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VARIABLE RATE LOANS INCREASE EFFICIENCY  
BUT NOT NECESSARILY BORROWERS'  
CONSUMPTION OF FINANCED GOODS

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Despite historically high nominal interest rates, housing activity grew rapidly between 1982 and 1984. Single family housing starts, for example, grew from a seasonally adjusted rate of 540,000 in January, 1982 to 995,000 in September, 1984, reaching a maximum of 1,336,000 in February, 1984. This growth occurred during a period in which effective interest rates on conventional, fixed rate mortgages fell from a high of 15.9 percent in April 1982,<sup>1/</sup> but never dropped below 12.2 percent (see Chart 1).

The rapid rate at which housing grew caught many analysts by surprise. For example, Goodman (p. 1) notes that:

"The average housing starts forecast for 1983 of 18 organizations polled by U.S. Gypsum company at the beginning of 1983 was 1.44 million units. The actual starts that year were 1.70 million. . . ." Also, consensus forecasts for housing starts reported by Blue Chip Economic Indicators were almost identical to the low forecasts mentioned above. Finally, a single family housing forecasting model we constructed for this study, which is described later in this paper, would have underpredicted seasonally adjusted, single family housing starts during both 1983 and 1984.

Widespread issuance of adjustable rate mortgages (ARMs) occurred concurrent with this unexpectedly strong housing market. An ARM is a mortgage whose interest rate and subsequent monthly payments are permitted to fluctuate when some predetermined index of interest rates fluctuates, unlike a fixed rate mortgage (FRM). There was at most an estimated 65.1 billion dollars of ARMs outstanding at major institutions by December,

Chart I

**Mortgage Interest Rates for ARMs and FRMs\***  
(July 1982 - October 1984)

Percent

17

16

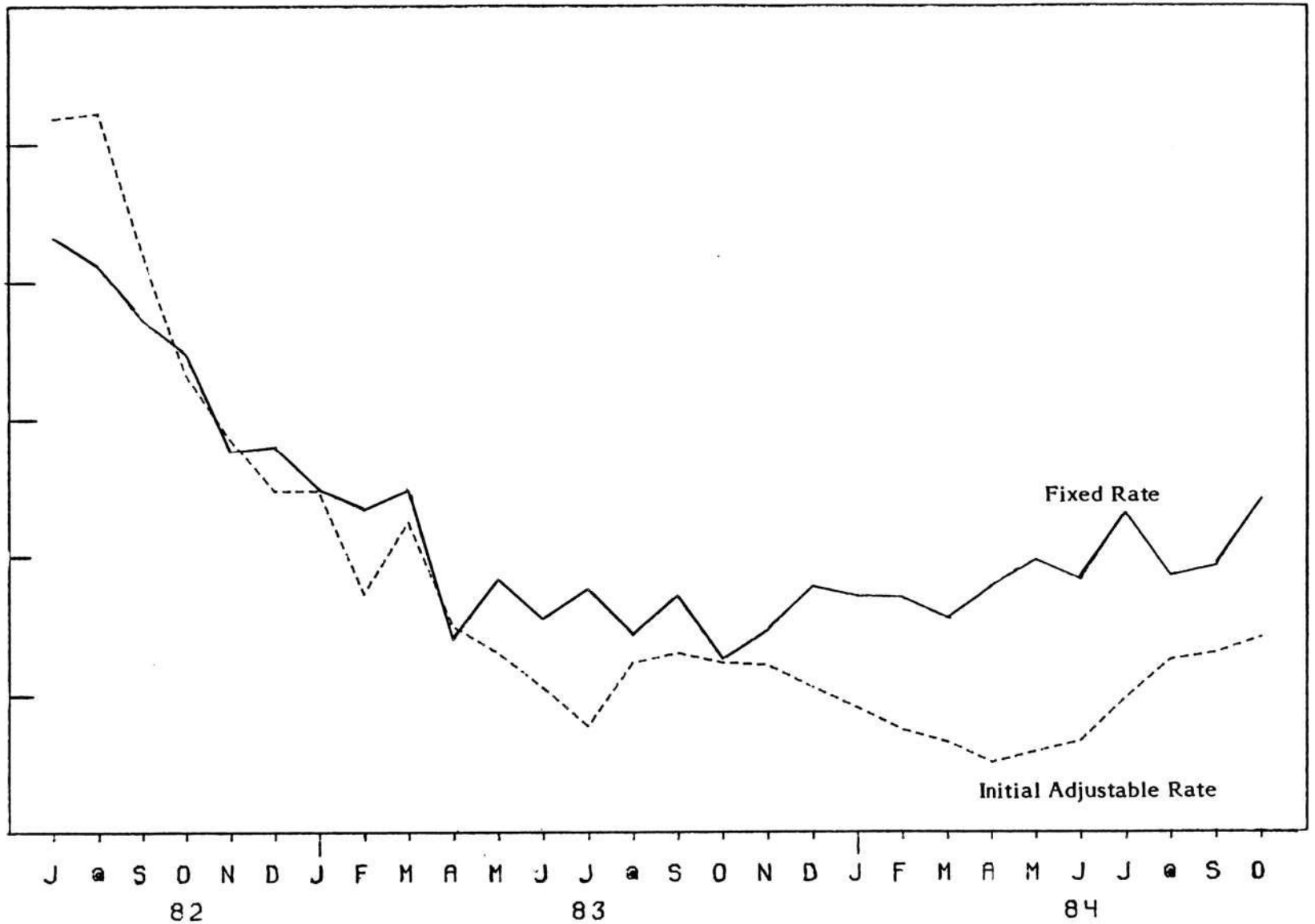
15

14

13

12

11



\* Average initial effective interest rates on conventional mortgages  
Source: FHLBB

1982. But in 1983 alone, an estimated additional 39.3 billion dollars ARMs were added to this stock, accounting for one third of the net growth of outstanding mortgage debt that year (see Nothaft, p. 448). ARMs continued to be used heavily in 1984, accounting for over 60 percent of all conventional home mortgage loans closed that year (see Chart 2).

Noting that the concurrent growth of ARM usage and the unexpectedly strong housing recovery occurred despite historically high FRM rates, some observers have concluded that the advent of ARMs was responsible for the strong housing recovery. Trade publications printed stories asserting that ARMs were and still are a major factor in increasing home sales. One typical story, entitled "ARMs Are The Muscle Behind Housing," included the following quote:

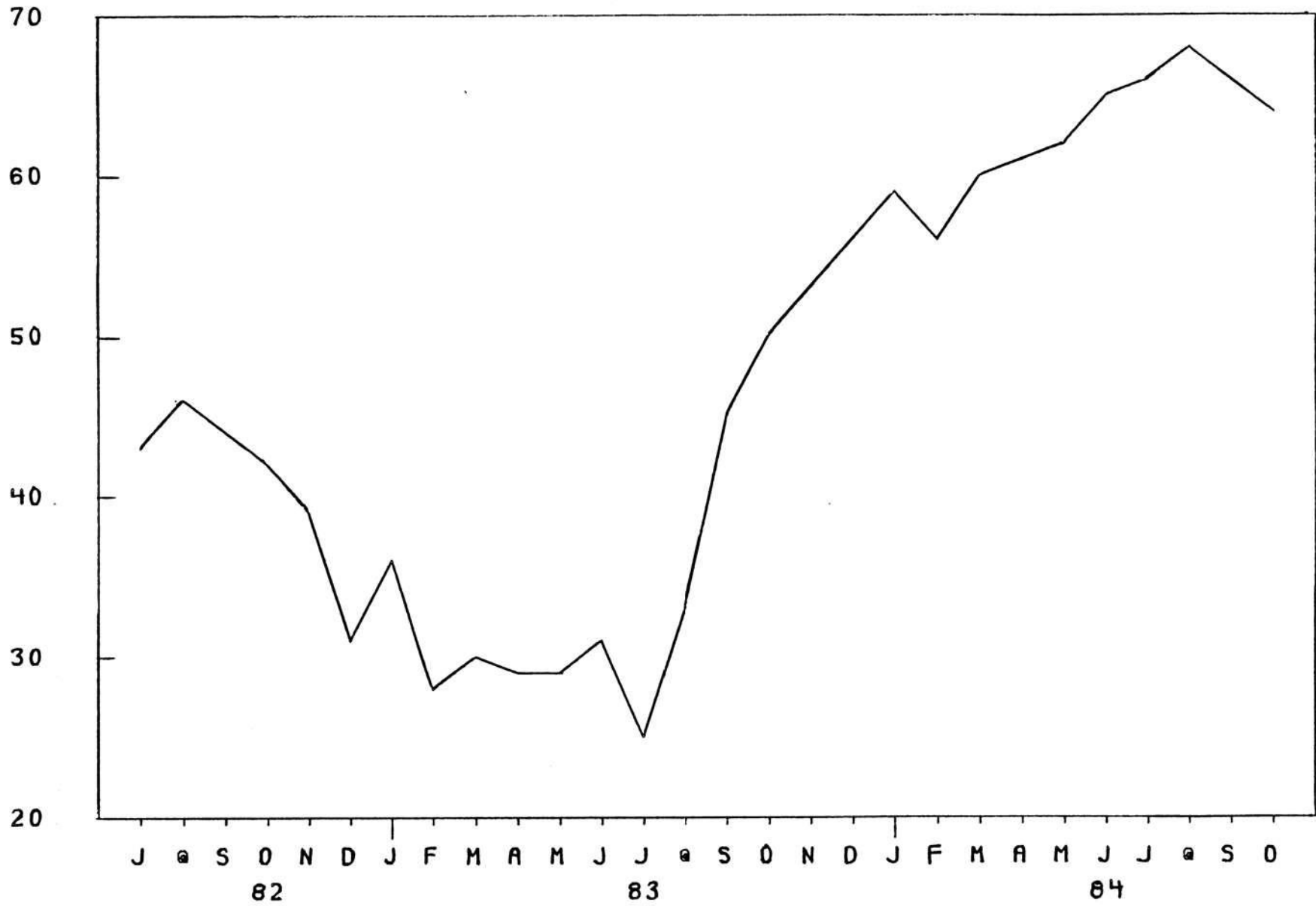
" 'There would be no housing activity to speak of without ARMs ' according to Paul W. Pryer, chairman of the US League of Savings Institutions. 'If home lending institutions could not make adjustable rate mortgages, we'd be forced to abandon housing just to survive in a deregulated savings world'." (See Savings Institutions, September 1984, p. 45.)

The major motivation behind regulators granting lenders permission to issue ARMs was not to stimulate housing consumption. Rather, it was to help lenders share the up-side interest rate risk with borrowers. Regulators employed implicit "invisible hand" logic to help justify authorizing ARMs. In doing so, the Federal Home Loan Bank Board said its actions permitting federal chartered savings and loan institutions to issue ARMs were to

Chart 2

**ARMs' Share of Conventional Mortgages Issued**  
(July 1982 - October 1984)

Percent



Source: FHLBB

"allow the lender and borrower the flexibility to agree upon terms that will best suit their individual needs." (See FHLBB Journal, p. 54.) Language like this suggests that Pareto improvement should result from allowing the issuance of mutually acceptable mortgage contracts with variable rate features.

In this paper, we argue that while the introduction of ARMs may have increased market efficiency, it didn't increase housing activity much. Our argument is in two parts. Section 1 presents a simple general equilibrium model in which the legalization and subsequent introduction of variable rate loans, like ARMs, improves market efficiency but doesn't increase borrowers' demands for a good financed with them, such as housing. This suggests the theoretical plausibility of the empirical results presented in Section 2, which utilize a Bayesian Vector Autoregression (BVAR)<sup>2/</sup> housing forecasting model we constructed. We use it to argue that the fall in fixed rate mortgage rates between 1982 and 1984 explains a good deal of the concurrent growth in housing activity. ARMs' contribution to that growth thus appears to have been relatively small, a result consistent with the theoretical result of Section 1 and recent results of Esaki and Wachtenheim (1984) and of Palash and Stoddard (1985). Had ARMs not been available, we suspect that the growth in housing activity would have been financed by a mixture of FRMs and "creative financing."<sup>3/</sup>

Our findings provide the first theoretical explanation of how ARMS have come to be widely used, yet still haven't increased aggregate housing consumption, heretofore viewed as para-

doxical (see Palash and Stoddard (1985)). The derived demand for ARMs results from their potentially enabling borrowers and lenders to more efficiently share risks associated with uncertain states affecting future income and consumption, rather than by enabling higher current housing expenditures.

Section 1: A General Equilibrium Model of Variable Rate Loans

The model is a 2-person, 2-period, exchange model of a Fisherian borrower and lender. The lender is endowed only in the first period, nonstochastically. The borrower is endowed only in the second period. The size of the borrower's endowment depends on the occurrence of one of two possible states of nature, denoted "L" and "H" (i.e., low and high), with respective probabilities  $\pi_L$  and  $\pi_H$ . Both borrower and lender are assumed to possess the same utility function of consumption, and are expected utility maximizers.

The notation used throughout is as follows:

	"Lender"		"Borrower"	
	<u>period 1</u>	<u>period 2</u>	<u>period 1</u>	<u>period 2</u>
Endowment	$C_1^e$	0	0	$C_{L2}^e, C_{H2}^e$
Consumption	$C_1^l$	$C_{L2}^l, C_{H2}^l$	$C_1^b$	$C_{L2}^b, C_{H2}^b$
Utility	$U(C_1^l)$	$\beta(\pi_L U(C_{L2}^l) + \pi_H U(C_{H2}^l))$	$U(C_1^b)$	$\beta(\pi_L U(C_{L2}^b) + \pi_H U(C_{H2}^b))$
Units of Asset "i" purchased	$w_1^l$	$w_2^l, w_3^l$	$w_1^b$	$w_2^b, w_3^b$

In subsections A and B, it is shown that competitive trading in claims on first period consumption, fixed rate loans, and variable rate loans provides complete markets, and hence the Pareto efficient Arrow-Debreu competitive equilibrium allocation for the specified endowment pattern.

Subsection C shows that legal restrictions prohibiting the issuance of variable rate loans result in incomplete markets. It is shown that the incomplete market-competitive equilibrium, resulting from competitive trading only in claims to first period consumption and in fixed rate loans, has the property that the borrower's first period consumption (financed solely by loans) is the same as in the complete markets case. In the presence of costless lump sum transfers, though, the Pareto efficiency of the complete market solution implies its potential Pareto superiority over the incomplete market solution, which is shown to be inefficient.

Subsection 1A: The Arrow-Debreu Competitive Equilibrium

Denoting the market prices of contingent claims to period 2 consumption by  $P_{L2}$  and  $P_{H2}$  in states L and H, we have:

Lender's Problem

$$(1) \quad \max_{C_1^L, C_{L2}^L, C_{H2}^L} U(C_1^L) + \beta(\pi_L U(C_{L2}^L) + \pi_H U(C_{H2}^L))$$

$$\text{s.t. } P_1 C_1^L + P_{L2} C_{L2}^L + P_{H2} C_{H2}^L = P_1 C_1^e$$



Borrower's Problem

$$(2) \quad \max_{C_1^b, C_{L2}^b, C_{H2}^b} U(C_1^b) + \beta(\pi_L U(C_{L2}^b) + \pi_H U(C_{H2}^b))$$

$$\text{s.t.} \quad P_1 C_1^b + P_{L2} C_{L2}^b + P_{H2} C_{H2}^b = P_{L2} C_{L2}^e + P_{H2} C_{H2}^e$$

Market Clearing

$$(3) \quad C_1^l + C_1^b = C_1^e$$

$$C_{L2}^l + C_{L2}^b = C_{L2}^e$$

$$C_{H2}^l + C_{H2}^b = C_{H2}^e$$

We also assume that utility is logarithmic,

$$(4) \quad U(C) = \ln(C)$$

a special case of the generalized logarithmic family  $\ln(A+C)$ , where A is a constant, forcefully advocated by Rubinstein (1976) as a good choice of utility function family. The familiar first-order conditions of either problem are (omitting the superscript for borrower or lender):

$$1/C_1 = \lambda P_1$$

$$(5) \quad \beta/C_{L2} = \lambda P_{L2} / \pi_L$$

$$\beta/C_{H2} = \lambda P_{H2} / \pi_H$$

i.e., the marginal utilities of consumption in each state (treating period 1 as a state) must equal the price/probability ratio of each state times the marginal utility of income (i.e., the Lagrange multiplier  $\lambda$ ).

Multiplying each equation in (5) by its consumption variable and adding equations yields:

$$(6) \quad 1 + \beta = \lambda [P_1 C_1 + P_{L2} C_{L2} + P_{H2} C_{H2}]$$

which, upon substitution of either the borrower's or lender's budget constraint yields:

$$(7) \quad \lambda = (1 + \beta) / E$$

where  $E$  is the value of the borrower's or lender's endowment.

Substitution of (7) into (5) yields the contingent claim demand functions:

$$C_1 = E / (1 + \beta) P_1$$

$$(8) \quad C_{L2} = \pi_L \beta E / (1 + \beta) P_{L2}$$

$$C_{H2} = \pi_H \beta E / (1 + \beta) P_{H2}$$

Respectively substituting the value of the borrower's and lender's endowments into (8) and the resulting demand functions into the market clearing conditions (3) yields the following relative price (choose first period consumption as numeraire,  $P_1 = 1$ ) determination matrix equation:

$$(9) \quad \begin{bmatrix} (1+\beta)C_{L2}^e - \pi_L \beta C_{L2}^e & -\pi_L \beta C_{H2}^e \\ -\pi_H \beta C_{L2}^e & (1+\beta)C_{H2}^e - \pi_H \beta C_{H2}^e \end{bmatrix} \begin{bmatrix} P_{L2} \\ P_{H2} \end{bmatrix} = \begin{bmatrix} \pi_L \beta C_1^e \\ \pi_H \beta C_1^e \end{bmatrix}$$

The determinant in (9) is nonzero as long as the borrower is endowed positively in both states. Under the maintained assumption that  $C_{H2}^e > C_{L2}^e$ , we need really only additionally assume that  $C_{L2}^e \neq 0$  to ensure a nonzero determinant, and hence a unique solution to (9).

Solving (9) via Cramer's rule then yields the market clearing prices:

$$(10) \quad \begin{aligned} P_{L2} &= \pi_L \beta C_1^e / C_{L2}^e \\ P_{H2} &= \pi_H \beta C_1^e / C_{H2}^e \end{aligned}$$

Substituting (10) and the respective values of the borrower's and lender's endowments into (8) then yields the Arrow-Debreu competitive equilibrium

$$(11) \quad \begin{aligned} C_1^l &= C_1^e / (1+\beta) \\ C_{L2}^l &= C_{L2}^e / (1+\beta) \\ C_{H2}^l &= C_{H2}^e / (1+\beta) \end{aligned}$$

and

$$\underline{C}^b = \underline{\beta C}^l, \text{ where the underlined symbols are vectors.}$$

By the fundamental theorem of welfare economics, the competitive equilibrium allocation (11) for the posited endowment pattern is Pareto efficient. For arbitrary endowments, by maximizing the lender's objective function in (1) subject to constraining the borrower's objective function in (2) to exceed a constant and to the feasibility constraints (3), one obtains the following condition for Pareto efficiency when utility is of log form (4):

$$(12) \quad c_1^e / c_1^l = c_{L2}^e / c_{L2}^l = c_{H2}^e / c_{H2}^l$$

Subsection 1B: Complete Asset Markets Via Fixed and Variable Rate Loans

The competitive equilibrium allocation (11) can also be attained by competitive trading of claims to first-period consumption and two additional assets. One of the two assets is a fixed rate loan, one unit of which yields the borrower one unit of the lender's first-period consumption at the expense of  $R_F$  units of consumption paid to the lender in period 2. The other asset is a variable rate loan, which differs from the fixed rate loan by mandating that  $R_L$  units of consumption must be paid to the lender when state L occurs, (i.e., when the borrower's endowment is low) and  $R_H$  must be paid otherwise, when state H occurs.

The payoff matrix, showing the asset payoffs (to the borrower) in each state is:

		Asset			
	State	1	2	3	
	period 1	1	1	1	
(13)	L	0	$-R_F$	$-R_L$	$= X$
	H	0	$-R_F$	$-R_H$	

The lender's payoff matrix is, of course,  $-X$ . Because  $\text{Det } X = R_F(R_H - R_L)$ ,  $X$  is nonsingular as long as  $R_H \neq R_L$ , which we assume.

The lender's joint portfolio-consumption problem is to:

$$\begin{aligned}
 (14) \quad & \max_{w_1^l, w_2^l, w_3^l} U(C_1^l) + \beta[\pi_L U(C_{L2}^l) + \pi_H U(C_{H2}^l)] \\
 \text{s.t.} \quad & C_1^l = w_1^l + w_2^l + w_3^l + C_1^e \\
 & C_{L2}^l = -R_F w_2^l - R_L w_3^l \\
 & C_{H2}^l = -R_F w_2^l - R_H w_3^l \\
 & w_1^l V_1 + w_2^l V_2 + w_3^l V_3 = 0
 \end{aligned}$$

where  $w_i^l$  is the number of units of the  $i$ th asset (i.e., column in  $X$ ) purchased (or sold, if  $w_i^l < 0$ ) and  $V_i$  is its unit price. The final constraint states that there is no initial wealth (other than  $C_1^e$ ). In matrix notation, the constraints in (14) are:

$$\begin{aligned}
 (15) \quad & \underline{X} \underline{w}^l = \begin{bmatrix} C_1^l - C_1^e \\ C_{L2}^l \\ C_{H2}^l \end{bmatrix} \\
 & \underline{V}^T \underline{w}^l = 0.
 \end{aligned}$$

The borrower's problem is

$$\begin{aligned}
 & \max_{w_1^b, w_2^b, w_3^b} U(C_1^b) + \beta[\pi_L U(C_{L2}^b) + \pi_H U(C_{H2}^b)] \\
 (16) \quad & \text{s.t.} \quad \underline{X}\underline{w}^b = \begin{bmatrix} C_1^b \\ C_{L2}^b - C_{L2}^e \\ C_{H2}^b - C_{H2}^e \end{bmatrix} \\
 & \underline{V}^T \underline{w}^b = 0
 \end{aligned}$$

and asset market equilibrium requires  $\underline{V}$  to adjust so that

$$(17) \quad \underline{w}^l = -\underline{w}^b.$$

Substituting (17) into the constraints (15) and (16) shows that any asset market equilibrium also satisfies the consumption feasibility constraints (3). Also, because we have assumed  $R_H \neq R_L$ ,  $X$  is nonsingular so both (15) and (16) can be solved to find portfolios  $\underline{w}^l$  and  $\underline{w}^b$  which would produce any prespecified right-hand side consumption vector. In particular, the three Arrow-Debreu pure state securities, which were denoted  $C_1$ ,  $C_{L2}$  and  $C_{H2}$  and traded in the previous section of this appendix, could be produced by three separate portfolios  $W = (\underline{w}^1, \underline{w}^2, \underline{w}^3)$ , found by solving

$$(18) \quad XW = I, \text{ or } W = X^{-1}$$

where  $I$  is the payoff (identity) matrix whose columns give the state-payoff vectors for the three pure-state securities. This so-called spanning or complete market property of  $X$ , when coupled

with the fact that asset markets are in equilibrium if pure-state consumption markets in  $C_1$ ,  $C_{L2}$  and  $C_{H2}$  are, guarantees that the Pareto efficient Arrow-Debreu competitive equilibrium (10) and (11) also yields the solution to the asset market equilibrium solving (14), (16), and (17).

Substituting (11) into (15) and (16), that solution is:

$$(19) \quad \underline{w}^l = X^{-1} \begin{bmatrix} -\beta C_1^e / (1+\beta) \\ C_{L2}^e / (1+\beta) \\ C_{H2}^e / (1+\beta) \end{bmatrix}$$

$$(20) \quad \underline{w}^b = X^{-1} \begin{bmatrix} \beta C_1^e / (1+\beta) \\ -C_{L2}^e / (1+\beta) \\ -C_{H2}^e / (1+\beta) \end{bmatrix}$$

and assuming the market eliminates arbitrage between the three assets and their respective duplicating portfolios ( $W = X^{-1}$ ) made of pure state securities  $C_1$ ,  $C_{L2}$  and  $C_{H2}$ ,

$$(21) \quad \underline{v} = X^T \underline{p} = X^T \begin{bmatrix} 1 \\ P_{L2} \\ P_{H2} \end{bmatrix}, \text{ or}$$

$$V_1 = P_1 = 1$$

$$V_2 = 1 - R_F (P_{L2} + P_{H2}) = 1 - \beta C_1^e (R_F \pi_L / C_{L2}^e + R_F \pi_H / C_{H2}^e)$$

$$V_3 = 1 - R_L P_{L2} - R_H P_{H2} = 1 - \beta C_1^e (R_L \pi_L / C_{L2}^e + R_H \pi_H / C_{H2}^e)$$

Complete Market Asset Prices

$P_{L2} + P_{H2}$  is the present value of a risk-free unit of consumption in period 2, i.e., the value of a portfolio of a unit each of two assets, one paying a unit of  $C_{L2}$  and the other paying a unit of  $C_{H2}$ . Thus,  $P_{L2} + P_{H2} = 1/R_2$ , the risk-free discount factor for period 2. The value of the fixed rate loan,  $V_2$  in (21), equals  $1 - R_F/R_2$ , its risk-free discounted present value. A similar interpretation can be given to  $V_3$  by defining two separate risk-specific discount factors for the states L and H.

By subtracting  $V_2$  from  $V_3$ , a little algebra shows that:

$$(22) \quad V_3 \begin{matrix} \leq \\ > \end{matrix} V_2 \text{ when } R_F(\pi_L/C_{L2}^e + \pi_H/C_{H2}^e) \begin{matrix} \leq \\ > \end{matrix} R_L \pi_L/C_{L2}^e + R_H \pi_H/C_{H2}^e$$

i.e., one loan is worth less than the other when its interest "cost," i.e., as computed on the expected marginal utility of the borrower's endowment, is greater. Furthermore, for  $V_3$  and  $V_2$  to be positive, their respective interest "costs" must be less than  $1/\beta C_1^e$ .

More insight into the nature of the complete market asset prices can be found by deriving the following, CAPM-like pricing representations of the variable rate loan's price  $V_3$ . Following the derivation in Varian (p. 311), use (21) to write:

$$(23) \quad V_3 = 1 - \pi_L R_L P_{L2} / \pi_L - \pi_H R_H P_{H2} / \pi_H$$

$$= 1 - E(R \cdot P_2 / \pi)$$



where  $R$  denotes the random payment variable, taking on the values  $R_L$  when the borrower's endowment is low and  $R_H$  when it is high, and the random price/probability ratio  $P_2/\pi$  is denoted similarly. The definition of covariance then yields:

$$\begin{aligned} V_3 &= 1 - \text{cov}(R, P_2/\pi) - E(R)E(P_2/\pi) \\ &= 1 - \text{cov}(R, P_2/\pi) - (\pi_L R_L + \pi_H R_H)(P_{L2} + P_{H2}). \end{aligned}$$

Representing  $P_{L2} + P_{H2} = 1/R_2$ , as described a few paragraphs earlier, yields

$$(24) \quad V_3 = 1 - E(R)/R_2 - \text{cov}(R, P_2/\pi).$$

i.e., the difference between its risk-free discounted expected present value and the covariance of payments with the price/probability ratio. Substituting (10) into (24) yields:

$$(25) \quad V_3 = 1 - E(R)/R_2 - \text{cov}(R, \beta C_1^e / C_2^e)$$

for the variable rate loan's value. From (21), we get the fixed rate loan's value

$$(26) \quad V_2 = 1 - R_F/R_2.$$

Comparing (25) and (26), we see that even if  $E(R) = R_F$ , i.e., if the expected payment on the variable rate loan equals the fixed rate loan's payment, the fixed rate loan is worth more to the borrower when the variable rate loan's payment is negatively correlated with the borrower's second period endowment (i.e., income)  $C_2^e$ . Ceteris paribus, the borrower will value the vari-

able rate loan less if she thinks that interest rates (and variable loan payments) will be high when her income is low, i.e., when it is "riskier."

Complete Market Asset Demands

From (13) and (20), use Cramer's rule to compute the complete market loan demands:

$$(27) \quad w_2^b = (R_H C_{L2}^e - R_L C_{H2}^e) / ((1+\beta)R_F(R_H - R_L))$$

$$w_3^b = (C_{H2}^e - C_{L2}^e) / ((1+\beta)(R_H - R_L))$$

Because of the maintained assumption that  $C_{H2}^e > C_{L2}^e$ , the "borrower" will actually borrow via variable rate loans, i.e.,  $w_3^b > 0$ , only when  $R_H > R_L$ , i.e., the return on the variable rate loan when the borrower's endowment is high exceeds its return when the borrower's endowment is low. That is, the borrower will borrow via variable rate loans only when the covariance between her endowment and interest rates is positive. If the covariance is negative, then the "borrower" will actually lend variable rate loans while borrowing with fixed rate loans.

Section 1C: Incomplete Markets Due to Legal Prohibition of Variable Rate Loans

If legal prohibitions prevent the issuance of variable rate loans, the asset payoff matrix becomes

$$(28) \quad X = \begin{pmatrix} 1 & 1 \\ 0 & -R_F \\ 0 & -R_F \end{pmatrix} \cdot$$

Because there are fewer assets than generalized states ("time 1," "L," and "H"), it is not possible for borrowers and lenders to construct any possible consumption pattern by a portfolio  $w = X^{-1}\underline{c}$ , because  $X^{-1}$  does not exist.

To see what restrictions are placed on possible consumption vectors for the borrower and lender, consider the lender's problem first. Reducing the lender's portfolio-consumption constraints (14) with  $w_3^l = 0$  to row echelon form, we see that a portfolio  $(w_1^l, w_2^l)$  exists which produces a vector  $(c_1 - c_1^e, c_{L2}^l, c_{H2}^l)$  if and only if:

$$(29) \quad c_{H2}^l = c_{L2}^l.$$

The vector is produced by the portfolio:

$$(30) \quad w_1^l = c_1^l - c_1^e + c_{L2}^l/R_F$$
$$w_2^l = -c_{L2}^l/R_F$$

Because a fixed rate loan pays back the same amount ( $R_F$ ) in both states L and H, and the lender is only endowed in period 1, the lender has no means of making  $c_{L2}^l \neq c_{H2}^l$ .

The borrower, however, does have an endowment in the second period. By reducing the borrower's constraints (16) with  $w_3^b = 0$  to row echelon form, we see that a portfolio  $(w_1^b, w_2^b)$  exists which produces a vector  $(c_1^b, c_{L2}^b - c_{L2}^e, c_{H2}^b - c_{H2}^e)$  if and only if

$$(31) \quad c_{H2}^b - c_{L2}^b = c_{H2}^e - c_{L2}^e$$

The vector is produced by the portfolio:

$$(32) \quad w_1^b = C_1^b + (C_{L2}^b - C_{L2}^e)/R_F$$

$$w_2^b = -(C_{L2}^b - C_{L2}^e)/R_F$$

Substituting the lender's incomplete market constraints (29) and (30) into the lender's objective function and zero initial wealth constraint  $V^T \underline{w}^l = 0$  in (14), obtain the lender's problem:

$$(33) \quad \max_{C_1^l, C_{L2}^l} U(C_1^l) + \beta U(C_{L2}^l)$$

$$\text{s.t.} \quad V_1 C_1^l + (V_1 - V_2) C_{L2}^l / R_F = V_1 C_1^e$$

with the solution demand functions:

$$(34) \quad C_1^l = C_1^e / (1 + \beta)$$

$$C_{L2}^l = \beta R_F V_1 C_1^e / ((1 + \beta)(V_1 - V_2)) = C_{H2}^l$$

with asset demands  $w_1^l$  and  $w_2^l$  found by substituting (34) into (30).

Substituting the borrower's incomplete market constraints (31) and (32) into the borrower's objective function and zero initial wealth constraint  $V^T \underline{w}^b = 0$  in (16), the incomplete market problem for the borrower is:

$$(35) \quad \max_{C_1^b, C_{L2}^b, C_{H2}^b} U(C_1^b) + \beta[\pi_L U(C_{L2}^b) + \pi_H U(C_{H2}^b)]$$

$$\text{s.t.} \quad v_1 C_1^b + (v_1 - v_2) C_{L2}^b / R_F = (v_1 - v_2) C_{L2}^e / R_F$$

$$C_{H2}^b - C_{L2}^b = C_{H2}^e - C_{L2}^e$$

Finally, the incomplete market's asset market equilibrium constraints require prices  $\underline{v}$  to adjust so (17) holds. As before, (17) ensures that exchange economy consumption constraints (3) also hold. Because of this,  $C_1^l = C_1^e / (1 + \beta)$  from (34) can be substituted in (3) to obtain:

$$(36) \quad C_1^b = \beta C_1^e / (1 + \beta)$$

in the incomplete market solution.

Comparing (36) with (11), we see that the borrower's first-period consumption in the incomplete market is the same as in the complete market. This is one of our two major results.

The second major result is that the incomplete market allocation isn't Pareto efficient. To see this, note that Pareto efficiency requires that (12) holds. But because incomplete markets force (29) to hold, i.e.,  $C_{L2}^l = C_{H2}^l$ , efficiency conditions (12) can't hold unless  $C_{H2}^e = C_{L2}^e$ , which has been ruled out by the maintained assumption that  $C_{H2}^e > C_{L2}^e$ . So, the incomplete market allocation is inefficient.

Furthermore, if costless lump sum transfers between borrower and lender are feasible, the Pareto efficient complete market allocation (11) is always at least potentially Pareto

superior to the inefficient incomplete market solution. The potential Pareto superiority brought about by the introduction of variable rate loans provides an alternative explanation for the growth of their use: they help improve the efficiency of risk-sharing arrangements, rather than increase borrowers' purchases of goods so financed.

Of course, the extreme result of no change in first period borrower purchases of the financed good is special to the particular specification of this two-period model. Nonlogarithmic utility, or a change in the endowment pattern making the lender's endowment stochastic, would change this result. Our purpose has been merely to establish the theoretical plausibility of the result, not its empirical likelihood. The latter is addressed in the next section.

## Section 2: The BVAR Housing Model

We included four monthly data series in our BVAR forecasting model, for the period January 1964 to October 1984. The first series, privately owned single family housing starts, was taken as a measure of single family housing market activity. The second series, the real median price of new single family houses, was used as an indicator of the inflation adjusted average price of new homes sold. Published in nominal form, it was deflated using the consumer price index (CPI). For an index of mortgage interest rates, we used the average effective interest rate for conventional (non-VA or -FHA) mortgages on new houses. We also made use of the separate series available for the fixed and ad-

justable rate components of this average. These separate component series first became available in July 1982. Prior to that date, we assumed that the average rate series and the fixed rate series coincide. The fourth and last series used was the percentage change (differences in natural logarithms) in the Consumer Price Index (CPI) as a measure of inflation. All data series except for the interest rate series were seasonally adjusted. (Further details concerning the data series are given in Appendix A.)

To help test for the possible effect of ARM financing on the level of housing starts, we made two comparisons using two sets of projections for housing starts and prices. Both sets of projections cover the period January 1982 through October 1984. The first set of projections are forecasts that could have been made on December 1981, i.e. which used only data dated December 1981 or earlier. The second set of projections used all the data available for the first set, plus the actual data on FRM rates between January 1982 and October 1984. Since ARMs were not widely available before 1982, it is doubtful that the first set of projections were influenced by the advent of ARMs. The second set of projections, then, shows how much better the first set of forecasts could have been had future (i.e. January, 1982-October, 1984) FRM rates been known. One comparison compares the second set of projections to the first set, to give an indication of the impact that falling FRM rates had on the post-1982 performance of housing starts and prices. The second comparison compares the

second set of projections to the actual data on housing starts and prices, yielding information about the effects that additional factors, including ARMs, may have had on housing starts and prices.

We refer to the first set of projections as unconditional forecasts, and to the second set as conditional forecasts. Using this terminology, our two comparisons are perhaps best illustrated through a simple equation. Let  $Y$  denote something that we are trying to forecast, e.g., housing starts in February 1983. Let  $U$  denote the unconditional forecast of  $Y$  while  $C$  denotes the conditional forecast of  $Y$ . Then, the error of the unconditional forecast, i.e.,  $Y - U$ , can be written as

$$Y - U = (C - U) + (Y - C).$$

That is, the error of the unconditional forecast is always equal to the difference between the conditional and unconditional forecasts ( $C - U$ ) plus the error of the conditional forecast ( $Y - C$ ). The former difference ( $C - U$ ) represents the impact of the additional information used to make the conditional forecasts. For our study, this difference is attributed to the impact of unanticipated changes in FRM rates. This is our first comparison. In our second comparison, we examine the latter difference, i.e. between the data and the conditional forecast ( $Y - C$ ). Due to the influence of additional factors other than FRMs, such as demographic factors, we would not expect ( $Y - C$ ) to be zero even if ARMs had never existed. Unusually large or persistent condi-



tional forecast errors, however, would suggest that ARMs, perhaps in conjunction with these other factors, had a significant impact on the housing market over the forecast period.<sup>4/</sup>

Because our study did not uncover particularly large or persistent forecast errors, as will now be demonstrated, we tentatively conclude that neither ARMs, nor other additional factors, were major contributors to the unexpectedly strong housing recovery.

#### Our Two Comparisons

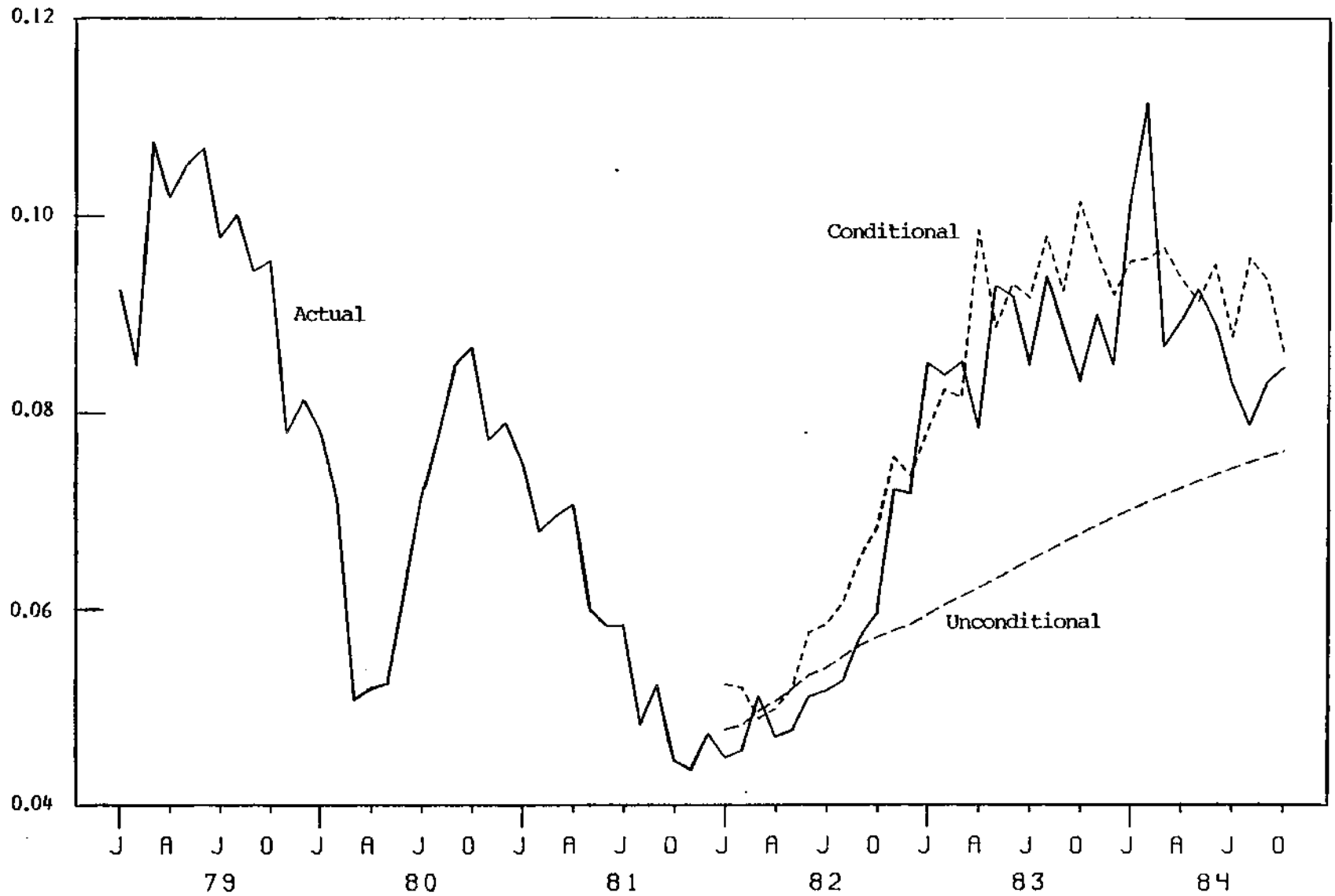
Our first comparison is made by comparing the year-end 1981 conditional (on actual post-1981 FRM rates) forecast to the unconditional forecast in Charts 3 and 4. Doing so shows how much higher and more accurately the BVAR model would have forecast both single family housing starts and median real home prices, had the actual, post-1981 declining path of FRM rates been known then. There is thus a lot of information useful for forecasting housing starts and prices in the unanticipated (i.e. not BVAR predictable) fall in FRM rates.

Our second comparison, between the same conditional forecasts and actual data in Charts 3 and 4 shows that the conditional forecast doesn't systematically overpredict or underpredict either housing starts or prices over the whole 1982-1984 period. Of course, the inclusion of other factors in the model may help to track housing starts and prices better. But further reduction in the already relatively modest error Y-C, earned by including other

Chart 3

The BVAR Model's Forecasts of Single-Family Housing Starts\*

Mil. Units

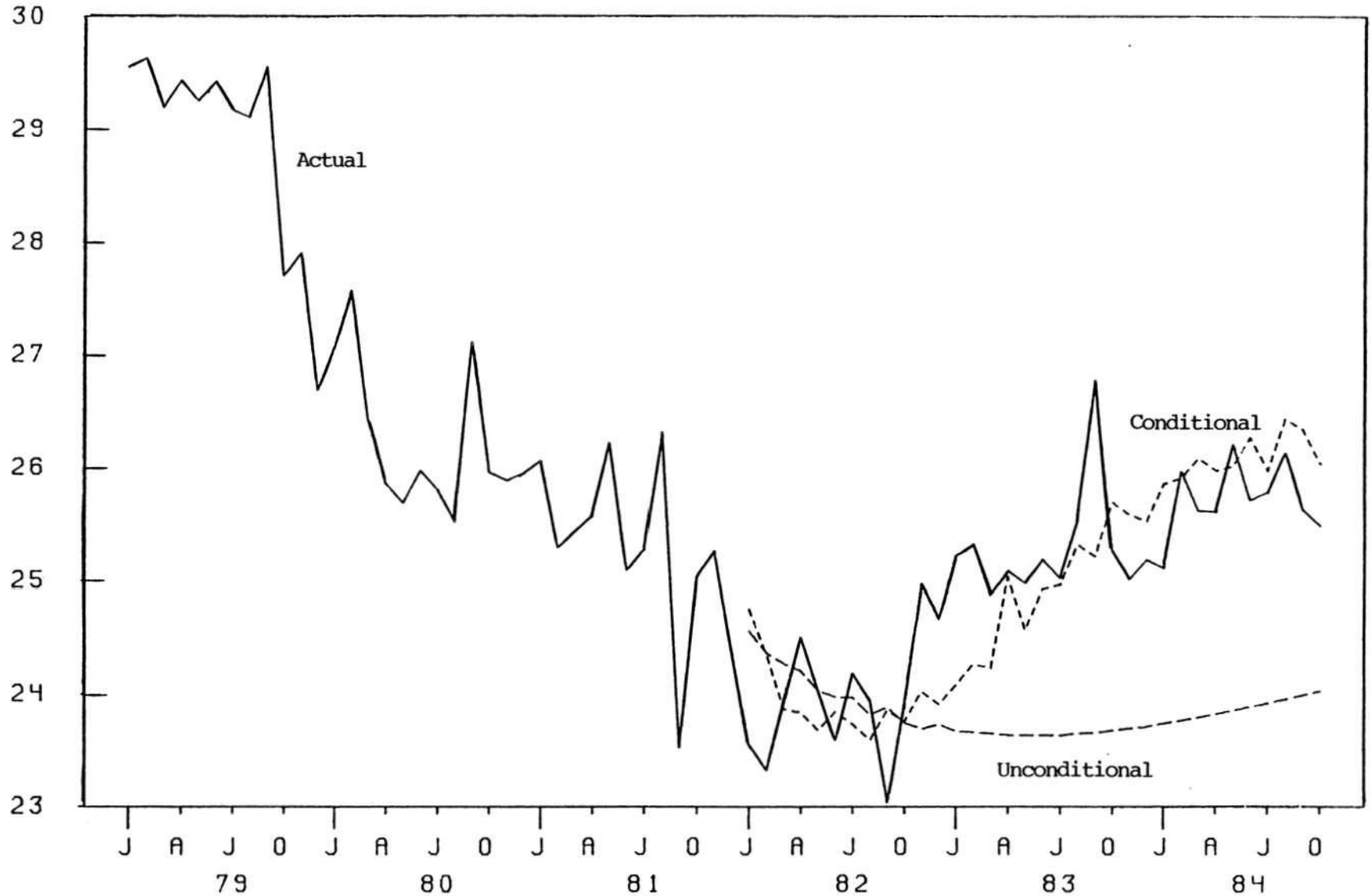


\* New privately owned single-family housing starts, seasonally adjusted monthly rate. The unconditional forecast uses pre-ARMs data from Jan. 1964 to Dec. 1981; the conditional forecast uses the same data plus actual data for FRM rates from Jan. 1982 to Oct. 1984.  
 Source of data: FHLBB

Chart 4

The BVAR Model's Forecasts of Housing Prices\*

Thou. \$  
(1967 \$)



\* Median sale price of new privately owned single-family houses, deflated by the consumer price index (1967=100). The unconditional forecast uses pre-ARMs data from Jan. 1964 to Dec. 1981; the conditional forecast uses the same data plus actual data for FRM rates from Jan. 1982 to Oct. 1984. Sources of data: FHLBB, U.S. Department of Commerce

factors, probably wouldn't justify attributing major significance to them.

For example, we exchanged U.S. personal income for the inflation rate (which we dropped to conserve degrees of freedom in estimation) in the BVAR model, reasoning that unanticipated growth in personal income, following the end of the nation's recession in November 1982, may have been in a factor in the subsequent strong housing market. In Chart 5, we see that the new conditional forecast for single family housing starts (conditioned on both FRM rates and personal income) is slightly lower and only somewhat more accurate than the old conditional forecast (conditioned only on FRM rates). We thus conclude that possibly higher than expected income growth probably wasn't a major factor in the unexpectedly strong housing market: knowledge of actual income growth helped better predict housing growth, but not substantially so.

### Conclusions

While the usage of adjustable rate mortgages (ARMs) and housing activity grew concurrently between 1982 and 1984, we cast doubt on claims in the trade literature that the former caused much of the latter. In the context of a simple, general equilibrium model of a Fisherian borrower and lender, we show that the legalization and introduction of variable rate loans, like ARMs, completes loan markets while leaving first period consumption of the borrower invariant. Thus, the demand for variable rate loans may derive from the desire to attain Pareto efficiency via com-

Chart 5

Including a Personal Income Series  
in the Model's Conditional Forecast of Housing Starts\*

Mil. Units

0.12

0.10

0.08

0.06

0.04

J A J O J A J O J A J O J A J O J A J O J A J O  
79 80 81 82 83 84

Conditioned on Fixed Rates

Conditioned on  
Fixed Rates and Personal Income

\*The housing start series and conditional forecast are the same as shown on Chart 3. The new conditional forecast is conditioned on FRM rates and U.S. personal income from Jan. 1982 to Oct. 1984. Sources of data: FHLBB, U.S. Department of Commerce

plete markets, rather than a desire to increase borrower consumption of goods so financed. Empirical evidence from a statistical housing forecasting model we construct supports the notion that ARMs didn't increase housing consumption much. Rather, the evidence suggests that the decline of fixed-rate mortgage rates between 1982 and 1983 stimulated much of the growth in housing activity.

Footnotes

1/This figure includes some adjustable rate mortgages, a separate series for which wasn't kept until July 1982.

2/For presentations of the BVAR approach, see Litterman (1985).

3/"Creative financing" is a number of techniques borrowers and sellers adopt to help finance the sale of homes, often in conjunction with fixed rate mortgages. Such techniques include contracts for deed and builder "buy-downs" of fixed-rate mortgage rates.

4/It is possible that the presence of ARMS may have caused FRM rates to be lower than they otherwise would have been. If so, the ARMs may also have (indirectly) influenced our first comparison, i.e., the difference C - U. But given the short length of the separate monthly interest rate series (post-July 1982), one can't reliably test the hypothesis that the ARMs' presence lowered FRM rates.

Appendix A

To construct our housing projections, we used a statistical technique known as "Bayesian Vector Autoregression (BVAR)." There are both advantages and disadvantages for use of this technique in studies such as ours.

The principal advantage is that BVAR techniques typically yield relatively good forecasts given relatively small inputs of human and computer time. For the purposes of our study, the major disadvantage is that BVAR is a purely statistical technique. While projections from BVAR models may be reasonably accurate, assigning unambiguous economic interpretations to these projections is often an elusive task.

Despite this serious limitation, we felt that BVAR techniques represented the best available methodology for the present study. Widely used alternative forecasting techniques include Box-Jenkins, (i.e. univariate autoregressive moving average time series models) and "structural" econometric models. Box-Jenkins models, while simple to construct and estimate, utilize data only on one time series. Such models are of no use in constructing conditional forecasts. Structural models, based on economic theory, are preferable to BVAR models in the sense that it is easier to give economic meaning to their projections. Constructing a both theoretically and empirically valid structural model, however, can be a difficult and time-consuming task. For the purposes of preliminary data analysis, the BVAR technique might be thought of as a practical compromise between the two alternative techniques.



Accordingly, we estimated a four-variable (or "four-equation") BVAR for the monthly series listed below:

- (1) Private single family housing starts, monthly rate.  
Source: Federal Home Loan Bank Board.
- (2) Median sale price of a new privately owned single family house, thousands of current dollars. Source: Federal Home Loan Bank Board.
- (3) Consumer price index, (1967 = 100). Source: U.S. Department of Commerce.
- (4) Nationwide average effective rate on conventional mortgages for purchase of newly-built homes, percent per year.  
Source: Federal Home Loan Bank Board. After June 1982, the fixed and adjustable rate components of this series are available separately.

Series (1), (2), and (3) were seasonally adjusted, using a procedure described in Amirizadeh (1985). Efficient seasonal adjustment of a quantity  $x_t$  (say, housing starts in January 1982) generally involves knowledge of future values of  $x_t$  (that is, housing starts in February 1982, March 1982, etc.). For this reason, the forecasts reported in our study would tend to be more accurate than those actually made over the forecast period covered (January 1982-October 1984). Included in each equation were 15 lags of all variables and a constant term. All series except the inflation series were first converted to natural logarithms. In the terminology of Doan and Litterman (1984), we placed a fairly

"tight" prior over the model parameters. These priors were chosen because we felt that (the logarithms of) each of the four series could be reasonably well approximated as a random walk, or as a random walk with drift.

The next step was to validate the model. To do so, we evaluated the out-of-sample (unconditional) forecasting performance of the model over the period January 1976 to December 1981. This was done for forecast intervals of 1 to 12 months. So as not to anticipate the effects of ARMs, no data dated January 1982 or later was used in this evaluation.

Results of this evaluation are reported in Table 1. Overall, they indicate that our model performs at an acceptable level. Of particular interest are the columns entitled "Theil U." These report the ratio of the root mean square of the forecast error (RMSE) of the model to the RMSE of a naive forecast of no change in the (natural logarithms of the) series. Except for housing starts, almost all of the reported Theil U statistics are below one. This indicates our model outperforms the naive forecasting procedure. Unfortunately, in the case of housing starts, our model performs only about as accurately as the naive procedure, at least in terms of unconditional forecasts. Another potential cause of dissatisfaction with our model is the relatively large mean absolute error of the mortgage rate equation. This statistic indicates that our model tended to underestimate mortgage rates over the 1976-1981 period.

Table 1

FORECAST STATISTICS FOR SERIES			NEWRATE (Fixed Mortgage Rates)		
STEP	MEAN ERROR	MEAN ABS. ERROR	RMS ERROR	THEIL U	N. OBS
1	.1341E-02	.8435E-02	.1728E-01	.9726	61
2	.3458E-02	.1488E-01	.2837E-01	.9882	60
3	.6232E-02	.2237E-01	.4023E-01	1.030	59
4	.8635E-02	.2779E-01	.4643E-01	1.009	58
5	.1059E-01	.3176E-01	.5029E-01	.9634	57
6	.1215E-01	.3358E-01	.5078E-01	.8749	56
7	.1386E-01	.3494E-01	.5000E-01	.7738	55
8	.1582E-01	.3668E-01	.5164E-01	.7124	54
9	.2258E-01	.3826E-01	.5060E-01	.6256	53
10	.2812E-01	.4214E-01	.5482E-01	.6095	52
11	.3346E-01	.4657E-01	.5984E-01	.6053	51
12	.3688E-01	.5115E-01	.6520E-01	.6083	50

FORECAST STATISTICS FOR SERIES			PCIU (Inflation)		
STEP	MEAN ERROR	MEAN ABS. ERROR	RMS ERROR	THEIL U	N. OBS
1	-.8950E-03	.1300	.1784	.8950	61
2	.4291E-02	.1577	.2094	.8980	60
3	.1082E-01	.1795	.2343	.8486	59
4	.1554E-01	.1881	.2473	.7866	58
5	.1965E-01	.1909	.2447	.7735	57
6	.2048E-01	.1867	.2428	.7479	56
7	.2195E-01	.1819	.2342	.7757	55
8	.2540E-01	.1897	.2355	.7776	54
9	.3103E-01	.1960	.2357	.8204	53
10	.3951E-01	.1985	.2446	.8155	52
11	.5206E-01	.2025	.2523	.8122	51
12	.6313E-01	.2033	.2601	.7646	50

FORECAST STATISTICS FOR SERIES RHSPRC (Real Median New House Prices)

STEP	MEAN ERROR	MEAN ABS. ERROR	RMS ERROR	THEIL U	N. OBS
1	.1560E-02	.1523E-01	.2095E-01	.9303	61
2	.2555E-02	.1851E-01	.2407E-01	.9351	60
3	.3261E-02	.1968E-01	.2673E-01	.9617	59
4	.3753E-02	.2384E-01	.3153E-01	.9658	58
5	.5073E-02	.2797E-01	.3596E-01	.9896	57
6	.5480E-02	.3174E-01	.4077E-01	.9880	56
7	.4899E-02	.3464E-01	.4384E-01	.9955	55
8	.4568E-02	.3807E-01	.4914E-01	1.000	54
9	.3078E-02	.4149E-01	.5298E-01	.9603	53
10	.1640E-02	.4269E-01	.5490E-01	.9398	52
11	.5866E-03	.4617E-01	.5920E-01	.9378	51
12	.6554E-03	.5287E-01	.6340E-01	.9577	50

FORECAST STATISTICS FOR SERIES NSFHS (Single Family Housing Starts)

STEP	MEAN ERROR	MEAN ABS. ERROR	RMS ERROR	THEIL U	N. OBS
1	.2526E-02	.7143E-01	.1036	1.014	61
2	.5149E-02	.1066	.1429	1.017	60
3	.5143E-02	.1278	.1722	1.031	59
4	.9868E-02	.1463	.1954	1.030	58
5	.1554E-01	.1549	.2109	1.018	57
6	.2058E-01	.1660	.2227	1.008	56
7	.2315E-01	.1751	.2341	1.001	55
8	.2237E-01	.1886	.2438	1.002	54
9	.1544E-01	.1890	.2498	1.002	53
10	.8103E-02	.1942	.2548	1.009	52
11	.1670E-02	.2074	.2646	1.021	51
12	.1650E-02	.2152	.2685	1.041	50

Note for Table 1

Let  $F$  be the forecast of a series and  $A$  its actual value. Then the mean forecast error is the average of  $(F-A)$  over the forecast period. The mean absolute error is the average of  $|F-A|$ . The terms "Root mean squared error" and "Theil U" are explained in the text. Except for inflation, all errors can be considered as percentages, divided by 100. This is because the equations are in logarithms of the original series.

Experimentation suggests that the performance of our model in predicting housing starts could be slightly improved by adding a short interest rate variable, such as a T-bill rate, as an "exogenous" variable in the VAR system. Our subsequent conditional forecasting experiments (see Appendix B for details on how the conditional forecasts were made) suggest, however, that this increase in forecast accuracy would largely be due to better (unconditional) forecasts of mortgage rates. It is thus unlikely our conditional forecasts would be strongly affected by this complication. With the goal of keeping our study inexpensive, simple, and easily duplicable, we decided against constructing a more complicated model.

Also, our study ideally would have covered a greater number of indicators of housing sector activity. Increasing the number of series to be modeled by BVAR techniques, however, makes both forecasting and interpretation of BVAR models more difficult. Construction of a large BVAR necessarily involves making some arbitrary modelling decisions inappropriate to the preliminary nature of our study. Hence, we restricted our attention to two of the most widely followed aggregates: single-family housing starts, and (real) median single-family home prices.

Appendix B: Unconditional and Conditional  
Forecasting in VAR Models<sup>1/</sup>

Consider a N-variate process  $x_t$  with known VAR representation

$$A(L)x_t = e_t,$$

where  $x_t$  is a N x 1 vector,  $A(L)$  a N x N polynomial matrix one-sided in nonnegative powers of L, and  $e_t$  a vector of white noise.

Denote by  $\Omega_t$  the set  $\{x_t, x_{t-1}, \dots\}$ . The "unconditional forecast" of  $x_{t+k}$ ,  $k > 0$ , is defined to be the linear least squares projection of  $x_{t+k}$  on  $\Omega_t$ , denoted

$$P[x_{t+k} | \Omega_t],$$

or  $\hat{x}_{t+k}$ . It is well known that  $\hat{x}_{t+k}$  can be derived recursively by the formula:<sup>2/</sup>

$$(B1) \quad \hat{x}_{t+k} = -\left[\frac{A(L)}{L}\right]_+ \hat{x}_{t+k-1},$$

where  $\hat{x}_{t+k} = x_{t+k}$  for  $k \leq 0$ .

Now let the vector  $x$  be defined by

$$[x'_{t+1} \dots x'_{t+K}]', K > 1.$$

The unconditional forecast  $P[x | \Omega_t] = \hat{x}$  can be obtained via successive applications of formula (B1).

We now consider the conditional forecasting problem. Suppose that, in addition to  $\Omega_t$ , it is known that

$$R'x = \underline{r},$$

where  $R'$  is a known  $J \times NK$  matrix of rank  $J$ ,  $J < NK$ , and  $\underline{r}$  is a known  $J \times 1$  vector. The conditional forecasting problem is one of computing

$$P[x | \Omega_t, R'x = \underline{r}].$$

Since  $\hat{x}$  is in the span of  $\Omega_t$ , this is equivalent to calculating

$$P[(x-\hat{x}) | \Omega_t, R'(x-\hat{x}) = \underline{r}-R'\hat{x}].$$

Defining  $v = x - \hat{x}$  and  $r = \underline{r} - R'\hat{x}$ , the projection problem to be solved is that of calculating

$$P[v | \Omega_t, R'v = r].$$

By the "projection theorem,"<sup>3/</sup> the unconditional forecast error  $v$  is uncorrelated with  $\Omega_t$ . Hence the above projection reduces to

$$P[v | R'v = r],$$

which we designate as  $u^*$ .

By definition,  $u^*$  solves the problem

$$\min_u E(v-u)'(v-u) \quad \text{s.t. } R'u = r,$$

$\Leftrightarrow$

$$\min_u Ev'v + u'u \quad \text{s.t. } R'u = r,$$

since  $u$  is nonstochastic and  $Ev = 0$ . Since  $Ev'v$  does not depend on  $u$ , the last problem reduces to

$$(B2) \quad \min_u u'u \quad \text{s.t. } R'u = r$$



which has solution

$$u^* = R(R'R)^{-1}r.$$

The conditional forecast of  $x$  may then be recovered as

$$P[x|\Omega_t, R'x = \underline{r}] = \hat{x} + u^*.$$

#### Additional Notes

The derivation given above is for a process  $x_t$  with mean zero. The formulas can be easily extended to processes with nonzero means.

To calculate our conditional forecasts, we used the RATS code given in Example 17.2 of Doan and Litterman (1984), which solves the programming problem (B2). Having estimated  $A(L)$  using data on  $x_t, x_{t-1}, \dots$  (with  $t = \text{December } 1981$ ), we then proceeded as if  $A(L)$  were known. Proceeding in this fashion ignores the impact of the additional information  $R'x = r$  on the estimate of  $A(L)$ , but saves significant amounts of computation time.

Appendix Notes

1/What follows is a somewhat bowdlerized version of a presentation in Doan, Litterman, and Sims (1984). The notation is that of Sargent (1979), chapter 10.

2/The notation  $[ \quad ]_+$  means "ignore negative powers of L." See Sargent (1979), chapter 12, for an extensive discussion of forecasting with time series models.

3/For a presentation of this theorem, see Luenberger (1969), chapter 3.

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