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A RECONSIDERATION OF THE PROBLEM OF SOCIAL COST: FREE RIDERS AND MONOPOLISTS

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Abstract

This paper examines the validity of one very special version of Coase's Theorem. The version we examine is that in any economy in which the property rights are fully allocated, competition will lead to efficient allocations. One repercussion of this result is that one way to "solve" the public goods problem would be to allocate property rights fully, transforming the economy to a private goods one and let markets do their work. This is particularly appealing due to its decentralized nature, but one must question the claim that the market will lead to efficient outcomes in this case. That is, the privatized economy created above is of a very special type which, as it turns out is highly susceptible to strategic behavior. We show that the "mechanism" suggested above is not likely to work well in economies with either pure public goods or "global" externalities. Basically, the free-rider problem manifests itself as one of monopoly power in this private goods setting. On the other hand, if the public goods or externalities are "local" in nature, there is reason to hope that this (and perhaps other) mechanism(s) will work well.

The work is related to the recent literature on the foundations of Walrasian Equilibrium in that it points up a relationship between the appropriateness of Walrasian equilibrium as a solution concept, the incentives for strategic play, the aggregate level of complementarities in the economy and the problem of coordinating economic activity.

Over the years, two approaches have evolved concerning the allocation of resources for the provision of public goods. The first, identified loosely with Coase (1960), holds that if there is a problem at all, it is that the resources of the economy are not fully allocated. That is, property rights are not fully assigned and moreover, if they were, competition would lead to an efficient allocation. This last statement has come to be known as Coase's Theorem.

The second approach, even more loosely associated with Samuelson [through his paper (1954)], espouses quite a different view holding that the "free rider" problem is both significant and important and that individuals will strategically misrepresent their true desires regarding the provision of public goods.

The contrast between the views on policy between these two schools of thought is equally striking. According to the one, nothing need be done save the development of a system for the full assignment of property rights (perhaps the courts are thought to serve this purpose). According to the other there is an important role for the government in actively participating in the provision of public goods.

This difference of opinion would not matter if it could be clearly identified that one or the other of the approaches was "right." The problem is that both approaches have intuitive appeal. In particular, one can think of examples in which it seems likely that the Coasian policy recommendation would be successful--Meade's apple orchard and beekeeper example as well as the classic example of the candy-maker next to the dentist--as well as examples in which it seems likely to fail--acid rain and national defense come to mind. Ideally one would like one theory which gives the intuitively correct prediction for all of these examples.

The purpose of this paper is to try to begin to develop a precise answer to the question of whether Coase or Samuelson is "right" (i.e., for which economies does the Coasian approach give the "right" answer, etc.). Of course, the ideal situation would be to have a taxonomy through which one could classify economic situations by which of the above approaches is appropriate. We will fall considerably short of that goal, but we hope our results will shed some light on the issue.

To do this will require several steps. We will have to adopt some convention as to the changes that take place when a public goods economy is "privatized." We will have to adopt some standards for what constitute a theory of decentralized exchange, etc. The techniques used borrow heavily from the literature on mechanism design, pioneered by Hurwicz (1972).

The remainder of this paper is organized as follows: Section 2 contains two illustrative examples. Sections 3 and 4 contain a more general approach to the free-rider problem in privatized public goods economies. Section 5 gives an initial view on the differences between "global" and "local" externalities. Section 6 offers concluding comments.

2. Examples

We begin with a simple but standard example. We follow the approach to the problem suggested above--we start with an economy with an obvious externality, add the needed markets, assign the property rights for these new goods, posit a form for competition and calculate the equilibrium.

Example 1: Consider a simple economy in which there is a town with a number of firms each producing the same final product for sale to an external market. Assume that each producer owns a factory which produces smoke in addition to the output. Assume that the smoke emanating from each factory spreads uniformly over all locations throughout the town and is (for simplicity) proportional to the output of the factory. Then, the total smoke over any location in the town is proportional to the total output of the industry. Assume that each firm can produce as much of the output as it likes at zero marginal cost and that demand for the final output is linear in price. We assume that the firms are perfect competitors in the output market. This is clearly something that we could formalize through either Bertrand price competition among any finite number of firms or approximate arbitrarily well by assuming quantity competition among the firms and letting the number of firms be taken to be very large. Assume that the demand for the final product is given by

$$D(p) = a - bp$$

where $a > 0$, $b > 0$, and $a > b$. Finally, we assume that there are n residents in the town. We assume for simplicity that the residents have utility functions over money and smoke consumption of the form $U(m,s) = m - (s/n)$ (we will see in a moment why it is s/n and not s).

For this example, in the absence of smoke production rights it is clear what the equilibrium will be: $p^* = 0$, aggregate output of both the final good and smoke will be a and the utility of the residents will be $m^* - a/n$ where m^* is their initial allocation of money. The example is standard enough--in their calculations of profit maximizing production plans, the firms have ignored the social cost of their production of smoke thereby "imposing" an externality on the residents. Thus, the market acts as if the marginal social cost of both output and smoke is zero while in reality it is \$1 (\$1/n of burden of each resident for each unit of smoke produced).

It is clear what the Coase camp would suggest for this economy--the problem is that no markets for pollution over the homesites exists. The solution is equally straightforward. Namely, introduce these markets and let competition proceed as usual. It is here that we begin to have problems. First, how do we formally treat these goods in both the production and consumption sides, and second, how are the endowments of these new goods to be assigned and what are their initial quantities? To handle these problems, we will have to be a little more formal and introduce a little more notation.

Let s_i be the amount of smoke consumed at location i. Let s denote the vector of smoke consumption. Then, we assume that resident j has a utility function over money and the two types of smoke given by:

$$U^j(m,s) = m - s_j/n, \ j = 1, 2, \ldots, n.$$

Thus, as is usual, residents only consume smoke at their own locations and so, there are no externalities in consumption. We endow each of the households with S units of smoke rights where S is a large positive number (at least as large as a). Note that smoke rights are a good rather than a bad from the consumers' point of view and that if the resident sells s units of his smoke rights (and they are all used to produce smoke), his utility would be

$$m - s/n = m + (S-s)/n - S/n.$$

Thus, utility is increasing in the consumption of smoke rights as we expected. (Note that it is as if the utility function has been reoriented by shifting it by - S/n, however.)

In keeping with the spirit of competitive product markets, we assume that the firms take the prices of the smoke rights as given when maximizing their profits. Note that from the firms' viewpoint it is as if the various types of smoke rights are inputs to the production process and if a firm wants to produce q units of the final good it must possess q units of <u>each</u> of the types of rights.

Before calculating what seems to be a reasonable equilibrium for this economy we should note what the perfectly competitive outcome is. It is straightforward to check that this is for each of the smoke rights to sell for a price $p_j = 1/n$, the final good sells for $p = 1$ and the quantities are $a - b$ of final product (assuming that $a > b$, which we have assumed) and of each of the two types of smoke. It is easy to see that this equilibrium is efficient (and is the Lindahl equilibrium of the original economy).

Given that the markets for the individual smoke rights are so thin, it is natural to question our assumption of price-taking behavior. For this reason, we examine a different, more strategic, notion of equilibrium. Note that this is not contrary to Coase's original intent in any way. In fact, Coase seemed to think that efficient allocations would arise out of strategic bargaining between the parties involved (although the article is sufficiently vague as to make any statements of this sort necessarily conjectural). Restated, what we are looking for is a strategic justification for Coase's optimism with regard to the equilibrium in this economy. This is the motivation for considering this and the ensuing examples.

For the moment we assume that the form of competition that takes place is for residents to set prices for their site specific smoke rights fully cognizant of the effects that this will have on the production of the final good and the incentives of his neighbor to price his smoke rights. Thus, if resident i prices his smoke rights at p_i, firms will act as if their marginal cost of production is $\sum_{i=1}^{n} p_i$. Hence, in this case, the price in the final goods market is (because of our assumption of perfect competition in the final good market) $\sum p_i$. Then, output is q(p) = a - b ($\sum p_i$) and revenues from sales for the households are r_i = $p_i q(p)$ giving utility m* + r_i - q/n. It is straightforward to check that given the price set by other residents, the best price for resident 1 is given by:

$$(2.1) \qquad p_1(p_{-1}) = \frac{1}{2b}\left[a + \frac{b}{n} - b \sum_{i \neq 1} p_i\right]$$

From this it is straightforward to calculate that the equilibrium prices for the smoke rights are given by

$$p_i = \frac{\frac{a}{b} + \frac{1}{n}}{n + 1} \text{ all i.}$$

Hence, from the firms point of view, the marginal cost of production (and hence the price of the final good as well) is $\frac{n}{n+1}\left[\frac{a}{b} + \frac{1}{n}\right]$.

As a point of comparison, it is useful to consider the case where the residents are not adversely affected by the smoke at all (i.e., utility is given by U(m,s)=m). In this case, an easy calculation shows that the equilibrium price of the smoke rights is p_i = a/(n+1)b. Hence the marginal cost of production of the final good (and its price) is given by $\frac{n}{n+1}\left(\frac{a}{b}\right)$. Note that this even exceeds the monopoly price of the output (which is a/2b). Thus, the equilibrium is even less efficient than monopoly using the standard producer plus consumer surplus measure.

A similar argument holds when we recognize that what the residents are selling to the firms is a productive input to the firm. It is as if each resident is selling an output which costs \$1/n per unit to produce (the dollar cost of the loss of one unit of smoke rights) and is maximizing profits. If these two residents could get together and sell their inputs to the firms jointly, they could do much better. In this case, it is easy to see that the best price for them to charge is $(1+a/b)/2(n+1)$ apiece giving the final product price of $(1+a/b)/2$. It is easy to see that this price is lower than that calculated as the equilibrium above (since $a/b > 1$).

It is probably not surprising that the equilibrium is inefficient since we have given the residents an element of monopoly power which they exploit. In light of this, a natural question to ask is what happens when the number of residents is large. From (2.1) we see that as the population size grows, the equilibrium price of the final product converges monotonically to the reservation price, a/b.

Note that through our choice of normalization, we have constructed a sequence of economies in which the Lindahl equilibrium is unchanged as a function of the population size, n, in the sense that the output and price of the final product are independent of n. Of course, the prices of the individual smoke rights do change with the population size as they are given by $1/n$ in the n-th economy. (Thus, we have adopted the normalization recommended by Milleron (1972). Roberts (1976) uses an alternative normalization in which the reservation price is na/b and $u = m - s$ for all n.)

This is not the end of the story, however, as the next example shows.

Example 2: This example is a slight modification of Example 1. Consider an economy like that considered above where the only difference is that there are two towns each with n residents and many price-taking producers of the final product. To make the problem as simple as possible, we abstract away from the fact that the final product is now being produced at different locations. That is, we will proceed as if transportation costs for the final product are zero and hence consumers treat output at the two locations as perfect substitutes. Roughly speaking, this is equivalent to assuming that neither town has a specific productive advantage over the other. This is, of course, a strong assumption, but greatly simplifies matters.

Assume that demand for the final good is exactly as in Example 1. Finally, assume that smoke from the factory in city 1 has no effect on the residents of city 2 and vice versa.

It is easy to see that there are many competitive equilibria for this economy which differ only in the proportion of final output produced in the two cities. Thus, in all of the equilibria, $p^* = 0$ and $q^* = a$, but any combination of quantities which sum to a can arise in equilibrium.

Now, let us introduce new individualized pollution markets as we did in Example 1. It is easy to see that there is a fundamental difference between this economy and the one considered earlier--in order to produce a unit of output a firm need not buy one unit of smoke rights from each household, rather it need only purchase one unit from each household in the city in which it plans to produce. It is easy to see how this change might have a substantial impact on the nature of the equilibrium--now firms may be able to get residents in the two cities to compete against one another in pricing their smoke rights. It is important to note that this is so even though in some sense markets are just as thin as they were before--individuals still have monopoly power over the smoke rights at their individual locations--but now these products are no longer so complementary. In fact, there is now a perfect substitute for each of the smoke rights and moreover a perfect substitute for an exhaustive list of all of the smoke rights of each city. That is, there is now a perfect substitute for each of the two production sites (i.e., cities).

Formally, let s_i be the amount of smoke consumed by consumer i, i = 1, ..., 2n, where for simplicity we will assume that residents 1 through n live in city 1 and the rest live in city 2. Then, we assume that

$$U_i(m,s_1,...,s_{2n}) = m - s_i/n.$$

It is immediate that the Lindahl equilibria (there are many) of this economy have each smoke right priced at 1/n and the final output priced at 1.

As before, we assume that competition proceeds by residents setting prices for their individual smoke rights, which are then taken as given by the producing firms. We assume that there are already producing firms in both cities (this is presumably irrelevant--we could have them choose their sites as a function of the announced smoke rights prices with the same result) and that as before, they behave as perfect competitors in that they charge marginal cost for their output. Thus, letting p_i be the price announced by resident i and Π_1 be the sum of the prices in city 1 and Π_2 the sum of prices in city 2 we see that the payoff to resident i as a function of the announced prices is given by:

$$U_i(p_1,\ldots,p_{2n}) = \begin{cases} m_i + (a-b\Pi_1)(p_i-1/n) & \text{if } \Pi_1 < \Pi_2 \\ m_i + (a-b\Pi_1)(p_i-1/n)/2 & \text{if } \Pi_1 = \Pi_2 \\ m_i & \text{if } \Pi_1 > \Pi_2 \end{cases}$$

if i = 1, ..., n and similarly if i = n + 1, ..., 2n.

Note that we have assumed that only output from the cheaper city is sold (i.e., the one with the lower cost of production Π) and that in case of ties, the market is split evenly. These are assumptions which are familiar in economics, we discuss them in more detail below.

It is easy to see that the Lindahl equilibrium (i.e., $p_i=1/n$ for all i) for this economy is a symmetric equilibrium of the game as given--if any individual raised his price, no production would take place in his city and hence he would be no better off than before, of course he could not lower it and hence this is an equilibrium. As it turns out, this is the only symmetric equilibrium for the game in which there is positive output of the final product. The argument for this is familiar--if all charge some price higher than 1/n any individual can, through lowering his own price only slightly, bring all of the production of the industry to his city. Since the marginal benefit of this change is 1 and the marginal cost is 1/n, the consumer will be better off by making this change.

Note that there at least two other types of equilibria as well. These arise due to a difficulty in coordinating price offers among residents in a given town. The first of these occurs when for all combinations of n - 1 residents of each town, the sum of the offered prices is larger than the choke-off price, a/b. In this case, it is easy to see that no individual resident can, by lowering his price unilaterally, lower the marginal cost of production in his town to the point that any firm could break even and sell a positive quantity. The second type occurs in a similar fashion. That is, if all of the residents of one town charge the one town equilibrium prices outlined in Example 1, and all groups of n - 1 residents of the other town charge prices summing to more than that of the first town, an equilibrium is obtained. Again, since no individual in the second town can lower his price enough unilaterally, there will be no production in that town.

Some comments

(1) Although we have proceeded as if this was all original, that is not quite true. Example 1 was in fact considered by Cournot in a quite different setting. Cournot (in chapter 9) framed the problem as one of complementary monopoly. He considered a model in which there is a monopolist producer of zinc, a monopolist producer of copper and a perfectly competitive market for the production and sale of the alloy, brass. He assumed that demand for the final product (brass) is linear and that the only possible use of copper and zinc is for the production of brass.

It is easy to see that this is formally equivalent to the example we considered above with two residents. (Note that Cournot considered the analogue of the case we outlined above in which the disutility from smoke on the part of the residents is zero.) In addition, he considered the n-input complementary monopoly problem analogous to our example.

(2) Many people will probably not find the above example too surprising. That is, even though the number of residents is increasing, the markets for smoke rights are highly individualized and hence intrinsically "thin." Thus, although it might be surprising that the problem gets worse as the number of residents grows, it should not be surprising that the inefficiency does not go away. This is clearly related to the view adopted by Arrow (1970), where he states that although you can get rid of the externality problem by creating markets for smoke rights, these markets are likely to be thin so that perfect competition may not be the correct notion of equilibrium to employ. One must be careful in making this judgement too hastily however. The fact that only one individual is selling in the market for the individualized smoke rights does not necessarily imply he has market power he can exploit. Indeed, the model examined in Mas-Colell (1975), Hart (1979), and Jones (1984) gives an example in which it is possible for each individual to have sole ownership of some good, yet due to the fact that good substitutes exist the individuals have no market power. In connection with the example presented above, the thin markets argument by itself would lead one to believe that individual residents have market power in the

local real estate market as well. It is easy to see that this will not be true if n is sufficiently large and potential residents have reasonable utility functions over land. That is, although individuals have monopolies over their specific sites, it is reasonable to expect that different plots are very good substitutes for one another. All this is not to say that the thin markets argument is wrong, just that it is much too subtle an issue to pass over without thought. In summary, both the monopoly power on the part of the individuals and the fact that the goods are complementary in the production process seem crucial in the example. Of course, example 2 highlights the importance of the complementarities even more.

(3) There is another reason that the results of example 1 might be expected. This will be familiar to all in public finance. A simple reinterpretation of the game we have outlined above will make this clear. Suppose that instead of assigning property rights and letting residents price these rights as they see fit, we had instead asked residents to announce their per unit damages due to the smoke over their property. They would then be paid the product of this amount with the total quantity of smoke produced. This program would be financed by "taxing" the final good by an amount (ad valorem) equal to the total per unit announced damage. If we maintain our assumption that firms in the output market are perfect competitors and that the residents fully and correctly anticipate the effects of their announcements on the output market, we see that the game that we get is formally equivalent to the one we have outlined above.

Of course, it should not surprise anyone the outcome of the game is inefficient when stated in this way. The reason is simple--there is a free-rider problem. This is exactly what seems to be captured by the strategic pricing formulation of the situation which we have presented above--each resident has an incentive to free-ride on the others honest revelations of their cost due to pollution (i.e., truth telling is not an equilibrium) and in so doing, they reach an outcome which is, in some ways, terribly inefficient. Thus, it should be no surprise that the inefficiency does not go away in large economies as this is exactly the situation in which the free-rider problem is commonly believed to be most severe.

Moreover, this allows us to see that the free-rider problem is formally equivalent to one of market power. This is a theme which will recur in the next section.

(4) It is interesting to note the role of the strategic formulation of the equilibrium problem considered here. The difference in the efficiency results between the perfectly competitive equilibria in the original public goods economy and the private markets version arises because of the differences in perceptions about the constraints faced by decision makers. That is, in the private markets version of the economy, agents act as if they could set smoke levels independently of the actions of the other agents. Thus, the <u>fact</u> that this is not possible is imposed only in equilibrium, not in the individual decisions. Introducing strategic play reintroduces these social constraints in the individual decision makers problems giving rise to the adverse effects on welfare.

(5) We should point out that the connection between public goods economies and perfect complementarities in private goods economies has been used earlier in Foley (1967), Milleron (1972) and Starrett (1972). In these papers, it was used primarily as a device to facilitate the proof of the existence of Lindahl equilibrium. However, the relationship between this equivalence and problems of market power was not exploited.

(6) The existence of complementarities in the associated private goods economy seems to capture precisely the notion common in the public goods literature of nonexclusion (that is complementarities in outputs rather than in inputs). This suggests a way of defining partially public goods as the degree of complementarity between the individualized goods in the associated private goods economy. This also raises the question of whether the results suggested by these examples hold up in this more general class of economies.

The problems caused by complementarities are recurrent themes in the literature on the foundations of perfect competition. Examples include Hart (1980), Makowski (1980), and Jones (1984) and (1985) in addition to the reference to Cournot mentioned above.

(7) Note that what we have done with Example 2 is the standard Bertrand trick--have two players compete through prices with constant costs (constant marginal disutility of smoke). Hence, the standard objections to this approach apply. That is, is efficiency with only two towns is too strong to be plausible? Might they collude, etc? This is explored in more detail in section 5.

(8) Note that if perfect exclusion was possible at the town level in that the factories could perfectly and costlessly limit the pollution in one city to only one resident's property the strategic form presented here would give rise to the Lindahl equilibrium with only one city and two residents due to the standard Bertrand argument. This is just further evidence that the problem here is the complementarities--with two residents and costless exclusion the two smoke rights become perfect substitutes. Of course, costless perfect exclusion is a very special case (this is exactly what we assumed by having two cities with no overlap in smoke) but it does serve as a reminder of the importance of the complementarities.

(9) In terms of the interpretation of Coase suggested earlier as describing a mechanism for the provision of public goods, we can now give a better (although still very imperfect) summary of its usefulness. It is clear from Example 1 that this mechanism will not work in full generality. Further, it is clear that the mechanism can do very badly in "large economies." However, Examples 1 and 2 taken together suggest that the problem with the large economies result of Example 1 is the high degree of complementarity. This suggests another definition of what it means for an economy to be large in the context of public goods production (i.e., as in Example 2). Further it suggests that there may well be an interesting class of large economies in which the Coase mechanism (and perhaps other mechanisms as well) perform the job of allocating the costs of externalities reasonably well. We should emphasize at this point that this seems (accord-

ing to the examples) to be limited to those economies in which both the externalities are local in nature and there are a large number of potential locations. Thus, it seems quite plausible that the mechanism will work for Meade's (1952) beekeeper and apple orchard example (if the price charged by the orchard owner is too high, go to another orchard). It seems reasonable to expect that it might work in Coase's example of candy maker and noise creation (move the candy factory). It seems hard to believe that it presents a reasonable solution to either the acid rain problem or the problem of allocating funds for national defense.

The reader will note that the force of this comment is very much in the spirit of Tiebout (1956). In fact, Tiebout is quite explicit (in his commentary) that he views his contribution as one of determining when Coase's argument is likely to be correct.

3. A More General Approach

We now provide a more general framework for analyzing public goods provision in a decentralized, or market-like environment. First we consider the global externalities problem. In section 5 we extend our analysis to local public goods. We transform the environment with externalities into an economy with well-defined property rights and a given technology. This allows us to analyze the outcomes of general market-like mechanisms. To this end consider an economy described as follows. The commodity space is R^{n+1}. The last commodity is interpreted as a numeraire consumption good. The remaining commodities, $i = 1, \ldots, n$, are interpreted as consumption of smoke rights. There are n consumers. Their consumption sets are given by $X_i = R_+^{n+1}$. The preferences of consumer i are given by

$$V_i(x) = x_{n+1} + u_i(x_i).$$

Notice that each consumer cares only about consumption of his smoke rights, that is smoke produced over his location. Let x_j^i denote the consumption of good j by consumer i. The endowment vector of consumer i, denoted by w^i is given by

$$w_i^i = \bar{q}_i > 0,\ w_{n+1}^i = \bar{m}_i > 0 \text{ for all i, and}$$

$$w_j^i = 0,\ i \neq j,\ i,\ j = 1,\ \ldots,\ n.$$

The technology set for this economy is given by

$$Y_n = \{y \in R^{n+1} | y_1 = y_2 = \ldots = y_n = -q, y_1, y_2, \ldots, y_n \leq 0, y_{n+1} \geq 0 \text{ and}$$

$$\text{and } R(q) \geq y_{n+1}\}.$$

Note that the description of the endowments (which are the property rights) conveys to each agent monopoly power over smoke produced at his location. The complementarities inherent in public goods are captured in the description of the technology. To relate this economy to the demand functions specified in the examples, we can set $R(q) = qD^{-1}(q)$.

A feasible allocation is defined by

$$\sum_{i=1}^{n} x^i - y = \sum_{i=1}^{n} w^i$$

and

$$y \in Y_n$$

Having completed markets and allocated smoke rights to individual residents we can now "let markets work." It is clear that competitive equilibria yield efficient outcomes for this economy. It is also clear that the monopoly power inherent in this economy makes competitive behavior a suspect assumption. Therefore we consider a more general description of the workings of a marketplace. Fix the number of players n. Each player chooses an action a_i from an action set A_i, $i = 1, \ldots, n$. Let a denote the vector of actions.

A mechanism for our economy is a collection of action sets and outcome functions x(a) and y(a) which map the vector of actions into the space of feasible allocations. We assume there is a class of allowable payoffs U_i. Let $u = (u_1, \ldots, u_n) \in U = (U_1 \times \ldots \times U_n)$. Let N(u) denote the Nash equilibrium correspondence given a mechanism.

Consider now an alternative mechanism with action sets for each player given by U. The interpretation is that each player reports the utility functions of all players in the economy. Let t_i (the "type" of player i) denote the vector of utility functions reported by player i and let t denote the vector of types reported by all players. A revelation mechanism is a collection of type sets and allocation functions $x^r(t)$, $y^r(t)$ which map reported types into the space of feasible allocations. We now show that the equilibrium outcomes of any mechanism can be implemented as equilibrium outcomes of a revelation mechanism. Let ψ denote a selection from the Nash equilibrium correspondence, N(u), of an arbitrary mechanism. Define the outcome function in the revelation mechanism by $x^r(t) = x(\psi_1(t_1),\psi_2(t_2),\ldots,\psi_n(t_n))$ and let $y^r(t)$ be defined similarly. Note that in this formulation, the action chosen for player i is the equilibrium action for the environment in which he claims to be playing. Since the vector of actions implied by ψ constitute an equilibrium for the original mechanism, it follows that truth-telling is also an equilibrium of the revelation mechanism and yields the same outcomes. We have proved the following theorem.

Theorem 1: (Revelation Principle)

Suppose the Nash correspondence for some mechanism is nonempty for all $u \in U$. Then there is a revelation mechanism for which truth telling is an equilibrium yielding the same outcomes.

We therefore restrict attention to revelation mechanisms. Exactly the same logic applies even with private information. In this case, a revelation mechanism yields the same outcomes as the Bayesian Nash correspondence of an arbitrary mechanism.

The space of possible utility functions we consider in the revelation mechanism is $U_i = \{$utility functions over smoke rights on $[0,Q]$ which are nondecreasing, with $u_i(0)=0\}$.

Associated with a revelation mechanism for our economy are outcome functions $x(t)$ and $y(t)$ which satisfy feasibility and $y(t) \in Y_n$. It will be convenient to let m_i denote the consumption of the numeraire consumption good by consumer i and to let q denote the amount of smoke produced. Then these outcome functions must satisfy

$$(3.1) \qquad \sum_i m_i(t) \le \sum_i \bar{m}_i + R(q(t))$$

and

$$(3.2) \qquad \sum_{i=1}^{n} x_j^i(t) = \bar{q}_j - q(t), \; j = 1, \ldots, n$$

for all $t \in U^{n \times n}$ where $\bar{m}_i$ and $\bar{q}_j$ denote the endowments of the consumption good and smoke rights respectively.

The payoffs in the revelation mechanism are then given by

$$V_i(\hat{t}) = m_i(\hat{t}) + u_i(x_i^i(\hat{t}))$$

for all $\hat{t} \in U^{n \times n}$. For notational convenience, let $q_i = x_i^i$. It will also be convenient to let $\tilde{u}$ denote the vector of truthful announcements.

We can define a sequence of mechanisms as the population size n changes. Note, of course, that the underlying commodity space and the spaces over which the outcome functions are defined also change. As we change the population size, we also allow the utility functions to change. We denote the utility function of consumer i by $u_i(q_i;n)$. We now prove that under a set of axioms, the equilibrium output of the revelation mechanisms converges to zero. Suppose therefore that the sequence of mechanisms satisfies

A1. Voluntary Trade: For all n, for all $u^n \in U^{n \times n}$

$$m_i(\tilde{u},n) + u_i(q_i(\tilde{u},n);n) \ge \bar{m}_i + u_i(\bar{q}_i;n).$$

A2. Continuity: For all $\delta > 0$, there exists $\varepsilon > 0$ such that for all n, for all i = 1, ..., n

$$|q(\tilde{u},n)-q(\tilde{u}_{-i},t_i,n)| < \delta$$

if

$$|t_{ij}(x)-u_j(x)| < \varepsilon \text{ for all } x \in [0,Q],$$

for all $j = 1, \ldots, n$.

Axiom A1 is one way of representing the idea that each consumer has a right not to be affected by smoke unless he consents. Axiom A2 (which plays a central role in the proof) requires that no mechanism punish deviations from truth-telling too severely. For example, mechanisms which simply impose efficient allocations and severe penalties for deviation are disallowed. One such example is a mechanism which gives each consumer his endowment if there is any disagreement among consumers about reported utility functions. If all consumers agree in their reports, the mechanism computes the Lindahl equilibrium for such an economy and gives each consumer the associated allocations. While this mechanism is extremely discontinuous, it is possible to construct similar mechanisms which are continuous, but punish deviations severely enough. A key feature of A2 is that we require that mechanisms be uniformly continuous across the sequence of economies. Thus the power of any individual to affect aggregate outcomes by small deviations is limited uniformly across the sequence of economies.

An alternative, and stronger, condition is that a small change in any consumer's report has a correspondingly small effect on the allocations received by every other consumer. This latter condition might perhaps be more suitable for environments where the notion of aggregate outcome is more difficult to define. Mechanisms satisfying A1 and A2 seem to us to capture two key features of market mechanisms. Hence we say that a sequence of mechanisms satisfying A1 and A2 is decentralized.

In section 5 we show that the set of decentralized mechanisms is nonempty. We consider a price-setting game as in example 1. It can be verified that the revelation mechanism satisfies A2.

We now state our main theorem

Theorem 2: (Decentralized Mechanisms yield zero output in the limit)

Consider a sequence of utility functions $u_1(1)$; $u_1(2)$, $u_2(2)$; $u_1(3)$, $u_2(3)$, $u_3(3)$; ... . Suppose the revenue function $R(q)$ is bounded above by $K < \infty$. Assume truth telling is an equilibrium of the revelation mechanism for each n and denote the equilibrium output level by $q^n = q(\tilde{u}(n),n)$. If A1 and A2 are satisfied $\lim_{n\to\infty} q^n = 0$.

Proof: Suppose not. Choose subsequences if necessary so that $\lim_{n\to\infty} q^{n_k} = d > 0$. In what follows, we drop the subscript k for notational convenience. Since $R(q)$ is bounded, using (3.1) we have that

$$\sum_i m_i(\tilde{u},n) \le \sum_i \bar{m}_i + K.$$

From equation (3.2) using $q(\tilde{u},n) \ge 0$ it follows that $q_i(\tilde{u},n) \le \bar{q}_i$. for all i. Hence from A1 we have that $m_i(\tilde{u},n) \ge \bar{m}_i$

for all n and for all i. This implies there is some sequence i_1, i_2, ..., such that

$$\lim_{n\to\infty} \left[m_{i_n}(\tilde{u},n)-\bar{m}_{i_n}\right] = 0.$$

From A1 therefore

$$\lim_{n\to\infty} u_{i_n}\left(q_{i_n}(\tilde{u})\right) = u_{i_n}(\bar{q}_{i_n}).$$

Choose $\delta > 0$ so that $d - \delta > 0$. From A2 we have that there exists $\varepsilon > 0$ such that

$$|q(\tilde{u},n)-q(\tilde{u}_{-i},t_i,n)| < \delta \text{ if } |t_{ij}-u_j| < \varepsilon \text{ for all } j.$$

Consider an alternative strategy for i_n given by truthful reporting of the utility functions of other players and

$$\hat{u}(x,n) = u_{i_n}(x,n) + \frac{\varepsilon}{Q}x.$$

The payoffs for i_n are then given by

$$V_{i_n}(\tilde{u}_{-i_n},\hat{u}) = m_{i_n}(\tilde{u}_{-i_n},\hat{u},n) + u_{i_n}\left(q_{i_n}(\tilde{u}_{-i_n},\hat{u},n)\right).$$

From A1 we have that

$$m_{i_n}(\tilde{u}_{-i_n},\hat{u},n) + \hat{u}_{i_n}\left(q_{i_n}(\tilde{u}_{-i_n},\hat{u},n)\right) \geq \bar{m}_{i_n} + \hat{u}_{i_n}(\bar{q}_{i_n}).$$

Using this fact, the difference in utilities between this alternative strategy and truth telling is given by

$$\begin{aligned}\Delta_n = V_{i_n}(\tilde{u}_{-i_n},\hat{u}) - V_{i_n}(u) \geq &\left[u_{i_n}\left(q_{i_n}(\hat{u},n)\right)-\hat{u}_{i_n}\left(q_{i_n}(\hat{u},n)\right)\right] \\ &+ \left[\bar{m}_{i_n}+\hat{u}_{i_n}(\bar{q}_{i_n})\right] \\ &- \left[m_{i_n}(u)+u_{i_n}\left(q_{i_n}(u)\right)\right]\end{aligned}$$

where we have suppressed the dependence on $\tilde{u}_{-i_n}$ for convenience. Adding and subtracting $u_{i_n}(\bar{q}_{i_n})$ to the right side we get after rearranging that

$$\Delta_n \geq [u_{i_n}(q_{i_n}(\hat{u},n))-u_{i_n}(\bar{q}_{i_n})] - [\hat{u}_{i_n}(q_{i_n}(\hat{u},n))-\hat{u}_{i_n}(\bar{q}_{i_n})]$$
$$+ [\bar{m}_{i_n}+u_{i_n}(\bar{q}_{i_n})-m_{i_n}(u)-u_{i_n}(q_{i_n}(u))].$$

We have already argued that the term in the last square brackets goes to zero. Consider the terms in the first two brackets. These are given from the definition of $\hat{u}$ by

$$\frac{\varepsilon}{Q}[\bar{q}_{i_n}-q_{i_n}(\hat{u})].$$

From feasibility we have that

$$\bar{q}_{i_n} - q_{i_n}(\hat{u}) \geq q(\hat{u},n).$$

From A2 we have that $q(\hat{u},n) > q(u,n) - \delta$. Since $q(u,n)$ converges to a positive constant d we have that

$$\lim_{n\to\infty} \Delta_n \geq d - \delta > 0.$$

Hence, the difference in utilities is strictly positive for large enough n. ◊

Monopoly Power and the Free Rider Problem:

We have transformed our environment into a private ownership economy and shown that decentralized mechanisms lead, in general, to extremely inefficient outcomes with a large enough population. In this formulation, monopoly power in the ownership of smoke rights plays a central role in generating inefficient outcomes. Alternatively, we could have formulated this as a mechanism design problem in an environment with externalities. In this case, the preferences of the agents over smoke are given by

$$m_i - v_i(q)$$

where q is the amount of smoke produced in the town and v_i is a nondecreasing function on $[0,Q]$. Analogously to the privatized economy, a revelation mechanism is a collection of type sets for agents and outcome functions m(t), q(t) which specify consumption vectors of the numeraire good and production of smoke respectively. Feasibility requires that a mechanism satisfy (3.1). Consider a sequence of mechanisms as the population size, n, changes. Suppose that the sequence of mechanisms satisfies

A1. Veto Power: For all n, for all $\tilde{v} \in U^{n\times n}$

$$m_i(\tilde{v},n) - v_i\big(q(\tilde{v},n);n\big) \geq \bar{m}_i - v_i(0;n)$$

A2. Continuity For all $\delta > 0$, there exists $\varepsilon > 0$ such that for all n, for all i = 1, ..., n

$$|q(\tilde{v},n)-q(\tilde{v}_{-i},t_i,n)| < \delta$$

if $|t_{ij}(x)-v_j(x)| < \varepsilon$ for all $x \in [0,Q]$ for all j = 1, ..., n.

The veto power, or individual rationality condition A1' makes more explicit that the voluntary trade axiom A1 is a description of the legal environment underlying the privatized economy. The obvious question of alternative legal environments is addressed in section 4 below.

It is easy to prove along the lines of theorem 2 that if the revenue function is bounded, the equilibrium output of smoke converges to zero. This formulation of the problem shows that our result of extreme inefficiency does not depend upon the particular way that we have privatized the public goods economy. In fact, the privatized public goods economy formulation is in some ways more general than the public goods formulation. To see this, consider the following privatized economy. Let the endowment of smoke rights for each agent be the same, say $\bar{q}$. Restrict attention to mechanisms which allocate positive consumption rights at location i only to consumer i. That is, let $x_j^i = 0$ if $i \neq j$. Define

$$u_i(x_i^i) \equiv - v_i(\bar{q}-x_i^i)$$

Clearly, theorem 2 continues to apply. In the public goods formulation, the free rider problem is made explicit. The privatized economy makes explicit the role of monopoly power. This is the sense in which the two problems are equivalent.

Normalization of Demand

We turn now to the particular normalization of demand we have chosen. As the population size n changes we have kept demand for the final good unaffected. Suppose now that demand for the final good grows at rate n. We show that the ratio of equilibrium output to the efficient level of output goes to zero as n gets sufficiently large. Thus, while output itself need not go to zero, it is arbitrarily far from the efficient level.

Suppose therefore that $R_n(q) = nR(q)$. Suppose that there is a number K such that for all n

$$R_n\big(q(\tilde{u},n)\big) \le nK.$$

Suppose as before that there is a truth-telling equilibrium of the revelation game associated with this mechanism. Consider the following transformed game. We use carets to denote the transformed game. The strategy spaces are unaltered. The outcome functions for the transformed game are defined by

$$\hat{m}_i(\tilde{u},n) = \frac{m_i(\tilde{u},n)}{n}$$

$$\hat{q}(\tilde{u},n) = \frac{q(\tilde{u},n)}{n}.$$

For the transformed game, the bounded revenues condition must also be changed. This now reads

$$R\big(\hat{q}(u,n)\big) \le K.$$

We now prove that if (m^n,q^n) is an equilibrium outcome of the original game $(\hat{m}^n,\hat{q}^n)$ is an equilibrium outcome of the transformed game. The only condition we need to verify is feasibility in the transformed game. But this is immediate since $R_n(q) = nR(q)$. Recall that for the transformed game, the equilibrium output level $\hat{q}$ converges to zero. Now, in general, in the transformed environment, the efficient level of output is uniformly bounded away from zero. Hence, in general, in the original environment, the ratio of equilibrium output to the efficient level of output converges to zero.

Private Information

It turns out that with appropriate modifications, we obtain results similar to theorem 2 when individuals have private information about their valuations for smoke. Of course, in this case the type of an individual is only his utility function. The revelation mechanism requires each individual to report his utility function. Suppose that $(u^1,u^2,\ldots)$ is a random variable drawn from some distribution an $U^1 \times U^2 \times \ldots$. The axioms now read:

A1'. Voluntary Trade: For all n, for all i, for all $u_i \in U_i$

$$E[m_i(u,n)|u_i] + E[u_i(q_i(u,n);n)|u_i] \geq \bar{m}_i + u_i(\bar{q}_i;n).$$

A2'. Continuity: For all $\delta > 0$, there exists an $\varepsilon > 0$ such that for all n

$$|E(q(u,n)|u_i)-E(q(u_{-i},\hat{u},n)|u_i)| < \delta$$

if $|\hat{u}(x)-u(x)| < \varepsilon$ for all $x \in [0,Q]$.

Given these axioms, it is straightforward to prove that $q^n \to 0$ in probability.

Theorem 3: Consider a random variable $(u^1,u^2,\ldots)$ drawn from a given distribution on $U^1 \times U^2 \times \ldots$. Assume truth telling is an equilibrium of the revelation game and denote the equilibrium output level by the random variable $q^1 \times q^2 \times \ldots$.

If A1' and A2' are satisfied $q^n \to 0$ in probability as $n \to \infty$.

Proof: Consider a draw of the random variable $(u^1, u^2, \ldots)$. The revelation mechanism must satisfy A1' and A2' for this realization of the random variable. The obvious modification of the proof of theorem 2 shows that for this realization, $q^n \to 0$. Since A1' and A2' hold with probability one, the result follows. ◊

When there is no private information, it is clear that mechanisms which satisfy our axioms lead, in general, to inefficient outcomes. However, with private information, interim efficient mechanisms [defined as in Holmstrom-Myerson (1983)] cannot punish individuals too severely for small deviations from truth-telling. Hence, it is possible that all interim efficient mechanisms yield zero output in the limit. Rob (1987) proves such a result with an indivisible public good and under the assumption that the density function from which the utility vector is drawn is bounded. We construct an example to show that interim efficient mechanisms need not yield zero output in the limit. The example also demonstrates the role played by uniform continuity in theorem 3.

Example 3: Suppose the utility functions are given by

$$U_i(m,q) = m - \theta_i q$$

where θ_i is identically, independently distributed across i. θ_i is distributed uniformly over [0,1/2n] with density p_n and uniformly over [1/2n,1] with density r_n given by

$$r_n = (p_n/2n)/\left(1 - \frac{1}{2n}\right).$$

The inverse demand function is given by

$$D^{-1}(q) = 1 - \frac{q}{4}.$$

Our aim is not to characterize incentive efficient mechanisms. Rather we construct a particular mechanism for which the sum of the expected utilities over all individuals is bounded away from zero with positive probability. Thus, we restrict attention to efficient mechanisms which maximize the sum of the expected utilities of the agents. If such efficient mechanisms yield zero output in the limit they yield zero utility. Then we have a contradiction and therefore the desired result.

Consider, therefore, the following mechanism. If all agents report $\theta_i \in [0,1/2n]$ then $q = 1$, otherwise $q = 0$. (For this example, the efficient output level under full information is 1 when $\theta_i = 1/2n$ all i.) If $q = 1$, each agent receives an equal share of the revenues.

It is clear that for any consumer i, if $\theta_i \leq 1/2n$ a dominant strategy is to report the true value of θ. Hence, in this case

$$E[V_i^n | \theta_i \leq \frac{1}{2n}] \geq \frac{1}{n} (p_n/2n)^{n-1} \frac{3}{4} - \theta_i$$

$$\geq \frac{1}{n} [\frac{3}{4} (p_n/2n)^{n-1} - \frac{1}{2}].$$

The sum of the expected utilities then satisfies

$$\sum_{i=1}^{n} EV_i^n \geq \frac{3}{4} (p_n/2n)^{n-1} - \frac{1}{2}.$$

It is clear that there are many sequences p_n which yield welfare levels bounded away from zero. For example, suppose p_n = $2n(1-1/n)$. Then

$$\lim_{n\to\infty} \sum_{i=1}^{n} EV_i^n = \frac{1}{4}.$$

Furthermore, the probability that $\theta_i \leq 1/2n$ for all i converges to unity. Efficient mechanisms must yield at least as high a utility level. Hence, the output of smoke cannot converge to zero. The axiom violated by efficient mechanisms in this example is A2'. The particular mechanism we consider is discontinuous at $\theta_i = 1/2n$. However, it is straightforward to prove that for fixed n, the efficient mechanism yields output levels which are continuous in θ. But this sequence of mechanisms does not yield output levels uniformly continuous in θ. ◊

4. Alternative Property Rights

Our results emphasize the role of monopoly power in producing inefficient outcomes. However, there are two sources of monopoly power in the economy considered in section 3. First the distribution of endowments, or property rights, gives each consumer monopoly power over smoke produced at his location. Second, each person cares only about smoke produced over his location. Thus there is a source of monopoly power arising from preferences. We wish to disentangle the effects of these two sources. A natural question (suggested by Coase's paper) is to examine the provision of public goods under alternative property rights distributions. In particular, since we wish to understand the problem caused by monopoly power arising from preferences it is convenient to endow individuals outside the town with smoke rights and have them behave competitively. This eliminates the monopoly power arising from ownership. One can think of the individuals who own the smoke rights as neither caring about smoke nor the final good. Alternatively, we can think of the government as auctioning off the smoke rights.

The simplest interpretation of what follows is that the government auctions off a fixed quantity of smoke rights. However, we consider more general mechanisms where the quantity of smoke rights issued depends upon the reported types of individuals.

We show that if there are many residents in the town, each of whom cares very little about the smoke (although the aggregate loss is possibly significant) the residents don't buy any of the smoke rights. As before, we restrict attention to revelation games. The strategy space for each player i is given by U_i consisting of nondecreasing, smooth functions on [0,Q]. These functions are interpreted as utility from smoke rights. Let $U^n = U_1 \times \ldots \times U_n$ where the number of residents in the town is n. We define a mechanism slightly differently here than earlier.

The outcome functions which constitute a mechanism are now described. The same quantity of smoke rights is issued for each location and is denoted by $\bar{q}(\tilde{u})$. The amount of smoke produced is denoted by $q(\tilde{u})$. Denote the consumption of smoke rights at location j by consumer i by $x_j^i(\tilde{u})$. Smoke rights not purchased by consumers are retained by the government or the demand sector of the economy and used to produce smoke. Consumers in the town pay a price $p_i(\tilde{u})$ per unit of smoke rights at location i. A mechanism is then defined as a set of outcome functions $p(\tilde{u})$, $x(\tilde{u})$, $q(\tilde{u})$, and $\bar{q}(\tilde{u})$ satisfying

$$(4.1) \qquad \sum_{i=1}^{n} x_j^i(\tilde{u}) = \bar{q}(\tilde{u}) - q(\tilde{u}),\ j = 1, \ldots, n$$

and

$$(4.2) \qquad \sum_{i=1}^{n} p_i(\tilde{u}) \sum_{j=1}^{n} x_i^j(\tilde{u}) + R\big(q(\tilde{u})\big) = \bar{q}(\tilde{u}) \sum_{i=1}^{n} p_i(\tilde{u}).$$

Equation (4.2) deserves some comment. Implicit in this feasibility condition is that the amount of smoke rights made available is valued by the government or the demand sector at the prices of smoke rights at each location. Thus, in effect, the mechanism does not permit price discrimination between the residents of the town and the owners of the smoke rights. Equation (4.2) is then derived by noting that the first term on the left is total expenditures by town residents on smoke rights and the second term is revenues from sale of the consumption good. Again, for convenience, let $q_i = x_i^i$.

We define a sequence of mechanisms as the population size changes exactly as earlier. To prove our next theorem we need stronger assumptions than those made earlier. Suppose that the sequence of mechanisms satisfies

B1. <u>Voluntary Trade</u>: For all n, for all i = 1, ..., n, for all $\tilde{u} \in U^{n \times n}$,

$$u_i\big(q_i(\tilde{u},n);n\big) - \sum_j p_j(\tilde{u},n) x_j^i(\tilde{u},n) \geq 0.$$

B2. Differentiable Outcome Functions: For all n, for all $u \in U^n$, for all i, $x^i(\tilde{u}_{-i},\alpha u)$, $p(\tilde{u}_{-i},\alpha u)$ are smooth functions of α when they are positive and there are real numbers k_1,k_2 such that

$$\left|\frac{dx_j^i}{d\alpha}\right| < k_2 < \infty \text{ and } \left|\frac{dp_i}{d\alpha}\right| > k_2 > 0$$

for all i and j = 1, ..., n.

A sequence of mechanisms which satisfies these axioms is said to be smooth and decentralized. We now prove a theorem that essentially states that consumers within a town do not purchase any smoke rights in the limit.

Theorem 4: (Smooth Decentralized Mechanisms Lead to Inefficient outcomes.)

Consider a sequence of smooth utility functions for consumers $u_1(q,n)$, ..., $u_n(q,n)$ with $\lim_{n\to\infty} du_i/dq = 0$ for all $q \in [0,Q]$. Suppose also that there is a number K such that $R(q)/q \le K$. Assume truth-telling is an equilibrium of the revelation game for each n and denote the equilibrium amount of smoke rights at location j bought by consumer i by $x(i,j,n) = x_j^i(\tilde{u},n)$. Denote the equilibrium price per smoke right by $p(i,n)$. If B1 and B2 are satisfied then

$$\lim_{n\to\infty} \sum_{i=1}^{n} p(i,n) \sum_{j=1}^{n} x(i,j,n) = 0.$$

Proof: Suppose not. As usual, we drop subscripts on subsequences and suppose that total expenditures converge to a positive constant. Clearly, total expenditures satisfy

$$\sum_i \sum_j p(j,n)x(i,j,n) \leq \sum_i \left[\sum_j p(j,n)\right]\left[\sum_j x(i,j,n)\right].$$

From (4.1), (4.2), and the assumption that $R(q)/q$ is bounded, the sum of the prices is bounded. By hypothesis, the left side converges to a positive number. Hence, there is some sequence of consumers in and a positive number d such that

$$\lim_{n\to\infty} \sum_j x(i_n,j,n) \geq d.$$

Consider the following deviation for i_n. Let this consumer report utility functions of the form

$$\hat{u}_{i_n} = \alpha u_{i_n} \text{ with } \alpha \geq 0.$$

Let

$$p(\alpha,n) = p(\tilde{u}_{-i_n}, \alpha u_{i_n}, n)$$

and

$$x(\alpha,n) = x^i(\tilde{u}_{-i_n}, \alpha u_{i_n}, n).$$

From B2 these are differentiable in a neighborhood of $\alpha = 1$. The utility of i_n if he reports αu_{i_n} is given by

$$u(\alpha) = \bar{m}_{i_n} + u_{i_n}\left(x_i(\alpha,n)\right) - \sum_j p_j(\alpha,n)x_j(\alpha,n). \tag{4.3}$$

Differentiating (4.3) and using the fact that truth-telling is an equilibrium, we have

$$u'(1) = u'_{i_n}\left(x_i(1,n)\right)x_i'(1,n) - \sum_j p_j'(1,n)x_j(1,n)$$
$$- \sum_j p_j(1,n)x_j'(1,n) = 0.$$

From the hypothesis in the theorem, u'^i_n converges to zero. We have already argued that the sum of the prices is bounded. Since $x'(\cdot)$ is bounded (see B2) the first and last terms converge to zero. Because $p'(\cdot)$ is uniformly bounded away from zero, the sum of smoke rights bought by consumer i cannot converge to a positive number. We have obtained the desired contradiction. ◊

We have shown that total expenditures on smoke rights converges to zero. This implies (given our assumption of finite reservation price) that either the sum of the prices converges to zero or purchases of smoke rights converges to zero. In the first case, from (4.2) it follows that production of smoke converges to zero. This is clearly inefficient in general. In the second case we have inefficient outcomes if the government auctions off a fixed quantity of smoke rights. We conjecture that even when the amount of smoke rights depends upon the reported types that outcomes are inefficient. Since consumers are not being compensated for suffering smoke, they have every incentive to overstate their true aversion to smoke.

We now prove that under slightly stronger assumptions about the sequence of utility functions, we can replace the differentiability axiom B2 by a weaker continuity axiom to obtain a theorem similar to theorem 4. Indeed, we can even drop the requirement of a finite reservation price. The theorem is closely related to theorem 1 in Roberts (1976). Let $q_i = x^i_i$. The continuity axiom is

B2'. <u>Continuity</u>: For all $\delta > 0$, there exists $\varepsilon > 0$ such that for all n and for all i if $|t_{ij}(x,n)-u_j(x,n)| < \varepsilon$ for all $x \in [0,Q]$, for all j, then

$$|q_i(\tilde{u}_{-i},u_i,n)-q_i(\tilde{u}_{-i},t_i,n)| < \delta.$$

We now state and prove a theorem that decentralized mechanisms lead to inefficient outcomes.

<u>Theorem 5</u>: (Decentralized mechanisms yield inefficient outcomes)

Consider a sequence of smooth utility functions for consumers. Assume that there is some $B < \infty$ such that $du_i(q;n)/dq \leq B/n$ for all i and for all n, for all $q \in [0,Q]$. Assume truth-telling is an equilibrium. Denote the equilibrium amounts of smoke rights at location j bought by consumer i by x(i,j,n) and the price per smoke right by p(j,n). If B1 and B2' are satisfied by a sequence of mechanisms then

$$\lim_{n\to\infty} \sum_j p(j,n) \sum_i x(i,j,n) = 0.$$

<u>Proof</u>: We first show that

$$\lim_{n\to\infty} n\left[\max_i \sum_j p(j,n)x(i,j,n)\right] = 0. \quad (4.4)$$

The theorem then follows immediately. Suppose, therefore, that (4.4) does not hold. Again, choose subsequences if necessary and drop the subscript on the subsequence. There is some sequence of consumers i_n whose expenditures are maximal over all consumers such that

$$\lim_{n\to\infty} \sum_j p(j,n)x(i_n,j,n) = d > 0.$$

Consider the following deviation for i_n. Let $\hat{u}_{i_n} \equiv 0$. From B1 we have that total expenditures by in are zero for all n. Hence, the difference in utilities between this deviation and truth-telling is given by

$$(4.5) \qquad \Delta_n = V_{i_n}(\hat{u};n) - V_{i_n}(\tilde{u};n)$$

$$= u_{i_n}(\hat{q}_i^n;n) - u_{i_n}(q_i^n;n) + \sum_j p(j,n)x(i_n,j,n)$$

where $\hat{q}_i^n = q_i(u_{-i_n},\hat{u}_{i_n},n)$ and q_i^n is defined similarly. If $\hat{q}_i^n$ is greater than q_i^n then (4.5) is positive and q_i^n cannot be an equilibrium outcome. Therefore $\hat{q}_i^n \le q_i^n$. Since the utility function is differentiable, there is some $Q^n \in (\hat{q}_i^n,q_i^n)$ such that

$$n\Delta_n = n(\hat{q}_i^n-q_i^n)\,\frac{du_{i_n}(Q^n;n)}{dq} + n\sum_j p(j,n)x(i_n,j,n).$$

Choose $\delta = d/2B$. Clearly, for large enough n, $|\hat{u}-u|$ is arbitrarily close to zero. Hence, for large enough n, $q_i^n - \hat{q}_i^n < \delta$. Using the fact that marginal utilities are bounded by B/n we have

$$n\Delta_n \ge \frac{d}{2} > 0.$$

This contradicts the supposition that (p,x) is an equilibrium outcome. ◊

Again, since total expenditures converge to zero, decentralized mechanisms lead to inefficient outcomes.

The Free Rider Problem Revisited

As the discussion following theorem 2 in section 3 indicates, our results are not fundamentally affected by the fact that we have chosen to address the public goods problem in a privatized economy. This is best seen by considering a public goods economy where the government auctions off a fixed quantity of smoke rights $\bar{q}$. Denote the amount of smoke rights bought and used for production by q. The remainder is allocated to the town residents. The preferences of consumers in this public goods economy are given by

$$m_i - v_i(q).$$

The individual rationality condition associated with B1 for this public goods economy is now

B1" Individual Rationality. For all n, for all i = 1, ..., n, for all $\tilde{v} \in U^{n\times n}$

$$m_i(\tilde{v},n) - v_i\big(q(\tilde{v},n);n\big) \geq \bar{m}_i - v_i(\bar{q};n).$$

In effect, in this public goods economy, reduction in the quantity of smoke requires unanimous consent. The relationship between the public goods and the private goods economy can be seen by simply defining

$$u_i(q) \equiv v_i(\bar{q}) - v_i(q).$$

Again, the public goods formulation makes clearer the free rider problem. The private goods formulation emphasizes the role of monopoly power. Because the results are the same, the two formulations are, in a sense, equivalent.

5. More Local Public Goods

Our aim in this section is to explore equilibrium outcomes of decentralized mechanisms in the context of a local public goods economy. The results are special in many ways and so they should be interpreted as suggestive only.

To do this, we first want to relax the assumption of constant marginal disutility of pollution used in Example 3. The reason for centering attention on this assumption is straightforward. The constant marginal disutility situation is qualitatively unrepresentative in the sense that in this case it does not matter, from a social point of view, what quantity of smoke is produced in which town. That is, quantities are indeterminate in the Lindahl equilibrium (although the sum of quantities is determinate). en more general situations, optimality will require positive production in all towns. In this case, the problem faced is more significant than simply getting the price right in the market as a whole, it requires setting the quantities right within towns as well.

Thus, our primary goal in this section is to relax the assumption of constant marginal disutility. To this end, we assume that individual utility functions are of the form $U(m,s) = m - u(s)$ where s represents the consumption of smoke by the individual and u is twice continuously differentiable. The major problem we face now is the choice of a strategic form for competition. The difficulty lies in the by now standard problem of giving a reasonable description of price competition with increasing costs. That is, how is the output to be shared among the competitors in the event of unequal costs? Typically this is handled through the introduction of some (by its nature arbitrary) rule for rationing consumers. Unfortunately, the resulting discontinuities usually preclude the existence of pure strategy equilibria in much generality and hence should be seriously questioned.

Fortunately, in our situation, there is a simple and natural way around the problem. This is to make the firms in the town perform an active strategic role. Formally, we model a two stage game in which residents simultaneously set prices for their pollution rights at the first stage. These first stage choices then determine (constant) marginal costs for firms (one per town for simplicity) located in the towns and these firms play a one-shot simultaneous move Cournot quantity setting game. It is easy to see that this structure smooths out the payoff discontinuities (as a function of prices) discussed above. For notational simplicity, we will assume that there are the same number of residents in each town. Let this number be n.

The experiments we consider consist of replicating both the number of towns and the size of aggregate demand for the final product in such a way as to hold the Lindahl equilibrium constant throughout the experiment. Thus, let $D^m(p) = mD(p)$ where $D(p)$ is given by $a - bp$ as in the examples. Assume that there are m towns, and let $U_{ij} = m_{ij} - u_{ij}(s_i)$ denote the utility function of resident j in town i, where s_i is the amount of production in town i. Assume that $u'_{ij}(0) = 0$ for all i and j (pollution is costless at the margin when you have none) and that u'_{ij} is strictly increasing.

Let p_{ij} be the price announced by resident j in town i and let q_i be the level of output chosen by the firm in town i. Let $Q = \sum_i q_i$ denote aggregate output and let $p_i = \sum_j p_{ij}$ denote the marginal cost of firm i. Then, the payoff to firm i given an array of strategies is given by

$$\pi_i = q_i\{\max(D^{-1}(Q),0)-p_i\}.$$

Payoffs to the residents are defined in the obvious way. Note that we have suppressed the superscript m for notational convenience.

Proposition 1: Given any array of positive prices p_{ij}:

1. There is a unique equilibrium in quantity choices by the firms.
2. These quantities depend continuously on the prices p_{ij}.
3. For arrays of prices such that q_i is positive, it is linear in p_{ij} for all j.
4. For arrays of prices such that q_i is positive, the slope of q_i (as a function of p_{ij}) is $b\{-m+m/(m+1)\}$.

This is a standard result and hence the proof will not be given. Clearly the result depends both on our assumption of constant costs and the assumption that the demand curve is linear.

Let $q_i(p)$ denote the equilibrium quantity choice of firm i given the first stage price choices by all residents in all towns. (Note that this may well be zero if p_i is high enough.) The proposition implies that q_i has all of the obvious properties: It is strictly decreasing in p_{ij} for all j as long as it is positive, it is strictly increasing in p_{kj} for all k and j as long as q_k is positive, etc. Most importantly, it follows that in the region where q_i is positive the derivative converges to infinity as the number of terms goes to infinity. All of this is just to support the intuitive argument that each town becomes more and more like a price taker in the output market as n goes to infinity. (Detailed proofs of these claims are available from the authors upon request.)

In fact, the results contained in proposition 1 are basically all we need to show that, in the limit, interior equilibria are approximately efficient at the individual town level. We have,

Theorem 6: Consider a sequence of games as outlined above with n fixed and $m \to \infty$.

1. Existence: There is a pure strategy equilibrium such that $q_i(p) > 0$ for all i.
2. Consider a sequence of equilibria with $q_i(p) > 0$ for all i, if u''_{ij} is bounded away from zero, p_{ij} converges to $u'_{ij}(q_i(p))$ as $m \to \infty$.
3. In the symmetric case the equilibrium price converges to the Lindahl price, the equilibrium per town quantity converges to the (one town) Lindahl level.

That is, in the limit, the coordination problem at the town level discussed above does not appear (hence it is both Pareto optimal and individually rational).

Proof:

(1) The argument that an equilibrium exists for this game is actually quite straightforward, it is guaranteeing that $q_i(p)$ is positive for all i that causes the difficulties. That is, as in the examples, having all residents in all towns set prices higher than the reservation price from the demand curve gives an equilibrium with $q_i(0) = 0$ for all i. To show that there is an equilibrium with positive output in all towns, we will define a new game in which the strategy sets of the individual residents depend on the actions of all other residents of all towns (i.e., a pseudogame). We will show that this new game has an equilibrium, that all equilibria of this game have positive activity in all towns and finally that any equilibrium of this new game is also an equilibrium of the original game as well.

From the proposition, it follows that given any array of strategy choices by all residents in all towns other than town i, there is a p* such that sales by firm i are a linear (and decreasing) function of p_i for $p_i \leq p^*$ and zero for $p_i > p^*$. Moreover, p* is a continuous function of the prices selected in the other towns. Let the permissible choice of strategies, ϕ_j be defined by:

$$\phi_j(p_{i1},\ldots,p_{ij-1},p_{ij+1},\ldots,p_{mn}) = \{p \mid p + \sum_{k \neq j} p_{ik} \leq p^*\}$$

if $\sum_{k \neq j} p_{ik} \leq p^*$ and $\phi_j = \{0\}$ otherwise.

It follows that ϕ_j is nonempty, compact, and convex valued for all price choices by all other residents and is continuous.

Consider the pseudogame in which each agent is restricted in his choice of strategies to ϕ_j. It follows from the linearity of demand on ϕ_j and the convexity of u_{ij} that the utilities are quasiconcave in each agents' own actions. Thus, it follows from Debreu's result (1962) that an equilibrium in pure strategies exists.

The argument that production is positive in all towns in equilibrium follows from our assumption that $u'_{ij}(0) = 0$.

To see that the equilibrium of the pseudogame is an equilibrium for the original game one only need check that no omitted strategies can do any better. This follows immediately, however, since all omitted strategies guarantee a payoff of zero.

(2) To see that this holds, first note that q_i is given by:

$$(5.1) \qquad q_i^m = \frac{m}{m+1} a + bp_i\left[\frac{m}{m+1} - m\right] + \frac{mb}{m+1} \sum_{k \neq i} p_k.$$

It follows that, in equilibrium, resident j in town i behaves like a monopolist facing a linear demand curve of the form:

$$q_{ij}^m = \alpha_{ij}^m - \beta_{ij}^m p_{ij}$$

where $\beta_{ij}^m \to \infty$ and $\alpha_{ij}^m/\beta_{ij}^m$ is bounded above by a/b.

The remainder of the proof consists of showing that a monopolist, facing a series of linear demand curves behaves more and more like a price taker and hence is omitted.

(3) The proof is immediate. ◊

Comments on Theorem 6:

(1) This result seems to substantiate our discussion above concerning the ease with which "good" allocations can be realized in local public goods economies. However, this is a little misleading since it is easily seen that the argument depends on our assumption of simultaneous price setting as our model of market competition. That is, the argument is: Since any one town is small relative to the market as a whole, town residents take the price of the final good as approximately given when they make their decisions. Since we have assumed simultaneous price-setting, Nash equilibrium requires that each resident act as if he takes the other residents' prices as given when setting his own. These two facts together, imply that in equilibrium each resident is approximately taking the price of his own smoke right as given. As we see below, mechanisms which do not imply price-taking behavior in the limit seem to lead to inefficiency.

(2) Undoubtedly symmetry is stronger than necessary for part 3 of the result to hold. Presumably the standard sort of boundedness conditions will allow us to make a similar conclusion.

Our next goal is to extend the intuition from this result to a more general class of theories of markets. From the above, it is clear there are two separate problems to be solved. First, we must show that as the number of towns grows, the demand facing any given town becomes infinitely elastic. Second, we must show that within each town, as demand becomes more elastic, output converges to the efficient level. Our focus here is on the second problem: Therefore, we consider the problem of a town which faces a sequence of demand curves which become more and more elastic. A more complete treatment, of course, requires that we consider the first problem as well.

As before, we restrict attention to revelation games. The number of residents in the town is n. The space of possible utility functions over smoke is U = {smooth, nonincreasing convex functions on $[0,\infty)$}. A mechanism is a set of outcome functions $p_i(\tilde{u})$, $q(\tilde{u})$ for i = 1, ..., n, which map $U^{n\times n}$ into prices per unit of smoke paid to each consumer and the amount of smoke produced respectively. Feasibility requires that any mechanism satisfies $\sum_{i=1}^{n} p_i(\tilde{u}) = D_m^{-1}(q(\tilde{u}))$.

We now define the highest price that consumer i can get consistent with also producing a given amount of output. To this end, fix the utility functions of the other players, u_{-i}. Let $A(u_{-i}) = \{p,q \mid \exists\, t_i$ such that $(p_i(\tilde{u}_{-i},t_i),q(u_{-i},t_i))=(p,q)\}$. Let $d_i(q,u_{-i})$ = sup p such that $(p,q) \in A(\tilde{u}_{-i})$. Let $U(q;u_{-i})$ denote the set of utility functions t_i which give rise to d_i. For each $u_i \in U(q;\tilde{u}_{-i})$ there is a set of prices for other consumers induced by the outcome functions $p(\tilde{u})$, $q(\tilde{u})$. We assume that it is possible to make a smooth selection from this set of prices. Denote this selection by $d_k\ (q,\tilde{u}_{-i})$ for $k \neq i$. Let $A_q(u_{-i})$ denote the set of quantities such that there is some price p with $(p,q) \in A(u_{-i})$.

As before, define a sequence of mechanisms as the inverse demand $D_m^{-1}(q)$ changes. We now state the axioms which characterize a market-like theory. They are

C1. <u>Voluntary Trade</u>: For each m, for all i = 1, ..., n, for all $u^n \in U^{n\times n}$,

$$p_i(\tilde{u},m)q(\tilde{u},m) - u_i\big(q(\tilde{u},m)\big) \geq 0.$$

C2. <u>Smooth Tradeoffs for Consumers</u>: For each m, for all i = 1, ..., n, for all $\tilde{u} \in U^{n\times n}$, there exists $\bar{q}(\tilde{u}_{-i},m)$ such that

$$A_q(\tilde{u}_{-i},m) = \big[0,\bar{q}(\tilde{u}_{-i},m)\big]$$

and

$$d_i\big(\bar{q}(\tilde{u}_{-i},m),\tilde{u}_{-i},m\big) = 0.$$

Furthermore, the prices $d_j(q,\tilde{u}_{-i},m)$ are smooth functions of output $q \in Aq(\tilde{u}_{-i},m)$ for all j = 1, ..., n.

C3. <u>Bounded Tradeoffs</u>: There is some number $K < \infty$ such that for all m, for all, i = 1, ..., n

$$\frac{\left|\frac{\partial d_i}{\partial q}\right|}{\left|\sum_{k=1}^{m}\frac{\partial d_k}{\partial q}\right|} \leq K.$$

Axiom C2 requires that the residual demand curve facing each consumer, given the actions of others be smooth. Furthermore, we require that this residual demand be smooth in the actions of the other players. Axiom C3 requires that if a consumer chooses an action resulting in lower output, the effect on the price paid to all residents in the town be of the same order of magnitude as the effect on his own price.

We now state a theorem that as the demand curve becomes more elastic, output in the town converges to the efficient level.

Theorem 7: (Market mechanisms lead to efficient outcomes)

Suppose $D_m^{-1}(\cdot)$ is differentiable, $mD_m^{-1'}(q) = D_1^{-1'}(q)$, $D_1^{-1'}$ is bounded and D_1^{-1} is bounded. Suppose also that $u_i''(\cdot)$ is bounded away from zero. Assume truth-telling is an equilibrium of the revelation game and denote the equilibrium outcome by $p_i(u,m)$, $q(u,m)$. Then, if a sequence of mechanisms satisfies C1 through C3,

$$\lim_{n\to\infty} \left[p_i(\tilde{u},m) - u_i'\left(q(\tilde{u},m)\right)\right] = 0.$$

Proof: Since $p_i(\tilde{u},m)$, $q_i(\tilde{u},m)$ is an equilibrium outcome, from C2 it follows that for each i

$$(5.2) \qquad q(u,m)\,\frac{\partial d_i\left(q(\tilde{u},m),\tilde{u}_{-i}\right)}{\partial q} + \left[p_i(\tilde{u},m) - u_i'\left(q(\tilde{u},m)\right)\right] = 0.$$

From feasibility and C2 we have that

$$(5.3) \qquad \sum_{j=1}^{n} \frac{\partial d_j\left(q(\tilde{u},m),\tilde{u}_{-i}\right)}{\partial q} = D_m^{-1'}\left(q(\tilde{u},m)\right).$$

Consequently, the left side of (5.3) converges to zero as m gets large. Using this fact and C3, we see that the first term in (5.2) converges to zero. Since u_i'' is bounded away from zero, q(u,m) is bounded. The theorem follows. ◊

We now construct an example which violates axiom C3 to demonstrate the role played by that axiom in the proof of theorem 7.

Example 4:

Consider a town with two residents who set the prices for smoke sequentially. Let $D^m(p) = ma - mbp$. Of course, $p = P1 + p_2$ where p_1 and p_2 are the prices set by the first and second resident respectively. The choice of p_2 must satisfy

$$(5.4) \qquad -[p_2 - u_2'(q)]mb + q = 0.$$

We use (5.4) to derive the derivative of resident 2's equilibrium reaction as a function of resident 1's price. This is given by

$$p_2'(p_1) = \frac{-bmv_2''(q) - 1}{2 + bmv_2''(q)}.$$

We can now derive resident 1's choice of p_1. This must satisfy

$$-[p_1 - u_1'(q)]mb(1+p_2'(p_1)) + q = 0.$$

It is easy to show that $mb(1+p_2'(p_1))$ converges to $1/u_2''(q)$. Note from (5.4) that for resident 2, price converges to this marginal disutility. However, this is not so for resident 1. The reason that the Lindahl allocation is not an equilibrium outcome is that asymptotically, an increase in resident 1's price is met by a decrease in resident 2's price of the same magnitude. Consequently, the price received by the town as a whole from the market is not changed. Note that this causes axiom C3 to violated.

In this section, we have constructed a particular mechanism which achieves efficiency. We have indicated the kinds of properties that decentralized theories of markets must have to achieve efficient allocations. Axiom C3 limits the power of an individual consumer to affect the prices received by other consumers. Finally, we have illustrated the role played by this axiom in a simple example.

6. Concluding Comments

(1) The appeal of the results in sections 3 and 4 is that they show that a large class of "theories" of the workings of markets give rise to the same prediction in large economies. Production of the public good converges to zero. Of course, as our discussion about the normalization of demand in section 3 suggests, there is nothing special about zero. Rather, our results suggest that, in general, in large economies the provision of public goods is far from the efficient level. Two assumptions play a key role in our results. First, we require that all trades be voluntary. Second, we require that the mechanisms be uniformly continuous in the actions of consumers. We call a sequence of mechanisms which satisfy these assumptions, a theory of decentralized markets. It seems clear that a theory of decentralized trades must satisfy voluntary trade. The continuity assumption rules out mechanisms that confer too much power to a single consumer or cause outcomes to be unduly sensitive to a consumer's actions. As we point out in section 3, even with private information the outcomes are often far from those obtained from interim efficient mechanisms. While decentralized theories lead to inefficient outcomes with pure public goods, the results in section 5 suggest that with local public goods it may be possible to

construct decentralized theories which yield efficient outcomes (see also comment 9).

(2) It is clear that the problems raised here are present in the context of pure public goods as well. As should be clear, one difference between the externalities case and the public goods case is that with public goods, the associated private goods economy contains perfectly complementary outputs rather than inputs.

In fact, it is easy to reinterpret our results in section 4 as applying to the case of pure public goods. (Indeed, this is the interpretation that Roberts (1976) gives in his work.) This can be done by setting initial endowments of smoke rights at zero (as in section 4) and reinterpreting R(q) as a cost function rather than a revenue function. It is interesting to note that this demonstrates one difference between public goods and externalities. This difference is that in a public goods economy there is no need to assign initial endowments. Consequently the source of market power which causes inefficiencies is due completely to the uniqueness of preferences.

(3) The interpretation that we have given to Coase is but one of many (rivaling Keynes we're sure). Of course, nothing as explicit as what we have done appears in Coase's paper. Certainly, Coase never mentions "adding" markets as the solution to the externalities problem. The notion that the problem is essentially one of missing markets appears explicitly as a definition in Heller and Starrett (1976) and is often attributed to Arrow (1970). It can probably be traced farther back at least in some

form. Something very close to this appears in Meade (1952) and is one interpretation of Lindahl (1919).

Of course, the contention that the whole problem with externalities is one of missing markets is not the view that we have adopted in this paper. Quite the contrary, the question we have considered is whether once property rights have been fully distributed (we do not deny the difficulty in doing this in the first place), is it "likely" that the outcome will be efficient. That is, do economies with public goods and/or externalities present a significantly more severe problem for decentralization through self interested voluntary exchange than economies with only private goods? The results of sections 3 and 4 suggest that the answer to this question is that they do. However, the results of section 5 suggest that these problems may not be universal.

Other interpretations of Coase are possible. For example, it has been said that Coase said that the distribution of legal rights does not have any effect on allocations. Clearly, this is the case in many of his examples but cannot be considered seriously as it clearly will not hold except in very special circumstances (e.g., the income distribution does not matter for equilibrium allocations). That is, the only reasonable interpretation is that all equilibria are efficient, but that different distributions of initial property rights may give rise to alternative efficient allocations.

Green (1982) gives a very interesting interpretation of Coase in terms of the efficiency of equilibria of legal structures. He gives conditions under which all efficient outcomes are equilibria of the legal system. In this framework, Coase's theorem would state that all equilibria of the legal system are efficient. This is an interesting approach to the problem but as of yet no attempt to prove Coase's Theorem as stated above has been undertaken to our knowledge.

(4) Note that the problem exposited in the examples of section 2 has another analog in economics. This is the problem of eminent domain (i.e., eminent domain is the commonly used solution). That is, suppose that a city decides to build a new civic center (or highway). To do this, it must displace all residents in the area of the proposed site. If the compensation of current owners is decided by having them announce the value of their lots it is easy to see that they have a strong incentive to overstate the value of their property to try and extract all the surplus from the arrangement. Again, the lots being considered are perfectly complementary given the site and so the same problem arises. Example 2 has a natural interpretation in this connection as well--if there are many equally opportune sites, competition among them should give rise to efficient outcomes.

(5) The mechanism design tradition [see Hurwicz (1972), Groves and Ledyard (1977), and Groves and Ledyard (1985) for a useful survey] has taken a related route to confronting the problems raised here. Discussing all the issues raised in this literature would take us too far afield. However, as Hurwicz (1979) and Groves and Ledyard (1977) suggest and Roberts (1976) demonstrates even more explicitly, mechanisms which are both decentralized and feasible (in the sense of balancing the budget) are, in general, not efficient.

(6) An obvious question to ask in this connection is why have we not had the problems suggested in Starrett's (1972) work. The answer to this probably lies in the fact that we have implicitly assumed that people can't move and sell all of their smoke rights. The ability to move would correspond naturally to Starrett's shutdown option. It would be of interest to see what happens if this is included as a possibility. That is, are there problems with nonexistence in this case? (See comment 9 as well.)

(7) Our approach to the problems considered has been to sacrifice complexity in the environments in favor of generality in the class of "theories" allowed. In doing this we have made many special assumptions concerning preferences, technologies, etc. It is natural to question the importance of these restrictions. In particular, it is of interest to know whether our results generalize along several dimensions. Obvious generalizations to consider include: (a) Dropping the special structure of preferences to allow income effects, etc. (b) Allowing for more goods both private and public. (c) Allowing for situations in which residents care about consumption of the produced good which is exchanged for money as well as the level of pollution. (d) Allowing for pollution technologies in which partial exclusion is possible.

(8) In our view, the most important generalizations of the results obtained to this point involve extensions of the results of section 5. Ideally, one would like a set of conditions under which "most" theories give rise to efficient outcomes in economies with local public goods and many potential "locations."

This leads one naturally to consider alternative formulations of Tiebout's hypothesis. Note that the result given in section 5 differs from the explanation offered by Tiebout in at least one important way. Mobility is extremely limited. That is, we do not allow agents to move at all (although output of the final good is allowed to move freely). This inability to move leaves open the possibility that the various residents of a given town may be able to exploit monopoly power relative to one another. This feature can give rise to inefficiencies even with local public goods.

These considerations lead one naturally to consider models in which town formation itself is endogenous (as Tiebout does). In this regard, the recent work by Scotchmer (1985), Wooders (1986), and Scotchmer and Wooders (1986) is likely to be quite useful.

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