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WHY DOES INVENTORY INVESTMENT FLUCTUATE SO MUCH?

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1. Introduction

Quarterly changes in inventory investment are on average about half the size of quarterly changes in GNP, even though inventory investment is on average only a tiny .6 percent of GNP. The observation that the magnitude of fluctuations in inventory investment is so out of proportion to their small average size has inspired much of the literature on inventory investment. It has moved some writers to speculate that understanding the reason for inventory fluctuations may provide the key to understanding the business cycle itself. An example of this is Samuelson's multiplier accelerator model, in which the structure of the inventory investment decision converts exogenous, uncorrelated economic shocks into serially correlated movements in GNP that resemble the business cycle. Another example along similar lines is provided by Blinder and Fischer ( ). Blinder, who has done much recently to bring the facts about inventory investment to the attention of economists, concludes (1981, p. 500): ". . . to a great extent, business cycles are inventory fluctuations."

To my knowledge, a quantitatively convincing explanation for the large observed fluctuations in inventory investment has not been provided. The purpose of this paper is to suggest one, and to provide an assessment of its empirical importance. The proposed explanation is based on two casual observations about the economy. First, decisions about fixed investment and employment for a given quarter have to be made before there is full information about the state of the economy in that quarter.[1.1] Second, consumption is a very smooth time series and appears to be relatively unresponsive to disturbances.[1.2] An implication of the first observation is that when something unexpected happens in a quarter, say a bad productivity shock, the burden of equating the economy's reduced output with the demands placed on it falls on consumption and inventory investment. The second observation suggests that most of the burden in fact will fall on inventories, which in this sense are a kind of residual. The results of this paper suggest that the residual role of inventory investment may account for the bulk of the observed volatility in investment.

I adopt a three stage strategy to assess the empirical significance of the above explanation. First, I present a model in which the intuition described above is formalized. The model is a simplified version of the equilibrium growth model in Kydland and Prescott [1982]. A key feature of the model is its assumption that employment and investment decisions by firms for a given quarter are based on an imperfect observation of that quarter's productivity and taste shocks. Household consumption decisions and inventory investment decisions, on the other hand, are made when full information about a quarter's state is available. Underlying these assumptions is the view that elements of precommitment are important in fixed investment and employment decisions, but minor in consumption decisions. The economic model is structured in such a way that, in equilibrium, households smooth consumption. As a result, one of the roles of inventory investment is to be a residual in the sense described above. More familiar roles for inventory investment are also present in the model. These include the role of inventories in buffering output from taste shocks and buffering consumption from technology shocks. In addition, inventories are modelled as playing a small direct role in production.

In the second stage, I assess the empirical plausibility of the model, and assign values to its parameters. The importance of this stage lies in the fact that the credibility of the results of the third stage are enhanced if the model can be shown to be consistent with the general features of aggregate post war U.S. data. In the third stage, I focus on the question that is the title of the paper. There I present a precise definition of "inventory investment volatility" and use the model to decompose the total measured volatility into two parts: one that reflects the residual role of inventories, and the other that reflects the other roles of inventories. The way I achieve this decomposition is simply by setting the variance of firms' observation errors to zero and calculating the resulting fall in the inventory volatility statistic.

Following is an outline of the paper. Section 2 presents a formal description of the model. Section 3 describes the method used to obtain an approximation to the solution of the model of Section 3. Since the model cannot be represented as the maximization of a quadratic function subject to a linear constraint, I used a modified version of the Kydland-Prescott quadratic approximation technique to obtain an approximate solution to the model. The novelty in my framework, relative to that of Kydland and Prescott is that I build growth explicitly into the model. As a consequence, the endogenous variables of the model do not possess a steady state, as the Kydland-Prescott technique requires. In order to apply the Kydland-Prescott technique anyway, I first transform the model into one in which the variables do possess a steady state. In executing this transformation, I make use of the model's property that its variables converge to a constant growth rate, which is the same across variables. Section 4 discusses the method I used to econometrically estimate the free parameters of the model. Sections 5 and 6 discuss the empirical results, which are not included in this draft. I will distribute them in a handout during the talk.

2. The Model

2.a Technology

The economy-wide resource constraint is given by

$$C_t + K_t - (1-\delta)K_{t-1} + I_t - I_{t-1} \le f(z_t, H_t, K_{t-1}, I_{t-1}) \quad (2.1)$$

where

- t time in quarters
- C_t aggregate, quarter t consumption
- K_t aggregate, end-of-quarter stock of capital
- H_t aggregate hours worked in quarter t
- I_t aggregate, end-of-quarter stock of inventories
- z_t shock to technology
- δ quarterly depreciation rate on capital
- f production function, linearly homogeneous in K, H, and I

I use the production function studied in Kydland and Prescott namely,

$$f(z_t, K_{t-1}, I_{t-1}) = [z_t H_t]^{(1-\theta)}[(1-\sigma)K_{t-1}^{-\nu} + \sigma I_{t-1}^{-\nu}]^{-\theta/\nu} \quad (2.2)$$

where $0 < \theta < 1$, $0 < \sigma < 1$, and $\nu > 0$.

This production function is used for several reasons. First, it is consistent with the observed small variation in labor's share of output in postwar, U.S. data. [2.1] Second, because the estimated value of σ turns out to be quite small, (2.2) is approximately a Cobb-Douglas production function in K_{t-1} and H_{t-1}. Consequently, the results in this paper can be compared with the many other studies that use this functional form. Third, because of the linear homogeneity of f, the aggregation theory underlying the production function is simple (see, e.g., Sargent [1979, pp. 6-10]).

Later, I end up assigning a very small value to σ, implying that inventory stocks play a minor direct role in production. Nevertheless, some motivation of why inventories play any direct role at all is warranted.[2.2] I allow a nonzero value for σ for two reasons. First, other things equal, larger inventory stocks probably do augment society's ability to produce goods. For example, large inventory stocks reduce the likelihood of unexpected stockouts, letting retailers bunch orders and thereby economize on labor inputs. Another reason for giving inventory stocks a direct role in production is that without this nonnegative constraints on i_t are binding, invalidating the solution technique I use for obtaining equilibrium decision rules for my model.

This paper works with per capita rather than aggregate quantities. Accordingly, I converted the resource constraint to per capita terms by dividing both sides of (2.1) by N_t, the total population. The result is

$$(2.3) \qquad c_t + k_t - \frac{(1-\delta)}{n}k_{t-1} + i_t - \frac{1}{n}i_{t-1} \leq [z_t h_t]^{(1-\theta)} n^{-\theta}[(1-\sigma)k_{t-1}^{-\nu} + \sigma i_{t-1}^{-\nu}]^{-\theta/\nu}$$

where lower case letters denote variables measured in per capita terms. Because the resource constraint is dynamic, converting to per capita quantities requires making an assumption about N_t/N_{t-1}. I simply assume this is a constant, equal to n. [2.3]

The discussion of society's production technology is completed by describing the statistical model for z_t. I assume

$$(2.4) \qquad z_t = z_{t-1} \exp(x_t),$$

where x_t is a stationary stochastic process, whose distribution is to be described below.

The reason for positing (2.4) is that it--in conjunction with the assumptions placed on preferences below--has implications consistent with the statistical properties of U.S. data on consumption, investment and output. As will be shown below, in the solution to this model, the latter variables are the product of z_t and a covariance stationary stochastic process. Consequently, the model implies that logarithmic first differencing is required to induce stationarity. United States data on consumption, investment and output are consistent with this implication (see, e.g., Nelson and Plosser [1982]).

2.b Preferences

As of the start of date 0, the representative consumer in this economy orders consumption and hours streams according to the following criterion:

$$(2.5) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \{\exp (u_t) \ln (c_t) - \gamma h_t\} \qquad \gamma > 0,$$

where u_t is a zero mean taste shock and E_t is the expectation operator conditioned on information available at the start of date t. This utility function was used by Gary Hansen [1985].

The particular form of the utility function in (2.5) plays an important role in this paper, and so warrants some discussion. The reason the logarithm of c_t appears in (2.5), rather than for example $\exp(u_t)(c_t^{\phi}/\phi)$, $\phi \neq 0$, is partly computational. The solution strategy I use to solve the model requires that the function relating consumption to instantaneous utility have the property that it convert multiplication into addition, as the logarithm does. A number of papers have attempted to estimate ϕ in settings where the variance of u_t is zero, and find that $\phi = 0$ is not a bad approximation (see, for example, Hansen and Singleton [1982,1983]). However, it is not clear how relevant these studies are for the present context, in which the variance of u_t is not zero.

G. Hansen [1985] showed that the linearity of h_t in (2.5) is important for explaining the large observed fluctuations in h_t given the magnitude of the technology shocks observed in the postwar United States, and assuming a framework very similar to the one in this paper. [2.4] There are at least two ways to interpret the linearity of h_t in (2.5). One is that it reflects the assumption that individual household utility functions take the form given in (2.5). This can be criticized on two grounds. First, it implies that individual hours worked are varied continuously over time. Heckman [1984] argues that this is at variance with the facts, since a large part of the variation in aggregate hours reflects fluctuations in employment rather than in hours per worker. For example, the standard deviation of average weekly hours in the manufacturing sector is a mere 37 minutes in the post war period, i.e., the time for one or two coffee breaks. [2.5] (See Economic Report of the President [1986, p. 300].) A second difficulty with this interpretation is that it implies that leisure at different dates is infinitely substitutable. This is inconsistent with empirical results found in panel data (see Altonji [1986] and MaCurdy [1981]).

G. Hansen [1985], drawing on work by work by Rogerson [1984], describes an alternative interpretation of the linearity of h_t in (2.5) that avoids both the criticisms described in the preceding paragraph. Unfortunately, these benefits do not come without cost. Under Hansen's interpretation, the utility function in (2.5) is consistent with any degree of intertemporal substitutability of leisure at the individual level. Moreover, Hansen's interpretation has the implication that average weekly hours are constant, and that all fluctuations in aggregate hours result from fluctuations in employment. This latter implication probably goes too far. For example, Heckman [1984, p. 212] argues that, at the quarterly level, variation in persons employed constitutes about half of total aggregate variation in hours. A literal reading of Hansen's interpretation also has other unfortunate implications; for example, that an individual's probability of being unemployed is independent from quarter to quarter. These and other difficulties with Hansen's framework are the subject of current research. Details on this approach, some of its shortcomings, and possible fixes are described in Hansen [1985], Rogerson [1984], and Prescott [1986].

2.c Stochastic Structure of Exogenous Shocks

I assume that the shocks to tastes and technology, $w_t \equiv (u_t, x_t)'$, have the following time series representation:

$$(2.6) \qquad w_t = a + Aw_{t-1} + \varepsilon_t,$$

where ε_t is a white noise with variance V, and uncorrelated with w_{t-s}, s > 0. Also, a is a 2 by 1 vector with first element zero, and A is a diagonal matrix, with eigenvalues less than one in absolute value. In this model, taste and technology shocks may be correlated to the extent that their innovations are correlated, but they do not Granger-cause each other. Under (2.6), the technology shock, z_t, has the following representation:

$$(2.7a) \qquad (1-A_{22}L)(1-L) \ln (z_t) = a_2 + \varepsilon_{2t}$$

so that

$$(2.7b) \qquad (1-L) \ln (z_t) \cong a_2/(1-A_{22}) + \varepsilon_{2t} + A_{22}\varepsilon_{2t-1}$$

where the approximation is good for A_{22} small. Here, L denotes the lag operator, and

$$(2.7c) \qquad a = \begin{matrix} 0 \\ a_2 \end{matrix} \qquad A = \begin{matrix} A_{11} & 0 \\ 0 & A_{22} \end{matrix}$$

Writing the representation for z_t in the form in (2.7) allows us to give A_{22} an interpretation. In particular, of a given one percent surprise upward move in z_t, $1+A_{22}$ percent is permanent, and $-A_{22}$ is temporary. Rough calculations reported in Prescott [1986], which assume $\theta = .36$, suggest a value of A_{22} in the neighborhood of $-.2$. Also, $100(1-\theta)a_2/(1-A_{22})$ is the average quarterly percent growth in productivity, or "technical progress." Solow [1957] estimated this to be .37 for the period 1909 to 1949, while Denison [1980] reports an estimate of .4 for the period 1929 to 1969.

2.d Information Structure

I assume that at the beginning of the quarter, when hours and capital investment decisions have to be made, only a noisy observation on w_t is available. After the hours and investment decisions are made, then w_t is observed exactly, at which time the consumption and inventory investment decisions are made. This information structure is intended to capture the idea that there is "momentum" in employment and capital investment decisions. Momentum exists in the sense that in the middle of a quarter, when w_t is observed exactly, firm managers have to go through with the production and capital investment decisions they made at the beginning of the quarter, even if these now look suboptimal. By contrast, there is no such momentum in consumption and inventory investment decisions.

I assume that the hours and capital investment decision is contingent on observing $\bar{w}_t$, where

$$(2.8) \qquad \bar{w}_t = w_t + v_t, \quad Ev_t v_{t-s}^T = \begin{matrix} V_v & s = 0 \\ 0 & s \neq 0 \end{matrix},$$

and $Ev_t\varepsilon_{t-s}^T = 0$ for all s. Given this structure,

$$(2.9) \qquad \hat{w}_t \equiv E[w_t | w_{t-s}, s>0; \bar{w}_t] = a + Aw_{t-1} + D(v_t+\varepsilon_t),$$

where $D = V[V_v+V]^{-1}$. Equation (2.9) follows from standard Kalman filtering arguments (see, e.g., Sargent [1979, p. 208]). Note from (2.9) that if there is no observation noise, i.e., $V_v = 0$, then $\hat{w}_t = w_t$. By contrast, if the observation noise is very large, then D = 0 and $\hat{w}_t$ is just the best predictor of w_t based on last period's disturbance, w_{t-1}.

2.e Equilibrium Quantities

Below I describe the equilibrium quantities of the model as the solution to a central planning problem. I do not take a stand on the decentralization mechanism that supports this equilibrium, since there are several, and they all have the same implication for aggregate, per capita quantities.[2.6] It is the latter that are of interest in this paper.

The problem at date t is to maximize (2.5) over state contingent plans for $\{c_{t+s}, k_{t+s}, i_{t+s}, h_{t+s}\}$, $s \geq 0$, subject to (2.3), the information structure, and the initial conditions: k_{t-1}, i_{t-1}, and w_{t-1}. The solution to this optimization problem is a set of functions, $c_t = c(\Omega_t)$, $i_t = i(\Omega_t)$, $h_t = h(\bar{\Omega}_t)$, $k_t = k(\bar{\Omega}_t)$. Here, $\Omega_t = \{\varepsilon_t, w_{t-1}, h_t, k_t, k_{t-1}, i_{t-1}\}$, and $\bar{\Omega}_t = \{\varepsilon_t+v_t, w_{t-1}, k_{t-1}, i_{t-1}\}$. Methods for obtaining c, i, h, k exactly and for doing this sufficiently rapidly to make econometric estimation feasible are not available, as far as I know. Instead, approximations to c, i, h, and k were calculated. This is described in the next section.

3. Approximate Solution to the Planning Problem

The solution procedure used is related to the one proposed in Kydland and Prescott [1982]. Their procedure cannot be applied directly in the model of this paper, since it requires that the certainty version of the model possess a steady state. (By "certainty version" of the model, I mean the one with $V = V_v = 0$.) This is not the case in my model because I specified the logarithm of the technology shock to be a random walk with drift. Although the endogenous variables of the model do not converge to a steady state, they do converge to a steady state growth path. It is possible, by exploiting this fact, to transform the model economy into another one (the "star economy") whose variables do possess a steady state. The first part of this section does this. The second subsection obtains a version of the Kydland/Prescott linear quadratic approximation to the decision rules for the star economy. Finally, the third subsection "unwinds" the approximate decision rules obtained for the star economy to get approximate decision rules for the problem of interest, namely, (2.10).

3.a The Star Economy

Define

$$(3.1) \quad k_t^* \equiv \log (k_t/z_{t-1}), \quad c_t^* \equiv \log (c_t/z_t), \quad i_t^* \equiv \log (i_t/z_t), \quad h_t^* \equiv \log (h_t)$$

Note that (3.1) is well defined because of the nonnegativity constraints on k_t, c_t, h_t, and because the posited probability model for z_t guarantees that this variable is always positive. Because of this, the resource constraint, (2.3), is unaltered if both sides of that equation are divided by z_t. Doing so, and substituting from (3.1) and (2.4),

$$(3.2) \quad \exp (c_t^*) + \exp (-x_t) \exp (k_t^*) - \frac{(1-\delta)}{n} \exp \left[-(x_t+x_{t-1})\right] \exp (k_{t-1}^*)$$

$$+ \exp (i_t^*) - \exp (-x_t) \frac{1}{n} \exp (i_{t-1}^*) \leq \exp (y_t^*)$$

where,

$$(3.3) \qquad \exp(y_t^*) = n^{-\theta} \exp\left[(1-\theta)h_t^*\right] \exp(-\theta x_t)$$

$$\times \left[(1-\sigma)\exp(\nu x_{t-1})\exp(-\nu k_{t-1}^*)+\sigma\exp(-\nu i_{t-1}^*)\right]^{-\theta/\nu}.$$

Here, $y_t \equiv z_t \exp(y_t^*)$ is per capita output.

Note that (2.5) can be rewritten as follows:

$$(3.4) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \{\exp(u_t)c_t^* - \gamma \exp(h_t^*) + \exp(u_t)\ln(z_t)\}.$$

Since (3.2) and (3.4) are identical to (2.3) and (2.5), respectively, it is immaterial to the planner whether (2.3), (2.5) is maximized directly for c_t, h_t, i_t, k_t, or instead (3.2), (3.4) is maximized first with respect to c_t^*, h_t^*, i_t^*, k_t^*, and then converted to c_t, i_t, k_t, h_t via (3.1). From my point of view, it is more convenient to work with (3.3) and (3.4), because the certainty version of that system possesses a steady state.

Note that the third term in braces in (3.4) is beyond the control of the planner and can be dropped from the analysis. Imposing the strict equality in (3.2) (reflecting non-satiation), use (3.2) to substitute out for c_t^* in (3.4) and denote the resulting function by r. In this way, the planning problem becomes to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t r(k_{t-1}^*, k_t^*, i_{t-1}^*, i_t^*, h_t^*, w_{t-1}, w_t),$$

subject to the information structure and the initial conditions. A way to present this problem which reflects explicitly the informational constraints is as the dynamic programming problem: solve

$$(3.5) \qquad v(k_{t-1}^*, i_{t-1}^*, w_{t-1}, \varepsilon_t+v_t) = \max_{k_t^*, h_t^*} E\{\max_{i_t^*} E[r(k_{t-1}^*, k_t^*, i_{t-1}^*, i_t^*, h_t^*, w_{t-1}, w_t)$$

$$+ \beta v(k_t^*, i_t^*, w_t, \varepsilon_{t+1}+v_{t+1}) | T_t] | \bar{T}_t\}$$

subject to (2.6). Here, $T_t = \{\varepsilon_t, w_{t-1}, h_t^*, k_t^*, k_{t-1}^*, i_{t-1}^*\}$ and $\bar{T}_t = \{\varepsilon_t + v_t, w_{t-1}, k_{t-1}^*, i_{t-1}^*\}$. The solution to this is a set of functions $i_t^* = i^*(T_t)$, $h_t^* = h^*(\bar{T}_t)$, $k_t^* = k^*(\bar{T}_t)$. The preceding, informal argument implies that $z_t \exp[i^*(T_t)] = i(\Omega_t)$, $z_{t-1} \exp[k^*(\bar{T}_t)] = k(\bar{\Omega}_t)$ and $\exp[h^*(\bar{T}_t)] = h(\bar{\Omega}_t)$.

My objective is to approximate i, k, c and h. My strategy for doing this approximates i^*, k^*, h^* and then uses (3.1) to deduce the approximations implied for i, k, h. The implied approximation for c can then be obtained by substituting the approximations for i, k, h into (2.3). The next subsection presents my strategy for approximating i^*, k^*, h^*.

3.b Computing Approximate Decision Rules for the Star Economy

The Kydland/Prescott approximation technique involves taking a second order Taylor series expansion of r about the steady state values of k_t^*, i_t^*, h_t^*, w_t. Denote the steady state value of a variable by replacing its time subscript by an s. Trivially, $w_s = (I-A)^{-1}a$. The three variables k_s^*, i_s^* and h_s^* are the unique solution to the following three first order conditions, which must obtain at an interior steady state: $r_2 + \beta r_1 = 0$, $r_4 + \beta r_3 = 0$, $r_5 = 0$. Here, r_j denotes the derivative of r with respect to its j-th argument, evaluated at $k_{t-1}^* = k_s^*$, $k_t^* = k_s^*$, $i_{t-1}^* = i_s^*$, $i_t^* = i_s^*$, $h_t^* = h_s^*$, $w_{t-1} = w_s$, $w_t = w_s$. The solution in Appendix D is shown to be

$$i_s^* = \log(\lambda) + k_s^*$$

$$k_s^* = \log(\psi) + h_s^*$$

$$\exp(h_s^*) = \frac{\exp(u)}{\gamma}(1-\theta)(\alpha_1/\alpha_2)$$

where

$$\lambda = \{\frac{\sigma[1-\beta(1-\delta)\ \exp\ (-x)/n]}{(1-\sigma)\ \exp\ [(\nu+1)x][1-\beta\ \exp\ (-x)/n]}\}^{(\frac{1}{1+\nu})}$$

$$\psi = \{\frac{n^{\theta}\exp[(\theta-\nu-1)x][1-\beta(1-\delta)\exp(-x)/n][(1-\sigma)\exp(\nu x)+\sigma\lambda^{-\nu}]^{\frac{\theta+\nu}{\nu}}}{\beta\theta(1-\sigma)}\}^{\frac{-1}{1-\theta}}$$

$$\alpha_1 = n^{-\theta}\ \exp\ (-\theta x)\psi^{\theta}[(1-\sigma)\ \exp\ (\nu x)+\sigma\lambda^{-\nu}]^{-\theta/\nu}$$

$$\alpha_2 = \alpha_1-\{\psi\ \exp\ (-x)[1-(1-\delta)\ \exp\ (-x)/n]+\lambda\psi[1-\ \exp\ (-x)/n]\}$$

$$(u,x) \equiv w_s'.$$

Denote the second order Taylor series expansion of r about steady state by R. The approximations to i*, k*, h* are then obtained by replacing r by R in (3.5) and solving the resulting quadratic dynamic programming problem subject to the linear constraints, (2.6). The details of this are described in Appendix B. The solution derived there is

$$(3.7)\qquad i_t^* = i_0^* + i_1^* \begin{matrix} k_{t-1}^* \\ i_{t-1}^* \end{matrix} + i_w^* w_{t-1} + i_\varepsilon^* \varepsilon_t + i_d^* \begin{matrix} k_t^* \\ h_t^* \end{matrix},$$

$$(3.8)\qquad \begin{matrix} k_t^* \\ h_t^* \end{matrix} = d_0 + d_1 \begin{matrix} k_{t-1}^* \\ i_{t-1}^* \end{matrix} + d_w w_{t-1} + d_\varepsilon D(\varepsilon_t + v_t).$$

A property of (3.7) is that $i_{w,1}^* = A_{11} i_{\varepsilon,1}^*$, where $i_w^* \equiv (i_{w,1}^* i_{w,2}^*)'$ and $i_\varepsilon^* \equiv (i_{\varepsilon,1}^* i_{\varepsilon,2}^*)'$. Using this, (3.7) can be written,

$$(3.7)'\qquad i_t^* = i_0^* + i_1^* \begin{matrix} k_{t-1}^* \\ i_{t-1}^* \end{matrix} + i_{w,2}^* x_{t-1} + i_{\varepsilon,2}^* \varepsilon_{2t} + i_d^* \begin{matrix} k_t^* \\ i_t^* \end{matrix} + i_{\varepsilon,1} u_t,$$

where $u_t = A_{11} u_{t-1} + \varepsilon_{2t}$ has been used (see [2.6]), and recall that $w_t = (u_t x_t)'$. The mapping from the sixteen structural parameters, β, θ, n, δ, γ, σ, ν, a, A, V, V_v to the reduced form parameters in (3.7) and (3.8) is given in Appendix B.[3.1]

3.c The Approximate Solution to the Planning Problem

To get the approximation to i, k, c, h, substitute (3.1) into (3.7) and (3.8) and rearrange, to get

$$(3.9)\qquad \log i_t = \log z_t + i_0^* + i_1^* \begin{matrix} \log k_{t-1} - \log z_{t-2} \\ \log i_{t-1} - \log z_{t-1} \end{matrix} + i_w^* w_{t-1} + i_\varepsilon^* \varepsilon_t$$

$$+ i_d^* \begin{matrix} \log k_t - \log z_{t-1} \\ \log h_t \end{matrix}$$

$$(3.10)\qquad \begin{matrix} \log k_t \\ \log h_t \end{matrix} = \begin{matrix} \log z_{t-1} \\ 0 \end{matrix} + d_0 + d_1 \begin{matrix} \log k_{t-1} - \log z_{t-2} \\ \log i_{t-1} - \log z_{t-1} \end{matrix} + d_w w_{t-1}$$

$$+ d_\varepsilon D(\varepsilon_t + v_t)$$

the equation generating output is

$$(3.11)\qquad y_t = [z_t h_t]^{(1-\theta)} n^{-\theta} [(1-\sigma) k_{t-1}^{-\nu} + \sigma i_{t-1}^{-\nu}]^{-\theta/\nu},$$

which can be computed recursively after (3.9) and (3.10). Consumption can then be derived from the national income identity, (2.3). This completes the discussion of approximating the solution to the planning problem.

4. Parameter Estimation

The underlying disturbances of the model are the ε_t's and v_t's. The joint Gaussian density of these is proportional to

$$f(\varepsilon_t, v_t) = [|V|V_v|]^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}\varepsilon_t' V^{-1} \varepsilon_t - \tfrac{1}{2} v_t' V_v^{-1} v_t\}.$$

Let $x_t = (\log k_t, \log h_t, \log y_t, \log i_t)'$. The objective is to express the density of the data sample, $\{x_1, \ldots, x_T\}$, in terms of $f(\varepsilon_t, v_t)$, $t = 1, \ldots, T$, and initial conditions.

Equations (3.9) - (3.10) can be written

$$x_t = \begin{cases} g(\varepsilon_t, v_t; x_{t-1}, x_{t-2}, \ldots, x_1, z_0, z_{-1}, k_0, i_0, w_0) & t = 2, 3, 4, \ldots \\ g(\varepsilon_1, v_1; z_0, z_{-1}, k_0, i_0, w_0) & t = 1. \end{cases}$$

Define $W = \partial g/\partial(\varepsilon_t', v_t')$. After some algebra, we find

$$W = \begin{bmatrix} d_\varepsilon D & d_\varepsilon D \\ A & B \end{bmatrix},$$

where

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

Here, A_i and B_i are 1 x 2 vectors, i = 1, 2, and

$$A_2 = i_\varepsilon + i_d^*(d_\varepsilon D), \; B_2 = i_d^*(d_\varepsilon D)$$

$$A_1 = (1-\theta)\left[(1,0)d_\varepsilon D+(0,1)\right], \; B_1 = (1-\theta)(1,0)d_\varepsilon D.$$

It is a standard result (see, eg., Dhrymes [1974, p. 10]) that the conditional density of x_t can be expressed as follows:

$$h(x_t | x_{t-1}, x_{t-2}, \ldots, x_1, z_0, z_{-1}, k_0, i_0, w_0)$$

$$= f(\varepsilon_t, v_t)|W|^{-1}.$$

Thus, conditional on the initial observations, z_0, z_{-1}, k_0, i_0, w_0, the density of $\{x_1, \ldots, x_T\}$, is

$$h(x_T, \ldots, x_1 | z_0, z_{-1}, k_0, i_0, w_0)$$

$$= h(x_T | x_{T-1}, \ldots, x_1, z_0, z_{-1}, k_0, i_0, w_0)$$

$$\cdot \; \cdot \; \cdot \; \cdot$$

$$h(x_1 | z_0, z_{-1}, k_0, i_0, w_0)$$

$$= |W|^{-T}[|V||V_v|]^{-\frac{T}{2}} \exp\{-\tfrac{1}{2}\varepsilon_t' V^{-1}\varepsilon_t - \tfrac{1}{2} v_t' V_v^{-1} v_t\}.$$

Finally, the Gaussian log likelihood is

$$(4.1) \qquad L = -T \log |W| - \frac{T}{2}\log |V| - \frac{T}{2}\log |V_v| - \frac{1}{2}\varepsilon_t' V^{-1}\varepsilon_t - \frac{1}{2} v_t' V_v^{-1} v_t.$$

Data on k_t, h_t, y_t, i_t for the 115 quarters 1955.3-1984.1 were used for estimation. For any admissible value of the parameter vector $\Gamma = (\beta,\theta,n,\delta,\gamma,\sigma,\nu,a,A,V,V_v)$, these data can be used to compute a set of 111 fitted disturbances $\hat{\varepsilon}_t$, $\hat{v}_t$ for the period 1956.3-1984.1 in seven steps:

1. Compute the reduced form parameters i_0^*, i_1^*, i_w^*, i_ε^*, i_d^*, d_0, d_1, d_w, d_ε, D in (3.7) and (3.8) using the formulas in Appendix B.
2. Compute a set of 114 $\hat{z}_t$'s for the period 1955.4-1984.1 using the production function, (3.11).
3. Given the $\hat{z}_t$'s, compute a set of $\hat{\varepsilon}_{2t}$'s for the period 1956.2-1984.1 using (2.7a).
4. Given the $\hat{z}_t$'s, compute a set of 113 technology shocks, $\hat{x}_t \equiv \hat{w}_{2t}$, for the period 1956.1-1984.1 using (2.4).
5. Use the $\hat{z}_t$'s and (3.1) to compute a set k_t^*, i_t^* for the period 1956.1-1984.1.
6. Solve equation (3.7)' for $\hat{u}_t$ for 1956.2-1984.1 using i_t^*, k_t^* for 1956.1-1984.1 and $\hat{\varepsilon}_{2t}$ for 1956.2-1984.1. Use the $\hat{u}_t$'s to compute a set of $\hat{\varepsilon}_{1t}$'s for the period 1956.3-1984.1 using (2.6) and (2.7c).
7. Using the $\hat{w}_t$'s and $\hat{\varepsilon}_t$'s, compute a set of $\hat{v}_t$'s from (3.8).

The $\hat{\varepsilon}_t$'s and $\hat{v}_t$'s computed above were substituted into (4.1) to get $L(\Gamma)$, the likelihood function of the data, conditioned on the initial observations. The data on which the likelihood is conditioned are the 1955.3-1956.2 observations on k_t and i_t and the 1955.4-1956.2 observations on h_t and y_t. It took roughly 2 cpu seconds to evaluate L once on an IBM 3033 computer.

My parameter estimator is defined by $\tilde{\Gamma}$ = argmax $L(\Gamma)$, where the maximization was carried out over admissible parameter values. Because of the dependence of $d_{\varepsilon}D$ on V and V_v, it was not possible to concentrate V and V_v out of the likelihood function. I did not search over β, η, δ, which were set to .99, 1.00324 and .018, respectively, and so deleted these from Γ. The values for η and δ were chosen based on calculations reported in Appendix A. The variance-covariance of the parameter estimator was estimated by minus the inverse of the second derivative of L, evaluated at $\Gamma = \hat{\Gamma}$.

5. Fitting the Model to Data

I used two strategies to assign numerical values to the parameters of the model. I call the first strategy "calibration," because it resembles in some respects the one proposed in Kydland and Prescott [1982]. A difference between my approach and that of Kydland and Prescott is that I do not utilize parameter estimates taken from other studies. My approach resembles theirs in that it places heavy emphasis on matching the first moment implications of the model with sample first moments. The second strategy attempts to match first and second moment implications of the model with the corresponding sample objects. This strategy corresponds to standard econometric estimation, in that the metric used is the conditional Gaussian likelihood function described in Section 4. Before presenting the results, I briefly describe the data. For a more detailed discussion, see Appendix A.

5.a The Data

My consumption measure is consumption of nondurables and services, plus the imputed rental value of the stock of consumer durables, plus government consumption. Investment was defined as fixed investment plus consumption of durable goods, plus government investment. A capital stock series which closely matches my investment concept was used. It includes the sum of fixed nonresidential capital (private and government), plus the stock of durable goods held by consumers, plus the stock of government and privately held residential capital. My measure of inventories is the sum of farm and nonfarm inventories. Inventory investment is the first difference of the stock of inventories. Gross output was defined as consumption plus investment plus inventory investment. The difference between my measure of GNP and the one published by the Survey of Current Business is that mine includes the imputed rental value of the stock of consumer durables, but does not include net exports.

Hours were measured in "efficiency units" by weighting hours worked by different age-sex categories by their average wage in the 1970's. The data were constructed by Gary Hansen. All data were converted to per capita terms by dividing by the total population between the ages of 15 and 65. These data were converted to efficiency units in the same way as the hours data. Reasons for working with population and hours data that are quality adjusted in this way are described in Denison [1979, chapter 3] and Darby [1984].

5.b Calibration Results

The first column in Table 5.1 reports the relevant averages for the data used in this project. The second column presents steady state values implied by the model at its "calibrated" parameter values. These are reported in column 1 in Table 5.2. A brief discussion of the way I chose these follows.

The parameter n was equated to the average growth in the quality adjusted, working age population for the period 1952-1984. Given this value of n, $\delta = .018$ is required if the gross investment series implied by $k_t - \frac{1-\delta}{n} k_{t-1}$ is to resemble the gross investment series supplied by the Department of Commerce. (See Appendix A for a further discussion.) The value of the parameter β was simply taken from Kydland and Prescott [1982]. The remaining parameters that determine steady state values are ν, θ, σ, $a_2/(1-A_{22})$. I chose these as follows. Given numerical values for ν, θ, σ, a z_t series can be computed from (2.3) using the historical data on h_t, k_t, and i_t. Values for a_2 and A_{22} were then obtained as the constant and slope terms, respectively, in the least squares regression of $\log(z_t/z_{t-1})$ on $\log(z_{t-1}/z_{t-2})$. I searched over alternative values of ν, θ and σ to make the model's steady state implications close to the corresponding averages listed in the first six rows of column 1 in Table 5.2. The values I chose for σ and ν coincide with the values used by Kydland and Prescott [1982], whose production function is identical to mine. This procedure implied a value of .0032 for $a_2/(1-A_{22})$. This is the steady state growth rate of z_t and therefore of all the variables of the model, except h_t, which does not grow in steady state.

Before studying the version of the model with observation noise, we need more parameter values. In particular, we need V and V_v, which enter the decision rules for k_t and h_t via D in (3.10). (D is defined after [2.9].) We also need a value for A_{11}. I obtained these by maximizing the likelihood function, (4.1), with respect to V, V_v and A_{11} and holding the remaining parameters fixed at their values reported in Table 5.2.

Tables 5.3 and 5.4 provide an analysis of the residuals of the calibrated model. The estimated innovations to productivity, ε_2, correspond well with the hypothesis of white noise. This reflects in part the fact that they are the residuals from a least squares regression, and in part that the growth rate of productivity, z_t, seems only to have significant autocorrelation of order one. (This is consistent with results in Prescott [1986].) The other residuals do not conform so well with the hypothesis of white noise. For example, the innovation to taste shocks has significant negative first order autocorrelation. In addition, there is very substantial positive autocorrelation in the estimated observation errors, v_t. The results in Table 5.3 represent strong evidence against the calibrated model.

Further evidence against the model shows up in a comparison of Tables 5.4 and 5.2. There we see that there is a major discrepancy between the estimated V and second moment properties of the fitted ε_t's. For example, according to the estimates reported in Table 5.2, the standard deviation of the innovation to $\log(z_t)$ is an astounding 14.5 percent. By contrast, the standard deviation of the estimated innovations to $\log(z_t)$ is a more reasonable 1.9 percent. If the calibrated model were a good representation of the data, these results would be similar. The numerically large correlation between the fitted v_1 and v_2, and fitted ε_2 constitutes further evidence against the model.

Because the estimates of the innovation variances in Table 5.2 seemed so unreasonable, for simulation purposes I replaced them by the values reported in Table 5.4. The likelihood function complained bitterly at this change, and dropped to -24112.100 from 971.503.

5c. Estimation Results

I estimated 13 parameters of the model by maximizing the conditional log likelihood function, (4.1). The free parameters where ν, σ, θ, γ, V, V_v, and the three parameters in a and A. The results are reported in the second column in Table 5.2. The parameter values of the estimated model differ from those of the calibrated model in several important respects. First, note the substantial reduction in the values of ν and σ relative to their values in column 1. The consequence of this is that inventories have been driven out of the production function. To see this, recall the well-known result that as ν goes to zero, $[(1-\sigma)k_{t-1}^{-\nu}-\sigma i_{t-1}^{-\nu}]^{-1/\nu}$ goes to $k_{t-1}^{(1-\sigma)}i_{t-1}^{\sigma}$. Then, as σ goes to zero, the latter term converges to k_{t-1}. In addition to driving inventories out of the production function, the estimated model reduces substantially the share of income going to capital. The model's estimate of .19 for this quantity is considerably less than what seems plausible for the U.S. economy (see footnote 2.1).

Another striking difference between the calibrated and estimated parameter values is that a_2 has a negative sign in the latter, implying negative average growth. The estimated value of $a_2/(1-A_{22})$ is exactly what is required for the steady state first order condition on inventories (eg., $r_4 + \beta r_3 = 0$) to hold with equality. As noted in section 3b, this condition is assumed in the steady state calculations.

The steady state implications of the estimated parameters are reported in column 3 of Table 5.1. Note that the capital-output ratio is about what it is in the calibrated model. This is because the effects on the capital-output ratio of a reduction in the value of θ and $a_2/(1-A_{22})$ are offsetting. The dramatic difference between the second and third columns lies in the absence of inventories and the negative growth rate in the latter. Both these implications are very much at variance with the U.S. data.

Tables 5.3 and 5.4 provide an analysis of the fitted disturbances in the estimated model. Table 5.3 shows that there is less serial correlation in the fitted disturbances than for the calibrated model. Nevertheless, there is still enough serial correlation in the fitted observation errors to make the null hypothesis that they correspond to an underlying white noise implausible. In addition, Table 5.4 shows that the correlations between $\hat{\epsilon}_t$ and $\hat{v}_t$ are numerically large, and probably not consistent with the model's assumption that ϵ_y and v_t are uncorrelated.

To summarize, there appears to be substantial evidence against the model. The estimated model yields implausible parameter estimates and its fitted disturbances depart from the model's assumptions. In addition, the fact that the parameters of the calibrated and estimated models differ so mcuh is a count against both of them. If the model were a good approximation to the U.S. data, then it should not make such a difference whether the parameters are chosen based on first moment considerations alone (as in the calibrated model) or based on first and second moments (as in the estimated model). Hopefully, the differences between the calibrated and estimated models supplies clues to help produce a diagnosis about what is wrong with the model. Providing such a diagnosis here is beyond the scope of this paper.

The dynamic response of two versions of the calibrated model to an innovation in technology is graphed in Figures 1-5. The version labeled "no noise" is the one in which $V_v = 0$. The version labeled "estimated noise" is the one in which V_v is assigned the values in Table 5.4. In both versions of the model, V is assigned the values in Table 5.4. The figures show how the calibrated model captures the intuition in the introduction.

The figures show the first 15 quarters' responses of z_t, h_t, dk_t, y_t, di_t, and c_t to ε_{2t} = .019 for t = 3 and 0 otherwise. The curves in the figures are the log deviation of these variables from a baseline scenario in which the variables are on a steady state growth path and $\varepsilon_{2t} = 0$ for all t. Note the "spike" in z_t. This reflects the fact the $A_{22} = -.076$, so that only 92 percent of the innovation in technology is permanent (see [2.7b]). The effect of the shock is to eventually put z_t on a growth path .18 percent higher than if the shock had not occurred. Because of the balanced growth property of the model, c_t, dk_t, di_t, and y_t also eventually end up .18 percent higher. There is no long run effect of the shock on h_t.

Note that in the "estimated noise" version of the model, there is little response in h_t--and hence y_t--and dk_t in the first quarter to the shock. This reflects the fact that the shock is only imperfectly observed at the time these decisions are made.

Footnotes

[1.1] The time-to-build technology described in Kydland and Prescott (1982) provides one reason why additions to plant and equipment may respond weakly to contemporaneous innovations in tastes and technology. In their model current additions to capital depend on decisions taken several periods earlier.

[1.2] For example, Deaton ( ) shows that the standard deviation of an innovation to consumption is a fraction of the standard deviation of an innovation to income.

[2.1] Under the assumption that employed workers are paid the value of their marginal product, (2.3) indicates that labor's share, $(1-\theta)$, is the ratio of the wage bill to total gross output. Although measurement problems make it difficult to measure labor's share exactly using aggregate data, there is some evidence that--regardless of how it is measured--labor's share is roughly constant. One way to measure labor's share under the assumption that employed people are paid the value of their marginal product is to take the ratio of employee compensation plus proprietor's income to total gross output. I measured total gross output as GNP plus the imputed rental value of the stock of consumer durables. (A time series on the latter was taken from the data base documented in Brayton and Manskopf [1985].) All other calculations in this footnote are based on data in the Economic Report of the President [1986].) The average value of annual observations on this ratio for the period 1948-1985 is 66 percent, with a standard deviation of only 1 percent. There is a slight downward trend in this series. In the 1950's it is around 67 percent, and by the late 1970's and 1980's it is about 64 percent. Nevertheless, as an approximation it seems fair to conclude that this series is roughly constant. On the other hand, there are at least two potential sources of measurement error in this estimate of labor's share. First, including the whole of proprietor's income in the numerator overstates labor's share, since some of this represents return on capital. On the other hand, the wage bill is understated by assigning the whole of the discrepancy between national income and net national product (NNP-NI) to capital's share. Alternately deleting proprietor's income, and then adding NNP-NI to my base measure of labor's share, I get measures of average labor's share of 57 (2) and 75 (1) percent, respectively. (Numbers in parenthesis are standard deviations.) Thus, although these--admitedly crude--calculations place labor's share somewhere in the rather large range of 57 to 75 percent, whatever method I use results in a roughly constant share time series. Solow [1957, Table 1] reports share data for the period 1909-1948. That series is also close to constant, and close to my guess of 66 percent.

2.2/It should be emphasized that the role of inventories in this model is not limited to the direct one via the production function. As will be evident later, inventories also play a role in smoothing the impact on production of taste shocks and in smoothing the impact of technology shocks on consumption. In addition to these roles, which are similar to the ones played by fixed capital, inventories play the role of a residual. This paper argues that the latter function of inventories is key to understanding the large fluctuations in inventory investment.

2.3/The assumption that employment growth is constant is not a good one for the post war United States. A detailed discussion of this appears in Appendix A.

2.4/These shocks are measured using the Solow [1957]'s method of decomposing output growth into parts due to growth in factor inputs and a residual. The residual is interpreted as a technology shock. For a recent discussion of this, see Prescott [1986].

2.5/The average of average weekly hours in manufacturing over this period was 40.2 hours per week and appears to be trendless. Of the variance in average weekly hours, a little over 70 percent is accounted for by variance in average overtime hours, which averages only three hours a week, and which is close to uncorrelated with straight time hours. (The standard deviation of overtime hours is about 31 minutes per week.) Average weekly hours of all nonagricultural workers has declined significantly since the 1940's, from 40 hours per week to about 35 in the 1980's. On the other hand, the standard deviation of this series's deviation from a trend line is also tiny. (The data underlying the discussion in this footnote were taken from economic Report of the President [1986, p. 300].)

2.6/More basically, I would first have to take a stand on which of the two interpretations of (2.5) described in the text to choose. If at this stage, G. Hansen's interpretation were chosen, then I would have to decide between at least two possible market structures. In Hansen's model, workers choose to insure themselves against the event of being laid off. Under one market structure, firms supply the insurance, while under the alternative, an insurance company does so. When firms supply unemployment insurance, the wage paid to employed persons is the value of their marginal product, minus the cost for insurance. When a separate insurance company supplies unemployment insurance, employed people are paid the value of their marginal product. Hansen shows that these two market structures have the same implications for equilibrium quantities.

3.1/In fact, Appendix B only covers the case in which $V_v = 0$, i.e., there is no observation error. In this case, $d_\varepsilon D(\varepsilon_t+v_t) = d_\varepsilon \varepsilon_t$. The presence of the $D(\varepsilon_t+v_t)$ term arises in the $V_v \neq 0$ case because this is the best estimate of ε_t, given $\bar{w}_t$ and w_{t-1}.

Table 5.1

Model and Sample Averages[1]

	Sample Average 56.3-84.1	Calibrated Model[2]	Estimated Model[3]
c_t/y_t	.72 (.017)[4]	.729	.91
$(k_t-.9788k_{t-1})/y_t$	.27 (.013)	.264	.09
$(i_t-i_{t-1}/1.00324)/y_t$	.006 (.007)	.006	0.
k_t/y_t	10.58 (.44)	10.85	10.57
i_t/y_t	.90 (.047)	.97	0.
h_t	320.4 (7.70)	321.6	465.0
$(c_t-c_{t-1})/c_{t-1}$	.0039 (.0056)	.0032	-.013
$(y_t-y_{t-1})/y_{t-1}$	.0041 (.0115)	.0032	-.013
$(k_t-k_{t-1})/k_{t-1}$	.0046 (.0021)	.0032	-.013
$(i_t-i_{t-1})/i_{t-1}$	.0042 (.0082)	.0032	-.013
$(h_t-h_{t-1})/h_{t-1}$	.00042 (.0150)	0.0	0.0

[1] Rows 1-3 do not add to 1 due to rounding.

[2] Parameter values underlying these results are reported in column 1, Table 5.2.

[3] Parameter values underlying these results are reported in column 2, Table 5.2.

[4] Numbers in parentheses are sample standard deviations.

Table 5.2

Parameter and Likelihood Values

	Calibrated Model[1]		Estimated Model[3]
	Parameter Value	(standard error)	Parameter Value
β	.99		.99
n	1.00324		1.00324
δ	.018		.018
ν	4		.00338
θ	.39		.19
σ	$.28 \times 10^{-5}$		$.92 \times 10^{-17}$
γ	.0026		.0019
A_{22}	-.076		-.216
a_2	.0035		-.016
A_{11}	.986	(.0025)	.957
V_{11}	$(.111)^2$	(.0015)	$(.030)^2$
V_{12}	-.014	(.0019)	-.00026
V_{22}	$(.145)^2$	(.0028)	$(.033)^2$
$V_{\nu,11}$	$(.063)^2$	(.00050)	$(.0070)^2$
$V_{\nu,12}$	-.0044	(.00060)	.0012
$V_{\nu,22}$	$(.082)^2$	(.00090)	$(.0173)^2$
L[2]	971.503		1785.94

[1]The first 9 parameter values obtained as described in Section 5.b. The next 7 parameter values maximize (4.1). The corresponding standard errors appear in parentheses.

[2]Value of (4.1) at parameter values reported above.

[3]The first 3 parameter values obtained as described in Section 5.6. The next 13 obtained by maximizing (4.1). Standard errors are not reported, since the estimate of σ lies on a boundary.

Table 5.3

Autocorrelations of Fitted Residuals[1/]

	Lag								
Variable	1	2	3	4	5	6	7	8	s.e.[3/]
Calibrated Model[2/]									
ε_1	-.25	-.00	.01	.01	-.21	.15	-.11	-.01	.09
ε_2	-.02	.07	.03	-.01	-.13	-.01	-.09	-.01	.09
v_1	.89	.76	.63	.49	.35	.25	.20	.17	.09
v_2	.96	.89	.81	.73	.66	.59	.53	.48	.09
Estimated Model[4/]									
ε_1	-.22	-.21	.10	-.00	-.01	.03	.03	-.05	.09
ε_2	-.02	-.05	.04	.03	-.08	-.04	-.06	.04	.09
v_1	.61	.60	.55	.51	.40	.31	.25	.26	.09
v_2	.67	.65	.60	.56	.46	.38	.32	.32	.09

[1/]Residuals are the values of ε_t and v_t for t = 1956.3-1984.1 implied by parameter values reported in Table 5.2 and by U.S. data on k_t, i_t, y_t, and h_t (see Section 4 for details).

[2/]Underlying parameter values reported in column 1, Table 5.2.

[3/]Standard error under the null hypothesis of white noise.

[4/]Underlying parameter values reported in column 2, Table 5.2.

Table 5.4

Covariance/Correlation Matrix
of Fitted Residuals[1/]

	ε_1	ε_2	v_1	v_2
		Calibrated Model[2/]		
ε_1	.015	-.60	.13	-.081
ε_2	-.00018	.019	-.469	.213
v_1	.00009	-.0004	.046	-.646
v_2	-.00009	.0003	-.0023	.076
		Estimated Model[3/]		
ε_1	.011	-.459	.364	.337
ε_2	-.00008	.016	-.635	-.596
v_1	.00004	-.00009	.009	.995
v_2	.00008	-.0002	.0002	.023

[1/]Numbers below the diagonal are covariances. Numbers above the diagonal are correlations. Numbers on the diagonal are standard deviations.

[2/]Underlying parameter values reported in column 1, Table 5.2.

[3/]Underlying parameter values reported in column 2, Table 5.2.

Table 5.5

Reduced Form Parameters[1/]

Calibrated Model[2/]	Estimated Model[3/]
$i_s^* = 7.261$	$i_s^* = -18.500$
$k_s^* = 9.675$	$k_s^* = 9.051$
$h_s^* = 5.773$	$h_s^* = 6.142$
$i_0^* = -.691$	$i_0^* = -21695598.018$
$i_1^* = (10.289,\ .923)$	$i_1^* = (68375777.93,\ .000073)$
$i_W^* = (-.358,\ -10.236)$	$i_W^* = (-4442264.18,\ -68795363.97)$
$i_\varepsilon^* = (-.363,\ -.694)$	$i_\varepsilon^* = (-464197.80,\ 1941875.76)$
$i_d^* = (-10.498,\ .572)$	$i_d^* = (-69589497.02,\ 5171975.92)$
$d_0' = (.790,\ 9.975)$	$d_0' = (.624,\ 12.35)$
$d_1 = \begin{matrix} .862 & .0753 \\ -.378 & -.0754 \end{matrix}$	$d_1 = \begin{matrix} .930 & .9 \times 10^{-12} \\ -.686 & -.2 \times 10^{-11} \end{matrix}$
$d_W = \begin{matrix} .039 & -.867 \\ 1.173 & .338 \end{matrix}$	$d_W = \begin{matrix} .018 & -.955 \\ 1.079 & .425 \end{matrix}$
$d_\varepsilon = \begin{matrix} .030 & .050 \\ .881 & .381 \end{matrix}$	$d_\varepsilon = \begin{matrix} -.0025 & .089 \\ .852 & .816 \end{matrix}$

[1/] The reduced form of the model is defined by (3.9) and (3.10).

[2/] Corresponds to parameter values in Table 5.2, column 1.

[3/] Corresponds to parameter values in Table 5.2, column 2.

Table 6.1

Standard Deviation of One Quarter Growth Rates (a) and Correlations of One Quarter Growth Rates With Output (b) for U.S. and Model

	U.S. Data		Calibrated Model[1/]	
	(a)	(b)	(a)	(b)
Output	.012		.015 (.001)	
Consumption	.007	.54	.007 (.0004)	.50 (.085)
Investment	.023	.69	.070 (.007)	-.10 (.088)
Hours	.015	.40	.015 (.001)	.61 (.058)

[1/] The underlying model parameter values are those in column 1, Table 5.2, except V, V_v, which were taken from Table 5.4.

Table 6.2

Average Absolute Quarterly Change
Relative to Output

	U.S. Data	-----------Calibrated Model----------		
		(1) Estimated Noise[1/]	(2) High Noise	(3) Low Noise
Output	.0098	.012 (.0009)[2/]	.012 (.0009)	.012 (.0008)
Consumption	.0046	.0046 (.0004)	.0046 (.0004)	.0046 (.0005)
Investment	.0046	.015 (.0012)	.015 (.0013)	.0095 (.0008)
Inventory Investment	.0050	.019 (.0017)	.019 (.0019)	.002 (.00018)
Likelihood Value	NA	-24112.10	-3057735211.3	$-\infty$

[1/]See note 3, Table 6.3.

[2/]Standard deviation across 100 model simulations.

[3/]Value of L in (4.1) at the indicated parameter values. Model (3) has $L = \infty$ since there is a stochastic singularity in this case.

Table 6.3

Volatility[1]

	U.S. Data	Calibrated Model[2]		
		(1) Estimated Noise[3]	(2) High Noise	(3) No Noise
Output[1]	.0098	.012 (.0009)[4]	.012 (.0009)	.012 (.0008)
Consumption	.65	.523 (.050)	.516 (.049)	.538 (.056)
Investment	1.74	4.57 (.49)	4.62 (.473)	3.09 (.24)
Inventory Investment	81.39	265.70 (74.09)	273.03 (76.75)	29.71 (7.23)
Likelihood Value[5]	NA	-24112.10	-3057735211.3	$-\infty$

[1] Row 1: Average absolute quarterly change in output relative to output. Rows 2-4: Let x denote one of the variables in rows 2-4 of the left column (i.e., x = consumption, investment, or inventory investment), and let y denote output. On a given model simulation, the volatility of x, V_x (say), was computed as the ratio of two statistics, denoted v_x and v_y. Here,

$$v_x = \sum_{t=1}^{T} [|x_t - x_{t-1}|/y_t] / \sum_{t=1}^{T} (x_t/y_t) \text{ and } v_y = \sum_{t=1}^{T} |y_t - y_{t-1}|/y_t.$$

Then $V_x \equiv v_x/v_y$.

[2] Underlying model parameter values are those given in Table 5.2; and V was taken from Table 5.4. In case (1), V_v was taken from Table 5.4. In case (2), $V_v = 500I_2$, where I_2 is the two dimensional identity matrix. In case (3), $V_v = 0$.

[3] High noise: $V_v = 500I_2$, V taken from Table 5.4. Estimated noise: V_v, V both taken from Table 5.4. No noise: $V_v = 0$, V taken from Table 5.4.

[4] Standard deviation across 100 model simulations.

[5] Value of L in (4.1) at the indicated parameter values. Model (3) has $L = -\infty$ since there is a stochastic singularity in this case.

Appendix A: Data

Following is a discussion of the data used in this project.

Quality Adjusted, Working Age Population

Data on the total male and female working age population were obtained from the Chase Econometrics U.S. Macroeconomic data base. The working aged population was defined as males and females aged 15 to 64. The Chase mnemonics for these data are ANPTMT1519, ANPTMT2024, ..., ANPTMT6064, and ANPTFT1519, ANPTFT2024, ..., ANPTFT6064, respectively, and they were most recently revised on April 28, 1986. The data are available on an annual basis, and represent estimates of the population on July 1.

The model of this paper abstracts from the effects of changes in human capital on labor productivity. However, the human capital of the average worker in the post-war period has not been constant. In an attempt to adjust for this, I obtained a quality adjusted working age population by weighting each age-sex group by its average wage in the 1970's. The weights, which were standardized on males aged 35 to 44, were taken from Hansen (1984), and are as follows:

Table A1

Ages	Males	Females
15-19	.44	.4
20-24	.61	.51
25-34	.86	.64
35-44	1.0	.63
45-54	1.01	.62
55-64		

The gross growth rate in the quality adjusted (GWEIT) and unadjusted (GUNWT) working aged populations are graphed in Figure 1A. The results there show that adjusting the data along age-sex lines does not have a substantial effect on the numbers. Darby (1984) argues that the data ought to be adjusted for education levels and immigration flows. This further adjustment may be worth exploring, however, I have not done so.

Quarterly observations on the quality adjusted working age population were obtained by log-linearly interpolating the annual data. The calculations were carried out treating the annual observations as third quarter observations.

Several features of the data stand out. First, as is plain from Figure 1A, they do not satisfy the constant growth assumption in the text. This is confirmed by the numbers in Table 2A.

Table 2A: Percent Annual Growth Rate, Working Age Population

	Quality Adjusted	Not Adjusted
1952-1961	.85 (.07)	.9 (.19)
1970-1984	1.6 (.19)	1.5 (.35)
1949-1984	1.3 (.36)	1.3 (.43)

Numbers in parenthesis in Table 2A are standard deviations, in percent terms. The growth rate of the quality adjusted working age population approximately doubled in the 1970's and 1980's over what it was in the 1950's. Interestingly, the results are basically the same for the unadjusted working aged population.

A second important feature of the working age population is that they behave very differently from data on the population as a whole. Data on the total population, including armed forces overseas, were obtained from the Chase Econometrics database (mnemonic NPT). From the period 1952 to 1961, this data display an average annual growth rate of 1.7 percent, with standard error .07 percent. For the period 1970 to 1984, the average growth rate was 1 percent with standard error .1 percent. Thus, the pattern of growth in the working age population is opposite to that of the population as a whole. This probably reflects the large number of births shortly after the war, which showed up in the total population immediately, but only with a lag in the working age population. Because of this, the time series behavior of economic variables in per capita terms are sensitive to the choice of population data used.

A third feature of this population data is that the growth in total population exhibits substantial seasonality, with growth being especially high in the first few months of the year. Obviously my interpolated quality adjusted working age population data do not exhibit such seasonality, although the actual working age population probably does. The absence of seasonality in my working age population data is consistent with the fact that all other data used in this project have been seasonally adjusted.

Capital Stock and Investment

Investment for the purpose of this project is defined as real consumer purchases of durable goods (ECD) plus real gross private fixed domestic investment (IFIXED), plus real government (federal, state and local) investment, including investment in the military (IGINVEST). ECD and IFIXED are as reported in Table 1.2 of the Survey of Current Business (SCB). Annual observations on IGINVEST were provided to me by John Musgrave of the Bureau of Economic Analysis. The IGINVEST data are a revised and updated version of the government investment data discussed in Musgrave [1980]. Quarterly observations in IGINVEST were obtained using the interpolation by related series method of Chow and Lin (1972). The related series used for this purpose were ECD, IGD82, a constant and a linear trend. (IGD82 is gross private domestic investment, as reported in Table 1.2 of SCB.)

The aggregate investment data were converted to per capita terms by dividing by the quality adjusted working age population.

Annual, end of year capital stock data were obtained from the January, 1986 SCB, Tables 4, 8, 12, 16, 20, found on pages 59-75. These data were used to obtain a time series on the 1982 dollar value of the net stock of capital. The data are the sum of fixed nonresidential capital (private, federal, state and local), plus the stock of durable goods held by consumers, plus the stock of government and privately held residential capital. For further details about this data, the reader is referred to the data source.

Some details about the composition of the capital stock are of interest. First, the average value of the capital to quarterly GNP ratio in the period 1955 QIII to 1984 QI is 12.8, with a standard deviation of .5. (Here, GNP is defined as GNP according to National Income account standards, plus the services of consumer durables, minus net exports.) At the end of 1984, the aggregate net stock of capital was $9,799 billion, in 1982 dollars. Of this, 35 percent was private equipment and structures, consumer durables were 13 percent, public and private residential capital was 32 percent and government equipment and structures was 20 percent.

Since the capital data in the SCB are annual, they had to be converted to quarterly observations. Quarterly observations on consumer durables and private equipment and structures were obtained from the MPS model data base. (This is documented in Brayton and Manskopf [1985].) Data on the private stock of residential capital were also obtained from the MPS data base. These data, together with a constant and linear trend were used to interpolate the annual public and private stock of residential capital data in the SCB. (The method of interpolation by related series due to Fernandez [1981] was used for this.) A quarterly series on government equipment and structures was obtained by log-linearly interpolating the annual data taken from the SCB.

Finally, a quarterly per capita data series on the aggregate stock of capital was obtained by adding the individual components and dividing the result by the quality adjusted, working age population.

The depreciation rate on capital, δ, plays an important role in this paper for two reasons. First, it is a parameter of the model so that the value it is assigned has implications for the average capital to output ratio and other endogenous quantities. Second, in order to deduce my model's implications for capital investment, I have to quasi first difference the capital stock series that it generates, using some value for δ.

Based on my examination of the capital stock data, I decided to set $\delta = .018$, which is 7.4 percent annually. This is lower than the numbers used by other researchers. (For example, Kydland and Prescott [1982] assume 10 percent annual depreciation.) The reason I did this was that key time series properties of the actual investment data coincide with investment data derived from my capital stock series using $\delta = .018$. This is not the case when a 10 percent annual depreciation rate is assumed.

The regression of per capita capital (k_t) minus per capita gross fixed investment (dk_t) on k_{t-1} produced a coefficient of .9787. The sample period was 1955 QIV to 1984 QI. In terms of the model in the text, this regression coefficient is to be interpreted as a measure of $(1-\delta)/n$, where n is the quarterly gross growth rate of the working age population and δ is the quarterly depreciation rate. With $n = (1.013)^{.25}$, this implies $\delta = .018$, an annual depreciation rate of about 7.4 percent.

Unfortunately, $k_t - .9787k_{t-1}$ and dk_t differ by a substantial amount. The average value of $100|k_t - .9787k_{t-1} - dk_t|/|dk_t|$ is five percent for the period 1955,4 - 1984,1. (Here, $|\cdot|$ denotes the absolute value operator.) Moreover, the discrepancy, $k_t - .9787k_{t-1} - dk_t$ is highly serially correlated throughout the sample, being strictly positive before 1970, strictly negative thereafter, and close to zero on average.

These results are not consistent with my model formulation, although there may be reason to believe that the consequences of this misspecification are not serious. This is because $k_t - .9787k_{t-1}$ shares several key time series properties of dk_t. First, both are on average 27 percent of gross output. Also, the mean of $100|(k_t-.9787k_{t-1}) - (k_{t-1}-.9787k_{t-2})|/y(t)$ and $100|dk_t-dk_{t-1}|/y(t)$ are roughly the same. The former is .46 with standard deviation .40, while the latter is .47 with standard deviation .39. If δ = .025 is used, then the derived investment series is shifted up by a large 496 dollars per person on average. This is just the product of (.025-.018)/n and the average value of the stock of capital, which is large relative to investment. As a result of this, the average share of gross output of this investment series is 34 percent, substantially higher than the actual, 27 percent figure. Because of this, I set δ = .018 in this study and did not use the more conventional δ = .025.

Inventories

Inventory investment was defined as the change in farm and nonfarm inventories in 1982 dollars as reported in Table 5.9 of SCB. The stock of farm and nonfarm inventories is as reported in Table 5.11 of SCB.

Quality Adjusted Hours Worked

Time series for hours worked for the period 1955 Q3 to 1984 Q1 were provided to me by Gary Hansen. The underlying data were obtained from the Current Population Survey, which is a survey of households. The data were then aggregated by age-sex groups using the weights reported in Table 1A. For further details about this data and the manner in which they were constructed, see Hansen (1984).

As I noted earlier, Darby [1984] argues that data ought to be further adjusted to reflect changes in education levels and immigration flows. Darby provides an annual hours series adjusted in this way for the period 1900-1979 (his mnemonic is QATHWP). The gross rate of change in this data (I call it GDARBY) and in Hansen's quality adjusted hours series (GHANSEN) appear in Figure 2A. The difference between these two series is not great, suggesting that my analysis is probably not sensitive to adjustments for immigration and education.

I obtained a per capita hours series by dividing quality adjusted hours worked by the quality adjusted working age population. These data are graphed in Figure 3A. My model implies a per capital hours series that fluctuates about a constant mean. The data, in fact, show very slight evidence of an increase in hours worked per capita in the post war period. Average growth in the per capita hours series is .16 percent annually. On the other hand, the standard deviation is an enormous 6 percent.

Consumption

The measure of consumption I used is consumption of nondurables plus consumption of services plus the imputed rental value of the stock of consumer durables, plus government consumption. All these components except the last two were taken from SCB. A measure of the imputed rental value of consumer durables was obtained from the data base documented in Brayton and Mauskopf [1985]. Government consumption is government purchases of goods and services minus IGINVEST.

Per capita consumption was obtained by dividing by the quality adjusted, working aged population.

Appendix B:

Solving the LQ Approximate Problem Under Complete State Information

This Appendix describes the algorithm used to find decision rules for i_t^*, k_t^* and h_t^* that solve the linear quadratic approximation to the model in the text. In this Appendix, I assume there is full state information, in the sense that $v_t \equiv 0$. The solution strategy is to first transform the problem into the form of the linear regulator problem in the engineering literature. (See, e.g., Kwakernaak and Sivan [1972]).

Define

$$
s_t = \begin{bmatrix} k_{t-1}^* \\ i_{t-1}^* \\ w_t \\ w_{t-1} \end{bmatrix}, \quad
d_t = \begin{bmatrix} i_t^* \\ k_t^* \\ h_t^* \end{bmatrix}, \quad
\phi_1 = \begin{bmatrix} \underset{2\times 2}{\tilde{0}} & \underset{2\times 2}{\tilde{0}} & \underset{2\times 2}{\tilde{0}} \\ \underset{2\times 2}{\tilde{0}} & \underset{2\times 2}{A} & \underset{2\times 2}{\tilde{0}} \\ \underset{2\times 2}{\tilde{0}} & \underset{2\times 2}{I} & \underset{2\times 2}{\tilde{0}} \end{bmatrix}, \quad
\phi_0 = \begin{bmatrix} \underset{2\times 1}{\tilde{0}} \\ \underset{2\times 1}{a} \\ \underset{2\times 1}{\tilde{0}} \end{bmatrix}
$$

(B.1)

$$
B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ \underset{4\times 1}{\tilde{0}} & \underset{4\times 1}{\tilde{0}} & \underset{4\times 1}{\tilde{0}} \end{bmatrix}, \quad
e_t = \begin{bmatrix} \underset{2\times 1}{\tilde{0}} \\ \underset{2\times 1}{\varepsilon_t} \\ \underset{2\times 1}{\tilde{0}} \end{bmatrix}, \quad
Ee_te_t^T = W = \begin{bmatrix} \underset{2\times 2}{\tilde{0}} & \underset{2\times 2}{\tilde{0}} & \underset{2\times 2}{\tilde{0}} \\ \underset{2\times 2}{\tilde{0}} & V & \underset{2\times 2}{\tilde{0}} \\ \underset{2\times 2}{\tilde{0}} & \underset{2\times 2}{\tilde{0}} & \underset{2\times 2}{\tilde{0}} \end{bmatrix}
$$

The return function may be written

$$
\text{(B.2)} \qquad R(s_t,d_t) = c + c_1^T d_t + c_2^T s_t + s_t^T R s_t + d_t^T Q d_t + 2s_t^T F d_t .
$$

Here, c is a scalar, $c_1 \sim 3 \times 1$, $c_2 \sim 6 \times 1$, $R \sim 6 \times 6$, $Q \sim 3 \times 3$, $F \sim 6 \times 3$. Also, (B.3) $$R = \frac{1}{2}\begin{matrix} r_{11} & r_{13} & r_{17} & r_{16} \\ r_{31} & r_{33} & r_{37} & r_{36} \\ r_{71} & r_{73} & r_{77} & r_{76} \\ r_{61} & r_{63} & r_{67} & r_{66} \end{matrix}, \quad Q = \frac{1}{2}\begin{matrix} r_{44} & r_{42} & r_{45} \\ r_{24} & r_{22} & r_{25} \\ r_{54} & r_{52} & r_{55} \end{matrix}$$

$$F = \frac{1}{2}\begin{matrix} r_{14} & r_{12} & r_{15} \\ r_{34} & r_{32} & r_{35} \\ r_{74} & r_{72} & r_{75} \\ r_{64} & r_{62} & r_{65} \end{matrix}$$

Here, r_{ij} denotes the cross derivative of r with respect to its ith and jth arguments, evaluated at $k^*_{t-1} = k^*_t = k^*_s$, $i^*_{t-1} = i^*_t = i^*_s$, $h^*_t = h^*_s$, $w_t = w_s$. Note that r_{ij} are scalars for all i, j = 1, ..., 5. However, r_{kj} and r_{ik} are 2 × 1 and 1 × 2, respectively, k = 6, 7, and r_{kk} is 2 × 2, k = 6,7.

Finally, consider the constant terms, c_1 and c_2. Let $z_1 = (r_4, r_2, r_5)^T$, $z_2 = (r_1, r_3, r_7^T, r_6^T)^T$. Here, r_j denotes the derivative of r with respect to its jth argument, evaluated at $k^*_{t-1} = k^*_t = k^*_s$, $h^*_t = h^*_s$, $w_{t-1} = w_t = w$. Note that r_7 and r_6 are 2 x 1 vectors, while r_j, j = 1, ..., 5 are scalars.

Then,

$$c_1 = z_1 - 2F^T s - 2Qd$$

$$c_2 = z_2 - 2Fd - 2Rs,$$

where s and d are the steady state values of s_t and d_t, respectively. In particular,

$$s = \begin{matrix} k_s^* \\ i_s^* \\ w \\ w \end{matrix}, \quad d = \begin{matrix} i_s^* \\ k_s^* \\ h_s^* \end{matrix}.$$

Analytic formulas for the r_j's and r_{ij}'s are provided in Appendix C.

In these terms, the LQ approximate problem is to choose a contingency plan for d_t to maximize

$$\text{(B.4)} \qquad E_0 \sum_{t=0}^{\infty} \beta^t \{c + c_1^T d_t + c_2^T s_t + s_t^T R s_t + d_t^T Q d_t + 2 s_t^T F d_t\}$$

subject to

$$\text{(B.5)} \qquad s_{t+1} = \phi_0 + \phi_1 s_t + B d_t + e_{t+1}.$$

The solution to this problem is obtained by iterating on the following functional equation in v:

$$\text{(B.6)} \qquad v'(s_t) = \max_{d_t} \{R(s_t, d_t) + \beta E_t v(s_{t+1})\}$$

subject to (B.5) and s_t given and observable. Here,

$$\text{(B.7)} \qquad v(s_t) = v_c + v_s^T s_t + s_t^T v_Q s_t.$$

I now describe one step in this iteration. Substituting (B.7) into (B.6), get

$$\text{(B.8)} \qquad v'(s_t) = \max_{d_t} \{\tilde{c} + \tilde{c}_1^T d_t + \tilde{c}_2^T s_t + s_t^T \tilde{R} s_t + d_t^T \tilde{Q} d_t + 2 s_t^T \tilde{F} d_t\},$$

where

(B.9a) $\tilde{c} = c + \beta v_c + \beta v_s^T \phi_0 + \beta \phi_0^T v_Q \phi_0 + \beta tr(v_Q W)$

(B.9b) $\tilde{c}_1^T = c_1^T + \beta v_s^T B + 2\beta \phi_0^T v_Q B$

(B.9c) $\tilde{c}_2^T = c_2^T + \beta v_s^T \phi_1 + 2\beta \phi_0^T v_Q \phi_1$

(B.9d) $\tilde{R} = R + \beta \phi_1^T v_Q \phi_1$

(B.9e) $\tilde{Q} = Q + \beta B^T v_Q B$

(B.9f) $\tilde{F} = F + \beta \phi_1^T v_Q B.$

The solution to the maximization in (B.8) is

(B.10) $d_t = K_0 + K_1 s_t,$

where

(B.11a) $K_0 = -\frac{1}{2} \tilde{Q}^{-1} \tilde{c}_1$

(B.11b) $K_1 = -\tilde{Q}^{-1} \tilde{F}^T$

Substituting (B.10) into (B.8) get

$$v'(s_t) = v_c' + (v'_s)^T s_t + s_t^T v_Q' s_t,$$

where

(B.12) $v_c' = \tilde{c} + \tilde{c}_1^T K_0 + K_0^T \tilde{Q} K_0$

(B.13) $v_s' = [\tilde{c}_1^T K_1 + \tilde{c}_2^T + 2K_0^T \tilde{Q} K_1 + 2K_0^T \tilde{F}^T]^T = K_1^T \tilde{c}_1 + \tilde{c}_2$

(B.14) $v_Q' = \tilde{R} + K_1^T \tilde{Q} K_1 + 2\tilde{F} K_1 = \tilde{R} - \tilde{F} K_1 + 2\tilde{F} K_1 = \tilde{R} + \tilde{F} K_1$

The solution to (B.4)-(B.5) is obtained by iterating on (B.6) to convergence.

The calculations just described can be simplified further by first iterating to convergence on v_Q and K_1 using (B.9d)-(B.9f), (B.11b), (B.14). Equations (B.9b), (B.9c), (B.11a), (B.13) can then be solved for v_s and K_0. The vector v_s is obtained by setting $v_s' = v_s$ in (B.13) and solving for v_s. This yields

$$v_s = [I-\beta(K_1^T B^T+\phi_1^T)]^{-1} \{K_1^T[c_1+2\beta B^T v_Q \phi_0]+c_2+2\beta\phi_1^T v_Q \phi_0\}.$$

The constant terms, $\tilde{c}$ and v_c, are not needed and so can be ignored. The solution to (B.6) when v' = v solves (B.4)-(B.5).

An interesting feature of this problem is that the matrix Q is of rank 2. The results in the Appendix C show that

$$-Q = \ell\ell^T + pp^T,$$

where

$$\ell = [\tfrac{1}{2}\exp(-u)]^{\frac{1}{2}} \begin{matrix} r_4 \\ r_2 \\ \gamma \end{matrix}^{2} \quad , \; p = \begin{matrix} 0 \\ 0 \\ [\frac{1}{2}\theta(1-\theta)\frac{\exp(u)}{\tilde{c}h^2}\tilde{y}]^{\frac{1}{2}} \end{matrix}$$

A practical consequence of this is that the iterations on v_Q cannot be started at $v_Q = 0$, since in this case (B.11a) has no solution. Instead, I started v_Q at the product of the identity matrix and a small number.

The feedback rule in (B.10) expresses the decision variable as a function of the current state. It is convenient to express the rule for i_t^* as follows:

(B.14a) $\quad i_t^* = i(\tilde{d}_t, s_t),$

where $\tilde{d}_t \equiv (k_t^*, h_t)^T$. The second two decision rules in (B.10) are written

(B.14b) $\quad h_t = h(s_t)$

(B.14c) $k^*_t = k(s_t)$.

To get (B.14a), carry out the maximization in (B.8) with respect to i^*_t, taking $\tilde{d}_t$ and s_t as given. Doing so, I get

$$(B.15) \quad i^*_t = \alpha_0 + \alpha_1^T\tilde{d}_t + \alpha_2^T s_t \equiv i(\tilde{d}_t, s_t),$$

where

$$(B.16) \quad \alpha_0 = -\tilde{c}_{11}/(2q_{11}),\ \alpha_1^T = -(q_{12}/q_{11}),\ \alpha_2^T = -\tilde{F}_1^T/q_{11},$$

$$\tilde{F} = [\underset{6\times 1}{\tilde{F}_1}\ \ \underset{6\times 2}{\tilde{F}_2}],\quad \tilde{c}_1 = \begin{bmatrix} \underset{1\times 1}{\tilde{c}_{11}} \\ \underset{2\times 1}{\tilde{c}_{21}} \end{bmatrix},\quad \tilde{Q} = \begin{bmatrix} \underset{1\times 1}{q_{11}} & \underset{1\times 2}{q_{12}} \\ \underset{2\times 1}{q_{12}^T} & \underset{2\times 2}{q_{22}} \end{bmatrix}$$

Thus, the solution to (B.4)-(B.5) under complete state information is given by (B.14)-(B.15).

It is convenient to express i^*_t, and $\tilde{d}_t$ in (B.14) in terms of ε_t, k^*_{t-1}, i^*_{t-1}. Doing so, we get

$$(B.17) \quad i^*_t = i^*_0 + i^*_1 \begin{bmatrix} k^*_{t-1} \\ i^*_{t-1} \end{bmatrix} + i^*_w w_{t-1} + i^*_\varepsilon \varepsilon_t + i_d \tilde{d}_t,$$

where

$$i^*_0 = (\alpha_0 + \alpha_{22}^T a),\ i^*_1 = \alpha_{21}^T,\ i^*_w = (\alpha_{23}^T + \alpha_{22}^T A)$$

$$i^*_\varepsilon = \alpha_{22}^T,\ i^*_d = \alpha_1^T.$$

Here, α_2^T has been partitioned as follows:

$$\underset{1\times 6}{\alpha_2^T} \equiv (\underset{1\times 2}{\alpha_{21}^T} \ \vdots\ \underset{1\times 2}{\alpha_{22}^T} \ \vdots\ \underset{1\times 2}{\alpha_{23}^T}).$$

Also,

$$(B.18)\qquad \tilde{d}_t = d_0 + d_1 \begin{bmatrix} k^*_{t-1} \\ i^*_{t-1} \end{bmatrix} + d_w w_{t-1} + d_\varepsilon \varepsilon_t .$$

To describe the construction of d_0, d_1, d_w, d_ε, I first need some notation. Let $\bar{K}_0$ denote the vector formed by deleting the first element of K_0 and let $\bar{K}_1$ denote K_1 minus its first row. Then $\tilde{d}_t = \bar{K}_0 + \bar{K}_1 s_t$. Partition $\bar{K}_1$ as follows:

$$\underset{2x6}{\bar{K}_1} = \{\underset{2x2}{\bar{K}_1^{(1)}} \vdots \underset{2x2}{\bar{K}_1^{(2)}} \vdots \underset{2x2}{\bar{K}_1^{(3)}}\}$$

Then,

$$d_0 = \bar{K}_0 + \bar{K}_1^{(2)} a,\ d_1 = \bar{K}_1^{(1)},\ d_w = (\bar{K}_1^{(3)} + \bar{K}_1^{(2)} A)$$

$$d_\varepsilon = \bar{K}_1^{(2)} .$$

Appendix C:
Analytic Formulas For Derivatives

The derivatives in (B.3) and those implied by c_1 and c_2 can be computed numerically and analytically. For checking purposes, it is convenient to do both. Accordingly, the analytic formulas provided below.

Denote

$$\tilde{\tilde{y}} \equiv \tilde{y}[(1-\sigma)\exp(\nu x)\tilde{k}^{-\nu} + \sigma\tilde{i}^{-\nu}]^{-1}.$$

Then,

$$r_2 = -\frac{\exp(u)}{\tilde{c}}\exp(-x)$$

$$r_1 = -r_2/\beta$$

$$r_4 = -\frac{\exp(u)}{\tilde{c}}$$

$$r_3 = -r_4/\beta$$

$$r_5 = 0$$

$$r_6 = 0$$

$$r_7 = -\frac{\exp(u)}{\tilde{c}}\left\{\theta(1-\sigma)\exp(\nu x)\tilde{k}^{-\nu}\tilde{\tilde{y}} + \frac{1-\delta}{n}\exp(-2x)\tilde{k}\right\}$$

$$r_8 = \exp(u)\ln\tilde{c}.$$

$$r_9 = \frac{\exp(u)}{\tilde{c}}\left\{-\theta\tilde{y} + \exp(-x)\tilde{k}[1-\frac{1-\delta}{n}\exp(-x)] - \exp(-x)\frac{1}{n}\right\}$$

The expressions for r_1, r_3, and r_5 exploit the steady state first order necessary conditions.

Next, turn to the second derivatives.

$$r_{11} = -r_1^2 \exp(-u) - \{\theta(1-\theta) \exp(\nu x)\tilde{k}^{-(\nu+2)} \exp(u)/\tilde{c}\}$$

$$\times\{(\nu+1)\tilde{\tilde{y}}-(\theta+\nu)(1-\sigma) \exp(\nu x)\tilde{\tilde{y}}\tilde{k}^{-\nu}\}.$$

where,

$$\tilde{\tilde{y}} = \tilde{y}[(1-\sigma) \exp(\nu x)\tilde{k}^{-\nu}+\sigma\tilde{i}^{-\nu}]^{-1}.$$

Also,

$$r_{12} = -r_1 r_2 \exp(-u)$$

$$r_{13} = -r_1 r_3 \exp(-u) + \frac{\exp(u)}{\tilde{c}} \theta(1-\sigma) \exp(\nu x)(\theta+\nu)\sigma\tilde{\tilde{y}}(\tilde{k}\tilde{i})^{-(\nu+1)}$$

$$r_{14} = -r_1 r_4 \exp(-u)$$

$$r_{15} = -r_1 \gamma \exp(-u) + \frac{\exp(u)}{\tilde{c}} \theta(1-\sigma) \exp(\nu x)(1-\theta)\tilde{\tilde{y}}\tilde{k}^{-(\nu+1)}$$

$$r_{16} = 0$$

$$r_{17} = -r_1 r_7 \exp(-u) + \frac{\exp(u)}{\tilde{c}} \{\nu\theta(1-\sigma) \exp(\nu x)\tilde{\tilde{y}}\tilde{k}^{-(\nu+1)}$$

$$- (\theta+\nu)\theta(1-\sigma)^2 \exp(2\nu x)\tilde{k}^{-(2\nu+1)}\tilde{\tilde{y}} - \frac{1-\delta}{n} \exp(-2x)\}$$

$$r_{18} = r_1$$

$$r_{19} = -r_1 r_9 \exp(-u) - \frac{\exp(u)}{\tilde{c}} \{\theta^2(1-\sigma) \exp(\nu x)\tilde{k}^{-(\nu+1)}\tilde{\tilde{y}}$$

$$+ \frac{1-\delta}{n} \exp(-2x)\}$$

$$r_{22} = -r_2^2 \exp(-u), \quad r_{23} = -r_2 r_3 \exp(-u)$$

$$r_{24} = -r_2 r_4 \exp(-u), \quad r_{25} = -r_2 \gamma \exp(-u)$$

$$r_{26} = 0$$

$$r_{27} = -r_2 r_7 \exp(-u)$$

$$r_{28} = r_2$$

$$r_{29} = -r_2 r_9 \exp(-u) + \frac{\exp(u)}{\tilde{c}} \exp(-x)$$

$$r_{33} = -r_3^2 \exp(-u) + \theta\sigma \frac{\exp(u)}{\tilde{c}} \left\{-(\nu+1)\tilde{\tilde{y}}\tilde{i}^{-(\nu+2)} + (\theta+\nu)\sigma\tilde{\tilde{\tilde{y}}}\tilde{i}^{-2(\nu+1)}\right\}$$

$$r_{34} = -r_3 r_4 \exp(-u)$$

$$r_{35} = -r_3 \gamma \exp(-u) + \frac{\exp(u)}{\tilde{c}} \theta\sigma\tilde{i}^{-(\nu+1)}(1-\theta)\tilde{\tilde{y}}/h$$

$$r_{36} = 0$$

$$r_{37} = -r_3 r_7 \exp(-u) - \frac{\exp(u)}{\tilde{c}} \theta\sigma (\theta+\nu)(1-\sigma)\tilde{i}^{-(\nu+1)} \exp(\nu x)\tilde{k}^{-\nu}\tilde{\tilde{\tilde{y}}}$$

$$r_{38} = r_3$$

$$r_{39} = -r_3 r_9 \exp(-u) + \frac{\exp(u)}{\tilde{c}} \left\{-\theta^2\sigma\tilde{i}^{-(\nu+1)} - \exp(-x)/n\right\}$$

$$r_{44} = -r_4^2 \exp(-u)$$

$$r_{45} = -r_4 \gamma \exp(-u)$$

$$r_{46} = 0$$

$$r_{47} = -r_4 r_7 \exp(-u)$$

$$r_{48} = r_4$$

$$r_{49} = -r_4 r_9 \exp(-u)$$

$$r_{55} = -\gamma^2 \exp(-u) - \theta(1-\theta) \frac{\exp(u)}{\tilde{c}} \tilde{y}/h^2$$

$$r_{56} = 0$$

$$r_{57} = \frac{\gamma}{\tilde{c}} \{\tilde{\tilde{y}}\theta(1-\sigma) \exp(\nu x)\tilde{k}^{-\nu} + \frac{1-\delta}{n} \exp(-2x)\tilde{k}\}$$

$$- \frac{\exp(u)}{\tilde{c}} (1-\theta)\tilde{\tilde{y}}\theta(1-\sigma) \exp(\nu x)\tilde{k}^{-\nu}/h$$

$$r_{58} = \gamma$$

$$r_{59} = -\gamma r_9 \exp(-u) - \frac{\exp(u)}{\tilde{c}} (1-\theta)\theta\tilde{y}/h$$

$$r_{66} = \ldots = r_{69} = 0$$

$$r_{77} = -r_7 r_7 \exp(-u) + \frac{\exp(u)}{\tilde{c}} \{\theta(1-\sigma) \exp(\nu x)\tilde{k}^{-\nu}\tilde{\tilde{\tilde{y}}}(\theta+\nu)(1-\sigma)$$

$$\exp(\nu x)\tilde{k}^{-\nu} + \frac{1-\delta}{n} \exp(-2x)\tilde{k}\}$$

$$r_{78} = r_7$$

$$r_{79} = -r_7 r_9 \exp(-u) + \frac{\exp(u)}{\tilde{c}} \{\theta^2(1-\sigma) \exp(\nu x)\tilde{k}^{-\nu}\tilde{\tilde{y}}$$

$$+ \frac{1+\delta}{n} \exp(-2x)\tilde{k}\}$$

$$r_{88} = r_8$$

$$r_{89} = r_9$$

$$r_{99} = -r_9^2 \exp(-u) + \frac{\exp(u)}{\tilde{c}} \{\theta^2\tilde{y} - \exp(-x)\tilde{k} + \frac{1-\delta}{n} \exp(-2x)\tilde{k}$$

$$+ \exp(-x)\tilde{i}/n\}.$$

Appendix D

Derivation of the Steady State Formulas in Section 3.b

In section 3.b I display formulas for the logarithm of the steady state of k_t/z_{t-1}, i_t/z_t, and h_t. These are derived in this appendix. It is convenient to first derive the formulas for the levels, and to convert to logs at the last step. Define

(D.1) $\tilde{y}_t = y_t/z_t,\ \tilde{c}_t = c_t/z_t,\ \tilde{k}_t = k_t/z_{t-1},\ \tilde{i}_t = i_t/z_t,$

and let $\tilde{y}$, $\tilde{c}$, $\tilde{k}$, $\tilde{i}$, h denote the steady state values of $\tilde{c}_t$, $\tilde{k}_t$, $\tilde{i}_t$, and h_t, respectively. In addition, let w denote the steady state value of w_t, so that, trivially, $w = (I-A)^{-1}a = (u,x)^T$. The link between (D.1) and the starred variables in the text is given by

(D.2) $\tilde{c}_t = \exp\,(c_t^*),\ \tilde{i}_t = \exp\,(i_t^*),$

$\tilde{k}_t = \exp\,(k_t^*),\ \tilde{y}_t = \exp\,(y_t^*),\ h_t = \exp\,(h_t^*).$

In terms of these variables, the planning problem is to maximize

(D.3) $E_0 \sum_{t=0}^{\infty} \beta^t \tilde{r}(\tilde{k}_{t-1},\tilde{k}_t,\tilde{i}_{t-1},\tilde{i}_t,h_t,w_{t-1},w_t),$

subject to the information structure and the initial conditions. Here,

(D.4) $\tilde{r}(\tilde{k}_{t-1},\tilde{k}_t,\tilde{i}_{t-1},\tilde{i}_t,h_t,w_{t-1},w_t)$

$\equiv r\big(\log\,(\tilde{k}_{t-1}),\ \log\,(\tilde{k}_t),\ \log\,(\tilde{i}_{t-1}),\ \log\,(\tilde{i}_t),\ \log\,(h_t),\ w_{t-1},w_t\big)$

and r is defined before (3.5).

The first order necessary conditions satisfied by $\tilde{k}$, $\tilde{i}$, h are, respectively,

$$\tilde{r}_2(\tilde{k},\tilde{k},\tilde{i},\tilde{i},h,w,w) + \beta\tilde{r}_1(\tilde{k},\tilde{k},\tilde{i},\tilde{i},h,w,w) = 0$$

$$\tilde{r}_4(\tilde{k},\tilde{k},\tilde{i},\tilde{i},h,w,w) + \beta\tilde{r}_3(\tilde{k},\tilde{k},\tilde{i},\tilde{i},h,w,w) = 0$$

$$\tilde{r}_5(\tilde{k},\tilde{k},\tilde{i},\tilde{i},h,w,w) = 0,$$

where $\tilde{r}_j$ denotes the derivative of $\tilde{r}$ with respect to its j-th argument. The first relation states that the utility cost of increasing the current (detrended) stock of capital must equal the discounted utility benefit from the resulting increase in (detrended) consumption in the next period. The second and third relations have analogous interpretations for the stock of inventories and hours, respectively. For my particular parametric example, the above formulas are:

(D.5a) $$\exp(-x) = \beta\{\theta(1-\sigma)\exp(\nu x)\tilde{k}^{-(\nu+1)} \tilde{y}[(1-\sigma)\exp(\nu x)\tilde{k}^{-\nu}+\sigma\tilde{i}^{-\nu}]^{-1} + \frac{1-\delta}{n}\exp(-2x)\}$$

(D.5b) $$1 = \beta\{\theta\sigma\tilde{i}^{-(\nu+1)}\tilde{y}[(1-\sigma)\exp(\nu x)\tilde{k}^{-\nu}+\sigma\tilde{i}^{-\nu}]^{-1} + \exp(-x)\frac{1}{n}\}$$

(D.5c) $$\frac{\exp(u)}{\tilde{c}}(1-\theta)\frac{\tilde{y}}{h} = \gamma.$$

Here,

(D.6a) $$\tilde{y} = n^{-\theta}h^{(1-\theta)}\exp(-\theta x)[(1-\sigma)\exp(\nu x)\tilde{k}^{-\nu}+\sigma\tilde{i}^{-\nu}]^{-(\theta/\nu)}.$$

Also, the steady state resource constraint yields

(D.6b) $$\tilde{c} = \tilde{y} - \exp(-x)\tilde{k} + \frac{1-\delta}{n}\exp(-2x)\tilde{k} - \tilde{i} + \exp(-x)\frac{1}{n}\tilde{i}.$$

Equations (D.5a) - (D.6b) represent five equations in the five unknowns, $\tilde{y}$, h, $\tilde{i}$, $\tilde{k}$, $\tilde{c}$. We proceed now to obtain their unique solution.

First, note from (D.5b) that

$$\tilde{y}[(1-\sigma)\exp(\nu x)\tilde{k}^{-\nu}-\sigma\tilde{i}^{-\nu}]^{-1} = \frac{1-\beta\exp(-x)/n}{\beta\theta\sigma\tilde{i}^{-(\nu+1)}}.$$

Substituting this into (D.5a), get

$$\exp(-x) = \beta\theta(1-\sigma)\exp(\nu x)\tilde{k}^{-(\nu+1)}[1-\beta\exp(-x)/n]/[\beta\theta\sigma\tilde{i}^{-(\nu+1)}]$$

$$+\ \beta\,\frac{1+\delta}{n}\exp(-2x)$$

$$= \left(\frac{1-\sigma}{\sigma}\right)\exp(\nu x)[1-\beta\exp(-x)/n]\left(\frac{\tilde{k}}{\tilde{i}}\right)^{-(\nu+1)}$$

$$+\ \beta\,\frac{1-\delta}{n}\exp(-2x).$$

Conclude,

$$\frac{\tilde{i}}{\tilde{k}} = \left\{\frac{\sigma[1-\beta(1-\delta)\exp(-x)/n]}{(1-\sigma)\exp[(\nu+1)x][1-\beta\exp(-x)/n]}\right\}^{\left(\frac{1}{\nu+1}\right)}$$

or

(D.7a) $\tilde{i} = \lambda\tilde{k}$

where,

(D.7b) $$\lambda = \left\{\frac{\sigma[1-\beta(1-\delta)\exp(-x)/n]}{(1-\sigma)\exp[(\nu+1)x][1-\beta\exp(-x)/n]}\right\}^{\left(\frac{1}{1+\nu}\right)}.$$

Substitute (D.7a) and (D.6a) into (D.5a) to get

(D.8a) $\tilde{k} = \psi h$

where

(D.8b)

$$\psi = \left\{\frac{n^{\theta}\exp[(\theta-\nu-1)x][1-\beta(1-\delta)\exp(-x)/n][(1-\sigma)\exp(\nu x)+\sigma\lambda^{-\nu}]^{\left(\frac{\theta+\nu}{\nu}\right)}}{\beta\theta(1-\sigma)}\right\}^{\left(\frac{-1}{1-\theta}\right)}.$$

Next, I use (D.6a), (D.6b), (D.7a), (D.8a) to transform (D.5c) into one equation in h. Begin by substituting (D.7a) and (D.8a) into (D.6a) and (D.6b):

(D.9) $\tilde{y} = hn^{-\theta} \exp(-\theta x)\psi^{\theta}[(1-\sigma)\exp(\nu x)+\sigma\lambda^{-\nu}]^{-(\theta/\nu)} = h\alpha_1$

(D.10) $\tilde{c} = \tilde{y} - h\,\{\psi \exp(-x)[1-(1-\delta)\exp(-x)/n]+\lambda\psi[1-\exp(-x)\frac{1}{n}]\} = h\alpha_2.$

Here,

$$\alpha_1 = n^{-\theta} \exp(-\theta x)\psi^{\theta}[(1-\sigma)\exp(\nu x)+\sigma\lambda^{-\nu}]^{-\theta/\gamma}$$

$$\alpha_2 = \alpha_1 - \{\psi \exp(-x)[1-(1-\delta)\exp(-x)/n]+\lambda\psi[1-\exp(-x)/n]\}.$$

Substituting these into (D.5c), get

(D.11) $h = \frac{\exp(u)}{\gamma}(1-\theta)(\alpha_1/\alpha_2).$

Equations (D.7a), (D.8a), and (D.11) are the steady state equations sought.

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