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PRICE STABILITY AND  
FUTURES TRADING IN COMMODITIES

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ABSTRACT

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In this paper, we characterize those situations in which after the introduction of futures markets there is either an unambiguous change in the volatility of spot prices or an unambiguous change in welfare. We provide examples of the usefulness of this approach by giving two alternative sets of sufficient conditions for price volatility to decline following the introduction of futures trading. We also provide a set of sufficient conditions for the introduction of futures trading to increase the welfare of all agents.

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## 1. Introduction

The role of speculation in a market economy is a question which has long been of interest to researchers in both economics and finance. In particular, the effect of speculators' activities in futures markets on spot prices is often a hotly debated issue. Presumably, the fundamental economic issue to be addressed is that of the welfare implications of speculative activity. However, due to the difficulties with analyzing this question directly, most researchers have approached the more tractable problem of the effects of speculation on the mean and variance of spot prices.

The underlying assumption is that stable, i.e., less volatile prices are socially desirable. Chamberlin [1938], for example, argues that speculation will more likely cause greater fluctuations in prices, whereas, Friedman [1953] argues that profitable speculation in foreign currencies is necessarily stabilizing.

Several examples of destabilizing speculation are known in the literature. Some of these examples date back to the fifties and rely on either imperfect competition or irrational behavior on the part of some of the market participants (see Baumol [1957], for an example). More recently, Kawai [1983a], Hart and Kreps [1984] have shown that speculation can be destabilizing even when all the market participants behave competitively, and have rational expectations.

Kawai considers the case of speculation through futures trading in storable commodities. He uses a mean-variable framework in which agents have homogenous beliefs. He finds that if the primary source of randomness in the commodity market is through disturbances to consumption demand, then the introduction of futures market will stabilize spot prices; on the other hand, if the inventory demand disturbance is the preponderant shock, futures trading tends to destabilize prices. In addition, he shows that if inventory holding is prohibited, and there are no shocks to production, futures trading always stabilizes spot prices.

Hart and Kreps consider speculation through holding inventories. They show by means of an example that speculation can destabilize prices when speculators have superior information. However, when shocks to consumption demand are independently and identically distributed over time and speculators have no foresight at all or a great deal of foresight, speculation will be stabilizing. Turnovsky [1983] obtains results similar to Kawai. As in Kawai [1983] and Turnovsky [1983], we limit our attention to speculation through futures trading.

In this paper we examine the effect of futures trading on spot prices when there is no uncertainty associated with production. We know that, even when there are no shocks to production, futures trading can destabilize spot prices. We can show that, even when futures trading reduces the variance of spot prices, some agents can be made strictly worse off. We first pre-

sem conditions under which futures trading will stabilize spot prices, for the case of risk neutral speculators. Second, we examine, through a series of examples the necessity of the assumptions in these results.

Although the assumption of risk neutral speculators is rather severe, it does seem like a reasonable place to begin the analysis. That is, one of the primary economic functions of futures markets is to provide an outlet for producers to purchase insurance. Given this, it is natural to assume that the speculators are less risk averse than the producers. Risk neutrality is just the extreme version of this assumption. The advantage of this assumption is that it simplifies the proofs. This in turn allows an approach which is more intuitive than methods used in earlier papers.

We first show that the following sets of conditions will be sufficient to ensure that speculation through futures trading will be stabilizing:

- (a) All agents have the same information
- (b) The shock to demand is additive
- (c) The marginal cost of production is a constant
- (d) Speculators are risk-neutral
- (e) The commodity cannot be stored

We also show that assumption (c) can be relaxed if

- (f) The inverse demand function is linear
- (g) Producers are risk-averse, and have constant absolute risk aversion

The marginal cost of the production can then be linear in output, provided it does not depend on past decisions. It is not, however, necessary that (i) the utility functions of the producers belong to the time separable negative exponential family, and (ii) the demand shocks are Gaussian, as in Kawai [1983] or Turnovsky [1983]. We are able to get weaker sufficient conditions because we assume that speculators are risk neutral.

We also examine the necessity of some of the assumptions. We give three examples where futures trading destabilizes spot prices. In the first, the cost function is not quadratic. In the second, agents have private information and the cost function of the producers depend on past decisions. In the third, the shock to demand changes the slope as well as the intercept of the demand function.

A natural question at this point is whether we should be interested in price stability, and whether futures trading leads to welfare improvement for all market participants. If the answer is not always yes, we must identify the effect of futures trading on the welfare of different agents. We examine this issue in Section 4, and present a set of sufficient conditions for futures trading to lead to welfare improvement.

The rest of this paper is organized as follows. In Section 2, we describe the underlying economic environment and obtain two sets of sufficient conditions for futures trading to stabilize spot prices when commodities can not be stored. These are given in Theorems 1 and 2. In Section 3, we provide examples

which violate the sufficient conditions, and where futures trading leads to increased variance of spot prices. In Section 4, we examine the welfare implications. We conclude in Section 5.

## 2. Price Stabilizing Speculation Through Futures Trading Technology:

The economy has one competitive producer. There is no uncertainty in the technology. Production commitments are made one period before the realization of output. If the producer decides at time  $t$  to produce  $q_{t+1}$  units in period  $t + 1$ , a cost of  $c_{t+1} = C(q_{t+1}, q_t, q_{t-1}, \dots)$  is incurred in period  $t + 1$ , where  $C(\cdot)$  denotes the cost function of the producer. The price  $p_{t+1}$  at date  $t + 1$  is determined through competitive market clearing. The good is nonstorable.

### Preferences

The inverse demand function at date  $t + 1$  is given by

$$(1) \quad P_{t+1} = P(q_{t+1}, \epsilon_{t+1}, \eta_t)$$

There are two shocks to demand, viz.,  $\epsilon_{t+1}$  and  $\eta_t$ . The producer observes  $\eta_t$  at date  $t$ , before taking the production decision. The shock  $\epsilon_{t+1}$  observed at date  $t + 1$ , i.e., the production decision has been made.

The producer cares only about profits  $\pi_t$ , which are given by,

$$(2) \quad \pi_t = p_t q_t - c_t$$

when there is no futures trading. The producer chooses  $q_t$  at date  $t - 1$ ,  $t = 1, 2, \dots$  so as to maximize expected profits given by,

$$(3) \quad E_{t-1} \left[ \sum_{s=t}^{\infty} \beta^s u(\pi_s) \right]$$

In the above expression,  $\beta$  denotes the subjective time discount factor,  $u(\cdot)$  the period utility function for profits, and  $E_t(\cdot)$  the expectations operator, based on the producer's information set at time  $t$ . In the economy with futures trading, we will use the superscript 'f' on variables to distinguish them from corresponding variables in the economy without futures trading. When futures trading is permitted, the producer's profit  $\pi_t^f$  at date  $t$  is given by

$$(4) \quad \pi_t^f = p_t^f q_t^f - m(t-1, t) [p_t^f - f(t-1, t)] - C(q_t^f)$$

where  $m(t-1, t)$  is the number of futures contracts that the producer bought at date  $t - 1$  for delivery at date  $t$ , and  $f(t-1, t)$  denotes the futures price at date  $t$  for delivery of one unit of the commodity at date  $t$ .

Speculators are assumed to be risk neutral. They buy and sell futures contracts so as to maximize their expected profits.

It may appear that we are expanding the number of agents in the economy when futures trading is permitted. This is not necessarily true. For example, consider the two good economies with one producer and one consumer both of whom behave competitively. The producer does not get any direct utility for good

two, which he produces using good one as input. He exchanges it for good one in the spot market. Neither good is storable. Borrowing and lending at the risk free rate in good one, which is the numeraire, is allowed. The consumer gets a random endowment of  $y$  units of good one at the beginning of each time period and decides to exchange part of it for good two in the spot market. The consumer chooses a stochastic consumption-investment plan so as to maximize her life time expected utility given by,

$$(5) \quad E_0 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} [A_{t-1} r_{1t} + (\eta_{t-1} + \epsilon_t) r_{2t} + w(r_{2t})] \right\}$$

subject to her budget constraints. In expression (5),  $r_{1t}$  and  $r_{2t}$  denote the consumption of good one and good two at date  $t$ , and  $w(\cdot)$  is a concave function. While  $A_t$  is known at date  $t$  to the consumer, it can vary over time in a stochastic fashion. It can be verified that the consumer is risk neutral in good one. The price of a unit discount bond which pays one unit of good one at date  $t + 1$  will be  $A_{t-1}/(\beta A_t)$  units of good one at date  $t$ . The interest rate in good one will therefore be a constant over time if  $A_t = A$ , a constant. It can be verified that the consumer's implicit demand function will be,

$$(6) \quad \eta_{t-1} + \epsilon_t - A_{t-1} p_t + \frac{dw}{dr_{2t}} = 0$$

Since  $w(\cdot)$  is only a function of  $r_{2t}$ , the shock to demand in (6) enters additively. Also, notice that the introduction of futures trading does not alter the demand function. If  $w(\cdot)$  is quadratic, then, (6) will in addition be linear. It must be emphasized that

no part of the analysis below depends upon the particular specification of the economy in equations (5) and (6). This is only one of the possible scenarios consistent with the assumption that speculators are risk neutral. In what follows, we will not specify the underlying economy that supports the assumptions explicitly.

Competitive equilibrium:

A competitive equilibrium with futures trading is,

- (i) a set of decision rules for the producer, which gives quantities to be produced, i.e.,  $q_{t+1}^f = Q^f(\eta_t, f(t, t+1))$ , and the number of futures contracts to be bought, i.e.,  $m(t, t+1) = M(\eta_t, f(t, t+1))$ ,
- (ii) a futures price function  $f(t, t+1) = F(\eta_t)$  and,
- (iii) a spot price function  $p_{t+1}^f = P^{f*}(\eta_t, \epsilon_{t+1})$ ,

such that,

- (a) the futures market clears. Since speculators are risk neutral, this implies that the futures price  $f(t, t+1) = E_t(p_{t+1}^f)$ .
- (b) the spot price  $p_{t+1}^f$  at date  $t + 1$  is the market clearing price at date  $t + 1$ , and,
- (c) The decision rules  $Q^f(\cdot)$  and  $M(\cdot)$  are optimal for the producer.

A competitive equilibrium without futures trading is analogously defined.

We are now in a position to state our main result. This result does not depend on one producer, or infinite horizon discounting, but only on the risk aversion of the producer.

Theorem 1

Assume that,

(i) the producer's cost function  $C(q) = cq$ , i.e., is linear in planned output, with a constant marginal cost of production  $c$ ,

(ii) the shocks to the inverse demand function are additive, i.e.,  $P_{t+1} = P(q_{t+1}, \epsilon_{t+1}, \eta_t) = G(q_{t+1}) + \epsilon_{t+1} + \eta_t$ , where,  $E_{t-1}(\epsilon_t) = 0$ ,  $\text{Var}_{t-1}(\epsilon_t) = \sigma_\epsilon^2$ , and  $\text{Var}_{t-1}(\eta_t) = \sigma_\eta^2$ .

Then,

(A)  $\text{Var}(p_t) > \text{Var}(p_t^f)$ , i.e., the unconditional variance of the spot price without futures trading will be at least as great as the variance of the spot price when futures trading is allowed.

If, in addition,

(iii) the producer is risk averse,

then,

(B)  $E(p(t)) > E(p^f(t))$ , i.e., the unconditional expectation of the spot price without futures trading will be at least as large as the unconditional expected value of the spot price when futures trading is allowed. The inequalities will be strict if the producer is strictly risk averse.

Proof

The first order condition to the producer's maximization problem, without futures trading is given by equation (7a) below.

$$(7a) \quad E_{t-1}[u'(\pi_t)(p_t - c)] = 0$$

In equilibrium, it follows that,

$$(7b) \quad \begin{aligned} \text{Var}(p_t) &= \text{Var}(G(q_t) + \epsilon_t + \eta_{t-1}) \\ &= \text{Var}(E_{t-1}(G(q_t) + \epsilon_t + \eta_{t-1})) \\ &\quad + E(\text{Var}_{t-1}(G(q_t) + \epsilon_t + \eta_{t-1})) \end{aligned}$$

From equation (7a), however, we see that  $\text{Var}_{t-1}(E_{t-1}(p_t)) > 0$ . If the producer is not risk neutral, the inequality will be strict. Hence, the first expression on the right-hand side of equation (7b) will be positive. It will be strictly positive if the producer is not risk neutral. This is because the supply function of the producer, as a function of the expected price next period will be upward sloping without the futures market. Since there is a nontrivial demand shock  $\eta_t$  at each date  $t$ , the conditionally expected price next period will not be the same each period, when there is no futures market. The second expression equals  $\sigma_\epsilon^2$ . Hence,  $\text{Var}(p_t) > \sigma_\epsilon^2$ , with strict inequality when the producer is not risk neutral.

The first order conditions to the producer's problem when futures trading is permitted are given by equations (8a) and (8b) below.

$$(8a) \quad E_{t-1} [u'(\pi_t^f)(p_t^f - c)] = 0$$

$$(8b) \quad E_{t-1} [u'(\pi_t^f)(p_t^f - f_{t-1,t}^f)] = 0$$

Substituting (8b) into (8a), we get the well known result that the producer, facing no production uncertainty, will choose the production level so as to equate the marginal cost of production to the futures price,<sup>2/</sup> i.e.,  $c = f(t-1,t)$ . Since we have assumed that the speculator is risk neutral, the futures market will clear if and only if  $c = E_{t-1}(p_t^f)$ . Hence,  $\text{Var}_{t-1}(E_{t-1}(p_t^f))$  is zero. Hence, the first term in the variance decomposition of the spot price with futures trading given in equation (9) below is zero.

$$(9) \quad \text{Var}(p_t^f) = \text{Var}(E_{t-1}(p_t^f)) + E(\text{Var}_{t-1}(p_t^f))$$

Since the demand shock is additive,

$$E(\text{Var}_{t-1}(p_t^f)) = \sigma_\varepsilon^2.$$

Clearly,

$$\text{Var}(p_t) > \text{Var}(p_t^f).$$

If the producer is risk averse, as in assumption (iii), then, from equation (7a), we see that  $E_{t-1}(p_t) > c = E_{t-1}(p_t^f)$ . The inequalities will be strict if the producer is strictly risk averse. Q.E.D.

An examination of the results in Theorem 1 immediately leads to the conclusion that even when futures trading leads to lower and less variable prices, some agents may become worse off. In this case, the risk averse producer loses when futures trading is introduced. Since the marginal cost of production is a

constant, and the speculator is risk neutral, the profit to the producer is zero with futures trading. When there is no futures trading, the producer is indifferent between producing and not producing one more unit only at the margin, whereas with futures trading he is indifferent to producing and not producing at all. The assumption that there is no production uncertainty is crucial to the results. To understand why, consider the case where the actual quantity produced  $\tilde{q}_t = q_t(1+\tilde{v}_t)$ ,  $E_{t-1}(\tilde{v}_t) = 0$ . The conditional variance of the spot price will be:

$$\begin{aligned} \text{Var}_{t-1}(p_t) = & \text{Var}_{t-1}(G(\tilde{q}_t)) + \text{Var}_{t-1}(\tilde{\epsilon}_t) \\ & + 2 \text{Cov}_{t-1}(G(\tilde{q}_t), \tilde{\epsilon}_t) \end{aligned}$$

If  $\tilde{v}_t$  and  $\tilde{\epsilon}_t$  are independent, then  $\text{Var}_{t-1}(p_t)$  will be an increasing function of  $q_t$  and the conditional variance of the spot price will increase whenever introducing futures trading increases the planned output. The results in Theorem 1 will, however, still be true if the production uncertainty is independent of the level of production.

In what follows, we relax the assumption that the marginal cost of production is a constant, and assume that the inverse demand function is linear.

Let the inverse demand function be given by:

$$(10) \quad P(q_t, \epsilon_t, n_{t-1}) = n_{t-1} - dq_t + \epsilon_t$$

Since the demand shock  $\epsilon_t$  is additive, the variance of the market clearing spot price  $p_t$  is given by

$$\begin{aligned}\text{Var}(p_t) &= \text{Var}(E_{t-1}(p_t)) + E(\text{Var}_{t-1}(p_t)) \\ &= \text{Var}(E_{t-1}(p_t)) + E(\delta_\epsilon^2)\end{aligned}$$

Hence, to compare the  $\text{Var}(p_t)$  between two regimes, we need only compare  $\text{Var}(E_{t-1}(p_t))$ . We will, therefore, find it convenient to work with the average inverse demand function, given by:

$$(11) \quad E_{t-1}(P(q_t, \epsilon_t, \eta_{t-1})) = \bar{p}_t = \eta_{t-1} - dq_t.$$

We will suppress the time subscripts from here on, to simplify the notation. Consider two different supply functions  $Q_1(\bar{p})$  and  $Q_2(\bar{p})$  with  $\frac{\partial Q_j}{\partial \bar{p}}(\bar{p}) > 0$  and  $\frac{\partial^2 Q_j}{\partial \bar{p}^2}(\bar{p}) > 0$ . Figure 1 gives two such supply functions. Let  $\bar{p}^1$  and  $\bar{p}^2$  be the expected market clearing prices corresponding to the two supply functions, and  $q^1$  and  $q^2$  be the corresponding quantities supplied. Let,

$$(12) \quad \bar{p}^1 = \bar{p}^2 + x$$

$\bar{p}^1$ ,  $\bar{p}^2$  and  $x$  will be functions of  $\eta$ , the intercept of the demand function. Hence,

$$(13) \quad \text{Var}(\bar{p}^1) = \text{Var}(\bar{p}^2) + \text{Var}(x) + 2 \text{Cov}(\bar{p}^2, x).$$

A sufficient condition for  $\text{Var}(\bar{p}^1)$  to be greater than  $\text{Var}(\bar{p}^2)$  will be  $\text{Cov}(\bar{p}^2, x) > 0$ . since the quantity supplied,  $Q_2(\bar{p})$ , is an increasing function of this will be the case if  $x$  is an increasing function of  $\eta$ , i.e., if

$$(14) \quad \left. \frac{\partial Q_1(\bar{p})}{\partial \bar{p}} \right|_{\bar{p}^1} > \left. \frac{\partial Q_2(\bar{p})}{\partial \bar{p}} \right|_{\bar{p}^2}$$

for all possible  $\bar{p}^1(\eta)$ ,  $\bar{p}^2(\eta)$ . Clearly, condition (14) will be satisfied if  $\frac{\partial Q_1(\bar{p})}{\partial \bar{p}} > \frac{\partial Q_2(\bar{p})}{\partial \bar{p}}$  for all  $\bar{p}$ , since  $Q(\cdot)$  is a convex function. We have thus proved the following.

Lemma

Suppose that

- (a) The demand function is linear, as given in equation (11)
- (b) The supply functions  $q_{jt} = Q_j(\bar{p}_t)$ ,  $j = 1, 2$  are such that  $Q'_j > 0$ ,  $Q''_j > 0$  and  $Q'_1 > Q'_2$  for all  $\bar{p}_t$ . Then,  $\text{Var}(P_t^1) > \text{Var}(P_t^2)$  where  $p^1$  and  $p^2$  are the market clearing prices corresponding to supply functions  $Q_1(\cdot)$  and  $Q_2(\cdot)$  respectively.

Consider now the case where the cost function is quadratic, i.e., marginal cost is linear. The supply function with futures trading will be the same as the marginal cost curve. In Theorem 2 below, we show that if the producer's utility function displays constant absolute risk aversion, the supply function without futures trading will be steeply.

Theorem 2

Assume that,

- (i) The inverse demand function is  $P(q_t, \varepsilon_t, \eta_{t-1}) = \eta_{t-1} - dq_t + \varepsilon_t$ ,
- (ii) The cost function is  $C(q_t) = aq_t + bq_t^2$ .
- (iii) The producer's utility function displays constant absolute risk aversion.

Then statements (A) and (B) of Theorem 1 remain true.

Proof

The first order condition to the producer's maximization problem, with futures trading is given by equation (15) below:

$$(15) \quad E_{t-1}[u'(\pi_t)(p_t - a - 2bq_t)] = 0$$

The first order conditions to the producer's problem with futures trading are given by equations (16a) and (16b) below.

$$(16a) \quad E_{t-1}[u'(\pi_t^f)(p_t^f - a - 2bq_t^f)] = 0$$

$$(16b) \quad E_{t-1}[u'(\pi_t^f)(p_t^f - f_{t-1,t}^f)] = 0$$

These first order conditions give the supply function of the producer as a function of the expected price next period,  $E_{t-1}(p_t)$  and  $E_{t-1}(p_t^f)$ . When futures trading is permitted, equations (16a) and (16b) together imply that the inverse supply function is given by  $f_{t-1,t}^f = E_{t-1}(p_t^f) = a - 2bq_t^f$ . When there is no futures market, the producer's inverse supply function is implicitly given by equation (15) above.

We will find it convenient to write equation (15) as:

$$E_{t-1}[u'(\pi_t)(\bar{p}_t + \varepsilon_t - a - 2bq_t)] = 0$$

where  $\bar{p}_t = E_{t-1}(p_t)$ , is taken as given by the producer. Totally differentiating the above equation, we get,

$$E_{t-1}[u'(\pi_t)(d\bar{p}_t - 2bdq_t)] + E_{t-1}[u''(\pi_t)(\bar{p}_t + \varepsilon_t - a - 2bq_t)^2 dq_t]$$

$$+ E_{t-1}[u''(\pi_t)(\bar{p}_t + \varepsilon_t - a - 2bq_t)q_t d\bar{p}_t] = 0.$$

The last term in the above expression is zero, since  $u'(\pi_t)$  and  $u''(\pi_t)$  differ only by a scale factor, when  $u(\cdot)$  exhibits constant absolute risk aversion. This gives us, after rearranging the terms,

$$(17) \quad \frac{d\bar{p}_t}{dq_t} = \left\{ 2b - \frac{E_{t-1}(u''(\pi_t)(\bar{p}_t + \varepsilon_t - a - 2bq_t)^2)}{E_{t-1}(u'(\pi_t))} \right\}$$

We know from equations (16a) and (16b) that

$$(18) \quad \frac{d\bar{p}_t^f}{dq_t^f} = 2b.$$

Since  $E_{t-1}(u''(\pi_t))$  is negative, the left side of equation (17) is greater than the left side of equation (18). Hence, the supply curve without futures trading is steeper than the supply curve with futures trading. Applying the Lemma, we get  $\text{Var}(p_t) > \text{Var}(p_t^f)$ .

The second part of the proof, i.e., that  $E(p_t) > E(p_t^f)$  follows from the observation that the supply is zero when the expected price next period equals  $a$ , the marginal cost at zero output and, hence, both the inverse supply curves start from the same point. Q.E.D.

As pointed out earlier, Theorems 1 and 2 relax the assumptions made by the other authors regarding preferences of the producers and the distribution of the demand shocks, but assumes that speculators are risk neutral. In the next section we explore what happens when some of the assumptions in Theorem 2 are violated. We will see that in that case, speculation through futures trading can increase the variance of spot prices.

The main methodological contribution in these theorems is the fact that the unconditional variance of prices can be decomposed into the variance of the conditional mean and the mean of the conditional variance. This decomposition is useful because in a variety of circumstances introduction of a futures market does not alter the conditional variance of the spot price. This is particularly so if demand shocks are additive. However, the variance of the conditional mean falls with the introduction of a futures market if marginal cost is a linear function of production.

### 3. Destabilizing Speculation Through Futures Trading

In Theorem 2, we assume that (i) the producer's utility function displays constant absolute risk aversion, (ii) the cost function is quadratic and (iii) shocks to demand are additive. In what follows, we show that when any of these conditions are violated, futures trading may increase the variance of spot prices, by means of three examples.

#### Example 1

In this example, we show the assumption that the producer's utility function displays constant absolute risk aversion is not innocuous. Let  $u(\pi_t) = \ln(\pi_t)$ ;  $\tilde{\epsilon}_t = +1$  with equal probability; and  $C(q) = q^2$ . Substituting for  $u'(\cdot)$ ,  $\pi_t$ ,  $\tilde{\epsilon}_t$ ,  $a$  and  $b$  in the first order condition to the maximization problem given in equation (15) and simplifying, we get

$$(19) \quad \bar{p}_t - 3\bar{p}_t q_t + 2q_t^2 - 2 = 0.$$

Totally differentiating (19) gives,

$$(20) \quad \frac{d\bar{p}_t}{dq_t} = \frac{3\bar{p}_t - 4q_t}{2\bar{p}_t - 3q_t}$$

We also know that  $\bar{p}_t > 2q_t$  since the marginal cost of production, is  $2q_t$ . Hence, substituting  $2q_t$  for  $\bar{p}_t$  in the right side of equation (20) gives us the following inequality:

$$(21) \quad \frac{d\bar{p}_t}{dq_t} < 2$$

It can be verified that for any finite  $q_t$ ,  $\frac{d\bar{p}_t}{dq_t} < 2$ . For example, if  $\bar{p}_t = 4$ ,  $q_t = 1.775$ , and  $\frac{d\bar{p}_t}{dq_t} = 1.832 < 2$ . If  $\bar{p}_t = 400$ ,  $q_t = 199.9975$ , and  $\frac{d\bar{p}_t}{dq_t} = 1.999975$ . The supply curve with futures trading is given by  $\bar{p}_t = 2q_t$ , i.e.,  $\frac{d\bar{p}_t}{dq_t} = 2$ , with futures trading. Applying the Lemma, we get the result that  $\text{Var}(p_t) < \text{Var}(p_t^2)$ .

### Example 2

Let the cost function  $C(\cdot)$  be given by,

$$C(q) = \begin{cases} q^2 & \text{if } q < 1 \\ -1 + 2q + \frac{k}{2} (q-1)^2 & \text{if } q > 1 \end{cases}$$

Let the period utility function of the producer be given by:

$$u(\pi_t) = E_{t-1}(\pi_t) - \text{Var}_{t-1}(\pi_t),$$

where, the profit at date  $t$ ,  $\pi_t$  is given by equation (2) when there is no futures trading, and by equation (4) when futures trading is possible. Let,  $\sigma_\epsilon^2 = 1$ , and the inverse demand function be given by,

$$p_t = \eta_{t-1} - 4q_t + \epsilon_t,$$

where,  $\eta_{t-1}$  is either 1 or 5, with equal probability. It can be verified that,

|            | Realization of $\eta_{t-1}$ |     |
|------------|-----------------------------|-----|
|            | 1.0                         | 5.0 |
| $E(p_t^f)$ | 0.8                         | 4.0 |
| $E(p_t)$   | 0.67                        | 3.9 |

Hence,  $\text{Var}_{t-1}(E_{t-1}(p_t)) = 2.56 < \text{Var}(E_{t-1}(p_t^f)) = 2.60$ . Although futures trading reduces the average spot price, the variability of the spot price increases. When the demand is on the average low, futures trading reduces the average spot price from 0.8 to 0.67, i.e., by 0.13 unit. However, risk reduction through the futures market is not very effective when the demand is high. It reduces the average spot price from 4.0 to 3.9, i.e., by only 0.10 units. This is because physical constraints on production become more important, at higher levels of production.

While the example is rather artificial, it does capture the flavor of industries in which there are factors of production which are fixed in the short run. Agriculture gives a simple example of this situation.

Note, however, that the average price is still lower with futures trading than without.

### Example 3

Consider the economy in Example 1. Suppose that the demand function is given by,

$$P_t = 5 - bq_t$$

where  $b$  is either 0.1 or 10 with equal probability. The cost function of the producer is given by,

$$C(q_t) = q_t^2.$$

It can be verified that the supply curves of the producer are given by,

$$E_{t-1}(p_t) = 4q_t, \text{ and } E_{t-1}(p_t^f) = 2q_t^f.$$

Hence,  $E_{t-1}(p_t)$  will be either 4.878 or 1.479, whereas,  $E_{t-1}(p_t^f)$  will be either 4.762 or 0.833. Clearly the unconditional variance of the spot price will be increased with the introduction of futures trading, although, the average price will fall.<sup>3/</sup>

#### Example 4

In this example, we consider the case where speculators have foresight and the cost of producing the quantity  $q_t$  at date  $t$  depends on earlier production decisions. We also assume that both the producer and the speculator are risk neutral. At time  $t - 1$ , the producer decides on the quantity  $q_t$  to be produced at date  $t$ . The cost incurred at date  $t$  is given by,

$$(19) \quad C(q_t) = c_t = \delta_0 q_t + 0.5 \delta_1 q_t^2 + 0.5 \delta_2 (q_t + \gamma q_{t-1})^2$$

Equation (14) says that the marginal cost of production at date  $t$  depends on the production  $q_{t-1}$  at date  $t - 1$ . Once again, agriculture provides a simple example. If cereals are grown time

after time, the productivity of the land falls, and the farmer will either have to leave the land idle for a period of time or plant some other crop such as legumes which may be less profitable. When there is no futures trading, the producer chooses a sequence  $\{q_{-t}\}$   $t = 0$ , so as to maximize,

$$(20) \quad E_0^P \left\{ \sum_{t=0}^{\infty} \beta^t [p_t q_t - c_t] \right\}$$

where,  $E_0^P(\cdot)$  is the expectations operator conditioned on the information available to the producer at date 0. The inverse demand function is given by,

$$(21) \quad p_t = u_t - \alpha q_t,$$

where  $u_t$  is known to the speculator at date  $t - 1$ , but observed by the producer only at date  $t$ . The Euler equations for the producer's problem are:

$$(22) \quad \beta^t \{ p_t - \delta_0 - \delta_1 q_t - \delta_2 (q_t + \gamma q_{t-1}) \} - \beta^{t+1} \gamma \delta_2 E_{t-1}^P (q_{t+1} + \gamma q_t) = 0,$$

$t = 1, 2, \dots$  with  $q_0$  given and the transversality condition is given by,

$$(23) \quad \lim_{T \rightarrow \infty} \beta^T \{ p_T - \delta_0 - \delta_1 q_T - \delta_2 (q_T + \gamma q_{T-1}) \} = 0$$

Note that  $p_t$  and  $q_t$  are in the information set of the producer at date  $t - 1$ . Using standard techniques (see Sargent [1979]), it can be shown that the solution to the Euler equations which satisfy the transversality condition is,

$$(24) \quad q_{t+1} = \lambda_1 q_t - \frac{1}{\lambda_2 \beta \gamma \delta_2} \sum_{i=0}^{\infty} \left( \frac{1}{\lambda_2} \right)^i \{ E_t (u_{t+1+i} - \delta_0) \},$$

where,

$$(25) \quad -(\lambda_1 + \lambda_2) = \frac{\alpha + \delta_1 + \delta_2 + \delta_2 \gamma^2 \beta}{\gamma \beta \delta_2}, \text{ and } \lambda_1 \lambda_2 = \frac{1}{\beta}$$

In the above expressions, both  $\lambda_1$  and  $\lambda_2$  are of the same sign but opposite of that of  $\gamma$ . For simplicity assume that the  $u_t$ 's are i.i.d., with mean  $\bar{u}$ , and variance  $\sigma_u^2$ . Then,

$$(26) \quad q_{t+1} = \frac{-1}{\lambda_2 \beta \gamma \delta_2} \{ \sum_{j=0} \lambda_1^j (u_{t+1-j} - \delta_0) \} + \text{a constant}$$

$$(27) \quad p_{t+1} = u_{t+1} + \frac{\alpha}{\lambda_2 \beta \gamma \delta_2} \{ \sum_{j=0} \lambda_1^j (u_{t+1-j} - \delta_0) \} + \text{a constant}$$

Hence,

$$(28) \quad \text{Var}(p_{t+1}) = \left\{ 1 + \frac{\alpha}{\lambda_2 \beta \gamma \delta_2} \right\}^2 \text{Var}(u_{t+1}) \\ - \frac{\alpha^2 \lambda_1^2}{(\lambda_2 \beta \gamma \delta_2)^2 (1 - \lambda_1^2)} \text{Var}(u_{t+1})$$

Consider now the regime with futures markets. Since both the producer and the speculator are risk neutral, the only equilibrium will be one in which the producer infers  $u_{t+1}$  (already known to the speculator at date  $t$ ) by observing the futures price. The equilibrium quantity of futures contracts is indeterminate. Let  $p_t^f$  denote the spot price at date  $t$  with futures trading, as before. It can then be shown that,

$$(29) \quad p_{t+1}^f = u_{t+1} + \frac{\alpha}{\lambda_2 \beta \gamma \delta_2} (u_{t+2} - \delta_0) + \frac{\alpha(1+\lambda)}{\lambda_2 \beta \gamma \delta_2} \sum_{j=0} \lambda_1^j (u_{t+1, j} - \delta_0)$$

where  $\lambda = \lambda_1/\lambda_2$ . It can be verified that a sufficient condition for  $\text{Var}(p_t)$  to be less than  $\text{Var}(p_t^f)$  is,

$$(30) \quad \alpha > (\delta_1 + \delta_2 + \delta_2 \gamma^2 \beta)$$

This condition is satisfied for reasonable choice of values for the various parameters. Equation (30) says that the absolute value of the slope of the demand curve should be larger than the absolute value of the slope of the supply curve. The intuition behind this is as follows. Since more information is available with futures trading, for any given demand shock  $u_t$ , the production  $q_t$  is more variable. This results in higher variability of  $p_t$ . However, since tomorrow's demand is known in advance, the need to adjust tomorrow's production becomes less, and consequently, tomorrow's price becomes less variable. When the demand curve is steeper than the supply curve, the first effect dominates.

#### 4. Welfare

As noted in the introduction, the question we would ideally like to address is whether introduction of futures trading leads to an improvement in the welfare. The results presented in Section 2 and 3 are useful in this regard but give only partial answers. That is, while the mean level and volatility of prices are related to welfare, this correspondence is imperfect.

The model in Theorem 1 gives one example in which introduction of futures markets do not lead to pareto improvement, as the producer is worse off. This is because the producer earns zero rent with futures trading. With constant marginal cost of production he is indifferent between producing and not producing

at all when there is a futures market, whereas, he is indifferent only at a margin when there is no futures trading. This example looks rather artificial, since we may not expect to find a monopolistic producer when marginal cost of production is a constant. In this section we relax the assumption that there is only one producer and permit free entry into production.

There are a countable number of risk averse producers. Each producer can produce at most one unit of an indivisible good. The cost of producing a unit is  $c_i$  for producer  $i$ ,  $i = 1, 2, 3, \dots, \infty$ , where  $c_i > c_j$  for  $i > j$ . Each producer acts as a price taker, and chooses to produce one unit if  $E\{u(\bar{p}-c_i)\} \equiv u(0)$ , where  $u(\cdot)$  is the producer's utility function for profits and  $E\{\cdot\}$  denotes the expectation operator. The supply curve is assumed to be downward sloping with parallel shocks to demand, as in Theorem 1. The mean demand curve which gives the expected price as a function of the quantity consumed will be as in Figure 2.

Since the producers face no risk when there is a futures market, the supply curve with futures trading is just the marginal cost curve. Let  $p_N$  and  $q_N$  denote the equilibrium expected price and quantity produced without futures market. Let  $p_n$  and  $q_n$  denote the corresponding variables with futures trading. When there is no futures trading, producer  $i$  is indifferent between producing one unit and not producing one unit at an expected price  $p_i = E\{\bar{p}\} = c_i + \pi$ , where  $\pi$  satisfies  $E\{\bar{p}-c_i\} = u(0)$ . The risk premium  $\pi$  does not depend on the producer type  $i$ , since the producers are assumed to have identical derived utility function

$u(\cdot)$  for profits. Hence, the supply curve with futures trading will be the same as the supply curve without a futures market shifted upward by the risk premium  $\pi$ , as in Figure 2.

It can be seen that the number of producers  $q_n$  with futures trading is more than the number of producers  $q_N$  without futures trading. The  $n$ 'th producer is clearly indifferent between producing and not producing and, hence, not affected by the introduction of futures trading. On the other hand the  $N$ 'th producer who was indifferent to producing and not producing when there was no futures market, is clearly better off due to the introduction of the futures market. Hence, producers  $N, N + 1, \dots, n - 1$  benefit due to futures trading. What about the other producers? We answer this question under the assumption that  $u(\cdot)$  is of the constant absolute risk aversion type in Theorem 3 below.

### Theorem 3

Assume that all the producers are identical and have derived preferences for profits which exhibit constant absolute risk aversion. Then all the producers are made better off due to the introduction of futures trading.

### Proof

Each producer is willing to give up  $\pi$  units to get rid of the uncertainty associated with the price. But the mean price declines by less than  $\pi$ , since the demand curve is downward sloping and the supply curve is upward sloping. Q.E.D.

If producers' preferences are not identical and exhibit constant absolute risk aversion, then some producers will be made worse off. For example, consider the case producers preferences exhibit constant relative risk aversion. Since absolute risk aversion decreases with wealth, the producer with the smallest cost of production (who earns the greatest profit) will be less averse to price uncertainty than the producer with relatively higher production cost, at the equilibrium production level. It is therefore possible that some low cost producers may be made worse off even though  $(p_N - p_n) < \pi$ , if the supply curve is sufficiently steep.

## 6. Conclusions

We examined the effect of introducing trading in futures contracts on spot prices of nonstorable commodities. We showed that, even in the case of a simple economy where the good is perishable and there is no uncertainty associated with the production technology, trading in futures contracts can increase the volatility of spot prices. We obtain a set of sufficient conditions for futures trading to reduce spot price variance. We show by means of examples that when these conditions are not met, opening of futures markets can lead to increased volatility of spot prices.

The results we obtain are more general along some dimensions than the ones known in the literature. We do not make any assumptions regarding the nature of the probability distribution of prices, preferences of producers, except that they be risk

averse. We, however, assume that speculators are risk neutral. This assumption simplifies that analysis and enables us to use more intuitive geometric methods in proving the results. Since one of the primary functions of futures markets is to provide an outlet for producers to purchase insurance, it is natural to assume that speculators are less risk averse than the producers. Risk neutrality on the part of the speculators is just the extreme version of this assumption.

The fundamental economic issue to be addressed is that of the welfare implications of trading in futures markets. We show that the connection between welfare and spot price volatility is rather tenuous. Even in the case where futures trading leads to a reduction in the volatility of spot prices, some agents can be made worse off. We need rather strong restrictions on the preferences of producers to ensure that all agents are better off due to trading in futures markets.

Footnotes

1/

2/In our case the futures and the forward contracts are identical.

3/Note the similarity between this example and that presented in Hart and Kreps [1984].

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