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SOME NOTES ON MONETARY ECONOMICS
IN A WORLD ECONOMY

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I. Real Open Economy Macroeconomics

In this chapter, we consider a simple macroeconomic model of a world economy consisting of two countries. We can use this model to illustrate several points.

- (1) optimal consumption behavior in an open economy
- (2) implications of (1) for the balance of payments and prices (interest rates)
- (3) effects of government spending and taxes on consumption behavior and prices and in particular, the irrelevance of the timing of taxes for a given path of government expenditures [The Ricardian Proposition]

A. The Two Period-Two Country Pure Exchange Model

Consider a world economy composed of two countries. Call the two countries "home" and "foreign," denote home variables by superscript "H" and foreign variables by superscript "F." Each of the two countries are composed of a large number of identical consumers. Since consumers are identical within a country (but not necessarily across countries) it will suffice to consider a representative from each.

1. A Representative Consumer for the Home Country

The consumer lives for two periods ($t = 1, 2$). In the first period the consumer is endowed with y_1^H units of the one good, in the second he is endowed with y_2^H units of the one good. It will be important to distinguish between goods and commodities. Consider the following: Imagine the consumer owns an apple tree. The tree produces y_1^H apples in period 1, y_2^H apples in period 2 then dies. The apples are not storable, so if they are not eaten in the period they are produced they become rotten.

Now how many goods are there in this economy? Only one apple. How many commodities are there? Two--date 1 apple and date 2 apples. In static microeconomic theory commodities are distinguished by physical types, apples are different from oranges because they are physically different. In dynamic theory, (that is, in a theory which explicitly includes a time dimension) commodities are also distinguished by time, apples at date 1 which are physically indistinguishable from apples at date 2 are still different commodities.

a. Consumer Preferences

Denote the consumption of the home consumer in the two periods as c_1^H and c_2^H , respectively. We assume the consumer values his consumption stream $\{c_1^H, c_2^H\}$ according to the utility function denoted $U^H(c_1^H, c_2^H)$. This function completely summarizes the consumers' preferences over all possible combinations of date 1 apples and date 2 apples.

We will make the following assumptions about the form of the utility function and its properties.

(U1) Additive separability and discounting:

$U^H(c_1^H, c_2^H) = u^H(c_1^H) + \beta_H u^H(c_2^H)$ where $0 < \beta_H < 1$. Here we call U^H the total utility function and u^H the period utility function. The two most important implications of this assumption are

(Additive separability) (i) there are no cross-consumption effects.

By this we mean the amount that total utility U changes as we change the consumption of good 2 depends only on good 2 and not on the level of consumption of good 1. Notice additive separability implies

$$\frac{\partial^2 U(c_1, c_2)}{\partial c_1 \partial c_2} = \frac{\partial^2 U(c_1, c_2)}{\partial c_2 \partial c_1} = 0.$$

(Discounting) (ii) the future is discounted relative to the present.

That is, assuming $0 < \beta_H < 1$ means the consumer is "impatient" to eat apples in the sense that if he had a choice between eating now (period 1) or later (period 2) he would always prefer to eat now. If the period utility function is monotone (see U2), then discounting implies

$$U^H(c_1^H + \Delta, c_2^H) > U^H(c_1^H, c_2^H + \Delta) \text{ for all } \Delta > 0, \text{ all } c_1^H, c_2^H.$$

We will call β_H the discount factor. The smaller is β_H the more impatient is the consumer.

(U2) Monotonicity [more is preferred to less]

$$\frac{\partial U^H}{\partial c_t^H} > 0 \text{ for } t = 1, 2.$$

If we assume (U1) then this becomes

$$\frac{\partial U^H}{\partial c_1^H} = \frac{du(c_1^H)}{dc_1^H} \text{ and } \frac{\partial U^H}{\partial c_2^H} = \beta_H \frac{du(c_2^H)}{dc_2^H}$$

Monotonicity basically means the consumer always prefer more consumption to less consumption. In particular it implies that consumers are never satiated. Note: We will use (U2) to guarantee the consumers' budget constraints will bind with equality.

(U3) Concavity [Averages are preferred to extremes]

$U^H(c_1^H, c_2^H)$ is a concave function of c_1^H and c_2^H , i.e., (drop superscripts for a while) let (c_1, c_2) and (c_1', c_2') be two consumption bundles. Consider a third bundle that is weighted average of these where the weight on first bundle is λ ($0 < \lambda < 1$) and weight on second bundle is $(1-\lambda)$. Then this third bundle can be written

$$(c_1'', c_2'') \text{ where } c_1'' = \lambda c_1 + (1-\lambda)c_1' \\ c_2'' = \lambda c_2 + (1-\lambda)c_2'$$

Then we have the following definition.

Definition: $U(c_1, c_2)$ is a concave function of (c_1, c_2) if

$$(*) \quad U(c_1'', c_2'') > \lambda U(c_1, c_2) + (1-\lambda)U(c_1', c_2') \text{ for all } \lambda \text{ s.t. } 0 < \lambda < 1.$$

Notice we have a strict inequality in (*). To be precise, we should really say U is then a strictly concave function.

Note: I will ask you in the first problem set to show that given U satisfies (U1) then if the period utility function $u(c)$ is concave in c , then $U(c_1, c_2)$ is concave in (c_1, c_2) .

Note: Concavity will be used to guarantee that the first-order conditions are both necessary and sufficient for a maximum.

(U4) Marginal utility goes to infinity as consumption of either good goes to zero

$$(**) \quad \frac{\partial U(c_1, c_2)}{\partial c_t} \rightarrow \infty \text{ as } c_t \rightarrow 0, t = 1, 2$$

(**) basically says that consumers greatly desire to have positive consumption. Graphically this will imply that indifferent curves becomes flat as one good goes to zero.

Note: This assumption is basically a technical assumption that we will use to guarantee consumers will choose strictly positive consumption of both goods.

(U2) Monotonicity

(see Figure 1)

Monotonicity implies any point to northeast of (\bar{c}_1, \bar{c}_2) is preferred to (\bar{c}_1, \bar{c}_2) , let $S(\bar{c}_1, \bar{c}_2) \equiv$ set of points to NE of (\bar{c}_1, \bar{c}_2) , for example, $(\bar{c}_1, \bar{c}_2) \in S(\bar{c}_1, \bar{c}_2)$ and if U is monotone then $U(\bar{c}_1, \bar{c}_2) < U(\bar{c}_1, \bar{c}_0)$.

(U3) Concavity

(see Figure 2)

Concavity implies any point on the line segment connecting (c_1, c_2) to (c_1', c_2') is preferred to either (c_1, c_2) or (c_1', c_2') .

(U4) Marginal utility goes to infinity as consumption of either good goes to zero means

(see Figure 3)

(U5) U is twice continuously differentiable

We have implicitly been assuming this in our earlier definitions since if we don't, then $\partial^2 U / \partial c_1 \partial c_2$ does not make sense.

This assumption rules out flats or kinks in indifference curves and basically it is a technical assumption which guarantees we can use the calculus to do our analysis.

B. Assets and Budget Constraints

Initially we will consider only one type of asset, a one-period real bond. At time 1, a consumer can buy a one-period bond giving up, say, b units of his apples at date 1 for a promise to get Rb units of apples at date 2. We will call

$R \equiv$ gross real one-period interest rate.

We may also let

$R \equiv 1 + r$ where

r net real one-period interest rate.

We use the term "real" because R measures the relative price of time 2 goods to time 1 goods and not time 2 "dollars" to time 1 "dollars." We will discuss this later relative price, call the nominal interest rate in a later handout.

The period budget constraints for the consumer are

$$(1.1) \quad (t = 1) \quad c_1 + b \leq y_1$$

$$(1.2) \quad (t = 2) \quad c_2 \leq y_2 + Rb$$

where on the left hand side (LHS) we have the uses of goods and on the right hand side (RHS) we have the sources of good.

Notice if $b > 0 \rightarrow$ consumer is saving

$b < 0 \rightarrow$ consumer is borrowing

We can reduce these two budget constraints to a single budget constraint as follows. Solve the second equation (1.2) for b to obtain

$$(1.3) \quad b \geq \frac{c_2}{R} - \frac{y_2}{R}$$

Substitute (1.3) into (1.1) and rearrange to get

$$(1.4) \quad c_1 + \frac{c_2}{R} \leq y_1 + \frac{y_2}{R}$$

(1.4) is then a single budget constraint for the consumer called the present value form of the budget constraint.

C. The Consumer's Maximization Problem

Each consumer (H&F) is going to choose his/her consumption and bond supply/demand to maximize his/her utility. There are two ways to write the consumer's problem depending if we want to use the period budget constraints (1.1) and (1.2) or the present value budget constraint (1.4). Both ways are the same, except that if we use the period budget constraints we explicitly include bonds and if we use the present value budget constraint the choice of bonds is only implicit.

(i) The Consumer's Problem with Period Budget Constraints

Formally, we have the following:

Given endowments $\{y_1, y_2\}$ and taking as given $\{R\}$ choose $\{c_1, c_2, b\}$ to solve:

$$\begin{aligned} \text{(CMI)} \quad & \max U(c_1, c_2) \\ & \text{s.t. } 1. \quad c_1 + b \leq y_1 \\ & \quad \quad 2. \quad c_2 \leq y_2 + Rb \\ & \quad \quad 3. \quad c_1, c_2 \geq 0. \end{aligned}$$

(ii) The Consumer's Problem with a Present Value Budget Constraint.

Given $\{y_1, y_2\}$ and $\{R\}$ choose $\{c_1, c_2\}$ to solve

$$\begin{aligned} \text{(CMII)} \quad & \max U(c_1, c_2) \\ & \text{s.t. } 1': \quad c_1 + \frac{c_2}{R} \leq y_1 + \frac{y_2}{R} \\ & \quad \quad 2': \quad c_1, c_2 \geq 0. \end{aligned}$$

The solution to these problems can be found by several methods. The easiest is to substitute the constraints into the utility function and reduce the problem to a one-variable maximization problem. Note that if we assume U is monotone (U2) we are assured the constraints 1.2 or 1' bind with equality. So given monotonicity, (CMI) is equivalent to choose {b} to solve

$$(CMI') \quad \max U(y_1 - b, y_2 + Rb)$$

and (CMII) is equivalent to choose {c₁} to solve

$$(CMII') \quad \max U(c_1, R(y_1 - c_1) + y_2)$$

Notice that in both (CMI') and (CMII') we have ignored the nonnegativity constraints on consumption, (3 and 2'). We we assume (U4) then we are assured that any solution to the above problem will automatically satisfy nonnegativity.

How can we characterize the solution to (CMI') or (CMII')? If we assume U is concave (U3) and U is differentiable (U5) then we know the first-order conditions to the above problem are necessary and sufficient conditions for a maximum. Consider (CMI')

$$(*) \quad \max_{\{b\}} U(y_1 - b, y_2 + Rb)$$

The first-order condition for this problem is found by differentiating (*) with respect to b.

$$(1.6) \quad \text{FOC: } b: - \frac{\partial U}{\partial c_1} + R \frac{\partial U}{\partial c_2} = 0$$

or rearranging (1.6) gives

$$(1.7) \quad \frac{\partial U / \partial c_1}{\partial U / \partial c_2} = R$$

which just says, choose bonds (savings) to equate the ratio of the marginal utility of good 1 to good 2 to the relative price of good 1 to good 2.

Now if we add the assumption of additive separability and discounting (U1) then (*) becomes (**)

$$(**) \quad \max_{\{b\}} U(y_1 - b) + \beta U(y_2 + Rb)$$

and the FOC are now

$$(1.8) \quad \text{FOC: } b: -\frac{\partial U}{\partial c_1} + R\beta \frac{\partial U}{\partial c_2} = 0$$

rearranging gives

$$(1.9) \quad \frac{\partial U / \partial c_1}{\partial U / \partial c_2} = R\beta$$

The first-order condition implicitly defines an optimal rule for choosing bonds. We call this rule the demand function for bonds and write it

$$(2.0) \quad b = \tilde{b}(R, y_1, y_2)$$

If we substitute (2.0) into the budget constraints of the consumer in (CMI), we can solve for the optimal rule for choosing consumption of good 1 and good 2. We call these rules the demand functions for good 1 and good 2 and denote them as follows

$$(2.1) \quad c_1 = \tilde{c}_1(R, y_1, y_2)$$

and

$$(2.2) \quad c_2 = \tilde{c}_2(R, y_1, y_2)$$

Now if the demand for a good rises when income rises we call that good a normal good. In this problem we have endowments y_1 and y_2 which at interest rate R give the consumer a lifetime present value of income of w

$$(2.3) \quad w = y_1 + \frac{y_2}{R}$$

We often call w the wealth of the consumer (\equiv the present value of lifetime income). From (2.3) it is clear that raising endowments of either commodity 1 or 2 will raise wealth. So here we call a commodity normal if raising the endowments of either commodity raises the consumption of both commodities. Mathematically this means

commodity 1 is a normal good iff $\frac{\partial \tilde{c}_1}{\partial y_1} > 0$ and $\frac{\partial \tilde{c}_1}{\partial y_2} > 0$

and

commodity 2 is a normal good iff $\frac{\partial \tilde{c}_2}{\partial y_1} > 0$ and $\frac{\partial \tilde{c}_2}{\partial y_2} > 0$.

How do we expect an increase in the interest rate to affect the demand for commodities 1 and 2? In general the answer is ambiguous and depends on the relative size of the income and substitution effects.

Substitution effect: A higher R means the relative price of good 2 to good 1 has declined (it takes fewer units of good 1 to buy the same amount of units of good 2). So one would expect a lower consumption of good 1 and a higher consumption of good 2.

Wealth effect (present value of income effect): The present value of income has fallen so one would expect a lower consumption of both goods (if both normal). However, there is always the possibility that when wealth falls the consumption of good 1 rises (good 1 is strongly inferior).

D. An Example

We will often work with a simple example for which we can actually calculate the demand functions. let

$$(2.4) \quad U(c_1, c_2) = \ln c_1 + \beta \ln c_2$$

Then (CMI') becomes

$$(2.5) \quad \max_{\{b\}} \ln(y_1 - b) + \beta \ln(y_2 + Rb)$$

The FOC for this problem is a special case of (1.8)

$$(2.6) \quad (\text{FOC}) \ b: \quad -\frac{1}{y_1 - b} + \frac{R\beta}{y_2 + Rb} = 0$$

Rearranging gives

$$y_2 + Rb = R\beta(y_1 - b)$$

Solving for b gives

$$bR[1+\beta] = R\beta y_1 - y_2$$

or

$$(2.7) \quad b = \tilde{b}(R, y_1, y_2) = \frac{1}{1+\beta} \left[\beta y_1 - \frac{y_2}{R} \right]$$

Substituting (2.7) into the budget constraints in (CMI) yield

$$(2.8) \quad c_1 = \tilde{c}_1[R, y_1, y_2] = \frac{1}{1+\beta} \left[y_1 + \frac{y_2}{R} \right]$$

$$(2.9) \quad c_2 = \tilde{c}_2[R, y_1, y_2] = \frac{\beta R}{1+\beta} \left[y_1 + \frac{y_2}{R} \right]$$

Notice for this example we have

$$\frac{\partial \bar{b}}{\partial R} = \frac{1}{1 + \beta} \frac{y_2}{R^2} > 0$$

(savings increases as the interest rate increases)

$$\frac{\partial \bar{b}}{\partial y_1} = \frac{1}{1 + \beta} > 0$$

(savings increases as endowment of good 1 increases)

$$\frac{\partial \bar{b}}{\partial y_2} = - \frac{1}{(1 + \beta)} \frac{1}{R} < 0$$

(savings decreases as endowment of good 2 increases)

$$\frac{\partial \tilde{c}_1}{\partial y_1} = \frac{1}{1 + \beta} > 0, \quad \frac{\partial \tilde{c}_1}{\partial y_2} = \frac{1}{1 + \beta} \cdot \frac{1}{R} > 0$$

(consumption of commodity 1 increases as endowment of either good increases)

$$\frac{\partial \tilde{c}_2}{\partial y_1} = \frac{\beta R}{1 + \beta} > 0, \quad \frac{\partial \tilde{c}_2}{\partial y_2} = \frac{\beta}{1 + \beta} > 0$$

(consumption of commodity 2 increases as endowment of either good increases)

$$\frac{\partial \tilde{c}_1}{\partial R} = - \frac{1}{1 + \beta} \frac{y_2}{R^2} < 0$$

$$\frac{\partial \tilde{c}_2}{\partial R} = \frac{\beta}{1 + \beta} y_1 > 0$$

(as R increases the consumption of commodity 1 falls and consumption of commodity 2 rises)

E. Competitive Equilibrium

So that there is absolutely no possibility of confusion we can rewrite the consumer's problem with the country superscripts H & F back in.

The home consumer solves:

Given endowments $\{y_1^H, y_2^H\}$ and taking as given $\{R\}$, choose $\{c_1^H, c_2^H, b^H\}$ to solve:

$$(2.10) \quad \max U^H(c_1^H, c_2^H)$$
$$\text{s.t. } 1. \quad c_1^H + b^H < y_1^H$$
$$2. \quad c_2^H \leq y_2^H + Rb^H$$
$$3. \quad c_1^H, c_2^H \geq 0$$

denote the solution to the problem as

$$c_1^H = \tilde{c}_1^H(R, y_1^H, y_2^H)$$
$$c_2^H = \tilde{c}_2^H(R, y_1^H, y_2^H)$$
$$b^H = \tilde{b}^H(R, y_1^H, y_2^H)$$

The foreign consumer solves:

Given endowments $\{y_1^F, y_2^F\}$ and taking as given $\{R\}$ choose $\{c_1^F, c_2^F, b^F\}$ to solve

$$(2.11) \quad \max U^F(c_1^F, c_2^F)$$
$$\text{s.t. } 1. \quad c_1^F + b^F \leq y_1^F$$
$$2. \quad c_2^F \leq y_2^F + Rb^F$$
$$3. \quad c_1^F, c_2^F \geq 0.$$

A Competitive Equilibrium for this economy is

- (i) an allocation $\{\hat{c}_1^H, \hat{c}_2^H, \hat{b}^H; \hat{c}_1^F, \hat{c}_2^F, \hat{b}^F\}$
- (ii) a price system $\{\hat{R}\}$

such that (E1) and (E2) hold:

(E1) Markets clearing.

- (i) date 1 market clears $\hat{c}_1^H + \hat{c}_1^F = y_1^H + y_1^F$
- (ii) date 2 market clears $\hat{c}_2^H + \hat{c}_2^F = y_2^H + y_2^F$
- (iii) bond market clears $\hat{b}^H + \hat{b}^F = 0$

(E2) Maximization

- (i) Home consumer max'n: $\{\hat{c}_1^H, \hat{c}_2^H, \hat{b}^H\}$ solves (2.10) given endowments $\{y_1^H, y_2^H\}$ at prices \hat{R}
- (ii) Foreign consumer max'n: $\{\hat{c}_1^F, \hat{c}_2^F, \hat{b}^F\}$ solves (2.11) given endowments $\{y_1^F, y_2^F\}$ at prices \hat{R} .

An example: Let us compute the equilibrium in a special case. Suppose as in Section D we have utility functions given by

$$U^H(c_1^H, c_2^H) = \ln c_1^H + \beta \ln c_2^H$$

$$U^F(c_1^F, c_2^F) = \ln c_1^F + \beta \ln c_2^F$$

Then our previous analysis (2.8) shows

$$(2.12) \quad \tilde{c}_1^H(R, y_1^H, y_2^H) = \frac{1}{1 + \beta} [y_1^H + y_2^H / R]$$

$$(2.13) \quad \tilde{c}_1^F(R, y_1^F, y_2^F) = \frac{1}{1 + \beta} [y_1^F + y_2^F / R].$$

We can compute an equilibrium by substituting on the demand functions for any of the equilibrium market clearing conditions (E1(i, ii, iii)) and finding that \hat{R} that solves the equation (this is an implication of Walras' Law--which we will discuss in a problem set). Let us do this for (E1(i)), i.e., date 1 market.

$$\tilde{c}_1^H(R, y_1^H, y_2^H) + \tilde{c}_1^F(R, y_1^F, y_2^F) = y_1^H + y_1^F$$

for this example becomes (using (2.12) and (2.13))

$$(2.14) \quad \frac{1}{1+\beta} \left[y_1^H + \frac{y_2^H}{R} \right] + \frac{1}{1+\beta} \left[y_1^F + \frac{y_2^F}{R} \right] = y_1^H + y_1^F$$

Now the \hat{R} that solves this equation will be the equilibrium price and we can then find \hat{c}_1^j , \hat{c}_2^j , \hat{b}^j for $j = H, F$ by substituting \hat{R} into the demand functions.

To solve (2.13) first multiply both sides by $(1+\beta)$

$$(y_1^H + y_1^F) + \frac{(y_2^H + y_2^F)}{R} = (1+\beta)(y_1^H + y_1^F)$$

Subtract $y_1^H + y_1^F$ from both sides then solve for R to get

$$(2.15) \quad \hat{R} = \frac{(y_2^H + y_2^F)}{\beta(y_1^H + y_1^F)}$$

Let (y_1, y_2) be the world endowment of good 1 and 2, respectively

$$y_1 \equiv y_1^H + y_1^F$$

$$y_2 \equiv y_2^H + y_2^F$$

Then (2.15) becomes

$$(2.16) \quad \hat{R} = \frac{y_2}{\beta y_1}$$

Now to solve for the equilibrium consumption and saving (\equiv bond demand) we substitute \hat{R} from (2.16) into the demand functions. For the home consumer we have

$$(2.17) \quad \hat{c}_1^H = \frac{1}{1 + \beta} \left[y_1^H + \frac{y_2^H}{\hat{R}} \right] = \frac{1}{1 + \beta} \left[y_1^H + \left(\frac{\beta y_1}{y_2} \right) y_2^H \right]$$

$$(2.18) \quad \hat{c}_2^H = \frac{\hat{\beta R}}{1 + \beta} \left[y_1^H + \frac{y_2^H}{\hat{R}} \right] = \frac{1}{1 + \beta} \frac{y_2}{y_1} \left[y_1^H + \left(\frac{\beta y_1}{y_2} \right) y_2^H \right]$$

$$(2.19) \quad \hat{b}^H = \frac{1}{1 + \beta} \left[\beta y_1^H - \frac{y_2^H}{\hat{R}} \right] = \frac{\beta}{1 + \beta} \left[y_1^H - \left(\frac{y_1}{y_2} \right) y_2^H \right]$$

Likewise for the foreign consumer we have

$$(2.20) \quad \hat{c}_1^F = \frac{1}{1 + \beta} \left[y_1^F + \left(\frac{\beta y_1}{y_2} \right) y_2^F \right]$$

$$(2.21) \quad \hat{c}_2^F = \frac{1}{1 + \beta} \frac{y_2}{y_1} \left[y_1^F + \left(\frac{\beta y_1}{y_2} \right) y_2^F \right]$$

$$(2.22) \quad \hat{b}^F = \frac{\beta}{1 + \beta} \left[y_1^F - \left(\frac{y_1}{y_2} \right) y_2^F \right]$$

Then equations (2.16)-(2.22) given an equilibrium for our example economy. Before we analyze this simple equilibrium let us introduce some of the terminology of Balance of Payments Accounting.

F. Balance of Payments Accounting for a simple economy

Let Gross Domestic Product of country j: = GDP_t^j
 ≡ value of production (or endowments) of country j at t

Here: $GDP_1^H = y_1^H$, $GDP_1^F = y_1^F$

$$GDP_2^H = y_2^H$$
 , $GDP_2^F = y_2^F$

Let Gross National Product of country j at t: ≡ GNP_t^j
 ≡ "income" of consumers of country j at t

In this model GNP equals GDP plus net interest payments

$$\text{GNP}_1^H = \text{GDP}_1^H \quad , \quad \text{GNP}_1^F = \text{GDP}_1^F$$

$$\text{GNP}_2^H = \text{GDP}_2^H + \text{rb}^H \quad , \quad \text{GNP}_2^F = \text{GDP}_2^F + \text{rb}^F$$

Let Trade Balance of country j at t: = TB_t^j

≡ value of exports minus imports

In this model this is just net exports of commodity

$$\text{TB}_1^H = y_1^H - c_1^H \quad , \quad \text{TB}_1^F = y_1^F - c_1^F$$

$$\text{TB}_2^H = y_2^H - c_2^H \quad , \quad \text{TB}_2^F = y_2^F - c_2^F$$

Let Current Account of country j at t = CA_t^j

≡ trade balance plus interest income on foreign assets

$$\text{CA}_1^H = \text{TB}_1^H \quad , \quad \text{CA}_1^F = \text{TB}_1^F$$

$$\text{CA}_2^H = \text{TB}_2^H + \text{rb}^H \quad , \quad \text{CA}_2^F = \text{TB}_2^F + \text{rb}^F$$

Now we can use the present value form of the budget constraint to derive a simple yet important restriction on trade balances. For country j we know (from 1.4)

$$(3.1) \quad c_1^j + \frac{c_2^j}{R} = y_1^j + \frac{y_2^j}{R}$$

Rearranging this we have

$$(3.2) \quad (y_1^j - c_1^j) + \frac{(y_2^j - c_2^j)}{R} = 0$$

Using the definition of trade balance we have

$$(3.3) \quad TB_1^j + \frac{TB_2^j}{R} = 0.$$

(3.3) is an important relation: It says that the present value of trade balance surpluses is zero. That is, if a country runs a trade balance surplus at date 1 it must run a trade balance deficit at date 2.

Now if we had a many period model ($t = 1, \dots, T$) as opposed to a 2 period model, then (3.3) would become

$$\sum_{t=0}^T \left(\frac{1}{R}\right)^t TB_t^j = 0.$$

So that a trade balance surplus at date 1 does not in general imply a trade balance deficit at date 2 but rather it implies eventually there must be trade balance deficits which match in present value terms the trade balance surpluses.

G. Fluctuations in the World Economy

We will illustrate how fluctuations in world outputs affects consumption and the balance of payments by means of several simple examples.

Suppose we have data on output for these two countries for $t = 1, 2$, say $\{y_t^j | t = 1, 2, j = H, F\}$. Let us define \bar{y}_t to be the mean world output at t , that is,

$$(3.4) \quad \bar{y}_t = (y_t^H + y_t^F) / 2 \quad t = 1, 2$$

Let us define θ_t^j to be the amount that country j 's output at t deviates from the mean world output at t , that is

$$(3.5) \quad \theta_t^j = y_t^j - \bar{y}_t, \quad j = H, F, \quad t = 1, 2$$

Then we can decompose each countries' output into two pieces,

$$(3.6) \quad y_t^j = \bar{y}_t + \theta_t^j, \quad j = H, F, \quad t = 1, 2$$

country j output at t = mean world output at t + country j 's deviation from mean output at t .

Think of \bar{y}_t as coming from (common) world fluctuations and θ_t^j as coming from some country-specific fluctuation.

We can then compute the equilibrium for three cases:

- (I) No world fluctuations and no country-specific fluctuations;
- (II) Country-specific fluctuations but no world fluctuations;
- (III) World fluctuations but no country-specific fluctuations. And examine the behavior of consumption and balance of payments in each case.

I. No World Fluctuations and No Country-Specific Fluctuations

Let

$$(3.7) \quad y_t = \bar{y} \quad t = 1, 2.$$

By no world fluctuation we mean the mean world endowment is constant over time

$$(3.8) \quad \theta_t^H = +\theta \quad t = 1, 2$$

$$\theta_t^F = -\theta \quad t = 1, 2$$

By no country-specific fluctuations, we mean the deviations of country j 's output from world mean endowment is constant over time. (3.7) and (3.8) imply

$$(3.9) \quad y_t^H = \left\{ \begin{array}{ll} \bar{y} + \theta & t = 1 \\ \bar{y} + \theta & t = 2 \end{array} \right\}, \quad y_t^F = \left\{ \begin{array}{ll} \bar{y} - \theta & t = 1 \\ \bar{y} - \theta & t = 2 \end{array} \right\}$$

and

$$(3.10) \quad y_t = y_t^H + y_t^F = 2\bar{y}$$

Then substituting (3.10) into (2.16) implies

$$(3.11) \quad R = 1/\beta.$$

Substituting (3.9) into (2.17), (2.18) and (2.20), (2.21) imply (using 3.11)

$$(3.12) \quad c_1^H = \frac{1}{1+\beta} [\bar{y} + \theta + \beta(\bar{y} + \theta)] = \bar{y} + \theta$$

$$(3.13) \quad c_2^H = \frac{1}{1+\beta} [\bar{y} + \theta + \beta(\bar{y} + \theta)] = \bar{y} + \theta$$

So consumption in home country is constant and equal to home output

$$(3.14) \quad c_1^F = \frac{1}{1+\beta} [\bar{y} - \theta + \beta(\bar{y} - \theta)] = \bar{y} - \theta$$

$$(3.15) \quad c_2^F = \frac{1}{1+\beta} [\bar{y} - \theta + \beta(\bar{y} - \theta)] = \bar{y} - \theta$$

Consumption in foreign country is

- (i) constant
- (ii) equal to foreign output
- (iii) lower than in home country.

Using the definitions of trade balance and current accounts we see

$$(3.16) \quad TB_t^j = CA_t^j = 0 \text{ for } j = H, F, \quad t = 1, 2.$$

Graphically, we have the following:

(see Figure 4)

II. Country-Specific Fluctuations but No World Fluctuations

Let

$$(3.17) \quad y_t = \bar{y}, \quad t = 1, 2. \quad \theta_t^H = \begin{cases} +\theta & t = 1 \\ -\theta & t = 2 \end{cases}, \quad \theta_t^F = \begin{cases} -\theta & t = 1 \\ +\theta & t = 2 \end{cases}$$

this implies

$$(3.18) \quad y_t^H = \begin{cases} \bar{y} + \theta & t = 1 \\ \bar{y} - \theta & t = 2 \end{cases}, \quad y_t^F = \begin{cases} \bar{y} - \theta & t = 1 \\ \bar{y} + \theta & t = 2 \end{cases}$$

and

$$(3.19) \quad y_t = y_t^H + y_t^F = 2\bar{y} \quad \forall t$$

Substituting (3.19) into (2.16) implies

$$(3.20) \quad R = 1/\beta$$

Substituting (3.18) into (2.17), (2.18) and (2.19), (2.20) implies

$$(3.21) \quad c_1^H = \bar{y} + \left(\frac{1-\beta}{1+\beta}\right)\theta$$

$$(3.22) \quad c_2^H = \bar{y} + \left(\frac{1-\beta}{1+\beta}\right)\theta$$

$$(3.23) \quad c_1^F = \bar{y} - \left(\frac{1-\beta}{1+\beta}\right)\theta$$

$$(3.24) \quad c_2^F = \bar{y} - \left(\frac{1-\beta}{1+\beta}\right)\theta$$

So consumption in both countries

- (i) is constant
- (ii) but not equal to own output

In particular

$$(3.25) \quad TB_t^H = \begin{cases} 2\beta\theta/1+\beta & t = 1 \\ -2\theta/1+\beta & t = 2 \end{cases}, \quad TB_t^F = \begin{cases} -2\beta\theta/1+\beta \\ +2\theta/1+\beta \end{cases}$$

$$(3.26) \quad CA_t^H = \begin{cases} 2\beta\theta/1+\beta & t = 1 \\ -2\beta\theta/1+\beta & t = 2 \end{cases}, \quad CA_t^F = \begin{cases} -2\beta\theta/1+\beta \\ +2\beta\theta/1+\beta \end{cases}$$

where, for example, $TB_1^H = y_1^H = y_1^H - c_1^H = (\bar{y} + \theta) - [\bar{y} + (\frac{1-\beta}{1+\beta})\theta] = \frac{2\beta\theta}{1+\beta}$ of course, it is true that here

$$(3.27) \quad b^H = TB_1^H \text{ and } b^F = TB_1^F$$

Graphing these variables gives:

(see Figure 5)

III. World Fluctuations but No Country-Specific Fluctuations

Let

$$(3.28) \quad \bar{y}_t = \begin{cases} \bar{y} & t = 1 \\ \alpha\bar{y} & t = 2 \end{cases}, \quad 0 < \alpha < 1 \text{ and } \theta_t^j = 0, \quad j = H, F, \quad t = 1, 2.$$

Then

$$(3.29) \quad y_t^H = \begin{cases} \bar{y} & t = 1 \\ \alpha\bar{y} & t = 2 \end{cases}, \quad y_t^F = \begin{cases} \bar{y} & t = 1 \\ \alpha\bar{y} & t = 2 \end{cases}$$

$$(3.30) \quad y_t = y_t^H + y_t^F = \begin{cases} 2\bar{y} & t = 1 \\ 2\alpha\bar{y} & t = 2 \end{cases}.$$

Substituting (3.30) into (2.16) implies

$$(3.31) \quad R = \alpha/\beta.$$

Substituting (3.30) into (2.17), (2.18), and (2.20), (2.21) and using (3.29) gives

$$(3.32) \quad c_1^H = \frac{1}{1 + \beta} \left[\bar{y} + \left(\frac{\beta}{\alpha}\right)(\alpha\bar{y}) \right] = \bar{y}$$

$$(3.33) \quad c_2^H = \alpha\bar{y}$$

$$(3.34) \quad c_1^F = \bar{y}$$

$$(3.35) \quad c_2^F = \alpha\bar{y}$$

So consumption in both countries is

- (1) not constant
- (2) but is equal to own output

Clearly

$$(3.36) \quad TB_t^j = CA_t^j = b^j = 0 \quad j = H, F \quad t = 1, 2.$$

Graphing these variables gives:

(see Figure 6)

The basic point of these examples is that optimal consumption behavior is to smooth consumption over time as much as possible. In particular, if there are only country-specific fluctuations, each country's consumption will be perfectly flat (constant) even if the country's output is fluctuating. To accomplish this the country must lend when output is high (run a trade balance sur-

plus) and borrow when output is low (run a trade balance deficit). In such a circumstance, if the country were forced to have trade balanced in every period it would have a much lower level of utility. On the other hand, if there are world fluctuations in output, each country (consumer) will attempt to smooth consumption but will be frustrated in the aggregate and optimal consumption will fluctuate up and down.

Punchline: smooth country-specific fluctuations in output but have to bear the world fluctuations in output.

H. Adding Governments to the 2-Period Model

So far we have examined an economy with only private agents, here consumers. We now introduce into this economy a home of foreign governments. These governments will (i) buy output from private consumers; (ii) levy lump sum taxes to raise revenues and (iii) sell government bonds to separate current purchases from current revenues.

In particular, the government of country j

- (i) buys output g_1^j at time 1 and g_2^j at time 2 from private consumers (per capita)
- (ii) levies lump sum taxes τ_1^j at time 1 and τ_2^j at time 2 (per capita) on country j 's residents
- (iii) sells government bonds \bar{b}_1^j at time 1

A fiscal policy of the government can be summarized by $\{g_t^j, \tau_t^j, \bar{b}_1^j; t=1,2\}$. The period budget constraints of the government j are:

$$(4.1) \quad (t = 1) \quad \bar{b}_1^j = g_1^j - \tau_1^j$$

$$(t = 2) \quad 0 = R\bar{b}_1^j + g_2^j - \tau_2^j$$

Solving the second equation for b_1^j and substituting into the first gives the present value form of the government budget constraint

$$(4.2) \quad g_1^j \frac{g_2^j}{R} = \tau_1^j + \frac{\tau_2^j}{R} \quad j = H, F.$$

In words, the present value of government spending must equal the present value of (income) tax collection. So far we have not specified what the government does with its spending. In particular, we have not specified how government spending affects private consumers. We will make one of the following two assumptions.

(G1) Government spending does not give utility to private consumers (basically, government buys private output and throws it in ocean).

(G2) The government uses the revenues to manufacture a public good (say, national defense) the services of which benefits its own residents.

Initially, we assume (G1). Under (G1), the consumer's problem can be written

$$(4.3) \quad \begin{aligned} & \max_{\{c_1^j, c_2^j, b_1^j\}} U^j(c_1^j, c_2^j) \\ \text{s.t.} \quad & 1. \quad c_1^j + b_1^j = y_1^j - \tau_1^j \\ & 2. \quad c_2^j = y_2^j - \tau_2^j + Rb_1^j \\ & 3. \quad c_1^j, c_2^j \geq 0 \end{aligned}$$

or

$$(4.4) \quad \max_{\{c_1^j, c_2^j\}} U^j(c_1^j, c_2^j)$$

$$\text{s.t.} \quad c_1^j + \frac{c_2^j}{R} = y_1^j + \frac{y_2^j}{R} - \left(\tau_1^j + \frac{\tau_2^j}{R} \right)$$

(4.3) is the consumer's problem with period budget constraints. (4.4) is the consumer's problem with a present value budget constraint. Notice these constraints are the same as before if we call $y_t^j - \tau_t^j$ the net (after-tax) endowment.

The definition of a competitive equilibrium is similar to that on p. I, 14 except now we include governments.

Definition: A competitive equilibrium for this economy given government policies $\{g_t^j, \tau_t^j, b^j\}$ is

- (i) an allocation $\{c_1^H, c_2^H; c_1^F, c_2^F\}$
- (ii) a set of asset demands and supplies $\{b^H, b^F; \bar{b}^H, \bar{b}^F\}$
- (iii) a price system \hat{R}

such that (E1), (E2) and (E3) hold.

(E1) Market Clearing

- (i) date 1 market clears: $\hat{c}_1^H + \hat{c}_1^F + g_1^H = y_1^H + y_1^F$
- (ii) date 2 market clears: $\hat{c}_2^H + \hat{c}_2^F + g_2^H + g_2^F = y_2^H + y_2^F$
- (iii) bond market clears: $\hat{b}^H + \hat{b}^F = \bar{b}^H + \bar{b}^F$

(E2) Maximization

$\{\hat{c}_1^j, \hat{c}_2^j, \hat{b}^j\}$ solves (4.3) given endowments $\{y^j, y_2^j\}$, and taxes $\{\tau_1^j, \tau_2^j\}$ at price \hat{R} for $j = H, F$.

(E3) Government Feasibility

$\{g_1^j, g_2^j, \tau_1^j, \tau_2^j\}$ satisfy

$$b_1^j = g_1^j - \tau_1^j \text{ and } 0 = \hat{R}b_1^j + g_2^j - \tau_2^j$$

Remark: Call $(g_t^j - \tau_t^j)$ the (net) government deficit of j at t (net because we haven't included service payments in the debt). Then the present value budget constraint of the government requires the present value of government deficits must equal zero that is, if the government spends more than it takes in the first period it must take in more than it spends the second by either raising taxes or cutting government spending relative to the first period.

In a T period model with constant one-period interest rate R the government budget constraint is

$$\sum_{t=0}^T \left(\frac{1}{R}\right)^t g_t^j = \sum_{t=0}^T \left(\frac{1}{R}\right)^t \tau_t^j + \sum_{t=0}^T \left(\frac{1}{R}\right)^t (g_t^j - \tau_t^j) = 0.$$

Budget Deficits [see Barro chapters 13 & 15]

A question one often hears is:

Q: "What are the effects of budget deficits?"

The economic answer to this is either "This question is not well-defined" or "This question is not stated in a precise enough form so that there is a possibility of giving an unambiguous answer."

In the context of our simple model we can state precise versions of this question and give precise answers. We can then qualify our answers and state precise conditions under which they will be different.

Consider the government of country H 's budget constraints (in period form) (from 4.1)

$$(4.5) \quad (t = 1) \quad \bar{b}_1^H = g_1^H - \tau_1^H$$

$$(t = 2) \quad 0 = R\bar{b}_1^H + g_2^H - \tau_2^H$$

Now the present value form of these constraints is

$$g_1^H + \frac{g_2^H}{R} = \tau_1^H + \frac{\tau_2^H}{R}$$

or

$$(4.6) \quad (g_1^H - \tau_1^H) + \frac{(g_2^H - \tau_2^H)}{R} = 0$$

$$[(\text{Budget deficit at 1}) + \frac{\text{Budget deficit at 0}}{R} = 0]$$

Now from (4.5) we have that one of two possible government actions can cause the budget deficit to increase at date 1. Either

- (A1) The government can cut taxes ($\tau_1^H \downarrow$) (keeping government spending fixed at g_1^H).
- (A2) The government can raise spending ($g_1^H \uparrow$) (keeping taxes fixed at τ_1^H).

Now, the present value form of the government budget constraint makes it obvious that if the government runs a deficit the first period it must run a surplus the second period (if it obeys its present value budget constraint). Notice that if it doesn't obey its budget constraint, it won't be able to pay off the government bonds (IOUs) it sold the first period.

So the government in the second period can run a surplus by either

- (B1) The government can raise taxes ($\tau_2^H \uparrow$) and keeping government spending at two fixed at g_2^H

or

(B2) The government can lower spending ($g_2^H \downarrow$) and keeping taxes at 2 fixed at τ_2^H

The government can of course do both (A1) and (A2) then (B1) and (B2) all at once but for conceptual reasons it is better to consider one at a time.

We will analyze the effects of government spending changes by:

Step 1: specifying an original set of government policies for both countries and solving for the original equilibrium.

Step 2: specifying a new set of government policies for both countries and solving for the new equilibrium (in fact, we will change only the policies of the home government).

Step 3: comparing the original equilibrium with the new equilibrium.

We will consider four possible changes in policy:

- I. (A1) & (B1): a tax cut at 1, balanced by tax increase at 2.
- II. (A2) & (B2): a spending increase at 1, balanced by a spending cut at 2.
- III. (A1) & (B2): a tax cut at 1, balanced by a spending cut at 2.
- IV. (A2) & (B1): a spending increase at 1, balanced by a tax increase at 2.

More precisely, I will consider cases I and II and you will consider cases III and IV (in a homework).

Step 1: Consider a two-period, two-country economy as in (I,27) with endowments $\{y_1^H, y_2^H, y_1^F, y_2^F\}$ and an original set of government policies $\{g_1^H, g_2^H, \tau_1^H, \tau_2^H, b^H\}$ for home government and $\{g_1^F, g_2^F, \tau_1^F, \tau_2^F, b^F\}$ for foreign government. Now

throughout we will change only the policies of the home government. Just as in I.14-I.16, we can solve for an equilibrium by replacing our original endowments

$$(4.7) \quad \begin{array}{cc} y_1^H & y_1^F \\ & \& \\ y_2^H & y_2^F \end{array}$$

with the after-tax endowments

$$(4.8) \quad \begin{array}{cc} y_1^H - \tau_1^H & y_1^F - \tau_1^F \\ & , \\ y_2^H - \tau_2^H & y_2^F - \tau_2^F \end{array}$$

(1A) We can compute the demand functions for these endowments. We will then get the analogues of (2.17)-(2.22).

$$(4.9) \quad \hat{c}_1^H = \frac{1}{1 + \beta} \left[y_1^{H-\tau_1^H} + \frac{(y_2^{H-\tau_2^H})}{R} \right]$$

$$(4.10) \quad \hat{c}_2^H = \frac{\beta R}{1 + \beta} \left[y_1^{H-\tau_1^H} + \frac{(y_2^{H-\tau_2^H})}{R} \right]$$

$$(4.11) \quad \hat{c}_1^F = \frac{1}{1 + \beta} \left[y_1^{F-\tau_1^F} + \frac{(y_2^{F-\tau_2^F})}{R} \right]$$

$$(4.12) \quad \hat{c}_2^F = \frac{\beta R}{1 + \beta} \left[y_1^{F-\tau_1^F} + \frac{(y_2^{F-\tau_2^F})}{R} \right]$$

$$(4.13) \quad \hat{b}^H = \frac{1}{1 + \beta} \left[\beta (y_1^{H-\tau_1^H}) - \frac{1}{R} (y_2^{H-\tau_2^H}) \right]$$

$$(4.14) \quad \hat{b}^F = \frac{1}{1 + \beta} \left[\beta (y_1^{F-\tau_1^F}) - \frac{1}{R} (y_2^{F-\tau_2^F}) \right].$$

Then we can solve for the equilibrium by using any one of the three market clearing conditions. Any one of the three will do (since we can show they are all equivalent by using the budget constraints of both consumers and government; this is called Walras' Law).

Before we do this notice that we can rearrange the home consumer's demand function to be

$$(4.15) \quad \tilde{c}_1^H = \frac{1}{1 + \beta} \left[y_1^H + \frac{y_2^H}{R} - \left(\tau_1^H + \frac{\tau_2^H}{R} \right) \right]$$

but we know that

$$(4.16) \quad \tau_1^H + \frac{\tau_2^H}{R} = g_1^H + \frac{g_2^H}{R}$$

by the home government present value budget constraint. Substituting (4.16) into (4.15) we have

$$(4.17) \quad \tilde{c}_1^H = \frac{1}{1 + \beta} \left[y_1^H + \frac{y_2^H}{R} - \left(g_1^H + \frac{g_2^H}{R} \right) \right]$$

We can do a similar thing to $c_1^F + c_2^F$, and we can use the foreign government budget constraint to do a similar thing to c_1^F and c_2^F . If we do, we have

$$(4.18) \quad \tilde{c}_2^H = \frac{R\beta}{1 + \beta} \left[y_1^H + \frac{y_2^H}{R} - \left(g_1^H + \frac{g_2^H}{R} \right) \right]$$

$$(4.19) \quad c_1^F = \frac{1}{1 + \beta} \left[y_1^F + \frac{y_2^F}{R} - \left(g_1^F + \frac{g_2^F}{R} \right) \right]$$

$$(4.20) \quad c_2^F = \frac{R\beta}{1 + \beta} \left[y_1^F + \frac{y_2^F}{R} - \left(g_1^F + \frac{g_2^F}{R} \right) \right]$$

Now these substitutions will make the algebra of solving for the equilibrium easier. Now substitute (4.17) and (4.19) into (E1(i)) to obtain

$$(E1(i)) \quad c_1^H + c_2^H + g_1 + g_2 = y_1 + y_1$$

$$(4.21) \quad \frac{1}{1+\beta} \left[y_1^H + \frac{y_2^H}{R} - \left(g_1^H + \frac{g_2^H}{R} \right) \right] + \frac{1}{1+\beta} \left[y_1^F + \frac{y_2^F}{R} - \left(g_1^F + \frac{g_2^F}{R} \right) \right] \\ + g_1^H + g_1^F = y_1^H + y_1^F$$

Now we want to solve (4.21) for R; this will be the equilibrium interest rate. To do so in 4.21:

- subtract $g_1^H + g_1^F$ from both sides
- multiply both sides by $(1+\beta)$
- subtract $y_1^H - g_1^H + y_2^H - g_2^H$

this gives

$$(4.22) \quad \frac{y_2^H - g_2^H}{R} + \frac{y_2^F - g_2^F}{R} = \beta \left[(y_1^H - g_1^H) + (y_1^F - g_1^F) \right]$$

Solving for R then gives

$$(4.23) \quad R = \frac{1}{\beta} \left[\frac{y_2^H - g_2^H + y_2^F - g_2^F}{y_1^H - g_1^H + y_1^F - g_1^F} \right]$$

Note 1: If we compare (4.23) to (2.16) we notice we have similar formulas except in (4.23) we have in the numerator the total world endowment at period 2 after the government take their share of apples and save in denominator for period 1.

Note 2: Notice the interest rate depends only on

- (i) private endowments (in both periods)
- and
- (ii) government spending (in both periods)
- but not on
- (iii) taxes in any period.

This last observation will be important for assessing the effects of tax induced-tax balance budget deficits.

Summary of Step 1

For the original set of government policies we have

- (A) The equilibrium interest rate is given by (4.23).
- (B) The equilibrium consumption allocations are given by (4.17 to 4.20) when we substitute the equilibrium interest rate (4.23).

Step 2: Consider the same economy as before, let the policy of the foreign government be unchanged, however, let the home government

- (a) cut taxes in period one by Δ_1 and raise taxes in period 2 by Δ_2
- (b) keep government spending the same each period.
- (c) assume the government still satisfies its budget constraint.

Home Government Policy

Original

$$\{g_1^H, g_2^H, \tau_1^H, \tau_2^H; b^H\}$$

vs.

New

$$\{g_1^H, h_2^H, \tau_1^{H'}, \tau_2^{H'}, b^{H'}\}$$

where

$$\tau_1^{H'} = \tau_1^H - \Delta_1 \quad (\text{tax cut of } \Delta_1)$$

$$\tau_2^{H'} = \tau_2^H + \Delta_2 \quad (\text{tax increase of } \Delta_2)$$

Budget Constraints

(Period B.C.)

$$\begin{aligned} (t = 1) \quad b^H &= g_1^H - \tau_1^H & \text{vs.} \quad b^{H'} &= g_1^H - \tau_1^{H'} \\ & & &= g_1^H - (\tau_1^H - \Delta_1) \\ & & &= b^H + \Delta_1 \end{aligned}$$

(budget deficit goes up in period 1 by Δ_1)

$$\begin{aligned} (t = 2) \quad 0 &= Rb^H + g_2^H - \tau_2^H & 0 &= Rb^{H'} + g_2^H - \tau_2^{H'} \\ & & 0 &= R(b^H + \Delta_1) + g_2^H - (\tau_2^H + \Delta_2) \\ & & 0 &= Rb^H + g_2^H - \tau_2^H + R\Delta_1 - \Delta_2 \end{aligned}$$

0 if original policy satisfies (t = 2) constraint)

so

$$0 = R\Delta_1 - \Delta_2$$

$$\Delta_2 = R\Delta_1$$

(where we have implicitly assumed interest rates don't change)

Present Value Budget Constraints

$$g_1^H + \frac{g_2^H}{R} = \tau_1^H + \frac{\tau_2^H}{R} \quad \text{vs.} \quad g_1^H + \frac{g_2^H}{R} = \tau_1^{H'} + \frac{\tau_2^{H'}}{R}$$

$$\begin{aligned} " &= (\tau_1^H - \Delta_1) + \frac{(\tau_2^H + \Delta_2)}{R} \\ " &= \tau_1^H + \frac{\tau_2^H}{R} - \Delta_1 + \frac{\Delta_2}{R} \end{aligned}$$

I wrote out the above just to make sure it's totally clear what is happening to the government budget constraints. Now, let us solve for the new equilibrium:

- (1) Examination of the formula for interest rate (4.23) makes it clear that since government spending didn't change then interest rates didn't change.
- (2) To solve for consumption of home and foreign consumers, substitute this formula for interest rates into the demand functions (4.17 to 4.20). However, nothing here is changed either, so consumption doesn't change either.

We summarize this in a proposition.

The Ricardian Proposition:

Budget deficits caused by current tax cuts that will be balanced by future tax increases have no effect on either consumption or interest rates.

Now we have demonstrated this proposition in the context of a simple example, with additively separable-two period-log utility, etc. It is straightforward to generalize this proposition to include • general utility functions, • many period-live consumer and governments, • production, • uncertainty, etc. However, there are two important sets of circumstances when this proposition will not hold.

II. Monetary Economics

In this paper, we modify the real macroeconomy of my earlier paper to include fiat money. Once we accomplish this, we can analyze the interactions of monetary and fiscal policy and the resulting implications for the open economy. In particular, we will be concerned with the effects of budget deficits on: (i) prices and inflation rates (in a one country world), (ii) exchange rates. We will also derive (iii) a modified Ricardian proposition.

Some Motiviation

Before we consider models with money, let us first review the standard stories behind the general equilibrium model. There are two different interpretations of the market structure of the general equilibrium model: the date 0 interpretation and the sequence interpretation. In the date 0 interpretation it is imagined that all agents convene in one grand market at the beginning of time (date 0). Agents sign contracts promising to deliver goods to others in the future and for promises of housing goods delivered to themselves. An example of a contract between two agents, A and B, may be as follows:

Agent A promises to deliver 10 apples at date 3 to Agent B in exchange for a promise from B to deliver 12 pears at date 7 to A.

It is important to notice a couple features of this market structure:

- (1) All markets meet once at the beginning of time.
- (2) All contracts involve the sale of goods for goods.

In the sequence of markets interpretation, it is imagined that agents meet at the beginning of every period, say t . At this time, two types of markets open: (i) spot markets, and (ii) asset markets.

A spot market is a market in which current goods are traded against other current goods. For example, Agent A may give Agent B four time- t apples for five time- t pears. As asset market is a market in which current goods are traded against future goods. For example, Agent A may sell a one-period bond to Agent B, promising to pay, say, two apples at time $t + 1$ in exchange for one apple today, at t .

Now go back to handout I, page 6-8. Notice that we wrote the budget constraints in two different ways. These two types of budget constraints correspond to the two market structures: The present value budget constraint corresponds to the date 0 interpretation. The period budget constraint corresponds to the sequence interpretation.

Let us consider a many-period generalization of the problem faced by the consumer. Let us first consider the present formulation. (At time 0) choose $\{c_0, c_1, \dots, c_T\}$ to solve:

$$(1.1) \quad \max \sum_{t=0}^T \beta^t u(c_t)$$

such that

$$c_0 + \frac{c_1}{R_{0,1}} + \frac{c_2}{R_{0,2}} + \frac{c_3}{R_{0,3}} + \dots + \frac{c_T}{R_{0,T}} = y_0 + \frac{y_1}{R_{0,1}} + \dots + \frac{y_T}{R_{0,T}}.$$

Notice that this problem is identical to a static problem:

$$(1.2) \quad \max U(c_0, \dots, c_T)$$

such that

$$c_0 + p_1 c_1 + \dots + p_T c_T = y_0 + p_1 y_1 + \dots + p_T y_T.$$

In fact, if we impose the assumption of additive separability and discounting and we also let the relative price of good t to good 0 be $p_t = 1/R_{0,t}$, then we have exactly the same problems. One can imagine the consumer solving (1.1) just as he/she would solve (1.2), as a static once and for all problem.

The many period version of the period budget constraint formulation is, at time t , to choose current consumption c_t and new bond holdings b_{t+1} :

$$(1.3) \quad \max \sum_{t=0}^T \beta^t u(c_t)$$

such that

$$\begin{aligned} c_0 + b_1 &= y_0 \\ c_1 + b_2 &= R_1 b_1 + y_1 \\ c_2 + b_3 &= R_2 b_2 + y_2 \\ &\vdots \\ c_T + b_{T+1} &= R_T b_T + y_T \end{aligned}$$

a "sequence" of budget constraints.

Now at time t , future markets have not met (i.e., spot and asset markets at $t + 1$ and after), however, to decide on what to do today (at t) the agent must have some belief about the future. We will assume the agent has rational expectations about the future which, in this deterministic context, means the future is perfectly foreseen.

In either of these interpretations, there is no need for agents to hold pieces of paper--called first currency or money. Why should they? They

can't eat the money (it doesn't taste very good), they don't need it to buy goods with (they can just trade goods for goods), they don't need it to save with (they can save in the date 0 structure by selling goods now for promises to get paid in goods later), and they can save in the sequence of markets structure by buying assets (like bonds).

In fact, within either of these market structures, not only is there no need for fiat currency, there is no room for fiat currency. By this I mean unless we change some features of the above environment, no one will want to hold pieces of this paper. In equilibrium they will have no value, they will be just worthless pieces of paper. Thus, in order to have a theory of money, we will have to deviate somehow from the simple "frictionless" trading arrangements of the above.

There are three basic approaches to this problem:

- (1) Try to sidestep the problem completely.
- (2) Build into the environment a role for money as an important means of saving (between periods).
- (3) Build into the environment a role for money as an efficient means of transacting.

Broadly speaking, there are three types of models corresponding to these three approaches:

- (1) money in the utility function,
- (2) overlapping generations,
- (3) cash-in-advance.

Money in the Utility Function Model

This approach simply asserts that money is just like any other good that people value. To model money, we just need to add another good into the utility function called "services from real cash balances," or simply, money. Say $U(c, m/p)$ in a one-period model or $\sum \beta^t u(c_t, m_t/r_t)$ in a many-period model. This approach has not led to many insights and will not be pursued here.

Overlapping Generations Model

This approach assumes agents are separated in time in such a way that it is impossible to trade goods for goods. Basically, the timing is such that there is not "double coincidence of wants," i.e., no two agents want something the other one has and is willing to sell. Neil Wallace covers these models in his intertemporal economics course in detail. We will not pursue them here.

Cash-in-Advance Model

This approach is driven by two main assumptions: (i) one needs cash to buy good, and (ii) it is impossible to get cash instantly and costlessly. Basically, the idea is that using cash is a more efficient way to transact than to sell goods for goods. The possible stories behind this are: (for (i)) sellers accept only cash if they have little information about your credit position, or (for (ii)) it is difficult and costly to find someone who has just what you want and is willing to take in payment just what you have.

The Cash-in-Advance Model ^{1/}

Consider an economy consisting of a large number of identical households, a firm, and a government. There is one physical good per period and a large number of periods. There are two assets in the economy: fiat money and (nominal) bonds. It is a requirement that all goods be paid for with money. In particular, one cannot use bonds to buy goods and one cannot buy goods on credit.

Each household is composed of a worker/shopper pair. In order to understand the rather delicate timing of this model, it is useful to consider a representative day for the economy. A representative day for the household is as follows:

- 9:00 a.m. Get up holding: (i) unspent cash balances from yesterday, (ii) one period nominal bonds bought/sold yesterday, and (iii) wages worker received at end of period yesterday. Immediately household pays lump sum taxes to government (in cash).
- 9:15 a.m. The worker goes to factory and supplies labor to produce goods which are sent to shopping mall (goods market) continuously. The shopper goes to bank (asset market) and: (i) obtains or deposits cash balances, (ii) collects on or pays off nominal bonds from last period, and (iii) purchases or sells new one period bonds.
- 10:00 a.m. Shopper goes to shopping mall and shops all day. At the shopping mall, one can purchase goods only with cash.

^{1/}Some parts of this chapter draw heavily from Sargent's chapter on "Cash in Advance Models," especially the propositions presented later.

4:00 p.m. Stores close and the shopper returns home. The worker receipts from this period's goods which were sold at stores and collected and wages are paid to workers and then return home.

The following diagram is suggestive of a physical layout.

Let us review the constraint in more detail.

Asset market

Entering the asset market the consumer has:

- (i) bonds bought yesterday at $t - 1$ asset market B_t which now are worth $I_t B_t$. (The convention is that bonds are indexed by the date they pay off.)
- (ii) wages received (in cash) at end of last period $p_{t-1}y_{t-1}$ which were received after goods market had closed.
- (iii) unspent cash balances from yesterday $m_{t-1} - p_t c_{t-1}$ (spent $p_{t-1}c_{t-1}$ on consumption at goods market out of total cash of m_{t-1}).
- (iv) current taxes paid to government in cash, T_t .

The total wealth brought into asset market at t

$$p_{t-1}y_{t-1} + I_t B_t - T_t + (M_{t-1} - p_{t-1}c_{t-1})$$

(wages + bonds - taxes + unspent cash).

In the asset market the consumer purchases: (i) new cash, M_t , for today's shopping trip, and (ii) saves (or borrows) by purchasing (or selling) new bonds B_{t+1} . Thus we have the asset market constraint:

$$(1.4) \quad M_t + B_{t+1} \leq p_{t-1}y_{t-1} + I_t B_t - T_t + (M_{t-1} - p_{t-1}c_{t-1}).$$

Goods market

After leaving the asset market, the shopper goes to the goods market where store/factory owners sell current goods (say apples) but they are only willing to accept cash in payment. In particular, they are not willing to accept bonds or I.O.U.s, etc. so if one wants to buy c_t apples and the price is p_t dollars per one apple, one needs $p_t c_t$ dollars on hand in cash. The goods market constraint is:

$$(1.5) \quad p_t c_t \leq M_t.$$

The Consumer's Problem

The problem of the consumer is to choose at each t , $\{c_t, m_t, B_{t+1}\}$ (consumption at t , new money at t , and new bond holdings at t) to solve:

$$(1.6) \quad \max \sum_{t=0}^T \beta^t u(c_t).$$

$$\text{(Asset market) } 1. \quad M_t + B_{t+1} \leq p_{t-1}y_{t-1} + I_t B_t - T_t + (m_{t-1} - p_{t-1}c_{t-1})$$

$$2. \quad p_t c_t \leq M_t$$

$$3. \quad B_0 = B_{T+1} = 0.$$

Condition 3 says the consumer starts out with no debt or savings $B_0 = 0$ and will end up with no debt/savings $B_{T+1} = 0$. We will often consider a two- or three-period version of this problem.

The Government

The government does several things:

- (i) Spends g_t on applies and throws them away.
- (ii) Taxes consumers T_t dollars per period.
- (iii) Prints money so that total money at t is \bar{m}_t .
- (iv) Sells bonds to finance its deficit \bar{B}_{t+1} .

At time t the government needs revenues to pay: (i) $I_t \bar{B}_t$ on principal plus interest to bond holders, and (ii) $p_t g_t$ for current government purchases of goods.

At time t the government raises revenues by: (i) collecting T_t dollars in taxes from its consumers, and (ii) printing new money $\bar{m}_t - \bar{m}_{t-1}$ (the amount of new money). If money supply yesterday was \bar{m}_{t-1} , and today the total is \bar{m}_t , the change in the money supply is $\bar{m}_t - \bar{m}_{t-1}$.

Now if the government receipts $(T_t + \bar{m}_t - \bar{m}_{t-1})$ fall short of its current payments $(I_t \bar{B}_t + p_t g_t)$ then it can borrow from private consumers by printing and selling more government bonds \bar{B}_{t+1} (which it will pay off next period at $I_{t+1} \bar{B}_{t+1}$). Putting these together gives us the government budget constraint:

$$(1.7) \quad \bar{B}_{t+1} = I_t \bar{B}_t + p_t g_t - T_t - (\bar{M}_t - \bar{M}_{t-1})$$

(the amount the government has to borrow = current expenses - current reserves).

We also assume

$$(1.8) \quad \bar{B}_0 = \bar{B}_{T+1} = 0,$$

the government starts out and ends up with no debt/saving.

The government is also subject to a cash constraint: to buy g_t apples the government must have $p_t g_t$ dollars. Letting M_t^g denote the cash the government has, we have government cash-in-advance constraint (good market constraint):

$$(1.9) \quad p_t g_t = M_t^g.$$

We now can summarize government policy by $\{g_t, T_t, \bar{M}_t, \bar{B}_{t+1}; M_t^g\}_0^T$.

Equilibrium for the Monetary Sequence Economy

Let us define an equilibrium for this economy: Given government policy $\{g_t, T_t, \bar{M}_t, \bar{B}_{t+1}; M_t^g\}_0^T$ an equilibrium for this monetary sequence economy consists of: (i) an allocation of consumption $\{\hat{C}_t\}_0^T$ for private agents, (ii) a set of asset demands $\{\hat{M}_t, \bar{B}_{t+1}\}_0^T$, and (iii) a set of prices $\{\hat{I}_t, \hat{p}_t\}_0^T$ such that:

(1) Markets clear:

- (A) goods market $\hat{C}_t + g_t = y_t$
- (B) currency market $\hat{M}_t M_t^g = \bar{M}_t$
- (C) bond market $\hat{B}_t = \bar{B}_t$

(2) Consumer Maximization. $\{\hat{C}_t, \hat{M}_t, \hat{B}_{t+1}\}_0^T$ solves the consumers problem (1.6) at prices $\{\hat{I}_t, \hat{p}_t\}_0^T$ and given endowment and tax pattern $\{y_t, T_t\}$.

(3) Government Feasibility. The government meets its budget constraints at the equilibrium prices $\{\hat{I}_t, \hat{p}_t\}$, $\bar{B}_{t+1} = \hat{I}_t \bar{B}_t + \hat{p}_t g_t - T_t - (\bar{M}_t - \bar{M}_{t-1})$, and $\hat{p}_t g_t = M_t^G$.

HOMEWORK: Add the following problems to end of last problem set:

Government Budget Constraint

I. Solve out the government's sequence of budget constraints to give one present value type budget constraint. Proceed as follows: We have

$$(1.1) \quad \bar{B}_{t+1} = I_t \bar{B}_t + p_t g_t - T_t - (\bar{M}_t - \bar{M}_{t-1}).$$

To save space we will let

$$(1.2) \quad H_t \equiv p_t g_t - T_t - (\bar{M}_t - \bar{M}_{t-1}).$$

Substituting (1.2) into (1.1) we have

$$(1.3) \quad \bar{B}_{t+1} = I_t \bar{B}_t + H_t.$$

Now we want to start at $T + 1$ and use (1.3) to keep substituting out the \bar{B}_t 's.

Start at $T + 1$ $\bar{B}_{T+1} = I_T \bar{B}_T + H_T$. Substitute out \bar{B}_T using (1.3) for

$$t = T - 1 \quad (\bar{B}_T = I_{T-1} \bar{B}_{T-1} + H_{T-1}).$$

$$\begin{aligned} \bar{B}_{T+1} &= I_T (I_{T-1} \bar{B}_{T-1} + H_{T-1}) + H_T \\ &= I_T I_{T-1} \bar{B}_{T-1} + I_T H_{T-1} + H_T. \end{aligned}$$

Now substitute out \bar{B}_{T-1} using (1.3) for $t = T - 2$ ($\bar{B}_{T-1} = I_{T-2}\bar{B}_{T-2} + H_{T-2}$). Keep going until you have only \bar{B}_0 (and I's and H's) on the right hand side. Divide by $I_T \cdot \dots \cdot I_0$ on both sides. Substitute back for (1.2) for the H_t 's. Simplify and interpret.

II. Under what conditions will the consumers good market constraint $p_t c_t \leq M_t$? Bind with equality for every date. (Hint: the condition is an interest rates.) Argue intuitively why this is true for situations meeting your condition and explain what happens if it's not met.

III. Suppose the consumers constraint in the goods market binds with equality. Use the equilibrium conditions to find a formula for the price level (in terms of primitives).

Instead of trying to characterize the monetary sequence equilibrium defined on page II.11 directly, we shall try an indirect route. In particular, we will reduce the consumers problem to: (i) a date 0 problem with money (assuming $I_t > 1$ for all t), and (ii) a date 0 problem without money. Reduce (i) to (ii), and then we will characterize (ii).

First, let us turn the consumer's sequence of asset market constraints into one date 0 constraint. Rewrite (1.6) (i) as

$$(1.10) \quad B_{t+1} = I_t B_t + p_{t-1} y_{t-1} - M_t - T_t - (M_{t-1} - p_{t-1} c_{t-1}).$$

Using the standard substitutions (1.10) can be written as

$$(1.11) \quad \sum_{t=0}^T \left(\prod_{s=0}^t \frac{1}{I_s} \right) [p_{t-1} y_{t-1} - M_t - T_t - (M_{t-1} - p_{t-1} c_{t-1})] = 0.$$

Now assume $I_t > 1$ for each t , then $p_t c_t = M_t$ for each t . Substituting this into (1.11) gives ($M_t = p_t c_t$ and $P_{t-1} - p_{t-1} c_{t-1} = 0$)

$$(1.12) \quad \sum_{t=0}^T \left(\prod_{s=0}^t \frac{1}{I_s} \right) [p_{t-1} y_{t-1} - p_t c_t - T_t].$$

Now let $I_{0,t} = \prod_{s=0}^t I_s$ then we can rewrite (1.12) as

$$(1.13) \quad \sum_{t=0}^T \frac{1}{I_{0,t}} p_t c_t = \sum_{t=0}^T \frac{1}{I_{0,t}} [p_{t-1} y_{t-1} - T_t].$$

Breaking the right hand side of (1.13) into two pieces we have

$$(1.14) \quad \sum_{t=0}^T \frac{1}{I_{0,t}} p_t c_t = \sum_{t=0}^T \frac{1}{I_{0,t}} (p_{t-1} y_{t-1}) - \sum_{t=0}^T \frac{1}{I_{0,t}} (T_t).$$

Now from the homework problem we have the government's budget constraint

$$(1.15) \quad \sum_{t=0}^T \frac{1}{I_{0,t}} p_t g_t = \sum_{t=0}^T \frac{1}{I_{0,t}} T_t + \sum_{t=0}^T \frac{1}{I_{0,t}} [\bar{M}_t - \bar{M}_{t-1}].$$

Solving (1.15) for discounted taxes gives, after combining sums

$$(1.16) \quad \sum_{t=0}^T \frac{1}{I_{0,t}} T_t = \sum_{t=0}^T \frac{1}{I_{0,t}} [p_t g_t - (\bar{M}_t - \bar{M}_{t-1})].$$

Substituting (1.16) into (1.14) and combining sums gives

$$(1.17) \quad \sum_{t=0}^T \frac{1}{I_{0,t}} p_t c_t = \sum_{t=0}^T \frac{1}{I_{0,t}} [p_{t-1} y_{t-1} + (\bar{M}_t - \bar{M}_{t-1}) - p_t g_t].$$

Now from the homework problem III we have: $\bar{M}_t = p_t y_t$ and $\bar{M}_{t-1} = p_{t-1} y_{t-1}$.

Substituting this into (1.17) gives

$$(1.18) \quad \sum_{t=0}^T \frac{1}{I_{0,t}} p_t c_t = \sum_{t=0}^T \frac{1}{I_{0,t}} \left[\frac{p_{t-1}}{y_{t-1}} + (p_t y_t - \frac{p_{t-1}}{y_{t-1}}) - p_t g_t \right].$$

Collecting terms gives

$$(1.19) \quad \sum_{t=0}^T \frac{1}{I_{0,t}} p_t c_t = \sum_{t=0}^T \frac{1}{I_{0,t}} [p_t (y_t - g_t)].$$

Let $R_{0,t} = \frac{p_t}{p_0} I_{0,t}$ be the real interest rate, then (1.19) becomes

$$(1.20) \quad \sum_{t=0}^T \frac{1}{R_{0,t}} c_t = \sum_{t=0}^T \frac{1}{R_{0,t}} (y_t - g_t).$$

That is the consumer's problem (1.6) is equivalent to the following:

$$(2.1) \quad \max_{\{c_t\}} \sum_{t=0}^T \beta^t u(c_t)$$

such that

$$\sum_{t=0}^T \frac{1}{R_{0,t}} c_t = \sum_{t=0}^T \frac{1}{R_{0,t}} \left[\frac{T_t}{p_t} + \left(\frac{\bar{M}_t - \bar{M}_{t-1}}{p_t} \right) \right]$$

where $p_t = \frac{\bar{M}_t}{y_t}$.

Let us review the steps in a two-period model. The consumer's problem is

$$\max u(c_0) + \beta u(c_1)$$

such that

$$(2.3) \quad M_0 + B_1 = 0 + I_0 B_0 - T_0$$

$$(2.4) \quad M_1 + B_2 = p_D y_D + I_1 B_1 - T_1 + (M_0 - p_0 c_0)$$

and

$$(2.5) \quad p_0 c_0 = M_0$$

$$(2.6) \quad p_1 c_1 = M_1.$$

From (2.4) we have

$$(2.7) \quad B_2 = I_1 B_1 + [p_0 y_0 - M_1 - T_1 + (M_0 - p_0 c_0)].$$

Substitute (2.5) and (2.6) into (2.7)

$$(2.8) \quad B_2 = I_1 B_1 + [p_0 y_0 - p_1 c_1 - T_1].$$

Now solving (2.3) for B_1 gives

$$(2.9) \quad B_1 = I_0 B_0 + [-M_0 - T_0].$$

Substituting (2.5) into (2.9) gives

$$(2.10) \quad B_1 = I_0 B_0 + [-p_0 c_0 - T_0].$$

Substituting (2.10) into (2.8) gives

$$(2.11) \quad B_2 = I_1 I_0 B_0 + I_1 [-p_0 c_0 - T_0] + [p_0 y_0 - p_1 c_1 - T_1].$$

Imposing $B_2 = B_0 = 0$ and dividing by $I_0 I_1$ gives

$$(2.12) \quad 0 = -\left[\frac{p_0 c_0 - T_0}{I_0}\right] + \left[\frac{p_0 y_0 - p_1 c_1 - T_1}{I_0 I_1}\right].$$

Rearranging terms and let $I_0 = 1$, $I_{0,1} = I_1$

$$(2.13) \quad p_0 c_0 + \frac{p_1 c_1}{I_{0,1}} = \frac{p_0 y_0}{I_{0,1}} - \left(T_0 + \frac{T_1}{I_{0,1}}\right).$$

Now the government's budget constraint can be written

$$(2.14) \quad p_0 g_0 + \frac{p_1 g_1}{I_{0,1}} = T_0 + \frac{T_1}{I_{0,1}} + \bar{M}_0 + \left(\frac{\bar{M}_1 - \bar{M}_0}{I_{0,1}}\right)$$

(we have put $M - 1 = 0$). Solving (2.14) for discounted taxes gives

$$(2.15) \quad T_0 + \frac{T_1}{I_{0,1}} = p_0 g_0 + \frac{p_1 g_1}{I_{0,1}} - \bar{M}_0 - \left(\frac{\bar{M}_1 - \bar{M}_0}{I_{0,1}}\right).$$

Substituting (2.15) into (2.13) gives

$$(2.16) \quad p_0 c_0 + \frac{p_1 c_1}{I_{0,1}} = \frac{p_0 y_0}{I_{0,1}} + \bar{M}_0 + \left(\frac{\bar{M}_1 - \bar{M}_0}{I_{0,1}}\right) - p_0 g_0 - \frac{p_1 g_1}{I_{0,1}}.$$

Now from the problem set we have $\bar{M}_0 = p_0 y_0$ and $\bar{M}_1 = p_1 y_1$. Substituting these into (2.16) gives

$$(2.17) \quad p_0 c_0 + \frac{p_1 c_1}{I_{0,1}} = \frac{p_0 y_0}{I_{0,1}} + p_0 y_0 + \left(\frac{p_1 y_1 - p_0 y_0}{I_{0,1}} \right) - p_0 g_0 - \frac{p_1 g_1}{I_{0,1}}.$$

Now canceling terms gives

$$(2.18) \quad p_0 c_0 + \frac{p_1 c_1}{I_{0,1}} = p_0 y_0 + \frac{p_1 y_1}{I_{0,1}} - p_0 g_0 - \frac{p_1 g_1}{I_{0,1}}.$$

Now dividing (2.18) by p_0 and letting $R_{0,1} = \frac{p_1}{p_0} I_{0,1}$ we have finally

$$(2.19) \quad c_0 + \frac{c_1}{R_{0,1}} = y_0 + \frac{y_1}{I_{0,1}} - g_0 - \frac{g_1}{R_{0,1}}.$$

Thus, the consumer can solve

$$(2.20) \quad \max_{\{c_0, c_1\}} u(c_0) + \beta u(c_1)$$

such that

$$c_0 + \frac{c_1}{R_{0,1}} = y_0 + \frac{y_1}{R_{0,1}} - g_0 - \frac{g_1}{R_{0,1}}.$$

Also, we can rewrite the government budget constraint as

$$(2.21) \quad g_0 + \frac{g_1}{R_{0,1}} = \frac{T_0}{p_0} + \frac{1}{R_{0,1}} \frac{T_1}{p_1} + \frac{M_0}{p_0} + \frac{1}{R_{0,1}} \left(\frac{M_1 - M_0}{p_1} \right).$$

Letting $\tau_t \equiv T_t/p_t$ we have

$$(2.22) \quad g_0 + \frac{g_1}{R_{0,1}} = \tau_0 + \frac{\tau_1}{R_{0,1}} + \frac{M_0}{p_0} + \frac{1}{R_{0,1}} \left[\frac{M_1 - M_0}{p_0} \right].$$

Consider the impact of budget deficits on this economy. Now go back to I,29 for a minute. Under equation (4.6), we considered the three ways to WORD a budget deficit at date 0 (replace 1 with 0). They are:

- (1) cut taxes at 0 ($\tau_0 \downarrow$)
- (2) raise spending at 0 ($g_0 \uparrow$)
- (3) cut money supply at 0 ($\bar{M}_0 \downarrow$).

Now given that the government runs a deficit the first period, it must run a surplus the second period (if it obeys its budget constraint). To run a surplus the second period (1) the government can:

- (1) raise taxes ($\tau_1 \uparrow$)
- (2) cut spending ($g_1 \downarrow$)
- (3) raise the money supply ($M_1 \uparrow$).

Now the changes involving spending and/or taxes are the same as before, we will concentrate on changing the money supply.

Let $\{g_0, g_1, \tau_0, \tau_1, \bar{M}_0, \bar{M}_1\}$ be the original policy. (Notice we have used real taxes $\{\tau_0, \tau_1\}$ vs. nominal taxes $\{T_0, T_1\}$.)

Now suppose the government cuts the money supply by Δ_0 in the period 0 and raises it by just enough in period 1 to keep its budget balanced, say, by Δ_1 .

Thus, if we let $\{g'_0, g'_1, \tau'_0, \tau'_1, M'_0, M'_1\}$ be new policy, then

$$g'_0 = g_0, g'_1 = g_1, \tau'_0 = \tau_0, \tau'_1 = \tau_1$$

$$M'_0 = M_0 - \Delta_0$$

$$M'_1 = M_1 - \Delta_1.$$

Now what happens to consumption, prices, and interest rates? Well, the consumer's budget constraint in (2.20) is unaffected by this change and the good market equilibrium conditions:

$$(2.22) \quad \text{date 0: } c_0 = y_0 - g_0$$

$$\text{date 1: } c_1 = y_1 - g_1$$

are also unaffected; therefore, the consumption is unaffected. Now we know from before that the real interest rate is given by

$$(2.23) \quad \frac{1}{R_{0,1}} = \beta \frac{U^1(c_1)}{U^1(c_0)}$$

Substituting the goods market equilibrium conditions into (2.23) gives

$$(2.24) \quad \frac{1}{R_{0,1}} = \beta \frac{U^1(y_1 - g_1)}{U^1(y_0 - g_0)}$$

(real interest rate is unaffected). So if the consumption of goods is unchanged, the real interest rate is unchanged.

Now the price levels in the two cases are

$$(2.25) \quad p_0 = \frac{\bar{M}_0}{y_0} > p'_0 = \frac{(\bar{M}_0 - \Delta_0)}{y_0}$$
$$p_1 = \frac{\bar{M}_1}{y_1} < p'_1 = \frac{\bar{M}_1 + \Delta_1}{y_1}$$

The price level falls in the period 0 (since there is a lower money supply) and rises in period 1 (since there is a higher money supply). Now the nominal interest rate is given by

$$(2.26) \quad I_{0,1} = \frac{p_1}{p_0} R_{0,1} < I'_{0,1} = \frac{p'_1}{p'_0} R_{0,1}$$

In summary, a cut in the money supply at 0, which is balanced (in terms of the government budget) by an increase in money supply at 1,

- does not affect consumption,
- does not affect real interest rates,
- lowers prices at 0, raises prices at 1, and
- raises the nominal interest rate.

So far, we have not specified how Δ_1 must relate to Δ_0 in order that the government budget constraint is met:

$$(2.27) \quad (a) \quad g_0 + \frac{g_1}{R_{0,1}} = \tau_0 + \frac{\tau_1}{R_{0,1}} + \frac{\bar{M}_0}{p_0} + \frac{1}{R_{0,1}} \left(\frac{\bar{M}_1}{p_1} - \bar{M}_0 \right)$$

$$(b) \quad g_0 + \frac{g_1}{R_{0,1}} = \tau_0 + \frac{\tau_1}{R_{0,1}} + \frac{\bar{M}_0 - \Delta_0}{p'_0} + \frac{1}{R_{0,1}} \left[\left(\frac{\bar{M}_1}{p'_1} + \Delta_1 \right) - (\bar{M}_0 - \Delta_0) \right].$$

Equations (2.27) (a)--old policy--and (b)--new policy--money supplies must satisfy

$$(2.28) \quad \frac{\bar{M}_0}{p_0} + \frac{1}{R_{0,1}} \left(\frac{\bar{M}_1}{p_1} - \bar{M}_0 \right) = \frac{\bar{M}_0 - \Delta_0}{p'_0} + \frac{1}{R_{0,1}} \left[\frac{(\bar{M}_1 + \Delta_1) - (\bar{M}_0 - \Delta_0)}{p'_1} \right].$$

Let us substitute out the prices using (2.25)

$$(2.29) \quad y_0 + \frac{1}{R_{0,1}} \left(\frac{\bar{M}_1 - \bar{M}_0}{\bar{M}_1} \right) y_1 = y_0 + \frac{\Delta_1}{R_{0,1}} \left[\frac{(\bar{M}_1 + \Delta_1) - (\bar{M}_0 - \Delta_0)}{\bar{M}_1 + \Delta_1} \right] y_1.$$

Now subtract y_0 from both sides and multiply by $R_{0,1}/y_1$ to obtain

$$(2.30) \quad \frac{\bar{M}_1 - \bar{M}_0}{\bar{M}_1} = \left[\frac{(\bar{M}_1 + \Delta_1) - (\bar{M}_0 - \Delta_0)}{\bar{M}_1 + \Delta_1} \right].$$

Solving (2.30) for Δ_0 gives

$$(2.31) \quad \Delta_0 = \left(\frac{\bar{M}_1 + \Delta_1}{\bar{M}_1} \right) (\bar{M}_1 - \bar{M}_0) - (\bar{M}_1 - \bar{M}_0) - \Delta_1.$$

Consider a two-period version of the two-country CIA model. Let the endowments of home consumer be $\{p_{H0}, y_{H0}\}$ and the endowments of foreign consumers be $\{p_{F0}, y_{F0}\}$. Notice that home consumers' endowments are in units of home currency and foreign consumers' endowments are in units of foreign currency.

The home consumers maximization problem is to choose $\{c_{Ht}, c_{Ft}, M_{Ht}, M_{Ft}, B_{Ht+1}, B_{Ft+1} | t=0, 1\}$ to solve

$$(2.1) \quad \max U(c_{H0} + c_{F0}) + \beta U(c_{H1} + c_{F1})$$

such that

$$(1) \quad M_{H0} + e_0 M_{F0} + B_{H1} + e_0 B_{F1} = 0 + I_{H0} B_{H0} + e_0 I_{F0} B_{F0} - T_0$$

$$(2) \quad M_{H1} + e_1 M_{F1} + B_{H2} + e_1 B_{F2} = p_{H0} y_{H0} + I_{H1} B_{H1} + e_1 I_{F1} B_{F1} - T_1$$

$$(3) \quad p_{H0} c_{H0} = M_{H0}, \quad p_{F0} c_{F0} = M_{F0}$$

$$(4) \quad p_{H1} c_{H1} = M_{H1}, \quad p_{F1} c_{F1} = M_{F1}$$

$$(5) \quad B_{H0} = B_{F0} = B_{H2} = B_{F2} = 0$$

where (1) equals asset market constraint in period 0.

(2) equals asset market constraint in period 1.

(3) equals cash-in-advance constraints in period 0.

(4) equals cash-in-advance constraints in period 1.

(5) equals initial and terminal conditions on debt.

The foreign consumers' problem is identical to the home consumers' except that there are asterisks on all quantities. Foreign consumers choose

$\{c_{Ht}^*, c_{Ft}^*, M_{Ht}^*, M_{Ft}^*, B_{Ht+1}^*, B_{Ft+1}^* | t=0, 1\}$ to solve

$$(2.2) \quad \max U(c_{H0}^* + c_{F0}^*) + \beta U(c_{H1}^* + c_{F1}^*)$$

such that

$$(1) \quad M_{H0}^* + e_0 M_{F0}^* + B_{H1}^* + e_0 B_{F1}^* = 0 + I_{H0} B_{H0}^* + e_0 I_{F0} B_{F0}^* - T_0^*$$

$$(2) \quad M_{H1}^* + e_1 M_{F1}^* + B_{H2}^* + e_1 B_{F2}^* = p_{F0} y_{F0}^* + I_{H1} B_{H1}^* + e_1 I_{F1} B_{F1}^* - T_1^*$$

$$(3) \quad p_{H0} c_{H0}^* = M_{H0}^*, \quad p_{F0} c_{F0}^* = M_{F0}^*$$

$$(4) \quad p_{H1} c_{H1}^* = M_{H1}^*, \quad p_{F1} c_{F1}^* = M_{F1}^*.$$

The home government's policy can be described by

$$(2.3) \quad \{g_{Ht}, \bar{M}_{Ht}, \bar{B}_{Ht+1}, T_t; M_{Ht}^g : t=0, 1\}.$$

The home government's policy is assumed to satisfy

$$(2.4) \quad (a) \quad \bar{B}_{H1} = I_{H0} \bar{B}_{H0} + p_{H0} g_{H0} - T_0 - (\bar{M}_{H0})$$

$$(b) \quad \bar{B}_{H2} = I_{H1} \bar{B}_{H1} + p_{H1} g_{H1} - T_1 - (\bar{M}_{H1} - \bar{M}_{H0})$$

(period budget constraints) along with the initial and terminal conditions

$$(c) \quad \bar{B}_{H0} = \bar{B}_{H2} = 0.$$

The home government is also subject to cash-in-advance constraints of the form:

$$(2.5) \quad p_{H0} g_{H0} = M_{H0}^g$$

$$p_{H1} g_{H1} = M_{H1}^g.$$

Likewise for the foreign government's policy can be described by $\{g_{Ft}^*, \bar{M}_{Ft}, \bar{B}_{Ft+1}, T_t^*; M_{Ft}^{g*}; t=0, 1\}$. The foreign government's policy is assumed to satisfy period budget constraint:

$$\bar{B}_{F1} = I_{F0} \bar{B}_{F0} + p_{F0} g_{F0} - T_0^* - \bar{M}_{F0}$$

$$\bar{B}_{F2} = I_{F1} \bar{B}_{F1} + p_{F1} g_{F1} - T_1^* - (\bar{M}_{F1} - \bar{M}_{F0}).$$

The foreign government's cash-in-advance constraints are of the form

$$p_{F0}g_{F0} = M_{F0}^g$$

$$p_{F1}g_{F1} = M_{F1}^g.$$

We are ready to define a monetary sequence equilibrium for this economy. Converting the consumers' sequence budget constraints into date 0 nominal constraints by solving (2.1) (1) for $B_{H1} + e_0 B_{F1}$ and substituting it into (2.1) (2), then imposing the conditions in (2.1) (3) and (4) in order to substitute out the money demands in terms of consumption. Impose conditions in (2.1) (5) and divide by $I_{H0}I_{H1}$. Use the notation $I_{H0} = 1$, $I_{0,1}^H = I_{H0}I_{H1}$.

Convert the consumers' sequence budget constraints into date 0 real constraints by using the Fisher relation between nominal and real interest rates $R_{0,1} = p_{H1}/p_{H0}(I_{0,1}^H)$. Use the same definition of taxes $\{\tau_t, \tau_t^M; t=0,1\}$ as in (A).

A monetary sequence equilibrium for above economy given government policies of home government $\{g_{Ht}, \bar{M}_{Ht}, \bar{B}_{Ht+1}, T_t; M_{Ht}^g | t=0,1\}$ and of foreign government $\{g_{Ft}^*, \bar{M}_{Ft}, \bar{B}_{Ft+1}, T_t^*; M_{Ft}^{g*} | t=0,1\}$ is: (i) a set of allocations $\{c_{Ht}, c_{Ft}; c_{Ht}^*, c_{Ft}^* | t=0,1\}$, (ii) a set of asset demands $\{M_{Ht}, M_{Ft}, B_{Ht+1}, B_{Ft+1}; M_{Ht}^*, M_{Ft}^*, B_{Ht+1}^*, B_{Ft+1}^*\}$, and (iii) a price system $\{I_{Ht}, I_{Ft}, p_{Ht}, p_{Ft}, e_t\}$ such that:

A. Markets clear

(i) goods market

$$\text{(home)} \quad c_{Ht} + c_{Ht}^* + g_{Ht} = y_{Ht} \quad t = 0, 1$$

$$\text{(foreign)} \quad c_{Ft} + c_{Ft}^* + g_{Ft} = y_{Ft}^* \quad t = 0, 1$$

(ii) money markets

$$\text{(home)} \quad M_{Ht} + M_{Ht}^* + M_{Ht}^G = \bar{M}_{Ht}$$

$$\text{(foreign)} \quad M_{Ft} + M_{Ft}^* + M_{Ft}^G = \bar{M}_{Ft}$$

(iii) bond markets

$$\text{(home)} \quad B_{Ht} + B_{Ht}^* = \bar{B}_{Ht}$$

$$\text{(foreign)} \quad B_{Ft} + B_{Ft}^* = \bar{B}_{Ft}$$

B. Consumers maximization

$\{c_{Ht}, c_{Ft}, M_{Ht}, M_{Ft}, B_{Ht+1}, B_{Ft+1} : t=0, 1\}$ solves (2.1) at equilibrium prices, and $\{c_{Ht}^*, c_{Ft}^*, M_{Ht}^*, B_{Ht+1}^*, B_{Ft+1}^* : t=0, 1\}$ solves (2.2) at equilibrium prices.

C. Government feasibility

$\{g_{Ht}, \bar{M}_{Ht}, \bar{B}_{Ht+1}, T_t; M_{Ht}^G\}$ satisfies (2.3) and (2.4)
 $\{g_{Ft}^*, \bar{M}_{Ft}, \bar{B}_{Ft+1}, T_t^*; M_{Ft}^G\}$ satisfies analogue of (2.3) and (2.4).

Now instead of trying to characterize this monetary sequence equilibrium directly, we will convert it to a simpler type of equilibrium. In fact, we can convert this equilibrium to a real date 0 economy. We proceed in several steps.

Converting the government's sequence budget constraints to date 0 nominal constraints by simply substituting (2.4) (b) into (2.4) (a) and imposing (2.4) (c).

Converting the government's sequence budget constraint to date 0 real constraints by using the Fisher relation between real and nominal interest rates. Then converting the commodity taxes T_0, T , to real taxes $t_0 = T_0/p_{H0}, t_1 = T_1/p_{H1}$. And then converting the revenues from money creation $\bar{M}_{H0}, \bar{M}_{H1} - \bar{M}_{H0}$ into "inflation taxes"

$$\tau_0^M = \frac{\bar{M}_{HO}}{P_{HO}}, \quad \tau_1^M = \frac{\bar{M}_{H1} - \bar{M}_{HO}}{P_{H1}}$$

and substituting these into step (i).

Converting the Monetary Sequence Economy to a Real Date 0 Economy

Convert the government's budget constraint to a date 0 constraint by rewriting the sequence budget constraints in (2.3) as follows

$$(2.6) \quad \frac{P_{HO}g_{HO}}{I_{HO}} + \frac{P_{H1}g_{H1}}{I_{HO}I_{H1}} = \frac{T_0}{I_{HO}} + \frac{T_1}{I_{HO}I_{H1}} + \frac{\bar{M}_{HO}}{I_{HO}} + \frac{(\bar{M}_{H1} - \bar{M}_{HO})}{I_{HO}I_{H1}}.$$

Put $I_{HO} = 1$ and $I_{O,1}^H = I_{HO}I_{H1}$

$$(2.7) \quad P_{HO}g_{HO} + \frac{P_{H1}g_{H1}}{I_{O,1}^H} = T_0 + \frac{T_1}{I_{O,1}^H} + \bar{M}_{HO} + \frac{(\bar{M}_{H1} - \bar{M}_{HO})}{I_{O,1}^H}.$$

Let $R_{O,1} \equiv P_{H1}/P_{HO} (I_{O,1}^H)$. Multiply (2.7) by $1/P_{HO}$ and use the definition of $R_{O,1}$ to obtain

$$(2.8) \quad g_{HO} + \frac{g_{H1}}{R_{O,1}} = \frac{T_0}{P_{HO}} + \left(\frac{T_1}{P_{H1}}\right) \frac{1}{R_{O,1}} + \frac{\bar{M}_{HO}}{P_{HO}} + \left(\frac{\bar{M}_{H1} - \bar{M}_{HO}}{P_{H1}}\right) \frac{1}{R_{O,1}}.$$

Now let $\tau_t \equiv \frac{T_t}{P_{Ht}}$ (real value of commodity taxes on home consumption at t) and let

$$\tau_0^M = \frac{\bar{M}_{HO}}{P_{HO}}$$

and

$$\tau_1^M = \frac{\bar{M}_{H1} - \bar{M}_{HO}}{P_{H1}}.$$

Then (2.8) can be written

$$(2.9) \quad g_{HO} + \frac{g_{H1}}{R_{O,1}} = \tau_0 + \frac{\tau_1}{R_{O,1}} + \tau_0^M + \frac{\tau_1^M}{R_{O,1}}.$$

Similarly, the budget constraint of foreign government can be written

$$(2.10) \quad g_{F1} + \frac{g_{F1}}{R_{0,1}} = \tau_0^* + \frac{\tau_1^*}{R_{0,1}} + \tau_0^M + \frac{\tau_1^{M*}}{R_{0,1}}$$

where

$$\tau_t^* \equiv \frac{T_t^*}{p_{Ft}}$$

$$\tau_0^{M*} = \frac{\bar{M}_{F0}}{p_{F0}} = \tau_1^{M*} = \frac{(\bar{M}_{F1} - \bar{M}_{F0})}{p_{F1}}$$

Using a similar method to handout II, we can write the consumers' budget constraints as home consumer

$$(2.11) \quad (c_{H0} + c_{F0}) + \frac{(c_{H1} + c_{F1})}{R_{0,1}} = y_{H0} + \frac{y_{H1}}{R_{0,1}} - \left(\tau_0 + \frac{\tau_1}{R_{0,1}} \right) - \left(\tau_0^M + \frac{\tau_1^M}{R_{0,1}} \right)$$

and foreign consumer

$$(2.12) \quad (c_{H0}^* + c_{F0}^*) + \frac{(c_{H1}^* + c_{F1}^*)}{R_{0,1}} = y_{F0} + \frac{y_{F1}}{R_{0,1}} - \left(\tau_0^* + \frac{\tau_1^*}{R_{0,1}} \right) - \left(\tau_0^{M*} + \frac{\tau_1^{M*}}{R_{0,1}} \right).$$

Notice we can express the inflation taxes τ_t^M , τ_t^{M*} in terms of primitives as follows:

$$(2.13) \quad \tau_0^M = \frac{\bar{M}_{H0}}{p_{H0}} = \frac{\bar{M}_{H0}}{\bar{M}_{H0}} y_{H0} = y_{H0}$$

$$\tau_1^M = \frac{\bar{M}_{H1} - \bar{M}_{H0}}{p_{H1}} = \frac{\bar{M}_{H1} - \bar{M}_{H0}}{\bar{M}_{H1}} y_{H1} = \left[1 - \frac{\bar{M}_{H0}}{\bar{M}_{H1}} \right] y_{H1}.$$

Similarly

$$(2.14) \quad \tau_1^{M*} = y_{F0}^*$$

$$\tau_2^{M*} = \left[1 - \frac{\bar{M}_{F0}}{\bar{M}_{F1}} \right] y_{F1}^*.$$

A real date 0 equilibrium associated with the above monetary equilibrium given government policies $\{g_{Ht}, \tau_t, \tau_t^M\}$ $\{g_{Ft}^*, \tau_t^*, \tau_t^{M*}\}$ is: (i) a set of allocations $\{c_t, c_t^*\}$, and (ii) and a price system $\{R_{0,1}\}$ such that:

A. Markets clear $c_t + c_t^* = y_{Ht} + y_{Ft}^*$

B. Consumer maximization

$$\{c_t\} \text{ solves } \max_{\{c_0, c_1\}} U(c_0) + \beta U(c_1) \text{ such that}$$

$$c_0 + \frac{c_1}{R_{0,1}} = y_{H0} + \frac{y_{H1}}{R_{0,1}} - \left(\tau_0 + \frac{\tau_1}{R_{0,1}}\right) - \left(\tau_0^M + \frac{\tau_1^M}{R_{0,1}}\right)$$

and $\{c_t^*\}$ solves $\max_{\{c_0^*, c_1^*\}} U(c_0^*) + \beta U(c_1^*)$ such that

$$c_0^* + \frac{c_1^*}{R_{0,1}} = y_{F0}^* + \frac{y_{F1}^*}{R_{0,1}} - \left(\tau_0^* + \frac{\tau_1^*}{R_{0,1}}\right) - \left(\tau_0^{M*} + \frac{\tau_1^{M*}}{R_{0,1}}\right).$$

C. Government feasibility

The home government is

$$g_{H0} + \frac{g_{H1}}{R_{0,1}} = \left(\tau_0 + \frac{\tau_1}{R_{0,1}}\right) + \left(\tau_0^M + \frac{\tau_1^M}{R_{0,1}}\right)$$

and the foreign government is

$$g_{F0}^* + \frac{g_{F1}^*}{R_{0,1}} = \left(\tau_0^* + \frac{\tau_1^*}{R_{0,1}}\right) + \left(\tau_0^{M*} + \frac{\tau_1^{M*}}{R_{0,1}}\right).$$

Consider a monetary sequence economy that originally has government policies. The home government is

$$(3.1) \quad \{g_{Ht}, \bar{M}_{Ht}, \bar{B}_{Ht+1}, T_t; M_{Ht}^g; t=0, 1\}$$

and the foreign government is

$$(3.2) \quad \{g_{Ft}^*, \bar{M}_{Ft}, \bar{B}_{Ft+1}, T_t^*; M_{Ft}^{g*}; t=0, 1\}.$$

Suppose for these sets of policies there is an equilibrium with:

(i) an allocation $\{c_{Ht}, c_{Ft}; c_{Ht}^*, c_{Ft}^*; t=0, 1\}$, and (ii) a price system $\{I_{0,1}^H, I_{0,1}^F; p_{Ht}, p_{Ft}, e_t; R_{0,1} | t=0, 1\}$. Now consider a monetary sequence economy identical to the above except the home government pursues a new set of policies, given by

$$(3.1)' \quad \{g_{Ht}', \bar{M}_{Ht}', \bar{B}_{Ht+1}', T_t'; M_t^G'; t=0, 1\}.$$

Suppose for policies (3.1)' and (3.2) there is an equilibrium with: (i) an allocation $\{c_{Ht}', c_{Ft}'; c_{Ht}'^*, c_{Ft}'^*; t=0, 1\}$, and (ii) a price system $\{I_{0,1}^H, I_{0,1}^F; p_{Ht}', p_{Ft}', c_t'; R_{0,1}' : t=0, 1\}$.

Now consider new policies of home government that lead to higher government deficits at home at date 0 (i.e., higher under new policies than under old). These deficits can arise for one of three reasons:

- (A1) a cut in commodity taxes in current period (0).
- (A2) a cut in current inflation taxes.
- (A3) a rise in current government spending.

If the government obeys its budget constraint these deficits must eventually be paid for by one of three methods:

- (B1) a rise in future commodity taxes.
- (B2) a rise in future inflation taxes.
- (B3) a decrease in future government spending.

Let

$$U(c_{Ht} + c_{Ft}) = \ln (c_{Ht} + c_{Ft}) \quad t = 0, 1$$

$$U^*(c_{Ht}^* + c_{Ft}^*) = \ln (c_{Ht}^* + c_{Ft}^*) \quad t = 0, 1.$$

Compute the equilibrium allocations and prices under original policies. Then compute the equilibrium allocations and prices under new policies for home government. (Assume the foreign country's government budget constraint is balanced in each period.) Characterize consumption levels and the price system given and: (i) (A1) and (B1), (ii) (A2) and (B2), and (iii) (A3) and (B3).

Real Exchange Rates and Purchasing Power Parity

In order to discuss real exchange rates we first must define two types of goods: nontraded goods and traded goods.

Nontraded Goods

A nontraded good of country i is some good which is not traded on the world market, that is, supplied and demanded only by country i residents. Often goods from service sectors are nontraded. (For example, haircuts in country i , dinners in country i , etc.) Empirically, the single most important nontraded good is housing services.

In the context of a simple exchange economy we will define a nontraded good in country (i) as any good such that:

- (N1) Only country i residents receive utility from consumption of the good.
- (N2) Only country i residents have a positive endowment of the good.
- (N3) Only the government of country i spends and taxes in terms of this good.

Traded Goods

A traded good in country i is any good that violates at least one of the above conditions N1, N2, N3.

Consider a simple two country exchange economy. The economy lasts for two periods ($t = 0, 1$) and there is one traded good and one nontraded good for each country. We index countries by $i = H, F$ and we index goods by subscript (T , traded and N , nontraded).

For concreteness, think of the home country as the U.S. and the foreign country as Germany. Think of the traded good as apples, the home nontraded good as haircuts in the U.S. and the foreign nontraded good as haircuts in Germany.

Home consumers have endowments $\{y_{T,t}, y_{N,t} | t=0,1\}$. Foreign consumers have endowments $\{y_{T,t}^*, y_{N,t}^* | t=0,1\}$.

Home consumers pay lump sum taxes $\{\tau_{T,t}, \tau_{N,t} | t=0,1\}$. Foreign consumers pay lump sum taxes $\{\tau_{T,t}^*, \tau_{N,t}^* | t=0,1\}$.

Home consumers consume $\{c_{T,t}, c_{N,t} | t=0,1\}$ and have preferences over this stream of consumption given by

$$(1.1) \quad U = [\alpha_T \ln c_{T,0} + \alpha_N \ln c_{N,0}] + \beta [\alpha_T \ln c_{T,1} + \alpha_N \ln c_{N,1}].$$

Foreign consumers consume $\{c_{T,t}^*, c_{N,t}^* | t=0,1\}$ and have preferences over this stream given by

$$(1.2) \quad U^* = [\alpha_T^* \ln c_{T,0}^* + \alpha_N^* \ln c_{N,0}^*] + \beta [\alpha_T^* \ln c_{T,1}^* + \alpha_N^* \ln c_{N,1}^*].$$

Before we define the budget constraint we need to define the price system. In a model with two or more goods per period there is no unique definition of "the" real interest rate.

In this model there are three goods per period (traded, home non-traded, and foreign nontraded). Possible real interest rates are:

- (i) the own rate of interest on the traded good.
- (ii) the own rate of interest on the home nontraded good.
- (iii) the own rate of interest on the foreign nontraded good.
- (iv) any index of (i), (ii), and (iii).

The "real" interest rate we compute by using the nominal interest rate minus the inflation rate is an index of type (iv). For this model, however, it is more convenient to use (i). Let $R_{0,1}$ equal the own rate of interest on the traded good. That is, one unit of the traded good at time 0 buys $R_{0,1}$ units of the traded good at time 1.

Let $\{q_{N,t} : t=0,1\}$ be the relative price between home nontraded goods at t and traded goods at t . That is, one unit of the home nontraded good at t buys $q_{N,t}$ units of the traded good at t .

Let $\{q_{N,t}^* | t=0,1\}$ be the relative prices between foreign nontraded goods at t and traded goods at t . One unit of the foreign nontraded good at t buys $q_{N,t}^*$ units of the traded good at t .

With this investment in notation we can write the home consumers' budget constraints (in present value form) as follows:

$$(1.3) \quad [c_{T,0} + q_{N0} c_{N0}] + \left[\frac{c_{T1} + q_{N1} c_{N1}}{R_{0,1}} \right] \leq [y_{T0} + q_{N0} y_{N0} - \tau_{T0} - q_{N0} \tau_{N0}] + \left[\frac{y_{T1}^* + q_{N1}^* y_{N1}^* - \tau_{T1}^* - q_{N1}^* \tau_{N1}^*}{R_{0,1}} \right].$$

Consumers Maximization Problem

Home consumers choose $\{c_{Ht}, c_{Nt} | t=0,1\}$ to maximize (1.1) subject to (1.3). Foreign consumers choose $\{c_{Ht}^*, c_{Nt}^* | t=0,1\}$ to maximize (1.2) subject to (1.4).

The home government's policy is assumed to satisfy the following present value constraint:

$$(1.6) \quad [g_{T0} + q_{N0} g_{N0}] + \frac{1}{R_{0,1}} [g_{T1} + q_{N1} g_{N1}] = [\tau_{T0} + q_{N0} \tau_{N0}] + \frac{1}{R_{0,1}} [\tau_{T1}^* + q_{N1}^* \tau_{N1}^*].$$

A date 0, equilibrium for this economy given government policies (1.5) and (1.7) is: (i) an allocation $\{c_{Tt}, c_{Nt}; c_{Tt}^*, c_{Nt}^* | t=0,1\}$, and (ii) a price system $\{R_{0,1}, q_{Nt}, q_{Nt}^* | t=0,1\}$ that satisfies A, B, and C below.

A. Market clear

- (a) traded goods $c_{Tt} + c_{Tt}^* + g_{Tt} + g_{Tt}^* = y_{Tt} + y_{Tt}^*$, $t = 0, 1$.
- (b) home nontraded goods $c_{Nt} = g_{Nt} = y_{Nt}$, $t = 0, 1$.
- (c) foreign nontraded goods $c_{Nt}^* + g_{Nt}^* = y_{Nt}^*$.

B. Consumer maximization

- (a) $\{c_{Tt}, c_{Nt} | t=0,1\}$ solves (1.1) subject to (1.3) at prices $\{R_{0,1}, q_{Nt}, q_{Nt}^* | t=0,1\}$.
- (b) $\{c_{Tt}^*, c_{Nt}^* | t=0,1\}$ solves (1.2) subject to (1.4) at prices $\{R_{0,1}, q_{Nt}, q_{Nt}^* | t=0,1\}$.

C. Government feasibility

- (a) The home government's policy (1.5) satisfies its budget constraint (1.6) at the equilibrium prices (ii).
- (b) The foreign government's policy (1.7) satisfies its budget constraint (1.8) at the equilibrium prices (ii).

Notice the difference between the traded goods equilibrium condition (1.9a) and the nontraded goods equilibrium conditions (1.9) (a) and (c).

The most straightforward way to compute this equilibrium is to solve for the demand functions of consumers. Then plug these demand functions into the market clearing conditions and solve for prices. Next plug these equilibrium prices back into the demand functions and solve for the equilibrium allocations.

We will use a similar technique, however, we will take account of the fact that having nontraded goods makes the problem simpler. From the equilibrium conditions for nontraded goods we can immediately derive the consumption of nontraded goods:

$$(2.0) \quad (a) \quad c_{Nt} = y_{Nt} - g_{Nt} \quad t = 0, 1$$

$$(b) \quad c_{Nt}^* = y_{Nt}^* - g_{Nt}^* \quad t = 0, 1.$$

Substituting this result into the consumers problem gives home consumer:

$$(2.1) \quad \max_{\{c_{Tt} | t=0,1\}} [\alpha_T \ln c_{T0} + \alpha_N \ln(y_{N0} - g_{N0})] + \beta [\alpha_T \ln c_{T1} + \alpha_N \ln(y_{N1} - g_{N1})]$$

such that

$$[c_{T0} + q_{N0}(y_{N0} - g_{N0})] + \frac{1}{R_{0,1}} [c_{T1} + q_{N1}(y_{N1} - g_{N1})] \leq \frac{1}{R_{0,1}} [y_{T1} + q_{N1}y_{N1} - \tau_{T1} - q_{N1}\tau_{N1}].$$

Let

$$\tilde{y}_0: y_{T0} - \tau_{T0} + q_{N0}(g_{N0} - \tau_{N0})$$

$$\tilde{y}_1: y_{T1} - \tau_{T1} + q_{N1}(g_{N1} - \tau_{N1}).$$

Then we can rewrite (2.2) as

$$(2.3) \quad \max_{\{c_{T0}, c_{T1}\}} \ln c_{T0} + \beta \ln c_{T1}$$

such that

$$c_{T0} + \frac{c_{T1}}{R_{0,1}} = \tilde{y}_0 + \frac{\tilde{y}_1}{R_{0,1}}.$$

Now this is identical in form to our previous one-good model, so the demand functions are given by:

$$(2.4) \quad (a) \quad c_{T0} = \frac{1}{1 + \beta} \left[\tilde{y}_0 + \frac{\tilde{y}_1}{R_{0,1}} \right]$$

$$(b) \quad c_{T1} = \frac{BR_{0,1}}{1 + B} \left[\tilde{y}_0 + \frac{\tilde{y}_1}{R_{0,1}} \right].$$

Similarly, we can substitute (2.0) (b) into the foreign consumers' problem. Using the same method as above, we can show the foreign consumers' problem becomes

$$(2.5) \quad \max_{\{c_{T0}^*, c_{T1}^*\}} \ln c_{T0}^* + \beta \ln c_{T1}^*$$

such that

$$c_{T0}^* + \frac{c_{T1}^*}{R_{0,1}} = \tilde{y}_0^* + \frac{\tilde{y}_1^*}{R_{0,1}}$$

where

$$\tilde{y}_0^* = y_{T0}^* - \tau_{T0}^* + q_{N0}^* (g_{N0}^* - \tau_{N0}^*)$$

$$\tilde{y}_1^* = y_{T1}^* - \tau_{T1}^* + q_{N1}^* (g_{N1}^* - \tau_{N1}^*).$$

The demand functions for this problem (2.5) are given by

$$(2.6) \quad (a) \quad c_{T0}^* = \frac{1}{1 + \beta} \left[\tilde{y}_0^* + \frac{\tilde{y}_1^*}{R_{0,1}} \right]$$

$$(b) \quad \tilde{c}_{T1} = \frac{\beta R_{0,1}}{1 + \beta} \left[\tilde{y}_0^* + \frac{\tilde{y}_1^*}{R_{0,1}} \right].$$

Now to solve for the equilibrium, plug the demand functions into the market clearing conditions and solve for prices. Plug (2.4) (a) and (2.6) (a) into the goods market at 0 for traded goods:

$$(2.7) \quad c_{T0} + c_{T0}^* = \frac{1}{1 + \beta} \left[\tilde{y}_0 + \frac{\tilde{y}_1}{R_{0,1}} \right] + \frac{1}{1 + \beta} \left[\tilde{y}_0^* + \frac{\tilde{y}_1^*}{R_{0,1}} \right] = y_{T0} + y_{T0}^* - (g_{T0} + g_{T0}^*).$$

Now assume for simplicity the governments of both countries tax nontraded goods just enough to cover their spending of nontraded goods; i.e., assume:

$$(2.8) \quad \tau_{Nt} = g_{Nt} \quad t = 0, 1$$

$$\tau_{Nt}^* = g_{Nt}^* \quad t = 0, 1.$$

Given this assumption, we can rewrite $\tilde{y}_t, \tilde{y}_t^*$ as follows:

$$(2.9) \quad \tilde{y}_t = y_{Tt} - \tau_{Tt}$$

$$\tilde{y}_t^* = y_{Tt}^* - \tau_{Tt}^*.$$

Plugging (2.8) into the governments budget constraint (1.6) and (1.8) gives

$$(2.10) \quad (a) \quad g_{T0} + \frac{1}{R_{0,1}} g_{T1} = \tau_{T0} + \frac{1}{R_{0,1}} \tau_{T1}$$

$$(b) \quad g_{T0}^* + \frac{1}{R_{0,1}} g_{T1}^* = \tau_{T0}^* + \frac{1}{R_{0,1}} \tau_{T1}^*.$$

Now using (2.9) and (2.10), we can rewrite (2.7) as

$$(2.11) \quad \frac{1}{1+\beta}[(y_{T0}-g_{T0}) + \frac{1}{R_{0,1}}(y_{T1}-g_{T1})] + \frac{1}{1+\beta}[(y_{T0}^*-g_{T0}^*) + \frac{1}{R_{0,1}}(y_{T1}^*-g_{T1}^*)]$$

$$= y_{T0} + y_{T0}^* - (g_{T0} + g_{T0}^*).$$

Solving for the interest rate gives:

$$(2.12) \quad \frac{1}{R_{0,1}} = \beta \frac{(y_{T0}-g_{T0} + y_{T0}^*-g_{T0}^*)}{(y_{T1}-g_{T1} + y_{T1}^*-g_{T1}^*)}.$$

Now the consumption allocations are given by

$$(2.13) \quad (a) \quad c_{Nt} = y_{Nt} - g_{Nt}$$

$$(b) \quad c_{Nt}^* = y_{Nt}^* - g_{Nt}^*$$

$$(c) \quad c_{T0} = \frac{1}{1+\beta}[(y_{T0}-g_{T0}) + \frac{1}{R_{0,1}}(y_{T1}-g_{T1})]$$

$$(d) \quad$$

$\frac{1}{4}Q^*$

$$(e) \quad c_{T0}^* = \frac{1}{1+\beta}[(y_{T0}^*-g_{T0}^*) + \frac{1}{R_{0,1}}(y_{T1}^*-g_{T1}^*)]$$

$$(f) \quad \text{*****}$$

where $R_{0,1}$ is given in (2.12).

Let W_{T0} = the present value of home consumers endowments of traded goods minus the present value of home government's spending of traded goods. Let ***** equal the present value of foreign consumers endowments of traded goods minus the present value of home governments spending of traded goods. That is

(3.0)

Then with this notation, we have

(3.1)

Now in any competitive equilibrium, we know prices are ratios of marginal utilities. The interest rate equals the rate of marginal utility of time 0 traded goods (or traded goods) to time 1 traded goods using home consumer:

(3.2) (a) *****

or using foreign consumer

(b) *****

The relative price of home nontraded goods at t to traded goods at t is q_{Nt} equals the ratio of the marginal utility of consumption of home nontraded goods at t to be the WORD of traded goods at t

(3.3) *****

Using (2.13) (a) and (3.1), we have

(3.3) (a) *****

(b) *****

Similarly, we have

(c) *****

(d) *****