Unemployment and Stabilization Policy in a Two-Sector, Two Country Aggregative Model

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I. Introduction

This paper analyzes the short-run influences exerted by monetary and fiscal policy and changes in other exogenous variables in a two-sector, two-country aggregative model with rigid money wages. The real side of the model is identical with that of the standard two-sector, two-country model of international trade extensively analyzed by Kemp [3],[1/] except that in each country one of the factors of production is explicitly identified as the stock of capital, with one industry in each country producing a flow of new capital goods.[2/] In contrast with prior studies, we assume that money wages are rigid in at least one of the countries. That assumption means that policy changes and other shocks are capable of producing movements in the rate of unemployment. So our analysis is in large part directed toward answering "Keynesian" questions about how one country's monetary and fiscal policies impinge both on its own level of employment and on its neighbor's. In this same Keynesian spirit, our analysis will be devoted exclusively to the "short-run", a period of time so short that the capital stock in each country is fixed. We regard time as continuous, so that for us "short-run" adjustments are those that occur instantaneously, our analysis being devoted to alternative momentary equilibria. But while both the time frame and the questions addressed by our analysis are "Keynesian" in character, the real side of our model is definitely not. In our model, there are within each country perfect

1/ For each good, the production function is linearly homogenous and identical across countries. Production functions for the two industries are different.

2/ The other good is a consumption good. This way of filling out the two-sector trade model is the standard one in analyses of growth and trade; e.g., see Uzawa [8] and Foley and Sidrauski [1].

markets in stocks of physical capital and labor, so that firms can instantaneously acquire the amounts of each factor that they desire. In contrast, the Keynesian model rules out instantaneous trading of capital by firms and, instead, posits flow investment demand schedules on the part of firms. It is known that in closed economy models, what one assumes about the presence or absence of a market in physical capital has important implications for the relative potency of monetary and fiscal policies (see Sargent and Wallace [6], and Henderson and Sargent [2]). Similarly, the properties of the open-economy model of this paper differ in important ways from those possessed by Robert Mundell's [4], a model that shares our assumptions about markets for paper claims, but that assumes "Keynesian" flow investment schedules rather than markets in stocks of physical capital. Like Mundell, we are interested in studying the implications of highly integrated financial markets, so that we follow him in making the extreme assumptions that "capital" (i.e., paper claims) is perfectly mobile and that households in each country regard the paper earning assets issued in the two countries as perfect substitutes.

In one respect, our model differs especially from Mundell's, since, contrary to his, our model, with money wages fixed in both countries, implies that the authorities cannot peg the exchange rate at an arbitrary level; for that would imply that one or the other of the two countries would be over-priced, implying incentives for both industries in that country to shut down and produce zero output. The possibility of such a situation has been described in a somewhat different context by Paul Samuelson [5].

Our model is but an open-economy version of the two-sector, closed-economy model described by Henderson and Sargent [2], the present model consisting of two different Henderson-Sargent economies operating side by side and interacting. In the closed-economy model, the assumption of two sectors with distinct capital-labor ratios was perhaps not central to the analysis, since the model continues to function in the limiting case in which capital intensities in the two industries are identical; in that case, the model collapses to a version of James Tobin's "Dynamic Aggregative Model" [7]. Here, however, the assumption of distinct capital intensities in the two sectors is essential, since the model would not possess a determinate solution were the capital intensities to be identical.

This paper describes two versions of the model. In the first version, money wages are fixed in both countries; here, the exchange rate must be flexible if both countries are to operate with positive levels of employment. In the second version, described briefly in the appendix, money wages are fixed in one country, while in the other they are flexible, being assumed to adjust instantaneously in order to assure full employment. In this version of the model, pegging the exchange rate is compatible with positive employment in both countries, so that we are free to analyze both flexible and fixed exchange rate regimes.

II. Description of Firms, Policy Making Authorities, and Households

Firms

Firms in the investment goods industry in country j (j=1,2) produce flows of I^j investment goods per unit time subject to the common linearly homogeneous production function

(1) $$I^j = N_I^j \, i(k_I^j) \qquad j = 1,2$$

where N_I^j is employment in the investment goods industry in country j, $k_I^j = K_I^j/N_I^j$, K_I^j is capital employed in the investment goods industry in country j; and $i(k_I^j)$ obeys $i'(k_I^j) > 0$, $i''(k_I^j) < 0$. Similarly, firms in the consumption goods industry in country j produce flows of consumption goods, C^j, subject to the common linearly homogeneous production function

$$(2) \qquad C^j = N_C^j c(k_C^j)$$

where N_C^j is employment in the consumption goods industry in country j, K_C^j is capital employed in the consumption industry in country j, $k_C^j = K_C^j/N_C^j$; and $c(k_C^j)$ obeys $c'(k_C^j) > 0$, $c''(k_C^j) < 0$. Notice that $i(\cdot)$ in (1) and $c(\cdot)$ in (2) are independent of j and so are identical across countries.

Within each country there are perfectly competitive factor markets in which firms can purchase or rent all the capital and labor that they desire at each moment. Firms in country j can hire all the labor they want at the money wage w^j, denominated in units of j's currency per man per unit time, and they can rent all the capital they want at the gross rental rate ϵ^j, measured in units of j's currency per unit of capital per unit time. Firms in the consumption goods industry in country j are perfect competitors who can sell all the output they want at the price P_C^j, which is denominated in j's currency per unit of consumption goods. Firms in the investment goods industry can sell all the investment goods they want at P_I^j. Capital and labor are perfectly mobile between industries within a country, but cannot move across borders. Firms in each industry in each country choose levels of employment and capital so as to maximize profits, which leads to the marginal conditions:

$$(3)\qquad \frac{w^j}{P_C{}^j} = c(k_C{}^j) - k_C{}^j c'(k_C{}^j) \qquad j = 1,2$$

$$(4)\qquad \frac{w^j}{P_I{}^j} = i(k_I{}^j) - k_I{}^j i'(k_I{}^j) \qquad j = 1,2$$

$$(5)\qquad \frac{\varepsilon^j}{P_C{}^j} = c'(k_C{}^j) \qquad j = 1,2$$

$$(6)\qquad \frac{\varepsilon^j}{P_I{}^j} = i'(k_I{}^j) \qquad j = 1,2$$

Equations (3), (4), (5), and (6) require equality between the marginal products of factors and the corresponding real rental rates in each industry in each country.

Firms in each industry in each country finance their investment by issuing equities and so have equities as their only liabilities. Households in both countries are assumed to view these equities as perfect substitutes for the bonds of both governments, which are themselves perfect substitutes from the viewpoint of households. This means that the real yield on equities in both countries must equal the common real yield on bonds, $r-\pi$, where r is the nominal rate of interest on government bonds, and where π is the public's expected rate of inflation, assumed to be the same for all prices and wages in both countries. That equality follows by virtue of the assumption that bonds of the two governments are perfect substitutes. Firms retain no earnings, paying out their entire net cash flows as dividends. Under these assumptions, the value of the equities of firms producing consumption goods in country j is

$$(7)\qquad V_C{}^j = \frac{P_C{}^j C^j - w^j N_C{}^j}{r-\pi}.$$

Arbitrage requires that

$$(8) \qquad \varepsilon^{j} = P_I^{\,j}(r-\Pi),$$

so that the rental rate on capital equals the real rate of interest on bonds or equities times the value of a unit of capital, since the latter is what a dealer in capital could afford to rent it at. Together with the marginal conditions and the linear homogeneity of the production function, (7) and (8) imply

$$(9) \qquad V_C^{\,j} = P_I^{\,j} K_C^{\,j} \qquad\qquad j = 1,2.$$

Similarly, the value of equities of investment goods producers satisfies

$$(10) \qquad V_I^{\,j} = P_I^{\,j} K_I^{\,j} \qquad\qquad j = 1,2.$$

Our assumptions that prices and interest rates are flexible instantaneously imply that in each country capital is fully employed,

$$K^{j} = K_C^{\,j} + K_I^{\,j}.$$

Since money wages are rigid, however, labor need not be fully employed.

We assume that there are very small transportation costs associated with the movement of a unit of newly produced investment goods across borders. Thus, if the price of investment goods produced in country one is the same as the price of investment goods produced in country two when both are measured in the same currency, there is no trade in flows of investment goods. This assumption will be seen to play no important role in our static analysis, as it only serves to allocate world investment between the two countries. However, in a dynamic analysis, such an assumption obviously would be a pivotal one (see Uzawa [8]).

Fiscal Authorities and Central Banks

The fiscal authority in country j has outstanding as liabilities a stock of bonds, $\bar{B}^j$, denominated in units of country j's currency. The bonds issued in both countries have a common variable nominal interest rate, r, and are fixed in nominal value, like savings deposits and call loans. Each fiscal authority makes purchases of consumption goods at the real rate per unit time, G^j, subject to the flow budget constraint

$$(11) \qquad P_C^{\ j} G^j = P_C^{\ j} T^j + \dot{\bar{B}}^j$$

where a dot over a variable denotes a derivative with respect to time, and T^j denotes total real tax collections, net of transfers, denominated in units of the consumption good per unit time.

The central bank in country j has as a liability the stock of money, M^j, which bears no nominal interest. As assets, the central bank of country j holds an amount of home government bonds, B^j, an amount, R^j, of international reserves denominated in the international unit of account (e.g., pounds of gold), and an amount of foreign government bonds, F^j. The value of the central banks' international reserves in units of domestic currency is $\sigma^j R^j$, while the value of its holdings of foreign bonds is $e^j F^j$, where $e^1 \equiv \sigma^1/\sigma^2$, and $e^2 = (e^1)^{-1}$. The variable e^j is the exchange rate, the price of foreign currency in terms of country j's currency. Central bank j's balance sheet appears in Table 1.

Assets	Liabilities
B^j	M^j
$\sigma^j R^j$	
$e^j F^j$	

Table 1

Central Bank Balance Sheet

At a point in time, the central bank can conduct stock exchanges or "open market operations" subject to the constraint

$$(12) \qquad dM^j = dB^j + \sigma^j dR^j + e^j dF^j$$

which in the absence of intervention in the foreign assets markets ($dR^j = dF^j = 0$) becomes the familiar closed economy constraint on open market operations, $dM^j - dB^j = 0$.

The central bank can also conduct flow exchanges subject to the constraint

$$(13) \qquad \dot{M}^j = \dot{B}^j + \sigma^j \dot{R}^j + e^j \dot{F}^j$$

Consolidating the flow constraints on the fiscal authority and the central bank of country j yields

$$(14) \qquad P_C(G^j - T^j) = \dot{M}^j + (\dot{\bar{B}}^j - \dot{B}^j) - \sigma^j \dot{R}^j - e^j \dot{F}^j$$

which states that the deficit of the fiscal authority can be financed by money creation, by selling bonds to an entity other than the central bank, or by a running down of the consolidated government's foreign assets.

Households

Households in countries 1 and 2 have nominal wealth, W^1 and W^2, each denominated in units of domestic currency, given by

$$W^1 = M^1 + \bar{B}^1 + P_I^1K^1 - B^1 - E^2 - F^2 + e^1E^1$$
$$W^2 = M^2 + \bar{B}^2 + P_I^2K^2 - B^2 - E^1 - F^1 + e^2E^2$$

where E^j is the amount, denominated in units of foreign currency of foreign bonds and equities held by residents of country j. Only residents of country j are assumed to want to hold its money. In each country, households regard both foreign and domestic bonds and equities all as perfect substitutes. Accordingly, portfolio equilibrium can be described by equality for each country between the supply of real balances, M^j/P^j, denominated in units of the consumptions good, and the demand, $m(r,Y^j)$,

(15) $$M^j/P_C{}^j = m(r,Y^j) \qquad m_r < 0,\ m_{Y^j} > 0,$$

where $Y^j = P^jI^j + C^j$ is real GNP denominated in units of the consumption good and $P^j = P_I{}^j/P_C{}^j$.

Households in each country also make a consumption decision, which we suppose can be described by a Keynesian consumption function $Z^j(Y^j-T^j)$ that shows the rate individuals in country j demand consumption goods per unit time. Equilibrium in the world market for consumption goods obtains when flow demand equals supply

(16) $$C^1 + C^2 = Z^1(Y^1-T^1) + Z^2(Y^2-T^2) + G^1 + G^2$$

There is free trade in consumption goods, which insures that

(17) $$P_C{}^1 = e^1P_C{}^2,$$

so that the price of consumption goods in terms of country j's currency does not depend on the country of origin of the goods.

III. Behavior of the Model

It is convenient to collect the equations of our model here in the following form:

(18) $C^1 + C^2 = Z^1(Y^1-T^1) + Z^2(Y^2-T^2) + G^1 + G^2$

(19) $Y^j = P^j I^j + C^j$ $\quad j = 1,2$

(20) $I^j = K_I{}^j/\beta_I{}^j$ $\quad j = 1,2$

(21) $\beta_I{}^j = k_I{}^j/i(k_I{}^j)$ $\quad j = 1,2$

(22) $C^j = K_C{}^j/\beta_C{}^j$ $\quad j = 1,2$

(23) $\beta_C{}^j = k_C{}^j/c(k_C{}^j)$ $\quad j = 1,2$

(24) $K^j = K_I{}^j + K_C{}^j$ $\quad j = 1,2$

(25) $M^j = P_C{}^j m^j(r,Y^j)$ $\quad j = 1,2$

(26) $w^j = P_C{}^j[c(k_C{}^j) - k_C{}^j c'(k_C{}^j)]$ $\quad j = 1,2$

(27) $w^j = P^j P_C{}^j[i(k_I{}^j) - k_I{}^j i'(k_I{}^j)]$ $\quad j = 1,2$

(28) $P^j(r-\pi) = c'(k_C{}^j)$ $\quad j = 1,2$

(29) $r-\pi = i'(k_I{}^j)$ $\quad j = 1,2$

(30) $P_C{}^1 = e^1 P_C{}^2$

This is a system of 24 equations in the 24 endogenous variables e^1, r, and for j = 1,2, C^j, I^j, Y^j, $P_C{}^j$, P^j, $\beta_I{}^j$, $\beta_C{}^j$, $k_C{}^j$, $k_I{}^j$, $K_I{}^j$, and $K_C{}^j$. The exogenous variables are M^j, w^j, K^j, G^j, T^j (for j = 1,2) and π.

Equations (26), (27), (28), and (29) for each j are four equations in the endogenous variables P_C^j, P^j, r, k_C^j, and k_I^j. In our earlier paper we showed how these four equations can be used to solve for P_C^j, P^j, k_C^j, and k_I^j each in terms of $r-\pi$ and w^j, thereby obtaining the following four relationships:

$$(31) \qquad k_I^j = k_I(r-\pi) \quad , \qquad k_I' = \frac{1}{i''} < 0$$

$$(32) \qquad k_C^j = k_C(r-\pi) \quad , \qquad k_C' = \frac{1}{c''} \; \frac{i}{c} \; \frac{(c')}{(i')^2} < 0$$

$$(33) \qquad P^j = P(r-\pi) \quad , \qquad P' = \frac{P^2}{c}(k_I - k_C) \gtrless 0 \text{ as } k_I \gtrless k_C$$

$$(34) \qquad P_C^j = P_C^j(r-\pi, w^j) \quad , \qquad P_{C1}^j = \frac{P_C^j P^j}{w^j} \; \frac{ik_C}{c} > 0, \; P_{C2}^j = \frac{P_C^j}{w^j} > 0$$

Since i(•) and c(•) are the same functions for each country, the four functions above are also the same for both countries; and since there is a common worldwide real yield, $r-\pi$, on all paper earning assets, equations (31), (32), and (33) imply that $k_I^1 = k_I^2$, $k_C^1 = k_C^2$, and $P^1 = P^2$. Henceforth, we will take advantage of these equalities and omit the superscripts from k_I, k_C, and P.

Combining (21) with (31) and (23) with (32), we obtain

$$(35) \qquad \beta_I^j = \beta_I(r-\pi) \quad , \qquad \beta_I' = \frac{w^j}{P_C^j P} \; \frac{1}{i^2} \, k_I' < 0$$

$$(36) \qquad \beta_C^j = \beta_C(r-\pi) \quad , \qquad \beta_C' = \frac{w^j}{P_C} \; \frac{1}{c^2} \, k_C' < 0.$$

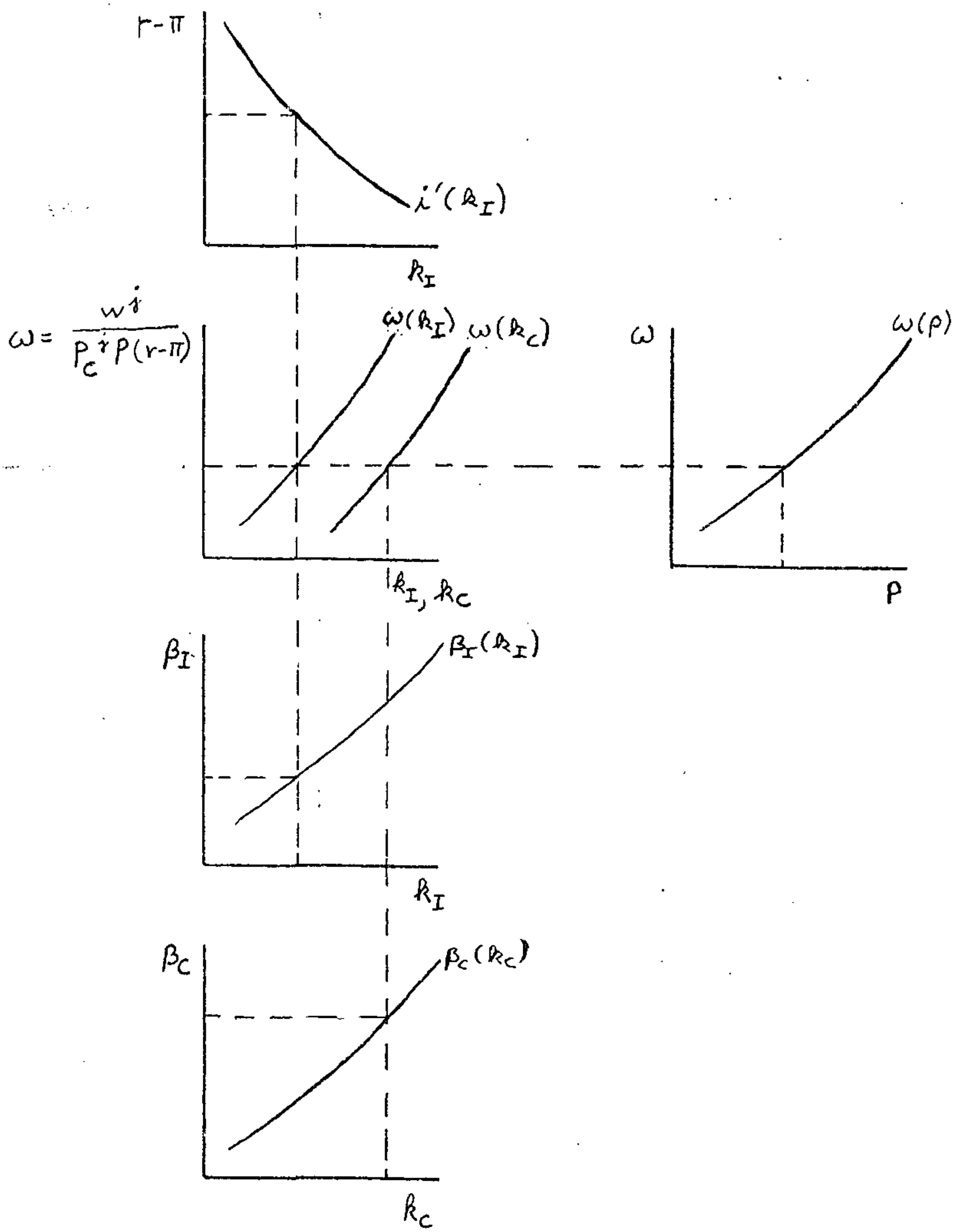

Figure 1

Again, since the functions in (35) and (36) are independent of j, the existence of a common worldwide real interest rate, $r-\pi$, implies that $\beta_I^1 = \beta_I^2$, $\beta_C^1 = \beta_C^2$, equalities that justify our dropping the superscripts from β_I and β_C subsequently.

The relationships leading to equations (31) through (36) are illustrated in Figure 1. Panel (a) plots equation (29) and depicts an inverse relationship between the real rate of interest and k_I. Panel (b) shows how the wage-rental ratio, $\omega = w^j/[P_C^j p^j(r-\pi)]$, must vary directly with both k_C and k_I, since an increase in k_I, for example, increases the marginal product of capital in the investment good industry. Panel (c) depicts the relationship between ω and P that must obtain for both industries just to be able to cover their costs, that is, to earn zero economic profits. The slope of this relationship depends on relative capital intensities. When the wage-rental ratio ω increases, the relative price of the more labor intensive good must rise if both industries are to continue to break even. Thus, the relationship in panel (c) slopes upward if $k_C > k_I$ and downward if $k_C < k_I$. Panels (d) and (e) record the fact that increases in capital-labor ratios lead to increases in capital-output ratios.

These five panels define one mapping from $r-\pi$ to k_I; another (composite) mapping from $r-\pi$ to ω to k_C; another from $r-\pi$ to k_I to ω to P; another from $r-\pi$ and w^j to P_C^j; another from $r-\pi$ to k_I to β_I; and finally, another from $r-\pi$ to ω to k_C to β_C. These six mappings are embodied in equations (31) through (36).

The existence of the mappings (32) and (34) prohibits using the model with the exchange rate e^1 as an exogenous variable pegged by one government. For as (32) shows, the existence of a single world-wide real interest rate $r-\pi$ implies identical capital-labor ratios k_C in the consumption goods industries. But then dividing (26) for j=1 by (26) for j=2 gives

$$\frac{w^1}{w^2} = \frac{P_C^{\ 1}}{P_C^{\ 2}} ,$$

which, when substituted into (30) shows that the exchange rate must obey

$$e^1 = \frac{w^1}{w^2} .$$

The above equation determines e^1 solely from reference to the arbitrage condition (30) and the marginal conditions (26)-(29). The governments do not have the option of choosing e^1 as one of their instruments, even though the model would continue to have 24 equations and 24 endogenous variables if e^1 were made exogenous and either M^1 or M^2 were made endogenous. For the above equations for e^1 cannot, in general, hold where w^1, w^2, and e^1 are all viewed as exogenous. Were one government to attempt to peg e^1 at a level violating the above equality, one country or the other would be overpriced in markets for both consumption and investment goods. Presumably, production in that country would cease. Samuelson [5] has alluded to this possibility in pointing out that under the conditions for factor price equalization, it is possible for a country to find itself priced out of all markets if its exchange rate is wrong.

We can write equation (24), the condition for equilibrium in the market for the existing stock of capital in country j, in the form

$$K^j = \beta_I(r-\Pi)I^j + \beta_C(r-\Pi)C^j \qquad j = 1,2$$

Solving (19) for I^j, substituting into the above equation, and rearranging, we obtain

$$(37) \qquad PK^j = \beta_I(r-\Pi)Y^j + [P(r-\Pi)\beta_C(r-\Pi) - \beta_I(r-\Pi)]C^j, \qquad j = 1,2$$

an equation that states that the supply of capital goods in country j, measured in units of the consumption good, must equal the demand for capital goods measured in the same units. Equation (37) shows the combinations of the endogenous variables Y^j, C^j, and $r-\Pi$, given K^j, that guarantee equilibrium in the market for the stock of physical capital in country j. In our earlier paper, we showed that

$$P\beta_C - \beta_I = \frac{w^j}{P_C{}^j}\ \frac{1}{ci}\ (k_C - k_I),$$

so that $P\beta_C - \beta_I \gtrless 0$ as $k_C \gtrless k_I$.

To obtain the combinations of r and Y^j consistent with both portfolio equilibrium and profit maximization in country j, we substitute (34) into (25) to obtain

$$(38)\qquad M^j = P_C{}^j(r-\Pi, w^j)m(r, Y^j). \qquad\qquad j = 1,2.$$

Equation (38) is identical with the MM curve of our earlier paper. Below we assume that the MM curves are upward sloping in the r, Y^j planes.

Equations (37) and (38) for j = 1,2, and equation (18), the equilibrium condition in the flow market for consumption goods, form a system of five equations in the five endogenous variables Y^1, Y^2, C^1, C^2, and r, a system that will, under certain circumstances, determine those five endogenous variables. For convenience, we rewrite these five equations here:

$$(37i)\qquad P(r-\Pi)K^1 = \beta_I(r-\Pi)Y^1 + [P(r-\Pi)\beta_C(r-\Pi) - \beta_I(r-\Pi)]C^1,$$

$$(37ii)\qquad P(r-\Pi)K^2 = \beta_I(r-\Pi)Y^2 + [P(r-\Pi)\beta_C(r-\Pi) - \beta_I(r-\Pi)]C^2,$$

(38i) $M^1 = P_C^{\ 1}(r-\Pi, w^1)m^1(r, Y^1)$,

(38ii) $M^2 = P_C^{\ 2}(r-\Pi, w^2)m^2(r, Y^2)$,

(18) $C^1 + C^2 = Z^1(Y^1-T^1) + Z^2(Y^2-T^2) + G^1 + G^2$.

It is easy to see that if $k_I = k_C$, implying that $P\beta_C - \beta_I = 0$, then the model possesses no solution. For when $P\beta_C - \beta_I = 0$, C^1 and C^2 fail to appear in equations (37i) and (37ii), respectively, causing our five equation system to decompose in an unfortunate way. When $P\beta_C - \beta_I = 0$, (37i) and (38i) become two equations in Y^1 and r that are capable of determining those two variables. Similarly, equations (37ii) and (38ii) become two equations in Y^2 and r that are capable of determining these two variables. The variable r is therefore over-determined since the two solutions for r are independent. On the other hand, C^1 and C^2 are both under-determined, since they only appear in equation (18), an equation which determines the sum $C^1 + C^2$, given Y^1 and Y^2, but which leaves both C^1 and C^2 undetermined individually. All of this indicates, once again, the dangers of naive equation counting as a device supposedly capable of ascertaining the capacity of n equations to determine n variables.

Where capital intensities are not equal in the two industries, so that $P\beta_C - \beta_I \neq 0$, the system formed by our five equations may possess a solution. To solve the model, we begin by adding together (37i) and 37ii), and use (18) to eliminate $C^1 + C^2$:

(39) $P(K^1+K^2) = \beta_I(Y^1+Y^2) + (P\beta_C - \beta_I)[Z^1(Y^1-T^1) + Z^2(Y^2-T^2) + G^1 + G^2]$,

where P, β_C and β_I are each functions of $(r-\Pi)$. Taking the total differential of equation (39), we obtain

$$\left\{P'(K_I^1 + K_I^2) - P\beta_I'(I^1 + I^2) - P\beta_C'(C^1 + C^2)\right\} (dr - d\pi) =$$

(40) $$\left[\beta_I + (P\beta_C - \beta_I)Z^{1'}\right]dY^1 + \left[\beta_I + (P\beta_C - \beta_I)Z^{2'}\right]dY^2$$

$$+ (P\beta_C - \beta_I)\left[dG^1 + dG^2 - Z^{1'}dT^1 - Z^{2'}dT^2\right] - P(dK^1 + dK^2).$$

Solving the total differential of the MM curve (38) for dY^j gives

(41) $$m_Y^j dY^j = \frac{dM^j}{P_C^j} - \left[m_r^j + \frac{M^j}{P_C^{j\,2}} P_{C1}^j\right] dr + \frac{M^j}{P_C^{j\,2}} P_{C1}^j\, d\pi - \frac{M^j}{P_C^{j\,2}} P_{C2}^j\, dw^j.$$

Substituting for dY^j from (41) into (40) gives the differential for the reduced form of $(r - \pi)$:

(42) $$\sum_{j=1}^{2} \left\{[\beta_I(1 - Z^{j'}) + P\beta_C Z^{j'}]\left[\frac{P'K_I^j - P\beta_I' I^j - P\beta_C' C^j}{(1 - Z^{j'})\beta_I + P\beta_C Z^{j'}} + \frac{m_r^j + \frac{M^j}{P_C^{j\,2}} P_{C1}^j}{m_Y^j}\right]\right\} (dr - d\pi)$$

$$= \sum_{j=1}^{2}\left(\frac{\beta_I(1 - Z^{j'}) + P\beta_C Z^{j'}}{m_Y^j}\right)\frac{dM^j}{P_C^j} + \left\{\sum_{j=1}^{2}\left(\frac{\beta_I(1 - Z^{j'}) + P\beta_C Z^{j'}}{m_Y^j}\right)\frac{M^j}{P_C^{j\,2}} P_{C1}^j\right\} d\pi$$

$$- \sum_{j=1}^{2}\left(\frac{\beta_I(1 - Z^{j'}) + P\beta_C Z^{j'}}{m_Y^j}\right)\frac{M^j}{P_C^{j\,2}} P_{C1}^j\, dw^j$$

$$+ (P\beta_C - \beta_I)(dG^1 + dG^2 - Z^{1'}dT^1 - Z^{2'}dT^2) - P(dK^1 + dK^2)$$

Since $1 > Z^{j'} > 0$, we know that $\beta_I(1 - Z^{j'}) + P\beta_C Z^{j'} > 0$. Now the remaining two terms in brackets on the left side of the above equation are related to the slopes of the single country MM and KK curves of our earlier paper. Indeed,

$$\frac{P'K_I^j - P\beta_I' I^j - P\beta_C' C^j}{(1 - Z^{j'})\beta_I + P\beta_C Z^{j'}} = \frac{1}{\left.\frac{d(r - \pi)}{dY_j}\right|_{\text{KK curve}}}$$

$$\frac{m_r^j + \frac{M^j}{P_C^{j^2}} P_{CI}^j}{m_Y^j} = - \frac{1}{\left.\frac{d(r-\pi)}{dY^j}\right|_{\text{MM curve}}}$$

In our earlier paper, we showed that a necessary condition for stability of the single-country model is

$$- \frac{1}{\left.\frac{d(r-\pi)}{dY^j}\right|_{MM}} + \frac{1}{\left.\frac{d(r-\pi)}{dY^j}\right|_{KK}} > 0,$$

a condition that implies, for example, that if both the MM curve and KK curve are positively sloped, the MM curve must be steeper. We assume that this single country stability condition is satisfied in each country, which is sufficient to imply that the coefficient on $(dr-d\pi)$ in equation (42) is positive. The coefficients on dM^1, dM^2, and $d\pi$ in (42) are unambiguously positive, while the coefficients on dw^1 and dw^2 are unambiguously negative. However, $P\beta_C-\beta_I \gtrless 0$ when $k_C \gtrless k_I$, so that the coefficients on dG^1, dG^2, dT^1 and dT^2 all depend on relative capital intensities.

Once $d(r-\pi)$ is determined from equation (42), the differentials of k_I, k_C, P, and P_C^j are determined from equations (31)-(34); Y^1 and Y^2 are determined by the MM curves (38i) and (38ii), respectively; then C^1 and C^2 are determined from (37i) and (37ii), respectively, while I^1 and I^2 can be obtained from (19).

We are now in a position to analyze the effects of changes in the exogenous variables:

a. <u>An increase in the money supply in country 1</u>: From the reduced form equation (42), an increase in M^1 drives $(r-\pi)$ upward. From the MM curves (38i) and (38ii), we have it that the rise in $(r-\pi)$ causes a rise in both Y^1 and Y^2, so long as both MM curves are upward sloping in the r,Y^j plane. The rise in $(r-\pi)$ causes k_I and k_C both to decline, by virtue of equations (31) and (32), while $P_C{}^1$ and $P_C{}^2$ both rise, by virtue of equation (34). The relative price of rises if $k_I > k_C$ and falls if $k_C > k_I$. The total differential of (37i) or (37ii) is

$$(P\beta_C - \beta_I)dC^j = - \beta_I dY^j + [P'K_I^j - P\beta_I'I^j - P\beta_C'C^j](dr - d\pi) + PdK^j$$

We suppose that the coefficient on $(dr-d\pi)$ is positive; but, since $P\beta_C - \beta_I \gtrless 0$ as $k_C \gtrless k_I$, and since the coefficients on dY^j and $d(r - \pi)$ are of opposite signs, the sign of the change in C^j is ambiguous. The sign of dI^j is therefore also ambiguous.

The pressures propelling the system from the old equilibrium to the new one can be summarized as follows. At the old levels of Y^1 and $r-\pi$, the increase in M^1 creates an excess supply of money in country 1. Households respond by bidding downward the real rate of interest $r-\pi$ on domestic and foreign paper earning assets. But that makes it profitable for firms in both industries and both countries to employ more capital intensive techniques. They respond by bidding for capital, causing the money prices of capital in each country $P_C{}^jP$ to rise. That creates upward pressure on $P_C{}^j$ also, if all firms are to continue just to cover costs at the fixed money wages. The rise in $P_C{}^j$ reduces the real wage and increases the level of employment, and therefore the output that profit maximizing firms want to produce. It also reduces the supply of real balances. Both of these effects tend to create an excess demand for real balances, causing $r-\pi$ to rise. But the rise in $P_C{}^j$ also increases the $r-\pi$ at which the markets for physical capital clear. For "stability" it is required that as $P_C{}^j$ and hence world output and employment increase, the portfolio balancing $r-\pi$ increase more rapidly than the capital market clearing $(r-\pi)$. That will occur if the coefficient on $(dr-d\pi)$ in the above differential of the reduced form for $(r-\pi)$, equation (42), is positive. This guarantees that the portfolio-balancing $r-\pi$ eventually "catches up" with the capital market clearing $r-\pi$, restoring equilibrium at a new and higher worldwide value of $r-\pi$. In the process, prices of both goods in both countries have been bid upward, implying general expansions in world output and employment.

While in our model a monetary expansion in one country leads to increases in output and employment in both countries, in models of the type explored by Mundell [4] a monetary expansion in one country results in a contraction abroad under flexible exchange rates. In a two-country, flexible exchange rate model with one world good and perfect capital mobility but with "Keynesian" flow investment schedules in both countries, it can be shown that an expansion in the money supply in one country lowers the world interest rate, depreciates the currency of the expanding country, raises the price level and economic activity in the expanding country, and lowers the price level and economic activity in the other country. The primary reason for the difference in results between our model and Mundell-type models is that we assume perfect markets in stocks of physical capital, thereby ruling out flow investment schedules.

b. <u>An increase in expected inflation, π</u>: An increase in π causes $r-\pi$ to rise, thereby causing Y^1 and Y^2 to rise if both MM curves are upward sloping. The effects on P, $P_C{}^j$, k_I, k_C, C^j and I^j all parallel those associated with an increase in M^j.

c. <u>An increase in the money wage in country 1</u>: As in the usual analysis of the "Keynes effect", an increase in w^1 operates exactly like a decrease in M^1, in this case sparking downward movements in Y^1, Y^2 and $r-\pi$. Cost push inflation in one country is "exported" to the other.

d. <u>An increase in government expenditures in country 1</u>: An increase in G^1 increases $(r-\pi)$ if $P\beta_C - \beta_I > 0$, i.e., if $k_C > k_I$ and decreases $(r-\pi)$ if the inequalities are reversed. With upward sloping MM curves the sign of the changes in Y^1 and Y^2 match the sign of the change in $r-\pi$. Thus, as in the closed economy model, whether or not an increase in government purchases is expansionary (i.e., causes rises in $(r-\pi)$, $P_C^{\,j}$, and Y^j) depends on relative capital intensities. This result is different from those obtained from Mundell type models in which an increase in government expenditure in one country is unambiguously expansionary both at home and abroad under flexible exchange rates. Notice that from (42) the effects of an increase in G^1 are identical with the effects of an increase in G^2 of identical size.

e. An increase in tax collections in country 1: An increase in $Z^{1'} dT^1$ operates exactly like a decrease in G^1 of equal magnitude.

There exists a convenient graphical apparatus for analyzing the model for the special case in which marginal propensities to consume are identical in the two countries, so that $Z^{1'} = Z^{2'}$. In this case, equation (39) becomes a worldwide KK curve in the $(Y^1 + Y^2)$, $r-\pi$ plane. Its differential is

$$d(Y^1 + Y^2) = (\beta_I + (P\beta_C - \beta_I)Z')^{-1}\{[P'(K^1 + K^2) - P\beta_I'(I^1 + I^2) - p\beta_C'(C^1 + C^2)]d(r-\pi)$$
$$+ P(dK^1 + dK^2) - (P\beta_C - \beta_I)(dG^1 + dG^2 - Z^{1'}dT^1 - Z^{2'}dT^2)\},$$

which is completely analogous to the differential of the single-country KK curve, with own-country magnitudes being replaced by worldwide totals. On our assumptions, the worldwide KK curve is upward sloping in the $Y^1 + Y^2$ plane, as shown in Figure 2.

The own country MM curves, equations (38) can be solved for Y^j and added together (horizontally in the $(Y^1 + Y^2, r-\pi)$ plane) to derive a worldwide MM curve. Its idfferential is

$$d(Y^1 + Y^2) = \sum_{j=1}^{2} \frac{1}{m_Y^{\,j}} \left[\frac{dM^j}{P_C^{\,j}} + (m_r^j + \frac{M^j}{P_C^{\,j\,2}} P_{C1}^{\,j}\, d(r - \pi) - m_r^j d\pi - \frac{M^j}{P_C^{\,j\,2}} P_{C1}^{\,j}\, dw^j\right].$$

The worldwide MM curve is depicted in Figure 2c, while the two individual country MM curves are in Figures 2a and 2b.

Finally, for fixed values of C^j, equations (37i) and (37ii) show the combinations of $r-\pi$ and Y^j that guarantee that capital in country j is fully employed, and so are in the nature of "capital equilibrium" curves. We dub these curves the KE curves in the Y^j, $r-\pi$ planes. Their differentials are given by

$$dY^j = \beta_I^{-1}(P'K_I^j - P\beta_I' I^j - P\beta_C' C^j)d(r - \pi) - \beta_I^{-1}(P\beta_C - \beta_I)dC^j - \beta_I^{-1}PdK^j.$$

On our assumptions, the curves are upward sloping; whether an increase in C^j shifts the curves upward or downward depends on the sign of $P\beta_C - \beta_I$, and so on relative capital intensities. The KE curves are plotted against the own country MM curves in Figures 2a and 2b.

The model works as follows: $Y^1 + Y^2$ and $r-\pi$ are determined at the intersection of the worldwide KK and MM curves, which appear in panel c. Then, given the $(r-\pi)$ so determined, Y^1 and Y^2 are read off the MM^1 curve and the MM^2 curve, respectively. Finally, C^1 and C^2 must adjust to make the KE^j curve at the $(r-\pi)$ and Y^j already determined. How the individual C^j's must adjust in response to any given disturbance depends partly on relative capital intensities as we have seen above.

This graphical apparatus helps emphasize why capital intensities must differ across industries if the model is to possess a solution. For if $(P\beta_C - \beta_I) = 0$, then C^j does not appear as a parameter of the KE^j curve, and there is available no device to move the KE^j curves through the equilibrium $r-\pi$, Y^j combination determined by the worldwide KK and MM curves and the MM^j curve.

IV. The Balance of Trade

In this model, the balance of trade is a residual and plays no active role in determining the current values of any endogenous variables. The model does

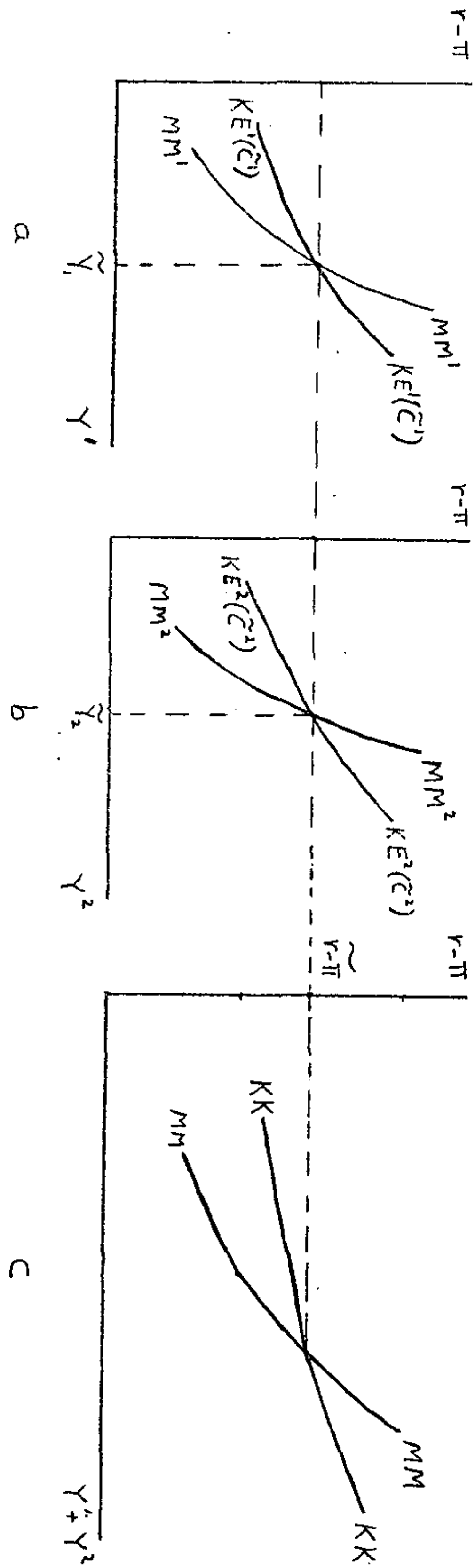

FIGURE 2

possess a device that guarantees that savers will be reconciled to accumulating foreign assets at just the rate of the balance of trade. Thus, households in country 1 desire to save at the rate

$$S^1 = Y^1 - Z^1(Y^1 - T^1) - T^1$$

$$S^1 = P^1 I^1 + C^1 - Z^1(Y^1 - T^1) - T^1.$$

Substituting for T^1 from the government budget constraint gives

$$S^1 = P^1 I^1 + \frac{\dot{M}^1}{P_C^{\,1}} + \frac{\dot{\bar{B}}^1 - \dot{B}^1}{P_C^{\,1}} - \frac{\sigma^1 \dot{R}^1}{P_C^{\,1}} - e^1 \frac{\dot{F}^1}{P_C^{\,1}} + \{C^1 - Z^1(Y^1 - T^1) - G^1\},$$

where the term in braces is country 1's balance of trade. For the case in which the authority in country 1 does not intervene in the foreign asset market, $\dot{R}^1 = \dot{F}^1 = 0$. The above equality asserts that in equilibrium the rate at which households desire to save equals the rate at which they are actually accumulating domestic assets plus the rate that they are actually accumulating claims on foreigners. Notice that by equating S^1 to $\dot{W}^1/P_C^{\,1}$ and rearranging, it follows that the balance of trade obeys the equality

$$P_C^{\,1}[C^1 - Z^1(Y^1 - T^1) - G^1] = \sigma^1 \dot{R}^1 + e^1 \dot{F}^1 + e^1 \dot{E}^1 - \dot{E}^2 - \dot{F}^2$$

V. Conclusion

This paper has analyzed the standard two-sector, two-country model of international trade, suitably extended with the addition of portfolio balance conditions, under the Keynesian condition that money wages are rigid so that there is less than full employment in each country. The model turns out to be quite "monetarist" in character. For one thing, the model embodies a pure purchasing power parity theory, since the exchange rate must equal the ratio of money wages in the two countries. Open market operations in one country create upward pressures on the real interest rate and cause prices in both countries to rise, thereby sparking expansions in world output and employment. By contrast, expansions in the government deficit in either country have ambiguous effects on interest, output, and employment. The nature of this model thus contrasts rather sharply with the Keynesian model described by Mundell [4]. A key difference between our model and Mundell's is that in ours perfect markets in the stocks of physical capital in each country ensure that the marginal productivity of capital equals the real rate of interest, thereby ruling out the possibility of a Keynesian investment demand schedule and hence a Keynesian "IS" curve. In our model, the market in physical capital occupies a key position in the mechanism by which policy variables impinge on the economy. Policies work by threatening to open gaps between the marginal product of capital and the real rental on capital, thereby prompting producers to bid the price of existing physical capital up or down.

APPENDIX

The Model with Full Employment in Country 1

In this appendix we sketch the structure of the model under the assumption that N^1 is exogenous and w^1 is endogenous, there being full employment of labor at all times in country 1. That is the only change that we make in the specification of the model.

Defining the output-labor ratios

$$\lambda_I = N_I/I = \frac{1}{i(k_I)}$$

$$\lambda_C = N_C/C = \frac{1}{c(k_C)}$$

it follows from (29) and (30) that λ_I and λ_C are both functions of $(r-\pi)$:

$$\lambda_I = \lambda_I(r-\pi) \qquad \lambda_I' = \frac{-\dot{i}k_I^1}{i^2} > 0$$

$$\lambda_C = \lambda_C(r-\pi) \qquad \lambda_C' = \frac{-\dot{c}k_C^1}{c^2} > 0 .$$

For full employment of labor in country 1, it must be so that

$$N^1 = N_I^1 + N_C^1 = I^1\lambda_I + C^1\lambda_C.$$

Using the definition of GNP, equation (19), to express I^1 in terms of C^1, Y^1, and P, and rearranging gives

$$(43) \qquad PN^1 = \lambda_I Y^1 + (P\lambda_C - \lambda_I)C^1,$$

which is one equation in $r-\pi$, Y^1, and C^1. It is easy to show that[1/]

$$P\lambda_C-\lambda_I = P(r-\pi)\lambda_I\lambda_C(k_I-k_C) \gtreqless 0 \text{ as } k_I \gtreqless k_C.$$

We can eliminate C^1 from (43) and (37i), (37i) being the full employment condition for capital in country 1, to obtain

$$(44) \quad Y^1 = g(r-\pi, N^1, K^1) ,$$

an equation that tells how much GNP is country 1 is associated with full employment of the stocks of factors, given that the marginal conditions are satisfied.

Equation (44), together with the following four equations, each of which is described above, describes our model with full employment in country 1:

$$(45) \quad Y^1 = \mu^1(r-\pi, w^1, M^1, \pi) \qquad (MM^1 \text{ curve})$$

$$(46) \quad Y^2 = \mu^2(r-\pi, w^2, M^2, \pi) \qquad (MM^2 \text{ curve})$$

1/ Euler's theorem implies product exhaustion, so that

$$Pi = \frac{w^j}{P_C^j} + \rho(r-\pi)k_I$$

$$c = \frac{w^j}{P_C^j} + \rho(r-\pi)k_C .$$

Subtracting the second from the first gives

$$Pi - c = P\frac{1}{\lambda_I} - \frac{1}{\lambda_C} = P(r-\pi)(k_I-k_C) .$$

Multiplying by $\lambda_I\lambda_C$ gives

$$P\lambda_C - \lambda_I = P\lambda_I\lambda_C(r-\pi)(k_I-k_C) .$$

(47) $P(K^1+K^2) = \beta_I(Y^1+Y^2)+(P\beta_C-\beta_I)[Z^1(Y^1-T^1)+Z^2(Y^2-T^2)+G^1+G^2]$ (Worldwide KK curve)

(48) $e^1 = \frac{w^1}{w^2}$

Equations (45)-(48) form a system of five equations in the five endogenous variables $r-\pi$, Y_1, Y_2, w^1 and either M^1 or M^2 or e^1. If e^1 is endogenous, then as before, M^1 and M^2 are exogenous. But if e^1 is to be pegged and so become exogenous, then either M^1 or M^2 must be endogenous.

We now describe the structure of the model under the three possible regimes.

a. The system with M^1 and M^2 exogenous.

With M^1 and M^2 exogenous, equations (44), (45) and (47) form a system of three equations that determine $r-\pi$, Y^1, and Y^2. Given these variables, equation (45) then determines w^1, while the exchange rate e^1 is determined by (48).

b. The system with M^2 and e^1 exogenous.

Here, equations (44), (46) and (47) again determine $r-\pi$, Y^1, and Y^2. Equation (48) then determines w^1, while (45) determines M^1.

c. The system with M^1 and e^1 exogenous.

Here equations (44), (45), and (48) form a system of three equations that determine $r-\pi$, w^1, and Y^1. Given values of those variables, equation (47) determines Y^2, and then equation (46) determines M^2.

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