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A MODEL OF "KEYNESIAN" UNEMPLOYMENT
RESULTING FROM ADVERSE SELECTION

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Men are involuntarily unemployed if, in the event of a small rise in the price of wage-goods relatively to the money-wage, both the aggregate supply of labour willing to work for the current money-wage and the aggregate demand for it at that wage would be greater than the existing volume of employment.

-John Maynard Keynes
(1936, p. 15)

Abstract

A model of a labor market is developed in which agents possess private information about their marginal products. As a result, involuntary unemployment may arise as a consequence of attempts by firms to create appropriate self-selection incentives. Moreover, employment lotteries may arise for the same reason despite the fact that, in equilibrium, there is no uncertainty in the model. When employment is random, this is both privately and socially desirable. Finally, it is shown that the unemployment that arises is consistent with (a) pro-cyclical aggregate real wages and productivity, (b) employment that fluctuates (at individual and aggregate levels) much more than real wages.

The traditional notion of (involuntary) unemployment in macroeconomics is the one suggested by Keynes (quoted above). In standard macro theory, this has been taken to mean that the level of employment (of some agents) is determined by firm behavior, with the preferences of (partially) unemployed agents playing no role in hours determination.^{1/} In practice, however, attempts to construct a general equilibrium foundation for this notion of unemployment have been both theoretically and empirically unsuccessful. In particular, it has proven difficult to construct theories in which firms set employment levels consistent with the facts that:

- 1) Average productivity is procyclical.
- 2) At an aggregate level, real wages appear to be procyclical (Prescott, et. al., (1983)).
- 3) Over the cycle, hours vary much more than productivity (Hodrick and Prescott (1981)).

In addition, many have argued that employment and unemployment seem too "weakly correlated" with real wages to be consistent with theories of unemployment offered thus far.

This paper is an attempt to construct a simple general equilibrium model of "Keynesian unemployment" in which hours of certain individuals can fluctuate widely at a constant (equilibrium) real wage rate. Moreover, this fluctuation will be consistent with the procyclical nature of the aggregate real wage, and with the procyclical nature of average productivity. Thus, the paper puts forth an equilibrium model of unemployment in which observed cyclical behavior is not anomalous.

Moreover, it will be argued that the unemployment which arises in this model is "involuntary" according to the standard Keynesian definition.

Other equilibrium models of unemployment cannot make this claim. In particular, there appear to be two other extant models which are capable of endogenously generating unemployment of labor. These are search and implicit contracting models. It is generally conceded that search theoretic settings will not confront the notion of "involuntary" unemployment. It is also typically granted that this notion cannot be confronted in standard implicit contracting models.

The context in which our results are derived is as follows. A model of a labor market is presented in which firms have access to a technology for converting labor into a single produced consumption good. Workers in the model are heterogeneous, varying in their (marginal) productivities in production, and in their preferences over consumption-leisure bundles. However, any worker's productivity is private information, and not directly observable. Then firms compete for workers' services by offering contracts which induce agents of different productivities to self-select by the type of contract accepted.

In order to induce this self-selection, firms might use contracts which result in the unemployment (underemployment) of certain sets of workers. It will be argued that this unemployment is involuntary, in the Keynesian sense. However, taking one step beyond this, it is also shown that firms may wish to offer contracts which consist of lotteries over various employment levels. These lotteries are used to induce self-selection of workers by type of contract selected, and also may result in some subset of workers being unemployed (underemployed) contingent on particular outcomes of the lottery.

It will be noted, then, that firms may offer employment lotteries even though there is no randomness in technology, preferences, endowments, population, government policy, etc. This highlights the fact that an adverse

selection problem in labor markets is, by itself, sufficient to give rise to involuntary unemployment, and to employment fluctuations that are unrelated to any variations of underlying parameters in the model. It will also be seen that this permits the model with adverse selection to confront the apparently anomalous observations listed above.

It is interesting to contrast this result with some other strands of literature. First, it will be noted that this approach gives rise to what Cass and Shell (1983) refer to as "sunspot equilibria," i.e., equilibria in which randomness in equilibrium allocations is not attributable to randomness in parameters of the economy. Second, it is useful to contrast this adverse selection setting with those arising in the literature on implicit contracts (possibly with asymmetric information). In the implicit contracting literature, firms seek to insure workers against fluctuations in parameters of the economy which might be (from the point of view of workers) unobservable (Hart (1983)). In contrast, in the model presented here, firms may wish to introduce (additional) randomness into workers' income streams in order to induce self-selection. Hence, firm motivations here are quite different from those in the implicit contracting literature. Moreover, it will be seen that when nondegenerate employment lotteries are used by firms in equilibrium, these are preferred by the workers affected to the best available certain income stream. Thus, this injection of randomness by firms is not only to each firm's advantage when it occurs, but is socially advantageous as well.

In the course of establishing the results described, two points are made regarding general adverse selection economies. First, it is shown that even when Nash equilibria exist for these economies in pure strategies, there may exist Pareto dominating mixed strategy equilibria. This appears not to have been explored in the context of Nash equilibria in these models.^{2/}

Second, it is shown that some economies with no Nash equilibrium in pure strategies have a Nash equilibrium in mixed strategies. This is of some interest, because mixed strategies do more than serve to provide convexification of Nash reaction correspondences here.

The format of the paper is as follows. Section I sets out the model. Section II defines a Nash equilibrium for it and describes some of its properties. Section III argues that unemployment in this economy is involuntary in Keynes' sense. Section IV describes mixed strategy equilibria, and partially characterizes when such equilibria will arise. It is also shown that there exist economies giving rise to unemployment in which firms use nondegenerate employment lotteries. This fact allows us to explain all of the anomalous observations noted at the outset. Section V indicates limitations on our ability to characterize the generality of the results obtained. Section VI concludes by relating this work to other attempts to generate unemployment as a feature of Nash equilibria in imperfectly competitive labor markets.

I. The Model

A. Description

In this section, a description of the simplest possible economy of interest is provided. This allows a straightforward presentation of arguments which also hold for more general versions of the model.

Consider an economy in which there are a fixed and finite number of firms, N , and a set of workers who are divided into two types, indexed by $i = 1, 2$.^{3/} Firms enjoy exclusive access to a technology for converting labor into a single produced consumption good, and workers are endowed with labor but are without direct access to such a technology.

The technology for converting labor into this consumption good is as follows. One unit of type i labor can be used to produce π_i units of the consumption good. The π_i are constants obeying $\pi_1 > \pi_2$, and the values π_i are assumed to be publicly known.

Let C_i denote consumption by workers of type i , and L_i denote hours worked by type i agents. Then the (common) preferences of type i agents over nonnegative (C_i, L_i) pairs are denoted by $U_i: \mathbb{R}_+^2 \rightarrow \mathbb{R}$, where the properties of U_i are standard:

- (i) $U_i \in C^2$
- (ii) $D_1 U_i(C_i, L_i) > 0$
- (iii) $D_2 U_i(C_i, L_i) < 0$
- (iv) U_i concave,

(where D_j denotes the partial derivative of what follows with respect to its j^{th} argument). All agents have von Neuman-Morgenstern preferences, and each worker is endowed with one unit of time to be divided between labor and leisure, and nothing else.

In order to complete the description of the economic environment, it remains to describe the nature of information, and the strategy space of firms, which call out sets of contracts to be accepted or rejected by workers. For purposes of defining the strategy space, it is assumed that there is a fixed set S of random events, with typical element s and $\#S$ finite. Below, we allow firms to choose the probability of a particular event s occurring. An example of the type of strategy space considered, then, is that the only randomizing devices available in an economy are coins, but that firms can use coins with arbitrary probabilities of the event "heads."

Finally, it remains to describe the nature of information in this economy. It is assumed that a worker's type (marginal product) is not directly observable by firms; i.e., the contribution of any individual to a firm's output cannot be observed. (Each agent knows his own type.) Thus, if firms wish to be able to discriminate between agents of different types, they must do this by inducing type 1 and 2 workers to accept different contracts, which in turn involves having $L_1(s) \neq L_2(s)$ for some $s \in S$. Firms in the model, then, call out contracts offered to type i agents which are merely sequences $\{[w_i(s), L_i(s)]\}_{s \in S}$ of wage-hours pairs offered in event s . Firms also offer to use randomizing devices which result in event s occurring with probability $p(s)$; $p(s) \geq 0 \forall s \in S$, and $\sum_S p(s) = 1$.

N firms, then, call out such contracts. Since all type i agents have the same preferences over contracts, all agents of type i accept the same contract. As production displays constant returns to scale, each firm accepts all comers at its offered contractual terms. There are, then, two possible states of affairs. Either type 1 and 2 workers accept different contracts (self-selection occurs), or not. If self-selection does occur, $L_1(s) \neq L_2(s)$ for some $s \in S$.

Suppose that self-selection does occur. Then agents are distinguishable by type, agents of type i receive wage rate $w_i(s)$ in state s , and firm profits per type i agent employed in state s are

$$[\pi_i - w_i(s)]L_i(s),$$

where the consumption good is chosen as the numeraire. On the other hand, if all workers choose the same contract, agents' types are not distinguishable. Then the marginal product of any agent selecting a contract is (from the point of view of the firm) a random variable drawn from the population distribution

of marginal products. Let θ be the fraction of the population with $i = 1$, and let $\bar{\pi}$ be mean population productivity: $\bar{\pi} \equiv \theta\pi_1 + (1-\theta)\pi_2$. In this case, expected firm profits per capita in state s would be

$$[\bar{\pi} - w(s)]L(s),$$

where $w(s)$ and $L(s)$ are the (common) values of $w_i(s)$ and $L_i(s)$.

Finally, it remains to say something about lotteries. It will be recalled that firms announce use of a randomizing device which assigns probability $p(s)$ to event s . It is assumed that only firms know the randomizing device actually used. Hence, in equilibrium it will be necessary that firms not have an incentive to announce sequences $\{p(s)\}_{s \in S}$ but use a set of lotteries with divergent true probabilities for various events. This, in turn, requires that $w_i(s) = \pi_i \forall s$ if self-selection occurs, and $w(s) = \bar{\pi} \forall s$ if it does not.

B. Self-Selection

We wish to construct an environment in which it is possible (at least in principle) for firms to induce self-selection of workers through the use of pure strategies alone. Therefore, we assume that

$$(v) \quad \forall (C, L) \in [0, \pi_1] \times [0, 1],$$

$$\left. \frac{\partial C}{\partial L} \right|_{dU_1=0} \neq \left. \frac{\partial C}{\partial L} \right|_{dU_2=0}.$$

This assumption implies a correlation between productivities and preferences which can be exploited by firms to induce self-selection. In order to justify the assumption, one might think of a model of home production giving rise to indirect utility functions $U_i(C, L)$.^{4/} These utility functions arise from underlying preferences over goods which are similar, but agents with different

productivities in the workplace also have different productivities at home. Typically, one would think of positive correlation as being the most natural case. Such correlation would entail

$$(v') \left. \frac{\partial C}{\partial L} \right|_{dU_1=0} > \left. \frac{\partial C}{\partial L} \right|_{dU_2=0},$$

but we will consider the general version (v) in what follows.

C. Firm Behavior

The behavior of workers in this economy is quite simple: firms announce contracts and each worker accepts the most preferred contract (or sets $L = 0$ if this the best alternative). However, it is necessary to describe in more detail the game played by firms here.

In order to do this, it is easiest to begin with a version of the game in which firms are restricted to use pure strategies (i.e., $\#S=1$, or there are no randomizing devices available to firms). Then firms compete for the services of workers by calling out contracts $\{(w_i, L_i)\}_{i=1,2}$, which are simply wage-hours pairs offered to each worker. If the announcement $(w_1, L_1) \neq (w_2, L_2)$ for any firm, the firm hopes to induce self-selection. A Nash equilibrium in pure strategies, then, is simply a set of announcements by each firm such that no firm has an incentive to call out a different contract, given the announcements of other firms. It will be noted that this is a simple labor market variant of the adverse selection insurance game of Rothschild and Stiglitz (1976), and we follow them in imposing an additional restriction on announced wage-hours pairs; each pair must earn nonnegative expected profits given the workers accepting it.

The game when mixed strategies are allowed ($\#S>1$) is similar, but now firms merely announce state-contingent wage-hours pairs $\{(w_i(s),$

$L_i(s) \}_{i=1,2; s \in S}$, and probabilities of event s occurring, $\{p(s)\}_{s \in S}$. In order for firms' announcements to be incentive compatible, it is necessary that $w_i(s) = \pi_i \forall s$ if self-selection occurs, and that $w_i(s) = \bar{\pi} \forall s$ otherwise. Again, we follow Rothschild and Stiglitz in imposing that each wage-hours pair $\{[w_i(s), L_i(s)]\}_{s \in S}$ must at least break even given the workers accepting it. A Nash equilibrium is then defined as above.

II. Equilibrium

A. Equilibrium Conditions

As indicated, a normal Nash equilibrium concept has been imposed, so that the game played by firms here is closely related to that played by firms in Rothschild-Stiglitz (1976). The equilibrium concept employed has several consequences, which are detailed here.

First, if self-selection of agents is to occur, type 1 agents must (weakly) prefer the lottery $\{[w_1(s), L_1(s)]\}_{s \in S}$ to $\{[w_2(s), L_2(s)]\}_{s \in S}$, and similarly for type 2 agents. Second, if self-selection does occur, then $[w_2(s), L_2(s)]$ must be maximal for type 2 agents over the set of lotteries consistent with the firms employing them breaking even, and consistent with self-selection. Otherwise, these agents could be bid away in a profitable fashion by other firms. Since (as will be proved below) considerations of self-selection affect only the offers $[w_1(s), L_1(s)]$, $\{[w_2(s), L_2(s)]\}_{s \in S}$ must be maximal for type 2 agents among the set of (w_2, L_2) pairs which at least break even for each s . Thus, given that $U_2(\)$ is concave, type 2 workers are offered degenerate lotteries if self-selection occurs.

Third, if self-selection does occur, type 1 agents must be offered the maximal lottery for them consistent with their employers at least breaking even, and consistent with self-selection. Otherwise, deviant firms could

attract these workers in a profitable manner. Thus, $\{[w_1(s), L_1(s), p(s)]\}_{s \in S}$ will be determined by this consideration.

Finally, if self-selection does not occur, all workers will accept the same contract. As all workers have convex preferences, there is then no reason for randomization (which serves only to induce self-selection). In such an (equilibrium) configuration, it would obviously be necessary to offer type 1 workers the maximal break-even (w, L) pair for them given that all workers accept the same contract. Otherwise, of course, some firm could attract all type 1 workers at the going wage, and thereby earn a profit.

The conditions that an equilibrium must satisfy, then, which we may take as a definition, are as follows.

Definition. A Nash equilibrium is a mapping $[w_1(s), w_2(s), L_1(s), L_2(s), p(s)] : S \rightarrow R_+^4 \times [0, 1]$ satisfying

$$(2.1) \quad \sum_S p(s) U_1[w_1(s)L_1(s), L_1(s)] \geq \sum_S p(s) U_1[w_2(s)L_2(s), L_2(s)]$$

$$(2.2) \quad \sum_S p(s) U_2[w_2(s)L_2(s), L_2(s)] \geq \sum_S p(s) U_2[w_1(s)L_1(s), L_1(s)]$$

$$(2.3) \quad w_i(s) = \pi_i \quad \forall s \in S \text{ if } L_1(s) \neq L_2(s) \text{ for some } s, \quad w_i(s) = \bar{\pi} \quad \forall s \in S \text{ if } L_1(s) \equiv L_2(s)$$

$$(2.4) \quad p(s) \text{ and } L_1(s); s \in S \text{ maximize } \sum_S p(s) U_1[w_1(s)L_1(s), L_1(s)]$$

subject to (2.2), (2.3), and

$$\sum_S p(s) = 1.$$

$$(2.5) \quad \text{if } L_2(s) \neq L_1(s) \text{ for some } s \in S, \text{ then } [C_2(s), L_2(s)] \text{ is maximal for type 2 agents over the set } \{[C_2(s), L_2(s)] : C_2(s) \leq \pi_2 L_2(s)\} \quad \forall s \in S.$$

$$(2.6) \quad \text{There does not exist an alternative mapping}$$

$[\hat{w}_1(s), \hat{w}_2(s), \hat{L}_1(s), \hat{L}_2(s), \hat{p}(s)]$ which

- a) attracts any workers, and
- b) given the workers it attracts, earns nonnegative profits.

Conditions (2.1) and (2.2) are the incentive compatibility restrictions, conditions (2.4) and (2.5) require that the $L_i(s)$ and $p(s)$ have the optimality properties stated above, (2.3) requires zero profits in each state, and (2.6) is the usual Nash equilibrium condition.

Some remarks on this definition are in order. First, for the usual reasons, the provision of employment insurance by third parties has been ruled out here. Second, we have used the fact, proved as Proposition 2 below, that (2.1) never holds with equality in any equilibrium with $L_1(s) \neq L_2(s)$. Third, as indicated above, this equilibrium concept is closely related to that employed by Rothschild and Stiglitz (1976). As alternate equilibrium concepts for the economy at hand have been proposed, this choice of equilibrium concept merits further comment.

In particular, two "problems" arise with this notion of equilibrium: a Nash equilibrium need not exist for a large set of economies in the class at hand, and moreover, even if such an equilibrium exists, it need not be Pareto optimal. The first problem is often resolved through use of the equilibrium concept due to Wilson (1977). This concept could also be adapted for use here. This is not done for the following reason. When a Nash equilibrium does exist, it is also a Wilson equilibrium. The phenomena of macroeconomic interest, such as unemployment and the use of employment lotteries, arise (in any meaningful sense) only in the equilibria which are also Nash equilibria. In short, the Nash equilibrium concept is simpler, and permits us to capture all the phenomena of interest here.

With regard to the second point, arrangements which support Pareto optimal allocations typically make use of side payments between players. Nash game forms which would support an optimum in such circumstances have yet to be designed. However, we could easily adapt our analysis to a study of optimal allocations, which would still display "Keynesian" unemployment and the use of employment lotteries in the allocation of resources. Thus, the discussion here can be taken as a quite general one.

As a final remark on the equilibrium concept employed, it will be noted that firms set employment here, i.e., firms call out wage-hours packages to be accepted or rejected by workers. This is generally in keeping with traditional Keynesian approaches to macroeconomics, and we will have more to say on this point below.

It remains to define some terms which will be employed below. We refer to an equilibrium in which $\#S = 1$ (i.e., in which firms are assumed not to have access to randomizing devices) as a nonstochastic equilibrium. We also refer to equilibria with nondegenerate employment lotteries as stochastic equilibria.

B. Properties of Equilibrium

This section discusses some basic properties of equilibria which will be familiar from Rothschild and Stiglitz (1976), Miyazaki (1977), and Spence (1978). These are that (a) in any equilibrium, self-selection occurs, (b) type 1 agents always strictly prefer their equilibrium allocation to that selected by type 2 agents, and (c) that an equilibrium allocation can be represented as the solution of a simple constrained optimization problem. For the interested reader, proofs of (a) and (b) are provided in this section. The results here, then, are not of direct interest, but are useful preliminary results to what follows.

The first such result is that, in equilibrium, there is no uncertainty regarding agents' types. This is

Proposition 1. Self-selection occurs in any equilibrium.

Proof: Suppose to the contrary that all agents select the same contract. Then, $w_1(s) = w_2(s) = \bar{w} \forall s$, and since preferences are convex, there is no need for firms to employ lotteries. Then let $w = \bar{w}$ and L be the common (non-random) wage and hours values. Since this is an equilibrium, by condition (2.6) there is no pair (w_1, L_1) preferred by type 1 agents and which results in nonnegative profits for some firm. In particular, there is no pair of "small" numbers ϵ and δ such that $U_1(\bar{w} + \epsilon, L + \delta) > U_1(\bar{w}, L)$, and such that firms earn nonnegative profits by offering ϵ additional units of the good in exchange for δ additional units of labor. However, suppose that $\epsilon, \delta > 0$, that

$$\left. \frac{\partial C}{\partial L} \right|_{\substack{U_1 \text{ constant} \\ (C,L) = (\bar{w}L, L)}} < \frac{\epsilon}{\delta} < \left. \frac{\partial C}{\partial L} \right|_{\substack{U_2 \text{ constant} \\ (C,L) = (\bar{w}L, L)}},$$

and that some firm offers ϵ units of additional consumption in exchange for δ units of additional labor. It is readily verified that type 1 workers accept any such offer, and that type 2 workers do not. Since any firm making such an offer attracts only type 1 workers, for any sufficiently small ϵ and δ it must earn positive profits. Thus, there exists an alternate wage-hours pair which results in increases in type 1 utility, and profits for some firm, contradicting the initial hypothesis. An identical argument with $\epsilon, \delta < 0$ applies if

$$\left. \frac{\partial C}{\partial L} \right|_{\substack{U_1 \text{ constant} \\ (C,L) = (\bar{w}L, L)}} > \left. \frac{\partial C}{\partial L} \right|_{\substack{U_2 \text{ constant} \\ (C,L) = (\bar{w}L, L)}}.$$

The resulting contradiction establishes the proposition.

The second result is most easily exposted by considering a non-stochastic equilibrium. By implication, then, it must also be true for a stochastic equilibrium, since firms always have the option of offering degenerate lotteries. This result is

Proposition 2. In a nonstochastic equilibrium, if one exists,

$$U_1(\pi_1 L_1, L_1) > U_1(\pi_2 L_2, L_2).$$

Proof: Let $L_i^*(\pi) = \operatorname{argmax} \{U_i(\pi L, L)\}$. Then obviously $U_1[\pi_1 L_1^*(\pi_1), L_1^*(\pi_1)] > U_1[\pi_2 L_2^*(\pi_2), L_2^*(\pi_2)]$. Since $L_2^*(\pi_2)$ must be the equilibrium value of L_2 (in light of Proposition 1), clearly $U_1(\pi_1 L_1, L_1) = U_1(\pi_2 L_2, L_2)$ in equilibrium only if (2.2) holds with equality. But then suppose an equilibrium could exist with $U_1(\pi_1 L_1, L_1) = U_1(\pi_2 L_2, L_2)$. This (as we have argued) requires that $U_2(\pi_1 L_1, L_1) = U_2(\pi_2 L_2, L_2)$ hold simultaneously. We show that this results in a contradiction, for suppose some firm offers all its employees (regardless of type) the (C, L) pair

$$(\hat{C}, \hat{L}) = [\lambda \pi_1 L_1 + (1-\lambda) \pi_2 L_2, \lambda L_1 + (1-\lambda) L_2]; \lambda \in (0, 1).$$

Then, by choosing

$$\hat{w}_i = \frac{\lambda \pi_1 L_1 + (1-\lambda) \pi_2 L_2}{\lambda L_1 + (1-\lambda) L_2}; \quad i = 1, 2,$$

which is the implicit wage associated with (\hat{C}, \hat{L}) , and letting λ approach zero, we see that

$$\lim_{\lambda \rightarrow 0} \hat{w}_i = \pi_2; \quad i = 1, 2.$$

Therefore, since \hat{w}_i varies continuously with λ , for λ sufficiently small, $\hat{w}_1 = \hat{w}_2$ is arbitrarily near π_2 . But then some firm could offer the

single contract (\hat{w}, \hat{L}) for some λ near zero, and if this attracts all workers, earn positive profits. But this offer will attract all workers, since by concavity of the U_i ,

$$U_i[\lambda\pi L_1 + (1-\lambda)\pi_2 L_2, \lambda L_1 + (1-\lambda)L_2] > U_i(\pi_1 L_1, L_1) = U_i(\pi_2 L_2, L_2).$$

However, this contradicts the assumption that the initial values constituted an equilibrium, and establishes the proposition.

The above propositions are important for two reasons. First, Proposition 1 establishes that, in equilibrium, all uncertainty regarding workers' types is resolved. In conjunction with the fact that there is no randomness affecting underlying parameters of the economy, this implies that nothing in the economic environment is uncertain in equilibrium. This will be useful to keep in mind in what follows.

Second, together Propositions 1 and 2 permit a simple characterization of equilibrium. In particular, self-selection considerations are never a factor in determination of the values $L_2(s)$. Hence, concavity of $U_2(\cdot)$ implies that L_2 will not be random and therefore, in equilibrium, $L_2 = L_2^* \equiv L_2^*(\pi_2) = \operatorname{argmax} \{U_2(\pi_2 L_2, L_2)\}$. (w_2, L_2) must be maximal for type 2 agents among the set of (w_2, L_2) pairs which at least break even, and $w_2 = \pi_2 \cdot$ Therefore, the only aspect of the problem which is influenced by adverse selection problems is determination of the values $\{[w_1(s), L_1(s), p(s)]\}_{s \in S}$. As we know self-selection always occurs, and since $w_1(s) = \pi_1 \cdot s$ in equilibrium, $L_1(s)$ and $p(s)$ must be chosen to maximize expected type 1 utility subject to self-selection being incentive compatible, $w_1(s) = \pi_1 \cdot s$, and an absence of side payments between agents.

Finally, when will an equilibrium not exist here? We know that the best pooling contract can always be dominated by a deviant firm, so that no

pooling equilibrium exists. Similarly, if the best arrangement consistent with self-selection can be dominated by a deviant firm, no equilibrium will exist. But obviously, taking the actions of other firms as given, the "best" contract consistent with self-selection can be dominated only by a pooling contract (one which attracts all workers). Thus, a Nash equilibrium must exist if we can compute the contracts which are consistent with self-selection (i.e., which leave no rent opportunities to any firms offering contracts which induce self-selection), which break even individually (in keeping with the Rothschild-Stiglitz (1976) convention), and if there are no pooling contracts which attract type 1 (all) agents.

In light of these arguments, it is clear that an equilibrium has $L_2 = L_2^*$, $w_i = \pi_i$; $i = 1, 2$, and values $\{[L_1(s), p(s)]\}_{s \in S}$ which solve^{5/}

$$(2.7) \quad \max_{\substack{L_1(s) \in [0,1] \\ p(s) \in [0,1]}} \sum_S p(s) U_1[\pi_1 L_1(s), L_1(s)]$$

subject to

$$(2.8) \quad EU_2[\pi_1 L_1(s), L_1(s)] \leq U_2[\pi_2 L_2^*, L_2^*]$$

$$(2.9) \quad EU_1[\pi_1 L_1(s), L_1(s)] \geq U_1[\pi_2 L_2^*, L_2^*]$$

$$(2.10) \quad EU_1[\pi_1 L_1(s), L_1(s)] \geq [\max_{\{L_1\}} U_1(\bar{\pi} L_1, L_1)]$$

$$(2.11) \quad \sum_S p(s) = 1.$$

If this problem has no solution, then no Nash equilibrium exists. Similarly, a nonstochastic equilibrium has $L_2 = L_2^*$, $w_i = \pi_i$, $i = 1, 2$, and L_1 a solution to

$$(2.12) \quad \max_{0 \leq L_1 \leq 1} U_1(\pi_1 L_1, L_1) \text{ subject to}$$

$$(2.13) \quad U_2(\pi_1 L_1, J_1) \leq U_2(\pi_2 L_2^*, J_2^*)$$

$$(2.14) \quad U_1(\pi_1 L_1, L_1) \geq \max_{\{L_1\}} U_1(\bar{\pi} L_1, L_1)$$

$$(2.15) \quad U_1(\pi_2 L_2^*, L_2^*) \leq U_1(\pi_1 L_1, L_1).$$

Given the association of equilibria with solutions to these constrained optimization problems, it is now easy to see that nonexistence of equilibrium is not due, in any conventional sense, to nonconvexities associated with incentive compatibility constraints. In particular, finding equilibrium values is equivalent to maximizing a continuous function over a compact set. Thus, nonexistence will be due to emptiness of the sets defined by (2.8)-(2.11) or (2.13)-(2.15). This will be useful to recall when we show below that some economies have stochastic, but no nonstochastic equilibria. The use of mixed strategies, then, clearly does more than just permit a convexification of Nash reaction correspondences.

III. An Equilibrium With "Keynesian" Unemployment

The main objective of the model of Section I, and of providing the equilibrium notion of Section II is to produce an environment which gives rise to unemployment which is involuntary according to Keynes' definition. In this section, we demonstrate that the economy at hand is capable of giving rise to such unemployment. Since for the purposes of this section it is sufficient to restrict consideration to nonstochastic equilibria, we focus here only on this case.

A. The Nature of Quantity Constraints

To begin, we may define what is meant by unemployment (underemployment) of labor. Let $L_i^* \equiv L_i^*(\pi_i) \equiv \operatorname{argmax}\{U_i(\pi_i L_i, L_i)\}$. Then L_i^* is the "notional supply of labor" of type i agents at the equilibrium real wage, and we will employ the following terminology.

Definition. An unemployment equilibrium is an equilibrium in which $L_i(s) < L_i^*$ for some $i = 1, 2$, for some $s \in S$ which occurs with positive probability.

Analogously, a nonstochastic unemployment equilibrium is one in which $L_i < L_i^*$ for some i .

An equilibrium with unemployment of labor is depicted in Figure 1. In this figure, the loci labelled \bar{U}_i are type i indifference curves, and the rays $C = \pi_i L$ are the zero profit loci. Point A is the equilibrium position for type 2 agents, which is merely the optimal (C, L) pair for these agents among those for which their employers "break even." Point B is the equilibrium position for type 1 agents. It is the maximal (C, L) pair for them among the set of pairs that break even for their employers, and that are not preferred to point A by type 2 agents (i.e., which are consistent with self-selection). It will be noted that the equilibrium value of L_1 is less than L_1^* , which indicates that this is an equilibrium with unemployment, as claimed.^{6/}

Several points should be noted about this equilibrium. First, (so long as parameters are such that self-selection constraints bind) the hours worked by type 1 agents, who are unemployed, are determined only by the intersection of \bar{U}_2 with the zero profit locus $C = \pi_1 L$. Thus, preferences of unemployed agents play no role in determination of their hours. (Moreover, this is true despite the fact that real wages here are at their competitive equilibrium levels.)

Second, it will be noted that it is workers with high marginal products who are unemployed here. This is a feature of any nonstochastic equilibrium with unemployment, and occurs because only the hours of type 1 workers need be constrained in order to induce self-selection. It may seem that this implication of the model is counterfactual. However, two remarks should be made at this point. First, this is an artifact of considering only pure strategies in a static setting. Smith (1983) shows in dynamic settings with firms using lotteries in equilibrium that economies otherwise identical to those discussed here can give rise to unemployment among both sets of workers. Thus, too much should not be made of this feature of the model.

Even so, however, the arrangement depicted in Figure 1 is not entirely at variance with postwar U.S. experience. In order to see this, consider the breakdown of data on average hours and average hourly earnings provided by the Economic Report of the President (1981, p. 273). Excluding agricultural workers and workers in wholesale and retail trades, we are left with production workers divided into two groups; manufacturing and construction. Workers in manufacturing typically earn 75 to 80 percent of the average hourly earnings of construction workers. In addition, the largest value for average weekly hours in construction in the postwar period is 38.9. The smallest such value in manufacturing is 39.1. Thus, this characteristic of an unemployment equilibrium, that workers with high wages work relatively few hours, is broadly consistent with U.S. data.

It is the case, then, that very simple versions of the model presented here deliver implications which are consistent with actual experience. We now turn to an examination of whether the unemployment which arises here is "Keynesian" in nature.

B. Is This Unemployment Involuntary?

According to Keynes (p. 289), there is involuntary underemployment of labor when "there are men unemployed [underemployed] who would be willing to work [more] at less than the existing real wage." A simple inspection of Figure 1 makes evident that at a real wage equal to $\pi_1 - \epsilon$, (ϵ "small"), type 1 workers would be willing to work more than they do. Thus, this Keynesian criterion is met by unemployed workers. We now show that unemployment in the model is consistent with Keynes' definition based on aggregates presented at the beginning of the paper.

In order to do this, it must be shown that an appropriately constructed reduction in the aggregate real wage will increase total hours worked, and will result in notional labor supply being greater than the initial level of employment. We now show that such a construction is possible.^{7/}

It will be noted (from Figure 1) that if $L_1 < L_1^*$, then L_1 is determined by the intersection of the indifference curve \bar{U}_2 with the zero profit locus $C = \pi_1 L$. Thus, in such a situation, L_1 solves

$$(3.1) \quad U_2[(\pi_1 - \epsilon)L_1, L_1] = U_2(\pi_2 L_2^*, L_2^*),$$

evaluated at $\epsilon = 0$, where the appearance of ϵ allows us to perform the thought experiment of reducing the real wage of type 1 agents.

Now consider the effect of increasing ϵ slightly. Clearly (again from Figure 1) this will not relax the self-selection constraint. Then the change in L_1 resulting from varying ϵ is given by (notice from the figure that since π_2 is unchanged, L_2 is unchanged)

$$(3.2) \quad \left. \frac{\partial L_1}{\partial \epsilon} \right|_{\epsilon=0} = \frac{L_1 D_1 U_2}{\pi_1 D_1 U_2 + D_2 U_2} > 0,$$

where the sign of this derivative is unambiguous since (again see the figure) $L_1 < \text{argmax}\{U_2(\pi_1 L, L)\}$. Thus, since L_2 is unchanged, an increase in ϵ raises total employment. Moreover, since self-selection constraints still bind after the change in w_1 , it is the case that the notional labor supply of type 1 agents still exceeds hours worked. Thus, this reduction in one of the real wages has the desired effect on total employment.

In order to complete the verification that our model is consistent with Keynes' definition, we must check that this rise in ϵ reduces the average per capita (i.e., aggregate) real wage. To this end, notice that the aggregate (per capita) real wage is given by

$$(3.3) \quad \bar{w} \equiv \left[\frac{\theta L_1}{\theta L_1 + (1-\theta)L_2} \right] (\pi_1 - \epsilon) + \left[\frac{(1-\theta)L_2}{\theta L_1 + (1-\theta)L_2} \right] \pi_2.$$

Therefore,

$$(3.4) \quad \left. \frac{\partial \bar{w}}{\partial \epsilon} \right|_{\epsilon=0} = \frac{-\theta L_1}{\theta L_1 + (1-\theta)L_2} + \left\{ \frac{(\pi_1 - \theta \pi_2)(1-\theta)L_2}{[\theta L_1 + (1-\theta)L_2]^2} \right\} \left. \frac{\partial L_1}{\partial \epsilon} \right|_{\epsilon=0}.$$

The sign of this derivative is generally ambiguous. However, if the effect of reducing w_1 on L_1 is not too large (e.g., if \bar{U}_2 is relatively steep at point B in Figure 1), an increase in ϵ will produce the desired effect on both total employment, and the aggregate real wage rate. Thus, the model presented here can be constructed in a way which is consistent with the Keynesian notion of unemployment.^{8/}

IV. The Desirability of Random Employment

We have now produced a model which gives rise to Keynesian unemployment. Moreover, in this model firms do not simply hire off of a downward sloping demand schedule for labor. It remains to show, however, that this model is

- (a) consistent with variations in employment that appear to have little to do with variations in individual real wages.
- (b) consistent with observed procyclical aggregate real wage and productivity movements.
- (c) consistent with hours varying more than average productivity.

In this section, we show that our model is consistent with each of these observations. This is in spite of the fact that there is no underlying parameter variation in the model, so that our explanation will arise out of the fact that, under a wide set of circumstances, firms will use employment lotteries in equilibrium.

The format of this section is as follows. We first provide sufficient conditions for firms to use lotteries in equilibrium. It is then shown that these conditions are nonvacuous, i.e., an economy is produced for which they are satisfied. It is then argued that the use of such lotteries permits our model to account for the three facts mentioned above. Finally, we show that employment lotteries can play a formal role in permitting existence of a Nash equilibrium.

A. Equilibrium Employment Lotteries

There are, in fact, a wide class of cases which give rise to the use of employment lotteries in equilibrium. To see this, we begin with the following proposition.

Proposition 3. Suppose type 1 preferences take the form $U_1(C,L) = C - \phi_1 L$, while $U_2(\)$ is strictly concave. Then (if a nonstochastic equilibrium exists and) if $L_1^* \neq L_1 > 0$ in a nonstochastic equilibrium, employment lotteries will arise if $\#S \geq 2$.

Proof. The conditions defining an equilibrium imply that all workers must be offered the contracts maximal for them which are consistent with self-selection, and with their employers at least breaking even. This implies that $L_2(s) = L_2^* \forall s \in S$. Moreover, having self-selection constraints bind in determination of L_1 ($L_1 \neq L_1^*$) implies that L_1 obey $U_2(\pi_1 L_1, L_1) = U_2^* \equiv U_2(\pi_2 L_2^*, L_2^*)$.

Now, given that a nonstochastic equilibrium exists, there exists a feasible, incentive compatible employment lottery $\{[\tilde{p}(s), \tilde{L}_1(s)]\}_{s \in S}$ with $\sum \tilde{p}(s) \tilde{L}_1(s) = L_1$, since by strict concavity of $U_2(\cdot)$,

$$(4.1) \quad \sum \tilde{p}(s) U_2[\pi_1 \tilde{L}_1(s), \tilde{L}_1(s)] < U_2[\pi_1 \sum \tilde{p}(s) \tilde{L}_1(s), \sum \tilde{p}(s) \tilde{L}_1(s)] = U_2^*.$$

Therefore, there exists some alternate lottery $\{[\hat{p}(s), \hat{L}_1(s)]\}_{s \in S}$ satisfying

$$(4.2) \quad \sum \hat{p}(s) U_2[\pi_1 \hat{L}_1(s), \hat{L}_1(s)] = U_2^*.$$

Finally, it is easy to verify that (if a nonstochastic equilibrium exists, and if $L_1 \neq L_1^*$) $D_L U_2(\pi_1 L_1, L_1) > 0$ holds. Therefore, since $U_2(\cdot)$ is strictly concave, (4.2) requires that $\sum \hat{p}(s) \hat{L}_1(s) > L_1$. But then

$$\sum \hat{p}(s) U_1[\pi_1 \hat{L}_1(s), \hat{L}_1(s)] = (\pi_1 - \phi_1) \sum \hat{p}(s) \hat{L}_1(s) > (\pi_1 - \phi_1) L_1 = U_1(\pi_1 L_1, L_1).$$

Therefore, if $\#S \geq 2$, there exists an incentive compatible employment lottery which type 1 agents prefer to the best incentive compatible certain employment contract which at least breaks even for their employers. Hence, any equilibrium with $\#S \geq 2$ will display lotteries. (Moreover, this equilibrium will exist since a nonstochastic equilibrium exists by assumption.)

It remains to show, then, that Proposition 3 is nonvacuous, i.e., that there exist economies for which it holds. This is now demonstrated by presenting an example with the following features. The economy has preferences as specified, and has a nonstochastic unemployment equilibrium. It is then demonstrated that randomization of employment over two states of nature

results in strict Pareto improvements. Thus, by implication, if $\#S > 1$ any equilibrium must display nontrivial employment variation across states of nature.^{9/}

Example 1. Preferences are given by $U_1(C,L) = qC - L$ and $U_2(C,L) = \phi C - (1/2)(L+1)^2$, and parameter values are $\phi = 2$, $q = 2/3$, $\pi_1 = 3$, $\pi_2 = 1$, and $\theta = 1/4$. In equilibrium, $L_2^* = \operatorname{argmax}\{U_2(\pi_2 L, L)\} = 1$. Also, if self-selection constraints did not bind in determination of L_1 , L_1 would solve

$$\max_{0 \leq L_1 \leq 1} (q\pi_1 - 1)L_1,$$

which results in $L_1 = 1$. But clearly since $w_1 > w_2$, having $L_1 = L_2$ cannot be consistent with self-selection. Thus, incentive compatibility constraints do bind in determination of L_1 . Therefore, the nonstochastic equilibrium value of L_1 must satisfy

$$(4.3) \quad \phi\pi_2 L_2^* - (1/2)(L_2^*+1)^2 = \phi\pi_1 L_1 - (1/2)(L_1+1)^2,$$

which (since $L_2^* = 1$, and $L_1 \leq 1$ must hold for feasibility) results in $L_1 = \frac{1}{10}$. Then $U_1 = (q\pi_1 - 1)(\frac{1}{10}) = \frac{1}{10}$.

This (candidate) equilibrium actually exists iff this allocation dominates the best pooling arrangement for type 1 agents. The best pooling arrangement (which need not involve lotteries) has $w = \bar{\pi}$, and has $L_1 = L_2 = L$ solving

$$\max_{0 \leq L \leq 1} (q\bar{\pi} - 1)L,$$

which again results in $L = 1$.^{10/} However, since $\bar{\pi} = 3/2$, this results in $U_1 = 0$. Therefore, type 1 agents cannot be lured away from their employers in a profitable fashion, so that $L_2 = 1$, $L_1 = \frac{1}{10}$ is an equilibrium as claimed.

If $\#S > 1$, however, this is not the allocation which will result. In order to show this, we demonstrate that a lottery exists which Pareto dominates the nonstochastic equilibrium allocation. In particular, consider a two-state lottery which offers $L_1(1) = 1$ with probability $p(1) = .105$, and $L_1(2) = 0$ with probability $p(2) = .895$. This is incentive compatible, since if type 2 agents selected the lottery they would derive expected utility

$$EU_2 = (.105)(\phi\pi_1 - 2) - (.895)(1/2) = -.03 < U_2(\pi_2 L_2^*, L_2^*) = 0.$$

It also provides expected utility $EU_1 = (.105)(q\pi_1 - 1) = .105$ to type 1 agents, so that any firm offering this lottery would attract all type 1 agents from firms offering the best nonrandom contract. It would also not attract any type 2 agents. This indicates that an equilibrium for this economy must involve employment lotteries.

Unfortunately, it does not appear possible to provide a useful general characterization of when employment lotteries will arise in equilibrium. This is discussed in greater detail below. However, a wide variety of examples can be constructed which result in the use of lotteries in equilibrium.

It is important to note that in the context of example 1, and in general when lotteries arise in this setting, firms are injecting randomness into the income streams of (some subset of) workers. This is in sharp contrast to the implicit contracts literature, where unemployment arises as a result of attempts by firms to insure workers against fluctuations in labor income. In addition, it will be noted that this injection of randomness into workers' income streams results in both private and social welfare improvement. In particular, unemployed agents in the model benefit from the presence of randomness regarding their employment and income levels.

It is the case, of course, that our model can now be used to confront several of the observations listed in the introduction. First, clearly in any equilibrium with lotteries average productivity will be procyclical. To see this, notice that L_2 does vary across states of nature. Thus, high employment states are ones where L_1 is relatively high. Hence, in such states (which are also high output states), hours worked by type 1 workers constitute a relatively large proportion of total hours. Therefore, average productivity is procyclical. For the same reason the aggregate real wage will be procyclical (since in this model the aggregate real wage is simply productivity per hour).

Third, we would like to confront the observation that there are people whose hours change (either rise or fall) while equilibrium wages (for the group in question) remain constant. Clearly when lotteries are used to resolve adverse selection problems, there are agents whose hours vary without changes in their (equilibrium) real wage. Hence, this observation is not anomalous in this setting.

Finally, there is the observation (Hodrick-Prescott (1981)) that hours vary more than (average) productivity. Average productivity here coincides with the average per capita real wage:

$$(4.4) \quad \bar{w} = \left[\frac{\theta L_1}{\theta L_1 + (1-\theta)L_2} \right] \pi_1 + \left[\frac{(1-\theta)L_2}{\theta L_1 + (1-\theta)L_2} \right] \pi_2.$$

The local variation of this measure as hours of type 1 agents change is given by

$$(4.5) \quad \frac{\partial \bar{w}}{\partial L_1} = \frac{(\pi_1 - \theta \pi_2)(1-\theta)L_2}{[\theta L_1 + (1-\theta)L_2]^2}.$$

Whether the variation in average per capita productivity induced by changes in L_1 is "large" or "small," then, depends on parameter values. However, for

certain sets of parameter values (for instance, which make π_1 small relative to $\theta\pi_2$), large changes in L_1 can induce only relatively small changes in \bar{w} . Hence, for such parameter values, it will be case that hours vary more than per capita productivity. Thus, the model developed is consistent with the observation that hours vary more widely than productivity per hour.^{11,12/}

Notice that we are able to confront these observations using a model in which the production technology is extremely simple, and from which all underlying parameter variation is scrupulously excluded. These simplifications are meant to underline the fact that nothing more than an adverse selection problem in labor markets is necessary to give rise to unemployment. Moreover, unemployment results from this adverse selection problem in a way that is consistent with a number of observations which are anomalous in the context of other static models of unemployment.

B. Lotteries and Existence of Equilibrium

Virtually all adverse selection economies that have received attention in the literature^{13/} are analogous to versions of our economy in which preferences take the form $U_1(C,L) = U(C) + \phi_1 V(L)$, with $\phi_1 > \phi_2$. As can readily be demonstrated, this implies that any equilibrium will not involve lotteries. Hence lotteries in the context of these adverse selection models have been little studied.^{14/} Therefore, in this section we establish that there exist economies for which employment lotteries are necessary to the existence of a Nash equilibrium. As again there appears to be little to be said about the generality of the result, this possibility is established by example.

The line of argument in the example is to show that a particular economy has no nonstochastic equilibrium. It is then demonstrated that there exists an arrangement with random employment which type 1 agents prefer to any

pooling arrangement. This will imply that there exists a stochastic equilibrium for the economy.

Example 2. Preferences and parameters are the same as for Example 1, with the single exception that we now set $\theta = .3285$. Consider first the candidate values for a nonstochastic equilibrium (i.e., impose $\#S = 1$). As in Example 1, self-selection constraints bind, so that L_1 must satisfy the restriction (4.3) with $L_2^* = 1$ as before. As noted in Example 1, the solution value for L_1 which is feasible (satisfies $L_1 \in [0,1]$) is $L_1 = \frac{1}{10}$. Then, as before, $U_1 = \frac{1}{10}$ under this allocation.

However, no equilibrium exists in pure strategies. To see this, notice that if all firms offered the contracts $(w_2, L_2) = (1,1)$, $(w_1, L_1) = (3, \frac{1}{10})$, a deviant firm could offer the contract $(w, L) = (\bar{\pi}, 1) = (1.657, 1)$, which all workers prefer to their existing contracts. (In particular, this results in $U_1 = (2/3)(1.657) - 1 = .1047$.) Thus, there is no equilibrium sorting contract, and we know from Proposition 1 that there is no equilibrium pooling contract. Hence, no nonstochastic equilibrium exists.

Notice that $(w, L) = (\bar{\pi}, 1)$ is the best contract for type 1 agents consistent with $w \leq \bar{\pi}$. Now consider a two-state lottery which offers $L_1(1) = 1$ with probability $p(1) = .105$, and $L_1(2) = 0$ with probability $p(2) = .895$. As in Example 1, this lottery is incentive compatible. It also results in expected utility for type 1 agents

$$E U_1 = (.105)[(2/3)3 - 1] = .105 > .1047 = \max_{0 \leq L \leq 1} U_1(\bar{\pi}L, L).$$

Therefore, the suggested lottery is preferred by type 1 agents to any pooling contract that at least breaks even. Hence, the optimal lottery cannot be improved upon (from the point of view of these agents) by any firm, so that a nondegenerate stochastic equilibrium exists for this economy.

It may not be clear from the example how lotteries are used in order to produce existence of equilibrium. The standard reason for nonexistence of a Nash equilibrium is that Nash "reaction correspondences" are not convex-valued. However, for our example, a nonstochastic equilibrium (if one exists), solves the maximization problem (2.12)-(2.15). Since this amounts to the maximization of a continuous function over a compact set, a solution (equilibrium) exists unless (2.13)-(2.15) define an empty set of L_1 values. In fact, for the economy of Example 2, (2.13) and (2.14) define an empty set of L_1 values.

The usual role for lotteries in supporting an equilibrium, then, would be to "convexify" reaction correspondences. Here, however, such convexification is not achieved. Rather, existence is obtained in the following way. For the economy of the example, type 1 agents are risk neutral, and type 2 agents are risk averse. Moreover, the expected utility of type 1 agents increases with $\sum p(s)L_1(s)$. Therefore, if this quantity can be increased while the values $L_1(s)$ are chosen "far enough apart," $EU_1[\pi_1 L_1(s), L_1(s)]$ can be increased without violating the self-selection constraint. If $\sum p(s)L_1(s)$ is increased sufficiently, (2.8)-(2.10) will define a nonempty set of values $L_1(s)$ and $p(s)$. For the parameters of the example, such choices are possible when $\#S = 2$, so randomization of employment permits an equilibrium to exist. Hence, for the set of economies at hand, lotteries play a formal role both in improving resource allocations, and in permitting existence of equilibrium.

V. When Will Lotteries Occur?

Obviously, it would be desirable to provide a fairly general characterization of when lotteries will arise in equilibrium. This characterization can be broken down into a series of three questions:

- (a) When will incentive compatibility constraints bind in the choice of contracts?
- (b) When will randomization be desirable (given that these constraints bind) in choice of contracts which induce self-selection?
- (c) When will such an equilibrium exist?

Unfortunately, it does not seem possible to provide economically meaningful general characterizations of answers to these questions. To illustrate this, in this section we completely characterize when lotteries will occur for one fairly simple class of economic environments. This characterization will serve to indicate that no economically meaningful restrictions are placed on underlying parameters of the economy.

To this end, then, we focus on the class of economies with preferences given by $U_1(C,L) = C - \phi_1 L$, $U_2(C,L) = C - (\phi_2/2)(L+1)^2$, and with $\pi_1 > \phi_1$, $\phi_2 > 0$. The reason for focusing on this class of environments is that, if self-selection constraints bind, lotteries will always arise in any equilibrium (from Proposition 3). We now proceed to answer the three questions listed above for this set of economies.

First, when will self-selection constraints bind in equilibrium? There are three cases to consider.

Case 1: $L_2^* = 0$. This requires $\pi_2/\phi_2 \leq 1$. Then, incentive compatibility constraints bind (since $L_1^* = 1$) if

$$\pi_1 - 2\phi_2 > -\phi_2/2,$$

or equivalently, if

$$(5.1) \quad \pi_1 > (3/2) \phi_2.$$

Case 2: $L_2^* = 1$. This requires $\pi_2/\phi_2 \geq 2$. Then self-selection constraints always bind since $L_1^* = 1$, and $w_1 > w_2$.

Case 3: $L_2^* \in (0,1)$. Then $L_2^* = (\pi_2/\phi_2) - 1$, with $\pi_2/\phi_2 \in (1,2)$. Then self-selection constraints bind when

$$(5.2) \quad 2\phi_2\pi_1 > (\pi_2 - \phi_2)^2 + 3\phi_2^2.$$

Second, when will lotteries occur? Since this class of economies falls within the class covered by Proposition 3, we know that they will occur whenever an equilibrium exists with $L_1(s) \neq 0$. This makes the analysis for this class of economies particularly simple, as no additional parameter restrictions are required to prove the desirability of lotteries.

Third, when will an equilibrium exist?^{15/} In order to simplify our answer to this question, let us restrict the strategies we allow firms to employ. In particular, we will allow firms to offer each type of agent either a single, nonrandom value $L_1 \in [0,1]$, or a lottery with either $L_1(s) = 1$, or $L_1(s) = 0 \forall s \in S$. Then let p denote the probability that $L_1(s) = 1$. With this convention, the candidate equilibrium value for p is determined as follows.^{16/}

Case 1: $L_2^* = 0$. Then the self-selection condition can be manipulated to obtain (if (5.1) holds) $p = 0$. But then an equilibrium exists only if type 1 agents would not wish to work at the wage rate $w = \bar{\pi}$, i.e., existence requires

$$(5.3) \quad \bar{\pi} - \phi_1 \leq 0.$$

If (5.3) holds, an equilibrium exists with no lotteries.

Case 2: $L_2^* = 1$. Then self-selection constraints bind and can be manipulated to obtain

$$(5.4) \quad p = \frac{\pi_2 - (3/2)\phi_2}{\pi_1 - (3/2)\phi_2}.$$

Then existence of an equilibrium requires

$$(5.5) \quad (\pi_1 - \phi_1) \left[\frac{\pi_2 - (3/2)\phi_2}{\pi_1 - (3/2)\phi_2} \right] > \bar{\pi} - \phi_1.$$

Case 3: $L_2^* \in (0,1)$. Then if (5.2) holds, we can solve for

$$(5.6) \quad p = \frac{(\pi_2 - \phi_2)^2}{2\phi_2[\pi_1 - (3/2)\phi_2]},$$

and existence requires

$$(5.7) \quad \frac{(\pi_1 - \phi_1)(\pi_2 - \phi_2)^2}{2\phi_2[\pi_1 - (3/2)\phi_2]} > \bar{\pi} - \phi_1.$$

We can now say when lotteries will arise in equilibrium (i.e., when lotteries occur and an equilibrium exists). These will occur if $1 < \pi_2/\phi_2 \leq 2$, and (5.2) and (5.7) hold, or if $\pi_2/\phi_2 > 2$, and (5.5) holds. Clearly, these constitute messy nonlinear restrictions across parameters, and are devoid of meaningful economic interpretation. Moreover, this is the case despite the simplicity of preferences here, and the simplification provided by requiring $L_1(s) = 1$ or $L_1(s) = 0$ in any nondegenerate lottery. Thus, the analysis serves to illustrate that further search for economically relevant restrictions which imply equilibrium lotteries is unlikely to be fruitful.

VI. Conclusions

A model has been presented which can give rise to Keynesian unemployment. Moreover, such unemployment arises in a way that is consistent with several features of observed unemployment that are anomalous in other "Keynesian models." It remains, then, to discuss the way in which the analysis here relates to other attempts to produce Keynesian unemployment in a general equilibrium setting, and to discuss some "problems" with the model that can be resolved by extending it.

With respect to the first point, we have modelled firms as calling out wage-employment contracts in a labor market in which they are imperfect (Nash) competitors. This is consistent with other recent approaches (e.g., Hart (1982)) that attempt to derive Nash equilibria with unemployment from economies with imperfectly competitive labor markets. However, here it was not necessary to impose exogenously the presence of certain parties in labor markets, such as unions (as in Hart (1982)), or anything other than the usual Nash conjectures about behavior in order to obtain unemployment equilibria.

With respect to the second point, perhaps the most troubling feature of the current analysis is that since $L_2 = L_2^*$, type 2 agents are never off of their notional supply curves of labor. However, this feature of the model is an artifact of its static nature. It is shown in Smith (1983) that dynamic versions of the model that allow for the presence of asset markets give rise to the possibility that $L_2 \neq L_2^*(s)$ in some states of nature. Thus, extension of the model to a dynamic context can resolve its most obvious shortcoming, and moreover such an extension permits derivation of a Phillips Curve with the proper slope from a model with the same basic features as the one treated here.

Thus, it is the case that very simple economies with no underlying randomness of parameters, and with no friction other than an adverse selection problem, can explain Keynesian unemployment and give rise to models with employment fluctuation. Moreover, even very simple versions of such economies do quite well in confronting a wide range of observations. This suggests that models such as the one presented here deserve to be taken seriously as models of macroeconomic phenomena.

Footnotes

1/Subject, of course, to their being in the labor force voluntarily.

2/Although it is shown by Cooper (1982) for the case of a monopolist, and by Stiglitz (1982) for the case of a government (Stackelberg leader) levying taxes.

3/It is not important to the analysis whether there are finite or countably infinite numbers of workers, or whether there is a continuum of workers. For some purposes, however, it is probably most natural to think of there being a measure space of workers. The discussion in the text could be easily adapted to accommodate such an interpretation.

4/I would like to thank Ron Michener for suggesting this interpretation.

5/The fact that equilibrium values for this economy can be associated with constrained optimization problems of this form is discussed in greater detail by Miyazaki (1977) and Spence (1978).

6/Existence of this equilibrium can be guaranteed here by an appropriate choice of θ .

7/Although, as will be clear from the argument, not all constructed declines in real wages will produce the desired result.

8/For the thought experiment just performed to be correct, it is necessary that an equilibrium continue to exist when type 1 productivity is reduced slightly. This can be guaranteed in Figure 1 by an appropriate choice of θ .

9/Moreover, such equilibria exist since they result in lotteries which are preferred by type 1 agents to the nonstochastic equilibrium allocation, and hence to the best allocation for type 1 agents under pooling.

10/Type 1 agents are indifferent over all values $L \in [0,1]$. However, type 2 agents prefer $L = 1$ to other values, so we have selected this.

11/Notice that equation (4.4) assumes a single firm economy. This is merely to simplify exposition.

12/Of course, nothing guarantees that changes in L_1 induced by equilibrium lotteries will be "small." Hence (4.5) should be viewed as a first order approximation to the changes in \bar{w} induced by lotteries.

13/Rothschild-Stiglitz (1976), Miyazaki (1977), Spence (1978), Wilson (1977), Prescott and Townsend (1981).

14/Although see Cooper (1982) and Stiglitz (1982) who consider contexts in which Nash equilibrium notions are not employed.

15/If a Wilson (1977) equilibrium concept were employed, an equilibrium would always exist. Under this alternate concept, we would rephrase this question to be when would a separating equilibrium exist. Since lotteries would not be required under a pooling arrangement (as they are not needed to induce sorting), the answer to this question is the same as the answer to the question in the text.

16/Notice that there will still always be lotteries in any equilibrium with $L_2^* > 0$ and in which self-selection conditions matter. To see this, note that setting $p = L_1 \in [0,1]$ (the nonstochastic solution to the self-selection constraint) results in $E U_1 = (\pi_1 - \phi_1)p = (\pi_1 - \phi_1)L_1 = U_1$ under the nonrandom allocation, and this relaxes the incentive compatibility constraint.

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