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TIME-TO-BUILD, DELIVERY LAGS,  
AND EQUILIBRIUM PRICING OF CAPITAL GOODS

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## Abstract

This paper characterizes the behavior of investment expenditures, optimal capital stocks, and real interest rates in the time-to-build model of investment. These results are used to show that the delivery lag model of investment fails to account for time lags in investment when constructing the cost of capital variable and hence, misspecifies the effects of interest rates on investment expenditures. Second, this paper derives equilibrium pricing relationships involving the prices of existing capital and uses these relationships to obtain simple tests of the underlying investment technology. Despite the widespread use of 'q' in the empirical investment literature, it is shown that the relationship between current investment and an appropriately defined measure of Tobin's 'q' contains no such testable implications. Finally, it is shown that the practice of using stock market data to measure the price of existing capital is invalid when time lags exist in the investment process.

## 1. Introduction

Much of the empirical investment literature has been based on the theory of the desired capital stock coupled with an investment equation which assumes a time lag between investment expenditures, on the one hand, and orders or purchases of actual capital goods, on the other. Dubbed the neoclassical model of investment, this framework originates with Jorgenson [1963] and Jorgenson and Hall [1967]. More recent examples include Clark [1979] and Abel and Blanchard [1983c]. One problem with this approach is that it ignores the effects of lags in the investment process on the optimal capital stock decisions of firms. However, if firms know that investment expenditures made in period  $t$  give rise to productive capital with some time lag, then their initial choice of capital should take these lags into account. Furthermore, much of this literature does not precisely specify the source of the so-called delivery lags in the investment process, assuming only that they arise from delays in "ordering, delivering, installing, and building" (see Hall [1977]).

An alternative model of investment which incorporates time lags in the investment process is provided by the time-to-build model. According to this model of investment, used recently by Kydland and Prescott [1982] and others, the source of the time lags lies directly in the production technology. In this case, the value of the optimal capital stock reflects the existence of such lags. This effect can be interpreted as occurring through a cost of capital variable which is derived under the assumption

that multiple time periods are required to build productive capital. Based on a model in which the output of the single good may be consumed or used to produce new capital goods, I show that the cost of capital with production lags of  $J$  periods depends on a weighted combination of  $j$ -period real interest rates for  $j = 1, \dots, J$ , these weights reflecting the fraction of resources allocated to the construction of incomplete capital goods in each period. Consequently, the distributed lag coefficients of an accelerator-like investment equation also determine the values of the optimal capital stocks, inducing a more complicated dynamic relationship among investment expenditures, capital stocks, and real interest rates than is assumed in the standard delivery lag models. In particular, when the cost of capital variable is specified to take into account the existence of investment lags, optimal capital stocks and investment expenditures all depend on the values of real interest rates having maturities  $J$  periods or less. By contrast, the delivery lag models based on Jorgensonian assumptions neglect the effect of changes in the term structure of interest rates on the investment decisions of firms.

These results are derived from the present-value maximization problem of competitive firms who trade in the market for existing capital, as well as investing their own output. Using pricing relationships which emerge from the necessary conditions of an individual firm's optimum problem, I also examine the equilibrium behavior of the shadow prices of existing capital and of Tobin's 'q.' This analysis is of interest because it provides a

way to determine whether different models of investment can be differentiated in terms of the relationship between Tobin's 'q' and the level of current investment. The practice of regressing the level of current investment on some measure of 'q' to obtain an investment equation was criticized by Sargent [1979] in the context of the optimal growth model with irreversible investment. The purpose of his analysis was to show that such a regression summarizes the joint behavior of two endogenously determined variables and consequently cannot by itself be used to predict the effects of any changes in policy.

The results of this paper further suggest that correlations between new investment and 'q,' as it is usually defined, are not useful for distinguishing among different economic models generating such correlations. On the other hand, I show that pricing relationships involving the shadow prices of existing capital do provide testable implications about the underlying investment technology. In order to use such restrictions, however, observable series on the shadow prices of the different types of capital are required. Consequently, I investigate whether the practice of using stock market data to measure such prices can be rationalized in the model of this paper. Unlike the analyses of Abel [1980] and Hayashi [1982], who consider a single capital good model with constant returns to scale production technology and adjustment costs in investment, I find that "average 'q'," defined from the value of an individual firm's equities and liabilities, does not provide a measure of the shadow prices

of existing capital or of "marginal 'q'." The difference arises from the existence of multiple types of capital. More precisely, the value of the firm depends not only on the value of its productive stocks of capital but also on the value attributed to the unfinished projects.

The remainder of this paper is organized as follows: Section 2 describes the model and sets up the firm's problem. Section 3 compares the time-to-build model with the delivery lag model of investment. This discussion is aided by the use of a specific example. Section 4 derives testable implications from the pricing relations of the time-to-build model. It also seeks observable measures of the prices of existing capital to give empirical content to these relations. Section 5 offers some concluding remarks.

## 2. The Model

This section describes the implications of the time-to-build model of investment for capital stocks, investment expenditures, and the shadow prices of existing capital. Use is made of first-order conditions of firms' optimum problems and the equilibrium conditions in the market for existing capital of a simple competitive economy.

Each firm in this economy has access to the same constant returns to scale production technology for producing the output of the single good, using labor and two types of capital. While the labor input is obtained from a competitive labor market, I assume that firms own the physical stocks of capital and use

part of their own output to produce new capital goods. In addition, each firm may increase its own capital stock by trading in the market for existing capital. The implied financial arrangement is straightforward: firms finance all investment from retained earnings and never issue any new shares or bonds.<sup>2/</sup> Hence, owners receive dividends from firms in each period according to their initial holdings of shares which are fixed at time zero. According to this setup, the objective of an individual firm is to maximize the value of owners' equity, which is the present value of the infinite stream of dividends paid by the firm.

This formulation of the firm's problem differs from the approach of Prescott and Mehra [1980] and Brock [1983], who assume that households own all the physical stocks of capital and rent or sell capital to firms on a period-by-period basis. In this case, all investment decisions are implemented by households and firms solve a static problem, choosing inputs of labor and capital to maximize current period profits. But this difference is only expositional. The framework adopted in this paper allows me to illustrate more easily the relationship between the prices of new and used capital goods and to provide a closer link with the existing investment literature.

The reason for omitting stock and bond markets is also expositional. Explicitly modeling the behavior of consumers and allowing trade in claims to the output of the different firms would allow me to derive the standard expression for share prices similar to that obtained by Lucas [1978] in a pure exchange econ-

omy or by Brock [1982] in a model with production and capital. Likewise, the introduction of a bond market would yield expressions for real interest rates. Aside from complicating the exposition, however, neither feature would add in substance to the contents of this paper.

Assume that there is a large number of identical firms and consider the problem of a typical firm. Each firm is endowed with a technology which enables it to produce the output of the single good according to the constant returns to scale production function given by

$$(2.1) \quad Q_t = \lambda_t f(n_t, k_{1t}, k_{2t}).$$

Here,  $n_t$  is the labor input measured as total man-hours,  $k_{1t}$  and  $k_{2t}$  denote the stocks of two types of capital and  $\lambda_t$  is a random shock to technology. According to (2.1), the stocks of the different types of capital and the services from such stocks are assumed identical. The function  $f(.,.,.)$  satisfies the usual Inada conditions in all its arguments and is further restricted to ensure the existence of a finite positive bound  $\gamma$  such that  $k_{1t} > \gamma$  and  $k_{2t} > \gamma$  implies  $f(n_t, k_{1t}, k_{2t}) < 2\gamma$  for  $n_t < N$ , where  $N$  is the total labor endowment of the economy. Finally,  $\{\lambda_t\}_{t=0}^{\infty}$  is defined as a sequence of random variables which take values in the real, strictly positive interval  $[\underline{\lambda}, \bar{\lambda}]$  with  $\bar{\lambda} < \infty$ .

Different assumptions characterize investment in the two types of capital  $k_{1t}$  and  $k_{2t}$ . Investment in the latter proceeds according to the simple neoclassical assumptions, where one unit



of investment in period  $t$ , denoted  $i_{2t}$ , yields an additional unit of the second type of capital in period  $t + 1$  without adjustment costs, production lags, etc. On the other hand, investment in the first type of capital is characterized by the time-to-build assumption as suggested by Kydland and Prescott [1982]. According to this technology, a unit of investment in period  $t$  yields productive capital with a lag of  $J$  periods, where  $J$  is exogenously specified. Let  $s_{Jt}$  denote the number of new projects initiated at time  $t$  in the first type of capital. Also let  $s_{jt}$ ,  $j = 1, \dots, J - 1$  denote the number of projects  $j$  periods from completion in period  $t$ . The laws of motion which describe the evolution of the incomplete projects are given by

$$(2.2) \quad s_{j,t+1} = s_{j+1,t} \quad j = 1, \dots, J - 1.$$

If it is assumed that a fixed fraction  $\phi_j$  of resources are expended in each period for the different incomplete projects, then total investment expenditures in the first type of capital are given by

$$(2.3) \quad i_{1t} = \sum_{j=1}^J \phi_j s_{jt}$$

with  $0 < \phi_j < 1$ ,  $j = 1, \dots, J$  and  $\sum_{j=1}^J \phi_j = 1$ . Although the aggregate capital stock can be increased only through new investment, an individual firm can alter its own stocks through purchases of existing capital. Let  $k_{1t}^d$  and  $k_{2t}^d$  denote the purchases of the two types of capital. Then the laws of motion for the capital stocks of an individual firm are given by

$$(2.4) \quad k_{1t+1} = k_{1t}^d + s_{1t}$$

$$(2.5) \quad k_{2t+1} = k_{2t}^d + i_{2t}$$

To complete the description of the firm's problem, the system of competitive prices must be specified, which in turn depends on the nature of trades individual agents can enter into. By abstracting from factors which make certain trades infeasible--such as private information and intergenerational restrictions--I can assume that this economy possesses a full set of contingent claims markets. In this case, all trades may be executed at time zero, with consumers and firms trading claims to output, and with capital goods and labor services to be delivered at each date and possible state of the economy. Alternatively, it is possible to give a sequential interpretation to this economy: first, agents trade claims to output from time zero forward, contingent on information available when such output is delivered. At each date and realized state, delivery takes place according to the contingent claims contracts. Then consumers and firms trade in spot markets for consumption and capital goods, and for labor services. I adopt the second interpretation in what follows because it illustrates how "stochastic discount factors" may be defined from the contingent claims prices of output. The second interpretation also permits a distinction to be drawn between the spot prices of labor and existing capital and their claims prices.

Some additional notation must be introduced before the competitive price system can be defined. Let  $(\Omega, I, P)$  be a probability space such that  $\Omega$  is the set of all sequences of real variables, i.e., a typical element of  $\Omega$  is given by  $\omega = \{\omega_t\}_{t=0}^{\infty}$ . In this case, the random variables  $\lambda_t$  may be defined as the  $t^{\text{th}}$  coordinate function of  $\Omega$ . In other words, for any  $\omega = \{\omega_t\}_{t=0}^{\infty} \in \Omega$ ,  $\lambda_t(\omega)$  is given by  $\lambda_t(\omega) = \omega_t$ . Furthermore,  $I$  denotes the  $\sigma$ -algebra of measurable sets or events in  $\Omega$ , and  $P$  is the measure which assigns probability to such events. Letting  $I_t$  be the information set or  $\sigma$ -algebra generated by realizations of the technology shock up to period  $t$ , i.e., by  $\{\lambda_s(\omega) : s \leq t\}$ , notice that  $\{I_t\}_{t=0}^{\infty}$  is a nondecreasing sequence of sub  $\sigma$ -algebras of  $I$ . More intuitively,  $I_t$  is the set, closed under complements and countable unions, of all events which may be verified to have occurred at date  $t$ . Finally, let  $P_t$  be the measure that assigns probability to events which are elements of  $I_t$ , i.e.,  $P_t$  is defined as the restriction of  $P$  to  $I_t$ .

Given this notation, the price of a contingent claims contract which promises to deliver one unit of output contingent on the event  $E \in I_t$  occurring at time  $t$  may be defined as

$$\int [p_t \chi_E dP_t] dP_0 = E[p_t \chi_E | I_0]$$

where  $\chi_E$  is the indicator function of  $E \in I_t$ .<sup>3/</sup> Likewise, the price of a sure claim to a unit of the single good at date  $t$  is  $E(p_t | I_0)$ . To define the contingent claims prices of the remaining goods, let  $d_t$ ,  $w_t$ ,  $q_{1t}$ , and  $q_{2t}$  be the spot value or price of

dividends, labor services, and the two types of existing capital. For example,  $w_t$  is expressed in terms of output per unit of labor and is defined for the particular event which is realized at date  $t$ . Hence, the value of a contingent claims contract for a sure unit of labor services at date  $t$  is given by  $E[p_t w_t n_t | I_0]$ . The value of contingent claims contracts for dividends and for existing capital are similarly defined.

Given this system of prices, the problem of an individual firm is now well-posed. At time zero, each firm maximizes the value of owners' equity  $V_0$ , taking as given the contingent claim prices  $\{p_t\}_{t=0}^{\infty}$  and the spot prices  $\{w_t\}_{t=0}^{\infty}$ ,  $\{q_{1t}\}_{t=0}^{\infty}$ , and  $\{q_{2t}\}_{t=0}^{\infty}$ :

$$\begin{aligned}
 (2.5) \quad & \text{maximize} & V_0 &= E \left[ \sum_{t=0}^{\infty} p_t d_t \mid I_0 \right] \\
 & \{s_{jt}\}, \{i_{2t}\}, & &= E \left\{ \sum_{t=0}^{\infty} p_t [\lambda_t f(n_t, k_{1t}, k_{2t}) + \right. \\
 & \{k_{1t}^d\}, \{k_{2t}^d\}, \{n_t\} & & \left. q_{1t} (1-\delta_1) k_{1t} + q_{2t} (1-\delta_2) k_{2t} - i_{1t} - \right. \\
 & & & \left. i_{2t} - w_t n_t - q_{1t} k_{1t}^d - q_{2t} k_{2t}^d \mid I_0 \right\}
 \end{aligned}$$

subject to

$$(2.5a) \quad k_{1t+1} = s_{1t} + k_{1t}^d$$

$$(2.5b) \quad k_{2t+1} = i_{2t} + k_{2t}^d$$

$$(2.5c) \quad s_{j,t+1} = s_{j+1,t} \quad j = 1, \dots, J - 1$$

$$(2.5d) \quad i_{1t} = \sum_{j=1}^J \phi_j s_{jt}$$

$$s_{Jt} > 0, i_{2t} > 0, k_{1t}^d > 0, k_{2t}^d > 0, n_t > 0 \text{ for all } t > 0$$

and given the initial stocks of capital  $s_{j0}$ ,  $j = 1, \dots, J - 1$ ,  $k_{10}$  and  $k_{20}$ . The firm is assumed to sell off its stocks of capital at the end of the period to other (identical) firms. Thus in (2.5), the second and third terms show the receipts from the sale of the two types of capital;  $\delta_1$  and  $\delta_2$  indicate the respective fractions lost during the period due to depreciation. To obtain capital for production in period  $t + 1$ , the firm can invest its output: if it undertakes investment in the second type of capital, it incurs the cost  $i_{2t}$ . On the other hand, investment in the first type yields productive capital with a lag of  $J$  periods. Hence, current investment expenditures include expenditures on new projects  $s_{Jt}$ , as well as expenditures for the incomplete projects. The term  $\sum_{j=1}^J \phi_j s_{jt}$  reflects such costs in (2.5), while  $w_t n_t$  is the current labor cost. Finally,  $q_{1t} k_{1t}^d$  and  $q_{2t} k_{2t}^d$  show the spot value of purchases of the two types of existing capital. Therefore,  $V_0$  is the expected discounted value of the net receipts of the firm at each date, with the sequence of contingent claims prices  $\{p_t\}_{t=0}^{\infty}$  expressed in terms of the price of output at date zero playing the role of "stochastic discount factors."

The solution to the present-value maximization problem described in (2.5) will be a set of optimal plans  $\bar{s}_J = \{\bar{s}_{Jt}\}_{t=0}^{\infty}$ ,  $\bar{i}_2 = \{\bar{i}_{2t}\}_{t=0}^{\infty}$ ,  $\bar{k}_1^d = \{\bar{k}_{1t}^d\}_{t=0}^{\infty}$ ,  $\bar{k}_2^d = \{\bar{k}_{2t}^d\}_{t=0}^{\infty}$ , and  $\bar{n} = \{\bar{n}_t\}_{t=0}^{\infty}$ , which specify the new investment, purchases of existing capital,

and labor input choices of the firm. To be consistent with the competitive price system described above, these plans must be restricted so that  $\bar{s}_{Jt}$ ,  $\bar{i}_{2t}$ ,  $\bar{k}_{1t}^d$ ,  $\bar{k}_{2t}^d$ , and  $\bar{n}_t$  for  $t \geq 0$  are measurable functions with respect to the information set  $I_t$  or, equivalently, depend only on information available at time  $t$ . In order to constitute a solution to (2.5), the optimal plans  $\bar{s}_J$ ,  $\bar{i}_2$ ,  $\bar{k}_1^d$ ,  $\bar{k}_2^d$ , and  $\bar{n}$  must also satisfy the Euler equations and transversality conditions to (2.5). Since (2.5) is a concave programming problem, the necessary conditions for a firm optimum are sufficient as well. Furthermore, the  $\bar{k}_1^d$  and  $\bar{k}_2^d$  sequences must satisfy the equilibrium conditions in the market for existing capital. With identical firms, these conditions may be expressed in terms of the individual firm's variables as

$$(2.6) \quad \bar{k}_{1t}^d = (1-\delta_1)k_{1t}$$

$$(2.7) \quad \bar{k}_{2t}^d = (1-\delta_2)k_{2t}$$

Equations (2.8) and (2.9) below provide a subset of the relevant optimality conditions, obtained by differentiating (2.5) with respect to  $s_{Jt}$  and  $i_{2t}$  for  $t \geq 0$  and assuming an interior solution:

$$(2.8) \quad E_t q_{1t+J-1} p_{t+J-1} = \\ E_t p_t \phi_J + E_t p_{t+1} \phi_{J-1} + \dots + E_t p_{t+J-1} \phi_1$$

$$(2.9) \quad E_t q_{2t} p_t = E_t p_t \cdot$$

These conditions imply that as long as new and used capital are perfect substitutes (for example, when there are no vintage effects), the prices of existing capital must adjust so that the firm is indifferent between investing a unit of its own output (thereby creating newly produced capital) and purchasing existing capital from other firms. Consequently, at the level of the individual firm, the values of  $s_{Jt}$  versus  $k_{1t}^d$  and  $i_{2t}$  versus  $k_{2t}^d$  will be indeterminate.<sup>4/</sup>

Nevertheless, conditions (2.8) and (2.9) show the implications for the equilibrium prices of existing capital  $\bar{q}_{1t}$  and  $\bar{q}_{2t}$ ,  $t \geq 0$ . (2.9) is easier to interpret: since  $p_t \in I_t$ , I can divide both sides of (2.9) by  $p_t$ . Then (2.9) shows that in equilibrium, the spot price  $q_{2t}$  must equal the spot price of output. Formally, the equilibrium sequence  $\bar{q}_2 = \{\bar{q}_{2t}\}_{t=0}^{\infty}$  must satisfy the condition  $q_{2t} = 1$  for all  $t$ . Intuitively, this implies that a unit of output can be costlessly transformed into a unit of the second type of capital.

Equation (2.8) shows the effect of the time-to-build assumption on the cost of obtaining additional capital. As before, divide both sides of (2.8) by  $p_t$ . This yields

$$E_t \frac{q_{2t+J-1} p_{t+J-1}}{p_t} = \phi_J + \phi_{J-1} E_t \frac{p_{t+1}}{p_t} + \dots + \phi_1 E_t \frac{p_{t+J-1}}{p_t}$$

Define the real risk-free interest rate between periods  $t$  and  $t + j$  as

$$\frac{1}{1 + r_t^j} \equiv E_t \frac{p_{t+j}}{p_t}$$

that is, as the ratio of prices of a sure claim to output at those dates. Then

$$(2.10) \quad E_t \frac{q_{1t+J-1} p_{t+J-1}}{p_t} = \phi_J + \frac{\phi_{J-1}}{1 + r_t^1} + \dots + \frac{\phi_1}{1 + r_t^{J-1}}.$$

This equation shows that the fraction of resources  $\phi_j$  which must be expended in each period until a given project is completed is discounted by the relevant interest rates between periods  $t$  and  $t + J - j$ . Hence, the marginal cost of new investment depends on a weighted combination of the  $j$ -period real interest rates prevailing at time  $t$ . Given the sequence of contingent claims prices  $\{p_t\}_{t=0}^{\infty}$ , (2.8) or (2.10) implicitly determine the equilibrium sequence  $\bar{q}_1 = \{\bar{q}_{1t+J-1}\}_{t=0}^{\infty}$ . With time lags in the investment process for the first type of capital, the spot price  $q_{1t}$ ,  $t > J - 1$  does not equal unity in equilibrium. This is in contrast to the behavior of  $q_{2t}$  under the simple neoclassical technology. Another consequence of the time-to-build assumption is that the equilibrium values of  $q_{1t}$ ,  $0 < t < J - 1$  are not determined. This occurs because the stocks of incomplete projects  $s_{j0}$ ,  $j = 1, \dots, J - 1$  are given according to the initial conditions of the firm's problem. Thus, there are no conditions analogous to (2.8) from which  $q_{10}$  to  $q_{1J-1}$  may be pinned down.

The remaining conditions which characterize the optimal plans are obtained by imposing the equilibrium conditions (2.6) and (2.7), and differentiating (2.5) with respect to  $k_{1t}^d$ ,  $k_{2t}^d$ , and  $n_t$  for  $t > 0$ :



$$(2.11) \quad q_{1t}p_t - E_t(1-\delta_1)q_{1t+1}p_{t+1} = \\ E_t p_{t+1} \lambda_{t+1} \frac{\partial f(n_{t+1}, k_{1t+1}, k_{2t+1})}{\partial k_{1t+1}}$$

$$(2.12) \quad q_{2t}p_t - E_t(1-\delta_2)q_{2t+1}p_{t+1} = \\ E_t p_{t+1} \lambda_{t+1} \frac{\partial f(n_{t+1}, k_{1t+1}, k_{2t+1})}{\partial k_{2t+1}}$$

$$(2.13) \quad w_t = \lambda_t \frac{\partial f(n_t, k_{1t}, k_{2t})}{\partial n_t}.$$

According to conditions (2.11) and (2.12), the firm must be indifferent between increasing next period's capital stock by a unit, on the one hand, and buying a unit of existing capital in period  $t$  and selling off the undepreciated portion the next period, on the other. Alternatively, (2.11) and (2.12) restrict the equilibrium sequences  $\bar{q}_1$  and  $\bar{q}_2$  such that the value of "pure speculation" in the market for existing capital cannot exceed the value of increasing the physical stocks. Finally, (2.13) relates the firm's choice of the optimal labor input to the spot wage.

In addition to the Euler equations (2.8), (2.9), (2.11), (2.12), and (2.13), the necessary conditions for a firm optimum include two transversality conditions which constrain the equilibrium value of the productive capital stocks, i.e.,

$$(2.14) \quad \lim_{t \rightarrow \infty} E_t p_{t+J-1} \bar{q}_{1t+J-1} \bar{k}_{1t+J} \rightarrow 0$$

$$(2.15) \quad \lim_{t \rightarrow \infty} E_t p_t \bar{q}_{2t} \bar{k}_{2t+1} \rightarrow 0.$$

These conditions ensure that the value of dividends  $\bar{d} = \{\bar{d}_t\}_{t=0}^{\infty}$  are finite in equilibrium, i.e.,  $E[\sum_{t=0}^{\infty} p_t \bar{d}_t | I_0] < \infty$ , and hence that the firm's present-value maximization is well-defined. One set of sufficient conditions for the transversality conditions to hold is that the optimal sequences of productive capital  $\{\bar{k}_{1t+J}\}_{t=0}^{\infty}$  and  $\{\bar{k}_{2t+1}\}_{t=0}^{\infty}$  are bounded and that the sequences  $\{p_{t+J-1} \bar{q}_{1t+J-1}\}_{t=0}^{\infty}$  and  $\{p_t \bar{q}_{2t}\}_{t=0}^{\infty}$  are decreasing towards zero. The existence of a maximum level of output ensures that the productive capital stocks remain bounded. With respect to the second requirement, notice from (2.8) and (2.9) that  $p_{t+J-1} q_{1t+J-1}$  and  $p_t q_{2t}$  for  $t > 0$  may be expressed solely in terms of current and past values of  $p_t$ . Hence, if  $\{p_t\}_{t=0}^{\infty}$  converges to zero, so will the equilibrium sequences  $\{p_{t+J-1} \bar{q}_{1t+J-1}\}_{t=0}^{\infty}$  and  $\{p_t \bar{q}_{2t}\}_{t=0}^{\infty}$ . Without explicitly modeling the consumer side of the economy, it is difficult to be precise about the conditions which ensure convergence of  $\{p_t\}_{t=0}^{\infty}$  to zero. As an example, however, this final requirement will be satisfied if consumers have time-separable preferences over consumption and leisure with bounded one-period utility functions and discount the future at the same rate  $0 < \beta < 1$ .

Finally, combining (2.11) and (2.12) with the transversality conditions, I can derive expressions for the spot prices of existing capital as

$$(2.16) \quad q_{it} = E_t \sum_{k=1}^{\infty} (1-\delta_i)^{k-1} \frac{p_{t+k}}{p_t} \lambda_{t+k} \frac{\partial f(n_{t+k}, k_{1t+k}, k_{2t+k})}{\partial k_{it+k}}$$

$$i = 1, 2$$

with (2.16) holding for  $t \geq J - 1$  for the first type of capital and for  $t \geq 0$  otherwise. These formulas are similar to those found in Sargent [1979] and Abel [1980] and express  $q_{1t}$  and  $q_{2t}$  as the discounted sum of the future marginal products of the two types of capital, the discount factors corresponding to the ratio of the contingent claims prices of output in period  $t + k$  relative to period  $t$ .

### 3. Comparison With the Delivery Lag Model of Investment

While the results of the previous section have interest in their own right, they are also useful for analyzing the implications of different models of investment and, in particular, for comparing the time-to-build model of investment with the delivery lag model.

First, to obtain a condition characterizing the determination of the optimal capital stock in the first type of capital  $k_{1t+J}$ , use condition (2.8) for periods  $t + J - 1$  and  $t + J$ , together with condition (2.11) for period  $t + J - 1$ . Jointly, they imply that

$$\xi_J p_t + \xi_{J-1} E_t p_{t+1} + \dots + \xi_0 E_t p_{t+J} =$$

$$E_t p_{t+J} \lambda_{t+J} \frac{\partial f(n_{t+J}, k_{1t+J}, k_{2t+J})}{\partial k_{1t+J}}$$

or

$$(3.1) \quad \xi_J + \frac{\xi_{J-1}}{1 + r_t^1} + \dots + \frac{\xi_0}{1 + r_t^J} =$$

$$E_t \frac{p_{t+J}}{p_t} \lambda_{t+J} \frac{\partial f(n_{t+J}, k_{1t+J}, k_{2t+J})}{\partial k_{1t+J}} \quad t \geq 0,$$

where  $g_J \equiv \phi_J$ ,  $g_{J-1} \equiv \phi_{J-1} - (1-\delta_1)\phi_J$ , ...,  $g_0 \equiv -(1-\delta_1)\phi_1$ . A similar condition for the second type of capital can be derived from (2.9) and (2.12) as

$$1 - E_t \frac{p_{t+1}}{p_t} + \delta_2 E_t \frac{p_{t+1}}{p_t} =$$

$$E_t \frac{p_{t+1}}{p_t} \lambda_{t+1} \frac{\partial f(n_{t+1}, k_{1t+1}, k_{2t+1})}{\partial k_{2t+1}}$$

or

$$(3.2) \quad 1 - \frac{(1-\delta_2)}{1+r_t^1} = E_t \frac{p_{t+1}}{p_t} \lambda_{t+1} \frac{\partial f(n_{t+1}, k_{1t+1}, k_{2t+1})}{\partial k_{2t+1}} \quad t > 0.$$

The left-hand sides of (3.1) and (3.2) represent cost of capital variables. For the neoclassical model, this is simply  $\frac{\delta_2 + r_t^1}{1 + r_t^1}$  and depends only on the short-term real interest rate  $r_t^1$ . These conditions also show that (3.2) is a special case of (3.1) when  $J = 1$ . In this case,  $g_J \equiv \phi_J = 1$  and  $g_{J-1} \equiv \phi_{J-1} - (1-\delta_1)\phi_J = 1 - \delta_1$ . Using the initial conditions  $k_{10}$  and  $s_{j0}$ ,  $j = 1, \dots, J-1$ , the values of  $k_{1t}$  for  $1 \leq t \leq J - 1$  may be determined from the laws of motion (2.5a) and (2.5c), together with the equilibrium condition (2.6) in the market for existing capital. Hence, conditions (3.1) and (3.2) and condition (2.13) linking the marginal product of labor to the spot wage for periods  $t$  to  $t + J$ ,  $t > 0$  can be jointly solved for the sequences of desired capital stocks and optimal labor inputs of an individual firm which takes as given all prices and the stochastic law of motion for the technology shock  $\lambda_t$ . Given the constant returns to scale property

of the production function, these factor demands must be interpreted for a given level of output, normalized here as one.

To derive relationships between investment expenditures desired, capital stocks and real interest rates, recall from Section 2 that current investment expenditures  $i_{1t}$  in the first type of capital are determined as  $i_{1t} = \sum_{j=1}^J \phi_j s_{jt}$ . However, using the laws of motion (2.5a) and (2.5c) and the equilibrium condition (2.6), this can be expressed as an accelerator-like equation in the desired capital stock,

$$(3.3) \quad i_{1t} = g(L)k_{1t+J}$$

$$= g_J k_{1t+J} + g_{J-1} k_{1t+J-1} + \dots + g_0 k_{1t}.$$

Expression (3.3), together with the joint solution to (2.13), (3.1), and (3.12), imply that investment expenditures in the first type of capital  $i_{1t}$  depend on current and past values of the  $j$ -period interest rates  $r_t^j$  for  $j = 1, \dots, J - 1$ . This occurs because the optimal values of  $k_{1t+J-j}$  for  $j = 0, \dots, J$  depend on multi-period interest rates through the cost of capital variable defined by the left-hand side of (3.1). Furthermore, the distributed lag coefficients  $g_j$  for  $j = 0, \dots, J$  which show the response of investment expenditures to changes in the desired capital stocks enter non-linearly into the "reduced form" expression for  $i_{1t}$ . Again, this occurs because such coefficients affect the values of the optimal capital stocks by weighting the  $j$ -period interest rates in the expression for the cost of capital.

Recall that investment in the second type of capital evolves according to the law of motion

$$(3.4) \quad i_{2t} = k_{2t+1} - (1-\delta_2)k_{2t} \\ = [1-(1-\delta_2)L]k_{2t+1}$$

with the value of the optimal capital stock  $k_{2t+1}$  determined from (2.13), (3.1), and (3.2). In this case, the cost of type 2 capital merely depends on the one-period real interest  $r_t^1$ . Provided the production function is not additive in the two types of capital, however,  $k_{2t+1}$  and  $i_{2t}$  will both depend on long-term interest rates  $r_{t-J+1}^j$  for  $j = 1, \dots, J$ . Furthermore, the distributed lag coefficients  $g_j$ ,  $j = 0, \dots, J$  will enter the expressions for  $k_{2t+1}$  and  $i_{2t}$  even though there exist no time lags in the investment process for the second type of capital.

By contrast, much of the investment literature which assumes the existence of such time lags ignores their effect in the determination of the optimal capital stock. Instead, an expression for the desired capital stock is derived from a relation such as (2.12). (See, for example, Jorgenson [1963] and Clark [1979].) An investment equation is then obtained by assuming that the actual capital stock partially adjusts toward the level defined by the desired stock, with this adjustment taking place according to arbitrarily specified distributed lag coefficients. One implication of this approach is that the relationship between current investment expenditures and real interest rates is not correctly specified. More precisely, the cost of capital

variable corresponding to the left-hand side of (2.12) is constructed by measuring the price of capital goods with some investment deflator and assuming that real interest rates are constant. Because an equilibrium condition corresponding to (2.8) is omitted, the cost of capital variable derived in this manner does not depend on long-term interest rates. With time lags in the investment process, firms take into account all real interest between periods  $t$  and  $t + j$  for  $j = 1, \dots, J$  when determining the value of the capital stock desired at  $t + J$ . The empirical investment literature based on the delivery lag model ignores the effects of such forward-looking behavior on the part of firms.

These points may be more easily illustrated by considering a certainty version of the model so that  $\{\lambda_t\}_{t=0}^{\infty}$  is a known sequence. The production function is assumed to be Cobb-Douglas with constant returns to scale. More specifically, let

$$f(n_t, k_{1t}, k_{2t}) = n_t^a k_{1t}^b k_{2t}^c$$

where  $a + b + c = 1$ . Then, the factor demands for  $n_t$ ,  $k_{1t+J}$ , and  $k_{2t+1}$  are given by

$$(3.5a) \quad n_t = \frac{n_1}{\lambda_t w_t^{1-a}} (\hat{r}_{t-J})^b (\delta_2 + r_{t-1}^1)^c$$

$$(3.5b) \quad k_{1t+J} = k_1 \frac{w_{t+J}^a}{\lambda_{t+J}} \frac{(\delta_2 + r_{t+J-1}^1)^c}{(\hat{r}_t)^{1-b}} \quad t > 0$$

$$(3.5c) \quad k_{2t+1} = k_2 \frac{w_{t+1}^a}{\lambda_{t+1}} (\hat{r}_{t-J+1})^b (\delta_2 + r_t^1)^{1-c}$$

where

$$n_1 = (a^{a-1} b^b c^c)^{-1}$$

$$k_1 = (a^a b^{(b-1)} c^c)^{-1},$$

$$k_2 = (a^a b^b c^{c-1})^{-1},$$

$$\hat{r}_t \equiv [g_J + g_{J-1} \frac{1}{1+r_t^1} + \dots + g_0 \frac{1}{1+r_t^J}](1+r_t^J)$$

and

$$\frac{1}{1+r_t^j} \equiv \frac{p_{t+J}}{p_t}, \quad j = 1, \dots, J.$$

Notice from (3.5b) that the value of the one-period interest in period  $t + J - 1$  helps determine the value of  $k_{1t+J}$  because the optimal choice for  $k_{1t+J}$  reflects future substitution possibilities between the different types of capital. These same substitution possibilities imply that both  $n_t$  and  $k_{2t+1}$  also depend on current and lagged values of one-period interest rates. In particular,  $n_t$  is a function of  $r_{t-J}^1$  and  $r_{t-1}^1$  while  $k_{2t+1}$  varies with  $r_{t-J+1}^1$  and  $r_t^1$ . Hence, factor ratios show a delayed response to changes in short-term real interest rates. Furthermore, all variables fluctuate with changes in the term structure of interest rates due to the presence of current and lagged values of the variable  $\hat{r}_t$ .

Using (3.5b) to substitute for  $k_{1t+j}$ ,  $j = 0, \dots, J$  in (3.3) shows the effects of time-to-build assumption for the behavior of investment expenditures. Provided  $g_j > 0$  for  $j = 0, \dots, J$ ,  $i_{1t}$  depends on current and lagged values of the  $j$ -period interest rates  $r_{t-s}^j$ ,  $j = 1, \dots, J$  and  $s = 0, \dots, J$ , as well as



$r_t^1, \dots, r_{t+J-1}^1$ . Also, the response of investment expenditures to real interest rates depends on the technology for producing new capital goods. This is because the distributed lag coefficients  $g_j, j = 0, \dots, J$  are themselves functions of the depreciation rate  $\delta_1$  and the parameters  $\phi_j, j = 1, \dots, J$ .

On the other hand, the delivery lag model displays none of these features. There, the cost of capital for both types of capital are determined as in the left-hand side of (3.2), and depend only on the one-period interest rate  $r_t^1$ . In this case, the expressions in (3.5a-c) are replaced by

$$(3.6a) \quad n_t = \frac{n_1}{\lambda_t w_t^{1-a}} (\delta_1 + r_{t-1}^1)^b (\delta_2 + r_{t-1}^1)^c$$

$$(3.6b) \quad k_{1t+1} = k_1 \frac{w_t^a}{\lambda_t} \frac{(\delta_2 + r_t^1)^c}{(\delta_1 + r_t^1)^{1-b}}$$

$$(3.6c) \quad k_{2t+1} = k_2 \frac{w_t^a}{\lambda_t} (\delta_1 + r_t^1)^b (\delta_2 + r_t^1)^{1-c}.$$

Investment expenditures in the two types of capital are determined by assuming that a fraction of expenditures required to close the gap between the actual and desired capital stocks are made in each period, i.e.,

$$(3.7) \quad i_{it} = w(L)(k_{it+1} - k_{it}) + \delta_i k_{it} \quad i = 1, 2$$

with  $w(L) = w_0 + w_1 L + w_2 L^2 + \dots$ . Equations (3.6a-c) and (3.7) show the differences with the time-to-build model. Since  $n_t, k_{1t+1}$ , and  $k_{2t+1}$  depend on at most single lagged values of the one-period interest rate, factor ratios do not display the delayed

response to changes in interest rates. Furthermore, neither capital stocks nor investment expenditures depend on long-term interest rates. This is because the cost of capital variable in the delivery lag model does not account for time lags in the investment process. This, in turn, has implications for evaluating the effects of exogenously induced changes in interest rates on the behavior of investment and capital stocks. Similarly, by ignoring forward-looking behavior on the part of firms, the delivery lag model potentially misspecifies the consequences of alternative tax or depreciation policies.

#### 4. Theoretical Measures of 'q' in the Time-to-Build Model

In this section, I show that the relationship between current investment and "Tobin's 'q'," as it is usually defined, is not useful for distinguishing between different models of investment. Furthermore, I show that the practice of using stock market data to measure the marginal valuation of new capital is not valid within the time-to-build model. These results have interest given the widespread use of 'q' in the empirical investment literature. Finally, using the pricing relationships of Section 2, I describe how to derive simple testable hypotheses about the existence of lags in the investment process.

Of these related issues, consider first the relationship between the level of investment and 'q'--in particular, that between the level of investment in the second type of capital,  $i_{2t}$ , and  $q_{2t}$ . Recall that  $q_{2t}$  shows the spot price of existing capital relative to the spot price of output. Without adjustment

costs or production lags, the replacement cost for the second type of capital equals the (spot) price of output. Hence,  $q_{2t}$  represents Tobin's 'q,' as the latter is usually defined. Recall from Section 2 that for an interior solution in which  $i_{2t} > 0$  for all  $t$ ,  $q_{2t}$  always equals one. When the equilibrium value of  $i_{2t}$  is zero for some  $t$ , such as the case of irreversible investment discussed by Sargent [1979], there will exist a relationship between  $i_{2t}$  and  $q_{2t}$ , though it is extremely complicated.

Now consider investment in the first type of capital. Due to the time-to-build feature, the spot price of existing capital  $q_{1t}$  does not equal one in equilibrium. However,  $q_{1t}$  does not correspond to the relative price hypothesized by Tobin as determining investment in new capital goods. The reason is that firms in this model undertake investment in new projects  $s_{Jt}$ , until condition (2.8) is satisfied. In this case, the relative price  $q_{1t}^+$  defined in (4.1) corresponds to Tobin's 'q':

$$(4.1) \quad q_{1t}^+ = \frac{E_t q_{1t+J-1} P_{t+J-1}}{\phi_J E_t P_t + \phi_{J-1} E_t P_{t+1} + \dots + \phi_1 E_t P_{t+J-1}}$$

Here firms compare the cost of new capital with the price of existing capital. However, since new investment becomes productive only with a lag of  $J$  periods, the relevant shadow price is the price of existing capital in period  $t + J - 1$  expected to prevail as of period  $t$ . The above discussion shows that one may talk about a 'q' theory of investment in the context of the time-to-build model. However, such a theory is vacuous, in that it

corresponds to a restatement of equilibrium conditions and does not lead to any testable implications about the underlying economic model. This is because  $q_{1t}^+$  displays the same behavior as  $q_{2t}$ . It differs from one only when investment in new projects  $s_{Jt}$  is zero. As Abel [1980] has shown, a similar result obtains for the adjustment cost model of investment when the denominator of 'q' is correctly defined to include costs incurred in adjusting the capital stock. Thus, the relationship between the level of new investment and an appropriately defined measure of Tobin's 'q' cannot be used to distinguish empirically between different models of investment.

Nevertheless, relationships involving the relative prices  $\frac{q_{1t+J-1}P_{t+J-1}}{P_t}$  and  $\frac{q_{2t+J-1}P_{t+J-1}}{P_t}$  do contain useful information about the underlying investment technology. Recall from conditions (2.8) and (2.10a,b) that

$$(4.2) \quad E_t \frac{q_{1t+J-1}P_{t+J-1}}{P_t} = \phi_J + \frac{\phi_{J-1}}{1 + r_t^1} + \dots + \frac{\phi_1}{1 + r_t^{J-1}},$$

when production lags exist in the investment process. With the no costs of adjustment neoclassical investment model, however,

$$(4.3) \quad E_t \frac{q_{2t+J-1}P_{t+J-1}}{P_t} = E_t \frac{P_{t+J-1}}{P_t} \equiv \frac{1}{1 + r_t^{J-1}},$$

where the last relation is derived using condition (2.9). Hence, if  $\frac{q_{1t+J-1}P_{t+J-1}}{P_t}$  and  $\frac{q_{2t+J-1}P_{t+J-1}}{P_t}$  can be measured, the restrictions in (4.2) and (4.3) can be used to test for the existence of time lags in the production of capital goods.

One way to perform such a test is to note that (4.2) and (4.3) may be rewritten as

$$(4.2a) \quad \frac{q_{1t+J-1}P_{t+J-1}}{P_t} = \phi_J + \frac{\phi_{J-1}}{1 + r_t^1} + \dots + \frac{\phi_1}{1 + r_t^{J-1}} + \epsilon_{1t}$$

and

$$(4.3a) \quad \frac{q_{2t+J-1}P_{t+J-1}}{P_t} = \frac{1}{1 + r_t^{J-1}} + \epsilon_{2t}$$

where  $\epsilon_{1t}$  and  $\epsilon_{2t}$  are forecast errors uncorrelated with variables in firms' information sets at time  $t$ , i.e.,

$$\epsilon_{it} = \frac{q_{it+J-1}P_{t+J-1}}{P_t} - E_t \left[ \frac{q_{it+J-1}P_{t+J-1}}{P_t} \right], \quad i = 1, 2.$$

Hence, using the generalized instrumental variables estimation method of Hansen and Singleton [1983], a test of the existence of time lags in the investment process for the first type of capital corresponds to a test involving the coefficients of a regression of  $\frac{q_{1t+J-1}P_{t+J-1}}{P_t}$  on a constant and  $\frac{1}{1 + r_t^j}$  for  $j = 1, \dots, J - 1$ . If the time-to-build assumption is true, all such coefficients should be positive and sum to one in the appropriate statistical sense. Notice that it is relatively straightforward to implement such a test at a disaggregated level where data on the prices of used capital goods exist. A difficulty arises in measuring the prices of existing capital when more aggregate levels of capital are considered. Consequently, I now turn to the question of finding empirical measures for these relative prices using valuation criteria such as stock prices.

Early applications of the 'q' theory of investment relied on stock market data to measure the price of existing capital. (See, for example, von Furstenberg [1977]). This practice was shown to be valid by Hayashi [1982] in a single capital good model with constant returns to scale production technology and adjustment costs in investment. Specifically, using the profit-maximization problem of an individual competitive firm, Hayashi [1982] shows that the shadow price of capital, or "marginal 'q'," will be identical to "average 'q'" defined from the value of the firm's equities and liabilities. In this case, the price of existing capital  $q_t$  can be measured as  $V_t/k_t$  where  $V_t$  denotes the value of the firm at time  $t$ .

To determine whether similar relationships can be derived in the model of this paper, notice first that with a constant returns to scale production technology, the value of the firm will merely reflect the value of its initial capital stocks. However, recall that firms endowed with the investment technology of Section 2 possess multiple types of capital: these include the two types of productive capital and the stocks of the unfinished projects  $s_{jt}$ ,  $j = 1, \dots, J - 1$ . It can be shown that the market value of the firm at date  $t$  is equal to the sum of the values of these different stocks at that date. To show this and to derive an explicit expression for the market value, I make use of the optimality conditions (2.8)-(2.15) at date zero. Multiplying (2.11) by  $k_{11}$  and (2.12) by  $k_{21}$  and adding yields

$$\begin{aligned}
 p_0(q_{10}k_{11}+q_{20}k_{21}) &= E_0[p_1k_{11}\lambda_1\partial f(n_1,k_{11},k_{21})/\partial k_{11} \\
 &+ p_1k_{21}\lambda_1\partial f(n_1,k_{11},k_{21})/\partial k_{21} \\
 &+ p_1\{(1-\delta_1)q_{11}k_{11}+(1-\delta_2)q_{21}k_{21}\}].
 \end{aligned}$$

Using the constant returns to scale property of the production function, and adding and subtracting  $E_0p_1[q_{11}k_{11}^d+q_{21}k_{21}^d]$  to the right-hand side above implies that it equals:

$$\begin{aligned}
 E_0p_1[Q_1+(1-\delta_1)q_{11}k_{11}+(1-\delta_2)q_{21}k_{21}-w_1n_1-q_{11}k_{11}^d-q_{21}k_{21}^d \\
 +q_{11}k_{11}^d+q_{21}k_{21}^d].
 \end{aligned}$$

Using the laws of motion (2.5a) and (2.5b) to eliminate  $k_{11}^d$  and  $k_{21}^d$  in the second line above yields

$$\begin{aligned}
 E_0p_1[Q_1+(1-\delta_1)q_{11}k_{11}+(1-\delta_2)q_{21}k_{21}-w_1n_1-q_{11}(k_{12}-s_{11})-q_{21}(k_{22}-i_{21})].
 \end{aligned}$$

Substituting recursively for  $p_t(q_{1t}k_{1t+1}+q_{2t}k_{2t+1})$  for  $t = 1, 2, \dots$  and using the transversality conditions, the above expression simplifies to

$$\begin{aligned}
 E_0 \sum_{t=1}^{\infty} p_t [Q_t+(1-\delta_1)q_{1t}k_{1t}+(1-\delta_2)q_{2t}k_{2t}-w_t n_t \\
 -q_{1t}k_{1t}^d-q_{2t}k_{2t}^d-q_{1t}s_{1t}-q_{2t}i_{2t}]
 \end{aligned}$$

Thus

$$\begin{aligned}
 (4.4) \quad & p_0(q_{10}k_{11}+q_{20}k_{21}) + E_0 \sum_{t=1}^{\infty} p_t(q_{1t}s_{1t} - \sum_{j=1}^J \phi_j s_{jt}) \\
 & + E_0 \sum_{t=1}^{\infty} p_t(q_{2t}i_{2t} - i_{2t}) = E_0 \sum_{t=1}^{\infty} p_t [Q_t + (1-\delta_1)q_{1t}k_{1t} \\
 & + (1-\delta_2)q_{2t}k_{2t} - w_t n_t - i_{2t} - \sum_{j=1}^J \phi_j s_{jt} - q_{1t}k_{1t}^d - q_{2t}k_{2t}^d]. \\
 & \equiv E_0 V_1.
 \end{aligned}$$

The right-hand side of (4.4) shows the expected value of the firm at time 1, conditional on information available at time zero. To simplify the left-hand side, recall that for an interior solution  $q_{2t} = 1$  for all  $t$  so that the last term on the left-hand side is zero. Also, using (2.8) and the laws of motion,  $s_{j,t+1} = s_{j+1,t}$ ,  $j = 1, \dots, J - 1$  repeatedly, the second term on the left-hand side "telescopes" to yield, for any  $t$ ,<sup>5/</sup>

$$\begin{aligned}
 (4.5) \quad E_{t-1} V_t &= p_{t-1}(q_{1t-1}k_{1t} + q_{2t-1}k_{2t}) + \\
 & (p_{t-J+1}\phi_J + p_{t-J+2}\phi_{J-1} + \dots + p_{t-1}\phi_2)s_{1t} + \\
 & (p_{t-J+2}\phi_J + \dots + p_{t-1}\phi_3)s_{2t} + \dots + p_{t-1}\phi_J s_{J-1,t}.
 \end{aligned}$$

To see how this expression arises, recall that a fraction  $\phi_j$  of resources is expended in each period on a project  $j$  periods from completion. Valuing the fraction spent in each period for a given project by the price of output in that period yields the terms involving the incomplete projects. Thus, the market value of a project two periods from completion is given by  $(p_{t-J+2}\phi_J + \dots + p_{t-1}\phi_3)s_{2t}$ . Since the fraction  $\phi_3$  of resources was expended in



period  $t - 1$ , the contingent price of output in period  $t - 1$  is used to value this part of the partially completed project. The market value of the firm also reflects the value of its productive capital, as is shown by the first two terms on the right-hand side of (4.5). The expression in (4.5) differs from similar expressions derived under the simple neoclassical model or one with adjustment costs due to the inclusion of terms involving the incomplete projects. (See, for example, Abel and Blanchard [1983a]).

Returning to the initial problem of indirectly measuring the relative prices defined on the left-hand side of (4.2) and (4.3) using data on firms' equities and liabilities (or aggregate valuation measures such as stock price indices), notice from (4.5) that, along the equilibrium path,

$$(4.6) \quad \frac{q_{1t+J-1}p_{t+J-1}}{p_t} = \frac{E_{t+J-1}V_{t+J}}{p_t k_{1t+J}} - \frac{q_{2t+J-1}p_{t+J-1}k_{2t+J} - \dots - p_{t+J-1} J^S_{J-1,t+J}}{p_t k_{1t+J}} = \frac{E_{t+J-1}V_{t+J} - p_{t+J-1}k_{2t+J} - \dots - p_{t+J-1} J^S_{J-1,t+J}}{p_t k_{1t+J}}.$$

The last step is derived using condition (2.9) to substitute for  $q_{2t+J-1}p_{t+J-1}$ . It is clear, however, that (4.6) is not particularly useful for giving empirical content to relations such as (4.2) and (4.3). This is because (4.6) involves unknown parameters such as the  $j$ , as well as prices  $p_{t+J-j}$ ,  $j = 1, \dots$ ,

J - 1. These are unobservable from the point of view of the econometrician.<sup>6/</sup>

Looked at in another way, (4.5) and (4.6) show that the practice of measuring the shadow price of existing capital by expressions such as  $\frac{E_{t-1}V_t}{P_{t-1}k_{1t}}$  (or  $\frac{E_{t-1}V_t}{P_{t-1}k_{2t}}$ ) is incorrect if there are lags in the investment process of the time-to-build variety. Even with a single type of productive capital, (4.5) provides an expression for the price of existing capital which includes the value attributed to the incomplete projects. One implication is that investment studies which use such measures of the price of existing capital and assume that investment in new capital proceeds with a time lag are built on mutually inconsistent assumptions.

## 5. Conclusion

This paper has used a model of firm optimization in a single good competitive economy to characterize the behavior of investment expenditures and capital stocks and to derive equilibrium relationships involving the prices of existing capital and real interest rates when there exist time lags in the investment process. One purpose behind such an analysis is to study the mutual consistency of the assumptions underlying other models of investment, such as the delivery lag model and those based on the 'q' theory. A second purpose is to derive testable implications about the investment technology from the relations involving the prices of new and used capital goods and real interest rates. It turns out such relations are not operational when considered in a

single good, aggregative economy. Subject to some modifications, however, the framework used in this paper may be extended to a more disaggregated level where measures of used capital goods prices exist. In this case, the pricing relations of Section 2 may be used to estimate the parameters of the model and to test its restrictions. But such an extension requires a multi-good environment, with different goods used for consumption and for producing new capital goods. This is a topic for further research.

FOOTNOTES

1. I am grateful to Ian Bain, Lars Peter Hansen, and Ramon Marimon for useful comments. Remaining errors are my own.
2. A slightly different scheme is adopted by Abel and Blanchard [1983a] who assume that replacement investment is financed out of retained earnings while net investment is financed by bonds.
3. This specification of the price system is similar to that described in Lucas and Prescott [1972].
4. This is in addition to the fact that with constant returns to scale, the scale of each firm or the size of its capital stock is also indeterminate.
5. Notice that to derive rigorously the expressions for  $V_0$  to  $V_J$ , an additional set of contingent claims markets would have to be opened in order to price the initial stocks of incomplete projects  $s_{j0}$ ,  $j = 1, \dots, J - 1$ . This problem in turn relates to the indeterminacy of the spot price  $q_{1t}$  for  $0 \leq t \leq J - 1$ .
6. An alternative approach to deriving empirical counterparts of the shadow prices of capital goods is provided by Abel and Blanchard [1983b]. Using a formula similar to (2.16), these authors derive an empirical 'q' series from its actual determinants, including future marginal projects and a stochastic "discount" factor, constructed as a weighted combination of the ex ante rates of return on corporate debt and equity. While possibly useful for characterizing the behavior of 'q,' such a series is constructed under some questionable assump-

tions--for example, the appropriate "discount factor" needed to value future marginal products of capital does not correspond to the measure used by these authors--and does not seem useful for testing the restrictions deriving from the investment technology.

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