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GOVERNMENT EXPENDITURES, DEFICITS, AND
INFLATION: ON THE IMPOSSIBILITY
OF A BALANCED BUDGET*

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"The government that does not have access to the printing press can, nonetheless, use emergency taxes or compulsory loans for emergency financing."

S. Fischer [1982, p. 297]

Abstract

A model is presented in which governments can select real expenditure levels which are feasible, but are sufficiently high that a balanced budget is impossible. Thus governments with large expenditures are committed to inflationary finance schemes. This is the case even though the governments in question have access to lump-sum taxes. In addition, the model can explain why poorer countries tend to make heavier use of the inflation tax than do wealthier countries, and can account for the existence of country-specific fiat monies.

A large literature considers the relationship between government deficits and inflation.^{1/} A second literature considers whether certain time paths of government deficits effectively commit governments to follow a policy of inflationary deficit finance.^{2/} A third literature focuses on questions of optimal taxation in monetary economies, and of the role of inflation taxes in optimal tax schemes.^{3/} Finally, a fourth literature concerns itself with the role for country specific monies in providing seignorage income.^{4/}

Each of these literatures proceeds under the (explicit or implicit) assumption that governments could, if they so desired, balance their budgets (or at least do so on average). However, in point of fact, it has been a rare historical experience in which governments with large revenue needs did not resort to inflationary financing schemes. A first reaction to this fact is that one might typically expect some use of the inflation tax to be part of any financing scheme that is not clearly suboptimal under such circumstances. However, it is not obvious that this argument bears closer scrutiny. In particular, this result does not emerge in all classes of optimal taxation models (see, e.g., Lucas and Stokey [1983]) and, moreover, it is based on the implicit assumption that the governments in question were either net debtors to their private sectors, or had no net position vis-a-vis this sector. If, however, a government is a net creditor to the private sector (owning nominal claims on it), the optimal taxation literature^{5/} suggests that the government in question should deflate in order to augment the value of its claims. In fact, it is easy to find instances in history of creditor governments with large revenue needs following inflationary finance schemes, and permitting their claims to be paid off in depreciated currency. Thus, it seems questionable whether observed behavior is to be well explained by models of optimal taxation in a competitive monetary economy. Moreover, in any casual reading

of economic history, one finds that governments with large revenue needs assert that it is impossible to meet these needs without resort to inflationary finance. In light of the fact that some such financing arrangements seem at first blush to be patently suboptimal, the question arises as to whether these claims that inflationary finance is essential deserve to be taken seriously.

This paper presents a class of economies in which such claims are correct. In particular, in the model presented here the following possibility will be demonstrated. A government may choose expenditure levels which are technologically feasible for the economy, and it may have a complete set of direct taxes (i.e., including lump-sum taxes) available to it. Nevertheless, if government expenditures are larger than some critical level (to be discussed), it will not be possible for the government to balance its budget. A deficit is a necessary feature of fiscal policy, given the chosen level of expenditures.

This, then, permits an answer to the question of why large government revenue needs are met through inflationary finance. The option of a balanced budget will not be open to a government under circumstances to be elaborated. The analysis also bears on the question of whether certain fiscal policy choices effectively commit monetary authorities to inflation. In fact, it suggests that not decisions to run deficits per se, but rather decisions merely to make sufficiently large expenditures can commit monetary authorities to inflate, as budget balance will be impossible. There is, of course, a corollary to this point, which is that under such circumstances a balanced budget cannot be obtained by raising taxes, but only by reducing expenditures.

The analysis also bears on two additional points which have been raised in the literature. The first is that LDCs typically resort more heav-

ily to inflationary finance than do more developed economies. It will be seen that an implication of the model to be presented is that if two economies are identical in all respects, except that population composition makes one poorer per capita, the poorer economy will have a larger minimum feasible inflation rate (if the budget cannot be balanced). Thus, the analysis suggests why a correlation is observed between per capita output and the use of seignorage.

The second is the question of why country specific currencies are observed. Existing literature has attributed this to the revenue gains to be had from seignorage. The analysis here goes somewhat further, however, and suggests that if government expenditures exceed some critical level, there is a minimum inflation rate which permits sufficient revenue to be raised. In the model of this paper, not only are seignorage gains an important reason for a national currency, but a government could not necessarily rely on other governments to inflate adequately if it employed the money of a different country. This is because, as will be seen, inflation rates are intimately linked with the feasible production of any economy in the class examined. Thus, if society is to produce an amount adequate to meet large government demands for resources, it may be necessary that the value of the currency it uses depreciate fairly rapidly. If other governments cannot be relied upon to inflate at a sufficiently high rate, it may be incumbent upon a government to employ its own currency. Or, in short, in the model presented here the opening quotation is false.

The results just outlined are obtained for a class of economies with the following features. There is a single produced commodity, and a heterogeneous labor force. Members of this labor force (workers) differ in their marginal products in production, and in their (indirect) preferences over alternate income-leisure bundles. What any worker's marginal product is,

however, is private information and not directly observable by any potential employer. It may be signalled, however, by the number of hours worked.

A set of firms, then, competes for the services of workers subject to this informational asymmetry. An equilibrium concept due to Wilson (1977) is then employed for the game played by firms. This concept has the feature that workers' productivities may either be revealed (a separating equilibrium) or not (a pooling equilibrium) in equilibrium.

If productivities are revealed in equilibrium, then hours worked will vary across agents. In a pooling arrangement, all individuals will work the same number of hours. However, for a separating arrangement to be viable in the face of the informational asymmetries involved, it will be the case that fairly severe restrictions are placed on the hours worked by more productive agents in the economy. These restrictions, which limit employment, are not in effect in a pooling arrangement. Thus, the productive capabilities of an economy may be much greater in a pooling than in a separating arrangement.

However, in the absence of distorting taxes, an economy may tend naturally to a separating equilibrium. If government resource needs are too large, then, it may be possible to meet these only in a pooling equilibrium. Hence, the government will need to use fiscal policy to force the economy into a pooling arrangement. Thus, the tax system will need to do two things: force a pooling equilibrium to result and raise sufficient government revenue.

It is shown below that if only direct taxes are available, these two objectives of taxation may not be feasible. However, for some (non-isolated) economies, the use of direct taxation coupled with an inflation tax will permit both objectives to be obtained. Thus, for these economies, it will be necessary that the budget not be balanced, or that some use of the inflation tax be made.

The scheme of the paper is as follows. Section I describes the economic environment to be studied. Section II defines an equilibrium, and (heuristically) characterizes the nature of an equilibrium for the economy at hand. Section III indicates when a balanced budget will be impossible, and Section IV provides a supporting numerical example. Section V discusses why LDCs resort to inflationary finance more than do wealthier countries. Section VI concludes by providing some casual empirical support for the analysis.

I. The Model

The model presented is the simplest one capable of illustrating the points of interest, and of providing a role for money. Thus, the economy considered consists of a set of two-period lived, overlapping generations. Time is discrete, and indexed by $t = 0, 1, \dots$. Finally, each generation (except the initial old, who may be ignored in what follows) is identical in size, and in the characteristics to be described. In light of this fact, only steady states are considered below.

At each date there are three commodities to be traded: money, a produced consumption good, and labor. All agents are endowed with labor only when young, and are retired when old. In addition, each young worker in the model is endowed with a single unit of time to be allocated between labor and leisure, and nothing else.^{6/} The initial old are endowed with all of the initial aggregate money stock, M_0 , and no other agents have an initial endowment of money.

Also, a subset of old agents is endowed with shares in firms (which are untraded).^{7/} These firms, in turn, are endowed with access to a technology for converting labor into the single consumption good. These firms hire young workers who are of two types. Types are indexed by $i = 1, 2$, and

for each unit of type i labor employed π_i units of the good can be produced. The π_i are scalar constants obeying $\pi_1 > \pi_2$. Thus, type 1 workers are more productive than workers of type 2. The proportion of type 1 agents in each young generation is θ , $\theta \in (0,1)$. θ and the values π_i are invariant over time.

It is not the case, however, that each worker's type is publicly known. Rather, each agent knows his own type, and is unaware ex ante of the type of any other agent. Moreover, no individual's contribution to production is directly observable. However, the number of hours worked by each agent is observable, and it is assumed that this is the only potentially observable difference between workers of different types.^{8/}

Firms in the model offer wage-hours packages to workers. They may offer more than one such package in an attempt to induce sorting (or separation) of workers by package selected. Let w_i denote the real wage offered to type i workers, and let L_i denote the employment offered to type i workers at the wage rate w_i . Then if type i agents and only type i agents offer to select the package (w_i, L_i) , self-selection has occurred, and agents' types are distinguishable (since $L_1 \neq L_2$). Firms compete with each other by offering different wage hours packages in attempts to attract workers in a profitable fashion.

Workers have preferences over nonnegative triples (C_1, C_2, L) , where C_1 is consumption when young, C_2 consumption when old, and L is hours worked in youth. The preferences of type i agents over such triples are described by the functions $U_i(C_1, C_2, L)$. For purposes of exposition, it is extremely useful to assume that the U_i take the form

$$(1) \quad U_i(C_1, C_2, L) = V_i(C_1, C_2) + \phi_i W(L) \equiv C_1 + \rho_i C_2 - \phi_i L,$$

with $\rho_1 \leq \rho_2$ and $\phi_1 > \phi_2$. Clearly, then, the forms taken by individual preferences have been severely restricted, so that it seems appropriate to explain in detail the reasons for the restrictions imposed.

First, the utility functions are assumed to be linear as this is necessary in order to obtain closed form expressions which describe when a balanced budget is impossible, and when a resort to inflationary finance schemes is required. The benefit associated with the ability to obtain such expressions much outweighs the loss in generality from considering only linear utility functions, as this restriction plays no important role in the analysis.

Second, the linear specification of preferences implies that in the absence of taxation, and under any of the tax schemes considered in detail below, there is no reason for firms in the model to employ consumption-hours lotteries in equilibrium. The ability to rule out such lotteries is important, as without it there would be little hope of characterizing when budget balance is impossible.

Third, the restrictions $\phi_1 > \phi_2$ and $\rho_1 \leq \rho_2$ should be discussed. These assumptions are, in fact, stronger than required, as below it is only necessary that

$$(2) \quad \frac{\phi_1}{\max[\rho_1, 1]} > \frac{\phi_2}{\max[\rho_2, 1]}$$

hold. The purpose of this restriction is as follows. Figure 1 depicts the indifference curves of type 1 and 2 agents in income-hours space. Given the assumed forms of utility functions, (2) guarantees that the indifference curves of type 1 agents through each point are more steeply sloped than those of type 2 agents through the same point in this space. This, in turn, is necessary for the existence of a separating equilibrium for any economy in the class under consideration where there is no taxation.

In light of the analytical importance of assumptions giving rise to (2), it seems appropriate to provide some justification for them here. First, then, there is a reasonable economic interpretation of (2) as follows. One could consider the utility functions appearing in equation (1) as indirect utility functions derived from a model of home production. In particular, agents could produce goods either at home or in the marketplace, with different agent types having similar preferences over home and market produced goods. Then (2), which implies that workers with high market productivity also have a relatively high opportunity cost of leisure, may be interpreted as asserting that workers with high market productivities also have relatively high productivities in the activity of home production. This positive correlation between productivity at home and in the marketplace is certainly a plausible assumption a priori.^{9/}

Second, if the linear specification of preferences (1) is relaxed, it is possible to produce versions of the model which closely reproduce observed labor market behavior (Smith [1983]). In such models, assumptions like (2) are necessary to produce under rather than over-employment of labor as an equilibrium outcome, and are also necessary in accounting for the observation that workers who on average earn relatively high wages also on average work relatively few hours. Thus, there is empirical support for an assumption such as (2) as well.

In order to complete our description of workers' behavior, then, it is necessary to describe their savings decisions. As is evident from a consideration of endowment patterns, there can be no borrowing or lending in this economy. All young agents who wish to do so save by acquiring money balances from the old.^{10/} As the consumption good has been selected as numeraire, let S_t denote the quantity of the consumption good purchasable with one unit of

money (the inverse price level) at t . In addition, let the demand for real balances of young type i agents be denoted $\psi_i\left(\frac{S_{t+1}}{S_t}, w_i L_i\right)$; i.e., the demand for real balances is a function of the intertemporal rate of return S_{t+1}/S_t , and of income when young, $w_i L_i$. The fact that L_i does not appear as a separate argument is a consequence of the form assumed for the U_i in (1). All agents are assumed to behave competitively in asset markets, and to have perfect foresight regarding the sequence $\{S_t\}$.

It remains to describe the final agent in this economy; the government. Government behavior is quite simple. The government has steady state (real) expenditure requirements of G per capita. Its expenditures may be financed via direct taxation, or through the inflation tax. Finally, it is assumed that the government does not observe hours worked.^{11/}

As there is only consumption and labor in the model, direct taxes need be levied on only one commodity. To this end, let $T(y)$ denote the income tax function levied by the government, where y is income. This function may take any arbitrary form, so that (a) lump-sum taxation has not been ruled out a priori, and (b) it is not necessary that there be two functions $T_i(y)$. One restriction is imposed on the function T , which is

$$(3) \quad T(y) \leq y \leq y.$$

In addition to levying direct taxes, the government may employ seignorage income. Again, in accordance with the focus on steady states, the government raises revenue by allowing the aggregate per capita money stock to evolve according to

$$(4) \quad M_t = (1+\sigma)M_{t-1},$$

where the choice of σ is public knowledge.

Finally, it is necessary to impose a number of parameter restrictions in order to insure that agents are willing to participate in market activities under the circumstances that arise below. These restrictions may be conveniently summarized by the assumptions that $\theta\pi_1 + (1-\theta)\pi_2 > \phi_1$ (which implies $\pi_1 > \phi_1$) and that $\pi_2 > \phi_2$ (which implies $\theta\pi_1 + (1-\theta)\pi_2 > \phi_2$).

This effectively completes the description of the environment under investigation. However, a word of interpretation may be in order. In what follows, reference is made to governments with "large" revenue needs. No statements are made as to why government expenditures might be as high as required. A natural interpretation of the model is of a government facing the need to finance wartime expenditures. However, as wars are not steady state events, and the focus here is on steady states, this interpretation cannot be taken too literally. Perhaps a more realistic environment would be one in which the government had large short-term revenue needs. This approach would not alter the basic tenor of the paper. Thus, in view of the simplifications provided by the focus on steady states, the less realistic version of a government with large steady state revenue needs is retained throughout.

II. Equilibrium

A. Firm Behavior

The world envisioned here is one in which several firms, each employing the same constant returns to scale production technology, compete against each other for the services of the workers described above. This is done merely by calling out a set of wage-employment vectors $\{(w_i, L_i)\}$, and hiring all workers who accept the firm's offer. This economy, then, is one in which firms determine employment levels.

An equilibrium for this economy is a set of wage-hours vectors $\{(w_i, L_i)\}$ announced by each firm such that no firm earns a rent, or believes it can earn a rent by calling out an alternate set of wage-hours packages. Thus, the description of firm behavior is completed by outlining firms' beliefs regarding the way in which competitors will respond to their actions.

In this paper, an equilibrium concept due to Wilson (1977) is adopted. This concept employs the following formulation of firm beliefs. As a consequence of any offer of a set of wage-hours packages by a firm, the offers of other firms may attract a set of workers which makes them unprofitable. These offers will therefore be withdrawn. As a result of unprofitable offers being withdrawn, some new set of workers may choose to seek employment with the firm in question. It is assumed that each firm is aware of the offers which will become unprofitable as a result of any action it takes, and also is aware of the set of workers it will attract as a result. Thus, calculations regarding the profitability of potential actions by firms are made under the assumption that (a) any offers which remain profitable for other firms are unchanged, and (b) any offers which become unprofitable are dropped. It is also assumed that firms take the sequence $\{S_t\}$ as parametric.

B. A Wilson Equilibrium

Given these behavioral assumptions on firms, it is now possible to define a Wilson equilibrium for this economy. The definition presented here differs from that presented by Wilson in that a condition requiring clearing of competitive asset markets is appended as follows.

Definition. A Wilson equilibrium for this economy is a set of nonnegative wage-hours packages offered by each firm at each date $\{(w_i, L_i)\}_{i=1}^2$, and a nonnegative sequence $\{S_t\}$, such that

- a) each package offered earns a nonnegative profit^{12/}
- b) no firm has an incentive to offer an alternate set of wage-hours packages, given its beliefs regarding the behavior of other firms and the $\{S_t\}$ sequence
- c) money supply equals money demand at each date, i.e.,

$$(5) \quad \theta \psi_1 \left(\frac{S_{t+1}}{S_t}, w_1 L_1 \right) + (1-\theta) \psi_2 \left(\frac{S_{t+1}}{S_t}, w_2 L_2 \right) = S_t M_t \quad \forall t.$$

Because of the presence of money and the possibility that incentive constraints need not bind in equilibrium, Wilson's (1977) proof of the existence of an equilibrium cannot be directly applied here. Since an exploration of existence issues would take us far afield, we leave open the general question of existence of an equilibrium here. However, as will be seen below, there certainly are families of economies in the class at hand for which equilibria do exist. These equilibria will not typically be unique, however, since $S_t > 0 \quad \forall t$ and $S_t = 0 \quad \forall t$ are both possible if $\rho_2 > 1 + \sigma$. In what follows, attention is restricted to the case in which $S_t > 0 \quad \forall t$, as this is clearly the only case in which the inflation tax may be employed.

In the remainder of this section, the characteristics of the various possible equilibrium configurations are explored. In particular, there are two possibilities as regards equilibrium wage-hours packages. These may either induce sorting ($L_1 \neq L_2$) or not. Following Wilson, call the first case an E1 equilibrium and the second an E2 equilibrium. Both equilibrium configurations can arise, under circumstances which are now elaborated.

Figure 1 depicts a situation in which an E1 equilibrium results. In this figure, the indifference curves of both type 1 and 2 workers are drawn, as are the zero profit loci for the various types of wage-hours packages which can be offered. The rays $\pi_i L$ denote the loci of zero profit combinations for

packages accepted by type i workers and only type i workers. The locus $\bar{\pi}L$ represents the zero profit locus for packages accepted by workers in proportion to their population fractions, i.e., $\bar{\pi} \equiv \theta\pi_1 + (1-\theta)\pi_2$. Finally, the loci labelled \bar{U}_i are indifference curves for type i agents derived in the following fashion. (For ease of diagrammatic exposition, the indifference curves are not shown as being linear, as they must be under assumption (1).) Recall that firms determine the employment levels of workers, and the real wage rates they receive. Upon accepting a wage-hours package, then, a young type i worker solves the problem

$$(6) \quad \max_{\psi_i} V_i(C_1, C_2) \text{ subject to}$$

$$C_1 \leq w_i L_i - \psi_i(-)$$

$$C_2 \leq \left(\frac{S_{t+1}}{S_t}\right) \psi_i(-)$$

taking $w_i L_i$, S_t , and S_{t+1} as given. Substituting the solution into a type i agent's utility function, an indirect utility function

$$V_i\left[w_i L_i - \psi_i\left(\frac{S_{t+1}}{S_t}, w_i L_i\right), \left(\frac{S_{t+1}}{S_t}\right) \psi_i(-)\right] - \phi_i L_i \equiv \Phi_i\left[wL, L, \frac{S_{t+1}}{S_t}\right]$$

is obtained. Using the fact that in steady state $\frac{S_{t+1}}{S_t} = \frac{1}{1+\sigma}$, one obtains utility as a function of income, $y = wL$, and labor alone. Level surfaces of these functions are shown in Figure 1.

In this figure, point A represents the equilibrium hours-income package for type 2 workers, and point B the equilibrium package for type 1 workers. The reason why this is the equilibrium is as follows. First, clearly each package offered breaks even. Second, given the sequence $\{S_t\}$, no firm could attract any workers in a profitable manner. To see this, notice that point A is maximal for type 2 workers among the set of feasible wage-

hours vectors earning nonnegative profits. Thus, type 2 workers cannot be bid away by any deviant firm in a profitable fashion unless type 1 workers are also attracted by its offer.

Now consider type 1 workers. Point B is the maximal income-hours vector for them among the set of such vectors satisfying

$$(7) \quad y_1 \leq \pi_1 L_1$$

$$(8) \quad \phi_2(w_1 L_1, L_1, \frac{1}{1+\sigma}) \leq \phi_2(w_2 L_2, L_2, \frac{1}{1+\sigma}).$$

Hence, no deviant firm could offer a wage-hours package which (a) is preferred to B by type 1 workers, (b) does not attract type 2 workers, i.e., is not (strictly) preferred to A by type 2 workers, and (c) earns nonnegative profits.

Any potential deviant, then, must offer a wage-hours package which attracts both types of workers (i.e., such that $(w_1, L_1) = (w_2, L_2)$). Any such package must generate an income-hours pair on or below the ray $\bar{\pi}L$ to (at least) break even. However, type 1 workers prefer B to all income-hours pairs in this region, and hence no such offer could attract both types of workers. Thus, A and B are equilibrium income-labor configurations as claimed.

An E2 equilibrium is depicted in Figure 2. Again points A and B are candidates for separating equilibrium values. However, in this case, a point such as C is preferred to A and B by both types of workers, and breaks even after the offers A and B have been dropped. Thus, in this case, any equilibrium must involve pooling.

The equilibrium (common) income-hours package in this figure is point C, the maximal income-hours combination for type 1 workers among the set which (at least) breaks even. This is the case since any equilibrium combination must just break even, and since any which is not maximal for type 1

agents will lose these agents (and hence all its workers) to any firm making a preferred offer such that $y < \bar{\pi}L$.

To see that C is an equilibrium, notice that to make a nonnegative profit a deviant firm must induce self-selection of workers. Also, clearly this firm must do so by attracting type 1 workers only, since C is preferred to any point such that $y < \pi_2L$ by type 2 agents.

Suppose, then, that some firm made an offer that only type 1 workers prefer to C. Clearly, this must be such that $w_1 > \bar{\pi}$. Now all type 1 workers will leave firms offering C to accept the offer of the deviant firm. But when only type 2 workers accept C it is unprofitable, so firms drop this offer. Then the deviant firm will in the end attract all workers. Since it offers a wage greater than $\bar{\pi}$, clearly this offer results in negative profits, so that C is an equilibrium as claimed.

III. Financing "Large" Government Expenditures

The set of circumstances which will be of interest here are those in which the government must force an economy which would naturally be in an E1 equilibrium into an E2 equilibrium. Therefore, attention is confined to this case. Also, without significant loss of generality, it is sufficient to consider the case in which only type 2 workers hold money. Thus, in this section we further restrict preferences so that

$$(9) \quad U_1(C_1, C_2, L) = C_1 - \phi_1 L$$

$$(10) \quad U_2(C_1, C_2, L) = C_2 - \phi_2 L.$$

This restriction is not essential in what follows.

A. An Equilibrium Without Government

As indicated, the case in which an EI equilibrium arises absent government intervention is the case of interest. It will be recalled that in such an equilibrium L_2 solves

$$\max_{L_2 \leq 1} \phi_2 \left[\pi_2 L_2, L_2, \frac{S_{t+1}}{S_t} \right].$$

Therefore, in light of (10), if $\left(\frac{S_{t+1}}{S_t}\right)\pi_2 - \phi_2 > 0$, $L_2 = 1$. Since in this section there is no activity on the part of the government, the money growth rate, σ , may be set to zero, so in steady state $S_{t+1} = S_t$. Then assuming $\pi_2 > \phi_2$, $L_2 = 1$ in any EI equilibrium.

Inspection of Figure 1 will indicate that it is also straightforward to solve for L_1 , as the EI equilibrium value of L_1 is merely the intersection of $\phi_2(\pi_2, 1, 1)$ with the zero-profit locus. Thus, L_1 is determined by

$$(11) \quad \phi_2(\pi_2, 1, 1) = \phi_2(\pi_1 L_1, L_1, 1),$$

or using (10), by

$$(11') \quad (\pi_2 - \phi_2)L_2^* = (\pi_1 - \phi_2)L_1^*,$$

where an asterisk denotes an equilibrium value. Then, since $L_2^* = 1$,

$$L_1^* = \frac{\pi_2 - \phi_2}{\pi_1 - \phi_2}.$$

Finally, for an EI equilibrium to exist, it is necessary that (see Figure 1)

$$(12) \quad \phi_1 \left[\pi_1 \left(\frac{\pi_2 - \phi_2}{\pi_1 - \phi_2} \right), \frac{\pi_2 - \phi_2}{\pi_1 - \phi_2}, 1 \right] > \max_{L \leq 1} \phi_1 [\bar{\pi}L, L, 1].$$

For the economy at hand,

$$\max_{L \leq 1} \phi_1 (\bar{\pi}L, L, 1) = \bar{\pi} - \phi_1,$$

so (12) is equivalent to (for this economy),

$$(13) \quad (\pi_1 - \phi_1) \left(\frac{\pi_2 - \phi_2}{\pi_1 - \phi_2} \right) > \bar{\pi} - \phi_1.$$

Satisfaction of this condition is assumed throughout the remainder of the paper.

B. An Economy With Taxation

Now consider an economy in which the government is required to make steady state per capita expenditures of G units (in real terms). (The government is viewed here as merely consuming resources.) The focus of the analysis is on government expenditures which are "large" in the following sense. The fact that an economy is in an E1 equilibrium places obvious restrictions on the employment levels of type 1 workers. As is clear from Figure 1, in the absence of taxation the income (production) of type 1 workers cannot exceed that of type 2 workers. This is also true in the presence of an arbitrary income tax function $T(y)$.^{13/} Hence, total per capita output in an E1 equilibrium obeys $\theta\pi_1 L_1 + (1-\theta)\pi_2 L_2 \leq \theta\pi_2 L_2 + (1-\theta)\pi_2 L_2 \leq \pi_2$. Therefore, if $G > \pi_2$, it is obviously necessary that the tax system be used to force the economy into a pooling equilibrium, which can be consistent with government expenditures being feasible so long as $\bar{\pi} > G > \pi_2$.

The first point of note is that a government with these "large" revenue needs cannot rely on lump-sum taxation alone. To see this, notice that (if the government uses only direct taxation for the time being, i.e., sets $\sigma = 0$) existence of an E2 equilibrium requires that

$$(14) \quad \hat{\phi}_1 [w_1 \hat{L}_1 - T(w_1 \hat{L}_1), \hat{L}_1, 1] < \max_{L < 1} \hat{\phi}_1 [\bar{\pi} L - T(\bar{\pi} L), L, 1],$$

where \hat{L}_1 is the candidate E1 equilibrium value of L_1 in the presence of taxes. (14) merely states that existence of an E2 equilibrium requires that type 1 workers prefer some pooling arrangement to the maximal wage-hours vector for them under self-selection. For the linear specification of preferences employed here, (14) becomes

$$(14') \quad (\pi_1 - \phi_1)\hat{L}_1 - T(\pi_1\hat{L}_1) < (\bar{\pi} - \phi_1)\hat{L} - T(\bar{\pi}\hat{L}),$$

where $\hat{L} \equiv \operatorname{argmax} [(\bar{\pi} - \phi_1)L - T(\bar{\pi}L)]$.

Now suppose that the government need employ only lump-sum taxes, i.e., $T(\pi_1\hat{L}_1) = T(\bar{\pi}\hat{L}) \equiv T(y) = T \neq y > T$. Then it is straightforward to check that $\hat{L}_1 = L_1^*$, i.e., the candidate value for an E1 equilibrium value of L_1 is unaltered. Therefore, (14') becomes

$$(14'') \quad (\pi_1 - \phi_1)L_1^* - T < (\bar{\pi} - \phi_1) - T.$$

However, (14'') is obviously false, as (14'') contradicts (13), which has been assumed to hold. Thus, strictly lump-sum taxation will be inadequate to force this economy into a pooling equilibrium, and hence inadequate to finance government expenditures.

It is the case, then, that governments with expenditures such that $G > \pi_2$ will need to resort to the use of distorting taxes. These taxes must force the economy at hand into a pooling equilibrium, and at the same time, raise G units of revenue. It will now be demonstrated that these two objectives may be incompatible using direct taxation alone.

As we focus on the use of direct taxation only, again set $\sigma = 0$. Then existence of an E2 equilibrium requires that (14') hold. Moreover, since type 1 and 2 workers are indistinguishable from the point of view of the government, they will pay the same taxes in an E2 equilibrium (with $\sigma = 0$).

Consider the implications of (14') for the function $T(-)$ then. Rearranging terms, (14') becomes

$$(15) \quad (\bar{\pi} - \phi_1)\hat{L} - (\pi_1 - \phi_1)\hat{L}_1 + T(\pi_1\hat{L}_1) > T(\bar{\pi}\hat{L}).$$

Now recall that $\hat{L} \leq 1$, and that $T(\pi_1\hat{L}_1) \leq \pi_1\hat{L}_1 \forall \hat{L}_1 \in [0,1]$. Using these facts, (15) implies that

$$(16) \quad (\bar{\pi} - \phi_1) + T(\pi_1\hat{L}_1) - (\pi_1 - \phi_1) \frac{T(\pi_1\hat{L}_1)}{\pi_1} > T(\bar{\pi}\hat{L}),$$

or that

$$(16') \quad (\bar{\pi} - \phi_1) + \left(\frac{\phi_1}{\pi_1}\right) T(\pi_1\hat{L}_1) > T(\bar{\pi}\hat{L}).$$

Finally, again using the fact that $\pi_1\hat{L}_1 > T(\pi_1\hat{L}_1)$, and that $\pi_1\hat{L}_1 \leq \pi_2\hat{L}_2 \leq \pi_2$ in any candidate E1 allocation, (16') becomes

$$(17) \quad (\bar{\pi} - \phi_1) + \phi_1 \left(\frac{\pi_2}{\pi_1}\right) > T(\bar{\pi}\hat{L}).$$

Thus (17) places an upper bound on the revenue which can be raised via direct taxation.^{14/} In order to interpret this bound, it is useful to derive a somewhat looser upper bound on $T(\bar{\pi}\hat{L})$ as follows.

Recall that in any E1 equilibrium, an obvious upper bound on per capita revenue is π_2 . Recall also that by (13), $(\pi_1 - \phi_1)L_1^* > \bar{\pi} - \phi_1$, and that $\pi_1 L_1^* \leq \pi_2$. Using these facts, (17) implies that

$$(\pi_1 - \phi_1) \left(\frac{\pi_2}{\pi_1}\right) + \phi_1 \left(\frac{\pi_2}{\pi_1}\right) = \pi_2 > T(\bar{\pi}\hat{L}).$$

Thus, even in moving to an E2 equilibrium, π_2 still places an upper bound on the amount of revenue which can be raised via direct taxation. Therefore, if $G > \pi_2$, a balanced budget will be impossible for the government in question. However, as will now be seen, it may still be possible for the government to

raise the necessary revenue through a combination of inflation and direct taxation.

C. An Economy With Inflation

Consider the case in which the government accepts a permanently unbalanced budget, so that $\sigma > 0$. Then in steady state $\frac{S_{t+1}}{S_t} = \frac{1}{1+\sigma}$, so that in any candidate for an E1 equilibrium,

$$(18) \quad \hat{L}_2 = \operatorname{argmax} \phi_2 [\pi_2 L - T(\pi_2 L), L, \frac{1}{1+\sigma}],$$

and \hat{L}_1 solves

$$(19) \quad \phi_2 [\pi_2 \hat{L}_2 - T(\pi_2 \hat{L}_2), \hat{L}_2, \frac{1}{1+\sigma}] = \phi_2 [\pi_1 \hat{L}_1 - T(\pi_1 \hat{L}_1), \hat{L}_1, \frac{1}{1+\sigma}],$$

where " $\hat{\cdot}$ " denotes a candidate equilibrium value in this regime with inflation.

The candidate value for L in any E2 equilibrium is, of course,

$$(20) \quad \hat{L} = \operatorname{argmax} \phi_1 \{ \bar{\pi} L - T(\bar{\pi} L), L, \frac{1}{1+\sigma} \}.$$

Suppose, to keep matters simple, that all government revenue needs can be met using only the inflation tax, and direct lump-sum taxation.^{15/} Then let T denote the lump-sum tax. The value of \hat{L}_2 determined by (18) solves

$$\max_{L_2 \leq 1} \left(\frac{\pi_2}{1+\sigma} - \phi_2 \right) L_2 - \frac{T}{1+\sigma},$$

so that $\hat{L}_2 = 1$ so long as $\frac{\pi_2}{1+\sigma} > \phi_2$, and $\frac{\pi_2 - T}{1+\sigma} - \phi_2 > 0$.^{16/} Then for the preferences here (19) becomes

$$\frac{\pi_2 - T}{1+\sigma} - \phi_2 = \left(\frac{\pi_1}{1+\sigma} - \phi_2 \right) \hat{L}_1 - \frac{T}{1+\sigma}$$

which implies

$$\hat{L}_1 = \frac{\pi_2 - \phi_2(1+\sigma)}{\pi_1 - \phi_2(1+\sigma)}$$

in any E1 allocation.^{17/} Finally, since all direct taxation is lump-sum and type 1 workers do not hold money, the candidate value of L in an E2 equilibrium configuration is obviously $\hat{L} = 1$.

Now since $G > \pi_2$, it is necessary that the tax system force an E2 equilibrium to result, i.e., that it force

$$(21) \quad (\pi_1 - \phi_1)\hat{L}_1 - T \leq \bar{\pi} - \phi_1 - T$$

or, using the expression for \hat{L}_1 ,

$$(21') \quad (\pi_1 - \phi_1) \left[\frac{\pi_2 - \phi_2(1+\sigma)}{\pi_1 - \phi_2(1+\sigma)} \right] \leq \bar{\pi} - \phi_1.$$

As the left-hand side of (21') is decreasing in σ , this places a well-defined lower bound on the rate of inflation that is required for the government to force an E2 equilibrium, and thus raise adequate revenue for its expenditures. In short, then, the analysis here indicates that (a) it may be impossible for the government to balance its budget, and (b) in light of this, there may be a minimum rate of inflation (which can be quite large) that permits the government to raise adequate revenue (i.e., to force an E2 equilibrium).

It remains to be shown that the scheme of combining inflationary and direct taxation actually allows the government to raise revenues equal to expenditures. Recalling that only type 2 workers hold money, and that they save all of their income, the demand for real balances (in per capita terms) in this economy is $(1-\theta)[\bar{\pi}-T(\sigma,G)]$, where $T(\sigma,G)$ is the level of direct lump-sum taxation required to meet expenditure needs of G if the money growth rate is σ . Notice that the fact that $\hat{L} = 1$ has been used, and that this requires both $\bar{\pi} - \phi_1 - T(\sigma,G) \geq 0$, and $\frac{\bar{\pi}-T(\sigma,G)}{1+\sigma} - \phi_2 \geq 0$.

Given this per capita demand for real balances, the government's per capita revenue from the inflation tax is $(1-\theta) \left(\frac{\sigma}{1+\sigma} \right) [\bar{\pi}-T(\sigma,G)]$, so that the government raises adequate revenue iff

$$(22) \quad T(\sigma, G) + (1-\theta)\left(\frac{\sigma}{1+\sigma}\right)[\bar{\pi}-T(\sigma, G)] = G.$$

This, in turn, implies that

$$(23) \quad T(\sigma, G) = \left(\frac{1}{1+\theta\sigma}\right)[G(1+\sigma)-\bar{\pi}(1-\theta)\sigma].$$

Hence, if G and σ are such that $G > \pi_2$, (21'), $\frac{\bar{\pi}-T(\sigma, G)}{1+\sigma} > \phi_2$ and $\bar{\pi} - \phi_1 - T(\sigma, G) > 0$ hold, it is impossible for the government to balance its budget, and yet possible for the government to raise revenue adequate to meet its expenditures through the use of inflationary taxation. It is demonstrated in Section IV that these conditions are nonvacuous, i.e., a set of economies is presented for which they hold.

D. Optimal Taxation

It is, of course, the case that optimal taxation exercises can be performed for the economy at hand. For the purposes of performing such an exercise, one might confine consideration to schemes where all direct taxation was lump-sum, with shortfalls met via the inflation tax. Then (since $\tilde{L} = 1$ will hold in any such E2 equilibrium)

$$(24) \quad U_1 = \bar{\pi} - \phi_1 - T(\sigma, G)$$

$$(25) \quad U_2 = \frac{\bar{\pi} - T(\sigma, G)}{1 + \sigma} - \phi_2.$$

This implies that if the government were to maximize, say, a utilitarian social welfare function (ignoring the initial old), it would solve

$$\max_{\underline{\sigma} \leq \sigma \leq \bar{\sigma}} \theta U_1 + (1-\theta)U_2,$$

where $\underline{\sigma}$ and $\bar{\sigma}$ are lower and upper bounds on σ , respectively, derived as follows. First, as already noted, (21') places a lower bound on σ , i.e., $\underline{\sigma}$ is that value of σ for which (21') holds with equality. Also, in order for type

2 agents to be in the workforce voluntarily, it is necessary that $\bar{\pi} - T(\sigma, G) - \phi_2(1+\sigma) > 0$ hold. Using the expression for $T(\sigma, G)$ given in (23), it is easily shown that this condition is equivalent to $(\bar{\pi}-G)/\theta\phi_2 - \theta^{-1} > \sigma$. Letting $\bar{\sigma}$ be defined by $\bar{\sigma} = (\bar{\pi}-G)/\theta\phi_2 - \theta^{-1}$, we have defined a compact interval over which this maximization is to take place.

The first order condition (for an interior optimum) associated with this maximization problem is

$$(26) \quad \theta \frac{\partial U_1}{\partial \sigma} + (1-\theta) \frac{\partial U_2}{\partial \sigma} = 0,$$

where from (24) and (25),

$$\begin{aligned} \frac{\partial U_1}{\partial \sigma} &= -T_\sigma \\ \frac{\partial U_2}{\partial \sigma} &= \frac{-T_\sigma}{1+\sigma} - \left[\frac{\bar{\pi} - T(\sigma, G)}{(1+\sigma)^2} \right]. \end{aligned}$$

As (from (23))

$$T_\sigma(\sigma, G) = \frac{(1-\theta)(G-\bar{\pi})}{(1+\theta\sigma)^2},$$

and as $\bar{\pi} > G$ for feasibility, $T_\sigma < 0$. For economies in the class under consideration, either corner or interior solutions may be optimal, so that in general optimal taxation questions are easily addressed here, and a unique (ignoring the initial old) optimal inflation rate exists. Thus, the economies of this paper lend themselves to fairly conventional optimal taxation exercises, although these are not pursued further in what follows.

IV. An Example

In this section a specific economy is presented for which the conditions discussed above hold. It is then argued that this is a robust example, i.e., that there exists an open set of economies in the class under

consideration for which budget balance is impossible, and yet in which inflationary finance makes it possible for the government to meet its revenue needs.

To this end, let $\pi_1 = 4$, $\pi_2 = 2$, $\phi_1 = 1$, $\phi_2 = .001$, $\theta = .24$ (so that $\bar{\pi} = 2.48$), and $G = 2.05$. Obviously, as $G > \pi_2$, government revenue needs are such that an E2 equilibrium must result. First, then, it is shown that absent taxation (or with only lump-sum taxation), this economy will be in an E1 equilibrium. Hence, the tax system will have to be used to force an E2 equilibrium.

To see this, then, note that absent taxation $L_2^* = 1$. Then

$$L_1^* = \frac{\pi_2 - \phi_2}{\pi_1 - \phi_2} = .4999$$

if an E1 equilibrium exists. But such an equilibrium does exist, as under the allocation above, type 1 workers obtain utility $U_1 = (\pi_1 - \phi_1)L_1^* = 1.4997 > \bar{\pi} - \phi_1 = 1.48$. Thus distorting taxation will be required to force this economy into an E2 equilibrium.

In addition, the upper bound on $T(\hat{\pi}L)$ under direct taxation only given by (17) is $T(\hat{\pi}L) < \bar{\pi} - \phi_1 + \phi_1 \left(\frac{\pi_2}{\pi_1} \right) = 1.98 < G = 2.05$. Thus, budget balance under an E2 equilibrium is also impossible, i.e., $\sigma > 0$ is required in equilibrium.

Suppose, then, that the government confines itself to direct lump-sum taxation at the level $T(\sigma, 2.05)$ given by (23), and to use of the inflation tax. Then to force an E2 equilibrium to result, it is necessary that

$$(\pi_1 - \phi_1) \left[\frac{\pi_2 - \phi_2(1+\sigma)}{\pi_1 - \phi_2(1+\sigma)} \right] < \bar{\pi} - \phi_1,$$

which for this example implies $\sigma > 51.6939$ must hold. Thus, a large inflation is required simply to force a pooling arrangement in this economy.

To show that an equilibrium can be attained for this economy, let $\sigma = 60$. From (23), $T(60, 2.05) = .7717$, and the per capita revenue from the inflation tax is $(1-\theta)\left(\frac{\sigma}{1+\sigma}\right)(\bar{\pi} - .7717) = 1.277$. Thus, to within rounding error, government revenue needs are met by this system.

Finally, it is necessary to check that several conditions are met. First, clearly $\tilde{L} = 1$, so that type 1 utility under this arrangement is given by $U_1 = \bar{\pi} - \phi_1 - T(\sigma, G) = .7083$, and type 2 utility by $\frac{\bar{\pi} - T(\sigma, G)}{1 + \sigma} - \phi_2 = .0270$. Thus, the proposed allocations do, in fact, dominate autarky, as required.

Second, the analysis here would be fairly trivial if the government simply levied lump-sum taxes which could never be paid in any separating arrangement, and in this way forced an E2 equilibrium to result. This is why it has been required that $T(y) < y$. Similarly, the analysis would be trivial if the only separating equilibrium was the autarky arrangement. Thus, it is now verified that neither of these situations arises here.

To see this, note first that the candidate E1 allocations are $\tilde{L}_2 = 1$, and

$$\tilde{L}_1 = \frac{\pi_2 - \phi_2(1+\sigma)}{\pi_1 - \phi_2(1+\sigma)} = .4923.$$

$T(\sigma, G) = .7717 < \min(\pi_2 \tilde{L}_2, \pi_1 \tilde{L}_1)$, so that clearly the government is not levying unaffordable taxes under an E1 arrangement. Also, it is easy to check that $\tilde{L}_2 = 1$ and $\tilde{L}_1 = .4923$ are both preferred to autarky. Hence, this example is innocent of the criticisms just suggested.

Thus, in the example economy, it is impossible for the government to balance its budget, and yet government expenditure levels are feasible. This demonstrates nonvacuousness of the preceding discussion. Also, as noted previously the economy at hand places an upper bound on inflation. This is

the case since if $\sigma > \bar{\sigma}$, type 2 workers will prefer not to enter the labor force. Their absence would then imply that per capita government expenditures are no longer feasible, so that the analysis here places both upper and lower limits on the feasible inflation rates available to the government.

Having presented this example, it may now be of value to provide some additional intuition regarding the role of inflationary finance in the model. In particular, under the assumptions (9) and (10) on preferences, increases in the inflation rate have the following effects on the indirect preferences depicted in Figure 1. So long as $\sigma \leq \bar{\sigma}$, point A remains the candidate equilibrium value for hours worked and gross of tax income on the part of type 2 agents. The effect of increasing σ is to rotate \bar{U}_2 (the indifference curve through A) counterclockwise, while type 1 preferences are left unaffected. This rotation of \bar{U}_2 moves the candidate equilibrium pair point B $(\pi_1 \hat{L}_1, \hat{L}_1)$ down the $\pi_1 L$ locus towards the origin. If σ is increased sufficiently that the type 1 indifference curve through B intersects the $\bar{\pi} L$ locus, a pooling equilibrium results.

Once the economy is in a pooling equilibrium, the restrictions on hours worked (and on total output) that arise due to the incentive constraints are relaxed. Hence, sufficiently large increases in σ permit larger levels of total output to be realized. Moreover, the seignorage revenue generated reduces the need for direct taxation. Thus, higher inflation rates help the government meet its revenue requirements in each of these ways.

A. A Remark

Having presented a sample economy which cannot balance its budget, it is natural to ask how robust this example is. Recall that the important features of the example were as follows: (a) distorting taxes are required to force an E2 equilibrium, (b) $G > \pi_2$, (c) the candidate F2 equilibrium values

are actually preferred to autarky, and (d) even in the candidate for an F1 equilibrium, all agents can meet their tax obligations, i.e., $T(\sigma, G) < \pi_1 \hat{L}_1$, with $T(\sigma, G)$ given by (23). Thus, the parameter values for economies in this class must satisfy (13), $G > \pi_2$, (21'), $\bar{\pi} - \phi_1 - T(\sigma, G) > 0$, $\frac{\bar{\pi} - T(\sigma, G)}{1 + \sigma} > \phi_2$, and

$$\left[\frac{\pi_2 - \phi_2(1+\sigma)}{\pi_1 - \phi_2(1+\sigma)} \right] (\pi_1 - \phi_1) - T(\sigma, G) > 0.$$

It will be noted that all of these conditions hold as strict inequalities for the example, and that each expression in the inequalities varies continuously as a function of the parameters of the economy. Hence, there exists an open neighborhood, containing the economy of the example, in which each economy displays all of the qualitative features of the example listed above. In short, then, the inability of the government of the example to balance its budget, and the necessity of resorting to inflationary finance are economic features which would be common to a nonnegligible class of economies.

V. Inflationary Finance and "LDCs"

One of the salient observed features of inflationary finance schemes is that they seem to be used more heavily by "less developed" than by "more developed" countries.^{18/} This is a fact which the model developed here can (partially) account for. To see this, recall that when budget balance is impossible, and inflationary finance coupled with lump-sum taxes are employed, for the government to force an F2 equilibrium (21') is required to hold. Rearranging terms in this equation, a lower bound for σ can be obtained:

$$(27) \quad \sigma > \frac{(\pi_1 - \phi_1)\pi_2 - (\bar{\pi} - \phi_1)\pi_1}{\phi_2(1-\theta)(\pi_1 - \pi_2)} - 1.$$

Now consider two economies with access to identical technologies, where type i agents are identical across the two, and where G is the same in each economy. The only difference between the economies, then, is that economy one has a higher value of θ , θ_1 , than does economy two (i.e., $\theta_1 > \theta_2$). Clearly, then, in per capita terms economy one is wealthier than economy two. Also, so long as $G > \pi_2$ for each economy, the only condition of importance here which is affected by θ is (27).

Consider the effect of varying θ on the right-hand side of (27) then. Rewriting the relevant expression as

$$(28) \quad B(\theta) \equiv \frac{(\pi_1 - \phi_1)\pi_2}{\phi_2(\pi_1 - \pi_2)(1-\theta)} - \frac{[\theta\pi_1 + (1-\theta)\pi_2 - \phi_1]\pi_1}{\phi_2(\pi_1 - \pi_2)(1-\theta)},$$

it is easy to check that $B'(\theta) < 0$.^{19/} Thus, since $\sigma > B(\theta) - 1$ for each economy, and since $B(\theta_1) < B(\theta_2)$, it is clear that the "less developed" country has a larger lower bound on its inflation rate than does the wealthier country. If two governments face identical expenditure needs, then, and have access to identical technologies but different populations, the poorer country faces a strictly smaller range of feasible inflation rates than does the richer one (so long as both restrict themselves to only the inflation tax and lump-sum taxation). In particular, it will be feasible for the wealthier country to finance its expenditures with less heavy reliance on the inflation tax. Moreover, it is not difficult to find circumstances where (in solving an optimal taxation problem) the wealthier country will make use of this option.

VI. Conclusions

It has been seen that in economies where adverse selection problems arise in labor markets, if government expenditures are sufficiently large (in real terms), it may be impossible for the government to balance its budget.

Thus, a decision on the part of the government to make large expenditures may, by itself, commit the government to inflationary finance schemes. Moreover, this is true even if the government may levy arbitrary lump-sum taxes. Thus, the statement quoted at the beginning of this paper is often false, at least for the economies considered here.

In addition, the analysis suggests why LDCs may tend to resort more heavily to inflationary finance than do more developed nations. It also suggests reasons for the existence of national currencies which are even stronger than the existence of seignorage gains. In particular, as has been seen, even when a government has access to lump-sum taxes, it may be necessary for there to be inflation in order for an E2 equilibrium to result. If an E2 equilibrium is essential for a government to finance its expenditures, there will then be a lower bound on the feasible rate of inflation in the country in question. If foreign monetary authorities cannot be trusted to inflate sufficiently, it will be absolutely necessary for the government in question to run its own monetary policy. Thus, the model here provides a cogent rationale for the presence of national currencies in addition to those already existing.

At this point a natural question arises, however. In particular, are these results simply theoretical possibilities, or is there some reason to think that the analysis performed here is capable of confronting real world phenomena. More specifically, one might wonder whether the informational asymmetries modeled here can be important at an aggregate level. This is really, of course, an empirical question as to whether models in the class at hand can confront empirical regularities which are anomalous in the context of other models, and do so in a way which is consistent with a broad class of observations. While a complete answer to this question is beyond the scope of this paper, Smith [1983] has shown that the class of adverse selection economies at hand is capable of giving rise to the following phenomena:

- (a) labor is underemployed.
- (b) in cross-sections, workers with relatively high average earnings work relatively few hours.
- (c) cross-occupational relative wages are important determinants of labor market behavior (Solow [1980]).
- (d) in aggregated data, hours worked respond strongly to real wage movements. In panel data hours worked respond very weakly to real wage movements for a large class of individuals.
- (e) real wage movements are pro-cyclical or acyclic, and average productivity is pro-cyclical.
- (f) over the business cycle, hours worked respond strongly to relatively minor variations in real wages. However, secular increases in real wages are not associated with similar trends in hours worked.

Thus, the model at hand is capable of confronting a broad class of observed labor market phenomena, some of which are anomalous in the context of other models. This suggests that models such as the one under consideration are not implausible as models of aggregate phenomena.

Are there some empirical reasons, then, for preferring the analysis here to other analyses of inflationary finance? There would seem to be at least two, which will be briefly touched on here. The first is that there are historical episodes where the optimal taxation literature suggests that the governments in question have followed dramatically suboptimal policies. In particular, Lucas and Stokey [1983] suggest that governments holding nominal claims on their citizens should optimally engineer deflations and finance their expenditures from the capital gains which accrue. In practice, however, it is not difficult to find creditor governments which have run deficits and resorted to inflationary finance schemes. As an example, Maryland resorted to

inflationary finance in the Revolutionary War despite its creditor position, and even enacted legislation permitting creditors to pay off their debts in depreciated currency (which they would otherwise not have been able to do).^{20/} While this policy may have been due to the difficulty of collecting debts, or to the fact that Maryland was part of a confederation which was resorting to inflationary finance, there is at least an open question as to whether the traditional optimal taxation literature can lend any insight into the reasons for running such policies. Moreover, as a more modern example, Fischer [1983] shows that in Israel "an increase in the money stock would reduce the present discounted value of government revenue," i.e., that there are revenue losses from use of the inflation tax. Nevertheless, the Israeli government has made little attempt to reduce its triple digit inflation rates. While such behavior is anomalous in the context of standard models of optimal taxation, this is a phenomenon readily accounted for by the analysis here.

Secondly, casual empiricism suggests that during wars, for instance, the composition of the workforce performing certain tasks tends to become much more heterogeneous. While again there are obviously other contributing factors, the analysis above suggested that large revenue needs on the part of the government would tend to force "pooling" arrangements in labor markets. It is an open question whether existing optimal taxation literature can address this phenomenon.

APPENDIX

In this appendix, we prove that any separating equilibrium, with any arbitrary income tax function $T(y)$ obeying $T(y) \leq y \forall y$, has the feature that $\pi_2 L_2 > \pi_1 L_1$. To begin the proof, suppose to the contrary that $L_1 = L_2$ (and hence that $\pi_1 L_1 > \pi_2 L_2$) in some separating equilibrium. Then, since $L_1 = L_2$, incentive compatibility of such an allocation requires equality of net of tax incomes:

$$(A1) \quad \pi_1 L_1 - T(\pi_1 L_1) = \pi_2 L_2 - T(\pi_2 L_2).$$

Moreover, existence of a separating equilibrium requires (under our assumptions on preferences) that

$$(A2) \quad (\pi_1 - \phi_1) L_1 - T(\pi_1 L_1) > (\bar{\pi} - \phi_1) \tilde{L} - T(\bar{\pi} \tilde{L}) \quad \forall \tilde{L} \in [0, 1].$$

However, consider a value \tilde{L} chosen such that $\bar{\pi} \tilde{L} = \pi_2 L_2$, which in turn implies that

$$(A3) \quad \bar{\pi} \tilde{L} - T(\bar{\pi} \tilde{L}) = \pi_2 L_2 - T(\pi_2 L_2) = \pi_1 L_1 - T(\pi_1 L_1)$$

since $\pi_2 L_2 = \bar{\pi} \tilde{L}$, and where we have used (A1). Then since, by hypothesis, $L_1 = L_2$, (A3) implies

$$\begin{aligned} \bar{\pi} \tilde{L} - T(\bar{\pi} \tilde{L}) - \phi_1 \tilde{L} &= \pi_1 L_1 - T(\pi_1 L_1) - \phi_1 \left(\frac{\pi_2}{\bar{\pi}} \right) L_2 = \\ \pi_1 L_1 - T(\pi_1 L_1) - \phi_1 \left(\frac{\pi_2}{\bar{\pi}} \right) L_1 &> (\pi_1 - \phi_1) L_1 - T(\pi_1 L_1). \end{aligned}$$

But this contradicts (A2), and hence the assumption that a separating equilibrium exists. Thus, a separating equilibrium with $L_1 = L_2$ is impossible.

Suppose, then, that $L_1 > L_2$ (and hence $\pi_1 L_1 > \pi_2 L_2$). The incentive compatibility constraints are

$$(A4) \quad (\pi_1 - \phi_2)L_1 - T(\pi_1 L_1) < (\pi_2 - \phi_2)L_2 - T(\pi_2 L_2)$$

$$(A5) \quad (\pi_1 - \phi_1)L_1 - T(\pi_1 L_1) > (\pi_2 - \phi_1)L_2 - T(\pi_2 L_2).$$

Subtracting (A5) from (A4),

$$(\phi_1 - \phi_2)L_2 > (\phi_1 - \phi_2)L_1.$$

But by assumption $\phi_1 > \phi_2$, implying $L_2 > L_1$. This contradicts the initial hypothesis, so that $L_1 > L_2$ is not possible.

Finally, then, it may be possible that $L_1 < L_2$, and yet that $\pi_1 L_1 > \pi_2 L_2$. Notice that since $L_1 < L_2$, this arrangement cannot be incentive compatible unless $\pi_1 L_1 - T(\pi_1 L_1) < \pi_2 L_2 - T(\pi_2 L_2)$. Notice also that if $\pi_1 L_1^* = \pi_2 L_2$ were to obtain, $\pi_1 L_1^* - T(\pi_1 L_1^*) = \pi_2 L_2 - T(\pi_2 L_2)$ would also hold. This would also imply $L_1^* = \frac{\pi_2}{\pi_1} L_2 < L_1$ (the hypothesized equilibrium value of L_1). Therefore, in the absence of incentive compatibility constraints being binding, type 1 agents would prefer the allocation associated with L_1^* to that associated with L_1 . Hence, incentive compatibility constraints must be binding in equilibrium, i.e.,

$$(\pi_1 - \phi_2)L_1 - T(\pi_1 L_1) = (\pi_2 - \phi_2)L_2 - T(\pi_2 L_2).$$

This, in turn, implies that

$$\pi_1 L_1 - T(\pi_1 L_1) = \pi_2 L_2 - T(\pi_2 L_2) - \phi_2(L_2 - L_1)$$

in any candidate equilibrium. Also, notice that in such an equilibrium,

$$U_1 = \pi_2 L_2 - T(\pi_2 L_2) - \phi_2(L_2 - L_1) - \phi_1 L_1 =$$

$$\pi_2 L_2 - T(\pi_2 L_2) - \phi_2 L_2 - (\phi_1 - \phi_2)L_1,$$

and also we know that in any separating equilibrium, $L_2 = L_2^* =$

$\text{argmax}\{(\pi_2 - \phi_2)L_2 - T(\pi_2 L_2)\}$. Therefore, in any separating equilibrium,

$$(A6) \quad U_1 = \pi_2 L_2^* - T(\pi_2 L_2^*) - \phi_2 L_2^* - (\phi_1 - \phi_2)L_1.$$

It will now be recalled that a value of L_1 satisfying $L_1 = \left(\frac{\pi_2}{\pi_1}\right) L_2^*$ is not consistent with self-selection. Also notice that if $L_2^* \neq 0$, then

$$(A7) \quad \pi_2 L_2^* - T(\pi_2 L_2^*) - \phi_2 L_2^* > T(0).$$

(A7) states that it would be incentive compatible to set $L_1 = 0$. Therefore, in light of these two facts, there exists a value $\lambda \in (0,1)$ such that

$$(A8) \quad \lambda T(0) + (1-\lambda) \left\{ (\pi_1 - \phi_2) \left(\frac{\pi_2}{\pi_1}\right) L_2^* - T\left[\pi_1 \left(\frac{\pi_2}{\pi_1}\right) L_2^*\right] \right\} = (\pi_2 - \phi_2) L_2^* - T(\pi_2 L_2^*).$$

(A8) may be interpreted as implying that there exists an incentive compatible lottery which offers $L_1 = 0$ with probability λ , and $L_1 = \left(\frac{\pi_2}{\pi_1}\right) L_2^*$ with probability $1 - \lambda$. Such a lottery generates expected utility

$$(A9) \quad \begin{aligned} EU_1 &= \lambda T(0) + (1-\lambda) \left\{ \pi_1 \left(\frac{\pi_2}{\pi_1}\right) L_2^* - T\left[\pi_1 \left(\frac{\pi_2}{\pi_1}\right) L_2^*\right] - \phi_1 \left(\frac{\pi_2}{\pi_1}\right) L_2^* \right\} \\ &= \lambda T(0) + (1-\lambda) \left\{ \pi_2 L_2^* - T(\pi_2 L_2^*) - \phi_2 \left(\frac{\pi_2}{\pi_1}\right) L_2^* \right\} - (1-\lambda) (\phi_1 - \phi_2) \left(\frac{\pi_2}{\pi_1}\right) L_2^* \\ &= (\pi_2 - \phi_2) L_2^* - T(\pi_2 L_2^*) - (1-\lambda) (\phi_1 - \phi_2) \left(\frac{\pi_2}{\pi_1}\right) L_2^*, \end{aligned}$$

where the latter equality follows from (A8).

Now, consider any value \hat{L}_1 satisfying $L_2^* > \hat{L}_1 > \left(\frac{\pi_2}{\pi_1}\right) L_2^*$, and which is incentive compatible, i.e., such that

$$(\pi_1 - \phi_2) \hat{L}_1 - T(\pi_1 \hat{L}_1) = (\pi_2 - \phi_2) L_2^* - T(\pi_2 L_2^*).$$

By hypothesis, such a value \hat{L}_1 exists. Then the value of U_1 associated with this value \hat{L}_1 is, from (A6), $U_1 = (\pi_2 - \phi_2) L_2^* - T(\pi_2 L_2^*) - (\phi_1 - \phi_2) \hat{L}_1$. But now notice that for any such \hat{L}_1 , $\hat{L}_1 > \left(\frac{\pi_2}{\pi_1}\right) L_2^* > (1-\lambda) \left(\frac{\pi_2}{\pi_1}\right) L_2^*$. Therefore, from

(A9), there exists an employment lottery which type 1 agents prefer to any certain value of employment which is incentive compatible, and which has $L_1 > (\pi_2/\pi_1)L_2^*$ (or $\pi_1 L_1 > \pi_2 L_2$). This contradicts the hypothesis that such an equilibrium could exist. Hence, there is no separating equilibrium with a nonstochastic value of L_1 such that $\pi_1 L_1 > \pi_2 L_2$. Moreover, the lottery constructed in (A9) has $E \pi_1 L_1 = (1-\lambda) \pi_1 \left(\frac{\pi_2}{\pi_1}\right) L_2^* = (1-\lambda) \pi_2 L_2^* < \pi_2 L_2^*$ (since $\lambda > 0$). Since any incentive compatible employment lottery has

$$EU_1 = (\pi_2 - \phi_2)L_2^* - \pi(\pi_2 L_2^*) - (\phi_1 - \phi_2)EL_1,$$

the lottery constructed in (A9) is also preferred to any incentive compatible employment lottery with $EL_1 > (\pi_2/\pi_1)L_2^*$. Hence, there is also no equilibrium employment lottery with $E\pi_1 L_1 > \pi_2 L_2^*$. In the text, we confine attention to the case where firms do not use employment lotteries, however.

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FOOTNOTES

1. See Miller [1983], and the references contained therein.
2. Sargent and Wallace [1981].
3. See Lucas and Stokey [1983], and the references therein.
4. See Fischer [1982], and the references he provides.
5. Lucas and Stokey [1983].
6. The assumptions of retirement when old, and of zero endowment of the consumption good in each period merely permit some economy of notation, and are otherwise inessential.
7. It is not necessary to describe share endowments in detail, as in equilibrium there are no profits to be distributed.
8. The only aspect of economic behavior other than hours worked which might be observed is savings behavior. However, since firms may attempt to sort workers before production occurs, and hence before they have been paid and have acquired a portfolio, the assumption in the text seems natural. It would not qualitatively affect the results if firms could observe portfolio behavior, however.
9. In order to see that such an interpretation is consistent with the analysis that follows, consider a version of the model in which home production can occur when young. Let C_{m1} denote consumption of the market produced good, and C_{h1} denote consumption of the home produced good (when young). Let all agents have identical preferences given by $U_i(C_{m1}, C_{h1}, C_2, L) = C_{m1} + \psi C_{h1} + \rho C_2$. The technology for producing goods at home obeys $C_{h1} = \eta_1 L$, with $\eta_1 > \eta_2$. Also, let L_{im} be the amount of labor devoted to market production, so that $1 - L_{im}$ is the amount of labor devoted to home production. All other aspects of the economy are as described in the text. Then $C_{h1} = \eta_1(1 - L_{im})$, so that agents' preferences can be equivalently expressed (see,

e.g., Ghez and Becker (1975)) as $U_i(C_{m1}, C_2, L) = C_{m1} + \rho C_2 + \psi \eta_i (1 - L_{im}) \equiv \phi_i + C_{m1} + \rho C_2 - \phi_i L_{im}$, with $\phi_1 > \phi_2$. Except for the appearance of the constant terms ϕ_i , these preferences are identical to those of equation (1). Since these constant terms do not affect the analysis, all of the arguments that follow are consistent with the interpretation of the model suggested above.

10. It is natural to ask what formal role the exclusion of assets other than money plays in the analysis that follows. The answer is that this provides considerable simplification without affecting the tenor of the argument. As an example, we could easily amend the analysis to include utility functions in which money was an argument (to proxy for transactions services provided). This would permit the easy incorporation of multiple assets. However, this would add a degree of additional complexity to the analysis without providing any additional insights into the workings of the model. Hence, we proceed with this simplified version of an asset market in what follows.

11. This is a standard assumption in the literature on optimal taxation with private information. See, e.g., Stiglitz [1982].

12. By this it is meant that there is no subsidization of type 2 workers by type 1 workers in any announced contract. This is a common restriction in these settings and is imposed by both Rothschild and Stiglitz [1976] and Wilson [1977].

13. This fact is perhaps not immediately obvious. Its proof, while not difficult, is tedious and is, therefore, left for the appendix.

14. Notice that if workers are free to leave the labor force, as we assume below,

$$(\bar{\pi} - \phi_1) \hat{L} - T(\bar{\pi} \hat{L}) > 0$$

must hold (as $T(0) \leq 0$) if \hat{L} is to be an equilibrium. This places a tighter restriction on $T(\bar{\pi}\hat{L})$ than does (17). Two remarks are therefore in order. First, (17) places an upper bound on tax revenue even if the government can prevent workers from leaving the labor force. Second (15) places a restriction on tax revenues which may (or may not) be tighter than either (17) or the condition above.

15. "Lump-sum taxation" here means that $T(w\hat{L}) \leq w\hat{L}$, and that $T(w\hat{L}) - T(w\hat{L}-\epsilon) = 0$ for all sufficiently small $\epsilon > 0$.

16. If the second condition failed, $\hat{L}_2 = 0$ would result since $T(0) \leq 0$.

17. It is also necessary that $(\pi_1 - \phi_1)\hat{L}_1 - T > 0$ for $\hat{L}_1 \neq 0$ to hold.

18. See Fischer [1982].

19. The fact that there exists a nonempty open set of economies for which our analysis holds implies that there also exists a nonempty set of economies for which θ can be varied locally in a way which satisfies all of the relevant conditions which make inflationary finance both necessary and feasible.

20. For a discussion of "monetary policy" in Maryland in the 18th century, see Behrens [1923].

Figure 1
An El Equilibrium

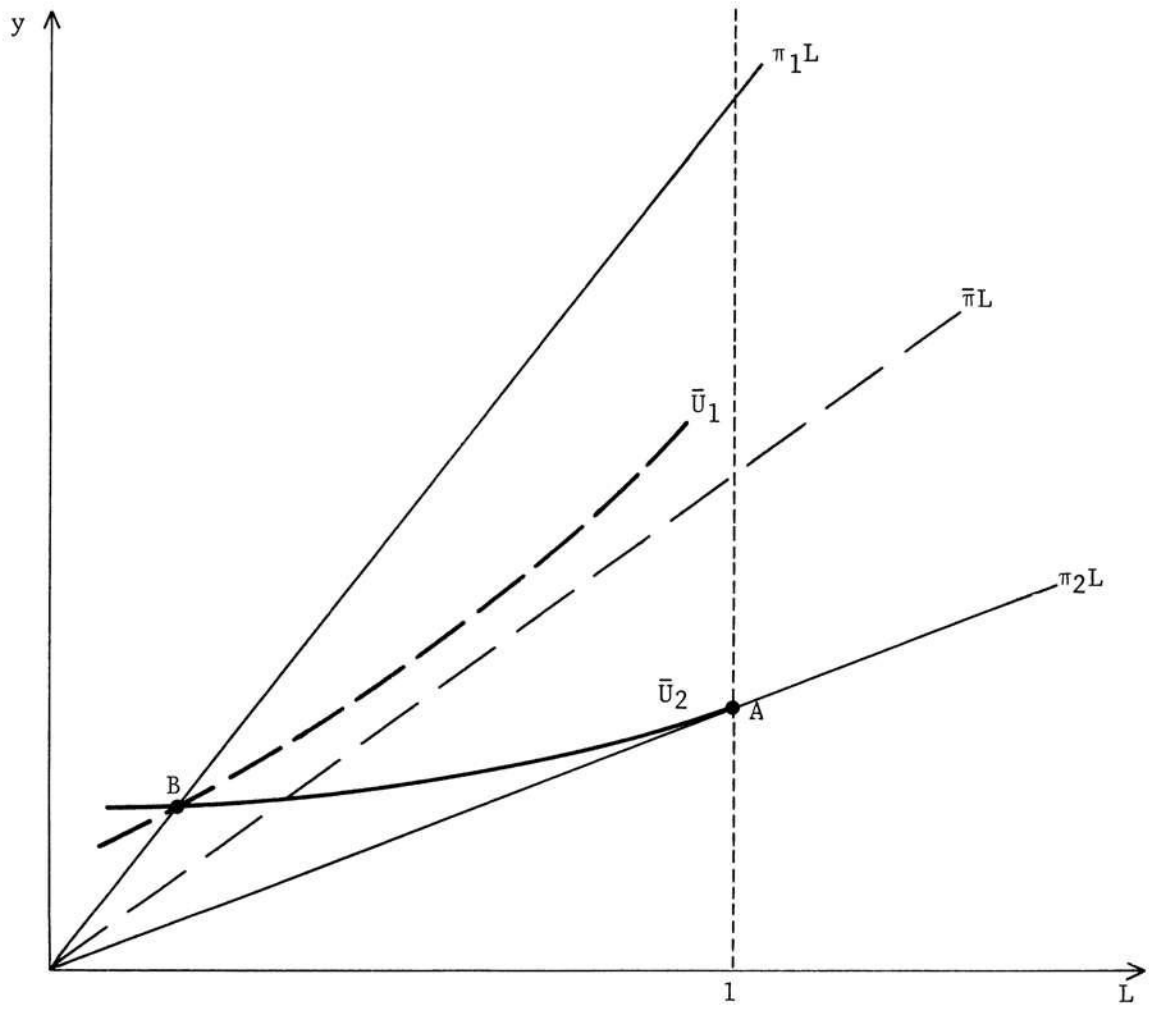


Figure 2
An E2 Equilibrium

