LIMITED ENFORCEABILITY AND
INTERNATIONAL DEFAULT

Patrick J. Kehoe*

.

*Graduate student, Harvard University.

Introduction

Recently creditors of Latin American debt have voiced fears of a repeat of the synchronized defaults which occurred in the recessions of the 1830's, 1870's, 1890's and 1930's. These fears have arisen partly because of the rapid deterioration of the Latin American debt situation. From the time of the first oil price shock in 1973 until the end of 1979 Brazil's external debt rose from 10 billion dollars (10b$) to 47b$; Mexico's debt from 7.2b$ to 11b$; Argentina from 3.5b$ to 11b$; Peru from 2.4b$ to 8b$. The large debts were explained by Latin American countries as a way of cushioning temporary shocks. In 1979-1980 two new shocks hit Latin America; the price of oil rose again sharply and the price of primary commodities fell drastically. Oil-producing countries such as Mexico borrowed freely against expected future oil earnings. Non-oil LDC's largely dependent on revenues from the export of primary commodities borrowed heavily to keep up current consumption with the expectation that commodity prices would soon rise. The result is that today total Latin American debt is over 300 billion dollars with Brazil owing about 90 billion, Mexico 80 billion, Argentina 38 billion, Venezuela 32 billion, and Peru 10 billion.

In the past year rumors of possible default by Mexico and Brazil have caused renewed attention to the problem of small country international debt. While there has been a proliferation of articles by authors purporting to explain the situation and offering rough-and-ready policy advice, there has been a surprising lack of desire to make the analysis

concrete in the context of an economic model[1] The purpose of this paper is to help fill this gap, that is, to provide a simple, admittedly artificial, economic environment which captures some of the crucial features of this situation.

In particular, the goal of the current analysis is to build a simple model of the international credit market with several features. First, the equilibrium exhibit <u>credit ceilings</u> and <u>recurrent defaults</u>. Second, the decision to default is <u>chosen by the borrower</u> and is not driven solely by lender's behavior. Third, the model endogenously determine the set of equilibrium loan contracts and these contracts be consistent with certain facts about actual contracts. Fourth, the equilibrium consist of a set of stationary stochastic processes, so that it is possible to compare the time series generated by the model to actual time series. Lastly, the model be consistent with modern contract theory.

II. Some Characteristics of International Credit Markets

The principal characteristics of the international credit markets explored   in the paper are:

(i)   International loan contracts made by private banks typically specify an amount borrowed and repayment schedule not contingent on the state of the borrower.

(ii)   Repayment schedules specify a predetermined margin or spread over the London Interbank Offered Rate (LIBOR).

(iii)   Banks set credit ceilings that vary across countries.

(iv)   Enforceability of contracts is severely limited. (This feature is considered by many as the crucial difference between international loans and other contracts).

(v)   Credit conditions vary over countries. The poorer LDC's both pay higher spreads over LIBOR and face lower credit ceilings than the richer ones.

(vi)   Credit conditions vary over time. In the century prior to 1980 countries paid larger percentage spreads than today.

(vii)   Synchronized defaults have historically coincided with downswings in the business cycle.

III. Explanations of Current Crisis and Proposals for Change

According to international financial analysts, during the past year the two largest holders of LDC debt, Mexico and Brazil, have come dangerously close to outright repudiation of debt. The two most popular explanations of the current crisis are the following. First, as claimed by the finance ministers of Mexico and Brazil, the debt crisis is the result of the lower price of commodity exports coupled with rising U.S. interest rates. These have caused the LIBOR to rise, the spread on loans to rise and credit limits to tighten.[2] Second, as claimed by several international bankers, the international debt problem is the result of prolifigate fiscal policy on the part of the LDC's.

A third explanation is the structure of bank insurance coupled with implicit guarantees of a bailout from a crisis by the U.S. government or the IMF distorted the incentives of the lending banks. These distortions decreased the riskiness of loans to LDC's making it optimal for the banks to lend at lower rates and larger amounts than they otherwise would have. The models presented in the paper shed some light on the first two explanations. Addressing the third explanation, however, would require a fairly different type of model and is left for future research.

Several prososals for resolving the cirsis have been made. One is that Mexico and Brazil should decrease their "wasteful" government spending. Indeed this seems to have been accepted by U.S. banks since a precondition for banks to extend new loans to Mexico last fall was that Mexico sign an austerity agreement with the IMF promising to reduce

government deficit from 16% of GDP in 82, to 8.5% in 83, to 3.3% in 85. A similar schedule for eliminating government deficits was a precondition for new loans to be made in Brazil. This raises the general question of what role does an international institution like the IMF play in the international credit market. Especially, how does committing to a plan of conditionality affect the creditworthiness of the borrowing country This idea is explored in sections (IX) and (X).

An alternative proposal made some years ago by Pranob Barhan is that small countries should institute a tax on foreign borrowing. His argument is basically that private agents in borrowing countries don't take into account the fact that their borrowing causes all agents in the country to face higher interest rates. Thus private agents borrow more than is socially optimal and the optimal policy is to tax foreign borrowing. This proposal is examined in section (VIII).

IV.   The Economic Environment

In the standard Walrasian equilibrium model, it is imagined that
all markets meet once at the beginning of time. At this time detailed
contracts are made by market participants. Each contract specifies an
amount of goods to be delivered to an individual at each future date
and state of the world in exchange for an amount of goods to be deli-
vered to the market at each date-state pair (with many entries perhaps
null). As time unfolds and uncertainty is resolved, these contracts
are imagined to be carried out precisely as specified at time zero.

Left unspecified in the standard model is what concrete arrangement
would support the Walrasian outcome. A particular arrangement one could
imagine is that each individual can verify every date-state pair and at
time zero a legal system is established which operates according to the
following rule: If a contract is signed at time zero and then violated
at some future date, say by nondelivery of goods, the violator is
assessed an infinite penalty by the market. In such an arrangement where
information is perfect, an infinite penalty rule for violating contracts
is established, and all agents are small, one would expect to see the
Walrasian outcome.

In order to explain many of the facts of international credit mar-
kets two modifications of the above paradigm must be made. First, the
doctrine of sovereign immunity prohibits the legal imposition of tremen-
dous penalties. In fact the limited enforceability of contracts can be
singled out as the most important way in which the arrangement supporting

the international credit market differs from the arrangement supporting the Walrasian market. Secondly, as opposed to the detailed delivery-payment schedule of the Walrasian model, international loan contracts are basically of a simple uncontingent type. One way of capturing the first idea is to assume the penalty from breaking the contract is relatively small. The simplest assumption is that each time an agent violates a contract he is penalized by the market a fixed percentage of current endowment. This penalty captures the loss of output suffered by the defaulter's country due to international reprisals -- disruptions of trade either by embargo or forfeit of credits and seizure of assets of the borrower held in the lender's country.

It may be somewhat bothersome not to model international retaliation in more detail, and instead use such a simple form. Experimenting with more detailed models of the penalty drives one to conclude the additional insights gained are small relative to the mess involved. Except for problems with information structure many of the results go through for any penalty that is increasing in current endowment or _effectively_ increasing in current endowment. Any penalty with the opposite property, that is, a penalty effectively decreasing in current endowment leads the model to predict one should see defaults when output in small countries is high and no defaults when output is low. This prediction is strongly at variance with the historical facts. An example of a penalty with this property is barring defaulters from the credit markets. Details are presented in appendix C.

As long as the _relative_ _openness_ of the economy under consideration

is fairly constant over short periods this type of penalty seems reasonable. Where one has to be careful in applying the intuitions is to countries whose relative openness is strongly countercyclical. To capture such dynamic effects clearly one would need a more detailed model.

Next, a standard way to structure environments in order to reduce the number of contingencies specified in equilibrium contracts is to assume some variable is the private information of one of the agents. Here it will be assumed the borrowers' endowment is private information. There are several reasons for making this assumption. First there is some evidence that LDC's economic situations are to a large extent their private information. For example, recently the Institute for International Finance (IIF) considered the Ditchley initiative which proposed that the IIF set up a team that gathers information about LDC's economic situation and gives this information to the lending banks so they can better evaluate country risks. An IMF official commented that he believed such a program would not accomplish much since much of the important information is confidential. Another example is that it has been widely quoted that for the case of Zaire much of the problem resulted from lack of complete information on the part of the bankers. Lastly, for the contract approach completely private information is easier to handle analytically than noisy information.

Specifically, consider a world economy consisting of a large number of small countries and one large country. Each of the small countries is populated by a large number of identical, competitive infinitely-lived

economic agents called borrowers. Each of these agents is endowed with θ_t units of the single consumption good at t, and knowledge of this is the private information of the borrower. These agents receive utility $U(c_t)$ from the consumption of c_t at date t.

The single large country is inhabited by a large number of agents called creditors. The creditors are risk neutral and their sole purpose is to maximize revenues by making loans. The creditors have the option of lending at home and earning a safe rate of return $\bar{R}$ or lending abroad by signing a contract with an agent in a small country at a rate to be determined in equilibrium. In order to model the limited enforceability of international contracts, it is assumed at time zero that a rule is established specifying that if a contract is defaulted on at time t, the breaker is penalized by the market $\lambda\%$ of current endowment θ_t.

V. The Model

Assumptions and Definitions:

1. The <u>endowments of borrowers</u> θ_t are independent and identically distributed random variables with finite state space $\Phi \equiv \{\theta(1),..,\theta(n)\}$, where $\underline{\theta} = \theta(1) < \theta(2) < ... < \theta(n) \equiv \bar{\theta}$ and θ has distribution function $F(\theta)$. We will often suppress the argument referring to state and refer to a generic element of Φ as simply θ.

2. A <u>one period loan contract</u>

$$s_t = (b_t, p_{t+1}(\theta)) = (b_t, p_{t+1}(\theta(1)),..., p_{t+1}(\theta(n))) \in R^{n+1}$$

is a number b_t and a schedule $p_{t+1}(\theta)$ (vector $p_{t+1}(\theta(1))..., p_{t+1}(\theta(n))$) specifying an amount b_t of consumption goods borrowed at t in exchange for a <u>promise</u> to deliver $p_{t+1}(\theta)$ units of consumption goods at date t+1 state θ.

The <u>set of original loan contracts</u> is

$$S = \{s = (b,p(\theta)) \in R^{n+1}| \ ||s||_\infty = \max\{|b|,| \ p(\theta(1))|,...,| p(\theta(n))|\} < A < \infty\}.$$

It is assumed the trivial contract belongs to the set of original loan contracts: $(0,...0) \in S$.

3. The <u>penalty function</u> is

$$h(\theta,r,p(\phi)) \equiv \begin{cases} 0 & r \geq p(\phi) \\ \lambda\theta & r < p(\phi). \end{cases}$$

Several comments about these assumptions. First, to understand the penalty function consider the following. Suppose a borrower at t-1 signs a contract $s_{t-1} = (b_{t-1}, p_t(\theta))$ receives the b_{t-1} units of good

at t - 1 and <u>promises to repay</u> $p_t(\theta)$ units of consumption good at date t state θ. Now suppose at t, $\theta_t = \theta(j)$ is the true state realized and the borrower does two things, first he lies and <u>claims the state is ϕ not</u> $\theta(j)$ in which he promised to repay $p(\phi)$, next he <u>actually repays an amount</u> r possibly not equal to $p(\phi)$. Then if the amount r he actually repays is greater than or equal to $p(\phi)$ he receives no penalty, if he repays an amount r less than $p(\phi)$ he is penalized $\lambda\theta(j)$.

Next, one might wonder how agents can be penalized according to true state θ when it cannot be verified. It is assumed that when borrowers actually do default creditors retaliate by, for example, disrupting trade. This disruption amounts to borrowers losing 10% of their <u>true</u> current endowment. The creditors don't know what the true current endowment is but they <u>do know</u> that disruption of trade always amounts to borrowers losing 10% of their current <u>true</u> endowment regardless of what state borrowers declare. For example consider a small country whose main income comes from the export of bananas. When the crop of current bananas is good a trade embargo severely reduces national income, while if current crop is poor an embargo reduces income to a lesser extent. For countries in which the relative degree of openness not vary strongly countercyclically the above example is germane.

Lastly, default is a legal term not commonly found in the contract literature. Narrowly applied to a loan contract between a borrower and a lender it is taken here to mean that all market participants can verify that actual repayment does not coincide with promised repayment. Now one can usually define the contract space so that the promised

agreement coincide with the actual agreement in equilibrium. For example, here one could record the penalty as part of the original contract by defining a contract to be $s_t = (b_t, p_{t+1}(\theta), A)$ where $A \subset \Phi$ is the default region with interpretation that $\theta \in A$ the borrower pays $\lambda\theta$ to the market and nothing to the borrower, if $\theta \in A^c$ the borrower pays nothing to market and $p(\theta)$ to the borrower. This approach is similar to the one used in Townsend ( ). The allocations under both formalizations will be the same, however, the first seems intuitively to correspond more closely to the legal concept.
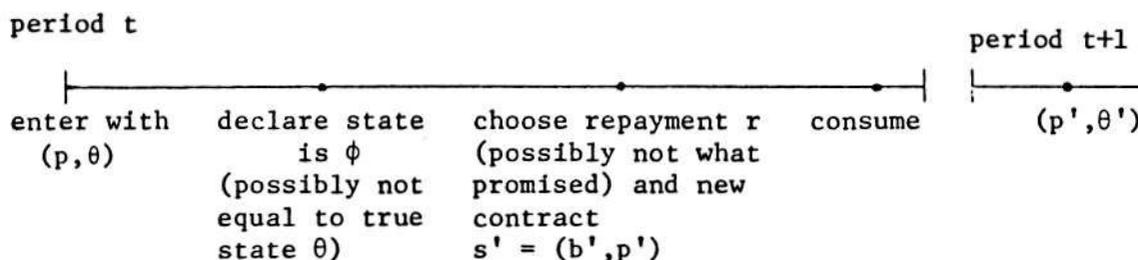
With this formalization the problem is the borrower is as follows. If the borrower is confronted with any closed subset of contracts $\tilde{S} \subset S$ he solves:

$$v(p, \theta | \tilde{S}) = \max U[c] + \beta \int v(p', \theta' | \tilde{S}) dF(\theta')$$

$$\phi \in \Phi, \quad r \geq 0, \quad s' = (b', p') \in \tilde{S}$$

$$s.t. \quad c = \theta + b' - r - h(\theta, r, p(\phi)) \geq 0$$

where (p, θ) are the state variables of the borrowers, S the set of contracts offered to the borrower, ϕ the state declared by the borrower (possibly not the true state θ), r the actual repayment made, $p(\phi)$ the promised repayment for the state ϕ the borrower declares and $h(\theta, r, p(\phi))$ the penalty assessed. Let T be the standard operator associated with above functional equation and $\phi(p, \theta | \tilde{S})$, $r(p, \theta | \tilde{S})$, $s'(p, \theta | \tilde{S})$ be the

optimal policy functions.

The timing of events can best be seen schematically:

period t

period t+1

enter with (p,θ)	declare state is ϕ (possibly not equal to true state θ)	choose repayment r (possibly not what promised) and new contract $s' = (b',p')$	consume

(p',θ')

Creditors choose that <u>subset</u> of the possible loan contracts which maximize revenues. For a given contract $s = (b,p(\theta))$ the creditors realize the borrowers optimal strategy will <u>not</u> necessarily be to repay according to schedule $p(\cdot)$ promised in the contract. With a conjecture $r^e(s', \theta'|\tilde{S})$ about borrowers actual repayment when borrower is confronted with $\tilde{S}$, the creditors solve:

$$J(r^e) = \max_{\tilde{S} \subset S} \; \max_{s' \in \tilde{S}} \left\{ -b' + \frac{1}{R} \int r^e(p',\theta'|\tilde{S}) \, dF(\theta') \right\}$$
$$\text{where } s' = (b',p')$$

Definition of equilibrium:

A <u>small country equilibrium</u> with private information for given S,h and R is a set of contracts $\hat{S} \subset S$, a value function $v(p, \theta|\tilde{S})$ and a triplet of optimal policy functions $\phi(p, \theta|\tilde{S})$, $r(p,\theta|\tilde{S})$, $s'(p, \theta|\tilde{S})$ such that:

(a) $\hat{S}$ is the set of all $s \in S$ that solve

$$\max_{S \subset S} \; \max_{s' \in S} \left\{ - b' + \frac{1}{R} \int r^e(p', \theta'|\tilde{S}) \, dF(\theta') \right\} \text{ where } s' = (b',p'(\theta))$$

s.t. 1. $-b' + \frac{1}{R} \int r^e(p', \theta'|\tilde{S}) \, dF(\theta') \leq 0$ [Free entry of creditors]

2. $r^e(p', \theta'|\tilde{S}) = r(p', \theta'|\tilde{S})$ [Rationality of creditors]

(b) $\phi(p, \theta|\tilde{S}), r(p, \theta|\tilde{S}), s'(p, \theta|\tilde{S})$ solve

$$v(p,\theta|\tilde{S}) = \max U[\theta - r - h(\theta,r,p(\phi)] + \beta\int v(p',,\theta'|\tilde{S})dF(\theta')$$

$$\phi \in \Phi, r \geq 0, s' \in S$$

s.t. $r + h(\theta,r,p(\phi)) \leq \theta + b'$

(c) $v(\cdot|\tilde{S})$ solves $T_S w = w$ for each $\tilde{S} \subset S$.

VI. <u>Reducing the Contract Space</u>

The main objective of this section will be to show one can reduce the original complicated state contingent contracts of S to much simpler ones without changing the equilibrium allocations. Since the result is very intuitive the uninterested reader may profitably proceed to the next section, realizing that this section shows the equilibrium contract space can be taken to be the set S_0 defined there. The reduction will consist of three steps. First, we reduce the contract space from the set of original contracts S to the set of contracts offered by the lenders in equilibrium $\hat{S}$. Next we show that many of the contracts in $\hat{S}$ are effectively copies of each other, by removing redundant copies we can restrict ourselves to a smaller contract space $\hat{S}_0$ without affecting equilibrium allocations. Finally we show that confronted with contracts in $\hat{S}_0$, borrowers will only ever choose a subset S_0. The final set S_0 called the set of equilibrium contracts will have a simple intuitive form.

Step 1: $S \rightarrow \hat{S}$

Since $0 \in S$ and free entry imply the equilibrium value of J is zero, $\hat{S}$ will be exactly the set of contracts that yield as actually fair rate of return. Since $r = r^e$ in equilibrium we can thus rewrite $\hat{S}$

$$\hat{S} = \{s \in S | \int r(p, \theta | \hat{S}) dF(\theta) = \bar{R}b\} .$$

In order to characterise this set we need to find $\phi(p, \theta | \tilde{S})$ and $r(p, \theta | \tilde{S})$.

That is, given that the borrower signed a contract s last period and the current state is ϕ, what state will the borrower declare and how much will he actually repay r.

We solve this problem in two steps. First given (p,θ), it is clear that for any $r \geq 0$ and $s' \in S$ it is optimal to declare the state $\phi*$ that solves min $p(\phi)$. Call $\phi*$ and $p_* = p(\phi*)$ the solution next, suppose $\phi*$
$\phi \in \Phi$
is chosen than for any fixed s', r must solve

$$\max_{r \geq 0} U[\theta + b' - r - h(\theta,r,p_*)]$$

$$\text{s.t.} \quad r + h(\theta,r,p_*) \leq \theta + b'$$

This maximization problem is equivalent to

$$\min_{r \geq 0} r + h(\theta,r,p_*)$$

$$\text{s.t.} \quad r + h(\theta,r,p_*) \leq \theta + b'$$

Ignoring for the moment the nonegativity constraint we obtain one of two possible for $r + h(\cdot)$:
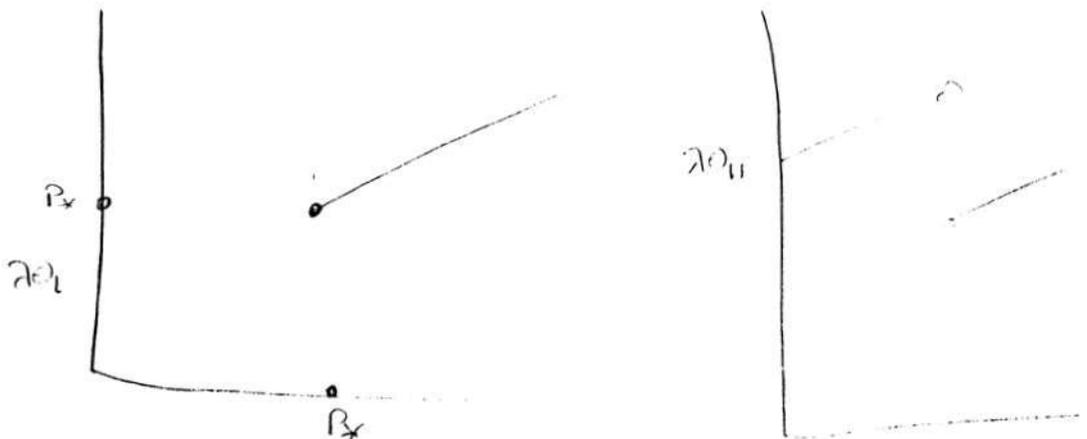
So the solution is

$$r(p,\theta|\tilde{S}) = \begin{cases} 0 \text{ if } \theta < \dfrac{P_*}{\lambda} \\[2ex] P_* \text{ if } \theta \geq \dfrac{P_*}{\lambda} \end{cases}$$

There are two points to verify. First, we show the nonnegativity constraint will not change the solution. Adding it could only change the solution by forcing a choice of $r = 0$ where without the constraint the choice would have been $r = p_*$. But this cannot happen because when the borrower chooses to repay without the constraint ($\theta \geq p_*/\lambda$), it is always feasible to repay since $\theta \geq p_*/\lambda \geq p_* - b'$. Second, the choice of repayment will not vary with the choice of s' because minimizing $r + h(\cdot)$ expands possible choices of s'. So the conjected solution is verified.

The above discussion implies

$$E_\theta r(p,\theta|\tilde{S}) = \int\limits_{\theta < P_x/\lambda} 0 \, dF(\theta) + \int\limits_{\theta \geq P_x/\lambda} P_* \, dF(\theta)$$

so we can write $\hat{S}$ as

$$\hat{S} = \{s = (b,p(\theta)) \in S \,|\, p_* Pr(\theta \geq p_*/\lambda) = \bar{R}b, \text{ where } p_* = \min_\phi p(\phi)$$

Now from the above it is clear for any two subsets S_A and S_B of S, $r(p,\theta|S_A) = r(p,\theta|S_B)$ and $\phi(p,\theta|S_A) = \phi(p,\theta|S_B)$ for any $s = (b,p) \in S_A \cap S_B$ (while it is not necessarily true that $s'(p,\theta|S_A) = s'(p,\theta|S_B)$). This eliminates the potentially nasty problem that the optimal repayment schedule and declared state schedule for the same set

of state variables may differ depending on which set of contracts is offered. This special feature is one of the great simplifying features of the above type of penalty, turning solving for $\hat{S}$ into a maximization problem instead of a much more difficult fixed point problem. So now we can drop the dependence of these functions on S and write simply $\phi(p,\theta)$, $r(p,\theta)$.

This shows <u>after</u> one realizes this special feature, for this structure the equilibrium can be defined more simply:

An equilibrium is a set of functions $\phi(p,\theta)$, $r(p,\theta)$, $s'(p,\theta)$, $v(p,\theta)$ which solve

$$v(p,\theta) = \max U[\theta + b' - r - h(\theta,r,p(\phi))] + \beta\int v(p',\theta')dF(\theta')$$

$$\phi \in \Phi \ , \ r \geq 0, \ (b',p') \in S$$

and

$Tw = w$ where T is the operator associated with the above functional equation $(T \equiv T_{\hat{S}})$ and $S = \{s = (b,p) \in S \mid \int r(p,\theta)dF(\theta) = \bar{R}b\}$

Step 2: $\hat{S} \rightarrow \hat{S}_0$

This form of the repayment schedule shows us that many different original contracts lead to the same repayment schedule. Indeed all those with the same minimum promised repayment will lead to the same actual repayment schedule and the same equilibrium allocation. If we group all contracts with the same minimum payment together and choose for each such group a single representative we would have for fewer contracts than before but still enough to achieve any possible equilibrium

allocation. A particularly simple set of such representatives for S are the ones that specify a constant repayment schedule. Thus

$$\hat{S}_0 = \{(b,p) \in S \mid p = (p_1, \ldots, p), \, p\Pr(\theta \geq p/\lambda) = \bar{R}b\}$$

or making the obvious identification.

$$\hat{S}_0 = \{(b,p) \in R^2 \mid p\Pr(\theta \geq p/\lambda) = \bar{R}b\}.$$

The above set can be conveniently depicted using a simple graph. For the purposes of the graph and later results it will be easier to work with repayment per dollar R/b rather than total repayment p. Let $R = p/b$ so that $(b,p) \equiv (b,Rb)$ then $s = (b,Rb) \in \hat{S}$ if and only if b and R solve

$$F\left(\frac{Rb}{\lambda}\right) = 1 - \frac{\bar{R}}{R}$$

Graphing these two function in Figure 1 we obtain three different cases.

One can read off the graphs for a given level b of borrowing what is the set of fair contracts (b,Rb) at that b.

Step 3: $\hat{S}_0 \to S_0$

By considering the viewpoint of the borrower we can make one final reduction to the set of contracts without affecting possible equilibrium allocations. Suppose a borrower were forced to choose one of several possible contracts on a given "slice" of $\hat{S}_0$ each with same b, but with different interest rates per dollar. It will be shown that the borrower
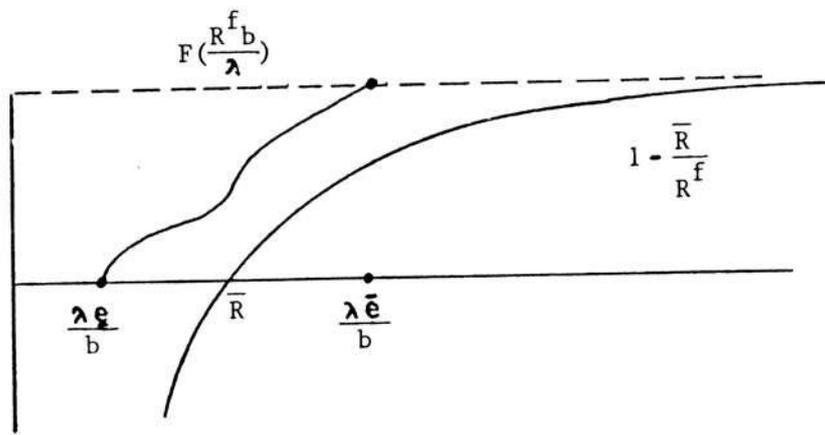
will always pick the schedule with the lower rate.

Suppose an agent were forced to choose between two contracts $(b, R_1 b)$ and $(b, R_2 b)$ both offering same b but the second has a higher rate, so $R_2 > R_1$. Ignore for a moment the fact that agents may choose different s' in two cases:
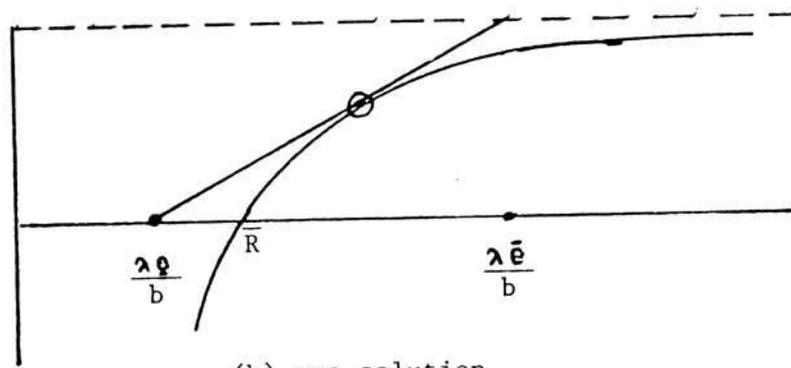
<div align="center">TWO CASES</div>

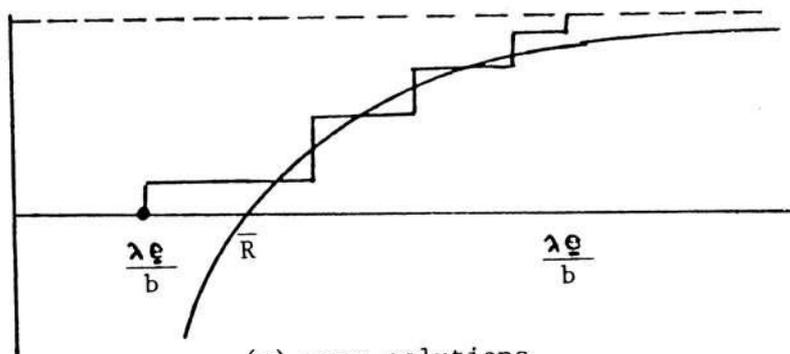|	R_1	R_2
I. $\theta < \dfrac{R_1 b}{\lambda} < \dfrac{R_2 b}{\lambda}$	$u(\theta - \lambda\theta + b')$	$u(\theta - \lambda\theta + b')$
II. $\dfrac{R_1 b}{\lambda} \leq \theta \leq \dfrac{R_2 b}{\lambda}$	$u(\theta - R_1 b + b')$	$u(\theta - \lambda\theta + b')$
III. $\dfrac{R_1 b}{\lambda} \leq \dfrac{R_2 b}{\lambda} \leq \theta$	$u(\theta - R_1 b + b')$	$u(\theta - R_2 b + b')$

In region I, both agents default and receive the same consumption. In region II, the agent repays under R_1 but defaults under R_2. Since under R_1 the agent also had the option to default it must be the case that not doing so yields more utility. In region III the agent repays under both, but repays uniformly less under lower rate. So the lower rate is revealed to be preferred. Since the feasible choice set for s' under the low rate includes the feasible choice set under the high rate, the choices of r and future s' are independent. Lastly, since an identical argument holds for any number of interest rates, the conclusion holds.

$F\left(\dfrac{R^f b}{\lambda}\right)$

$1 - \dfrac{\bar{R}}{R^f}$

$\dfrac{\lambda \underline{e}}{b}$ $\bar{R}$ $\dfrac{\lambda \bar{e}}{b}$

(a) no solution

$\dfrac{\lambda \underline{e}}{b}$ $\bar{R}$ $\dfrac{\lambda \bar{e}}{b}$

(b) one solution

$\dfrac{\lambda \underline{e}}{b}$ $\bar{R}$ $\dfrac{\lambda \underline{e}}{b}$

(c) many solutions

FIGURE 1

V. Characterizing the Equilibrium Contracts

In this section we characterize how the set of equilibrium contracts S_0 varies with changes in the amount borrowed b, safe interest rate $\bar{R}$, distribution function $F(\cdot)$ and degree of penalty λ. It is intersting to attempt to match up these characteristics with the stylized facts of Section II.

$$S_0 = \{(b,R) \in R^2 \mid 0 \le b \le \bar{b}, \ R = \inf(z \mid F(\frac{zb}{\lambda}) = 1 - \frac{\bar{R}}{z} \ \text{for some } z \in R\}$$

(A) **How contracts vary with the amount of the loan:**
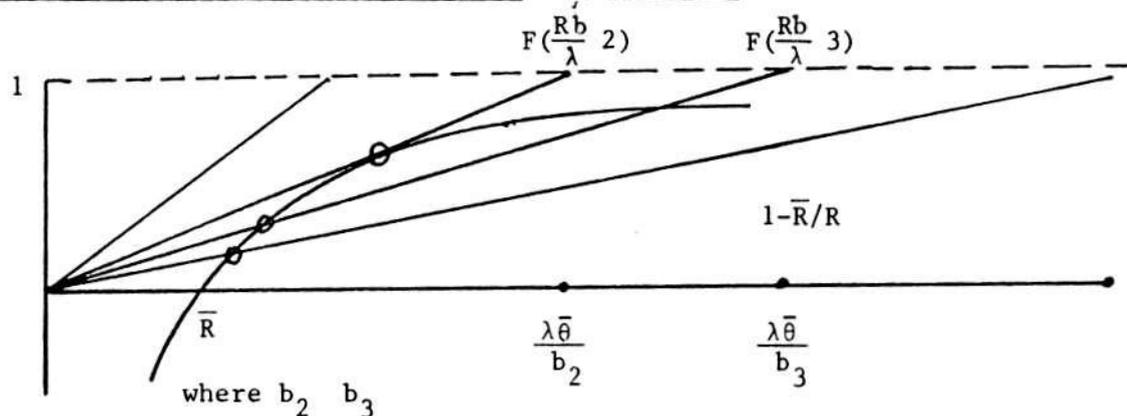
Thus the set of contracts S_0 has the following shape
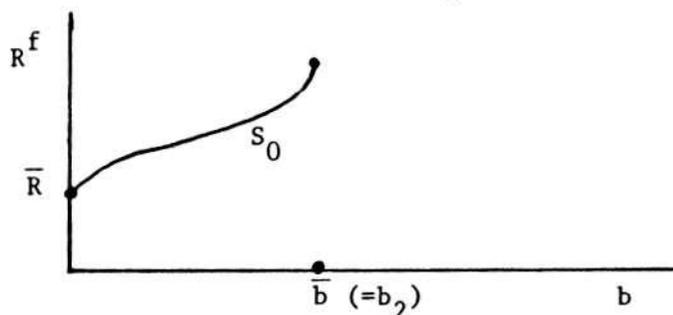
The two results are that:

(1) The interest rate per dollar changed on loans is an increasing function of total debt.

(2) There is a maximum possible amount, say $\bar{b}$, beyond which there is no possible interest rate R for which the resulting effective contract will be actuarily fair.

Here $\bar{b} \equiv \bar{b}(\lambda, \bar{R}, F(\cdot)) \equiv \sup\{b \mid F(\frac{Rb}{\lambda}) = 1 - \frac{\bar{R}}{R} \ \text{for some } R \in R\}$
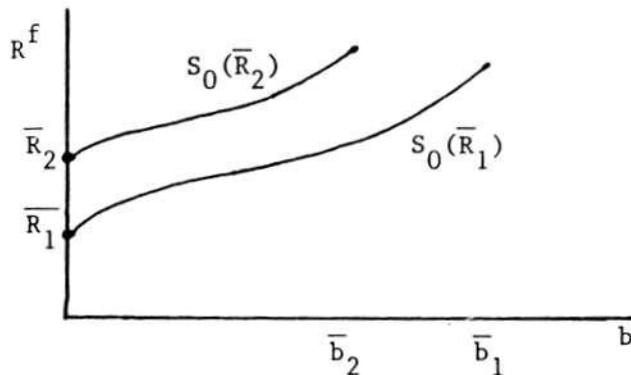
(B) <u>How contracts vary with the safe rate $\overline{R}$</u>

The result is that interest rates per dollar R increase and credit ceilings tighten with higher $\overline{R}$.

Let $\overline{R}_1 < \overline{R}_2$.
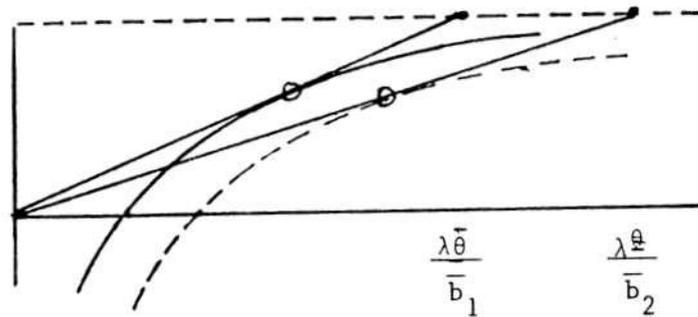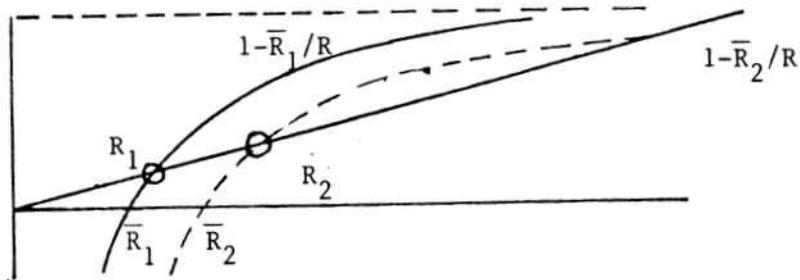
So we obtain the loan schedules $S_o(\overline{R}_1)$ and $S_o(\overline{R}_2)$


This result accords well with the fact that recent increases in US interest rates cause a severe tightening of credit conditions facing developing countries.

(C) <u>Comparison of credit conditions across types of countries</u>

Call country 1 richer then country 2 if the endowment distribution F_1 of the first country dominates F_2 of the second country in the sense of first order stochastic dominance (5) we can compare the equilibrium contracts faced by the two countries.
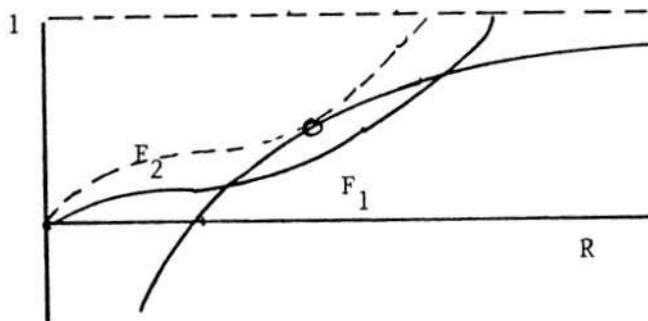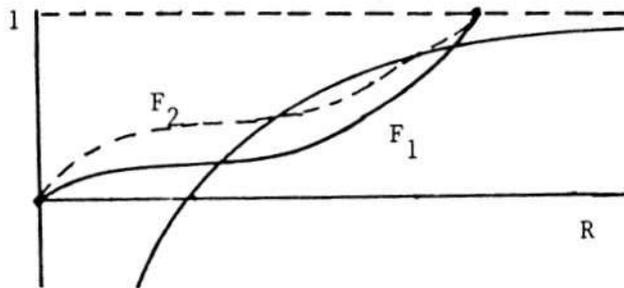

So we obtain the loan schedules $S_o(F_1)$ and $S_o(F_2)$
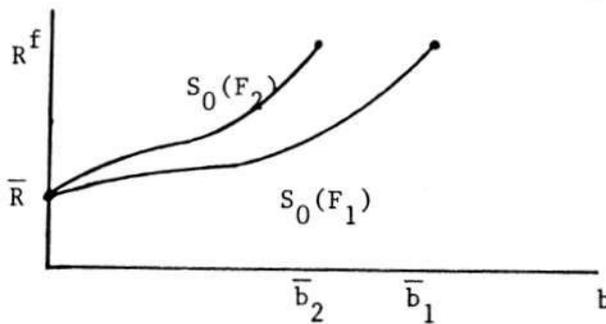
This matches up well with the fact that poor countries like Zaire face both uniformly higher rates per dollar borrowed than do richer countries like Mexico and Brazil.

(D) Comparison of credit conditions across regimes

Consider two regimes characterized by different degrees of enforceability of contracts: the first with a relatively low penalty λ_1 for default and the latter with relatively high penalty λ_2 ($>\lambda_1$) for default. Denoting the equilibrium contracts in the first regime $S_0(\lambda_1)$ and the latter $S_0(\lambda_2)$ we can obtain:
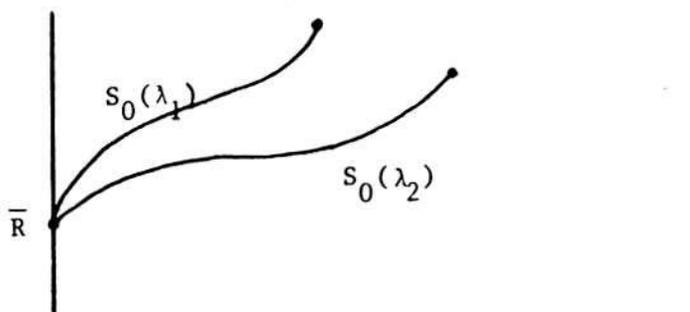
Thus credit conditions both in terms of percentage spreads and credit ceilings are better in regime with higher penalties. This matches Borchard's observation that spreads in post-1930's were much lower than in the pre-1930's. He claims the growth in world trade caused trade disruption associated with an international default to impose a relatively greater penalty on the defaulting country.

VIII. <u>Decentralizing the Optimum</u> [Optional]

In several models of international borrowing it has been suggested that when the country as a whole faces an upward sloping interest schedule the optimal policy of the small country is to institute a tax on borrowing. This section analyses the proposal in the context of the current model and argues in one interpretation the optimal policy is no tax, while under another it is to institute a nonlinear tax on borrowing the rate of which increases with the level.

The borrower's problem of the previous section is that of a small country planner facing an upward sloping supply of funds and credit limits. For this section only let B and P denote mean aggregate borrowing and promised repayment. Let $B' = \hat{B}(P,\theta)$ be the planner's optimal borrowing. Let $\hat{Z}(P,\theta)$ be planner's default decision where $Z = 0$ means "default" and $Z = 1$ means "fully repay." Then from earlier section:

$$Z(P,\theta) = \begin{cases} 0 & \theta < P/\lambda \\ 1 & \theta \geq P/\lambda \end{cases}$$

Likewise $B' = \hat{B}(P,\theta)$ and $P' = \hat{P}(P,\theta)$ solve

$$V(P,\theta) = \max_{(B',P')} U[\theta + B' - Z(P,\theta)P - (1 - Z(P,\theta))\lambda\theta] + \beta\int V(P',\theta')dF/\theta')$$

where $P' = R(B')B'$

and $B' \in [0,\bar{B}]$

Now suppose in the small country there are n identical consumers.

Letting b_i and p_i denote the borrowing and promised repayment of individual i, $B = \frac{\Sigma \, b_i}{n}$, $P = \frac{\Sigma \, p_i}{n}$. Assume all loans are made from the creditors directly to the planner who then must decide what interest rate to charge the borrowers when distributing these funds so as to give the borrowers the proper incentives. Assume also within the small country loans between the planner and individual borrowers are perfectly enforceable and endowments are public information. Now, the decision to default is only taken at the aggregate level by the planner on loans between the planner and lenders according to the $\hat{Z}(P,\theta)$ schedule. When the country defaults ($Z = 0$) all individuals lose $\lambda\%$ of current endowment.

With what interest rate and tax schedule must the planner confront the borrowers to ensure they borrow the optimum amount? It is claimed the optimal policy of the planner is to confront each borrower with the _identical_ upward sloping interest rate schedule and credit limit he faces. Taking the aggregate default schedule $Z(P,\theta)$ and borrowing schedule $P' = \hat{P}(B,\theta)$ as given, if the planner confronts the borrower with $R(b_i)$ (the same function $R(\cdot)$), and credit limit $\bar{b}_i$ ($\equiv \bar{B}$) the representative borrower solves:

$$v(p_i,\theta',\hat{Z},\hat{P}) = \max U[\theta + b_i' - Z(P,\theta)p_i - (1 - Z(P,\theta))\lambda\theta]$$
$$+ \beta\int v(p_i',\theta';\hat{Z},\hat{P})dF(\theta')$$

$$b_i' \in [0,\bar{b}_i]$$
$$p_i' = R(b_i')b_i$$

This can be written:

$$v(p_i,\theta;\hat{Z},\hat{P}) = \begin{cases} \max_{(b_i',P_i')} U[\theta+b_i'-\lambda\theta]+\beta\int v(p_i',\theta';\hat{Z},\hat{P})dF(\theta') & \text{if } \theta \leq P/\lambda \\ \\ \max_{(b_i',P_i')} U[\theta+b_i'-p_i']+\beta\int v(p_i\theta';\hat{Z},\hat{P})dF(\theta') & \text{if } \theta \geq P/\lambda \end{cases}$$

This will yield schedules $b_i = \hat{b}_i(p_i,\theta;\hat{Z},\hat{P})$ are $p_i' = \hat{p}(p_i,\theta;\hat{Z},\hat{P})$ such that:

$$\hat{B}(P,\theta) = \Sigma\hat{b}_i(p_i,\theta;\hat{Z}(P,\theta),\hat{P}(P,\theta))/n$$

$$\hat{P}(P,\theta) = \Sigma\hat{p}_i(p_i,\theta;\hat{Z}(P,\theta),\hat{P}(P,\theta))/n$$

holds identically in all (P,θ).

The main point is the planner should confront the individual with the identical interest rate and credit limit the country as a whole faces. If for some reason one wanted to, we could always write the interest schedule as a flat interest rate $\bar{R}$ plus a nonlinear tax $\tau(b)$ such that $R(b) \equiv \bar{R} + \tau(b)$ but this just seems to complicate matters. In models such as these when the country borrows more it pays a high interest rate not because it is "big" in some sense but because the type of good it is selling (promise to repay) changes with different level of promises. If the representative consumer is confronted with a schedule that takes account of this there is no need to impose an additional tax.

IX. Government Spending

Committing to an IMF high conditionality loan typically enhances the creditworthiness of small countries. For the recent cases of Mexico and Brazil such a commitment was a prerequisite for commercial banks to grant new loans. In this section we add government spending to the model and examine how reductions in the path of such spending can enhance creditworthiness.

Government spending can be classified into spending on productive capital and spending on services. We are concerned here with only the latter type, that is government spending which does not increase output but does produce utility-yielding services. Modify the model to include two goods, a consumption good c and government services G. For simplicity, let government spending be a fixed linear function of current output, $G = g \cdot \theta$ where $g \in (0,1)$. Let the one period utility function be $U^c(c) + U^G(G)$. The timing of the events are the same except now government spending is undertaken and services provided <u>immediately</u> <u>after</u> the new endowment θ is realized. This reduces the effective endowment to $(1-g)\theta$ for the repayment and new borrowing stage. Since the borrower's declared state will always be the one with minimum promised repayment we subsume this.

With these modifications the model becomes:

$$v(p,g,\theta) = \max U^c[(1-g)\theta + b' - r - h(\theta,g,r,p)] + U^G(g \cdot \theta) + \beta \int v(p',g,\theta')dF(\theta')$$

$$r \geq 0, \ (b',p') \in S$$

where $h(\theta,g,r,p) = \begin{cases} 0 & r \geq p \\ \lambda(1-g)\theta & r < p \end{cases}$
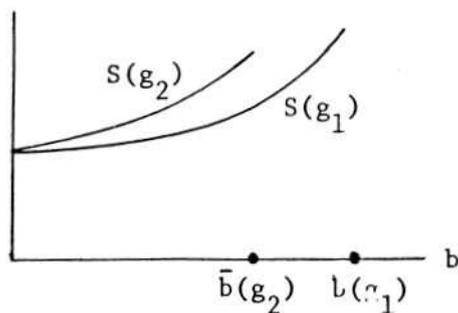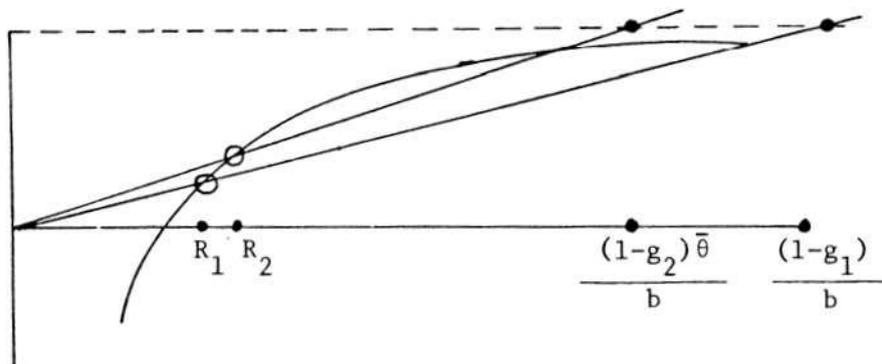
The optimal repayment schedule is:

$$r(p,g,\theta) = \begin{cases} 0 & \theta < p/\lambda(1-g) \\ p & \theta \geq p/\lambda(1-g) \end{cases}$$

Letting $R \equiv p/b$, the set of equilibrium contracts for a given g are:

$$S(g) = \{(b,Rb) \in R^2 | 0 \leq b \leq \bar{b}, R = \inf \{z| F(\frac{zb}{\lambda(1-g)}) = 1 - \frac{\bar{R}}{R} \text{ for some } z \in R\}$$

where $\bar{b} = \bar{b}/g; \lambda, \bar{R}, F) = \sup \{b| F(\frac{Rb}{\lambda(1-g)}) = 1 - \frac{\bar{R}}{R} \text{ for some } R \in R\}$

Now consider the effects of lowering the path of government spending on the equilibrium loan contracts. Letting $g_1 < g_2$ we obtain for a fixed b:

Thus a lower path of government spending enhances creditworthiness in the sense it lowers the interest rate per dollar on loans and increases the debt limit. However, the effect on the welfare of the small country is ambiguous, since there is a tradeoff between the benefits of lower government spending arising from enhanced creditworthiness and the costs of lowering government services in terms of foregone utility. Optimal government spending will balance these effects.
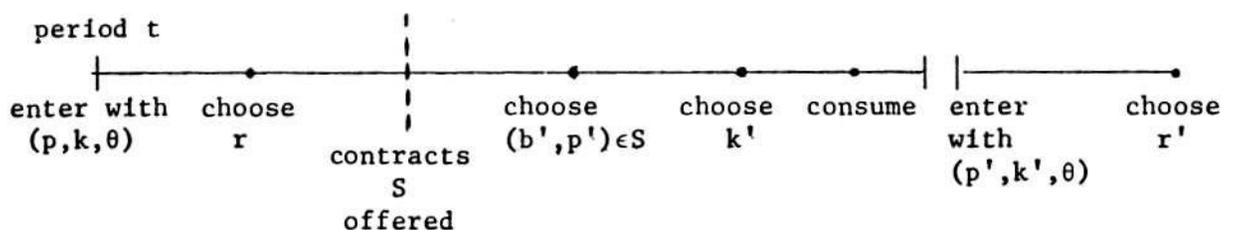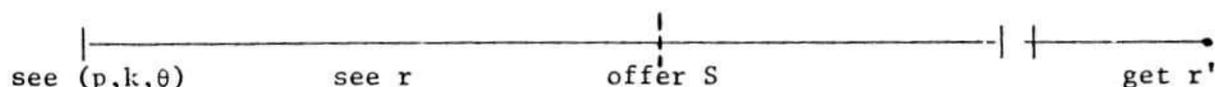
X. The Model with Capital

Expanding the model to include capital enriches the analysis by making the lender's inference problem more interesting and by pointing to a potential role for an IMF-like institution. Modify the model by replacing the stochastic endowment θ with a production function buffeted by productivity shocks, $\theta f(k)$, and consider two possible information structures. In the first, suppose the productivity shock is known only by borrower. In the second, suppose the shock is known both by borrowers and by lenders but if contracts are written contingent as the shock and then broken, the lenders cannot prove it to a third party.

These two information structures lead to identical conclusions for the endowment case, so the analysis of the first part of the paper applies equally to either case. For the capital case, however, the two information structures lead to fairly different equilibria. In this section we consider only the second case and sketch the details of the first in appendix . For simplicity, in all of what follows the optimal declared state rule has been solved out and redundant contracts deleted.

The timing of decisions with capital is as follows:

Borrowers

period t

enter with (p,k,θ) — choose r — contracts S offered — choose $(b',p')\epsilon S$ — choose k' — consume — enter with (p',k',θ) — choose r'

Lenders

```
|—————————————————————————————|————————————————————————————|  |—————————————→•
see (p,k,θ)        see r           offer S                       get r'
```

Similarly to previous analysis, the lenders know if the borrower chooses contract (b',p') and capital stock k' at t then at t+1 if state θ' is realized borrowers will repay according to:

$$r(p',k',\theta') = \begin{cases} 0 \text{ if } \theta \le p'/\lambda f(k') \\ p' \text{ if } \theta \ge p'/\lambda f(k') \end{cases}$$

But here since the capital decision is made <u>after</u> the contract decision the lender must <u>infer</u> what capital stock will be chosen given his information (p,k,θ,r;b',p') up to that point. Here (p,k,θ) is sufficient for (p,k,θ,r) in the sense that r can be deduced from (p,k,θ), so the decision default/not default yields no new information to the lender. (It is in this aspect that the two cases differ. See appendix .)

If lenders offer a given set of contracts $\tilde{S} = \{\tilde{S}(p,k,\theta)\}$, substituting out both the declared state and repayment rule, the borrower solves: Choose contract (b',p') and capital stock k' to solve:

$$v(p,k,\theta) = \max_{(b',p') \in S(p,k,\theta)} \left\{ \max_{k'} U[\theta+b'-k'-\min(p,\lambda\theta f(k))] + \beta\int v(p',k',\theta')dF(\theta') \right\}$$

Lenders know that if borrowers choose (b',p'), new capital stock will be chosen to solve:

$$\max_{k'} U[\theta + b' - \min(p,\lambda\theta r(k))] + \beta\int v(p',k',\theta')dF/\theta')$$

Call this solution $k' = \tilde{k}(p,k,\theta|b',p')$ and note that k will vary

with offered contracts $\tilde{S}$. Now, given $\tilde{S}$ are offered the set of actuarily

fair contracts will be $\hat{S} = \{\hat{S}(p,k,\theta)\}$ where:

$$\hat{S}(p,k,\theta) = \{(b',p') \in \tilde{S}(p,k,\theta)|\int r[p',\tilde{k}(p,k,\theta|b',p'),\theta']dF(\theta') = \bar{R}b'\}$$

The above indicates a map, say f, that maps the set of subsets of

S into itself. In particular, f maps offered contracts $\tilde{S}$ into actuarily

fair contracts $\hat{S}$ associated with that $\tilde{S}$. A fixed point of this map will

be the set of equilibrium loan contracts. The reason we cannot avoid

the fixed point problem in this case is that lenders must predict future

decisions of the borrower and these future decisions vary across offered

contracts.

Consider setting up in this environment an institution to which

agents can commit themselves to future policies. Indeed suppose borrowers

can commit themselves to a level of new capital stock <u>before</u> entering

into new contracts. If k' is committed to, contract (b',p') is chosen

at t and state θ' is realized at $t+1$ borrowers repay according to:

$$r(p',k',\theta') = \begin{cases} 0 & \text{if } \theta < p/\lambda f(k') \\ p' & \theta \geq p/\lambda f(k') \end{cases}$$

Thus with commitment lenders information about current state varia-

bles is irrelevant for predicting future repayment. Lenders offer

contracts which depend on the only relevant information, the committed

level of capital stock k'. For a set of offered contracts $\{\tilde{S}(k')\}$

borrowers solve:

$$v(p,k,\theta) = \max_{k'} \begin{cases} \max U[\theta f(k)+b'-k'-\min(\lambda\theta f(k),p)] + \beta\int v(p',k',\theta')dF(\theta') \\ (b',p') \in S(k') \end{cases}$$

So for a given level of commitment k' and offered contracts $\tilde{S}(k')$, the set of actuarily fair contracts will be

$$\hat{S}(k') = \{(b',p') \in \tilde{S}(k') | \int r(p',k',\theta')dF(\theta') = \bar{R}b'\}$$

Since the repayment schedule doesn't vary across offered contracts we avoid the fixed point problem and can rewrite this set. Letting $R' = p'/b'$ we obtain:
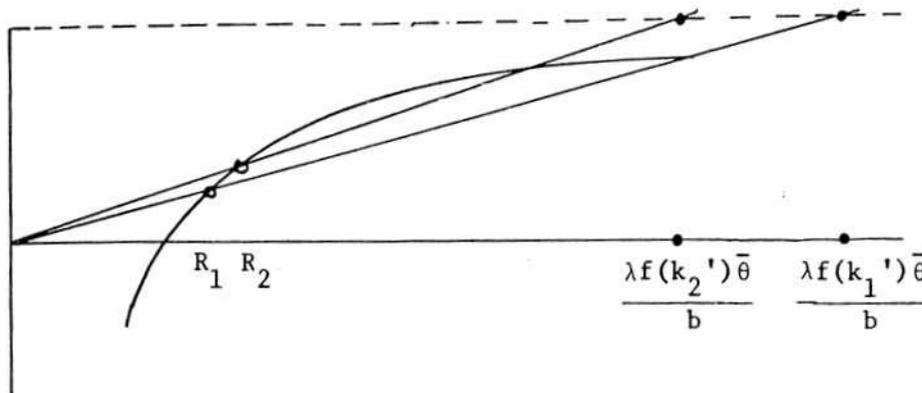
$$\hat{S}(k') = \{(b',R'b') \in S | 0 \le b' \le \bar{b}, R = \inf \{z | F(\frac{zb}{\lambda f(k')}) = 1 - \frac{\bar{R}}{z} \text{ for some } z \in R\}$$

where $\bar{b} = \bar{b}(k';\lambda,\bar{R},F) = \sup \{b | F(\frac{Rb}{\lambda f(k')}) = 1 - \frac{\bar{R}}{R} \text{ for some } R \in R\}$

In this case we obtain the four results analogous to the endowment case replacing "richer" by "more productive" whenever necessary. In addition we obtain a further result:

(E) Comparison of credit conditions for levels of commitment

Consider two levels of commitment k_1' and k_2' with $k_1' > k_2'$. We obtain
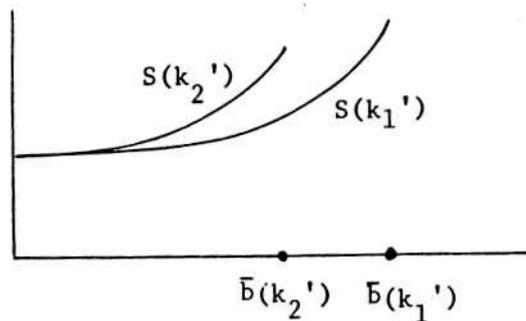
So for higher commitment levels of capital stock, (lower commitment levels of consumption) borrowers face uniformly better credit conditions in the sense that interest rates per dollar are lower and credit limits higher. We can summarize the contract sets with a schedule $R(b',k')$ with $R_{b'} > 0$, $R_{k'} < 0$ and credit limit $\bar{b}(k')$ with $\bar{b}_{k'} > 0$.

Now the value of the program with the opportunity to optimally commit, to say $\hat{k}(p,k,\theta)$, will strictly exceed the value of the program without that opportunity. Suppose now, instead of letting the borrower choose any level of commitment to new capital, the borrower is confronted with only one possible level, say k_0, and must decide to commit to that level or not commit at all. For any given set of state variables (p,k,θ) there will be an interval around the optimal commitment level k say

$$[\hat{k}(p,k,\theta) - \epsilon_1, \ \hat{k}(p,k,\theta) + \epsilon_2] \quad \text{where } \epsilon_i = \epsilon_i(p,k,\theta)$$

such that the country will prefer to commit to any k_0' in this interval to not committing. So allowing for the possibility of commitment will in general increase the welfare of the borrower. However, if a third party picks the single allowable commitment level k_0, there may well be a conflict between the objective of maximizing creditworthiness subject to constraint the level is preferred to no commitment and choosing a commitment level to maximize welfare of the borrowers.

XI. Conclusion

This paper displays a simple economic environment in which compli-
cated state contingent contracts are allowed. In equilibrium, however,
the complicated contract space can be reduced to a much simpler one of
a state in contingent nature. This set of simpler contracts was shown
to match up in a stylized way with those seen in international loan
markets.

It is also found the claim that the optimal policy in borrowing
countries facing upward sloping interest rate schedules depends on just
how the interest rate schedule is written. Lastly, a potential role
for an IMF-like institution as a means of committing to certain policies
is suggested. However, and this is crucial, if the borrowers are con-
fronted with only one possible commitment level on a take-it-or-leave-it
basis then there will in general be a conflict between the objectives
of maximizing the creditworthiness of the borrower and maximizing utility.

This paper is a member of the class of "borrower-chosen repudiation
models" of the Eaton and Gersovitz type. Alternatively, one could
imagine a whole class of imperfect information models in which the deci-
sion to default is driven by a contraction of the lenders supply of
funds arising from lenders gaining "bad" information about the borrower's
state either directly or by inferring it either from the actions of the
borrower or other lenders. Indeed, at least intuitively, such "lender-
driven default models" seem to be the most promising alternative to the
type of model considered here. However, the successful working out of
a theoretically tight model of this sort, allowing for entry into lending
poses a formidable challenge.

Appendix A

In both section ( ) of the current paper and in the Eaton and
Gersovitz paper, the problem of the borrower is stated directly in
dynamic programming form. The question of legitimacy naturally arises;
that is, are these problems bonafide <u>Stationary Discounted Dynamic
Programming Problems</u> (SDDPP) in the sense of Blackwell. This appendix
defines a SDDPP and shows the former meets its requirements while the
latter does not.

A SDDPP consists of five objects $(W, A, q, d,)$ where

W = the <u>set of possible states</u> of the system.

    W is a nonempty Borel subset of a complete separable metric space

    (i.e., a Polish pace)

A = the <u>set of feasible actions</u> in each period.

    A is a nonempty compact Borel Subset of a Polish space.

q = the <u>law of motion</u> or transition function of the system q is a

    probability measure on W given $W \times A$, that is,

    (i)   for each $(w,a) \in W \times A$, $q(\cdot | w,a)$ is a probability measure on W

    (ii) for each Borel subset B of w, $q(B | \cdot)$ is a Baire function on W.

where $q(w' | w,a)$ is the conditional probability the next state is w'
given the current state is w and the current action is a. To guar-
antee certain conditional expectations are continuous an additional
continuity assumption is made. One such condition is:

    For any sequence $\{w_n, a_n\}$ in $W \times A$ that converges to $(w,a) \in W \times A$,

    $q(w' | w_n, a_n)$ converges weakly to $q(w' | w,a)$

d = the one period return function.

In most applications d is taken to be a continuous real-valued
function on w x A x w bounded in the sup norm (that is,

$$||d||_\infty \equiv \sup_{(w',a,w)} |d(w,a,w')| < \infty)$$

d(w,a,w') is the current return if the current state is w, the
current action is a and next period's state is w'. Most often d
does not depend on w!

β = the discount factor

   $0 < \beta < 1$.

Now to state the original maximization problems of the borrower some
additional definitions are needed. A plan π is a sequence $(\pi_1, \pi_2, \ldots)$,
where π_t is a conditional probability on W given the history
$h_t \equiv (a_1, w_2, a_2, \ldots, w_n, a_n)$ of previous states and actions of the system
up to time t. Thus $\pi_t(a|h_t)$ is the probability of choosing the action
a in period t given the history is h_t.

Now any plan π along with the law of motion q defines a distribution
on all possible futures of the system $h_\infty = (a_1, w_2, a_2 \ldots)$ conditional on
initial state w_1. This conditional distribution is denoted e_π, so
$e_\pi(h_\infty|w_1)$ is interpreted as the probability the future of the system
will be h_∞ given the current state is w_1. Associated with any plan π
is the value of the plan π $I_\pi(\cdot)$ where $I_\pi(w_1) = E_{e_\pi}\left(\{ \sum_{t=1}^{\infty} \beta^t (w_t, a_t, w_{t+1}) | w_1 \right)$

is interpreted as the expected discounted return from plan π, given the
initial state is w_1. The problem of the decision maker is to choose an

optimal plan $\pi*$ which maximizes the value of I for any initial state w_1. that is achieves the <u>optimal value</u> $I(w_1) \equiv \sup_\pi I_\pi(w_1)$.

Blackwell shows that one can restrict the search for optimal plans to those plans with degenerate conditional probabilities which are time invariant functions of the current state, that is, to <u>stationary plans</u> $\pi = (f, f, \ldots)$. For any stationary plan $(f, f, \ldots)$ denoted $f^{(\infty)}$, $f(w)$ is interpreted as the planned action at any stage given the state at that stage is w. Ashok Maitra showed with the compactness and continuity assumptions made above, there always exists an optimal stationary plan.

With the above definitions and notation in place it is almost imme-diate to show:

I. The problem of the borrower on page ( ) is a SDDPP.

Consider the following identifications:

$W = (\text{proj } \tilde{S}) \times \Phi \subseteq R^{n+1}$, where proj $\tilde{S}$ is the projection of $\tilde{S}$ onto the last n coordinates.

$A \subset \Phi \times [0, \bar{r}] \times \tilde{S} \subseteq R^{n+3}$ where $A = \{(\phi, r, b', p') | \phi \in \Phi, r \geq 0,$ b', p') $\in S$ and $r + h(\theta, r, p(\phi)) \leq \theta + b'\}$ and the bound r comes from the feasibility constraint.

$d(w; a; w') = d(p, \theta; \phi, r, b', p'; p', \theta') \equiv u(\theta + b' - r - h(\theta, r, p(\phi)))$

$q(w' | w; a) = q(p', \theta' \ p, \theta; \phi_1 r_1 b', p') = q_1(p') \times q_2(\theta')$

where q_1 is degenerate on p' and $q_2(\theta_j) = F(\theta_j) - F(\theta_j - \theta_{j-1})$ and $q_2(\theta_1) = F(\theta_1)$.

Endowing R^k with the sup norm, it would be clear that all of the assumptions of SDDPP are satisfied except for the fact that penalty

function is not continuous. However, the discussion on page ( ) shows
we can work with an equivalent problem replacing r as a choice variable
with z, where $z \in \{0,1\}$, $z = 1$ means "default" and $z = 0$ means "repay."
The current period return function becomes $u(\theta + b' - z\lambda\theta - (1 - z)p(\theta))$
and the action space becomes:

$$A \subseteq \Phi \times \{0,1\} \times \tilde{S} \subseteq R^{n+3}$$

where $A = \{(\phi,z,b',p') \mid \phi \in \Phi, z \in \{0,1\}, (b',p') \in \tilde{S}$ and
$z\lambda\theta + (1 - z)p(\phi) \leq \theta + b'\}$

Now R^k is Polish, W and A are Borel sets, A is a closed subset of
a compact set so A is compact and u is continuous. Since F is a distri-
bution function q is a probability measure and the discreteness of ϕ
ensures the measurability and continuity assumptions on q are trivially
satisfied.

II. The borrowers problem in Eaton and Gersovitz is <u>not</u> a SDDPP.

Following my notation as closely as possible the Eaton and Gersovitz
model is similar to the one in the paper except for the penalty of
default. In particular, the main assumptions in which the model differs
are:

(A1) If the borrower defaults at t;

    a) Borrowers cannot borrow after t, i.e., $S_\tau = \{0\}$ all $\tau \geq t$

    b) Borrowers are penalized P_τ units of consumption for all $\tau \geq t$

(A2) θ_t has time varying distribution function F_t and support $[0,\bar{\theta}_t]$

The problem of the borrower is:

$$v^D(\theta_t) = \sum_{t=\tau}^{\infty} \int \beta^{t-\tau} u(\theta_t - P_t) dF_t(\theta_t)$$

$$v^R(P_t, \theta_t) \equiv \max \ u(\theta_t + b_t - P_t) + \beta \int v(\theta_{t+1}, P_{t+1}) dF(\theta_{t+1})$$

$$v(P_t, \theta_t) = \max \ [v^D(\theta_t), \ v^R(P_t, \theta_t)]$$

where v^D is the value if default, v^R is the value if repay and v is the optimal value. Now assumptions (A1)b and (A2) guarantee this is <u>not</u> a SDDPP. First, F_t is not Markov of some finite order (or iid). Second, the penalty can vary with (absolute) time. In particular, this problem cannot be mapped in the structure laid out by Blackwell, Blackwell's Theorems 2 and 6 do not apply, and one cannot summarize the state of the system by (θ_t, P_t). Instead, one needs to record the whole hisotry $h_t = (\theta_1, P_2, \theta_2, \ldots, P_t, \theta_t)$ and consider value functions and policy functions defined over such histories. To see the difficulty imagine solving the finite time problem and then driving the horizon to infinity. One will not be able to show limits of the associated value functions converge to the value function needed to write the problem in this form. Also, no attention is paid to the nonegativity constraint on consumption given the borrower defaults. Indeed the only value of P_τ which ensures $c_\tau(\theta_\tau) = \theta_\tau - P_\tau$ is positive for all $\theta_\tau \in [0, \bar{\theta}_\tau]$ is P_τ identically equal to zero, for all $\tau \geq t$.

Two modifications of the assumptions will remedy these difficulties:

(A2)' θ_t has time invariant distribution function F and support $[\underline{\theta}, \bar{\theta}] \subseteq R_+$

(A1)'b $P(t, \tau)$ is a time homogenous function in sense $P(t, \tau) = \tilde{P}(\tau - t)$

for some $\tilde{P}$ where $P(t,\tau)$ is the penalty at τ for default at t $(P(t,\tau) = 0$ for $\tau < t)$. Moreover $\hat{P}(\tau - t) < \underline{\theta}$. If it is desired that the penalty vary with endowment one could assume $P(t,\tau,\theta) = \tilde{P}(\tau - t,\theta)$ and $\hat{P}(\tau - t,\theta)$ $< \theta$. In particular taking $\hat{P}(\tau - t,\theta) = \lambda\theta$ for all $\tau \geq t$ and 0 for all $\tau < t$ would work.

Appendix B

This appendix establishes the existence and properties of the value function. Let:

$$X \equiv [0,\bar{b}] \times \Phi$$

$$C_B(X,R) = \{w: X \to R \mid w \text{ is continuous and } ||w||_\infty = \sup_{x \in X} |w(x)| < \infty\}$$

That is C_B is the set of continuous functions from X to R bounded in the sup norm. Endow C_B with addition and scalar multiplication and denote the resulting linear space $L = L(X,R)$. L is well known to be a Banach space. Let $A(X,R)$ be the linear space of all real-valued functions from X to R.

Define the operator T as follows:

$$T: L(X_1 R) \to A(X_1 R)$$

$$\forall w \in C_B, \quad (Tw)(p,\theta) \equiv \max_{b' \in [0,\bar{b}]} U[\theta + b' - \min(\lambda\theta,p)] + \beta \int v(p',\theta')dF(\theta')$$

$$\text{where } p' = R(b')b'$$

Lemma 1 $T(L) \subseteq L$, that is, T maps bounded continuous functions into themselves

P. The solution to the above functional equation is bounded since u and w are bounded and the sum and integral of bounded functions are bounded. The objective function is a continuous function of the choice variable and the constraint set is clearly a continuous compact-valued correspondence in the choice variable since it is constant. Berge's

Maximum Theorem therefore ensures the solution is a continuous function of the state variables. #

Lemma 2   (i) The functional equation has exactly one bounded continuous solution, say $v \in L$

         (ii) Iterations on the value function converge at a geometric rate for any initial value function $v_0 \in L$, in the sense that:

$$\|T^n v_0 - v\|_\infty \le \beta^n \|v_0 - v\|_\infty \quad \text{for all } n$$

P. The proof amounts to a straightforward verification of the assumptions of the following two theorems:

I. The Contraction Theorem:

If (a) (Y,d) is a complete metric space and (b) $T: Y \to Y$ is a contraction of modulus β in the sense that $d(Ty,Ty') \le \beta d(y,y')$ for all $y,y' \in Y$ for some $B < 1$

  (i) $Tw = w$ has exactly one solution $v \in Y$

  (ii) For any $v_0 \in Y$ and all positive integers n,
      $d(T^n v_0, v) \le \beta^n d(v_0, v)$

II. Blackwell's sufficient conditions for a contraction

(Theorem 3, Blackwell (   ))

For the special case that Y is the Banach space of continuous functions bounded in the sup norm the following two conditions suffice to show T is a contraction of modulus β.

(a) <u>T is monotone</u>:

$$v,w \in L, \quad v(x) \leq w(x) \rightarrow (Tv)(x) \leq (Tw)(x) \text{ all } x \in L$$

(b) <u>T discounts at β</u>:

$$v,\gamma \in L \text{ where } \gamma(x) \equiv \bar{\gamma} \text{ for some } \bar{\gamma} \in R$$

$$T(v + \gamma)(x) = (Tv)(x) + \beta\bar{\gamma}$$

Thus to demonstrate the lemma it suffices to verify the assumptions (a) and (b) of II:

(a) If $v,w \in L$ and $v(x) \geq w(x)$ for all $x \in X$ than $(Tv)(x)$ is the maximized value of a <u>uniformly</u> higher objective function than is $(Tw)(x)$, so the value is higher

(b) $T(v + \gamma)(p,\theta) = \max\limits_{b \in [0,\bar{b}]} U[\theta + b' - \min(\lambda\theta,p)] + \beta\int (v(p',\theta') + \bar{\gamma})dF(\theta')$

$\qquad = \max\limits_{b' \in [0,b]} U[\theta + b' - \min(\lambda\theta + p)] + \beta\int v(p',\theta')dF(\theta') + \beta\bar{\gamma}$

$\qquad = T(v)(p,\theta) + \beta\bar{\gamma}$ #

<u>Lemma 3</u>   The unique solution v of $Tw = w$ is decreasing in p and increasing in θ.

P.   The fact that U is decreasing in p and increasing in θ implies for <u>any</u> function $w \in L$, Tw is decreasing in p and increasing in θ. But by definition $Tv = v$, so v is decreasing in p and increasing in θ.   #

However, like many other bankruptcy problems, the optimal value function is not concave in p. To see this let $G(p) = U[\theta + b' - \min(\lambda\theta, p)]$ then for fixed b' and θ, we can write G as the maximum of two concave functions, which is not necessarily concave:

$$G(p) = \max [u(\theta + b' - \lambda\theta), u(\theta + b' - p)]$$
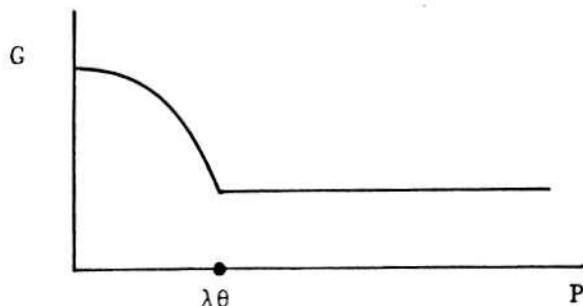
In order to elicit additional information about the value function and policy functions a more subtle approach is required. One approach is to find a problem for which solution of the above problem is both feasible and optimal which is concave in some set of set variables. One attempt is to use more general artificial contracts of the form (b,p,z) where $z = z(\phi) = $ the fraction of the penalty imposed and solve the problem

$$v(p,z,\theta) = \max U[\theta + b' - p(\phi) - z(\phi)\lambda\theta] + \beta\int v(p',z',\theta')dF(\theta')$$

$$(b',p',z') \in S'$$

where S' is the set of all contracts that satisfy the following:

(Incentive Compatibility)

1. $u[\theta + b' - p(\theta) - z(\theta)\lambda\theta] \geq U[\theta + b' - p(\phi) - z(\phi)\lambda\theta]$ all $\phi, \theta \in \Phi$

(Nonnegative Consumption)

2. $p(\theta)+z(\theta)\lambda\theta \leq \theta + b'$

(Actuarial Fairness)

3. $\int p(\theta)dF(\theta) \geq \bar{R}b'$

4. $0 \leq z(\theta) \leq 1$

and show the optimal z can only take on values 0 or 1 and interpret states such that $z(\theta)$ = default. I am currently working on such approaches.

Appendix C

This appendix establishes for the endowment model, if the penalty of default is a single period ban from the loan market defaults will occur only when output is high.

Follow my notation wherever possible, and let v^R and v^D denote the optimal value of repaying and defaulting respectively:

$$v^D(\theta) = U(\theta) + \beta \int v(0,\theta')dF(\theta')$$

$$v^R(p,\theta) = \max_{(b',p') \epsilon S} U(\theta + b' - p) + \beta \int v(p',\theta')dF(\theta')$$

where

$$v(p,\theta) = \max \{v^D(p,\theta), v^R(\theta)\}$$

Let the optimal policy functions be $b' = \hat{b}(p,\theta)$, $p' = \hat{p}(p,\theta)$. Clearly, v^R is decreasing in p, p^R and v^D are both increasing in θ. Fix $\theta = \theta_0$ and let p_0 be that level of repayment such that $v^D(\theta_0) = v^R(p_0,\theta_0)$. Substituting $\hat{b}_0 = \hat{b}(p_0,\theta_0)$, $\hat{p}_0 = \hat{p}(p_0,\theta_0)$ we obtain

$$U(\theta_0 + \hat{b}_0 - p_0) + \beta \int v(\hat{p}_0,\theta')dF(\theta') = U(\theta_0) + \beta \int v(0,\theta')dF(\theta')$$

For $\hat{p}_0 > 0$, since v is decreasing in p we obtain

$$\beta \int v(\hat{p}_0,\theta')dF(\theta') < \beta \int v(0,\theta')dF(\theta')$$

Thus $U(\theta_0 + \hat{b}_0 - p_0) > U(\theta_0)$ which by monotonicity of $U(\cdot)$ implies $\theta_0 + \hat{b}_0 - p_0 > \theta_0$. So with state variables (p_0,θ_0) the borrower is just indifferent between defaulting and repaying. Now consider decreasing

θ from θ_0 to θ_1, initially holding choice of contract $(\hat{b}_0, \hat{P}_0)$ fixed. Neither of future terms will change but the current V^D will decrease more than $V^R | \hat{b}_0, \hat{P}_0$
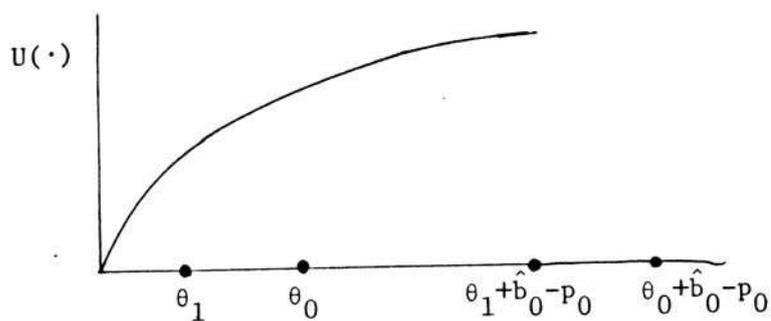
Now letting the new contract adjust optimally only increases the value of not defaulting. This establishes the conclusion.

FOOTNOTES

(1) Notable exceptions include the recent articles of Eaton and Gersovitz ( ), Sachs and Cohen( ) and Kahn ( ). This paper builds on the first two, especially the second. In particular, the form of the penalty function and the idea that IMF conditionality can be a means by which LDC's commit themselves to higher penalties are analogous. The latter idea has close ties to some ideas found in the literature on dynamic games.

(2) The spread on loans has risen perhaps less than might be expected. One possible explanation is that the spread on loans from one bank to a LDC is public information and could be interpreted by other banks as a signal about the riskiness of the loans to that country. Thus if some event occurs which causes loans to LDC to become more risky and this information is private to holders of LDC debt, then these holders may wish to increase total repayment on loans without signalling this information to other banks. By increasing some of the various fees of the loan, information about which is private to the LDC's and holders of current LDC's debt, this may increase total repayment without signalling increased riskiness. This suggests some of the information structure it would be interesting to include in a model of lender-driven default. An alternative explanation is that the banks believe they will probably be bailed out if a crisis occurs so such events in LDC's do not significantly decrease their expected repayment, and so don't significantly affect the riskiness of the loan and hence there is a need for only a small increase in spread.