respond positively to real wage movements. This is true as $L_1(1) > L_1(2)$, and $L_2(1) = L_2(2)$. Thus, the adverse selection setting can easily explain the apparent anomaly of small individual responsiveness, but larger aggregate responsiveness of hours worked to real wage movements.

IV. Some More General Results

The examples of the preceding section exploited several features of the model related to the equilibrium behavior of the hours and wage rates of type 1 workers:

- (a) hours worked by type 1 agents repond negatively to changes in their own wage rate
- (b) hours worked by type 1 agents respond positively to changes in the wage rate of type 2 workers.
- (c) type 1 workers are unemployed.

Also, a corollary of assumption (v) is that if

$$\max_{\{s\}} \pi_2(s) < \min_{\{s\}} \pi_1(s),$$

type changes within any agent's lifetime will be associated with opposite movements in hours and real wages if $\Phi_2(-)$ is not too large in absolute value. A second corollary is that cross-sectionally, hours and real wages will be negatively correlated. (On both points see Figure 1.)

In this section we show that if self-selection constraints bind in all states, and if an equilibrium exists, these results hold generally under

our assumptions. However, one should not overstate the generality of the results obtained since equilibria need not exist, and since self-selection constraints need not bind in all states. Unfortunately, there is little of generality to be said on these points, and we do not expand on them here.

A. Unemployment of Type 1 Agents

If self-selection constraints do bind in each state, then the $L_{21}^j(\hat{s},s)$ values are determined as solutions to the problems

$$\max U[\pi_1(s)L_{21}^j(\hat{s},s) + R(\hat{s})\phi_j(-)] + \phi_1 V[L_{21}^j(\hat{s},s)]; j, s, \hat{s} = 1, 2,$$

subject to

$$(14) \quad U[\pi_1(s)L_{21}^j(\hat{s},s) + R(\hat{s})\phi_j(-)] + \phi_2 V[L_{21}^j(\hat{s},s)] = U[\pi_2(s)L_{22}^j(\hat{s},s) + R(\hat{s})\phi_j(-)] + \phi_2 V[L_{22}^j(\hat{s},s)]$$

$$(15) \quad L_{21}^j(\hat{s},s) \leq 1,$$

where $L_{22}^j(\hat{s},s)$ is determined as above. If (14) and (15) have only one interior solution then this must be associated with unemployment of type 1 labor, i.e.,

$$(16) \quad L_{21}^j(\hat{s},s) < \operatorname{argmax} \{U[\pi_1(s)L_{21}^j(\hat{s},s) + R(\hat{s})\phi_j(-)] + \phi_1 V[L_{21}^j(\hat{s},s)]\}.$$

This is true since at any point in Figure 1 (given $R(\hat{s})$ and $\phi_j(-)$), the indifference curve of type 1 agents through that point is steeper than the indifference curve of type 2 agents through the same point. However, suppose (14) and (15) have two solutions. Then (16) still holds, for the following reason. Abbreviating equilibrium values as $L_1(s)$ and $L_2(s)$, clearly

$$\begin{aligned}
 U[\pi_1(s)L_1(s) + R(\hat{s})\phi_j(-)] + \phi_1 V[L_1(s)] &= U[\pi_1(s)L_1(s) + \\
 R(\hat{s})\phi_j(-)] + \phi_2 V[L_1(s)] + (\phi_1 - \phi_2)V[L_1(s)] &= U[\pi_2(s)L_2(s) + \\
 R(\hat{s})\phi_j(-)] + \phi_2 V[L_2(s)] + (\phi_1 - \phi_2)V[L_1(s)]. &
 \end{aligned}$$

Therefore, letting W_1^* and W_2^* denote equilibrium values of type 1 and 2 (old age) utility,

$$W_1^* = W_2^* + (\phi_1 - \phi_2)V[L_1(s)].$$

As $\phi_1 > \phi_2$ and $V' < 0$, clearly the smallest value of $L_1(s)$ satisfying (14) and (15) is the equilibrium value, implying an equilibrium with unemployment as claimed. A similar argument holds for young agents.

Since type 1 agents experience unemployment, then, the negative cross-sectional relation of wages and hours follows immediately, i.e., $L_1(s) < L_2(s)$ (obvious from Figure 1) and $w_1(s) > w_2(s)$. Also, if

$$\min_{\{s\}} w_1(s) > \max_{\{s\}} w_2(s),$$

then type changes imply negative correlations between wage and hours changes if savings are not too large. Thus, this feature of the examples is a general one for these economies.

B. The Response of Type 1 Hours to Wage Changes

So long as self-selection constraints bind in all states, hours for old type 1 agents are determined by which solution to (14) and (15) they prefer (if there is more than one). Under the assumption that incentive constraints always bind, we may solve (14) implicitly for $L_1(s)$ as a function of $w_1(s)$, $w_2(s)$, and $R(\hat{s})\phi_j(-)$ (with an obvious abbreviation of notation). Letting

$$L_1(s) = L[w_1(s), w_2(s), R(\hat{s})\phi_j(\)],$$

it is easy to show that

$$\frac{\partial L_1(s)}{\partial w_1(s)} = \frac{-L_1(s)U'[\pi_1(s)L_1(s)+R(\hat{s})\phi_j]}{\pi_1(s)U'[\pi_1(s)L_1(s)+R(\hat{s})\phi_j]+\phi_2V'[L_1(s)]}.$$

Since $L_1(s) < \operatorname{argmax} \{U[\pi_1(s)L_1(s) + R(\hat{s})\phi_j(\)] + \phi_2V[L_1(s)]\}$ (see Figure 1) the denominator of this expression is positive. Thus $L_1(s)$ and $w_1(s)$ move in opposite directions, ceteris paribus, or put otherwise, if $w_2(1) = w_2(2)$ and $w_1(1) > w_1(2)$, then $L_1(1) < L_1(2)$.

Now consider the derivative of $L(\)$ with respect to $w_2(s)$. Again one easily derives

$$\frac{\partial L_1(s)}{\partial w_2(s)} = \frac{L_2(s)U'[\pi_2(s)L_2(s)+R(\hat{s})\phi_j]}{\pi_1(s)U'[\pi_1(s)L_1(s)+R(\hat{s})\phi_j]+\phi_2V'[L_1(s)]},$$

so that L_1 responds positively to changes in w_2 with w_1 held constant.

It remains, then, to consider the hours of young type 1 agents. But, by an envelope theorem argument,

$$\frac{\partial L_{11}(s)}{\partial w_{11}(s)} = \frac{-L_{11}(s)U'[\pi_1(s)L_{11}(s)-\phi_1]}{\pi_1(s)U'[\pi_1(s)L_{11}(s)-\phi_1]+\phi_2V'[L_{11}(s)]}$$

$$\frac{\partial L_{11}(s)}{\partial w_{12}(s)} = \frac{L_{12}(s)U'[\pi_2(s)L_{12}(s) - \phi_2]}{\pi_1(s)U'[\pi_1(s)L_{11}(s)-\phi_1]+\phi_2V'[L_{11}(s)]}.$$

Thus, hours of young type 1 workers respond in qualitatively the same way to wage changes as do hours worked by old type 1 agents. Therefore, the examples of Section 3 displayed responses of type 1 hours to wages which are general in nature.

It is harder to show that the aggregate behavior of wages and hours is generally of the nature obtained in examples 1 and 2. This is because the response of hours worked by type 2 workers to own wage movements is ambiguous. Since aggregate hours movements are just simple functions of individual movements in hours worked, it is typically not possible to sign the effect of changes in π_1 and π_2 on aggregate (per capita) hours. Thus, the aggregate responses shown in Section 3 should be taken as more example specific than are the individual responses to changes in the real wages observed in that section.

In short, then, the behavior displayed by type 1 agents in the examples is quite general. Type 2 agents (and aggregate values) may behave in any number of ways, but clearly the examples of Section 3 are not in any way pathological. In light of these facts it seems that this class of models is deserving of serious consideration as a general model of labor market behavior.

V. Conclusions

The objective of this paper was to demonstrate that simple models with private information could confront a broad range of observations on the cyclical behavior of labor markets, and to do so in a way which was consistent with microeconomic evidence on labor supply behavior. As Section III of the paper shows, this is quite easy to do, and moreover, it can be done using artificial economies where the specification of preferences, as well as the choices of all parameter values, are governed either by time series observations, or by existing empirical evidence on microeconomic behavior. Moreover, as shown in Section IV, the important features of the examples which arise due to the presence of private information are fairly general in these economies.

The previous sections, of course, report only on attempts to generate "real business cycles" using adverse selection models. More specifically, the exercise performed above is a natural version for an economy with private information of the exercises performed by Kydland and Prescott (1982), Long and Plosser (1983), or King and Plosser (1984) for competitive economies with (essentially) complete information. In particular, all of these exercises show that a particular set of models can confront observed business cycles on the basis of shocks which impinge on the production technology of an economy. We have shown here that private information models can do this in a manner which is consistent with micro-evidence on labor supply, and without relying on the empirically questionable intertemporal substitution hypothesis that underlies real theories of the cycle based on competitive equilibrium approaches.

This is not to suggest, of course, that monetary factors are not important. In fact, it is quite easy to incorporate money into the class of models at hand. This is done in Smith (1984), which shows that in a slightly altered version of the economies above, a subset of workers may choose to sign contracts which set out prespecified nominal wage rates. This is the case even though fully-indexed contracts (paying the same expected real wage) are available to these agents. Then, given the presence of nominal contracts, it is shown that a Phillips curve can arise due solely to monetary factors. Hence, these economies can generate either real or monetary business cycles.

Finally, it is natural to comment on some features not included in the model at hand. First, no issues related to "persistence" have been treated here. This is largely for simplicity, as the model does, in fact, contain mechanisms for generating persistence of disturbances. One is simply that shocks to technology can be persistent. This possibility was not in-

cluded in our examples, as $p_s = 1/2$; $s = 1, 2$, was one of our specifications. Moreover, such an assumption is used to generate persistence in competitive models, e.g., Kydland and Prescott (1982). Also, a second means of generating persistence is that with saving not ruled out, last period's savings generally affect determination of this period's hours for old agents. This was omitted in the examples for simplicity of presentation.

A second feature that is omitted here is entry into and exit from the labor force. While entry and exit can be accommodated into our model,^{23/} it is not clear that this would add to the analysis. In particular, the model accounts for the cyclical variability of employment as is, and micro-evidence suggests that allowing for entry and exit does not greatly affect estimated earnings functions, for instance.^{24/} Hence, for a first attempt at empirically implementing a model with private information, it does not seem unreasonable to abstract from entry and exit.

In closing, it seems appropriate to relate the modeling strategy employed here to some other strategies employed in macroeconomic theory. First, our analysis employs a model of an imperfectly competitive labor market in which firms call out price-quantity pairs for potential employees, rather than just prices. Hence, the approach here is similar in spirit to that of Hart (1982), who develops a "Keynesian" model based on an imperfectly competitive labor market. Second, our approach views firms as setting the hours levels of employees. In this respect the model is similar to traditional textbook models which view firms as setting employment levels (within certain limits). Thus the modeling strategy employed here, while building on more recent developments in the theory of private information, is not dramatically different in spirit from more traditional approaches in macroeconomics.

Footnotes

1/This literature is far too large to permit an exhaustive list of efforts here. Hart (1983) provides a valuable survey and an extensive reference list on the topic.

2/See in particular Rothschild and Stiglitz (1976) and Wilson (1977).

3/See Prescott (1983), Tables 2 and 3.

4/Lucas and Rapping (1969) produce estimates which, when converted into raw levels, give an elasticity of aggregate hours worked with respect to real wage rates of 2.12. See Hotz, Kydland, and Sedlacek (1982) for this calculation. See also Ashenfelter (1983) and Prescott (1983) for further evidence on this point.

5/See below.

6/See Lilien (1982), especially page 779.

7/See, e.g., Reder (1962) or Freeman (1973).

8/For three expositions of this argument see Dunlop, Keynes (1964), and Solow (1980).

9/Again an exhaustive list of the evidence would be extremely long. A good survey is contained in Ashenfelter (1983), who also provides references.

10/See Prescott and Townsend (1984a,b) for a description of the role of such lotteries in models with privately informed agents.

11/As will be clear shortly, savings also depend on future wage distributions. This is subsumed in our notation.

12/The reason why $\phi_i(\)$ and L_{1i} are not thought of as simultaneously determined is that workers choose the $\phi_i(\)$, while the L_{1i} are set by firms (see below).

13/Last period's identified type is a sufficient statistic for hours worked and savings. It should be noted that the assumption of observable portfolios is inessential to any results and serves only to economize on notation.

14/I am grateful to Ron Michener for providing this interpretation.

15/The fact that for old workers last period's type is an observable attribute also makes the analysis similar to that carried out by Hoy (1982).

16/The same end result could be accomplished by choosing values for the discount rates (β_i) such that in equilibrium $\phi_i = 0$, $i = 1, 2$. To avoid the calculations involved, we make the simpler assumption in the text.

17/A formal argument to this effect appears in Section IV below for the general case which permits nonzero savings.

18/See Hotz, Kydland, and Sedlacek (1982).

19/Prescott (1983).

20/Economic Report of the President (1981), p. 273.

21/Type 1 agents prefer the negative to the positive root.

22/By equivalent here we mean "lead to the same equilibrium values."

23/See Judd (1983) for a similar adverse selection model with entry and exit.

24/See Hausman and Wise (1979).

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